NEW PROBLEMS

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The "New Problems" department presents interesting, novel problems for use in undergraduate physics courses beyond the introductory level. We will publish worked problems that convey the excitement and interest of current developments in physics and that are useful for teaching courses such as Classical Mechanics, Electricity and Magnetism, Statistical Mechanics and Thermodynamics, Modern Physics, or Quantum Mechanics. We challenge physicists everywhere to create problems that show how their various branches of physics use the central, unifying ideas of physics to advance physical understanding. We want these problems to become an important source of ideas and information for students of physics. Submit materials to Charles H. Holbrow, *Editor.*

Nuclear scattering

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I. SCOPE

This problem asks for an analysis of the angular distribution of 800 MeV protons scattered from lead nuclei. The analysis confirms the de Broglie wavelength of the protons, gives a measure of the radius of the lead nucleus, and demonstrates that the interference pattern is present even though only one particle at a time is in the range of the nuclear force, i.e., the particle interferes with itself.

The problem is suitable for courses in modern physics, waves, optics, or nuclear physics. It requires familiarity with diffraction from a single slit or a circular aperture. It makes use of the relativistic connection between kinetic energy and momentum of a particle.

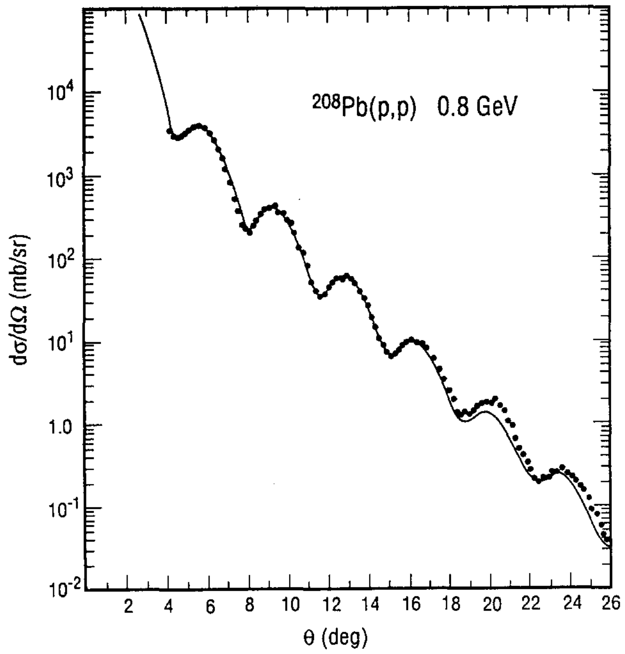

Fig. 1. Angular distribution for elastic scattering of 800 MeV protons from the ground state of ^{208}Pb. The solid line was calculated from theory and fitted to the data.

II. THE PROBLEMS

Figure 1 shows the experimentally determined[1] differential cross section for the elastic scattering of 800 MeV protons from ^{208}Pb. The pattern is analogous to that of Fraunhofer diffraction of light from a circular opaque obstacle.

A. de Broglie wavelength. Find the de Broglie wavelength λ of the 800 MeV protons.

B. Nuclear radius. Using the de Broglie wavelength and the measured angular separations of the diffraction minima, find the radius of the ^{208}Pb nucleus.

The radius R of a nucleus of mass A can be expressed as

$$R = R_0 A^{1/3}, \qquad (1)$$

where R_0 is a constant. Use the value of the radius you obtained from the data to determine the value of R_0. Give your answer in femtometers (fm).

C. Diffraction from a circular object. Show that the first minimum of the diffraction pattern in Fig. 1 is located at the angle expected for a circular obstacle.

III. SOLUTIONS

A. de Broglie wavelength

To find λ it is necessary first to find the momentum p of the protons. Then you can use $\lambda = h/p$. The protons have a kinetic energy of $K = 800$ MeV. Because this is 0.85 of the proton rest energy of $m_p c^2 = 937$ MeV, it is necessary to use relativistically correct equations.

$$pc = \sqrt{2Km_pc^2 + K^2}$$
$$= \sqrt{2 \times 800 \times 937 + 800^2} = 1463 \text{ MeV}.$$

The corresponding de Broglie wavelength is then

$$\lambda = \frac{hc}{pc} = \frac{1240}{1463} = 0.85 \text{ fm}.$$

B. Nuclear radius

The angular separation $\Delta\theta$ between the minima of a Fraunhofer diffraction pattern is given by $\Delta\theta=\lambda/a$, where a is the diameter of the circular aperture—the nucleus in this case. From the graph you can read off that the minima occur at about 4.4°, 8°, 11.5°, 15°, and 18.8°. The corresponding intervals are 3.6°, 3.5°, 3.5°, and 3.3°. The average of these intervals is 3.5° or 6.1×10^{-2} rad. Therefore,

$$R=\frac{\lambda}{2\Delta\theta}=6.9 \text{ fm}.$$

Using this result in Eq. (1) gives

$$R_0=\frac{R}{A^{1/3}}=\frac{6.9}{208^{1/3}}=1.2 \text{ fm}.$$

This value is in excellent agreement with results from other methods of measuring nuclear radii.

C. Diffraction from a circular object

The angular separation between the minima in a diffraction pattern is essentially the same for a slit and for a hole. However, the angular location of the first minimum differs in the two cases. For the slit it is equal to $\theta=\lambda/a$ where a is the width of the slit; for the hole or opaque circle—as per Babinet's principle—$\theta=1.22\lambda/a$. (This is the origin of the Rayleigh criterion for the diffraction limited resolution of a circular aperture.)

In the data of Fig. 1 the first minimum is at 4.4°. Multiplying 1.22 times the average angular separation of 3.5° yields 4.3°—remarkably good agreement given the simplicity of assumptions about the nucleus.

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[1] G. S. Blanpied *et al.* "Proton elastic and inelastic scattering at 0.8 GeV from ^{12,13}C and ^{208}Pb," Phys. Rev. C **18**, 1436–1446 (1978).

WORKING WITH EINSTEIN

The main thing that occurred to me as I worked with Einstein was that what I had been taught as to the nature of science was all wrong. I have the strong impression that science, as done by an Einstein at least, is one of the creative arts. ... You could see that Einstein was motivated not by logic in the narrow sense of the word but by a sense of beauty. He was always looking for beauty in his work. Equally he was moved by a profound religious sense fulfilled in finding wonderful laws, simple laws in the universe. It was really a religious experience for him, of the most profound sort, even though he did not believe in a personal god. ... I asked him once about a theory and he said, "When I am evaluating a theory, I ask myself, if I were God, would I have made the universe in that way?" If the theory did not have the sort of simple beauty that would be demanded of a God, then the theory was at best only provisional.

Banesh Hoffmann, in *Some Strangeness in the Proportion*, edited by Harry Woolf (Addison-Wesley, Reading, MA, 1980), p. 476.

NEWTON'S RECKLESSNESS

It is only by bringing into the open the inherent contradictions, and the metaphysical implications of Newtonian gravity, that one is able to realize the enormous courage—or sleepwalker's assurance—that was needed to use it as the basic concept of cosmology. In one of the most reckless and sweeping generalizations in the history of thought, Newton filled the entire space of the universe with interlocking forces of attraction, issuing from all particles of matter and acting on all particles of matter, across the boundless abysses of darkness.

Arthur Koestler, *The Sleepwalkers* (1959). (Reprinted by Grosset & Dunlap, New York, 1963), p. 504.