TRANSVERSE MOMENTUM DISTRIBUTIONS IN INTERMEDIATE-ENERGY HEAVY-ION COLLISIONS

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Transverse momentum distributions are calculated with the Boltzmann equation for different nuclear equations of state and nucleon–nucleon cross sections. Due to the increased importance of compressional potential energy and two-body collisions at higher bombarding energies, the mean transverse momentum changes sign from negative to positive as the bombarding energy is increased above $E/A=100$ MeV. The sensitivity of the mean transverse momentum to the uncertainties in the nucleon–nucleon cross section is comparable to the sensitivity to the nulear equation of state.

Momentum transfer in heavy-ion collisions is an important observable that reflects the balance between the mean field and the collisional dynamics. This balance evolves with bombarding energy. At low energies ($E/A=10$ MeV), the mean field is attractive and two-body collisions are suppressed by the Pauli exclusion principle. At higher energies ($E/A=400$ MeV), two-body collisions are frequent and the attract ve mean field is lost. Experimental observations will reflect this evolution. Indeed, the nucleons emitted in low-energy reactions have transverse momenta consistent with an attractive interaction [1,2]. In contrast, the transverse momentum distributions from high-energy reactions are generally interpreted in terms of a repulsive momentum transfer [3].

In this letter, we examine the sensitivity of nucleon transverse momentum distributions to nucleon–nucleon collisions and to the nuclear equation of state. The Boltzmann equation, given below, describes the time evolution of the Wigner function $f(\boldsymbol{r}, \boldsymbol{k}, t)$ in phase space [4–6]:

$$\frac{\partial f_1}{\partial t}+\boldsymbol{v}\cdot\nabla_r f_1-\nabla_r U\cdot\nabla_p f_1 = \frac{4}{(2\pi)^3}\int \mathrm{d}^3k_2\,\mathrm{d}^3k_3\,\mathrm{d}\Omega\,\sigma_{\mathrm{nn}}(\Omega) \times v_{12}[f_3f_4(1-f_1)(1-f_2) - f_1f_2(1-f_3)(1-f_4)] \times\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_3-\boldsymbol{k}_4)\ . \qquad (1)$$

Here, $\sigma_{\mathrm{nn}}(\Omega)$ and v_{12} are the cross section and relative velocity for the colliding nucleons, and U is the mean field potential approximated by

$$U=-a_1\rho/\rho_0+a_2(\rho/\rho_0)^\gamma\ . \qquad (2)$$

For simplicity, $\sigma_{\mathrm{nn}}(\Omega)$ was taken to be a constant and isotropic. Numerical solutions to eq. (1) were obtained by propagating test particles according to newtonian mechanics. The mean field and the Pauli blocking factors in the collision integral were calculated with distribution functions which were ensemble averaged over 100 parallel simulations in calculations for targets with mass 40 and 100. Since the amount of computer time required to do each calculation increases with total number of nucleons in the ensemble, for the heaviest target ($A=197$), 50 simulations were used. Reduction in the number of parallel simulations for the heavy target increases the

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fluctuations in the numerical evaluation of the nuclear mean field. Comparison between these calculations and corresponding calculations with 100 parallel ensembles indicates that the uncertainties in $\langle P_x \rangle$ introduced by these fluctuations are less than 3 MeV/c. Other details concerning the solution of this equation are contained in ref. [6].

Calculations were performed with the mean field potentials and nucleon–nucleon cross sections given in table 1. Both mean field potentials produce proper saturation of nuclear matter and are about 50 MeV deep at normal density, $\rho_0 = 0.17\ \mathrm{fm}^{-3}$. They differ in their compressibility with the stiff potential having a compressibility of $K = 375$ MeV and the soft potential $K = 200$ MeV. Both nucleon–nucleon cross sections in table 1 are less than the low-energy free nucleon–nucleon scattering cross section. Some reduction in the cross section must occur at low energies because some of the intermediate states in the nucleon–nucleon t-matrix expansion are Pauli blocked. Even at high energies, higher-order many-particles effects may lead to collision rates less than those computed with eq. (1) using free cross sections [7].

Collisions between mass 40 projectiles and mass 40, 100, and 197 targets were calculated for bombarding energies of $E/A = 60$, 100, 200, and 400 MeV and impact parameters of 3, 5, 7, and 9 fm. Here we report results concerning the momentum distributions of emitted nucleons. The nucleon mean transverse momentum, $\langle P_x \rangle$, was calculated by averaging over the distribution of nucleon momenta in the reaction plane perpendicular to the beam momentum [‡1]. To avoid cancellations between the transverse momenta of nucleons emitted parallel and antiparallel to the projectile momentum, nucleons emitted at enter-of-mass angles greater than 90° were excluded in the determination of $\langle P_x \rangle$. We examined the phase space distribution of nucleons at time intervals of 50 fm/c, 100 fm/c and 200 fm/c from the start of the collision and chose a time interval long enough for the final state residues to be well separated. Evaporation from the target-like residue was suppressed by excluding nucleons located within a sphere of radius $R = 1.4 A_{\mathrm{tgt}}^{1/3}$ about the target-like residue and by excluding nucleons with laboratory energies less than 15 MeV. The results are relatively insensitive to details concerning the exclusion of the target-like residue. For the results presented here, nucleons contained in the projectile-like residue were *not* excluded; we will return to this point later.

The results are shown in fig. 1. Consider first the calculations for the symmetric mass 40 system shown on the left-hand side of the figure. Calculations for the soft equation of state and $4\pi\sigma_{\mathrm{nn}}(\Omega) = 41$ mb (parameter set 1 of table 1) are indicated by the solid circles. At low energies, attractive momentum transfers dominate; the majority of energetic nucleons are deflected by the attractive mean field and emerge with negative transverse momenta. This is consistent with recent measurements of the sign of the emission angle for non-equilibrium light particles for ^{14}N-induced reactions on ^{154}Sm at $E/A = 35$ MeV [2]. The mean transverse momentum increases monotonically with bombarding energy. This is particularly evident at smaller impact parameters where $\langle P_x \rangle$ changes sign and goes rapidly positive for $E/A > 100$ MeV. A similar trend was reported in ref. [8]. Positive momentum transfers are consistent with the interpretation of momentum distributions measured with the Plastic Ball at the Lawrence Berkeley Laboratory [3]. These positive transverse momenta arise from the scattering of nucleons by the repulsive nuclear mean

[‡1] To facilitate discussion we designate the z-axis to be parallel to the projectile linear momentum; the x-axis is chosen to lie in the reaction plane. For non-zero impact parameters, the projectile is displaced to positive values of x; positive $\langle p_x \rangle$ (negative $\langle P_x \rangle$) corresponds to emission to the same (opposite) side of the target as the initial impact.

Table 1
Nuclear mean field parameters and nucleon–nucleon cross sections used in the calculations.

Set	Mean field	a_1 (MeV)	a_2 (MeV)	K (MeV)	γ	$4\pi\sigma_{\mathrm{nn}}(\Omega)$ (mb)
1	soft	356	303	200	7/2	41
2	stiff	124	70.5	375	2	41
3	soft	356	303	200	7/6	20
4	stiff	124	70.5	375	2	20

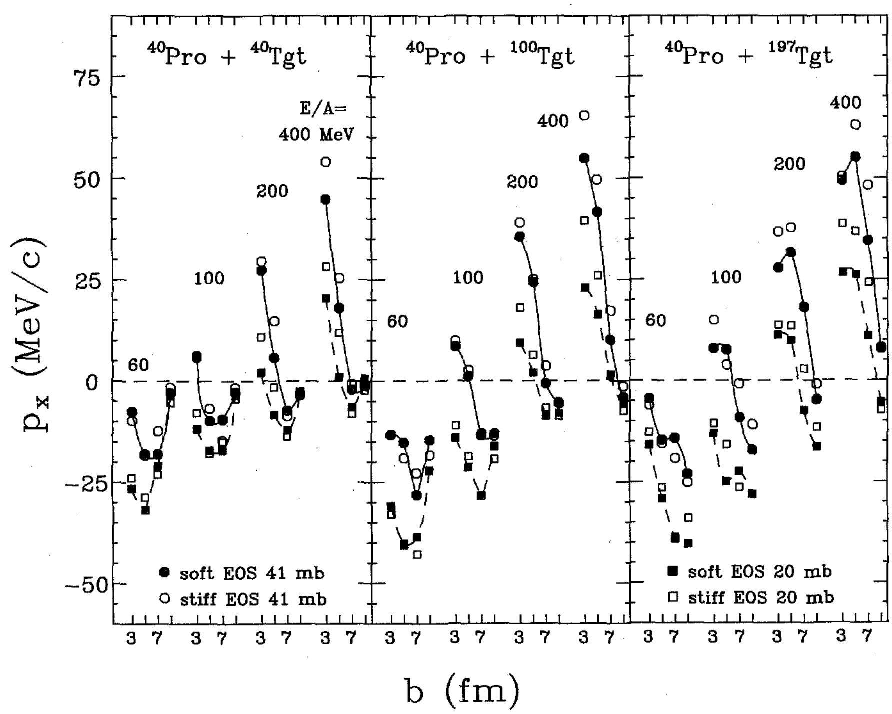

Fig. 1. Mean transverse momenta calculated for mass 40 projectiles and mass 40, 100 and 197 targets, impact parameters of 3, 5, 7 and 9 fm, and bombarding energies of $E/A=60$, 100, 200, and 400 MeV. The calculations are indicated by the solid circles and the solid lines for parameter set 1, the open circles for calculations with parameter set 2, the solid squares and dashed lines for parameter set 3 and the open squares for parameter set 4.

field present during the high density stage of the collision and from the kinetic pressure caused by the incoherent nucleon–nucleon collisions represented by the collision term.

We now examine the sensitivity of $\langle P_x \rangle$ to the equation of state and the nucleon–nucleon cross section. The sensitivity to the equation of state is observed by comparing calculations with the same nucleon–nucleon cross section but different equations of state. The stiff equation of state (open points) provides slightly more positive values of $\langle P_x \rangle$ than the soft equation of state (closed points) at small impact parameters. $\langle P_x \rangle$ is more significantly altered by keeping the equation of state constant and changing the nucleon–nucleon cross section. For example, the calculations for the soft and stiff equations of state for the same σ_{nn} differ by $\Delta\langle P_x \rangle \cong 10$ MeV/c at $b=3$ fm and $E/A=400$ MeV, while calculations for the same equation of state but different σ_{nn} differ by $\Delta\langle P_x \rangle \equiv 25$ MeV/c. The increase in $\langle P_x \rangle$ with increasing σ_{nn} is consistent with cascade calculations which showed an increase flow angle with increasing σ_{nn} [9]. Except for trivial geometrical effects in the impact parameter dependence, these trends are essentially preserved in the calculations with the heavier targets. Clearly, σ_{nn} must be accurately known before measurements of $\langle P_x \rangle$ can provide information concerning the equation of state [10].

Some caution must be employed when comparing these results to experimental data. The long-ranged Coulomb interaction, neglected in these calculations, will tend to make $\langle P_x \rangle$ more positive at larger impact parameters. In addition, projectile-like residual nuclei are produced in these calculations at most of the large impact parameters; therefore the momentum carried by these residues must be included if one wishes to construct comparable experimental values for $\langle P_x \rangle$. We have explored the effects of excluding the projectile-like residue and find that it does not alter the qualitative behavior of $\langle P_x \rangle$. However, the exclusion of the residues can affect $\langle P_x \rangle$ by as much as 10–30 MeV/c at large impact parameters. It then becomes important to have a good estimate of the number of evaporated nucleons, which is difficult to do with our present Boltzmann code. Sequential decay does not appear to be so serious a problem for calculations of the small impact parameter collisions.

A recent analysis of the momentum flow from a compiled set of experimental data suggests that the transverse momentum changes rapidly for lab energies below 200 MeV/nucleon [11]. This is consistent with our theoretical finding that the mean transverse momentum changes sign as the bombarding energy is increased above 100 MeV/nucleon due to the weakening of the attractive nuclear mean field and to the increased importance of nucleon–nucleon collisions at higher energies. Unfortunately for the prospects of measuring the nuclear equation of state, the calculations also indicate that the mean transverse momentum is as sensitive to uncertainties in the nucleon–nucleon cross section as it is to the nuclear equation of state.

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