

## Exotic Radioactivity as a Superfluid Tunneling Phenomenon

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The recently discovered radioactive decay of heavy nuclei by emission of carbon and neon is described by a model containing elements from both the theory of  $\alpha$  decay and that of fission. A key ingredient is the inertial mass, which depends on the superfluid pairing gap of the nucleus. The potential energy is estimated from macroscopic considerations. Theory and experiment fit together in a reasonable way to give an overall picture of exotic decay.

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Exotic radioactivity, the decay of nuclei<sup>1</sup> by the emission of particles heavier than the  $\alpha$ , poses a challenge to explain the process within the existing framework of theory.<sup>2,3</sup> On one hand, in the Gamow theory of  $\alpha$  decay,<sup>4</sup> one first determines the probability to form the  $\alpha$  particle by calculating the overlap of wave functions. The  $\alpha$  particle is then propagated in a potential which is dominated by the Coulomb field, giving a tunneling formula for the decay rate.

On the other hand, the theory of spontaneous fission is quite different,<sup>5</sup> although it is nothing more than radioactivity by emission of a heavy nucleus. The dynamics of barrier penetration now occurs not as a particle in a Coulomb field, but by shape changes of a many-particle system through shapes that are energetically very unfavored. Here, not only is the energy of a shape configuration needed but also the inertia, that is, the coupling of the different shapes to each other by the quantum-mechanical Hamiltonian.

We view the exotic-decay process as composed of two main steps: First, the parent nucleus deforms, leading to a shape corresponding to the two daughter nuclei in contact; second, the two daughter nuclei tunnel through the external Coulomb barrier without further change in shape. The second step is straightforwardly calculated, given the potentials acting between the daughter nuclei.

To treat the first step, we describe the wave function in the internal region  $\psi_i$  as a linear combination of determinants  $\phi_k$ ,  $\psi_i = \sum_k a_k^i \phi_k$ . In the simple model discussed below, determinants describing neighboring configurations will differ in the crossing of two single-particle levels, implying that the state of two particles is changed. The deformation is assumed to be adiabatic, so that level crossings do not lead to excitation but simply bring the

nucleus from one local minimum to the next. The mixing is due to the residual interaction, which has mostly a pairing character.<sup>6</sup> Although the pairing in nuclei profoundly affects the inertia of the system with respect to large shape changes, it is not strong enough for the system to reach the classical inertia of the superfluid limit.

In our model we will consider explicitly the mixing of configurations of different shapes. We first enumerate the configurations, starting from the ground-state equilibrium shape and going to the configuration of touching daughter nuclei. We then estimate the interaction strength connecting adjacent (nearest neighbor) configurations. To make contact with macroscopic descriptions, a continuous coordinate  $\xi$  could be defined along a path in the space of deformations, although, of course, the local minima described by the set of determinants  $\phi_k$  are discrete points. The separation of the Hamiltonian into internal and external parts is shown in Fig. 1.

A procedure for counting the states in an approximate way is given by Bertsch.<sup>7</sup> It is based on the behavior of the nucleon distribution function in phase space when the nuclear shape is changed. One deforms the system in small steps, each of them chosen so as to give an integral number of particles moved across the Fermi surface. At these points the particles are put back below the Fermi surface, restoring the spherical shape of the Fermi surface. The number of steps corresponds to the number of level crossings in a more exact treatment. For example, in the case of quadrupole deformation, the collective variable  $\xi$  coincides with the quadrupole deformation parameter  $\beta$ . The number of particles  $m$  moved in a configurational change of a nucleus of mass  $A$  is

$$m = \Delta\beta(5/12\pi)^{1/2}A. \quad (1)$$

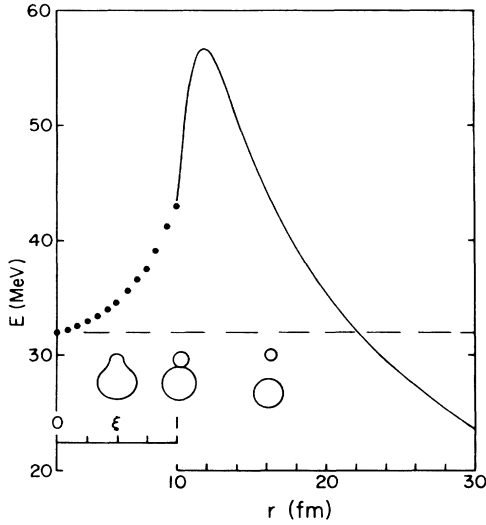


FIG. 1. Potential-energy plot of the decay  $^{223}\text{Ra}$  to  $^{14}\text{C}$  +  $^{209}\text{Pb}$ . On the left are shown the configurational energies of the thirteen shape configurations in our model. These are joined smoothly at 10 fm to the potential energy of the separating daughter nuclei. The dashed line indicates the total energy of the system.

For the case of pairing,  $m=2$ . This procedure has been checked by a comparison with microscopic determinations of the configurational changes along a deformation coordinate. Namely, we take a single-particle energy-level diagram and count the level crossings at the Fermi surface. In particular Figs. 7 and 8 of Bolsterli *et al.*<sup>8</sup> show 29 crossings between  $y=0$  and  $y=0.2$  [ $\beta=2y(\frac{4}{5}\pi)^{1/2}$ ] counting the levels of both protons and neutrons ( $Z=95$ ,  $A=245$ ). Equation (1) gives 32 jumps, which only differs by 10%.

In the present problem the deformation field must carry an initial spherical shape into a final shape that matches the daughter configuration of two touching spheres as closely as possible. We construct the field from a sum of pure multipole fields, so that the transformation preserves the local density. The coefficients are determined to match all multipole moments up to  $\lambda=10$ . The generalization of Eq. (1), now obtained numerically, then gives the number of transitions  $n$ . The number of steps<sup>9</sup> needed to reach the touching-daughter configuration starting from the parent ground state is 13 in the case of  $^{14}\text{C}$  emission, 23 in the case of  $^{24}\text{Ne}$  emission, and 30 in the case of  $^{32}\text{S}$ . Thus to form a  $^{14}\text{C}$  on the surface of  $^{223}\text{Ra}$  one has to move 26 particles.

The next question is the value of the pairing matrix element between nearest-neighbor configurations. Under the influence of this force, pairs of particles are removed at one deformation and added at an adjacent deformation. In the BCS approximation, each of these processes

is associated with an amplitude of the type

$$\langle \psi_{\text{BCS}} | \sum_v a_v^\dagger a_v^\dagger | \psi_{\text{BCS}} \rangle = \langle \psi_{\text{BCS}} | \sum_v a_v^\dagger a_v | \psi_{\text{BCS}} \rangle = \sum_v U_v V_v = \Delta/G, \quad (2)$$

where the operator  $a_v^\dagger a_v^\dagger$  creates a pair of particles in time-reversed states. The quantities  $U$  and  $V$  are the BCS occupation numbers,  $\Delta \approx 12/\sqrt{A}$  is the pairing gap,<sup>10</sup> and  $G \approx 25/A$  MeV is the pairing strength<sup>11</sup> for a nucleus of mass  $A$ . The pairing matrix element between paired states at adjacent deformations can now be written as

$$v = -\frac{1}{4} G (\sum_v U_v V_v)^2 = -(\Delta_p^2 + \Delta_n^2)/4G, \quad (3)$$

with both neutrons and protons contributing. The factor  $\frac{1}{4}$  arises because only one of the four possible types of pair jumps among upward and downward sloping levels changes the deformation in the desired direction. Using the estimates quoted above one obtains  $v=2.9$  MeV, essentially independent of  $A$  for the nuclei under discussion.

If we make a Hamiltonian of a continuous coordinate  $\xi$ , these off-diagonal elements lead to a kinetic term

$$T = -\frac{\partial}{\partial \xi} (\Delta \xi)^{-2} \frac{\partial}{\partial \xi}, \quad (4)$$

and thus to an inertial mass

$$D = \hbar^2 \frac{2G}{\Delta_n^2 + \Delta_p^2} (\Delta \xi)^{-2}. \quad (5)$$

The dependence on the pairing gap displayed by this inertial mass is well known from the work of Brack *et al.*<sup>6</sup> A detailed derivation is provided by Bertsch.<sup>12</sup>

To construct the Hamiltonian matrix we also need the diagonal energy as a function of deformation. The energy cost to get to the emission point is

$$\Delta E = U_C + U_N - Q + 2v, \quad (6)$$

where  $U_C$  and  $U_N$  are the Coulomb and nuclear potentials at the touching configuration and  $Q$  is the energy released by the exotic decay. We determine the nuclear field from the heavy-ion potential in Eqs. (40)–(43) of Broglia and Winther.<sup>13</sup> For the case of  $^{14}\text{C} + ^{208}\text{Pb}$ ,  $\Delta E$  is equal to about 7 MeV, showing that the energy changes with shape are rather small. We shall assume that the energy varies quadratically with deformation to get for the  $k$ th diagonal element in the Hamiltonian matrix

$$v(\xi_k) = \Delta E (k/n)^2. \quad (7)$$

With these ingredients the Hamiltonian defined by Eqs. (4)–(7) is diagonalized to get the amplitude of the daughter configuration in the ground state.

We couple to the continuum states using Fermi's "golden rule," treating the pairing interaction between the last configurations as the perturbation. The formula

for the absolute decay constant is then

$$\lambda = \frac{2\pi}{\hbar} \langle \psi_E | \phi_n \rangle^2 v^2 (a_{n-1}^0)^2 \frac{dn}{dE}, \quad (8)$$

where  $a_{n-1}^0 = \langle \psi_0 | \phi_{n-1} \rangle$  is the amplitude of the next to last configuration in the ground state. The wave function  $\phi_n$  represents the final configuration of touching daughter nuclei. Its overlap with the continuum wave function,  $\langle \psi_E | \phi_n \rangle$ , requires it to be decomposed into a product of internal daughter wave functions multiplied by a function of the relative coordinate. In the harmonic-oscillator shell model, the latter is an oscillator function.<sup>14</sup> We approximate it by a Gaussian function centered at the touching distance of the daughter nuclei [ $R = 1.2(A_d^{1/3} + A_b^{1/3})$  fm], with an oscillator frequency given by the usual formula,<sup>15</sup>  $\hbar\omega = (41/A_b^{1/3})$  MeV. The continuum wave function is obtained by our solving the Schrödinger equation for the relative coordinate using the above-mentioned ion-ion potential.

The computed daughter formation probabilities range in magnitude from  $5 \times 10^{-18}$  to  $7 \times 10^{-8}$ . The smaller probabilities are obtained for the decays which give larger daughter products, because the tunneling distance is greater when more particles are moved. This shows the importance of internal tunneling to produce the trends with mass  $A_b$  found in the final results. It may be seen in the continuum limit that the formation probability depends on the Hamiltonian parameters exponentially,  $a_n^0 \sim \exp[-n(\Delta E/2v)^{1/2}]$ . Thus a 10% change in  $n$  and a 20% change in  $\Delta E$  or  $v$  would produce 1 or 2 orders of magnitude change in  $a_n^0$ . The final absolute transition rates are shown in Fig. 2. The radium decay rates

are reproduced to within 2 orders of magnitude, within the range of accuracy of the model. The uranium decays are underpredicted, but we have ignored the initial deformation of the parent nucleus, which undoubtedly shortens the tunneling path.

Finally, the data show an odd-even staggering when compared to the predicted rates, which is easily understood from the superfluid tunneling physics. As is well known in fission theory, the odd-mass nuclei have lower transition rates because the odd particle cannot jump easily to other orbits. Also, only a few orbits are involved in the pairing, and the blocking of one of them in an odd-mass system significantly lowers the gap for that species of nucleon. With a higher inertia the transition rate is reduced. The odd-even staggering might also be due to the angular momentum carried off by the ejectile, but we estimate this effect to be small.

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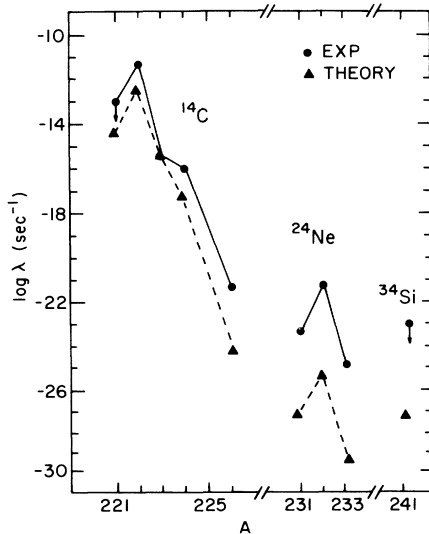


FIG. 2. Logarithm of the decay constant vs the parent mass number. The experimental data (Ref. 1), shown with circles, are for the decays  $^{221,222,223,224,226}\text{Ra}$  to  $^{14}\text{Pb} + ^{14}\text{C}$ ,  $^{231}\text{Pa}$  to  $^{207}\text{Tl} + ^{24}\text{Ne}$ ,  $^{232,233}\text{U}$  to  $^{14}\text{Pb} + ^{24}\text{Ne}$ , and  $^{241}\text{Am}$  to  $^{207}\text{Tl} + ^{34}\text{Si}$ . The calculation from Eq. (8) is shown with triangles.

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<sup>9</sup>A simple argument why the number of orbital jumps is nearly the same as the number of particles in the lighter daughter,  $A_b$ , can be made as follows: To get the touching configuration, one tries to move  $A_b$  particles through the original spherical surface to make the daughter, and an additional  $A_b$  particles through the other side to preserve the center of mass. Deformations in coordinate space are correlated with

distortions in momentum space, and so the same number of particles is moved through the Fermi surface. These particles are then relocated below the Fermi surface by the orbital jumps.

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<sup>11</sup>A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2, Eqs. (6)–(591), and p. 645; R. A. Broglia, D. R. Bes, and B. S. Nilsson, Phys. Lett. **50B**, 213

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