CORRELATION EFFECT ON UNIQUE FORBIDDEN DECAYS

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Abstract: The hindrance in some unique first-forbidden beta decays is related to the repulsive $T = 1$ particle-hole force Calculations are performed for ^{17}N, ^{39}Ar and ^{41}K The qualitative trend agrees with experiment, but not the numerical results.

1. Introduction

The question of how correlations are produced and how they affect physical properties is basic to the understanding of nuclear structure. With the shell model as a starting point, the wave function is a pure configuration which has relatively weak correlations Stronger correlations are built up by adding more configurations to the pure shell-model wave function.

The correlations can be divided into two kinds. One first thinks of the short-range correlations involving very highly excited configurations, produced by the singular behavior of the nuclear force at short distances. In this paper we will be concerned with some longer range correlations, which depend on the average properties of the nuclear force. These correlations will be evident in the measurement of operators connecting the physical states To make quantitative calculations, we should choose nuclei which can be described easily in the shell model.

The shell model works quite well in the neighborhood of doubly magic nuclei, especially if all the active particles and holes interact in relative $T = 1$ states. The force in configurations of maximum isospin is generally weak enough so that one configuration dominates in a calculation with configuration mixing [see for example ref. [12]]. From this we would expect that the correlations would not be too strong for such nuclei. In fact one particular model, the pairing vibrational model described in ref [1]), successfully explains much data in high-T nuclei assuming no particle-hole

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correlations at all. As will be seen in this paper, configuration mixing can and does produce substantial particle-hole correlations for $T = 1$ pairs. The main empirical question is to find operators that will be sensitive to the relative positions of particles and holes; the operators considered in ref [1]) do not have the required sensitivity

An ideal operator meeting this specification is the local operator which annihilates a $T = 1$ particle-hole pair. It is such an operator that enters the transition amplitude for forbidden beta decays. Thus we expect to gain information on particle-hole correlations by studying beta transitions such as

$$|1p - 1h, T = 1\rangle \xrightarrow{\beta} |T = 0\rangle, \tag{1}$$

$$|2p - 1h, T = \tfrac{3}{2}\rangle \xrightarrow{\beta} |1p, T = \tfrac{1}{2}\rangle,$$

$$|2h - 1p, T = \tfrac{3}{2}\rangle \xrightarrow{\beta} |1h, T = \tfrac{1}{2}\rangle. \tag{2}$$

A theoretical study of some transitions of type (1) has been made by Warburton and collaborators [2,3]). In this paper we shall investigate some transitions of type (2).

The physics of particle-hole correlations is very simple and follows from the fact that the $T = 1$ particle-hole force is repulsive. The eigenfunctions tend to have the particle and hole separated as much as possible in the nucleus. The transition matrix element of a local operator is hindered, since it is proportional to the amplitude of the hole at the particle coordinate.

The hindrance in the matrix elements of beta decay operators is very well known and appears in all regions of nucleides, whenever there are several active particles. The allowed Gamow-Teller operator σ has been most extensively studied [4,5]. We will consider the first forbidden unique beta decay operator,

$$M_1 = \sum_t \tau_\pm r_t [Y'(\hat{r}_t)\sigma]^{L=2}. \tag{3}$$

This operator has been calculated for nuclei in the calcium region by Oquidam and Jancovi [6]). These authors diagonalized a shell-model Hamiltonian, using only $f_{\tfrac{7}{2}}$ and $d_{\tfrac{3}{2}}$ in the wave functions. A later treatment [7]) included more configurations, but the wave functions were not calculated by Hamiltonian diagonalization. Hindrances were found in both cases. Besides the calculations of Warburton *et al.*, there is a recent study [8]) in a shell-model framework on first forbidden decays in the region around ^{208}Pb The correlation effects in the nuclei considered in this study do not have the same character as in the decays (1) and (2) above, and there is no hindrance. Further discussion of this point is deferred to the concluding section.

We shall consider two unique first-forbidden decays, ^{39}Ar(β^-) ^{39}K and ^{17}N(β^-) ^{17}O(g.s.), and one unique first-forbidden electron capture process, ^{41}Ca(ec)^{41}K. The ft value for unique transitions is given by eq (7.34) of Konopinski's treatise [9]),

$$f_n t_{\frac{1}{2}} = \frac{D\lambda_C^{2n}[(2n+1)!!]^2}{4\pi\langle M_n\rangle^2}, \tag{4}$$

where λ_C is the Compton wavelength of the electron, D is Fermi's universal time constant [10])

$$D = \frac{2\pi^3\hbar^7 \ln 2}{g_A^2 m_e^5 c^4} = \frac{6250}{g_A^2} = 4131 \text{ sec} \tag{5}$$

and the beta moment is defined as the square of the matrix element for the transition $J_i \to J_f$:

$$\langle M_n \rangle^2 = \langle [(J_f)M_n]^{J_i}|J_i\rangle^2. \tag{6}$$

The experimental $f_1 t$ values and derived β-momenta are listed in table 1.

TABLE 1

Experimental beta moments for first-forbidden decays

Nuclear decay	Partial half-life	$f_1 t_{\frac{1}{2}}$ (ns)	M^2 (fm^2)
$^{17}N \to {}^{17}O(\text{g s.})$	268 s$\pm$40 % [ref. [13])]	3 56	0.12
$^{39}Ar \to {}^{39}K$	269 y	1.12	0.04
$^{41}Ca \to {}^{41}K$	8×10^4 y	2.72	0 016

The beta moments obtained are all smaller than the order of magnitude we would guess from the dimensions of the nucleus. As we shall see in the next section, the moments are also much smaller than the pure shell-model estimates.

2. Nuclear physics

To evaluate the nuclear matrix elements, we want to include the important correlations, but otherwise keep the wave functions as simple as possible. Pure configurations are used for the single-particle state. For the 2p-1h and 2h-1p states, we would like to include all configurations keeping the particles and holes in the same major shell. This truncation of basis is natural for the shell model; other configurations would require the excitation of a particle across the major shell gap. Wave functions with mixing in the major shells have been published for two of the nuclei we consider, namely ^{41}K [ref. [15])] and ^{17}N [ref [16])]. Only the largest components of these wave functions are quoted in the references. In using these wave functions, much of the interferences effect from the small components will be left out. Therefore we also computed wave functions ourselves using the G-matrix force of Kuo and Brown [17]). A matrix diagonalization was performed for the largest components of the wave functions These are listed in appendix A. For computing the beta moment, other configurations were added to the lowest eigenstate in perturbation theory.

Two criticisms can be made at this point which throw doubt on the validity of the truncation we have made of the shell-model space. First, polarization effects involving the excitations of closed shells could easily change the single-particle matrix elements by 10 %. In fact the allowed Gamow-Teller decays in mirror nuclei do show rate changes greater than this. Second, recent calculations [14]) show that the shell gap could actually be much smaller than we estimate from separation energies. A small gap would be associated with strong configuration mixing across the closed shells

It is worth presenting some details of the Hamiltonian dynamics to make the interference effect more understandable. Given the two-body interaction between shell-model states,

$$\langle j_1 \, j_2 | V | j_3 \, j_4 \rangle_J,$$

the Hamiltonian matrix is

$$\langle [(j_1 \, j_2)^L j_3^{-1}]^J | H | [(j_1' \, j_2')^{L'} j_3'^{-1}]^{J'} \rangle = \varepsilon_{123} + \langle V_{\text{pp}} \rangle + \langle V_{\text{ph}} \rangle, \tag{7}$$

where we have defined single-particle energies,

$$\varepsilon_{123} = \delta_{11'} \delta_{22'} \delta_{33'} \delta_{LL'} (\varepsilon_1 + \varepsilon_2 - \varepsilon_3), \tag{8}$$

the interaction between particles

$$\langle V_{\text{pp}} \rangle = \delta_{33'} \delta_{LL'} \langle j_1 \, j_2 | V | j'_1 \, j'_2 \rangle_L \tag{9}$$

and finally the particle-hole interaction,

$$\langle V_{\text{ph}} \rangle = \frac{(1 - (-1)^{j_1 + j_2 - L} P_{12})}{\sqrt{1 + \delta_{12}}} \frac{(1 - (-1)^{j'_1 + j'_2 - L'} P_{1'2'})}{\sqrt{1 + \delta_{1'2'}}} \delta_{11'}$$
$$\times \sum_K U(j_1 \, j_2 \, J j_3; LK) U(j'_1 \, j'_2 \, J j_3'; LK) \langle j_2 \, j_3^{-1} | V | j_{2'} \, j_{3'}^{-1} \rangle_K \tag{10}$$

In the last equation the Racah coefficients are in Jahn's unitary form. The main part of the dynamics comes from the particle-hole interaction. The $T = 1$ particle-hole interaction can be expressed in terms of the n-p particle interaction by the Pandya relation

$$\langle j_1 \, j_2^{-1} | V | j_3 \, j_4^{-1} \rangle_K = - \sum_J \sqrt{\frac{2J+1}{2K+1}} (-1)^{j_1 + j_2 + j_3 + j_4}$$
$$\times U(j_1 \, j_2 \, j_4 \, j_3, KJ) \langle j_1^n \, j_4^p | V | j_3^n \, j_2^p \rangle_J \tag{11}$$

To compare phases of the interaction with the phases of the beta moments it is useful to write the particle-hole interaction in L-S coupling,

$$\langle j_1 \, j_2^{-1} | V | j_3 \, j_4^{-1} \rangle_J = \sum_{LS} \langle (l_1 \tfrac{1}{2})^{j_1} (l_3 \tfrac{1}{2})^{j_2} | (l_1 \, l_2)^L (\tfrac{1}{2} \tfrac{1}{2})^S \rangle_J \langle (l_3 \tfrac{1}{2})^{j_3} (l_4 \tfrac{1}{2})^{j_4} | (l_3 \, l_4)^L (\tfrac{1}{2} \tfrac{1}{2})^S \rangle_J$$
$$\times \langle l_1 \, l_2^{-1} | (-1)^S \tfrac{1}{2} V_{\text{singlet}}^{\text{np}} - (\tfrac{3}{2} - S) V_{\text{triplet}}^{\text{np}} | l_3 \, l_4^{-1} \rangle_L. \tag{12}$$

For a short-range force this L-S coupled matrix element is

$$\langle l_1 \, l_2^{-1} | V | l_3 \, l_4^{-4} \rangle_L = (l_1 \, 0 \, l_2 \, 0 | L 0)(l_3 \, 0 \, l_4 \, 0 | L 0) \times \text{radial integral.} \tag{13}$$

The beta decay matrix element connecting a 2p-1h state to the single-particle bears a relation to the matrix elements of the force between 2p-1h states, written out in the preceding two equations. The beta matrix element from appendix B is

$$\langle\{(j_1 j_2)^L j_3^{-1}\}^J | \{r^L (Y^L \sigma)^K j_1\}^J\rangle = (-1)^{j_1 + K - J} U(j_1 j_2 J j_3; LK)$$

$$\times \langle (l_2 \tfrac{1}{2})^{j_2} (l_3 \tfrac{1}{2})^{j_2} | (l_2 l_3)^L (\tfrac{1}{2}\tfrac{1}{2})' \rangle \sqrt{\frac{2(2l_2+1)(2l_3+1)}{4\pi(2L+1)}} \; (l_2 0 l_3 0 | L0)\langle \phi_2 | r^L | \phi_3 \rangle. \quad (14)$$

Now compare eq (14) with (10), (12) and (13) keeping only the L-multipole of the particle-hole force. As far as the angular momentum factors go there is a proportionality between products of beta-decay matrix elements for two configurations and the nuclear interactions between the configurations. We also expect the radial integrals to keep a phase relation: single-particle wave functions peak on the surface [11]) and the phase of integrals is essentially determined by the phase on the surface.

As is well known in other collective phenomena, this phase coherence implies that the beta moment will be enhanced or hindered depending on the sign of the particle-hole force in the appropriate multipole. Since the $T = 1$ force is repulsive in all natural parity multipoles, we get a hindrance. This is the microscopic mechanism of the effect. We now return to numerical results.

TABLE 2
Theoretical beta moments with shell-model wave functions

Nucleus	Pure configuration M^2 (fm^2)	Hindrance with configuration mixing		Experimental hindrance
		quoted wave functions	our wave functions	
^{17}N	0 96	1/2 0	1/2 4	1/8
^{39}Ar	1 11		1/500	1/35
^{41}Ca	0.55	1/35	1/1000	1/35

The theoretical beta moments are summarized in table 2. The first column shows the pure shell-model prediction for the beta moment. This prediction uses the lowest configuration with the valence pair coupled to $J = 0$. Such a wave function is a good representation of the pairing vibrational model. The radial integrals were evaluated with harmonic oscillator wave functions, using size parameters $V = 0.27$ and 0.33 for the Ca region and N respectively [18]). As compared with Woods-Saxon wave functions, this is accurate to within 5 % for all cases except with the loosely bound 2s particle in ^{17}N. Since this orbit did not play a great role in the beta moment, we preferred to stay with the simple harmonic oscillator model

The hindrance factor with respect to pure configurations,

$$h = \langle M \rangle^2_{\text{mixed}} / \langle M \rangle^2_{\text{pure}} \tag{15}$$

is given for the published configuration-mixed wave functions [15,16]) in the second column The third column shows the hindrance with our wave functions, including small components The cancellation is only moderate for ^{17}N, but near perfect for the two nuclei near ^{40}Ca Experimentally there is more hindrance for ^{17}N than we predict. The experimental hindrance is stronger in the heavier nuclei. but there is not the near-perfect cancellation of the matrix element that we predict

3. Concluding remarks

We have seen in the previous section that the unique forbidden beta decay process is particularly sensitive to the role of correlations in nuclear structure. The hindrance in the decay rate is essentially due to the repulsion between particles and holes in the $T = 1$ channel of p-h isospin coupling. On the basis of correlations it is possible to establish the hierarchy of table 3 for hindrances for various types of decays, explained

TABLE 3

Qualitative behavior on hindrances for various types of shell-model transitions

Type of transition	Hindrance
(1) $\lvert 1p \rangle \rightarrow \lvert 1p \rangle$	none
(2) $\lvert 2p \rangle \rightarrow \lvert 2p \rangle$	none
(3) $\lvert 1p\text{-}1h \rangle \rightarrow \lvert 1p\text{-}1h \rangle$	?
(4) $\lvert 1p\text{-}1h \rangle \rightarrow \lvert 0 \rangle$	weak
(5) $\lvert 2p\text{-}1h \rangle \rightarrow \lvert 1p \rangle$	
$\lvert 2h\text{-}1p \rangle \rightarrow \lvert 1h \rangle$	strong

below. Transitions of the first three kinds were studied by Damgaard et al. [8]). There are no strong correlations in the first case and no hindrance. In the second case there are correlations between valence particles due to the attraction between particles. However with similar correlations in parent and daughter there is no net effect. In the third case they studied, ^{208}Tl$(5^+) \rightarrow {}^{208}Pb(5^-)$, one transition is between particle-hole states of quite different kinds. The parent nucleus has a $T = 1$ particle hole pair with a repulsive interaction, while the daughter nucleus has a vibration-like particle-hole state which would be correlated [19]) We therefore expect a hindrance. In fact the authors did find a cancellation in matrix element to the collective state; however the correlation in the parent nucleus was just too weak to ascribe the effect to spatial correlations.

The fourth case, studied by Warburton et al , has definite correlation effects. Hindrances of $\frac{1}{2}$ for ^{16}N and $\frac{1}{2}$ for ^{40}K were reported. This cancellation is moderate and would be stronger with a stronger force. Instead of a stronger force, we can get better cancellation by turning to case (5). When two valence particles are well-

correlated, they will coherently repel the hole and double the effective strength of the repulsion. We will not attempt to trace this idea through the Racah algebra, but merely mention that it seems to work in the Ca region. In the O region it does not work well. The ^{15}C beta decay seems to have practically full single-particle strength [†]. The trouble with the O region is that the holes are primarily $p_{\frac{1}{2}}$. This makes it a special case because correlations are not easy to build. The $p_{\frac{1}{2}}$ holes in ^{15}C extend uniformly over the nuclear surface and cannot coherently repel the particle.

Other operators are used also to test correlations. Most important are the electromagnetic operators, which with electron scattering can probe the correlations as a function of excitation energy. However measurement of the large amplitudes, such as to the giant dipole state, does not provide as stringent a test of the nuclear forces. We have seen that the small amplitudes to the ground states of higher isospin are very sensitive to the strength of the force. The electromagnetic analog of the first forbidden unique beta decay is the M2 operator. This has been studied by Kurath and Lawson who found sizable hindrances in transitions between low states [20]. They used aligned coupling wave functions which readily provide anticorrelations between particles and holes. Shell-model hindrance has been discussed by one of us [21]) for another operator, the two-particle transfer operator in the reactions

$$\text{projectile} + |2p, T = 1\rangle \rightarrow |1p\text{-}1h, T = 1\rangle + \text{reaction product.} \tag{16}$$

As might be imagined, it is not easy to extract information about particle-hole correlations from such a complicated process as (16). Another experiment which tests p-h correlations is muon capture [22]) such as in

$$\mu^- + |0\rangle \rightarrow |1p\text{-}1h\rangle + \nu.$$

If feasible, the following π-capture process

$$\pi^- + |1p\rangle \rightarrow |2p\text{-}1h\rangle + \gamma$$

would provide an interesting experimental test of correlation.

In conclusion we wish to stress that even if the physics of the hindrance of the forbidden beta decay is understood, to get a numerical agreement with experimental values requires a much more refined description of the nuclear structure itself. For example, correlations in the closed shell core will also interfere destructively with the single-particle transition amplitude. Taking into account all the uncertainties, accurate reproduction of the experimental data cannot be expected.

[†] Not only is this decay nonunique, but the standard approximation is not valid. We hesitate to discuss it further

Appendix A

Wave functions were computed for ^{17}N, ^{39}Ar and ^{41}K with eqs. (8), (9) and (10). Single-particle energies, eq. (11), were determined from separation energies of one-particle and closed-shell nuclei. The interaction in eqs. (9) and (10) was taken to be the "bare" G-matrix interaction of Kuo and Brown.

The wave functions are quoted with the coupling $(h(pp)^L)^J$ or $(p(hh)^L)^J$, with the orbits in each major shell identified by j.

^{17}N $(J = \frac{1}{2}, T = \frac{3}{2})$:

$|\psi[\text{ref. }^{16})]\rangle = 0.900|\frac{1}{2}(\frac{5}{2}\frac{5}{2})^\circ\rangle + 0.353|\frac{1}{2}(\frac{1}{2}\frac{1}{2})^\circ\rangle + 0.182|\frac{1}{2}(\frac{3}{2}\frac{3}{2})^\circ\rangle + 0.159|\frac{3}{2}(\frac{5}{2}\frac{5}{2})^2\rangle,$

$|\psi(\text{KB})\rangle = 0\ 946|\frac{1}{2}(\frac{5}{2}\frac{5}{2})^\circ\rangle + 0.156|\frac{1}{2}(\frac{1}{2}\frac{1}{2})^\circ\rangle + 0\ 196|\frac{1}{2}(\frac{3}{2}\frac{3}{2})^\circ\rangle + 0.193|\frac{3}{2}(\frac{5}{2}\frac{5}{2})^2\rangle$
$+ 0.040|\frac{3}{2}(\frac{5}{2}\frac{1}{2})^2\rangle + 0.053|\frac{3}{2}(\frac{5}{2}\frac{5}{2})^2\rangle - 0.015|\frac{1}{2}(\frac{5}{2}\frac{3}{2})^1\rangle.$

^{39}Ar $(J = \frac{3}{2}, T = \frac{3}{2})$:

$|\psi(\text{KB})\rangle = 0.848|\frac{7}{2}(\frac{3}{2}\frac{3}{2})^\circ\rangle - 0.485|\frac{7}{2}(\frac{3}{2}\frac{3}{2})^2\rangle + 0.200|\frac{7}{2}(\frac{3}{2}\frac{1}{2})^2\rangle - 0.073|\frac{3}{2}(\frac{3}{2}\frac{3}{2})^2\rangle$
$+ 0.003|\frac{3}{2}(\frac{1}{2}\frac{3}{2})^2\rangle - 0.062|\frac{7}{2}(\frac{5}{2}\frac{2}{2})^2\rangle - 0.039|\frac{5}{2}(\frac{3}{2}\frac{3}{2})^2\rangle.$

^{41}K $(J = \frac{3}{2}, T = \frac{3}{2})$:

$|\psi[\text{ref. }^{15})]\rangle = 0.81|\frac{3}{2}(\frac{7}{2}\frac{7}{2})^\circ\rangle - 0.43|\frac{3}{2}(\frac{7}{2}\frac{7}{2})^2\rangle + 0.21|\frac{1}{2}(\frac{7}{2}\frac{7}{2})^2\rangle - 0.13|\frac{3}{2}(\frac{7}{2}\frac{3}{2})^\circ\rangle,$

$|\psi(\text{KB})\rangle = 0.825|\frac{3}{2}(\frac{7}{2}\frac{7}{2})^\circ\rangle - 0.491|\frac{3}{2}(\frac{7}{2}\frac{7}{2})^2\rangle + 0.211|\frac{1}{2}(\frac{7}{2}\frac{7}{2})^2\rangle + 0.030|\frac{5}{2}(\frac{7}{2}\frac{7}{2})^2\rangle$
$- 0.060|\frac{3}{2}(\frac{7}{2}\frac{3}{2})^2\rangle + 0.022|\frac{1}{2}(\frac{7}{2}\frac{3}{2})^2\rangle - 0.018|\frac{3}{2}(\frac{7}{2}\frac{5}{2})^2\rangle + 0.002|\frac{3}{2}(\frac{7}{2}\frac{5}{2})^1\rangle.$

Appendix B

CALCULATION OF BETA MOMENTS

Standard Racah techniques such as described by Edmonds [23] are sufficient to derive a formula for the beta moments between the simple shell-model states we use. However the expression with reduced matrix elements is not very transparent. Such a formula is quoted in ref [2], their eq (A17), with an error of phase[†]. We therefore provide an alternate derivation [24] which is easy to carry out and does not have the phase ambiguity of reduced matrix elements,

$$\langle j_1||\theta||j_2\rangle = (-1)^{j_1 - j_2}\langle j_2||\theta||j_1\rangle. \tag{B.1}$$

The strategy is to write matrix elements in terms of overlaps of angular momentum eigenstates. The matrix element is broken down into smaller pieces by factoring an overlap of a product of wave functions into products of overlaps:

$$\langle(\phi^L\Psi^K)^J|(\phi'^L\psi'^K)^J\rangle = \langle\phi^L|\phi'^L\rangle\langle\psi^K|\psi'^K\rangle. \tag{B.2}$$

† Nevertheless, the authors used the correct phase in numerical work.

We now apply these principles to the beta moment for the decay

$$|[(j_1 \, j_2)^L j_3^{-1}]^J\rangle \to |j_1\rangle$$

Writing the beta moment as an overlap,

$$M = \langle [(j_1 \, j_2)^L j_3^{-1}]^J | r^L([Y^L\sigma]^K j_1)^J\rangle \tag{B.3}$$

we first bring it into a form where the odd particle may be factored out. This requires a recoupling of the 2p-1h state which is conveniently done with Jahn's Racah transformation matrix

$$M = \sum_Q (-1)^{j_1+K-J} U(j_1 \, j_2 \, J j_3; LQ)\langle [j_1(j_2 \, j_3^{-1})^Q] | r^L[j_1[Y^L\sigma]^K]^J\rangle. \tag{B.4}$$

Next we factor out the odd particle in the matrix element and integrate

$$M = \sum_Q (-1)^{j_1+K-J} U(j_1 \, j_2 \, Jj_3; LQ)[\langle j_1|j_1\rangle\langle (j_2 \, j_3^{-1})^Q | r^L[(Y^L\sigma)^K\rangle \tag{B.5}$$

$$= (-1)^{j_1+K-J} U(j_1 \, j_2 \, Jj_3; LK)\langle (j_2 \, j_3^{-1})^K | r^L(Y^L\sigma)^K\rangle. \tag{B 6}$$

The remaining 1p-1h matrix element is further reduced with an LS-jj recoupling coefficient:

$$\langle (j_2 \, j_3^{-1})^K | r^L(Y^L\sigma)^K\rangle = \langle (l_2 \tfrac{1}{2})^{j_2}(l_3 \tfrac{1}{2})^{j_3} | (l_2 \, l_3)^L(\tfrac{1}{2}\tfrac{1}{2})^1\rangle_K$$
$$\times \langle (l_2 \, l_3^{-1})^L | Y^L\rangle\langle (\tfrac{1}{2}\tfrac{1}{2})^1|\sigma\rangle\langle \phi_2|r^L|\phi_3\rangle. \tag{B.7}$$

This is as far as the matrix element can be reduced by factoring; the remaining overlaps must be explicitly computed.

$$\langle (l_2 \, l_3^{-1})^L | Y^L\rangle = \sqrt{\frac{(2l_2+1)(2l_3+1)}{4\pi(2L+1)}}(l_2 \, 0 \, l_3 \, 0|L0), \tag{B 8}$$

$$\langle (\tfrac{1}{2}\tfrac{1}{2})^1|\sigma\rangle = \sqrt{2}. \tag{B 9}$$

Combining all these equations we get the result eq. (20) of the text.

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