Relativity in Exclusive and Semi-Inclusive Scattering

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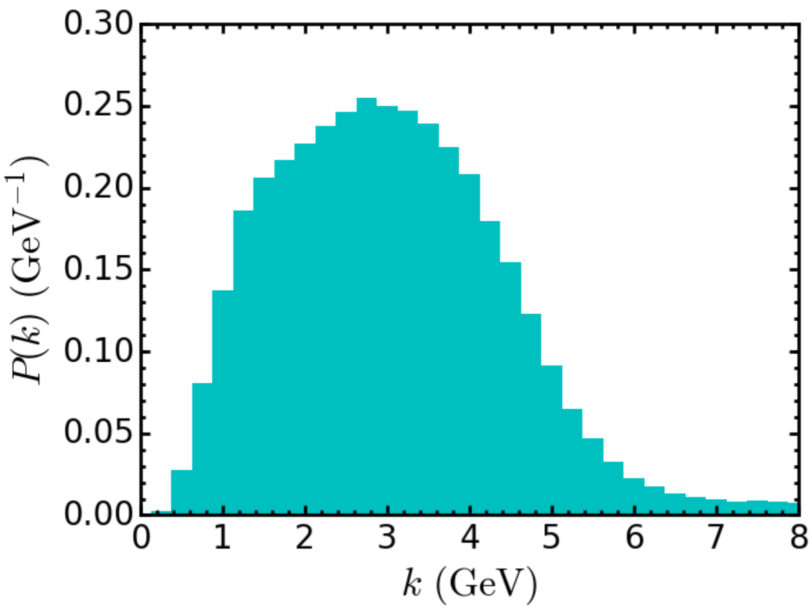
Relativity

Why do we need it?



DUNE flux as a probability density.

Poincare Group

$$[\hat{J}^i, \hat{J}^j] = i\epsilon^{ijk}\hat{J}^k \qquad [\hat{K}^i, \hat{K}^j] = -i\epsilon^{ijk}\hat{J}^k$$

$$[\hat{J}^i, \hat{K}^j] = i\epsilon^{ijk}\hat{K}^k$$

$$[\hat{P}^\mu, \hat{P}^\nu] = 0$$

$$[\hat{K}^i, \hat{P}^0] = -i\hat{P}^j \qquad [\hat{J}^i, \hat{P}^0] = 0$$

$$[\hat{K}^i, \hat{P}^j] = i\delta^{ij}\hat{P}^0 \qquad [\hat{J}^i, \hat{P}^j] = i\epsilon^{ijk}\hat{P}^k$$

Galilean Group

$$[\hat{K}^i, \hat{K}^j] = 0$$

$$[\hat{K}^i, \hat{P}^j] = i\delta^{ij}\hat{M}$$

There are two possible approaches to constructing covariant models.

1. Quantum Field Theory:

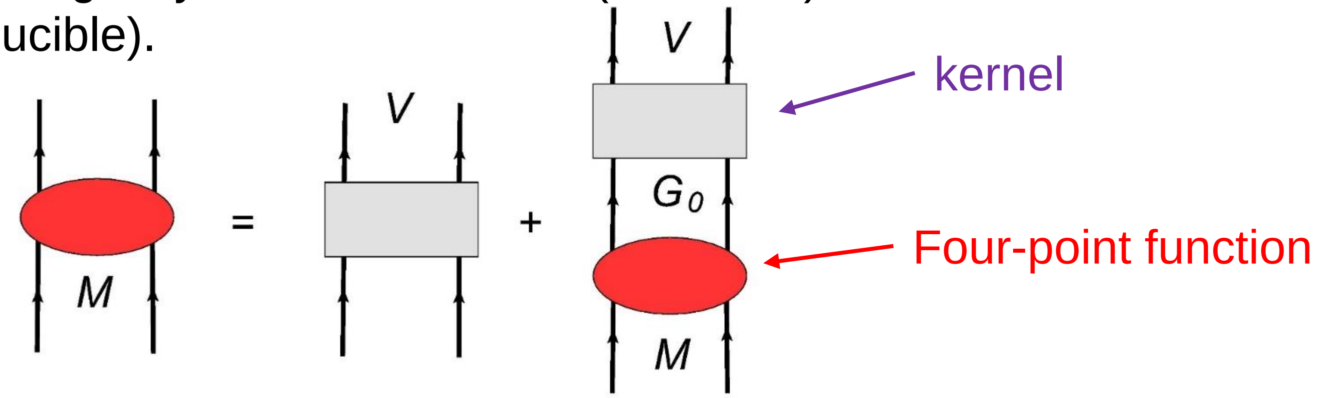
    a) Effective models based on field theory but with nucleon and meson degrees of freedom, form factors and nonrenormalizable couplings.
    b) Current operators can be constructed from effective interactions.
    c) Quasipotential equations.

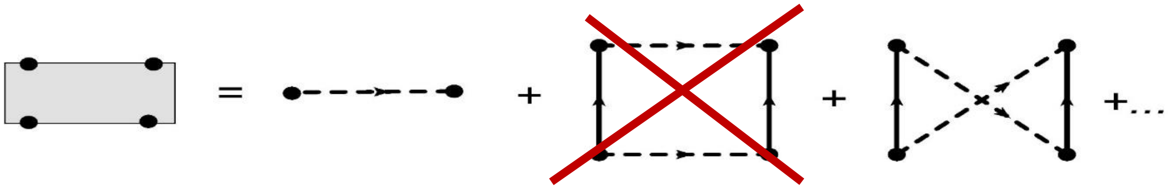
2. Dirac Constraint Dynamics:

    a) Operators satisfy Poincare Group.
    b) The number of particles is fixed.
    c) Wave functions are eigenvectors of Mass Operators containing instantaneous interactions.
    d) Models are required for constructing interaction currents consistent with interactions.

The Two-Nucleon Bethe-Salpeter Equation

- Based on a re-summation of all possible Feynman diagrams for the four-point functions using the model the model.
- Diagrams are separated by separating contributions that can be separated by cutting only two nucleon lines (reducible) from those that cannot be (irreducible).

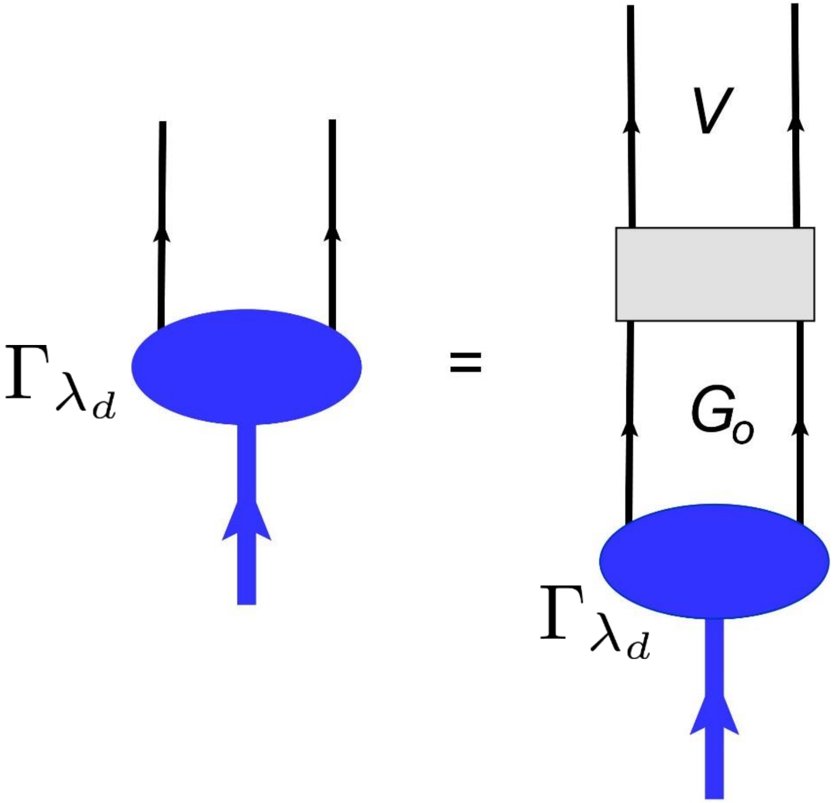




- All inelasticities other than those from the elastic cut are contained in the interaction kernel.

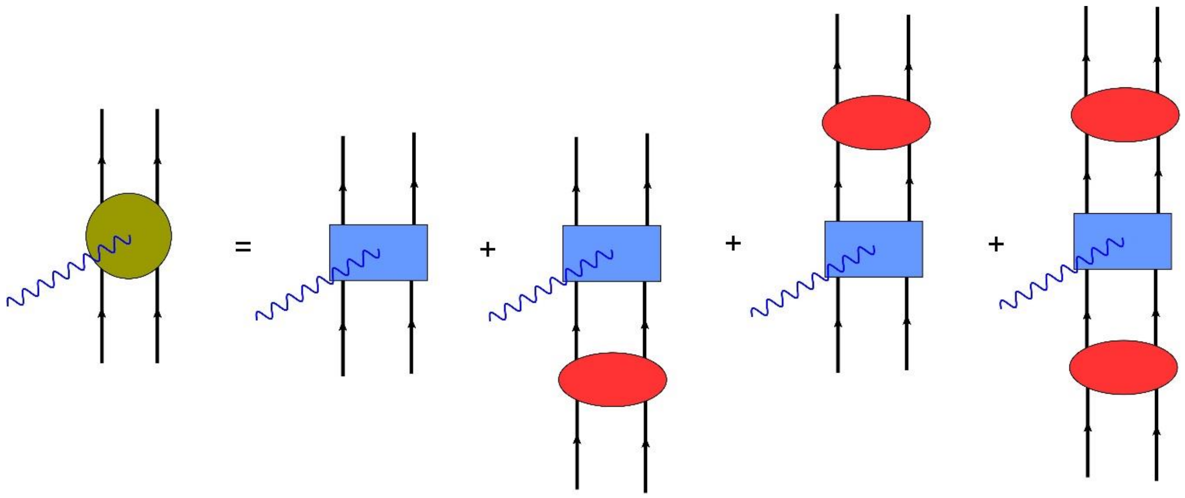
The Deuteron Vertex Function

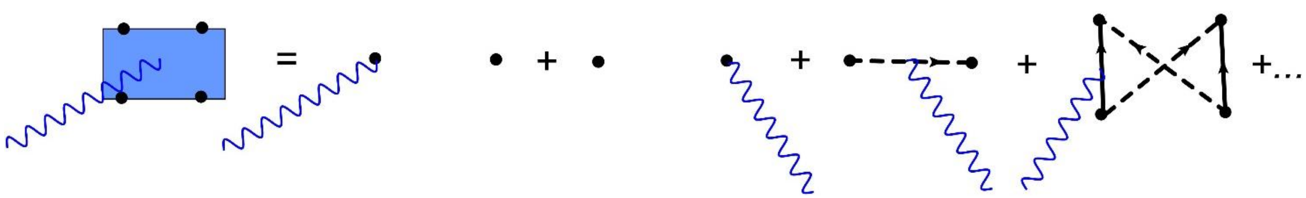
The bound state vertex function is related to the residue of the bound state pole of the four-point functions.



The Current Operator

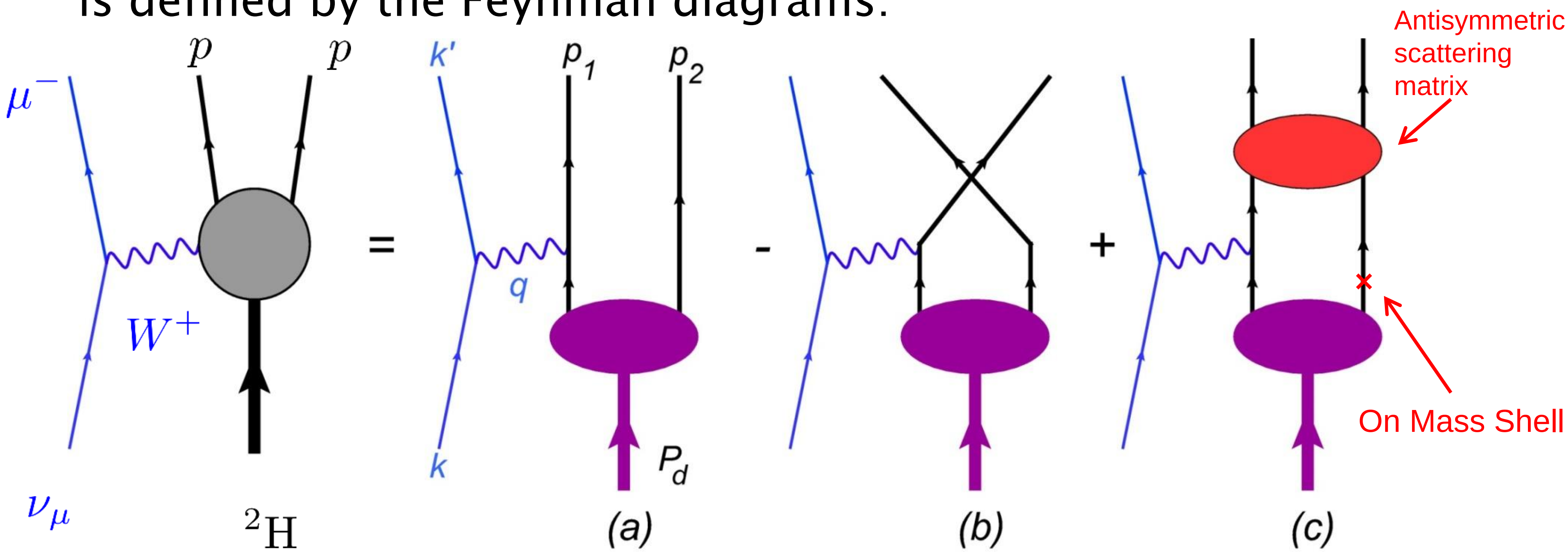
The current operator is determined from a similar analysis of the five-point function with one boson line.





The Impulse Approximation

The impulse approximation to CCν scattering from the deuteron is defined by the Feynman diagrams:

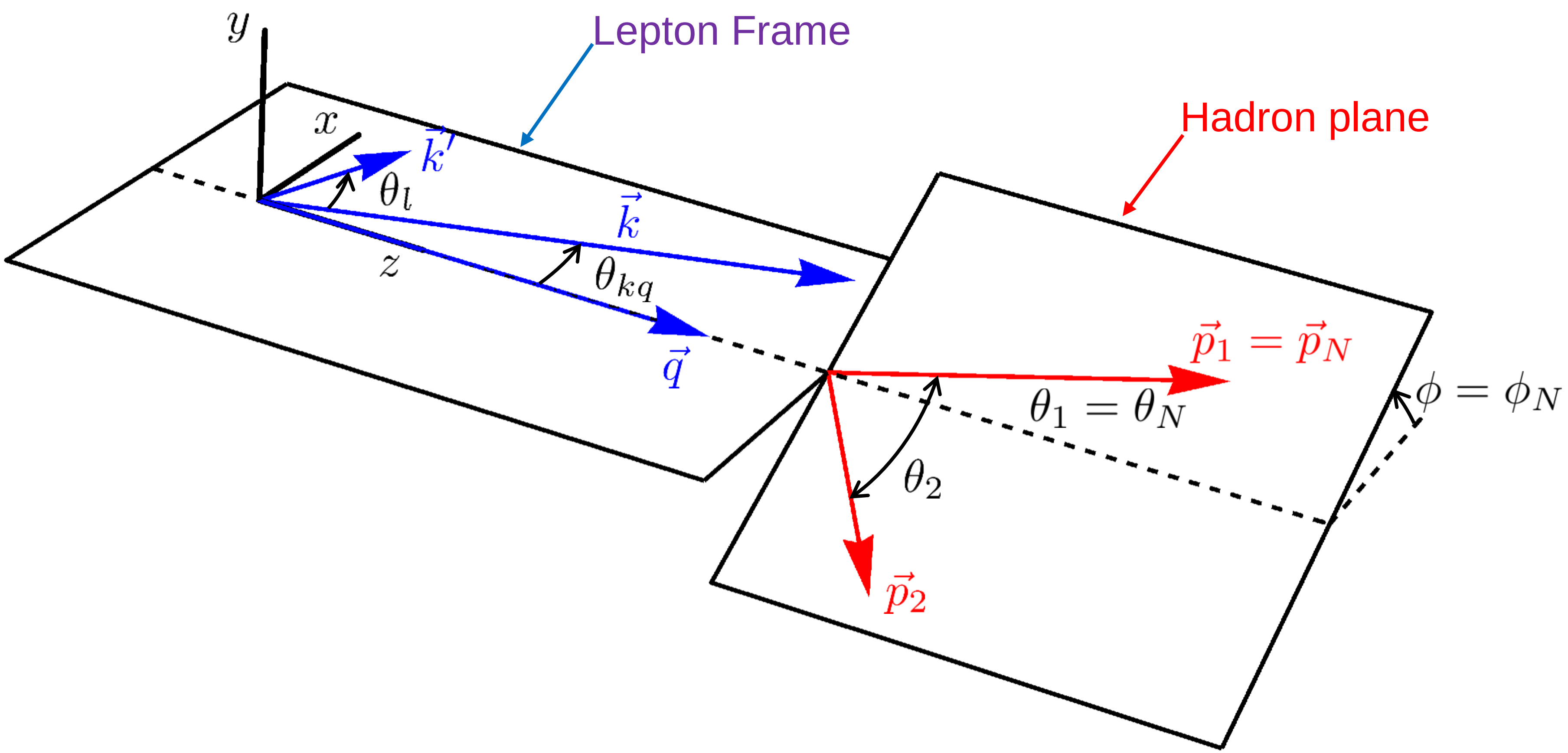


We will show three approximations:

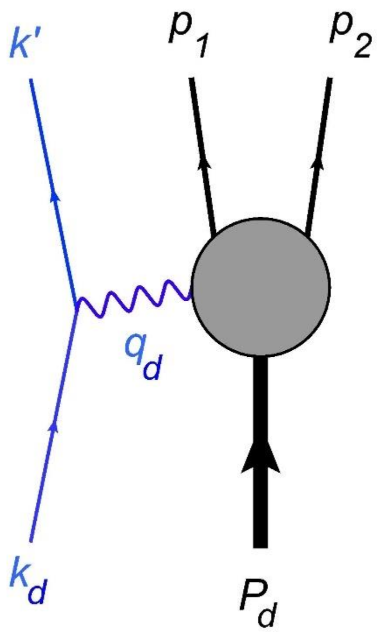
1. Plane Wave Impulse Approximation (PWIA): diagram (a) only.
2. Plane Wave Born Approximation (PWBA): diagrams (a)+(b).
3. Distorted Wave Born Approximation (DWBA): diagrams (a)+(b)+(c).

Kinematics

Fixed *q* frame variables



The Number of Free Kinematical Variables



5 four-momenta	+20
On-shell conditions	-5
Four-momentum conservation	-4
Choose z-axis	-2
Choose scattering plane	-1
Choose rest frame	-3
Independent degrees of freedom	**5**

Exclusive Deuteron Cross Section

$$\frac{d^5\sigma}{d\varepsilon' d\Omega_{k'} d\Omega_{p_1}} = \frac{G^2 \cos^2\theta_c \, m_p^2 |\boldsymbol{k}'||\boldsymbol{p}_1| v_0}{2(2\pi)^5 |\boldsymbol{k}| M_d \left(1 + \frac{\omega|\boldsymbol{p}_1| - E_1|\boldsymbol{q}|\cos\theta_1}{M_d|\boldsymbol{p}_1|}\right)} \mathcal{F}_\chi^2$$

where

$$v_0 = (\varepsilon + \varepsilon')^2 - \boldsymbol{q}^2$$

and

$$\begin{aligned}
\mathcal{F}_\chi^2 = \; & \hat{V}_{CC} W^{CC(I)} + 2\hat{V}_{CL} W^{CL(I)} + \hat{V}_{LL} W^{LL(I)} + \hat{V}_T W^{T(I)} \\
& + \hat{V}_{TT}\left(W^{TT(I)} \cos 2\phi + W^{TT(II)} \sin 2\phi\right) \\
& + \hat{V}_{TC}\left(W^{TC(I)} \cos\phi + W^{TC(II)} \sin\phi\right) \\
& + \hat{V}_{TL}\left(W^{TL(I)} \cos\phi + W^{TL(II)} \sin\phi\right) \\
& + \chi\Big(\hat{V}_{T'} W^{T'(II)} + \hat{V}_{TC'}\left(W^{TC'(I)} \sin\phi + W^{TC'(II)} \cos\phi\right) \\
& + \hat{V}_{TL'}\left(W^{TL'(I)} \sin\phi + W^{TL'(II)} \cos\phi\right)\Big)
\end{aligned}$$

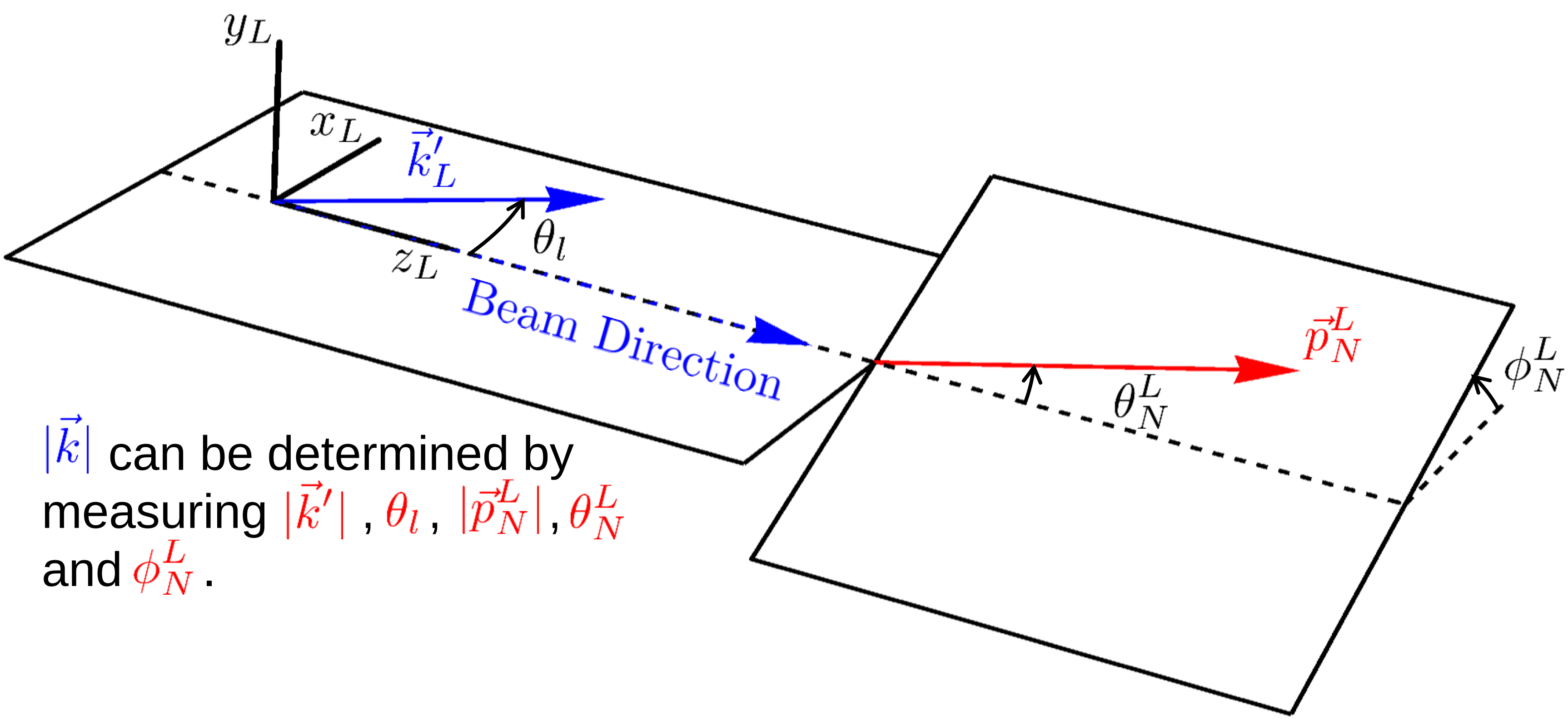
$\chi = 1$ for neutrino scattering

$\chi = -1$ for anti-neutrino scattering

O. Moreno, T. W. Donnelly, W. P. Ford and J. W. Van Orden, Phys. Rev. D **90**, 013014 (2014).

Kinematic Variables in the "Lab" Frame

Since the objective is to determine the incident neutrino energy to study neutrino oscillations and given that the beam direction is known but not the incident momentum, it is best to consider the frame.



$|\vec{k}|$ can be determined by measuring $|\vec{k}'|$, θ_l , $|\vec{p}_N^L|$, θ_N^L and ϕ_N^L .

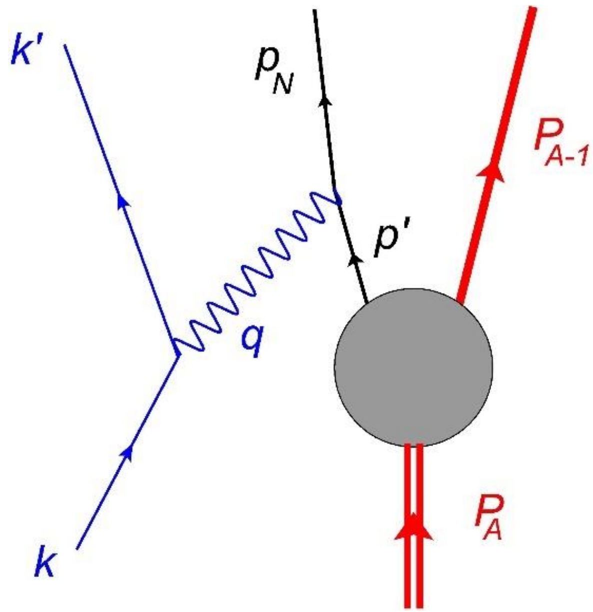
The deuteron cross section in the Lab frame can be written as

$$\frac{d^4\sigma}{dk' d\Omega_{k'} dp_N \Omega_N^L} = \frac{G^2 \cos^2\theta_c m_N \varepsilon {\boldsymbol{k}'}^2 {\boldsymbol{p}_N}^2 v_0}{2(2\pi)^5 \varepsilon' E_N \left| |\boldsymbol{k}_d| E_2 + \varepsilon_d \cos\theta_{kq} (|\boldsymbol{p}_N| \cos\theta_N^L - |\boldsymbol{q}_d|) \right|} \mathcal{F}_\chi^2 \delta(|\boldsymbol{k}_d| - k)$$

and the statistical average over the neutrino distribution as

$$\left\langle \frac{d^4\sigma}{dk' d\Omega_{k'} dp_N \Omega_N^L} \right\rangle = \int_0^\infty dk \frac{d^4\sigma}{dk' d\Omega_{k'} dp_N \Omega_N^L} P(k)$$
$$= \frac{G^2 \cos^2\theta_c m_N \varepsilon_0 {\boldsymbol{k}'}^2 {\boldsymbol{p}_N}^2 v_0}{2(2\pi)^5 \varepsilon' E_1 \left| |\boldsymbol{k}_d| E_2 + \varepsilon_d \cos\theta_{kq} (|\boldsymbol{p}_N| \cos\theta_N^L - |\boldsymbol{q}_d|) \right|} \mathcal{F}_\chi^2 P(|\boldsymbol{k}_d|)$$

Semi-inclusive Scattering from Nuclei with *A*>2



Spectral function model

where:

$$k = (\sqrt{\boldsymbol{k}^2 + m^2}, \boldsymbol{k}) = (\varepsilon, \boldsymbol{k})$$

$$k = (\sqrt{\boldsymbol{k}'^2 + m'^2}, \boldsymbol{k}') = (\varepsilon', \boldsymbol{k}')$$

$$P_A = (M_A, \vec{0})$$

$$p_N = (\sqrt{\boldsymbol{p}_N^2 + m_N^2}, \vec{p}_N)$$

$$P_{A-1} = (\sqrt{\boldsymbol{p}^2 + W_{A-1}^2}, -\boldsymbol{p})$$

$$p' = P_A - P_{A-1} = (M_A - \sqrt{\boldsymbol{p}^2 + W_{A-1}^2}, \boldsymbol{p})$$

The Semi-inclusive cross section for the spectral model in the Lab frame is

$$\frac{d^4\sigma}{dk' d\Omega_{k'} dp_N d\Omega_N^L} = \frac{G^2 \cos^2\theta_c m_N \boldsymbol{k'}^2 \boldsymbol{p}_N{}^2 v_0}{8(2\pi)^6 |\boldsymbol{k}| \varepsilon' E_N} \widetilde{\mathcal{F}}_\chi^2 \mathcal{S}(p,E)$$

Note that since the invariant mass of the residual *A-1* system is not fixed, *k* is not fixed.

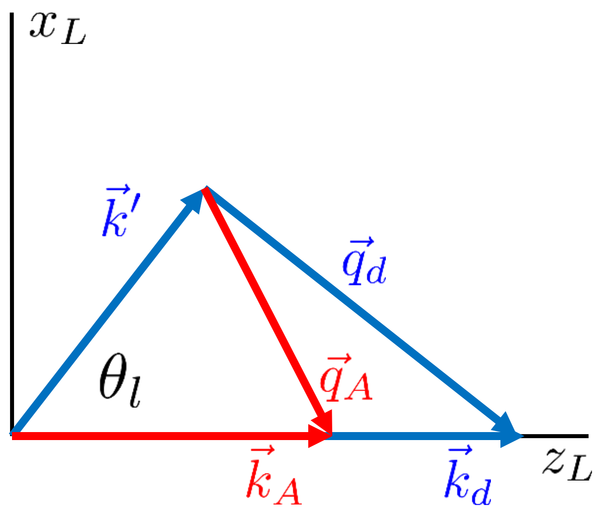
The statistical average over the neutrino flux is then given by

$$\left\langle \frac{d^4\sigma}{dk' d\Omega_{k'} dp_N \Omega_N^L} \right\rangle = \int_0^\infty dk \frac{G^2 \cos^2\theta_c m_N \boldsymbol{k'}^2 \boldsymbol{p}_N{}^2 v_0}{8(2\pi)^6 |\boldsymbol{k}| \varepsilon' E_N} \widetilde{\mathcal{F}}_\chi^2 \mathcal{S}(p,E) P(k)$$

The spectral function is normalized such that

$$n(p) = \int_0^\infty dE S(p,E) \quad \text{and} \quad \frac{1}{(2\pi)^3} \int_0^\infty dp p^2 n(p) = A - Z$$

$^{2}H_{2}{}^{16}O$
Kinematics



Optimize kinematics for the deuteron

$$s_d = (M_d + \omega)^2 - \boldsymbol{q}^2$$

$$y = \frac{(M_d + \omega)\sqrt{s(s - 4m_N^2)}}{2s} - \frac{|\boldsymbol{q}|}{2}$$

$$Y = y + |\boldsymbol{q}|$$

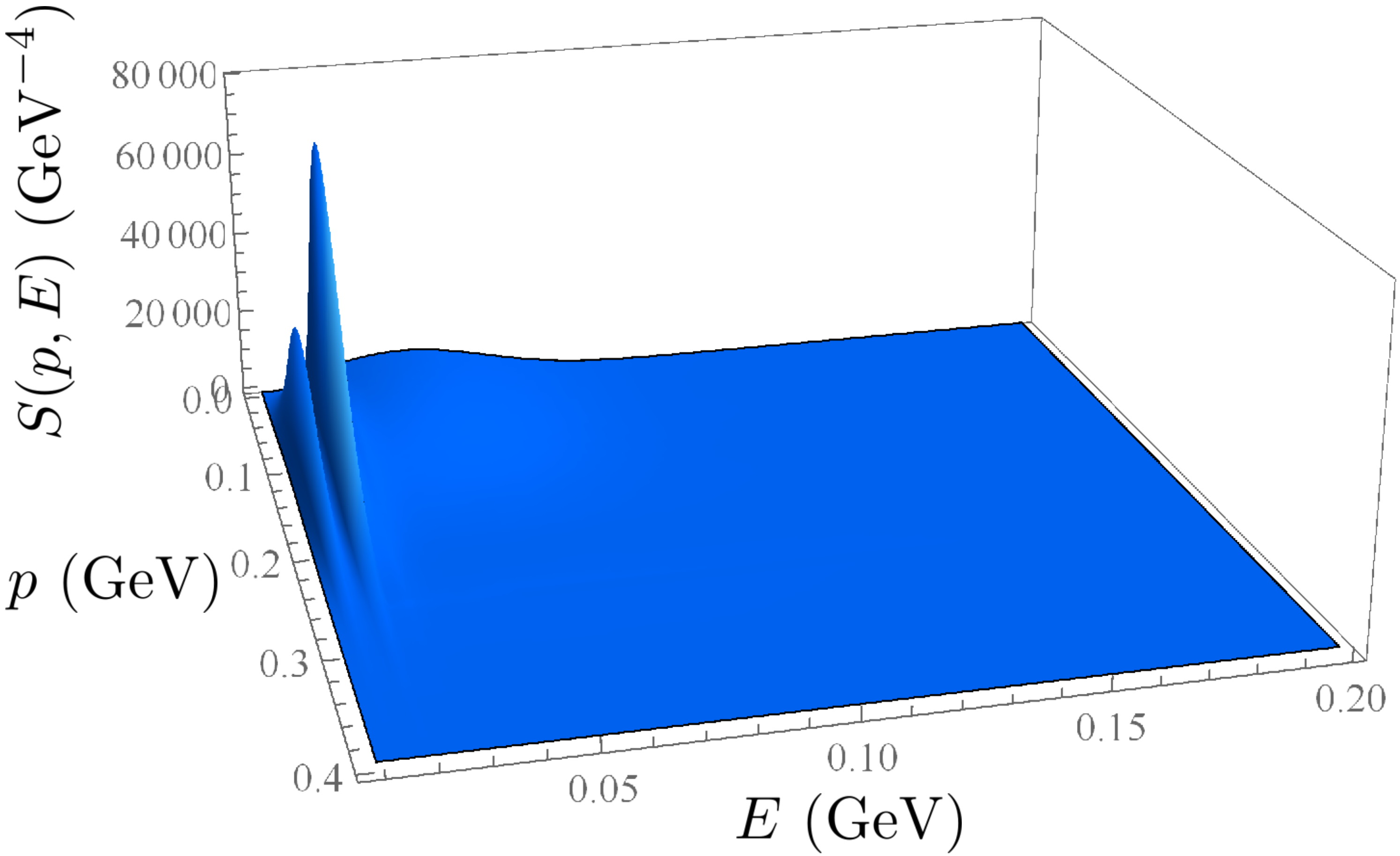
$$|y| \leq p \leq Y$$

Given $|\boldsymbol{k}'|$ and θ_l, choose $|\boldsymbol{k}_d|$ such that $y = 0$.

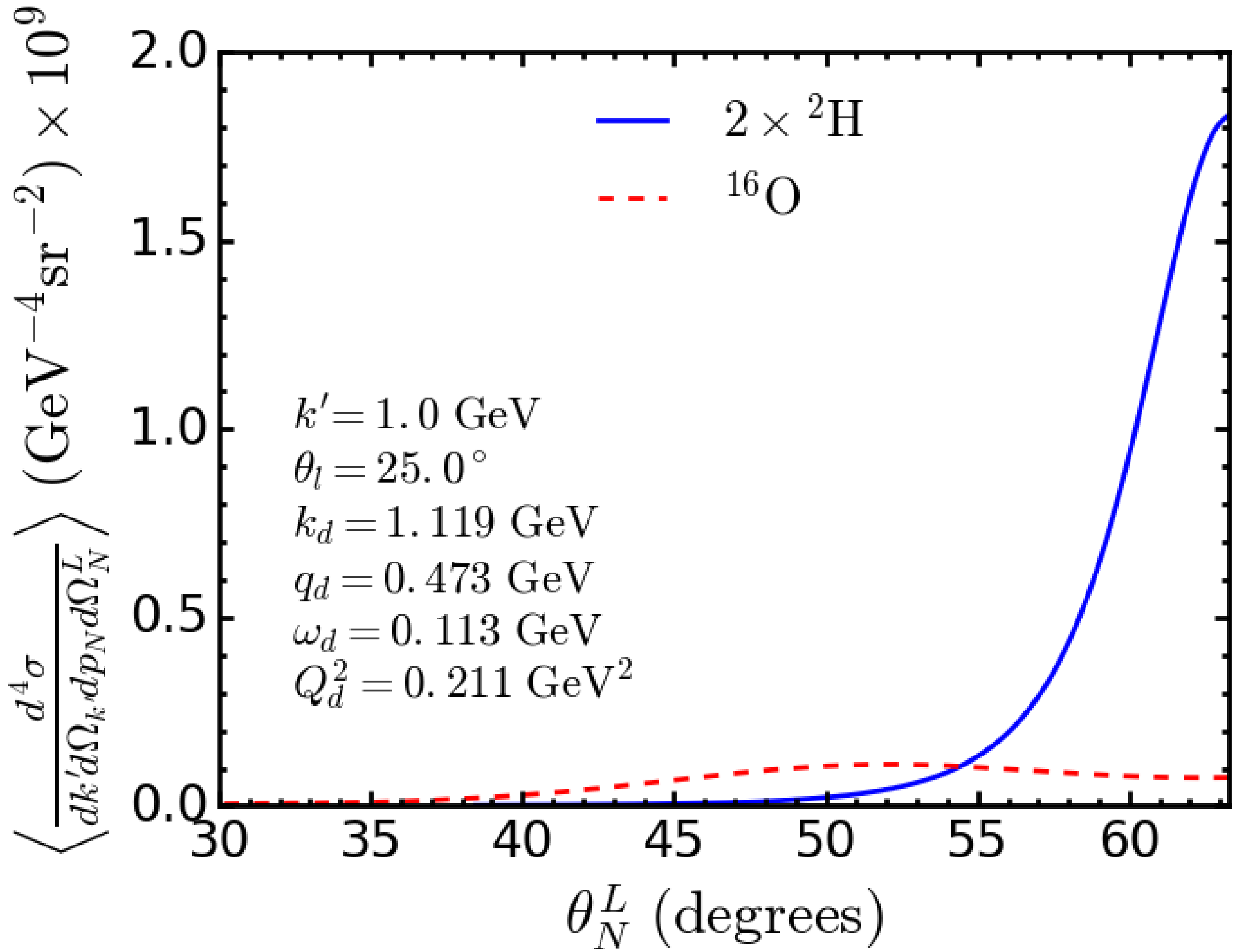
Then $|\boldsymbol{p}_N|$, θ_N^L and ϕ_N^L can be determined as functions of $|\boldsymbol{p}_N - \boldsymbol{q}_d|$.

The cross section for semi-inclusive scattering from $^{16}\mathrm{O}$ can then be determined as a function of $|\boldsymbol{k}_A|$ and $|\boldsymbol{p}_N - \boldsymbol{q}_d|$.

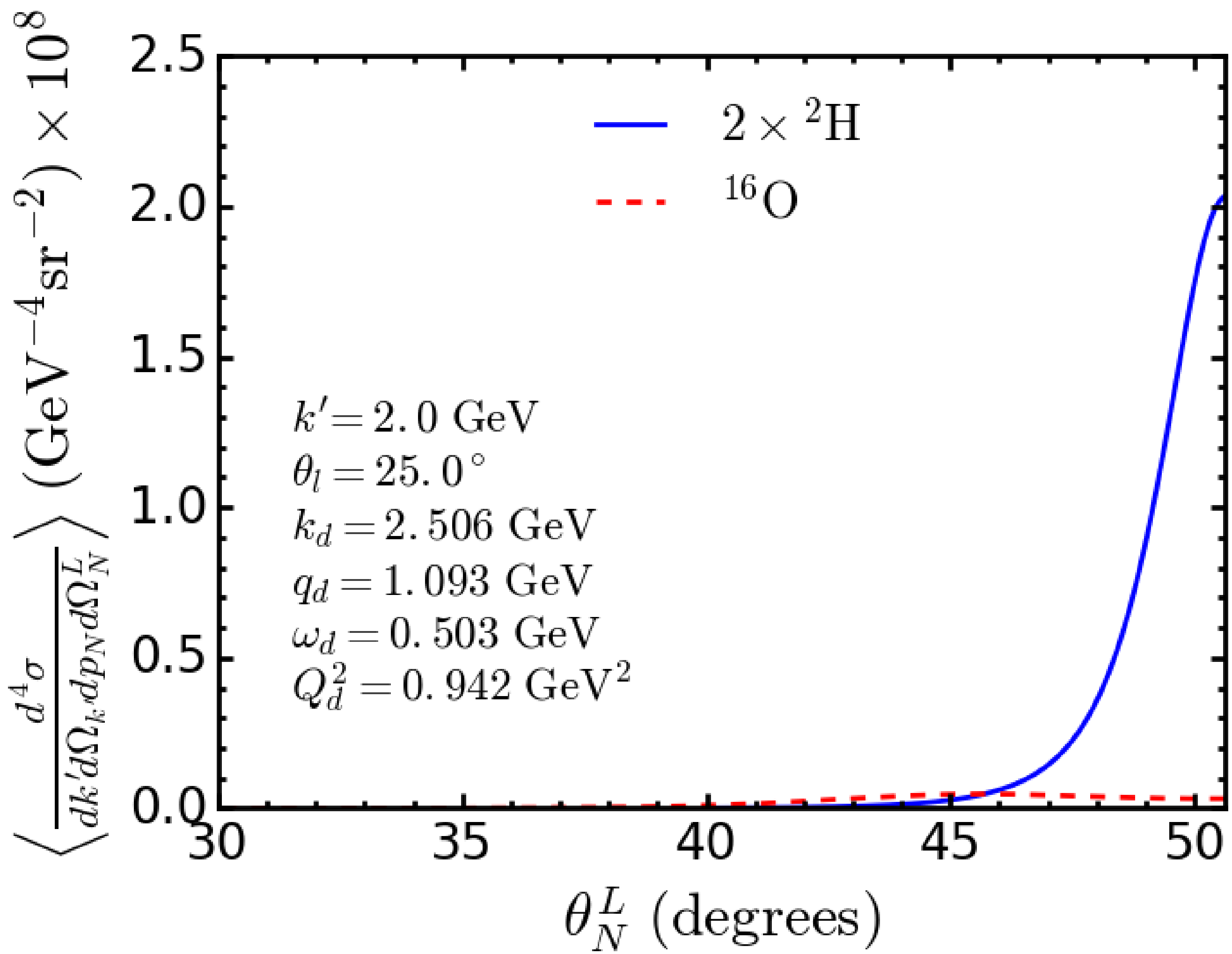
Spectral Function for ^{16}O



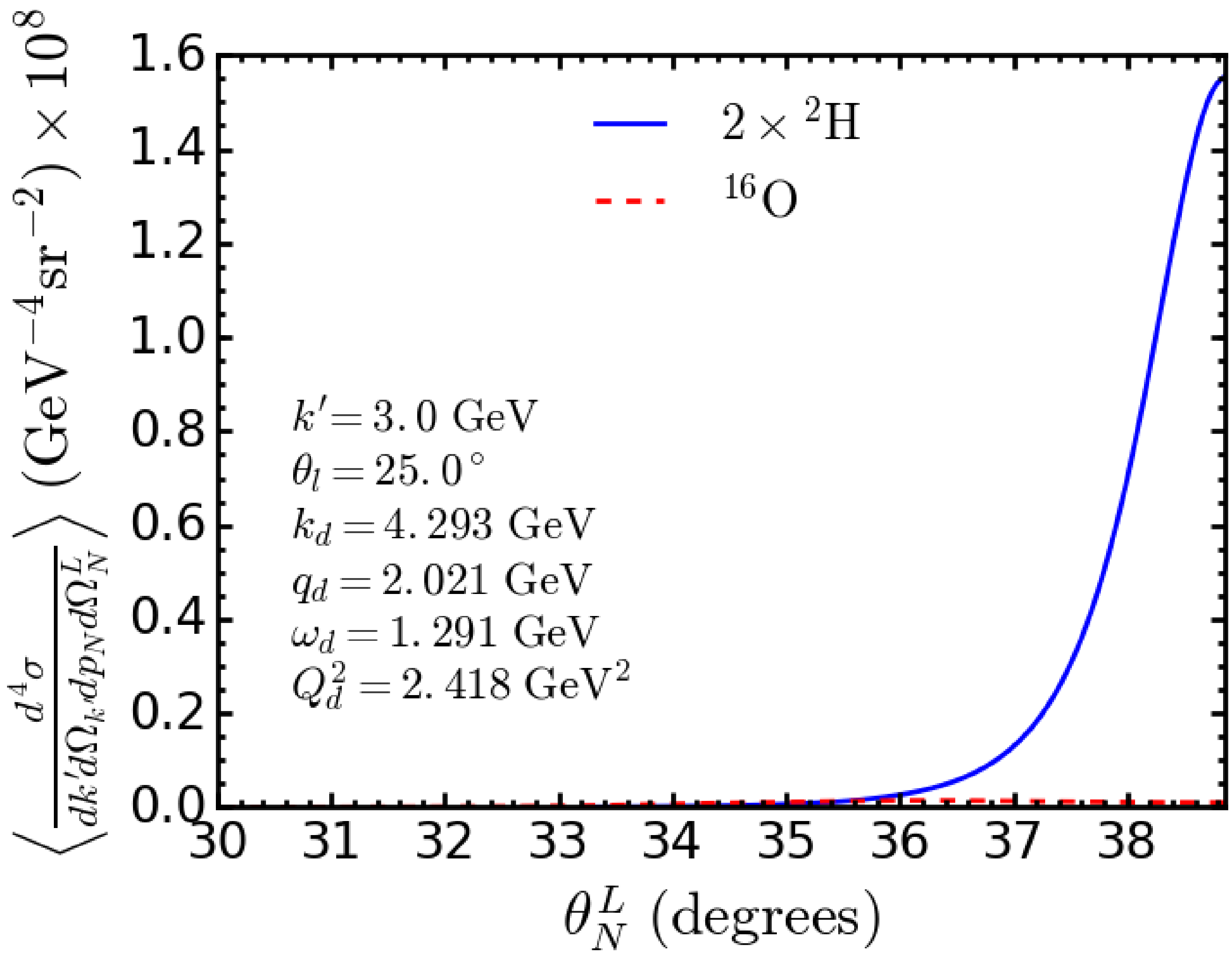
Courtesy of Omar Benhar



$2\times{}^2\mathrm{H}$
${}^{16}\mathrm{O}$
$k'=1.0$ GeV
$\theta_l=25.0^\circ$
$k_d=1.119$ GeV
$q_d=0.473$ GeV
$\omega_d=0.113$ GeV
$Q_d^2=0.211$ GeV^2
2.0
1.5
1.0
0.5
0.0
30
35
40
45
50
55
60
θ_N^L (degrees)
$\left\langle \frac{d^4\sigma}{dk' d\Omega_k d p_N d\Omega_N^L} \right\rangle$ (GeV^{-4}sr^{-2}) $\times 10^9$

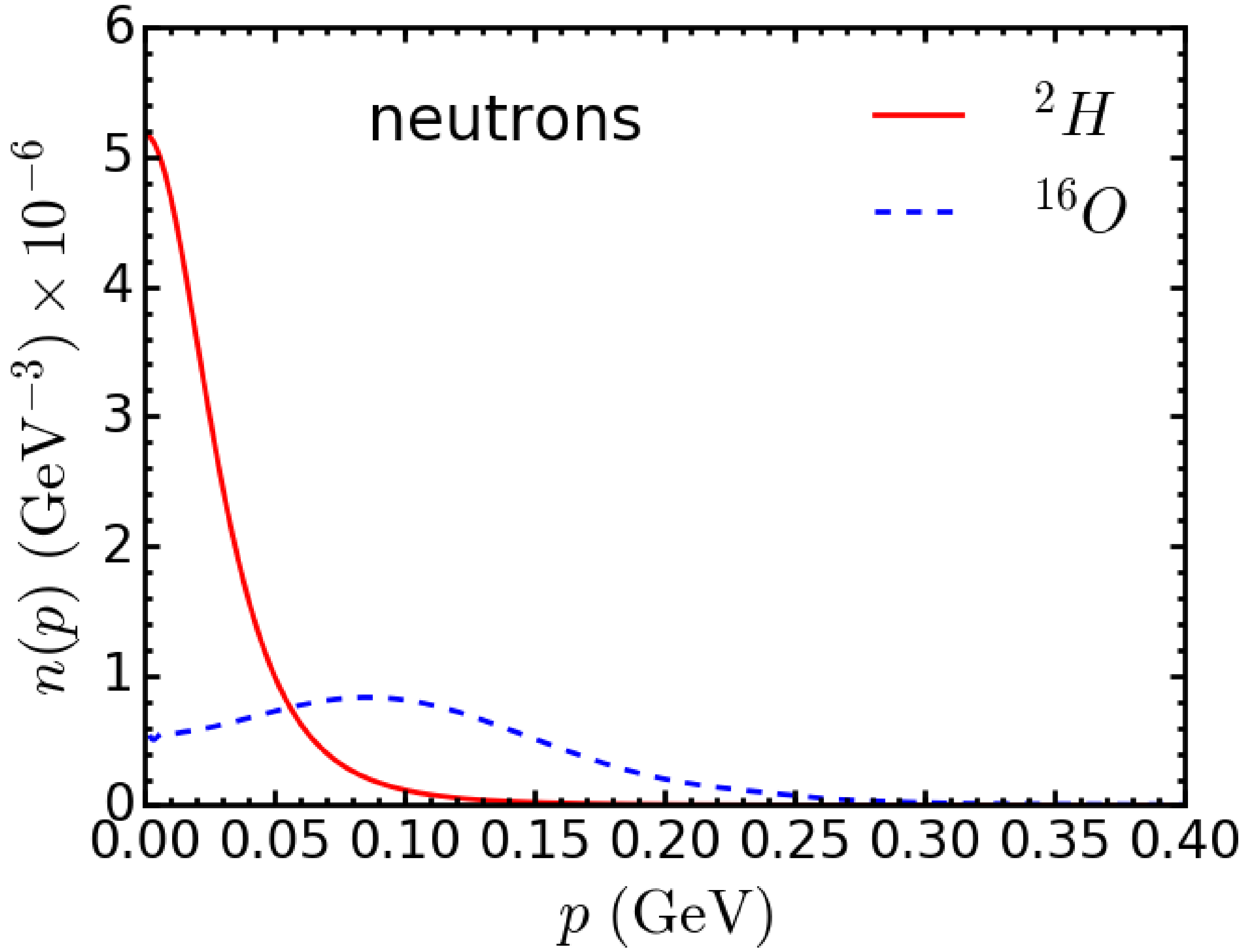


$\left\langle \frac{d^4\sigma}{dk' d\Omega_k dp_N d\Omega_N^L} \right\rangle$ (GeV^{-4}sr^{-2}) $\times 10^8$
2.5
2.0
1.5
1.0
0.5
0.0
$2 \times {}^2$H
^{16}O
$k' = 2.0$ GeV
$\theta_l = 25.0^\circ$
$k_d = 2.506$ GeV
$q_d = 1.093$ GeV
$\omega_d = 0.503$ GeV
$Q_d^2 = 0.942$ GeV^2
30
35
40
45
50
θ_N^L (degrees)

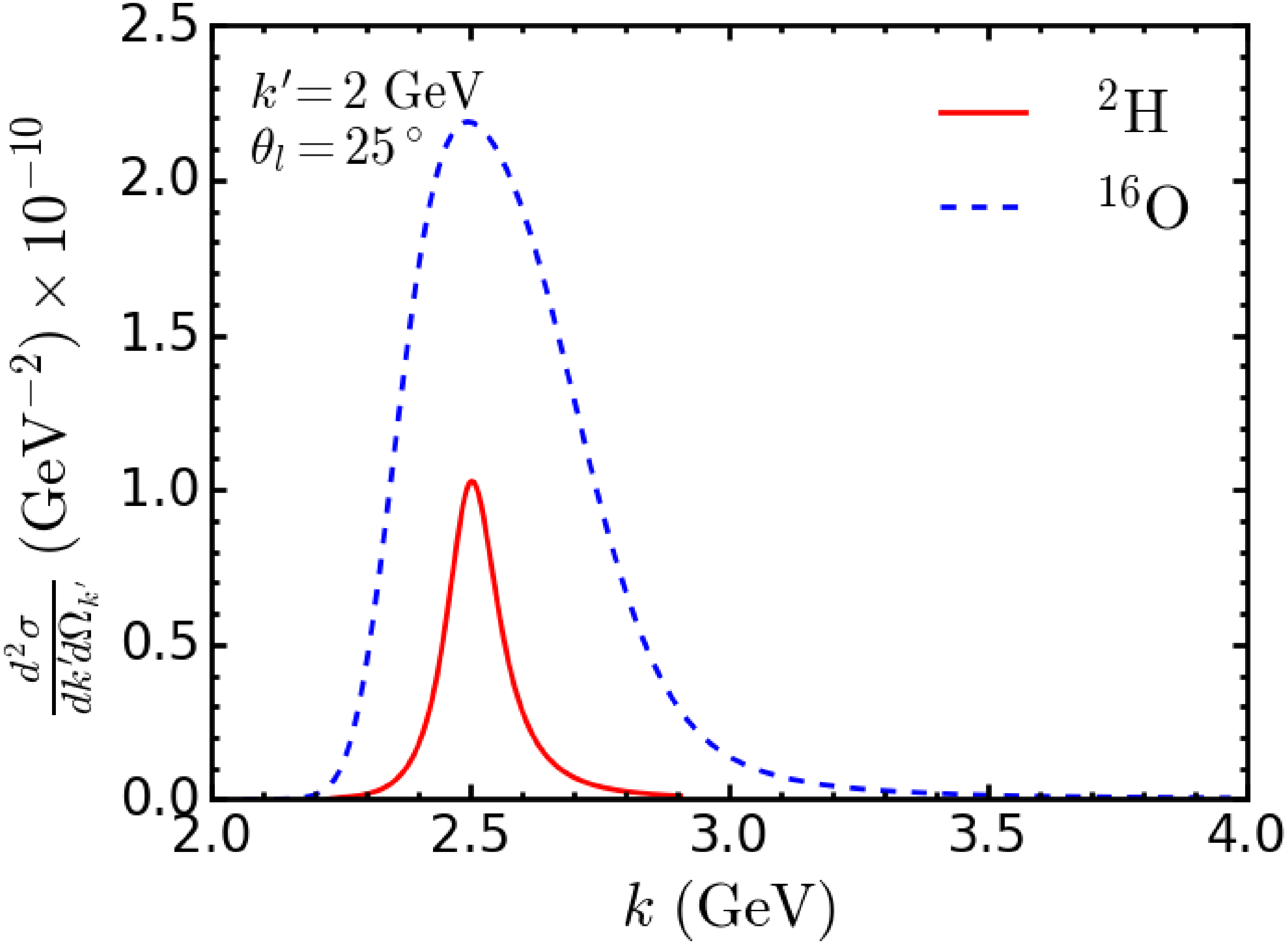


$\left\langle \frac{d^4\sigma}{dk' d\Omega_k dp_N d\Omega_N^L} \right\rangle$ $(\mathrm{GeV}^{-4}\mathrm{sr}^{-2}) \times 10^8$
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
30
31
32
33
34
35
36
37
38
θ_N^L (degrees)
$2 \times {}^2\mathrm{H}$
${}^{16}\mathrm{O}$
$k' = 3.0$ GeV
$\theta_l = 25.0^\circ$
$k_d = 4.293$ GeV
$q_d = 2.021$ GeV
$\omega_d = 1.291$ GeV
$Q_d^2 = 2.418\ \mathrm{GeV}^2$

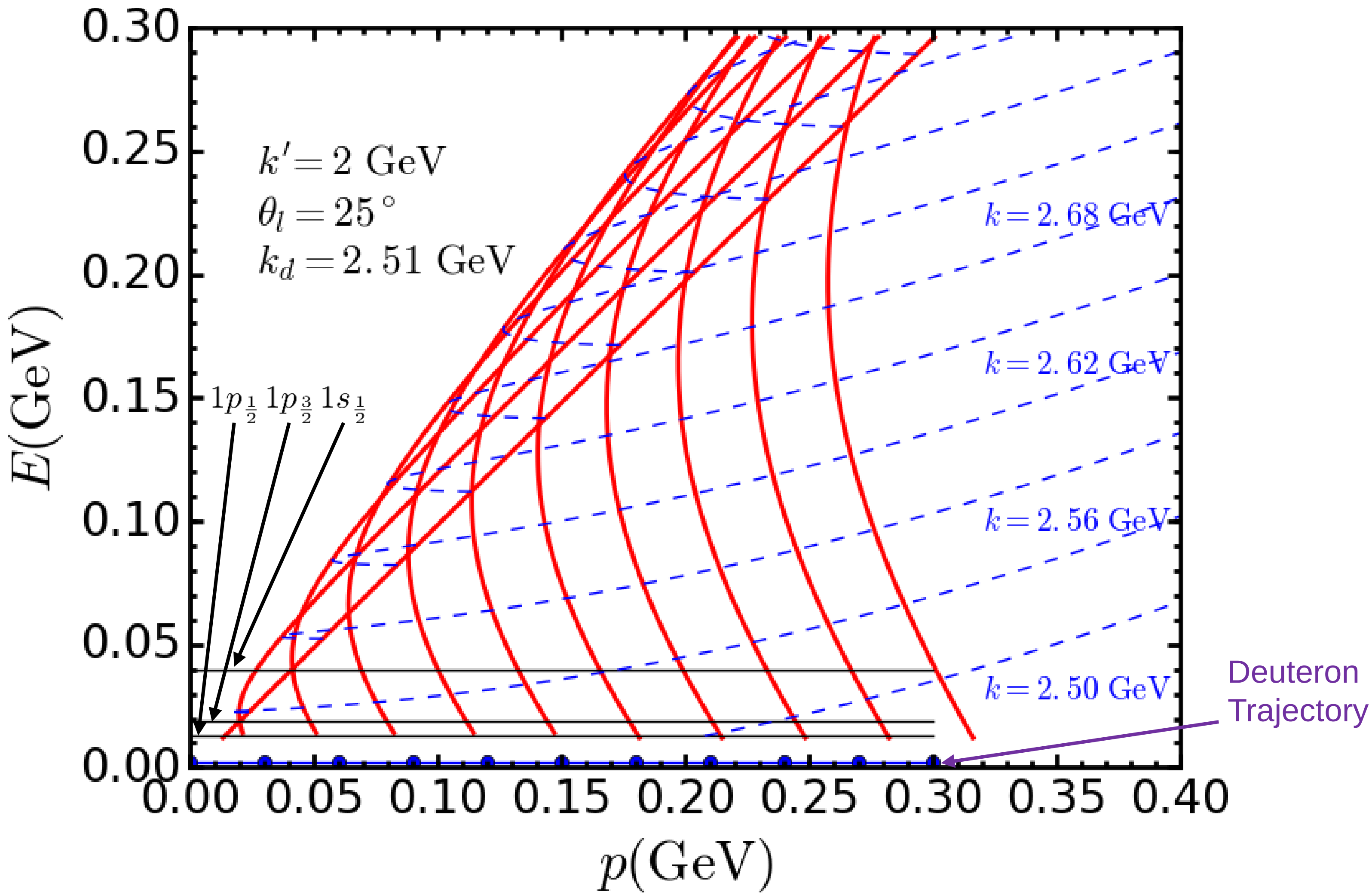
Neutron Momentum Distributions



Integrated Cross Sections



Trajectories in p and E



Summary

- For the deuteron the incident neutrino momentum can be determined exactly by measuring the muon and proton three-momenta.

- For semi-inclusive scattering, the incident neutrino momentum cannot be fixed in similar manner.

- The exclusive deuteron cross section is much larger than the semi-inclusive cross section for ^{16}O for scattering from heavy water.

- Semi-inclusive scattering may be able decrease the uncertainty in the neutrino momentum from nuclei.

References to the deuteron model and semi-inclusive CCν scattering:

- “A new calculation for $^2H(e,e'p)n$ at GeV energies,” S. Jeschonnek and J. W. Van Orden, Phys. Rev. C **78**, 014007 (2008).
- “Target polarization for $2\vec{H}(e,ep)n$ at GeV energies,” S. Jeschonnek and J. W. Van Orden, Phys. Rev. C **80**, 054001 (2009).
- “Ejectile polarization for $^2H(e,e'\vec{p})n$ at GeV energies,” S. Jeschonnek and J. W. Van Orden, Phys. Rev. C **81**, 014008 (2010).
- “Exclusive Scattering from Unpolarized and Polarized Deuteron,” S. Jeschonnek and J. W. Van Orden, Few Body Syst. **49**, 65 (2011).
- “A Regge Model for Nucleon-Nucleon Spin Dependent Amplitudes,”, W. P. Ford and J. W. Van Orden, Phys. Rev. C **87**, 014004 (2013).
- “^{2}H$(e,e'p)$ observables using a Regge model parameterization of final state interactions”,W. P. Ford, S. Jeschonnek and J. W. Van Orden, Phys. Rev. C **87**, 054006 (2013).
- “Off-shell extrapolation of Regge-model NN-scattering amplitudes describing final-state interactions in ^{2}H$(e,e'p)$”, W. P. Ford and J. W. Van Orden, Phys. Rev. C **88**, 054004 (2013).
- “Semi-inclusive charged-current neutrino-nucleus reactions,” O. Moreno, T. W. Donnelly, J. W. Van Orden and W. P. Ford, Phys. Rev. D **90**, 013014 (2014).
- “Coincidence charged-current neutrino-induced deuteron disintegration”, O. Moreno, T. W. Donnelly, J. W. Van Orden and W. P. Ford, Phys. Rev. D. **92**, 053006 (2015).

$$\cos\theta_N = \cos\theta_N^L \cos\theta_{kq} - \cos\phi_N^L \sin\theta_N^L \sin\theta_{kq}$$

$$\cos\phi_N = \frac{\cos\phi_N^L \sin\theta_N^L \cos\theta_{kq} + \cos\theta_N^L \sin\theta_{kq}}{\sin\theta_N}$$

$$\sin\phi_N = \frac{\sin\phi^L \sin\theta_N^L}{\sin\theta_N}$$

$$\cos\theta_N^L = \cos\theta_N \cos\theta_{kq} + \cos\phi \sin\theta_N \sin\theta_{kq}$$

$$\cos\phi_N^L = \frac{\cos\phi \sin\theta_N \cos\theta_{kq} + \cos\theta_N \sin\theta_{kq}}{\sin\theta_N^L}$$

$$\sin\phi_N^L = \frac{\sin\phi \sin\theta_N}{\sin\theta_N^L}$$