THE CLASSICAL LIMIT OF THE SURFACE RESPONSE IN FERMI LIQUIDS

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The RPA theory of the surface response of a self-bound Fermi liquid reduces to the classical diffusion equation in the low frequency, long wavelength limit. The dynamic equation for the surface position includes a restoring force identifiable with the surface tension and a viscous force identifiable with the usual wall friction.

We recently developed a model of the surface response of Fermi liquids based on RPA with self-consistent separable interactions [1]. The low-frequency limit of this model was exhibited in eq. (4) of ref. [2]. The functional form in this limit is quite simple, and in fact is just the Green function for the classical diffusion equation. We had identified one of the terms physically in terms of the surface tension of the liquid. Noting here the analogy with the diffusion equation, we are able to identify the other term with a classical friction force, and find that its magnitude agrees well with the wall formula [3]. While this work was in progress, a related study by Abrosimov and Randrup came to our attention [4]. These authors start from the Landau kinetic equation rather than the RPA response function. They invoke self-consistency in a different way from ours, but come to a similar conclusion.

We first define some notation. Let us assume that we have a solution of the mean field theory for a semi-infinite Fermi liquid with a surface at the $z = 0$ plane. The particle density in the ground state will be denoted by $\rho_0(z)$ and the self-consistent potential binding the ground state by $V_0(z)$. The RPA theory of surface excitations can be solved in a model with a self-consistent interaction of the separable form

$$v(r_1, r_2) = \int \frac{d^2K}{(2\pi)^2}\, \kappa(K) \exp[iK \cdot (r_1 - r_2)] \times V_0'(z_1) V_0'(z_2)\,. \tag{1}$$

Here V_0' denotes the derivative of V_0 with respect to z, K is a momentum vector along the surface of the liquid, and κ is an interaction strength. With this interaction the RPA response to an external field

$$f(r) = \exp(iK \cdot r) V_0'(z) \exp(-i\omega t)$$

is given by

$$\Pi^{\mathrm{RPA}}(K, \omega) \equiv \int dz\, dz'\, V_0'(z)\, \Pi^{\mathrm{RPA}}(K, \omega, z, z') V_0'(z') = \Pi_0(K, \omega)[1 - \kappa(K)\Pi_0(K, \omega)]^{-1}\,, \tag{2}$$

where Π^{RPA} and Π_0 are the RPA and free polarization propagators. In ref. [1] we found a parameterization of Π_0 which may be used to obtain the following formula for Π^{RPA},

$$\Pi^{\mathrm{RPA}}(K, \omega) = 2N\omega_0[(\omega + i\gamma/2)^2 - \omega_0^2 - 2\kappa N\omega_0]^{-1} \tag{3}$$

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Here N, ω_0 and γ are simple functions of K^2 defined in eq. (3.3) of ref. [1]. We next make use of the self-consistency condition

$$1/K(0) = \Pi_0(0, 0) = -2N\omega_0[\omega_0^2 + (\gamma/2)^2]^{-1}, \quad (4)$$

and expand eq. (3) to lowest order in ω and K^2. The result has the form

$$\Pi^{\mathrm{RPA}}(K, \omega) \simeq [\Pi_0(0, 0)]^2(ia\omega - \sigma K^2)^{-1}, \quad (5)$$

where the coefficients σ and a are obtained by the Taylor series expansion

$$\sigma = -\partial\Pi_0(K, 0)/\partial K^2 - [\partial\kappa(K)/\partial K^2]\Pi_0^2(0, 0), \quad (6)$$

$$a = -\mathrm{Im}\,\partial\Pi_0(0, \omega)/\partial\omega . \quad (7)$$

Eq. (6) is just the surface tension of the liquid, decomposed into kinetic and potential contributions [5]. The numerical value based on the parameters in eq. (3) comes out very close to the empirical surface tension in nuclear matter when the K dependence of κ is determined by a particle–particle interaction having a range of 1 fm.

The coefficient a in eq. (7) was not given a physical interpretation. This will now be done by writing down the differential equation that has a Green function of the form eq. (5). This is a diffusion equation for the surface position, involving derivatives with respect to x, y and t,

$$a\partial z/\partial t = \sigma\nabla_{x,y}^2 z . \quad (8)$$

The right-hand side expresses the pressure on the surface due to its curvature and the surface tension. The left-hand side has the dimensions of force per unit area, but since it is linearly proportional to the velocity of the surface, $\mathrm{d}z/\mathrm{d}t$, it is a friction force.

The magnitude of the friction constant, as determined from the parameterization in eq. (3), is

$$a = 2N\gamma\omega_0[\omega_0^2 + (\gamma/2)^2]^{-2}$$
$$= 0.157/\mathrm{fm}^4 = 31\ \mathrm{MeV}/c\ \mathrm{fm}^3 . \quad (9)$$

This may be compared with the friction constant proposed in the classical piston model of Gross [6], expressed in the wall formula of Blocki et al. [3]. Here the dissipative force is also proportional to the velocity, and is given by

$$F/A = \tfrac{3}{4}m\rho v_{\mathrm{F}}\,\mathrm{d}z/\mathrm{d}t , \quad (10)$$

where v_{F} is the Fermi velocity. The coefficient has a numerical value of

$$\tfrac{3}{4}m\rho v_{\mathrm{F}} \simeq \tfrac{3}{4}(938\ \mathrm{MeV}/c^2)(0.16\ \mathrm{fm}^{-3})(0.28c)$$
$$= 32\ \mathrm{MeV}/c\ \mathrm{fm}^3 ,$$

very close to eq. (9). In fact we should have anticipated this correspondence. An analytic demonstration that the free response limit eq. (7) is given by eq. (10) is contained in ref. [7], making use of eqs. (22) and (39) of ref. [8]. But it is not obvious a priori that a simple global approximation to the response, such as eq. (3), will achieve this analytic limit quite accurately.

Having made this identification, it becomes quite clear what the wall friction describes. It can be properly used as an ingredient in a model of the low- to moderate-frequency surface response for surface wavelengths that are small compared to the size of the system but large compared to the Fermi wavelength. This permits a useful domain in nuclear physics, for example for momentum transfers of the order of 100 MeV/c in heavy nuclei. On the other hand, the wall friction and associated response eq. (5) have no applicability to high-frequency phenomena such as giant resonances. In fact, the frequency-weighted sum rule for eq. (5) diverges, showing that the high-frequency behavior is incorrect.

Classically, inertial effects make the frequency-weighted sum finite, suggesting that the response eq. (5) could be improved by including in eq. (7) a force proportional to acceleration. This would give a surface response function having the form

$$\Pi(K, \omega) = \Pi_0^2(0, 0)(b\omega^2 + ia\omega - \sigma K^2)^{-1} . \quad (11)$$

In fact the parameterization eq. (3) already contains an ω^2 dependence, which would give the following value for the parameter b in eq. (11):

$$b = a/\gamma = 0.006\,\hbar^2/\mathrm{MeV\ fm}^4 .$$

With this value, the response (11) satisfies the frequency-weighted sum rule. It might be interesting to see if there is a physical basis for such an inertial parameter. Abrosimov and Randrup [4] also find a non-vanishing inertial parameter, but theirs is much larger and strongly K dependent.

For numerical applications, it is preferable to use eq. (3) directly, which is more accurate than (5) or (10) without being very complicated.

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