COLLECTIVE CONTRIBUTION TO ESCAPE WIDTH OF ANALOG RESONANCES*

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The collective isovector monopole state decreases escape widths of analog states by 10–20%. Thus spectroscopic factors determined in previous calculations are too small by this amount. The monopole has a much greater influence on the spreading widths; the reason it is less important in the decay width is that the transition density is more external to the nucleus.

The first calculations of the decays of analog states neglected many-body effects, and their successes made it appear that nothing further was needed [e.g. 1]. Later, more detailed calculations showed that in fact predicted spreading widths were much too large [2–4]. By spreading width is meant the excess width of the analog over the sum of the partial widths due to proton decays. It arises from the mixing of the analog with more complicated nuclear states. The problem was resolved by Mekjian who noted that the Coulomb interaction induces spreading mainly via an isovector monopole state [5]. As a result of the collectivity of this state, spreading is much reduced.

The importance of the collective isovector monopole excitation was first stressed by Bohr, Damgaard and Mottelson [6], in the question of corrections to superallowed beta decay. Other physical phenomena which may be influenced by this state include isotope shifts [7] and Coulomb energies [8], although the effect is smaller than ref. [8] indicates [9]. We here treat the effect of the monopole on escape widths of analogs. It will be seen that the monopole collectivity reduces the widths by 10–20%. This is in contrast to the situation with spreading widths, which are reduced by a factor of five or so. The reason for the difference is that the transition density for particle decay is peaked several fm from the nuclear surface, while the monopole transition density and the damping transition density are concentrated at the surface.

To derive the decay width of analogs, we shall use the following expression for decay amplitude:

$$\gamma = \langle \phi_c | V_c | A \rangle + \frac{\langle \phi_c | V_N | M \rangle \langle M | V_c | A \rangle}{E - E_M - i\Gamma_M/2} \tag{1}$$

In this equation $|A\rangle$ is the analog state, defined with the isospin lowering operator on the parent state $|\pi\rangle$ as $|A\rangle = (T^-/\sqrt{2T})|\pi\rangle$. The state $|M\rangle$ is the isovector monopole of the residual nucleus, with a mean energy E_M and width Γ_M. The interaction V_c is the one-body central Coulomb field, and V_N is the two-body nuclear interaction. Finally, ϕ_c is the continuum wavefunction of the ejected particle plus the residual nucleus. Eq. (1) may be derived as an approximation from the general theory of analog states given in ref. [4]. The relation between the amplitude in eq. (1) and the observed escape width is given in the same reference. The continuum wavefunction ϕ_c may be determined from an optical potential, and should be orthogonal to the single particle orbits in the state $|A\rangle$.

For the calculation we shall need the transition density between the monopole state and the residual nucleus ground state,

$$\delta\rho(r) \equiv \langle M | \tau_z \psi^+(r)\, \psi(r) | \rangle . \tag{2}$$

We shall also need the transition potential associated with the monopole state,

$$\delta V(r) = \sum_i \langle M | V_N(r - r_i) | \rangle \tag{3}$$

For a short-range interaction and neglecting exchange, δV is proportional to $\delta\rho$. A simple estimate of the ef-

fect of the monopole may be constructed without detailed knowledge of these transition densities assuming only that the Coulomb field has the same shape as the monopole field, i.e., that the Coulomb potential may be approximated by

$$V_c \approx a + b \delta V \qquad (4)$$

with

$$b = \frac{\int V_c \delta V d^3 r}{\int \delta \rho \delta V d^3 r}.$$

We now express all quantities in terms of δV and $\delta \rho$. The monopole energy E_M is given by an unperturbed energy $2\hbar\omega$ plus a shift δV due to the interaction:

$$\Xi_M = 2\hbar\omega + \Delta V \equiv 2\hbar\omega + \int \delta \rho \delta V d^3 r \qquad (5)$$

The direct escape amplitude is given by

$$\langle \phi_c | V_c | A \rangle \approx \frac{b}{\sqrt{2T}} \int \phi_c \phi_b \delta V dr^3 \qquad (6)$$

where ϕ_b is the orbit of the particle in A which is ejected from the nucleus. The matrix elements appearing in the monopole escape amplitude are given by

$$\langle M | V_c | A \rangle = \frac{1}{\sqrt{2T}} b \int \delta \rho \delta V d^3 r \qquad (7)$$

$$\langle \phi_c | V_N | M \rangle = \int \phi_c \phi_b \delta V d^3 r.$$

Substituting in eq. (1), we find the escape amplitude to be

$$\gamma \approx \langle \phi_c | V_c | A \rangle \left(1 - \frac{1}{(1+2\hbar\omega)/\Delta V} \right) \qquad (8)$$

We see that the larger ΔV is, the more closely the monopole contribution cancels the direct contribution to the escape. Reasonable estimates [6, 10] of the ratio $2\hbar\omega/\Delta V$ range from 1 to 2. Thus, with our assumption (5), the monopole collectivity decreases the escape width by a factor 2–3, just as the spreading width is decreased. However, as we will now show, eq. (3) is a poor approximation and in fact important parts of V_c lie beyond the range of δV. For this we have to look at the detailed structure of the monopole. The transition density $\delta \rho$ was determined previously from a sum rule [9], and found to be:

$$\delta \rho \sim 3\rho + r d\rho/dr. \qquad (9)$$

This is just the density change one would obtain by uniformly expanding the nucleus. It matches quite closely the $\delta \rho$ determined from Hartree-Fock calculations [11]. With this form for $\delta \rho$ and δV, we have calculated the various integrals arising in the case of the analog states of Pb isotopes. The various functions required for the integrals appear in figs. 1(a) and 1(b). We use optical wavefunctions with parameters given in the figure caption, orthogonalized by the method of ref. [4].

It may be seen that the transition density peaks 3–4 fm from the nuclear surface. Thus an important part of V_c is not included when it is approximated by a function which varies only in the immedi-
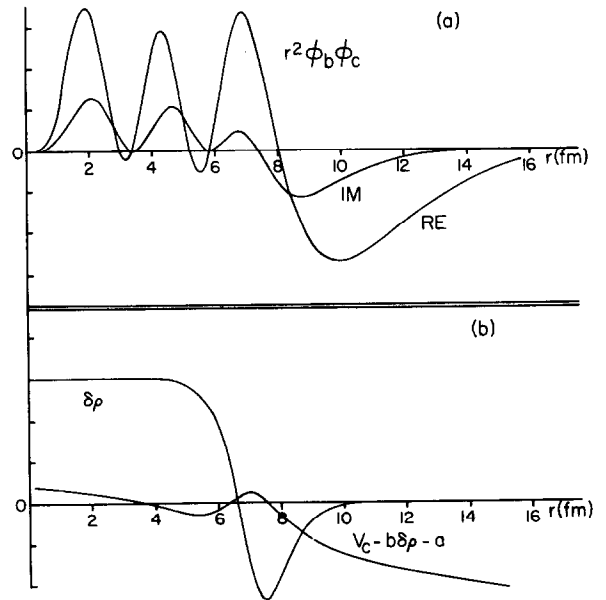

Fig. 1. (a) Upper curve shows the transition density $r^2 \phi_b \phi_c$ for a $p_{3/2}$ proton in ^{208}Pb. The bound state wavefunction ϕ_b is determined with a Woods-Saxon well of shape parameters $r_0 = 1.27$ fm, $a = 0.65$ fm, and depth to reproduce the empirical neutron separation energy. The complex continuum wavefunction is determined from a Woods-Saxon well of parameters $r = 1.17$ fm, $a = 0.72$ fm, $V = 55.3$ MeV, $W_D = 10$ MeV, and has an energy of 10.6 MeV (lab) corresponding to the empirical resonance energy. The phase of the wavefunction is the same as in ref. [4].
(b) The lower curve shows the functions $\delta \rho$ and $V_c - a - b\delta\rho$, where ρ is parametrized by a Fermi function fitting the empirical shape of the potential well. The latter quantity is small in the nuclear interior and surface region, but becomes large outside the nucleus, where the escape transition density peaks.

ate vicinity of the surface. We find that the direct matrix element is about five to six times larger than the result obtained using approximation (4).

We have compared these matrix elements for the analog resonances of ^{89}Y, ^{139}La and ^{209}Bi. We find that the ratio is relatively independent of the nucleus and orbit considered. Thus, in eq. (8), the second term should be reduced by a factor of five. This implied that the escape amplitudes are decreased by about 7%–10%, and the partial escape widths by about 15%.

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