Rotating Superfluidity in Nuclei.

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Summary. — The limits of angular momentum in rotating nuclei may permit superfluidity with nonzero angular momentum, but the excitation energy appears too high to have observable consequences.

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Nuclei in their ground state can be viewed, in general, as a condensate of pairs of nucleons coupled to angular momentum J equal to zero (1). Evidence for the existence of multipole (nonzero J) pairing has also been found in a variety of nuclear properties (2). Because the parity of two-particle states around the Fermi energy is positive, essentially only even multipolarities are allowed. A successful description of the variety of pairing correlations is obtained in terms of the Hamiltonian

$$H = H_{sp} + H_{p}, \tag{1}$$

sum of a single-particle term

$$H_{sp} = \sum_{\nu} \varepsilon_{\nu} a_{\nu}^{\dagger} a_{\nu}, \tag{2}$$

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and of a multipole pairing interaction

$$H_p = -\sum G_J \sum_{\substack{\nu_1\nu_2\\ \nu_3\nu_4}} \{[a^\dagger_{\nu_1} a^\dagger_{\nu_2}]_J [a_{\nu_3} a_{\nu_4}]_J\}_0 . \tag{3}$$

As usual $[a^\dagger_{\nu_1} a^\dagger_{\nu_2}]_J$ denotes coupling of the pair operators to total angular momentum J, and similarly for $\{\ \}_0$.

Restricting the single-particle states ν_i to the valence orbitals, the pairing coupling constant can be written as

$$G_J \sim G \sim \frac{27}{A}\,\text{MeV}, \tag{4}$$

where A is the mass number of the nucleus.

Empirically, the d-state pairing energy of a single pair is about half that of a monopole pair. The reduction is due to the decrease in phase space for valence pairs with higher J. This is shown schematically in fig. 1. The effect is even more marked for $J = 4$. Consequently, near the ground state, we do not expect metastability for $J \neq 0$.

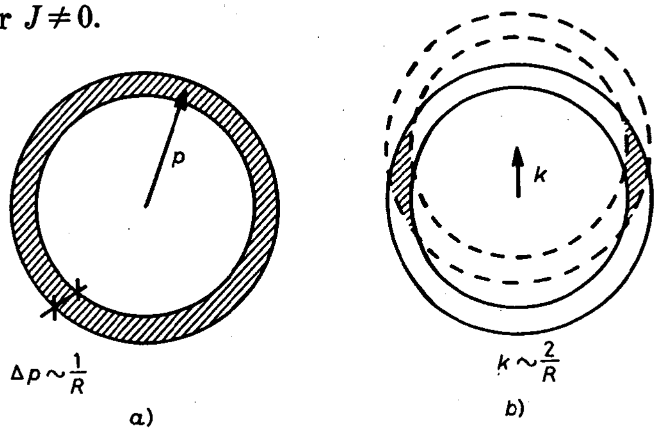

Fig. 1 – Phase space for particles in paired wave functions. The available momenta for valence particles in a finite Fermi system are shown in a). All momenta are allowed for a particle in a pair with total momentum zero. When the pair momentum in nonzero, the valence phase space is reduced as indicated in b).

This situation may be rather different for rapidly rotating nuclei. In this case large values of the angular momentum can be built by using a coupling scheme where both valence and core particles couple pairwise to angular momentum J. Because of the strong modifications suffered by the single-particle states, two-particle states with negative parity are now readily available close to the Fermi energy. The lowest multipolarity different from zero to which pairs of particles can couple is $J = 1$.

Under these circumstances, Galilean invariance allows one to redefine the phase space where dipole pairing acts, so that the resulting phase space is nearly the same as for $J = 0$ pairing.

In this scheme, the lowest rotating superfluid state has angular momentum $I = A/2$. The pairing energy will be $E_n = (\Delta_n^2 + \Delta_p^2)/2d \sim 2.5\,\text{MeV}$, where Δ_n and Δ_p are the protons and neutrons pairing gaps ($\sim 1\,\text{MeV}$). The quantity d is the distance between single-particle levels, which, for medium-heavy nuclei is $0.4\,\text{MeV}$ (cf. p. 653 of ref. (3)).

Metastability arises because transitions with $I \to I - \Delta I$ require breaking of pairs, which may be energetically forbidden.

For this scheme to work, two important questions have to be answered:

1) Does the nucleus allow spins as high as $I \sim A/2$?

2) How does the energy cost to form a vortex compare with the energy gain of pairing?

It has been calculated that a cold drop of Fermi liquid with $A \sim 150$ can accommodate up to about 80 units of angular momentum (4). Experimental studies of discrete lines in the quadrupole gamma-decay spectrum of strongly rotating nuclei have identified, in this mass region, rotational bands which carry up to 60 units of angular momentum (5).

In what follows we give a simple answer to the second question, making use of a schematic model. We assume the system to be a cylinder of height H and radius R, and constant density ρ_0 which rotates around the symmetry axis.

Because of the $J \neq 0$ superfluidity, a vortex forms with a cylindrical hole along the axis of rotation. The velocity of the vortex can be written $v_\phi = g/r$, where $g = \hbar/2m$ for $J = 1$ vorticity. The energy of the vortex consists of a rotational part and a part associated with the surface energy of the role. The rotational energy is estimated as

$$E_{\text{vortex}} = \frac{1}{2}\rho_0 \int_0^H \mathrm{d}z \int_a^R v_\phi^2 2\pi r\,\mathrm{d}r = \frac{1}{R^2 M} L^2 \ln\frac{R}{a}, \tag{5}$$

where a is the radius of the cylinder, M is the mass of the nucleus, and L is the angular momentum,

$$L = \rho_0 \int_0^H \mathrm{d}z \int_a^R v_\phi 2\pi r\,\mathrm{d}r \cong \rho_0 H \pi R^2 g. \tag{6}$$

We estimate from the surface tension σ the energy cost to make the vortex

$$E_{\text{int}} = 2\pi H a \sigma. \tag{7}$$

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Minimizing the total energy with respect to a, we find that $a = 0.7$ fm. To obtain the excitation energy of the state, we compare with the energy of rigid rotation, which is $E_{\text{rigid}} = L^2/MR^2$. For a nucleus of mass $A \sim 150$, rotating with angular momentum 75, the excitation energy is then

$$\Delta E = \frac{L^2}{MR^2}\left(\ln\frac{R}{a} - 1\right) + 2\pi H\sigma a \approx 70\,\text{MeV}\,, \tag{8}$$

where the values $\sigma \sim 1\,\text{MeV fm}^{-2}$, $R = 1.2A^{1/3}$ fm and $a = 0.7$ fm were used. The energy (8) is much larger than the energy gain from pairing.

We conclude that the vortex state could be metastable in nuclei, but its statistical weight would be too small to be observed.

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● RIASSUNTO

I limiti per il momento angolare di nuclei ruotanti sembrerebbe permettere l'esistenza di superfluidità con momento angolare diverso da zero, anche se l'energia di eccitazione è troppo elevata per avere conseguenze sperimentali.

Резюме не получено.