PHYSICAL REVIEW C VOLUME 37, NUMBER 5 MAY 1988

Pion interferometry in ultrarelativistic heavy-ion collisions

G. Bertsch, M. Gong, and M. Tohyama
Cyclotron Laboratory and Physics Department, Michigan State University, East Lansing, Michigan 48824
(Received 12 January 1988)

Pion correlations are calculated for a recently proposed model of ultrarelativistic heavy-ion collisions. Ingredients of the model are evaporation of globs of quark-gluon plasma followed by pion scattering in the final state. The model predicts strong correlations between the longitudinal momentum and position of the source. The predicted transverse source size is smaller than preliminary findings from a recent experiment.

A major goal in the study of ultrarelativistic heavy ion collisions is to identify the putative quark-gluon phase of matter. Signatures for the phase tend to be ambiguous, so much work is needed to study detailed models relating the high energy-density behavior to the observables in the final state. We have recently constructed a model to this end.[1] In Ref. 1 we applied the model to discuss theoretical questions such as entropy conservation and freezeout conditions, as well as to calculate one observable function, the dependence of transverse momentum on the particle multiplicity. In this paper we will apply the model to study the pion correlation function and compare it with the first data, reported in conferences,[2] from experiments at CERN (the European Organization for Nuclear Research).

We first briefly describe our model for hadronization in ultrarelativistic collisions. The initial condition is a boost-invariant cylinder of quark-gluon plasma, as proposed by Bjorken.[3] We assume that at the beginning of the phase transition the plasma becomes unstable and breaks up into droplets (which we call globs). The phase transition proceeds as globs evaporate by emitting thermal pions. The pions can be absorbed by the globs and they also scatter from each other. Thus the model gives a microscopic description of momentum-energy transport from the plasma phase to the hadronic phase. The initial condition of the system is specified by two parameters, the radius of the cylinder and its entropy density, which then determine the rapidity density of the pions dN/dy in the final state. Details of the model are described in Ref. 1.

The pion-pion correlation function provides specific information about the space-time evolution of the source of points.[4] It is defined by the ratio of two-pion to one-pion inclusive yields,

$$R(\mathbf{p}_1,\mathbf{p}_2)=\frac{\dfrac{d^6N}{dp_1^3dp_2^3}}{\dfrac{d^3N}{dp_1^3}\dfrac{d^3N}{dp_2^3}} . \tag{1}$$

For a chaotic pion field this may be calculated from the pion source function $D(x,p)$ describing the production of a pion at the space-time point x with four-momentum p. The formula is[5]

$$R(\mathbf{p}_1,\mathbf{p}_2)=1+\frac{\int d^4x\, d^4x' D\left[x,\dfrac{p_1+p_2}{2}\right] D\left[x',\dfrac{p_1+p_2}{2}\right]\cos(p_1-p_2)(x-x')}{\int d^4x\, d^4x' D(x,p_1)D(x',p_2)} . \tag{2}$$

This formula takes into account only the boson symmetry of the many-pion wave function, neglecting interactions between the poins. In the cascade model of Ref. 1, all particles are treated classically, and the only momentum correlation is the very weak one imposed by overall energy-momentum conservation. The classical source function resulting from the calculation may be expressed as

$$D(x,p)=\sum_{i=1}^{N}\delta^4(x-x_i)\delta^4(p-p_i) . \tag{3}$$

Here N is the total number of pions in the final state, and x_i, p_i are the position and momentum of the ith pion at its last interaction point. If the pion suffers no collisions, the coordinates are determined at the emission point from the glob. In applying Eq. (3), we shall also make an ensemble average by summing over a number of runs of the cascade program.

The classical source function is too singular to display interference effects. In any quantum treatment, the source function would depend smoothly on x and p. We shall be forced to smooth the source function in any case because of the limited statistics associated with the cascade simulation. We consider two procedures. One is to develop an analytic parametrization of the source function. R can then be obtained by integrations. This has

 ©1988 The American Physical Society

the advantage that one can compare with other assumed parametrizations, such as the one proposed by Kolehmainen and Gyulassy.[6] However, a simple analytic parametrization may not describe all the important correlations, as was emphasized by Pratt.[5] We also model the source function by smoothing the momentum dependence in the cascade result, Eq. (3), with a Gaussian convolution, $\exp\{-[(p-p')/p_0]^2\}$. The correlation function will not be sensitive to the smoothing provided the width of the Gaussian is not so large as to affect the spread in momentum.

Our calculations were performed for initial conditions simulating oxygen collisions on a heavy target. The radius of the cylinder is taken to be 3 fm, which is slightly larger than the oxygen radius. The initial entropy density was chosen to give a rapidity density of pions (all charges) in the final state satisfying $dN/dy \approx 80$. These are too few pions to give good statistics in the source function, so we combined data from 10 collisions to make the cascade function. This smooths out fluctuations in the source due to the localization of the globs, reducing the spatial correlation somewhat. Our model of the phase transition assumes a transition temperature of 200 MeV and a ratio of entropy densities in the two phases equal to 13. With these conditions, the hadronization in the model beings at proper time $\tau=0.7$ fm/c. This may be compared with the usual 1 fm/c assumed equilibration time. In fact, very few final state pions are produced at early times so the results are insensitive to this detail. Pion production is largest between 5 and 10 fm/c, and then quickly drops off. The time evolution is governed mainly by the evaporation rate of the globs; pion rescattering is relatively insignificant.

A boost-invariant source may be parametrized by a function of transverse position r_{perp}, transverse momentum p_{perp}, the difference between space-time and momentum rapidities $y_r - y_p$, and the proper time τ. The rapidities are defined as $y_r = \tanh^{-1}(z/t)$ and $y_p = \tanh^{-1}(p_z/E)$, and the proper time is $\tau = [(t^2 - z^2)]^{1/2}$. We first consider the proper time dependence. The production rate of pions is shown in Fig. 1. Conventionally, the source is parametrized by an exponential function in time, but from Fig. 1 a much better function would be $\tau\exp[-(\tau/\tau_0)^2]$. A fit to this function is shown in the figure, with the lifetime parameter $\tau_0 = 9$ fm/c. This time scale is essentially set by the time to make the phase transition in a one-dimensional hydrodynamic expansion.[3]

The transverse spatial distribution of the source is shown in Fig. 2. Note that the source is weak outside the physical dimensions of the initial cylinder. As discussed in Ref. 1, this could be anticipated from the hydrodynamic behavior of rarefaction shocks.[7] The quark-gluon plasma does not expand into the vacuum; rather, a rarefaction wave of hadronization moves into the plasma cylinder. As it turns out, the density of pions emerging from the cylinder is not large enough to shift the source outside by rescattering. The transverse spatial dependence of the model's source may be fit to the usual Gaussian parametrization, $\exp[-(r_{\text{perp}}/R)^2]$. A fit of this form is shown in the figure; the size parameter is $R=3.3$ fm.

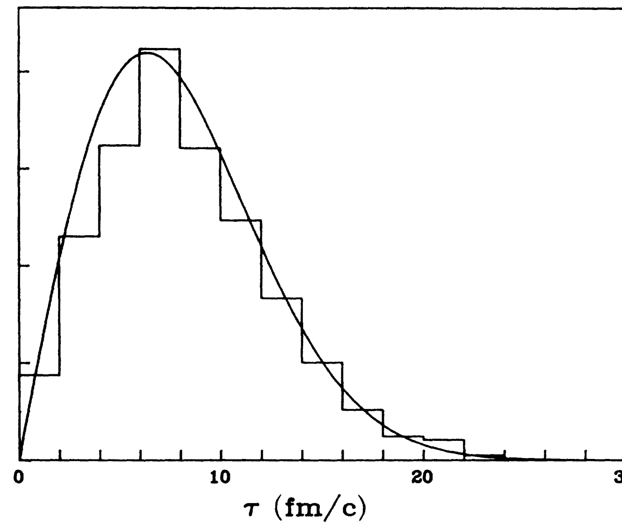

FIG. 1. Pion production rate as a function of time. In the cascade model, the initial cylinder radius is $R=3$ fm and the entropy density corresponds to a final state pion multiplicity $dN/dy=80$ (all pions). The solid line shows a fit with the function $\tau\exp[-(\tau/\tau_0)^2]$ and $\tau_0=9$ fm/c.

The momentum distribution in the source is close to a thermal distribution because the evaporation of the pions from the globs is assumed to be statistical. Given the transition temperature of 200 MeV, the average transverse momentum is 480 MeV/c, which is hardly changed by the rescattering of the pions. The detailed shape of the momentum distribution is not important if correlations between transverse momentum and position are ignored. Then the transverse momentum source function

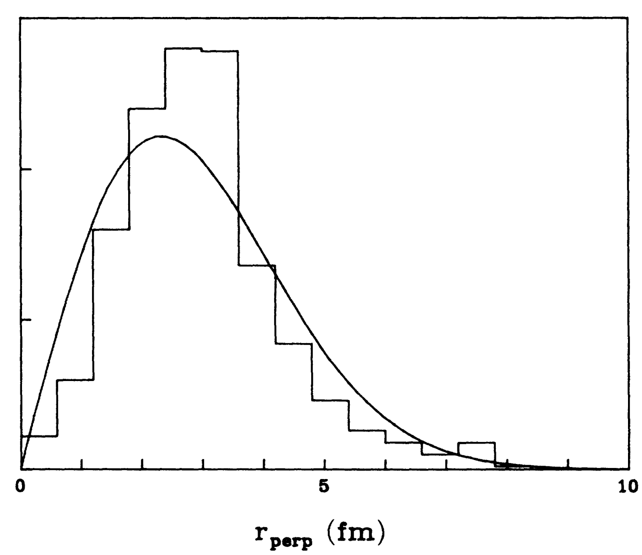

FIG. 2. Spatial origin of pions for the same conditions as in Fig. 1. The fit with a Gaussian $\exp[-(r_{\text{perp}}/R)^2]$ and $R=3.3$ fm is shown as the solid line.

approximately cancels out in Eq. (2) since $D^2((p_1+p_2)/2)\approx D(p_1)D(p_2)$. The longitudinal momentum is strongly correlated with the rapidity of the local frame, and this must be included in any parametrization. We show in Fig. 3 the distribution of sources as a function of the rapidity difference y_r-y_p. It is reasonably fit by a Gaussian of the form $\exp[-(y_p-y_r)^2/y_0^2]$ with $y_0^2=0.58$. This may be compared with the thermal parametrization

$$D\approx\exp(-e/T)=\exp[-m_{\mathrm{perp}}\cosh(y_r-y_p)/T]$$

proposed by Kohlemainen and Gyulassy.[6] Expanding the exponent in power of y_r-y_p, the width parameter in the thermal model is $y_0^2=2T/\langle m_{\mathrm{perp}}\rangle\approx2\times100/480\approx0.8$, somewhat broader than the cascade result. Evidently, the pion interactions reduce the dispersion in longitudinal momentum. To summarize, a rough characterization of the source in our model is the following:

$$\begin{aligned}&D(x,p)=f(p_{\mathrm{perp}})\tau e^{-(\tau/\tau_0)^2}e^{-(r_{\mathrm{perp}}/R)^2}e^{-(y_r-y_p)^2/y_0^2},\\&\tau_0=9\ \mathrm{fm}/c,\\&R=3.3\ \mathrm{fm},\\&y_0=0.76\ .\end{aligned}\tag{4}$$

We now examine the predicted pion correlation function. Using the analytic source function, Eq. (4), the integration over transverse position may be performed to obtain

$$R(p_1,p_2)=1+e^{-(Q_{\mathrm{perp}}R)^2/2}\int d\tau\,d\tau'dy\,dy'\frac{4\tau\tau'}{\tau_0^4\pi y_0^2}\exp\left[-\frac{\tau^2+\tau'^2}{\tau_0^2}\right]\exp\{-[(y_{(p_1+p_2)/2}-y)^2+(y_{(p_1+p_2)/2}-y')^2]/y_0^2\}$$
$$\times\cos[(p_{1_z}-p_{2_z})(z-z')-(E_1-E_2)(t-t')]\ ,\tag{5}$$

where $Q_{\mathrm{perp}}=|\mathbf{p}_{1_{\mathrm{perp}}}-\mathbf{p}_{2_{\mathrm{perp}}}|$. The correlation goes to 2 when the momentum difference is zero, as is required for any source in Eq. (2). The dependence on the transverse momentum is essentially a Gaussian in the sidewards direction [perpendicular to $(\mathbf{p}_{1_{\mathrm{perp}}}+\mathbf{p}_{2_{\mathrm{perp}}})$], and measures directly the transverse size of the source. In Fig. 4 we compare the sidewards Q_{perp} dependence of the parametrized source Eq. (4) and the actual cascade source, Eq. (3), convoluted with a Gaussian as described above. The parametrization gives a correlation reasonably close to the cascade source, which, as mentioned, has very little transverse expansion.

When the transverse momentum difference is along $\mathbf{p}_{\mathrm{perp}}$, which we call outward, the correlation is considerably reduced. This may be seen in Fig. 5. An effect of this sort is proposed in Ref. 5, but the reason seems to be quite different from what is suggested there. The distribution in time of emission in our model is considerably larger than the transverse size of the source. The energy of the particles vary with the outward momentum difference, so this observable is sensitive to the time correlation, as was proposed in Ref. 5.

We next examine the dependence of R on the longitudinal momentum difference. Again, the correlation in ra-

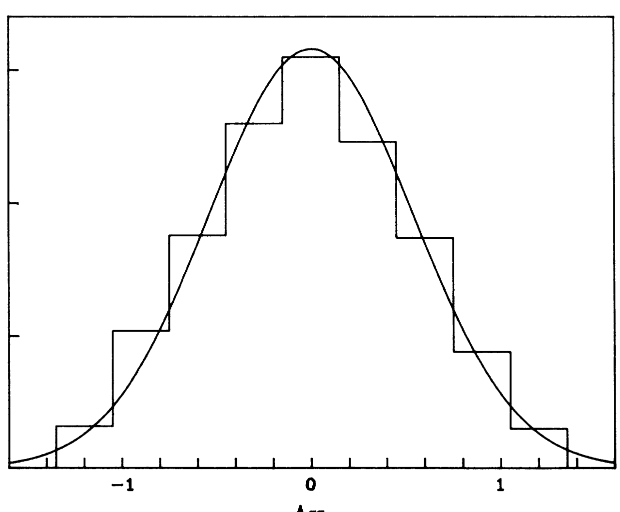

FIG. 3. Correlation between spatial and momentum rapidity of the pion source, for the same conditions as in Fig. 1. The fit with a Gaussian $\exp[-(\Delta y/y_0)^2]$ and $y_0=0.76$ is shown as the solid line.

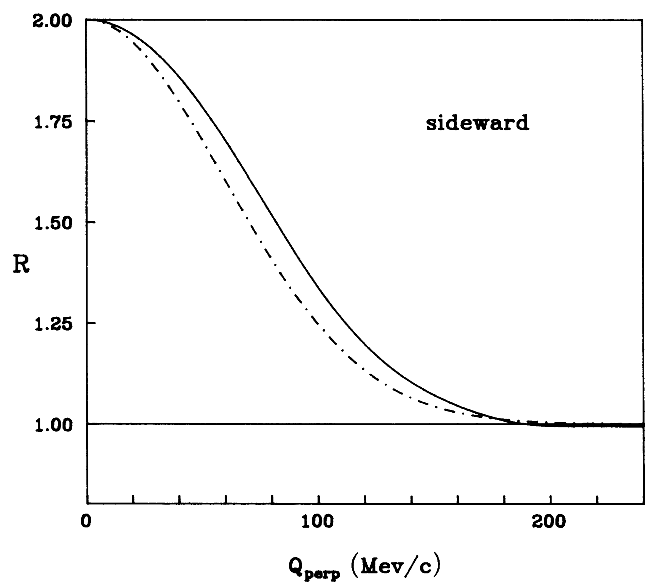

FIG. 4. Sideward transverse momentum correlation between pions, with outward $Q_{\mathrm{perp}}=0$ and longitudinal momentum difference $Q_{\mathrm{long}}=0$. Solid line is the cascade result, and the dashed line is the semianalytic parametrization.

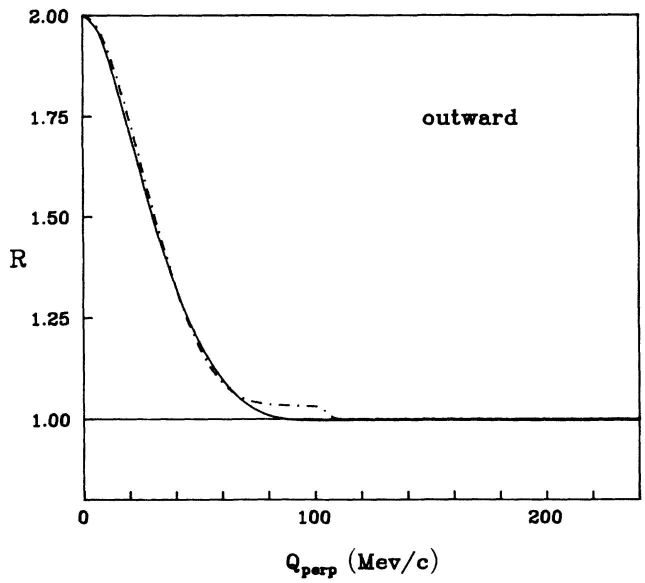

FIG. 5. Outward transverse momentum correlation between pions, with sideward $Q_{\text{perp}}=0$ and $Q_{\text{long}}=0$. Solid line is the cascade result, and the dashed line is the semianalytic parametrization.

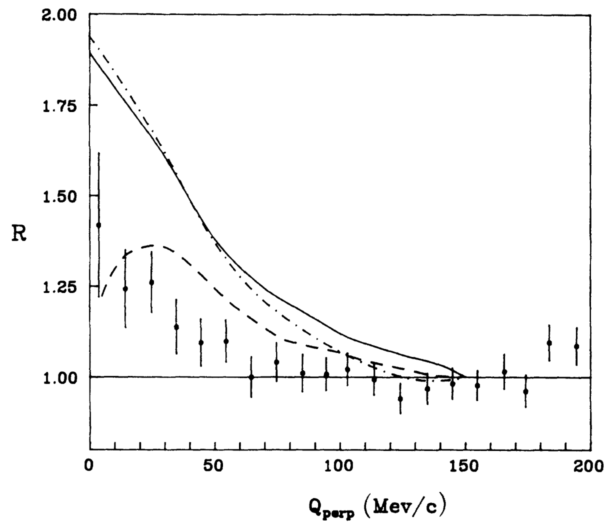

FIG. 7. Correlation between pions with the cuts $|Q_{\text{long}}| < 100$ MeV/c, $2<y<3$, as a function of magnitude of $\mathbf{Q}_{\text{perp}}$. Solid line is cascade result, and the dot-dashed line is the semianalytic parametrization. The data are from Ref. 2. The long-dashed curve is the cascade result renormalized by the Coulomb and strong interactions.

pidity and the delay in time of emission makes the longitudinal extension of the source quite large. This results in a very narrow correlation function, as may be seen in Fig. 6.

Finally, we examine the correlation integrating over variables to facilitate comparison with experiment. In the reported experiment,[2] the correlation is made using Q_{long} rather than Δy_p as a variable. This makes a difference when p_{perp} is different for the two particles. In Fig. 7 we show the correlation for the reported cut,

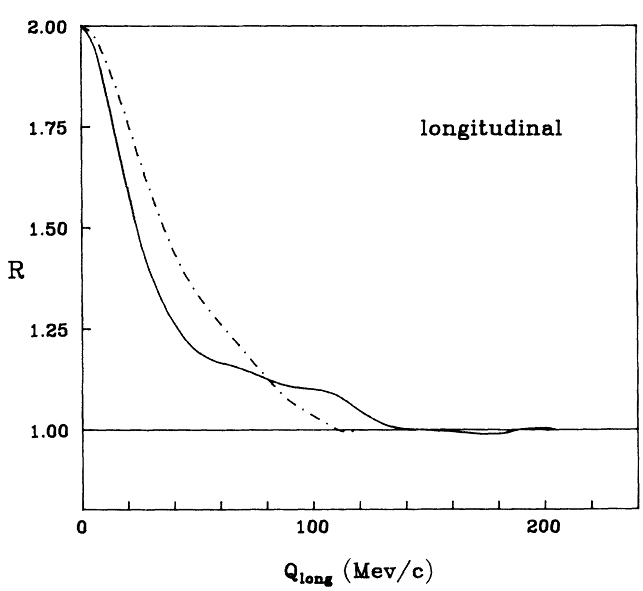

FIG. 6. Longitudinal momentum correlation between pions, with $Q_{\text{perp}}=0$. Solid line is the cascade result, and the dashed line is the semianalytic parametrization.

which is $2<y_p<3$, $|Q_{\text{long}}|<100$ MeV/c, and fixed $|Q_{\text{perp}}|$. Since the direction of $\mathbf{Q}_{\text{perp}}$ is allowed to vary, the measured correlation will be somewhere between the correlations associated with transverse size and with time. The maximum is slightly reduced from 2 by the integration over Q_{long}. The preliminary data from the NA35 experiment at CERN (Ref. 2) is also shown in Fig. 7. The maximum of the data is much less than predicted, and this is a major disagreement. The model neglects interactions between the particles which, in the case of nucleons, strongly affects the low momentum correlation function.[8] It is common to neglect the strong interaction and treat the Coulomb by the penetrability factor $C_0^2=2\pi\eta/e^{2\pi\eta}-1$.[4] We have examined the effects of finite source size and of the strong interaction by using numerically calculated wavefunctions.[9] We find that the finite source size practically does not affect the Coulomb distortion, but that the strong interaction effects are significant, assuming a strong interaction scattering length of $0.12/m_\pi$ (Ref. 1) and a source size of $R_0=3$ fm. For example, the renormalization factor N at $Q=50$ MeV/c is as follows: Coulomb penetrability, $N=0.94$; Coulomb with the finite source, $N=0.96$; strong interaction, 0.93; combined distortion, $N=0.90$. We show on Fig. 7 the predicted correlation (dashed curve) multiplied by the renormalization factors calculated as above. The theoretical correlation remains much higher than the data. The alternative explanation of the reduction, invoking a coherent source, would be a remarkable finding.

Turning to the shape of the correlation, it appears that the experimental correlation does not extend to as high a momentum, showing a larger source than the model predicts. There are several effects that could change the source size that we have not included. For the experiments in heavy targets, there is considerable spectator matter which conceivably scatters the outgoing pions, making the source more extended. We have also ignored other mesons that could be emitted. The ω meson, for example, would decay outside the cylinder since its width is only 10 MeV. With a thermal source at a temperature of 200 MeV, only a small fraction of the emitted particles are higher mesons, but with their decays into multiple pions they might have a significant effect. A theoretical calculation by Faessler *et al.*[10] finds that 20% of the $p\bar{p}$ annihilation produces ω mesons. This point is now under study.

We acknowledge support by the National Science Foundation under Grant No. PHY 87-14432. We thank L. Czernai, and L. McLerran for helpful conversations, and D. Boal for help with Coulomb corrections as well.

[1]G. Bertsch, M. Gong, L. McLerran, V. Ruuskanen, and E. Sarkkinen, Phys. Rev. D (to be published).

[2]T. Humanic, *Pion Interferometry with Ultrarelativistic Heavy-Ion Collisions from the NA35 Experiment,* Quark Matter 1987 Conference, Nordkirchen, West Germany, Z. Phys. (to be published).

[3]J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).

[4]A general discussion and references to the earlier literature may be found in M. Gyulassy, S. Kauffmann, and L. Wilson, Phys. Rev. C **20**, 2267 (1979).

[5]S. Pratt, Phys. Rev. Lett. **53**, 1219 (1984).

[6]K. Kolehmainen and M. Gyulassy, Phys. Lett. **180B**, 203 (1986).

[7]H. von Gersdorff, L. McLerran, M. Kataja, and P. V. Ruuskanen, Phys. Rev. D **34**, 794 (1986); M. Kataja, P. V. Ruuskanen, L. McLerran, and H. von Gersdorff, *ibid.* **34**, 2755 (1986).

[8]S. E. Koonin, Phys. Lett. **70B**, 43 (1978).

[9]D. H. Boal and J. C. Shillcock, Phys. Rev. C **33**, 549 (1986).

[10]S. Furui, A. Faessler, and S. Khadikar, Nucl. Phys. **A424**, 495 (1984).