Nuclear rotations

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I. SCOPE

These problems are about the production of nuclei with large angular momentum, and the analysis of the gamma-ray spectrum emitted when such a nucleus de-excites. The analysis uses the quantum mechanical formula for rotational energy:

$$E_J=\frac{\hbar^2 J(J+1)}{2I}, \tag{1}$$

where E is the nucleus' excitation energy, J is its angular momentum quantum number and I is its moment of inertia. The only specific nuclear physics knowledge called upon is for nuclear dimensions, that the radius of a nucleus is approximately given by $R = 1.2 \times A^{1/3}$ fm where A is its mass number, and familiarity with units used in nuclear physics, MeV for energy and fm for length. In these units $\hbar = 197.3$ MeV fm/c, $e^2/(4\pi\epsilon_0) = 1.44$ MeV fm, and the mass M_A of a nucleus of mass number A is $931A$ MeV/c^2. The problems are suitable for courses in quantum mechanics, modern physics, or nuclear physics.

II. ROTATING NUCLEI

Nuclei that are not spherically symmetric exhibit gamma-ray spectra analogous to those of rotating molecules. The 1975 Nobel Prize in Physics was awarded to Aage Bohr (Niels Bohr's son), Ben Mottelson, and James Rainwater for their work on the rotational behavior of nuclei.[1] With the development of heavy-ion accelerators nuclear physicists have produced highly deformed nuclei carrying a large amount of angular momentum, of the order of 50 $\hbar$ and higher. The gamma-decay spectrum of such a nucleus has a very simple appearance—it consists of uniformly spaced lines.

III. THE PROBLEMS

Just such a pattern of evenly spaced peaks appears in the gamma-ray spectrum of the nucleus ^{152}Dy shown in Fig. 1.[2] The pattern strongly indicates that the transitions are between levels in a rotational band.[3] These nuclei were produced in a rapidly rotating state by fusing 205-MeV nuclei of ^{48}Ca with ^{108}Pd in the heavy-ion reaction[4]

$$^{48}_{20}\mathrm{Ca}_{28}+{}^{108}_{46}\mathrm{Pd}_{62}\rightarrow{}^{152}_{66}\mathrm{Dy}_{86}+4n.$$

A. Angular momentum

To see the effectiveness of heavy-ion collisions in producing high-spin states in nuclei, estimate the angular momentum brought in by the above reaction by answering the following three questions:

(i) What is the total energy of the nuclei in the center-of-mass system given that the beam energy in the experiment was 205 MeV?

(ii) What are the Coulomb energy of the ion pair and the energy associated with the angular momentum, where you take the separation of the nuclei to be r and the angular momentum of the relative motion to be $\hbar J$?

(iii) Assuming that the two nuclei fuse on a trajectory which brings them just into contact (which you can show corresponds to a closest distance of approach equal to $r = 10$ fm), what are the Coulomb energy, the energy of angular motion, and finally the angular momentum (in units of $\hbar$)?

B. Estimating the rotational energy

To get a sense of the magnitude of energy associated with a rotating nucleus, estimate the rotational frequency of the lowest rotational state and assume that frequency corresponds to one quantum of rotational energy. For specificity do this for dysprosium-152, ^{152}Dy, ignoring its shape by taking the moment of inertia to be MR^2 and assuming it possesses one unit of angular momentum, $J=1$.

C. Transition energies

From Eq. (1), derive the formula for the energy of the gamma ray emitted when the nucleus goes from level E_J to: (a) level E_{J-1}; (b) level E_{J-2}.

D. Spacing of gamma energies

Now find the energy spacing between successive gamma rays by comparing the energies of successive transitions, again with the two assumptions: (a) J takes on successive integral values; (b) J changes by two units with each transition.

E. Numerical evaluation

Evaluate the spacing in MeV for ^{152}Dy assuming the moment of inertia is that of a rigid sphere, $I = 2MR^2/5$. Use

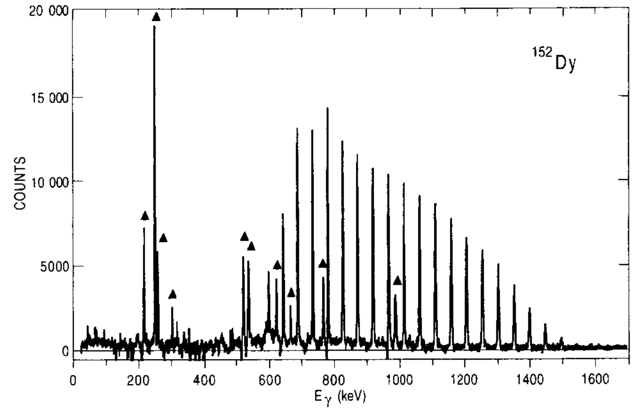

Fig. 1. The gamma-ray spectrum from the nucleus ^{152}Dy, first reported in Ref. 4. The horizontal axis shows the energy of the gamma ray and the vertical axis the number of gamma rays detected. The peaks marked with the triangle sign correspond to transitions which are *not* members of the rotational band.

your result to decide whether $\Delta J = 1$ or $\Delta J = 2$ better explains the pattern of decays shown in Fig. 1. Why might you expect the moment of inertia of this rotating nucleus to be larger than that of a rigid sphere?

IV. SOLUTIONS

A. Angular momentum

(i) The relation between center-of-mass (c.m.) and lab energy is

$$E_{\text{c.m.}} = \frac{M_T}{M_T + M_P} E_{\text{lab}},$$

where the projectile and target are denoted by the subscripts p and T, respectively. This yields $E_{\text{c.m.}} = 142$ MeV.

(ii)

$$E_{\text{coul}} = \frac{Z_T Z_p e^2}{4\pi\epsilon_0 r}, \quad E_{\text{rot}} = \frac{\hbar^2 J(J+1)}{2\mu r^2},$$

where $\mu = M_T M_p/(M_T + M_p)$ is the reduced mass.

(iii) $E_{\text{coul}} = 132.5$ MeV; $E_{\text{rot}} = E_{\text{c.m.}} - E_{\text{coul}} = 9.5$ MeV;

$$J \approx \sqrt{\frac{E_{\text{rot}} 2\mu r^2}{\hbar^2}}$$

$$\approx \sqrt{\frac{9.5 \text{ MeV} \times 2 \times 33.2 \times 931 \text{ MeV}/c^2 \times (10 \text{ fm})^2}{(197.3 \text{ MeV fm}/c)^2}}$$

$$= 39.$$

B. Estimated rotational energy

The angular frequency corresponding to one unit of angular momentum is estimated from $I\omega = \hbar$ from which $\omega = \hbar/I$. The quantum of energy corresponding to ω is $\hbar\omega = \hbar^2/I$. Taking $I \sim MR^2$ where $M = 152 \times 931$ MeV$/c^2$ and $R = 152^{1/3} \times 1.2$ fm gives

$$\hbar\omega = \frac{197^2 \text{ MeV}^2 \text{ fm}^2/c^2}{152 \times 931 \text{ MeV}/c^2 \times 152^{2/3} \times 1.2^2 \text{ fm}^2}$$

$$\approx 0.007 \text{ MeV} = 7 \text{ keV},$$

an energy much smaller than that characteristic of single-particle states in nuclei.

C. Transition energies

$$\Delta E_a = E_J - E_{J-1} = \frac{\hbar^2 J}{I},$$

$$\Delta E_b = E_J - E_{J-2} = \frac{\hbar^2(2J-1)}{I}.$$

D. Spacing of gamma energies

$$\Delta\Delta E_a = \Delta E(J) - \Delta E(J-1) = \frac{\hbar^2}{I}, \tag{2}$$

$$\Delta\Delta E_b = \frac{4\hbar^2}{I}. \tag{3}$$

Notice that in both cases the spacing is constant like the uniform "picket fence" pattern you see in Fig. 1. The result is convincing evidence for the presence of rotation.

E. Numerical evaluation

In nuclear units, the quantity $\hbar^2/I$ is given by:

$$\frac{\hbar^2}{I} = \frac{(197.3)^2}{(0.4)(931)(152)(1.2)^2(152)^{2/3}} \text{ MeV}$$

$$= 0.0168 \text{ MeV}. \tag{4}$$

This is the predicted spacing for $\Delta J = 1$ transitions; the prediction for $\Delta J = 2$ transitions is $\Delta\Delta E_b = 0.067$ MeV. In Fig. 1 there are 18 peaks in the interval 690–1490 keV, so the experimental spacing is $\Delta\Delta E \approx 0.048$ MeV. This is clearly too large for $\Delta J = 1$ transitions, and it is somewhat smaller than the predicted spacing for $\Delta J = 2$. In fact, $\Delta J = 2$ is correct: The rotational band of this nucleus has only even values of J. The moment of inertia is larger than the above estimate mainly because the nucleus is not spherical but is strongly deformed (called "superdeformed" in the literature, see, for example, Ref. 5. These states were first discovered in 1986 by Twin *et al.*[4]).

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[1]A. Bohr, "Rotational motion in nuclei," Rev. Mod. Phys. **48**, 365–374 (1976); B. Mottelson, "Elementary modes of excitation in the nucleus," *ibid.* **48**, 375–383 (1976); J. Rainwater, "Background for the spheroidal nuclear model proposal," *ibid.* **48**, 385–391 (1976).
[2]We thank Dr. C. W. Beausang (Univ. of Liverpool) for sending us his latest data obtained with the Eurogam II spectrometer.
[3]J. O. Newton, "Spinning nuclei," Contemp. Phys. **30**, 277–299 (1989).
[4]P. J. Twin, B. M. Nyako, A. H. Nelson, J. Simpson, M. A. Bentley, H. W. Cranmer-Gordon, P. D. Forsyth, D. Howe, A. R. Mokhtar, J. D. Morrison, J. F. Sharpey-Schafer, and G. Sletten, "Observation of a discrete-line superdeformed band up to 60ℏ in ^{152}Dy," Phys. Rev. Lett. **57**, 811–814 (1986).
[5]P. E. Hodgson, "Superdeformed nuclei," Contemp. Phys. **28**, 365–382 (1987).