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Channel coupling effects in subbarrier fusion of oxygen with oxygen

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Isotopic differences in the fusion cross sections for O + O, observed by Thomas *et al.*, are interpreted with a coupled-channel model. We find that most of the subbarrier enhancement of $^{18}O+^{16}O$ over $^{16}O+^{16}O$ is attributable to the excitation of the low 2^+ state in ^{18}O. However, the model predicts the enhancement to persist above the barrier, contrary to experimental findings.

The role of intrinsic degrees of freedom in subbarrier fusion reactions has become a subject of considerable interest. An overview of the present experimental and theoretical status of subbarrier fusion is given in the proceedings of a recent topical conference.[1] Strong isotopic dependences are found for the cross sections, particularly in heavy nuclei. The calculation of the cross sections is numerically difficult for heavy nuclei, since many channels must be included. This point was brought out in the semiclassical treatment of Ref. 2. Theory has not been entirely successful so far in reproducing the cross sections for heavy systems,[3] so it is important to find simpler cases more amenable to complete theoretical treatment. A recent study[4] comparing fusion cross sections of ^{18}O and ^{16}O on ^{16}O targets provides a nice case that we will examine here (Fig. 1). For the $^{16}O+^{16}O$ system, the internal degrees of freedom are relatively unimportant because of the high excitation energies of inelastic channels and particle transfer channels. The nucleus ^{18}O, on the other hand, has a 2^+ state at 1.98 MeV excitation with a transition strength from the ground state of 16.6 Weisskopf units.

The experimental finding of Ref. 4 is that the ^{18}O fusion cross section is enhanced over ^{16}O in the subbarrier region. We show this in Fig. 2 with the cross section ratio plotted as a function of the center-of-mass energy. The enhancement is small compared to that found in heavier nuclei, but there is certainly an effect present. In Ref. 4 it is suggested that the difference is mainly due to changes in the potential barrier going from one system to another. So, as a first step in our analysis, we will examine the expected effect of potential changes adding two nucleons to ^{16}O. We also expect a significant effect from the low 2^+ state, which may be roughly estimated from the frozen approximation of Esbensen.[5] According to Eq. (24) of Ref. 2, the enhancement in the subbarrier fusion may be estimated as

$$P=\exp\left[\frac{1}{2}\left(\frac{2\pi Z_1 Z_2 e^2}{\hbar\omega_0 R_B^2}\sigma\right)^2\right] . \quad (1)$$

Here σ is the deformation length of the transition density, related to the usual deformation parameter β by $\sigma=\beta R/\sqrt{4\pi}$. The parameter ω_0 is the oscillator constant of

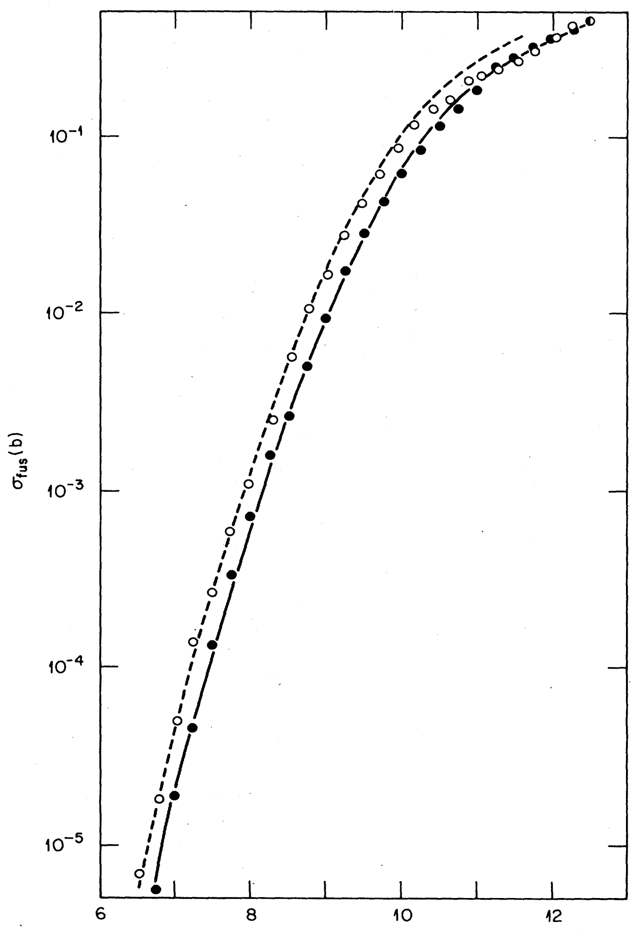

FIG. 1. The measured fusion cross section for $^{18}O+^{16}O$ and $^{16}O+^{16}O$, from Ref. 4 is shown by the experimental points. The potential model fit to the $^{16}O+^{16}O$ is shown as the solid line, and the coupled-channel model with the rescaled potential is shown by the dashed line.

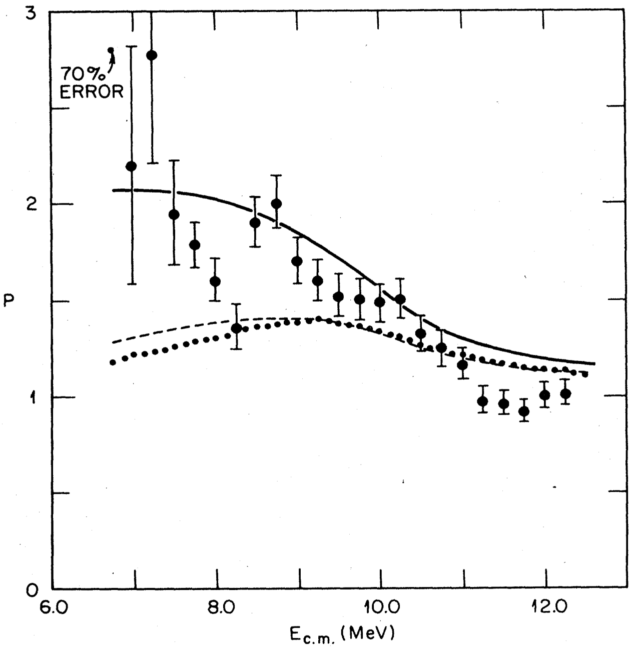

FIG. 2. Enhancement factors extracted by log-linear interpolation of the data from Ref. 4. The dashed line is the prediction of the rescaled potential, Eq. (2). The dotted line is the prediction of the potential Eq. (4). The solid line is the prediction of the coupled-channel model with the rescaled potential.

an equivalent inverted parabolic barrier, and R_B is the separation of the nuclei at the barrier top. The constants in Eq. (1) may be estimated as follows for the $^{18}O+^{16}O$ system. The deformation length associated with the $0^+ \rightarrow 2^+$ transition is determined from the transition strength by Eq. (11) of Ref. 5. This yields a value $\sigma = 0.33$ fm. From our potential model fit to the barrier described below, we find a barrier height of 9.7 MeV at a radius of $R_B = 8.9$ fm. The thickness of the barrier at an energy of 6.75 MeV is 6 fm, which corresponds to a parabolic barrier having an oscillator frequency of $\hbar\omega = 1.8$ MeV. The predicted enhancement with these parameters is 2.5, indicating that the excited state in ^{18}O may be significant to subbarrier fusion.

Our detailed calculations employ the coupled channel technique to calculate the fusion cross section, much as was done in Ref. 2. The potentials in each channel are purely real, and an incoming wave boundary condition is imposed at a small radius, which we take to be 4.1 fm. The coupled-channel equations are solved by matrix inversion, which is more reliable than iterative techniques below the barrier. Following Esbensen,[5] we use a nuclear potential proportional to the error function together with a simple Coulomb potential.

$$V(r) = V_N(r) + V_C(r) = \frac{V_0 R_1 R_2}{R_1 + R_2} \int_{(r-R_1-R_2-s_0)/a}^{\infty} e^{-x^2} dx + \frac{Z_1 Z_2 e^2}{r} . \quad (2)$$

The two nuclear radii, R_1 and R_2, are determined by the formula

$$R_i = 1.233 A_i^{1/3} - 0.98 A_i^{-1/3} . \quad (3)$$

We find that Eq. (2) fits the fusion of $^{16}O+^{16}O$ quite well with the parameter set $V_0 = -22.4$ MeV/fm, $s_0 = 0.45$ fm, and $a_0 = 1.65$ fm. Since Eq. (2) is of the proximity form, we can extrapolate to an $^{18}O+^{16}O$ potential by changing one of the radii according to Eq. (3). The rescaled potential is wider and deeper than the original ^{16}O potential, giving more subbarrier fusion. However, the enhancement is rather small, as may be seen by the dashed line in Fig. 2. Of course, the proximity scaling may not be a reliable way to extrapolate the influence of the valence nucleons on the potential, so we also examined their effect in a folding model. Namely, we determined the $^{18}O+^{16}O$ potential according to

$$V(^{18}O+^{16}O) = V(^{16}O+^{16}O) + \int \rho(r') U(r-r') d^3r' . \quad (4)$$

Here $U(r)$ is a Woods-Saxon potential with parameters given in Ref. 6, and ρ is the valence neutron density determined from the d-wave bound state in that Woods-Saxon potential. This method also gives a small enhancement, which is shown by the dotted line in Fig. 2. Thus, we are inclined to reject the potential barrier variation as the main mechanism for producing the ^{18}O enhancement.

The potential matrix for the coupled-channel calculation is constructed by a macroscopic model of the excitation. We assume the 2^+ state to be a harmonic vibration of a surface deformation coordinate s, and evaluate the matrix elements of the potential Eq. (2) with r replaced by $r-s$,

$$\tilde{V} = \begin{bmatrix} \langle 0|V_N(r-s)|0\rangle + V_c & \langle 0|V_N(r-s)|1\rangle \\ \langle 0|V_N(r-s)|1\rangle & \langle 0|V_N(r-s)|0\rangle + V_c + 1.98 \end{bmatrix} . \quad (5)$$

In writing the above equation, we have neglected the angular momentum transfer, in effect treating the combined system in a rotating frame approximation. The integral over s is easily performed for the error function, giving

$$\langle 0|V_N(r-s)|0\rangle = V_N(r)\Big|_{a_0' = (a_0^2 + 2\sigma^2)^{1/2}} ,$$

$$\langle 1|V_N(r-s)|0\rangle = \sigma \frac{d}{dr} \langle 0|V_N(r-s)|0\rangle .$$

The wave function for each entrance channel angular momentum is found by integrating the coupled equations out from the inner boundary. For the energies we study here, the sum over angular momentum can be truncated at $l = 12$, keeping an accuracy of 1%. Our predicted enhancement including potential and coupled-channel effects is shown as the solid curve in Fig. 2. As expected from the frozen approximation, inclusion of channel-coupling substantially enhances the cross section in the subbarrier region. However, at energies above the barrier, theory predicts the enhancement to persist, while experimentally equal cross sections are found. We will return to this later.

The coupled-channel model of Eq. (5) is oversimplified in several respects, and the assumptions need to be examined and justified. First, the higher excited states in ^{18}O might increase the cross section even more. In fact, such couplings are implicit in the harmonic model used to derive the sudden approximation, Eq. (1). However, the $B(E2)$ from the first 2^+ state to the triplet of states near 4 MeV is lower than that from the ground state to the 2^+, while the harmonic model requires a larger $B(E2)$. We find that including these channels in a five-dimensional matrix has a negligible effect on the fusion. Another oversimplification of our model is the neglect of the Coulomb excitation of the

2^+ state. It is well known for heavy systems that the Coulomb part of the coupling interferes destructively with the nuclear part and reduces the subbarrier enhancement.[7] Indeed, we find that including the Coulomb excitation according to Eq. (4) of Ref. 5, the cross section is reduced by as much as 7% at the lowest energy, becoming less important as the energy goes up. As mentioned above, our coupled-channel model treats the system in a rotating frame approximation. The physical justification for this is that we expect axially symmetric deformations to provide the dominant path to fusion. We tested the angular momentum transfer assumption with the code PTOLEMY, using it to compare cross sections treating the excited state as $L=2$ or 0. The cross section at subbarrier energies comes out larger with angular momentum transfer, by an amount comparable to the Coulomb correction. Since both effects are small and of opposite sign, we are justified in neglecting them in the present context. Nucleon transfer effects are also not included in our formalism. One might expect that the two-neutron transfer, which has a zero Q value, plays a dominant role. However, for such processes, the contributions to the potential due to even and odd partial waves have opposite signs and exactly cancel in the $Q=0$ limit.

Above the barrier, the cross section is reduced by the angular momentum transfer, but not enough to equalize the cross sections of the two reactions. The experimental observation of equal cross sections is inexplicable to us. A channel coupling under the barrier will always enhance the cross section. For the low partial waves that go over the barrier, we do not see how the extra couplings in the $^{18}O+^{16}O$ system could reduce the transmission factor. It appears to be a general phenomenon found also for other systems that the fusion cross sections above the barrier come out too large when the model is fit to the subbarrier region.[8]

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