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Pion–pion cross section in a dense and hot pionic gas

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We examine the in-medium scattering cross section in a dense and hot pionic gas. We show that the increase of the scattering rate associated with the Bose occupancy factors is largely compensated by a drop in the cross section, when medium effects on the pionic cross section are consistently taken into account. Implications for calculating the pion spectrum at low transverse momentum, in ultrarelativistic heavy ion collisions, are discussed.

1. Introduction

Ultrarelativistic nucleus–nucleus collisions deposite large amounts of energy in the central spatial and rapidity region, appearing ultimately as pions in the final state. The reaction may evolve through an intermediate stage with a dense pion gas, and if so it is important to understand pion gas dynamics better. One feature of the observed pion spectrum, the peak at low transverse momentum, has been associated with the dense pion zone [1,2].

An important aspect of the gas dynamics is the thermalization rate [3] [#1], which should be calculable from a Boltzmann equation with Bose statistics included in the collisional integral. Modelling with the bosonic Boltzmann equation has, indeed, recently been attempted [6,7]. Our purpose is to examine the effects of Bose statistics in the collision rate of a pion gas in more detail.

Naively, the Bose statistics will increase the collision rate because of the additional phase space factors $(1+f)(1+f)$ in the collision integral. This can lead to the condensation of pions [6]. However, in a self-consistent description the cross section is also affected by the medium. Thus, a large phase space occupancy may be compensated by a smaller effective cross section of the pions in the medium.

To examine the medium effects, we shall solve the Bethe–Goldstone equation for $\pi\pi$ scattering including the occupation factors in the intermediate states. The resulting in-medium cross sections are then used to calculate the collision rates for a pure pionic gas.

2. Cross section calculation

The transport equation for pions may be derived microscopically using the nonequilibrium Green fucntion technique (see refs. [8–10] and references therein). In the t-matrix approximation, the scattering cross section is given, up to a factor, by the square of the scattering matrix that is obtained by summing up repeated interactions in a channel. The t-matrix equation for the channel α is of the familiar Lippmann–Schwinger form and reads

[#1] Another aspect is the mean field [4,5], which we do not consider here.

$$t_\alpha = V^\alpha_{\pi\pi} + V^\alpha_{\pi\pi} G^0_{\pi\pi} t_\alpha \,. \tag{1}$$

Taking the in-medium effects into account, the free $\pi\pi$ propagator in eq. (1) is replaced by

$$G^0_{\pi\pi} \to G^{\text{in-medium}}_{\pi\pi} = \frac{1}{\omega_k} \frac{1+f_k+f_{-k}}{s-4\omega_k^2+\mathrm{i}\epsilon} \,. \tag{2}$$

Here $s=E^2-\boldsymbol{P}^2$ is the center of mass energy of the pion pair having a total momentum $\boldsymbol{P}$ and $\omega_k^2 = m_\pi^2 + k^2$ is the pion dispersion relation. The resulting t-matrix equation is then of the Bethe–Goldstone type (see also ref. [11]); it describes the properties of a pair of pions embedded in a pure pionic medium. The influence of the medium is accounted for through the phase space occupancy factors f_k and f_{-k} for pions with momenta $\boldsymbol{k}$ and $-\boldsymbol{k}$ in the center of mass.

The solution of the Bethe–Goldstone equation is greatly simplified when the $\pi\pi$ amplitude is parameterized in the separable form. We shall assume that in each channel α characterized by isospin I and angular momentum l the pions interact by forming a heavy virtual meson of mass M_α that further decays into two pions (see also refs. [12,13]). The potential is in this case

$$V^\alpha_{\pi\pi} = \langle \pi\pi | V | \alpha \rangle \frac{1}{s-M_\alpha^2+\mathrm{i}\epsilon} \langle \alpha | V | \pi\pi \rangle \,. \tag{3}$$

Good fits to the $\pi\pi$ scattering phase shifts are obtained with the following parameterizations. For the $I=0$, s-wave channel we take the meson mass $M_0 = 940$ MeV and a form factor

$$V_0(k) = 4\pi\omega(k)\sqrt{2M_0}\, g_0 \frac{1}{1+k^2/k_0^2} \,, \tag{4}$$

with $g_0 = 0.60 m_\pi^{-1/2}$ and $k_0 = 2.71 m_\pi$. For the $I=1$, p-wave channel we take $M_1 = 826.7$ MeV and

$$V_1(k) = 4\pi\omega(k)\sqrt{2M_1}\, g_1 \frac{k}{k_1} \frac{1}{(1+k^2/k_1^2)^2} \,, \tag{5}$$

with [#2] $g_1 = 0.6684 m_\pi^{-1/2}$ and $k_1 = 3.34 m_\pi$. In the $I=2$, $l=0$ channel the total cross section is of the order of 4 mb (see also ref. [14]) and is disregarded in the present calculations.

To facilitate the calculation of the in-medium t-matrix equation, we assume that the phase space occupancy factors can be expressed by the Bose functions

$$f(T,\mu) = \left[\exp\left(\frac{\omega-\mu}{T}\right) - 1 \right]^{-1} \,, \tag{6}$$

where T and μ are respectively the temperature and chemical potential of the pionic medium and ω is here the energy of the pions in the frame of the thermal distribution. Furthermore, we apply angular averaging for the statistical factor for intermediate states. The angular averaging of the occupancy factors for the pions can be done analytically. For the center of mass of a pair moving with velocity β in the frame of thermal distribution the result is [#3]

$$\langle 1+f_k+f_{-k} \rangle = \frac{T}{\gamma\beta k} \ln\left(\frac{\sinh((1/2T)\{\gamma[\omega(k)+\beta k]-\mu\})}{\sinh((1/2T)\{\gamma[\omega(k)-\beta k]-\mu\})} \right) . \tag{7}$$

The total momentum of a pair in the thermal frame is $P=\sqrt{s}\,\gamma\beta$, where $\gamma = 1/\sqrt{1-\beta^2}$. In doing so we end up with the following expression for the t-matrix in the channel α:

[#2] Note that the parameters differ from those given in ref. [13] which do not fit phase shifts. We have obtained a corrected value of p-wave coupling constants from one of the authors (P.S.) of that work. We refitted the s-wave parameters ourselves.

[#3] In the case of fermions the hyperbolic sine has to be replaced by the hyperbolic cosine.

$$t_\alpha(k,k',\boldsymbol{P},s)=V_\alpha(k)V_\alpha(k')\left(s-M_\alpha^2-\frac{1}{4\pi^2}\int_0^\infty \mathrm{d}k\,k^2\frac{V_\alpha^2(k)\langle 1+f_k+f_{-k}\rangle}{\omega(k)[s-4\omega^2(k)+\mathrm{i}\epsilon]}\right)^{-1}. \tag{8}$$

The differential cross section in a channel reads [#4]

$$\frac{\mathrm{d}\sigma_\alpha}{\mathrm{d}\Omega}=\frac{1}{256\pi^2\omega_k^2}\,|t_{l_\alpha}|^2(2l_\alpha+1)^2P_{l_\alpha}^2\cos\theta\,. \tag{9}$$

The total cross section for the channel α has the structure

$$\sigma_\alpha(\boldsymbol{P},s)\propto\frac{1}{(s-M_\alpha^2-a)^2+b^2\langle 1+f_{k_s^+}+f_{k_s^-}\rangle^2}, \tag{10}$$

where k_s^+ and k_s^- are the momenta of the pions in the laboratory frame and a, b come from the integral in eq. (8). This qualitative form for σ_α suggests that the cross section for the $\pi\pi$ scattering in the medium can drastically be diminished compared to the unaffected one due to the in-medium effects. This is illustrated in fig. 1, where the s- and p-wave in-medium cross sections for $T=150$ MeV and $T=200$ MeV are compared to the free cross section. The chemical potentials have been chosen to obtain an entropy per pion of $S_\pi=2.85$ corresponding to the parameterization of ref. [1], and the pion densities are 0.36 fm^{-3} and 0.76 fm^{-3} for the two temperatures, respectively.

We observe a substantial reduction of the cross section when the phase space occupancy is taken into account. In the s-wave channel the cross section is reduced by a factor of 6–8 at low energies.

For the p-wave scattering the cross section is reduced by a factor of about 2 at the resonance energy. The resonance becomes broader, because the phase space occupancy factors influence the width too.

The total isospin averaged cross section is given by

$$\sigma=\tfrac{1}{9}\sigma^0_{I=0}+\tfrac{1}{3}\sigma^1_{I=1}+\tfrac{5}{9}\sigma^0_{I=2}\,,$$

[#4] The differential cross section contains the proper factors due to symmetrization and the total cross section is given by integration of the differential cross section over the angle 2π.

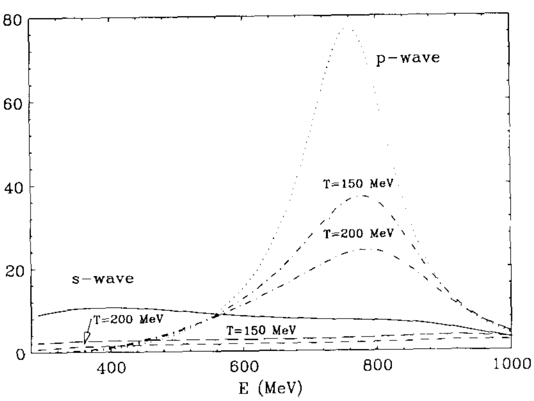

Fig. 1. The in-medium cross sections for s-wave (dashed lines) and p-wave (dash-dotted lines) scattering calculated at $T=150$ and 200 MeV, respectively, are shown for $\boldsymbol{P}=0$ as a function of energy. The entropy per pion is $S=2.85$ and the chemical potentials are $\mu=125$ and 130 MeV corresponding to the pion density of 0.36 fm^{-3} and 0.76 fm^{-3}, respectively. The free s- and p-wave cross sections are shown by solid and dotted lines, respectively. The cross sections contain the factors due to the isospin averaging.

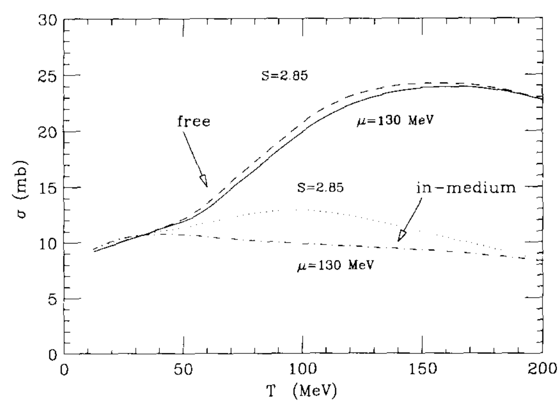

Fig. 2. The thermally averaged total cross section for free $\pi\pi$ scattering and in-medium $\pi\pi$ scattering. The curves are labelled by the values of the chemical potential or the entropy per pion.

where the last term of the order of 4 mb is ignored. In fig. 2 we show the thermally averaged total cross section

$$\langle\sigma\rangle = \frac{\langle f_1 f_2 v_{12}\sigma(1+f_3)(1+f_4)\rangle}{\langle f_1 f_2 v_{12}(1+f_3)(1+f_4)\rangle}, \tag{11}$$

as a function of temperature for a constant chemical potential $\mu = 130$ MeV and for a constant entropy per pion $S_\pi = 2.85$, respectively. Since the values of entropy per pion in the range $S = 2$–3 are associated with a chemical potential close to the pion mass for a wide range of temperatures (see also ref. [7]), thermal averaging at constant entropy or constant chemical potential give nearly the same results.

The thermal and isospin averaged in-medium cross section is about half the free one. This is a big effect and has to be taken into account in the kinetic equations tailored for describing the collision process. One has, however, to regard that, in particular, the low $p_\perp$ values are sensitive to the occupancy of the phase space density at low momenta, consequently the use of a thermally averaged cross section would imply that a too big cross section is used in the kinetic equations for the low momentum region. In principle, one has to take the full momentum dependence of the in-medium cross section into account. To get an orientation we show in fig. 3 the ratio of the collision rates,

$$R = \frac{\langle f_1 f_2 v_{12}\sigma_{\text{in-medium}}(1+f_3)(1+f_4)\rangle}{\langle f_1 f_2 v_{12}\sigma_{\text{free}}\rangle}, \tag{12}$$

for $\pi\pi$ scattering affected and not affected by the pionic medium. It is seen that for lower temperatures the bosonic rate exceeds the rate calculated without the effect of statistics by up to a factor of 2. For high temperatures the rates become nearly equal because the enhancement of the rate due to the final state factors is compensated by a drop of the cross section. That means that the enhancement of the collision rate due to the Bose phase space factors in the Boltzmann equation, which preferentially would drive the system to a regoin of high phase space density, is drastically reduced with proper account of medium effects on the cross section.

Our calculation of in-medium $\pi\pi$ cross sections includes the effect of the rho and sigma mesons only through their influence on $\pi\pi$ scattering phase shifts. The neglect of medium modifications to the bare propagators of the particles may be questioned. In a model of the $\pi\rho$ dynamics motivated by vector domiannce [15], the mean-field effect would inrrease the energy of the ρ and thereby lower the effective $\pi\pi$ cross section even more. On the other hand, Shuryak [5] considered the effect of higher resonances on the $\pi\rho$ interaction, and found an attraction. However, this attraction is unlikely to alter our qualitative conclusion that the thermally averaged cross sections are reduced by substantial factors with respect to the free cross sections. With our reduction factors, the

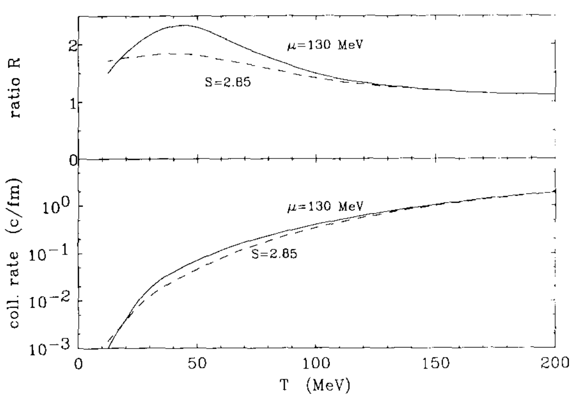

Fig. 3. Upper part: The ratios R of the collision rates calculated with Bose final state factors and in-medium cross sections to rates disregarding the Bose statistics as a function of the temperature T for constant entropy $S = 2.85$ per pion and for constant chemical potential $\mu = 130$ MeV, respectively. Lower part: The collision rates of a pion in a gas calculated with Bose occupancy factors as a function of the tmeperature T at constant entropy per pion and at constant chemical potential, respectively.

overall collision rate in a Boltzmann transport equation appears not to be enhanced compared to the usual one containing the unaffected cross section and no Bose enhancement factors.

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