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DOUBLE GAMMA DECAY IN LIGHT NUCLEI

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(Received September 5, 1972)

The $0^+ \to 0^+$ double gamma decay is calculated for the nuclei C^{12}, O^{16}, Ca^{40}, Ge^{72} and Zr^{90}, using a model of the states with spherical and deformed components. Due to the similarity in the structure of the components, double gamma decay is always greatly hindered. However, for O^{16}, the calculated transition matrix is too large by at least a factor of 3; in Ca^{40} there is agreement with a recently reported measurement.

INTRODUCTION

A recent measurement of double gamma decay [1] in the transition $Ca^{40}(0_2^+ \to 0_1^+)$ makes a review of the theory of nuclear double gamma decay worthwhile. In the previous theoretical literature [2-6], estimates of transition rates are either highly inaccurate or otherwise unreliable. An improved estimate is derived below, making use of the energy-weighted dipole sum rule.

The $0^+ \to 0^+$ transitions between low-lying levels are interesting because there is still much uncertainity about the structure of excited 0^+ states. The decay normally proceeds by pair production or by internal conversion, which depend on the transition matrix element

$$M_{EO} = \sum_p r_p^2 \qquad (1)$$

This matrix element is of the order of a few fm^2 which is what one would expect from a single-particle transition. Double gamma decay is proportional to the second order E1 matrix element,

$$M_{2\gamma} = \sum_{p,p',i} z_p \frac{|i\rangle\langle i|}{E_i - E_o} z_p \qquad (2)$$

Experimentally it is very weak and the limits are much less than the prediction of a single-particle model. Parenthetically, it may be noted that atomic $0^+ \to 0^+$ double photon decay has been observed [7,8] and can be accounted for quantitatively with simple atomic theory [9,10].

Before deriving the estimate for double-gamma nuclear decays, it is valuable to understand physically why the matrix element should be small between low-lying states. In 4N nuclei, the low-lying states tend to have maximum spacial symmetry. On grounds of maximizing the potential energy, it seems reasonable that the low excited states have similar relative wavefunctions for fully symmetric pairs, and differ mostly in the relative wavefunctions of nonsymmetric pairs. If this is so, the photon operator appearing in the double gamma decay, $\sum_i \tau_z z_i$, cannot easily connect the two states in second order. The first application of the operator must break a fully symmetric pair, since it produces a T=1 state from a T=0 state. The second operator can either restore the symmetry of the broken pair, or break a further symmetric pair, but in neither case would the wavefunction have a significant overlap with the orthogonal 0^+. This argument is especially easy to follow with the alpha particle model. If two states are constructed from orthogonal wavefunctions of the coordinates of the alpha particles, they cannot be connected by a succession of E1 operators.

THEORY

The two photon decay rate for a $0^+ \to 0^+$ transition in a T=0 nucleus is given by

$$W = \frac{1}{\hbar} \frac{4}{105\pi} \frac{1}{(\hbar c)^4} \left(\frac{e^2}{\hbar c}\right)^2 (E_i - E_f)^7 M_{2\gamma}{}^2 \qquad (3)$$

This is the formula of Oppenheimer and Schwinger [11], corrected by a factor of two.

For the nuclear model, we assume that the wavefunctions are mixtures of deformed and spherical states,

$$|0_1^+\rangle = a|\text{spherical}\rangle + b|\text{deformed}\rangle$$
$$|0_2^+\rangle = b|\text{spherical}\rangle - a|\text{deformed}\rangle \qquad (4)$$

More detailed specification of the states and the mixing amplitude can be found in the calculations of Brown and Green [12] for O^{16}, and Gerace and Green [13] for Ca^{40}. We shall use the empirical monopole matrix element to fix the admixtures coefficients, and so eliminate some of the dependence on the many assumptions that went into the above-mentioned calculations.

The transition matrix element can be written

$$M_{2\gamma} = \sum_n ab \left(\frac{\langle \text{spherical}|z|n\rangle^2 - \langle \text{deformed}|z|n\rangle^2}{E_i - E_n}\right)^2 + \sum_n (a^2 - b^2)\left(\langle \text{spherical}|z|n\rangle\langle n|z|\text{deformed}\rangle / E_i - E_n\right) \quad (5)$$

The second term in this expression is small, as will be shown below. To evaluate the first term, we consider only the giant dipole state as an intermediate state. It certainly seems safe to neglect nearby 1^- states, since they have very small dipole strength. The matrix element is then

$$M \approx ab \left(\frac{\langle \text{spherical}|z|\text{ spherical dipole}\rangle^2}{E_i - E_{\text{spherical dipole}}} - \frac{\langle \text{def.}|z|\text{ def. dipole}\rangle^2}{E_i - E_{\text{def. dipole}}}\right) \quad (6)$$

The two terms in this expression very nearly cancel. In fact, with oscillator wavefunctions they cancel perfectly. To assess the residual in the difference, we make use of the Bethe-Levinger dipole sum rule [14].

$$\sum_{n,p} \langle i|z_p|n\rangle^2 (E_i - E_n) = \frac{\hbar^2}{2M}\frac{NZ}{A}(1+x) \quad (7)$$

where x is a constant ~0.4.

We shall apply this to the spherical and deformed components separately. Theory suggests that the dipole state of a deformed nucleus is split into two or more components, the energy of a component depending inversely on the length of the principle axis [15]. Suppose we describe the intrinsic deformed state by a charge distribution with

$$\langle z^2\rangle \sim R_0^2 (1 + \epsilon/3)^2$$
$$\langle x^2\rangle = \langle y^2\rangle \sim R_0^2 (1 - \epsilon/6)^2 \quad (8)$$

The parameter ϵ, defined in ref. [15], is connected with the usual deformation parameter β by

$$\beta \approx 2/3 \, \frac{\pi}{5} \, \epsilon$$

Then the dipole matrix element along these axes will satisfy

$$\begin{aligned} &\langle \text{def. } |z| \text{ def. dipole} \rangle^2 \sim \frac{1}{E_z} \sim R_0(1+\epsilon/3) \\ &\langle \text{def. } |x| \text{ def. dipole} \rangle^2 \sim \frac{1}{E_x} \sim R_0\,(1+\epsilon/6) \end{aligned} \tag{9}$$

Inserted in Eq. (6), we find for the leading term in the two-photon transition operator,

$$M_{2\gamma} = ab \frac{\hbar^2}{2M} \frac{NZ}{A} (1+x) \left(\frac{2}{3} \frac{1}{E^2_{\text{dipole}}} \right) (\epsilon/3)^2 \tag{10}$$

We now consider the monopole matrix element for the same transition, which in the present model is given by

$$M_{EO} = ab \left(\langle \sum_p r^2 \rangle_{\text{spherical}} - \langle \sum_p r^2 \rangle_{\text{def.}} \right) \tag{11}$$

Evaluation of this difference requires detailed knowledge of the structure of the intrinsic states. The most reliable model is probably the harmonic oscillator-SU(3) model developed in ref. [12] and [13]. In this model the deformed component is primarily a 4p-4h state, each particle having one more oscillator quantum than the hole. We shall also assume that when the particles are promoted to the next oscillator shell, the core will deform to keep the interior density constant. Multiplying the valence contribution to the monopole matrix element by a factor of two to account for the core deformation, the following expression for the monopole matrix element is obtained.

$$M_{EO} = ab \, 4/\nu \tag{12}$$

Here ν is the oscillator size parameter which appears in the oscillator wavefunctions as $e^{-\frac{1}{2}\nu r^2}$. To relate this to the two-photon transition matrix element, we also need the deformation of the intrinsic state. This is determined from the expectation of r^2 along the three principle axes of the intrinsic wavefunction,

$$\frac{\langle z^2\rangle_{def.}}{\langle x^2,y^2\rangle_{def.}} = \frac{R_0^2(1+\epsilon/3)^2}{R_0^2(1-\epsilon/6)^2} = \left(\frac{n/3+\lambda}{n/3-\mu/2}\right)^2 \tag{13}$$

In the last equation, the ratio is evaluated in the SU(3) model, with a core deformation given by Arima and Yoshida [21]. Here (λ,μ) are the SU(3) quantum numbers of the deformed state, and n is the total number of oscillator quanta in the spherical state. In the table below, we use Eqs. (10), (12), and (13) to extract double gamma matrix elements from the experimental monopole rates. For one nucleus in the table, Zr^{90}, we have modified the model, since the 0^+ states differ only by a proton pair [20]. For the spherical state, the fp shell is assumed to be closed. In the deformed state, two protons are promoted from the $p_{1/2}$ orbit to the $g_{9/2}$ orbit.

The calculation of the monopole matrix element required knowledge of the core polarization, which is not well understood. We therefore mention two other models for the mean square radius of a state. The liquid drop model assumes that the nucleus is an incompressible spheroid. Then the mean square radius is related to the deformation by

$$\langle r^2\rangle \approx 3/5\, R^2 + \frac{7}{75}\, R^2\, \epsilon^2 \tag{14}$$

with R the nuclear radius. This model has been used in the discussion of isotope shifts [24] and Coulomb energies [25] in the rare earth region.

In the light nuclei, however, much too large a deformation is required to explain the monopole matrix element. The reason for the failure of the model is that the actual deformed state has large density fluctuations in relatively small regions near the surface. The liquid drop model has a small density fluctuation spread out over all the surface, so it produces a change in $\langle r^2\rangle$ only of order ϵ^2.

Another model for the density distribution of the deformed state can be constructed by removing an alpha particle from just below the surface of a spherical density distribution and placing it just above the surface on a different part of the nucleus. To maintain the density as nearly uniform as possible, the radius vector of the alpha particle must be increased by ~2 fm. This construction gives a monopole matrix element in agreement with the microscopic model for O^{16}, as indeed should be expected from the known success of the alpha particle model here [22].

However, for heavier nuclei, this construction yields a monopole matrix element about twice the SU(3) value.

We now turn to the second term in eq. (5), the second-order matrix element between spherical and deformed components. To estimate this we need the full shell model description of the states, including two-particle two-hole components of the wavefunction. Since the two-photon operator can only change the shell occupation number by two, the spherical component will not connect directly with the 4p-4h component. Furthermore, it turns out that the matrix element is hindered by the selection rule in the asymptotic quantum numbers of the deformed orbits. Rather than try to determine from calculated wavefunctions the degree of hindrance, we look at the systematics of low transitions in nearby nuclei. For example, for the $\frac{1}{2}^- \rightarrow \frac{1}{2}^+$ transition in F^{19}, the matrix element $\langle z \rangle$ is only 3×10^{-2} fm. In Ar^{39}, the E1 decay of $3/2^+(1.56) \rightarrow 3/2^-(1.26)$ has a matrix element 10^{-2} fm. We then estimate the two-photon matrix element as

$$M_{2\gamma} \sim 10^{-3} \times 1/3 \times \tfrac{1}{2} \sim 2 \times 10^{-4}$$

The first factor is a squared E1 matrix element. The second factor is a reasonable energy denominator for a 1p-1h or 3p-3h deformed state, and the third factor is the wavefunction overlap $\langle 0p0h | 2p2h \rangle \langle 2p2h | 4p4h \rangle$, with wavefunctions of ref. [12,13]. The result is much smaller than the contribution diagonal in particle-hole number, so it is neglected.

RESULTS

As may be seen from the table, the predicted two-photon matrix element decreases sharply in heavier nuclei. This is a consequence of the smaller deformations in the heavier nuclei. The Ca^{40} number agrees with the reported measurement of a branch of 4×10^{-4} in this nucleus. However, the same model predicts a rate 10 times the measured limit in O^{16}. The more detailed three-component models will yield a two-photon matrix element about half as big, since the 2p-2h state has only half the deformation. Improvement of the model with triaxial deformations has negligible effect. Thus we conclude that, while the smallness of the two-gamma matrix element is understood in a qualitative way, it is still not possible to make a quantitative calculation.

Nucleus	Exper M_{EC}
C^{12}	5.5
O^{16}	3.8
Ca^{40}	2.6
Ge^{72}	2.8
Zr^{90}	1.6

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several discussions

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Nucleus	Experimental $M_{EO}(fm^2)$	(λ, μ)	ϵ	ab	Theoretical $M_{2\gamma}$ (fm^2/MeV)	Experimental $M_{2\gamma}$
C^{12}	5.5 [16]	(12, 0)	1.95	0.47	2×10^{-2}	
O^{16}	3.8 [17]	(8, 4)	1.50	0.32	1×10^{-2}	$<3 \times 10^{-3}$ [23]
Ca^{40}	2.6 [18]	(12, 6)	0.66	0.18	5×10^{-3}	6×10^{-3} [1]
Ge^{72}	2.8 [19]	(16, 8)	0.54	0.18	9×10^{-3}	
Zr^{90}	1.6 [18] ×	(8, 0)	0.15	0.09	4×10^{-4}	$<6 \times 10^{-3}$ [22]

ACKNOWLEDGEMENT

The author thanks E. Beardsworth for prompting this investigation and for several discussions.

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*. Supported in part by the National Science Foundation.

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TABLE

Decay
$^{20}Ne(5.785) \rightarrow {}^{16}O(g.$
$^{19}F(10.136) \rightarrow {}^{15}N(5.3$
$^{19}F(10.136) \rightarrow {}^{15}N(5.2$

Refs. [3,4].
Ref. [1].