GAUSSIAN DISTRIBUTIONS OF SHELL-MODEL EIGENVECTOR COMPONENTS

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The distribution of basis-vector amplitudes in realistic sd shell-model eigenfunctions has been calculated and compared with the gaussian distribution expected from the theory of random matrices. The states in the region of high level density show the expected behavior. The largest deviations from gaussian are found for the states at the ends of the spectrum where the level density is low compared to the off-diagonal hamiltonian matrix elements. It is essential to use basis vectors with good J and T rather than M-scheme vectors in order to obtain a gaussian distribution.

The theory of random matrices is a powerfool tool for studying the statistical properties of nuclear excited states and nuclear reactions [1]. One of the most fundamental and useful properties of the eigenvectors $|\alpha\rangle$ in the random matrix theory is that the amplitudes $c_{i,\alpha}$ associated with the basis vector $|i\rangle$ are distributed on the average simply as a gaussian, known as the Porter–Thomas distribution [2]. The normalized probability distribution may be expressed

$$P(|c_{i,\alpha}|) = (2N/\pi)^{1/2} \exp(-N|c_{i,\alpha}|^2/2) , \qquad (1)$$

where

$$|\alpha\rangle = \sum_i c_{i,\alpha}|i\rangle , \quad \sum_i |c_{i,\alpha}|^2 = 1 ,$$

and where N is the dimension of the basis, with $N \gg 1$. Empirically, the reduced widths of transitions satisfy this distribution for levels of mid-shell nuclei at moderate excitation energy, although other aspects of the widths may appear nonrandom [3].

From the point of view of nuclear theory, it is important to know to what extent a realistic nuclear hamiltonian can be viewed as just a representative of a random ensemble. There have been a number of studies carried out on the s–d shell hamiltonian. In ref. [1], fig. 2c, it is seen that by one indicator of random matrix theory, the spacing of the levels, the hamiltonian shows the same behavior as the random ensemble. However, previous shell-model studies of the configuration amplitudes [4,5] suggest that the realistic distributions are quite different from the simple gaussian. In ref. [4] the basis was defined by configurations in the M-scheme which do not have good angular momentum and therefore may not reflect the behavior of an ensemble that is devoid of any symmetries. Distributions of configurations of good J were calculated in ref. [5], again finding strong deviations from gaussian, when the lowest few eigenstates of the spectrum were examined. These authors surmised that the deviations from gaussian were due to the single-particle part of the hamiltonian.

The domain of applicability of the theory of random matrices was studied recently with a model hamiltonian consisting of a fixed diagonal part to establish the level density, and a random part that is primarily off-diagonal [6]. The authors conclude that the spectra take on characteristics of the random matrix theory only when the average off-diagonal matrix element is comparable to the level spacing or larger. It is therefore of interest to examine the amplitude distribution

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for states at high excitation, where the level density is larger. Our study shows that in the middle of the spectrum the basis vector distribution is quite close to gaussian, providing the basis is in the J-scheme rather than the M-scheme.

We chose our basis to be the complete set of $(s-d)^5$ configurations for $T = 1/2, J = 1/2$, which has 109 states. These states were constructed using the shell-model program OXBASH [7]. This program starts with the M-scheme basis (there are 1933 states with $M = 1/2$ and $T_z = 1/2$) and projects out states with $J = 1/2$ and $T = 1/2$ from 109 of these M states selected such that the projected states are linearly independent. The resulting basis states are represented as a linear combination of M-scheme states $|m\rangle$,

$$|J, T, i\rangle = \sum_m b_{m,i}|m\rangle . \qquad (2)$$

The hamiltonian matrix was then calculated using a realistic interaction which reproduces binding energies and excitation energies in the sd shell. For our discussion we use the results obtained with the "universal sd" interaction of Wildenthal [8], however, our conclusions would be the same with the Chung–Wildenthal [9] or Preedom–Wildenthal [10] interactions.

In fig. 1, we show the histogram for the distribution of basis vector amplitudes $|c_{i,\alpha}|$ in groups of eigenstates as a function of excitation energy. The width of each bin is $N^{-1/2}/20$ and distribution is normalized to unit area. In these units eq. (1) is given by the universal curve $P(|x|) = [(2/\pi)^{1/2}/20] \times \exp(-x^2/800)$ which is also plotted in fig. 1. The distributions are shown for states 1–10 (E_x = 0–10.7 MeV), states 11–20 (E_x = 11.3–14.6 MeV), 31–60 (E_x = 17.3–24.6 MeV) and 100–109 (E_x = 35.9–51.7 MeV). The distributions are found to be close to gaussian in the middle of the spectrum (31–60) but deviate from gaussian toward the ends. The reason for this is probably associated with the diagonal single-particle terms in the hamiltonian. In the extreme case where the single-particle energies are much larger than the average two-body matrix element, it is clear that the lowest eigenstates will be dominated by those basis states involving the lowest orbit ($d_{5/2}$ in our case) and will have an abnormally large number of small amplitudes associated with the basis states involving the other orbits. To test this idea we repeated the calculation using degenerate single-particle energies. The results shown in fig. 2 for eigenstates 1–10 and 31–60 are indeed much closer to gaussian.

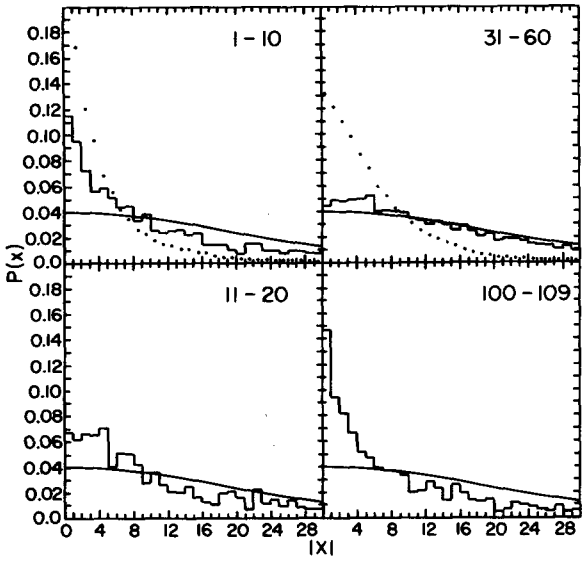

Fig. 1. Distribution of amplitudes of configurations in the $(s-d)^5$, $J = 1/2$, $T = 1/2$ states of the s–d shell model space. The smooth line shows the gaussian (Porter–Thomas) distribution from eq. (1). The solid histogram is the calculated distribution of the realistic hamiltonian in a basis with good J. The dotted histogram is calculated in the M-scheme basis.

It is suggested in ref. [6] that a useful parameter for determining the applicability of random matrix results is the ratio (λ) of the rms value of the off-diagonal matrix element to the average level spacing. When λ is of the order of unity or larger, the individual level positions lose memory of their location in the diagonal part of the hamiltonian. For our example the rms matrix element value is 0.64 MeV, and the average level

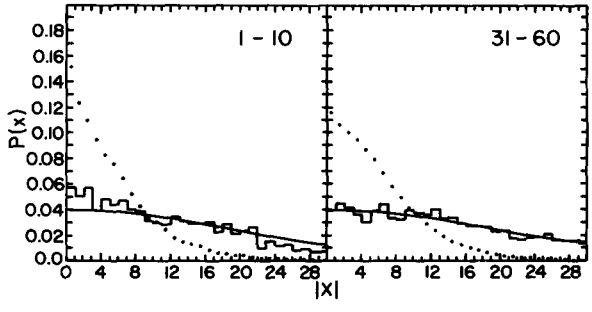

Fig. 2. The same as fig. 1, omitting the single-particle part of the hamiltonian in the calculation of the eigenvectors.

spacing varies from 1.1 MeV for the first 10 states to 0.24 MeV for states 30–61. Thus, λ varies from 0.6 to 2.4, going well into the regime where the random part of the hamiltonian dominates the local level positions. Of course, the shell-model hamiltonian does not look at all like a representative of the random ensemble, having about half zeros in our basis.

In the study of the amplitude distribution by Whitehead et al. [4], the calculations were performed in the M-scheme basis. That is, the basis contains all states of a definite M quantum number. The M-scheme states themselves do not have a definite J–T value, but since the hamiltonians used are invariant with respect to rotations in the space and spin coordinates, the eigenstates do have definite J–T values. By combining eqs. (1) and (2) we can easily obtain our eigenfunctions in terms of the M-scheme basis. The amplitudes $|c_{i,\alpha} b_{m,i}|$ are plotted in histogram form and compared with the previous results in figs. 1 and 2. It is clear that the M-scheme amplitudes have a very different distribution from those in the J-scheme and do not follow a gaussian even in the middle of the spectrum. This result may not be too surprising if we consider the fact that the M-scheme basis can be trivially rotated by using the J and T projection operators so that 1933 minus 109 of the basis states have zero overlap with the $J = 1/2$ $T = 1/2$ eigenstates. Lest there be any question that our gaussian results are due to our choice of hamiltonian or the choice of A, J and T we examined, we also checked with our hamiltonian the case studied in ref. [4], namely $A = 9$, $J = 1/2$, $T = 1/2$, and found agreement with their results for the lowest states.

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