Universality in Short-Range Correlations Or Hen – MIT

Hen Lab

Laboratory for Nucle Science @

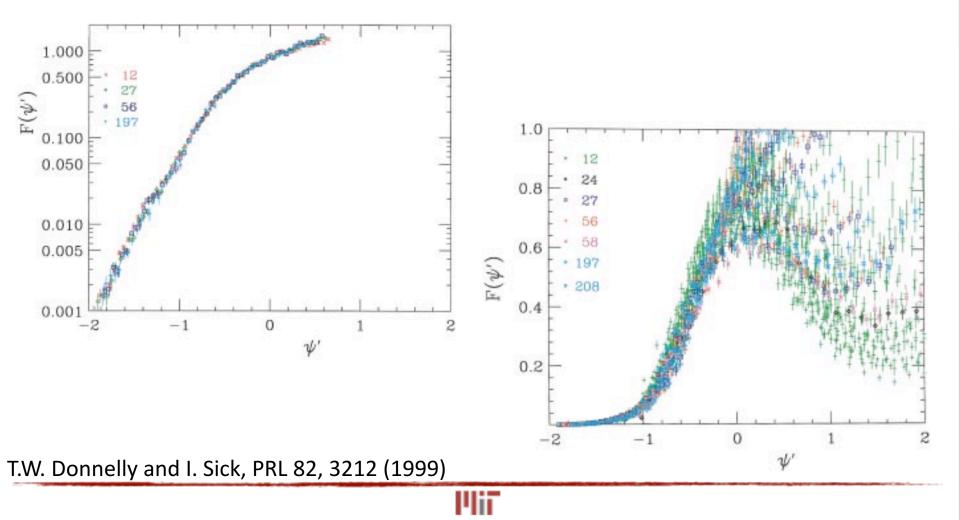
"One model to rule them all; One parameter to fit it all"

Theoretical Developments in Neutrino-Nucleus Scattering, INT, December 8th 2016





When discussing "universality" one should separate the ground state from the reaction.





My focus (today) is on universality of the ground state:

- 1. Non interacting fermi gases are universal. One parameter (k_F) essentially determines their properties.
 - Nuclei in strongly interacting systems...
- Dilute two-component fermi gases with a short-ranged interaction are also universal. Two parameters (kF and C) essentially determines their properties.
 - Nuclei are not dilute...

My goal is to convince you that nuclei are quite similar to case (2) and present the implications of this understanding.



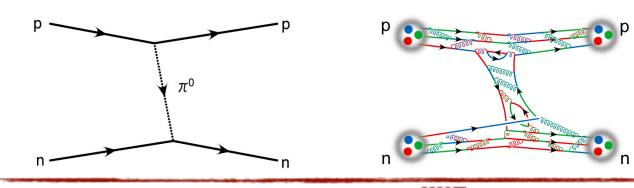


Many-body Schrödinger Equation

$$\sum_{i} \left\{ -\frac{\hbar^2}{2m_i} \nabla_i^2 \Psi(\vec{r}_1, \dots, \vec{r}_N, t) \right\} + U(\vec{r}_1, \dots, \vec{r}_N) \Psi(\vec{r}_1, \dots, \vec{r}_N, t) = i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}_1, \dots, \vec{r}_N, t)$$

Main Challenges:

- No 'fundamental' Interaction => residual interaction between quarks that makeup the nucleons.
- Phenomenological parametrizations are complex! (over 18 operators)

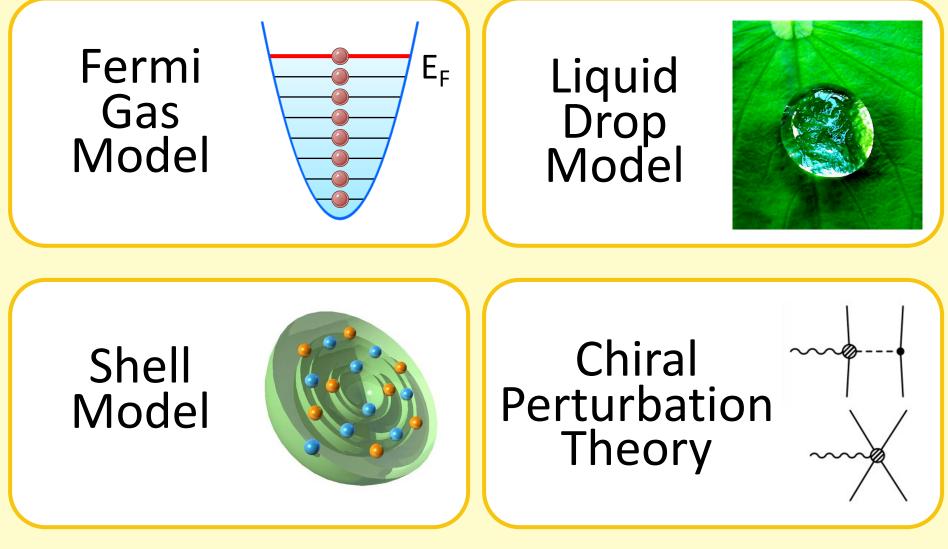






Solution: Effective Theories







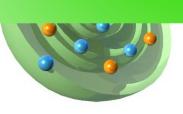




All effective theories relay on the same common idea:

Scale separation of long and short range dynamics in nuclei

Nioaei



Theory



Long-range (low-momentum) structure of nuclei was studied for many years and mean-field approaches give a good description of this part of the nuclear ground state.

- Two big questions:
 - How important are details of the short-range structure that are neglected / smeared out?
 - Is there an effective, *universal*, way to add the short-range dynamics for a global description?





Whole is different from the sum of parts! $n_{2N}(k_1, k_2) \neq n_N(k_1) \cdot n_N(k_2)$ $\rho_{2N}(\vec{r_1}, \vec{r_2}) \neq \rho_N(\vec{r_1}) \cdot \rho_N(\vec{r_2})$

Specifically, in coordinate space: SRC: $\rho_{2N}(\vec{r}_1, \vec{r}_2) \neq 0$ for $|\vec{r}_1 - \vec{r}_2| \approx R_N$ LRC: $\rho_{2N}(\vec{r}_1, \vec{r}_2) \neq 0$ for $|\vec{r}_1 - \vec{r}_2| \approx R_A$

(Some) Interesting questions:

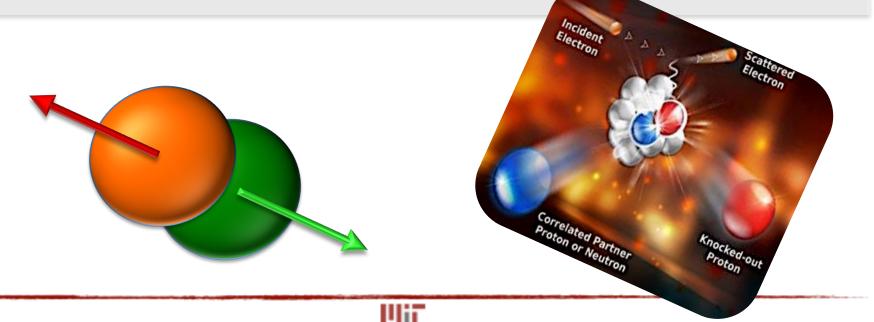
Is there a way to factorize the two-body density? Can we separate the 'mean-field' and 'SRC' effects? Are the SRC effects universal?





SRC are pairs of nucleon that are close together in the nucleus (wave functions overlap)

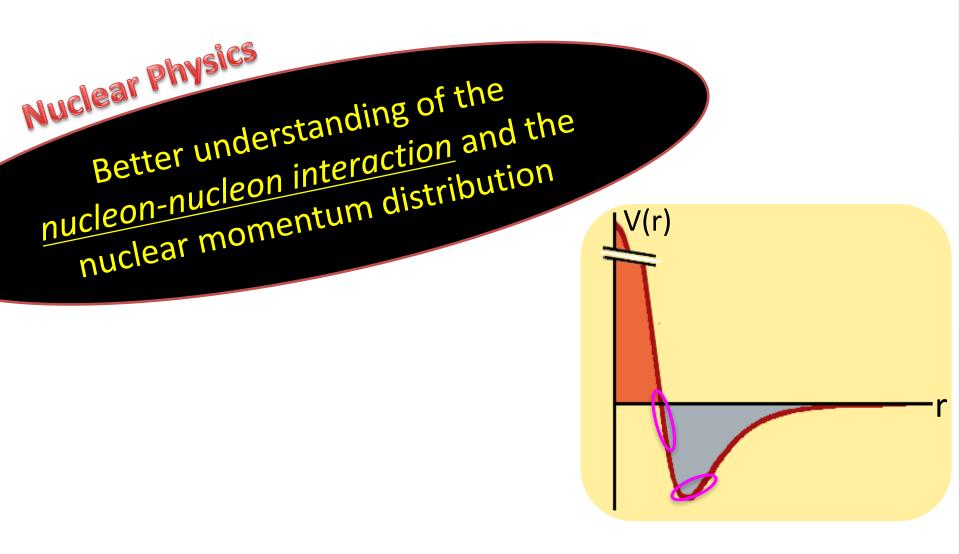
=> Momentum space: pairs with <u>high relative</u> <u>momentum and low c.m. momentum</u> compared to the Fermi momentum (k_F)





Why SRC?



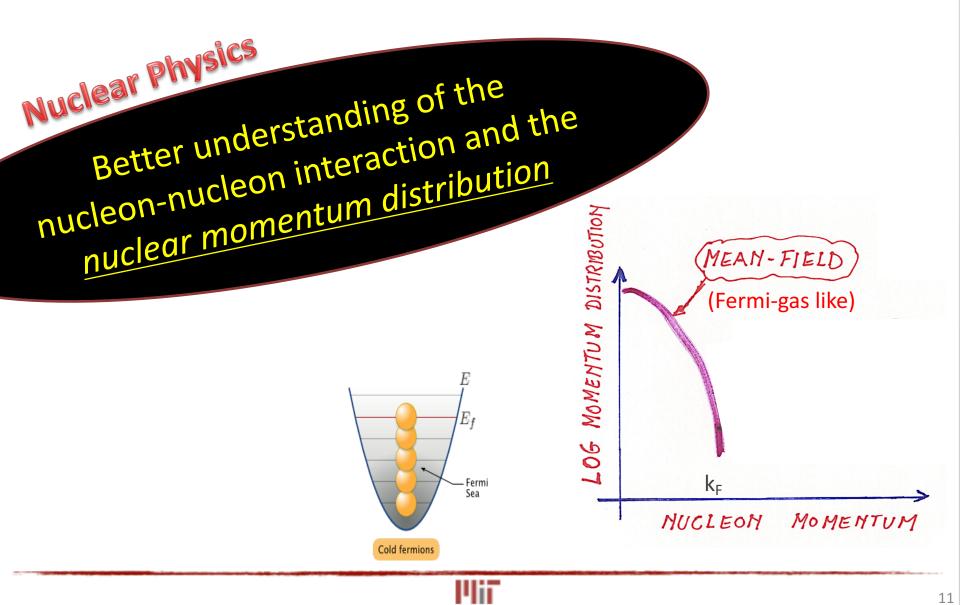


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Why SRC?

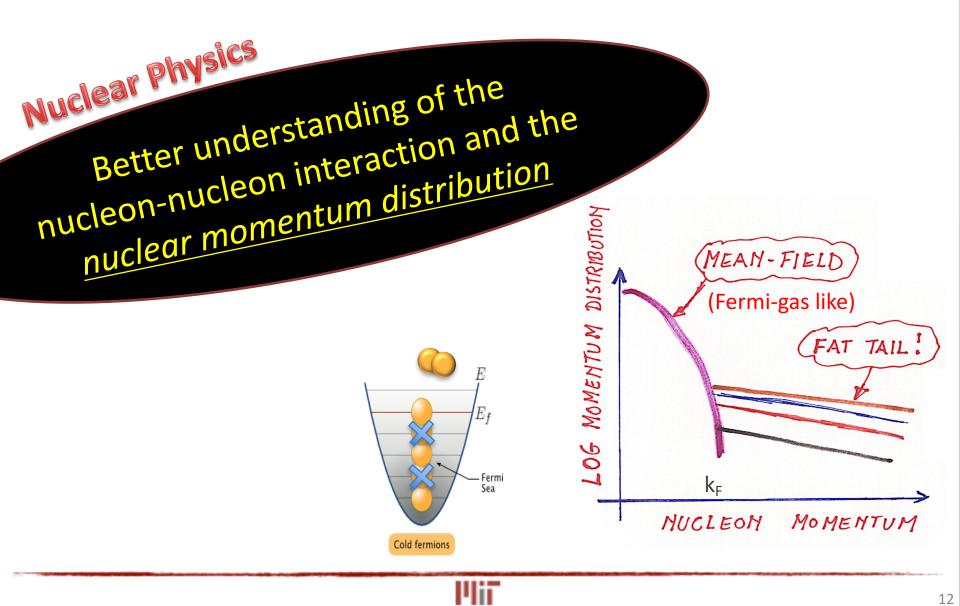






Why SRC?











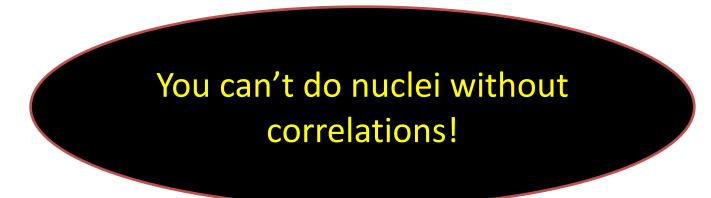
You can't do nuclei without correlations!







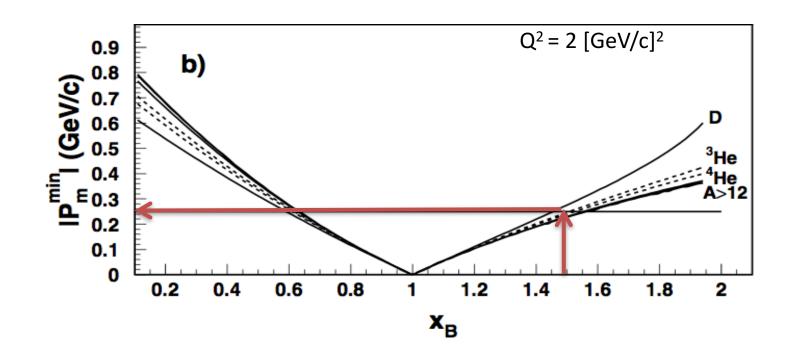
Today: (short) overview of SRC and a presentation of an effective theory for SRC in nuclei







(e,e') cross section at different kinematics is sensitive to different 'parts' of the nucleus.

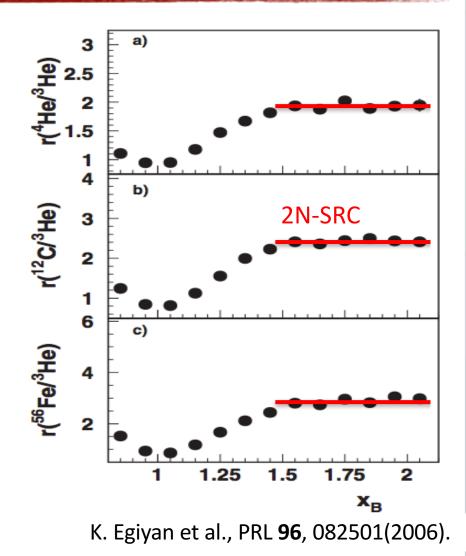




- (e,e') cross section ratios sensitive for A/d sensitive to n_A(k)/n_d(k)
- Observed scaling in for $x_B \ge 1.5$.

 $n_A(k>k_F) = a_2(A) \times n_d(k)$







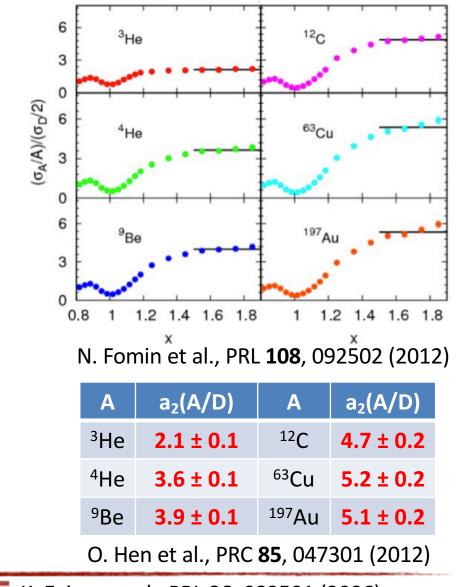




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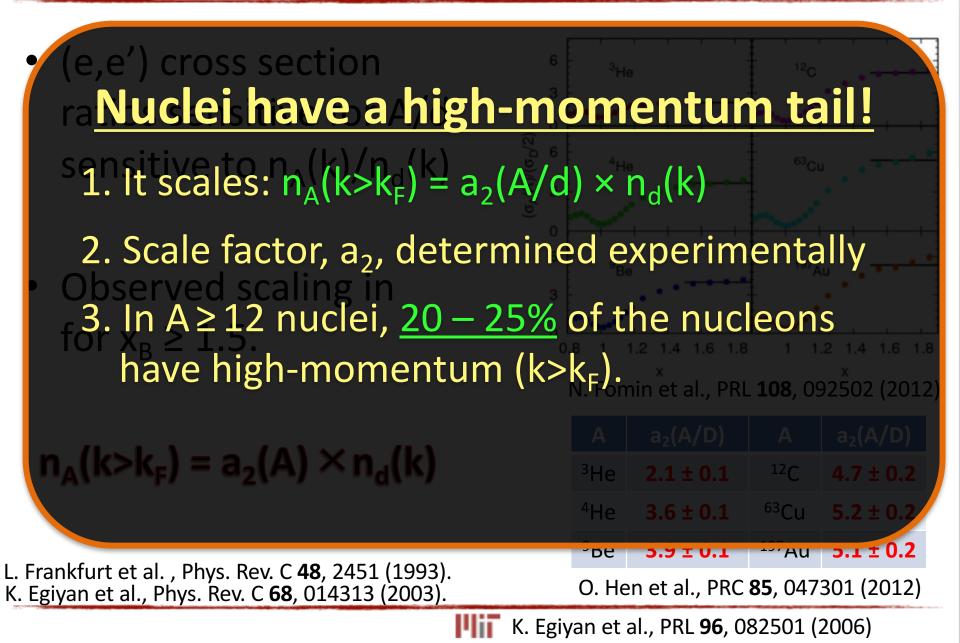
L. Frankfurt et al. , Phys. Rev. C **48**, 2451 (1993). K. Egiyan et al., Phys. Rev. C **68**, 014313 (2003).



K. Egiyan et al., PRL **96**, 082501 (2006)









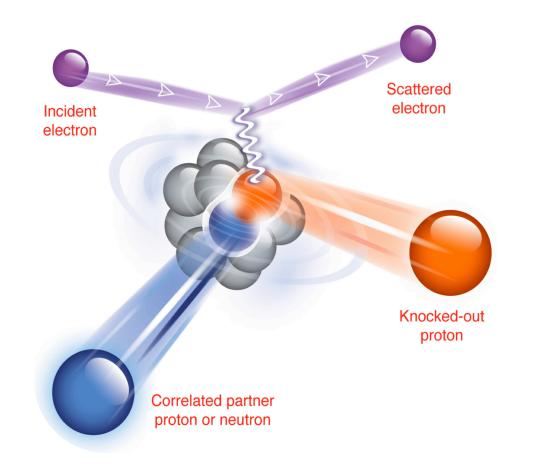


Nuclei have a high-momentum tail! s1. It scales: $n_A(k > k_F) = a_2(A/d) \times n_d(k)$ 2. Scale factor, a₂, determined experimentally 3. In A \geq 12 nuclei, 20 - 25% of the nucleons have high-momentum $(k>k_{F})$. Fomin et al., PRI **Do ALL high-momentum nucleons come in** pairs? What kind of pairs? 3.1 ± 0.2 ±°′Au 2.2 I U.I ье L. Frankfurt et al. , Phys. Rev. C 48, 2451 (1993). O. Hen et al., PRC 85, 047301 (2012) K. Egiyan et al., Phys. Rev. C 68, 014313 (2003). K. Egiyan et al., PRL 96, 082501 (2006)





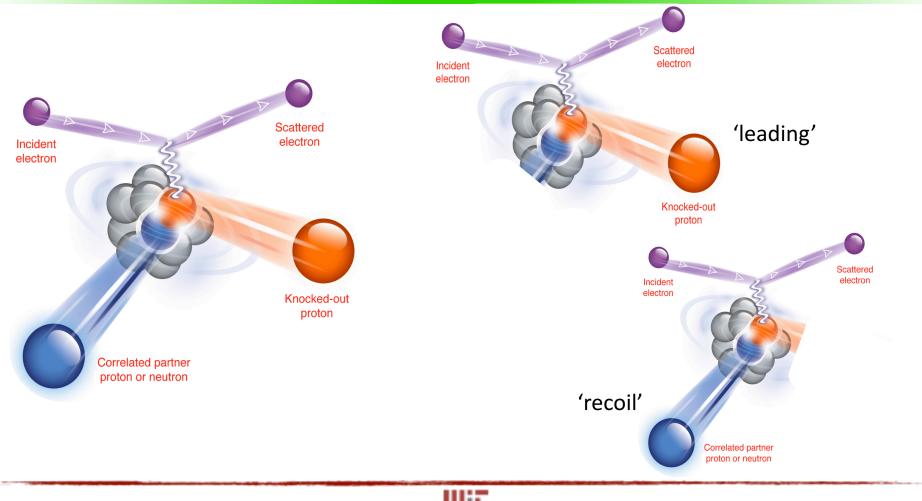
Breakup the pair => Detect both nucleons => Reconstruct 'initial' state



(semi) Exclusive 2N-SRC Studies



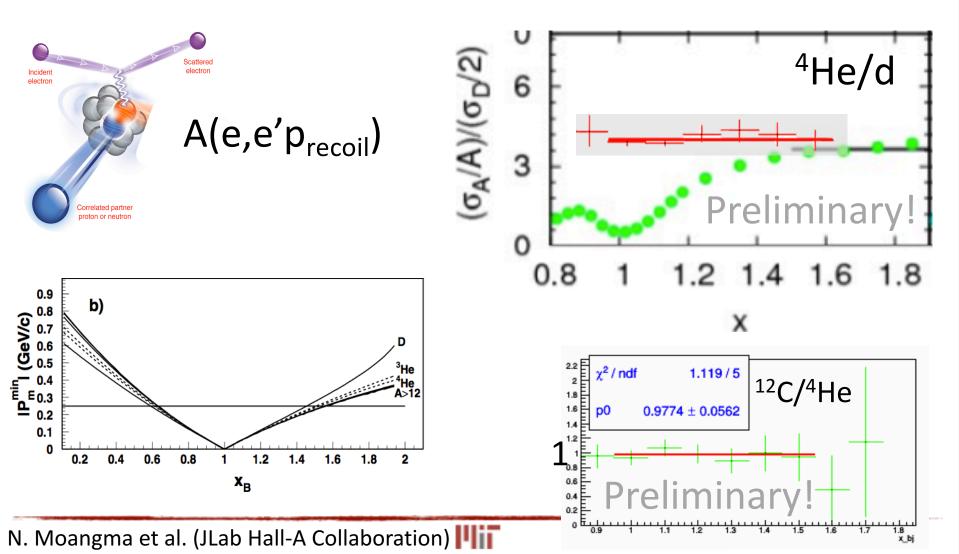
Breakup the pair => Detect both nucleons => Reconstruct 'initial' state

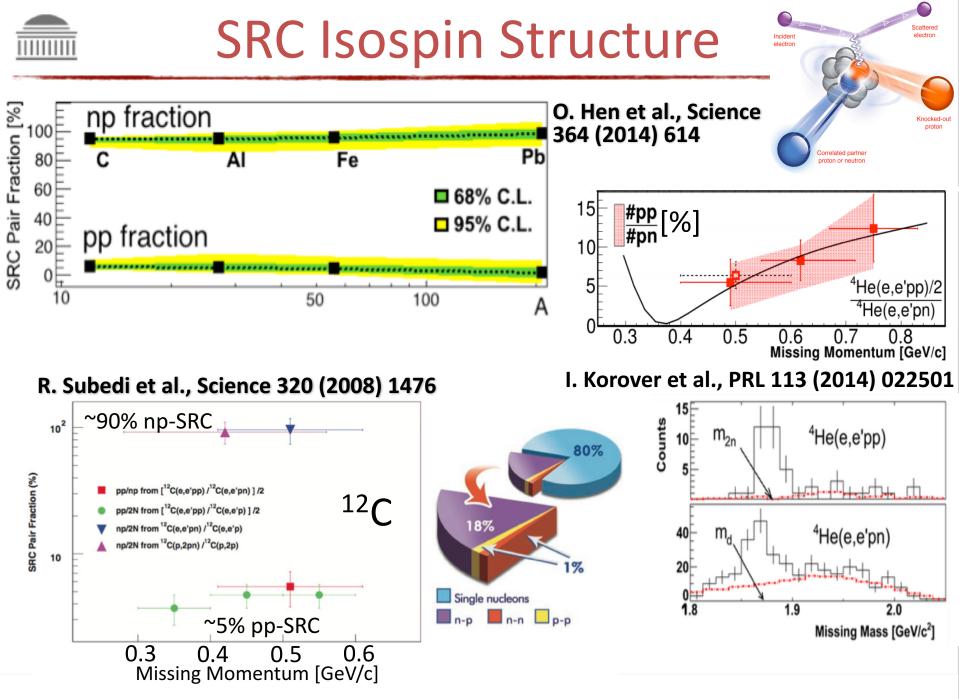






Counting SRCs using spectator tagging. [Can also be done @ Minerva? (I think so!)]





A. Tang et al., PRL (2003);

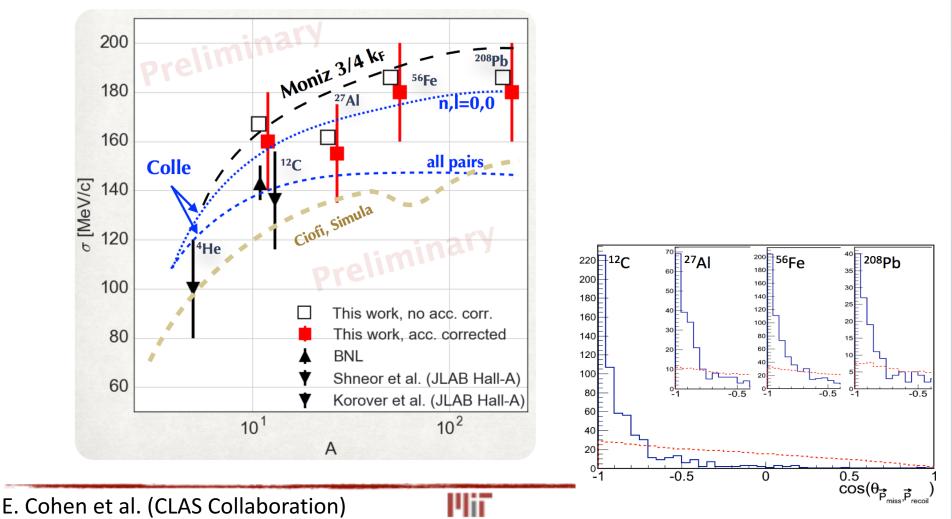
E. Piasetzky et al., PRL (2006);

R. Shneor et al., PRL (2007)





"... high relative momentum and <u>low c.m. momentum</u> compared to the Fermi momentum (k_F)"







Bottom Line:

• SRCs account for:

20% of the nucleons in nuclei.
~100% of the high-p (k>k_F) nucleons in nuclei.

- Predominantly due to np-SRC.
- Universal for A = 4 208 nuclei.
- <u>Tensor force</u> dominance at short distance.

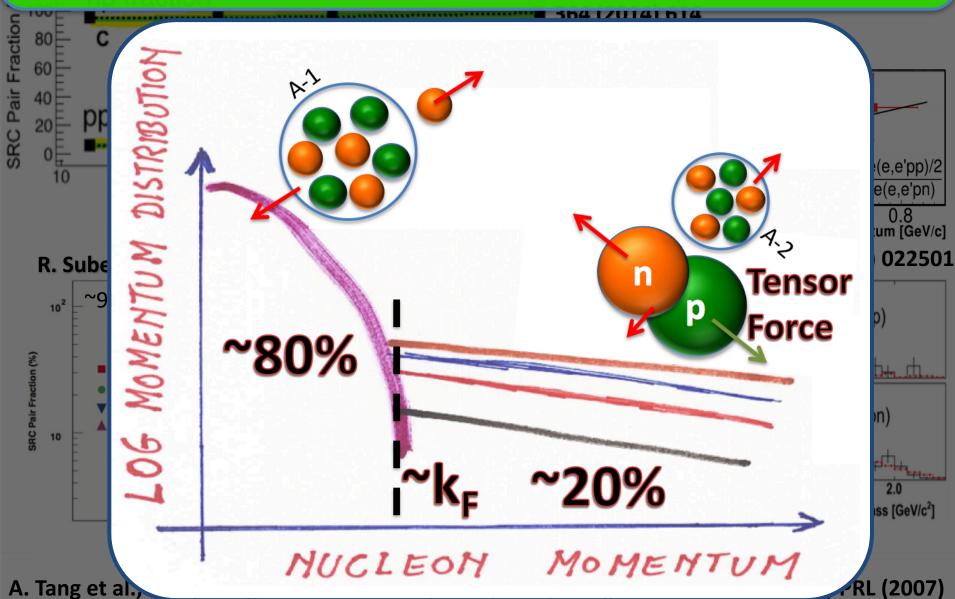
0.3 0.4 0.5 0.6 Missing Momentum [GeV/c]

A. Tang et al., PRL (2003); E.

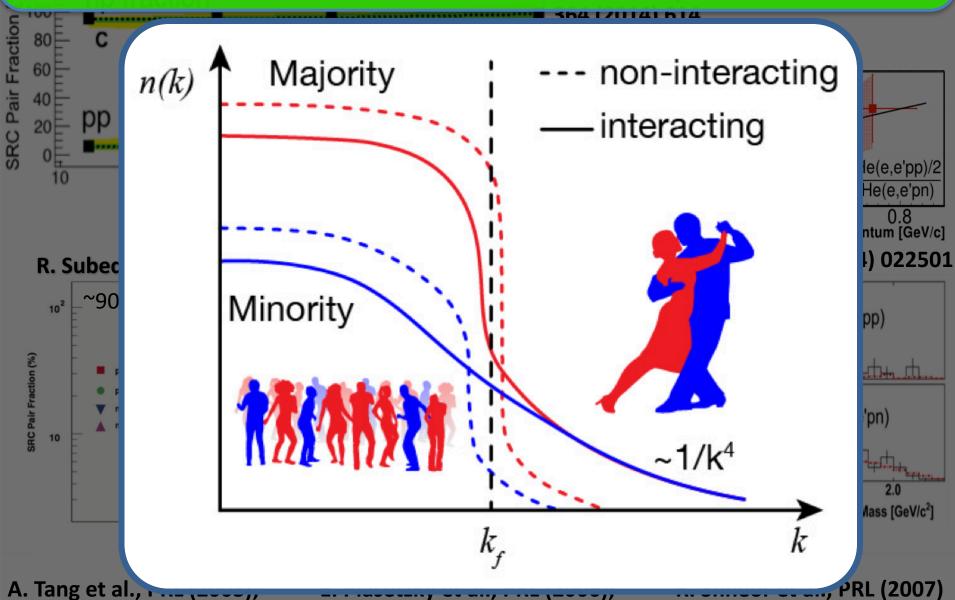
E. Piasetzky et al., PRL (2006);

R. Shneor et al., PRL (2007)

Universal structure of nuclear momentum distributions



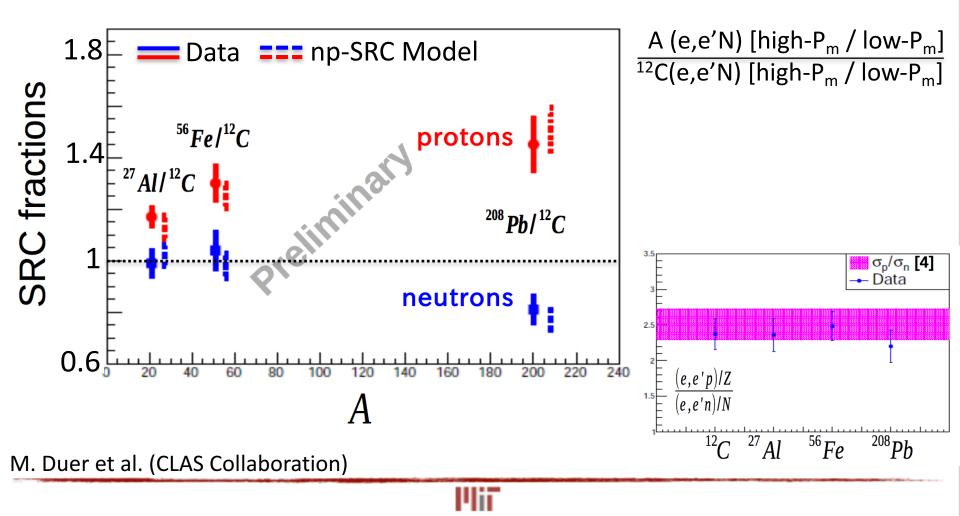
Universal structure of nuclear momentum distributions







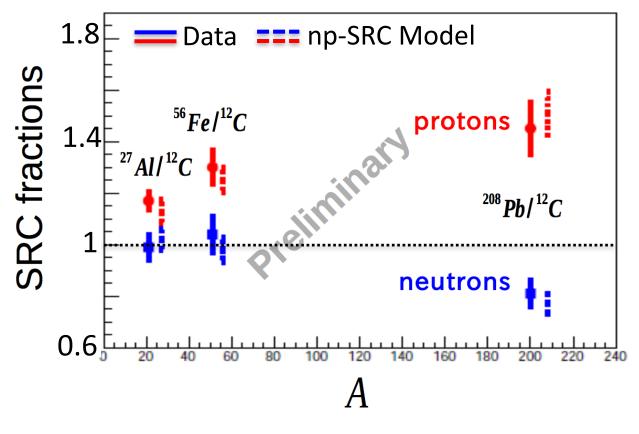
Fraction of high-momentum nucleons in asymmetric nuclei







Fraction of high-momentum nucleons in asymmetric nuclei



Protons in neutron rich nuclei have higher SRC probability!!

M. Duer et al. (CLAS Collaboration)



SRC 101

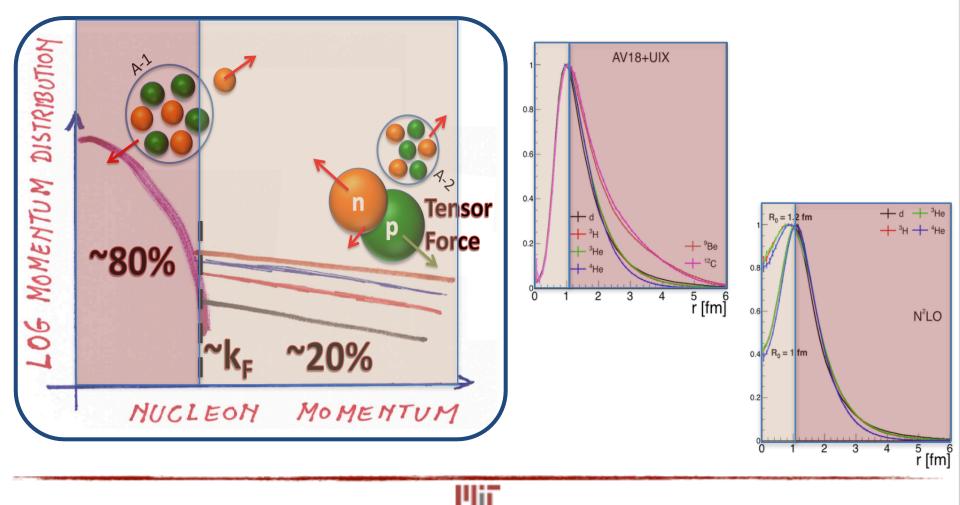


Consistent set of (e,e'), (e,e'p), (e,e'pN) and (p,2pn) measurements, on a variety of nuclei, allow quantifying SRCs with unprecedented accuracy!

- 1. SRC Exist in Nuclei (!) and account for:
 - ~ 20% of the nucleons in nuclei.
 - ~100% of the high-p (k>k_F) nucleons in nuclei.
- 2. Have large relative momentum and low c.m. momentum.
- 3. Predominantly due to np-SRC.
- 4. Universal for A = 4 208 nuclei.
- 5. np-SRC create a larger fraction of high-momentum protons in neutron rich nuclei!
- 6. <u>Tensor force</u> dominance at short distance.

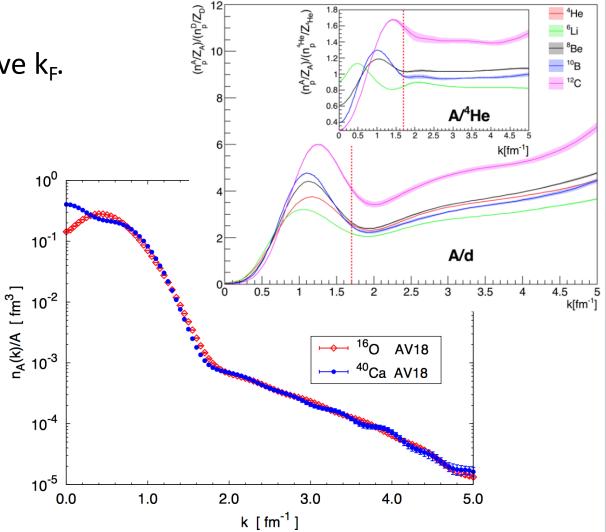


Can universality help describe the SRC phase of the nucleus in both coordinate and momentum space WITHOUT relaying on many-body calculations? (seems like the answer is YES)



Hints from Many-Body (VMC) ?

- One body momentum distribution scales above k_F.
- Good scaling relative to ⁴He NOT deuteron.
- ⇒ Importance of nondeuteron pairs? c.m. motion? Both?
- ⇒Why the (e,e') data for A/d scale but the calculations don't?





Two-component interacting Fermi systems

The contact term







Concept developed for dilute two-component Fermi systems with a short-range (δ -like) interaction.

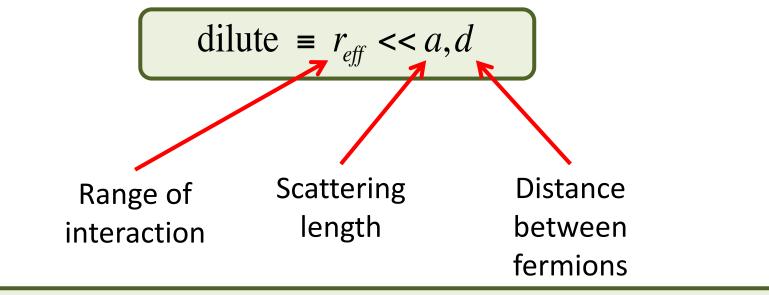
dilute =
$$r_{eff} << a, d$$

Dilute System

S. Tan Annals of Physics 323 (2008) 2952, ibid 2971, ibid 2987



Concept developed for dilute two-component Fermi systems with a short-range (δ -like) interaction.



Range of interaction much smaller than the other relevant length scales in the problem

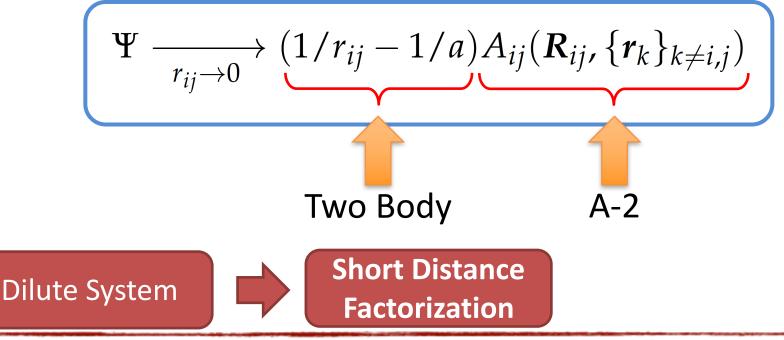


S. Tan Annals of Physics 323 (2008) 2952, ibid 2971, ibid 2987



Contact interaction is represented through a boundary condition

Imposing this B.C. on the Schrödinger equation yields an asymptotic wave function when two fermions get very close



S. Tan Annals of Physics 323 (2008) 2952, ibid 2971, ibid 2987

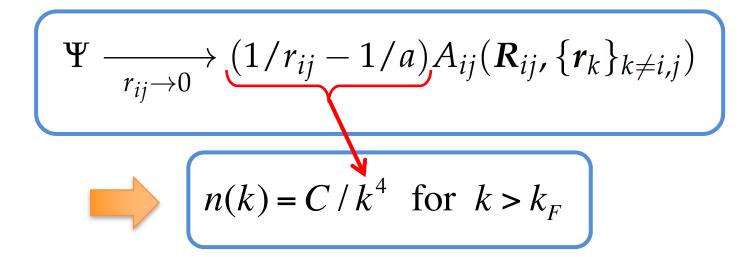


$$\Psi \xrightarrow{r_{ij} \to 0} (1/r_{ij} - 1/a) A_{ij}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j})$$

$$n(k) = C/k^4 \text{ for } k > k_F$$







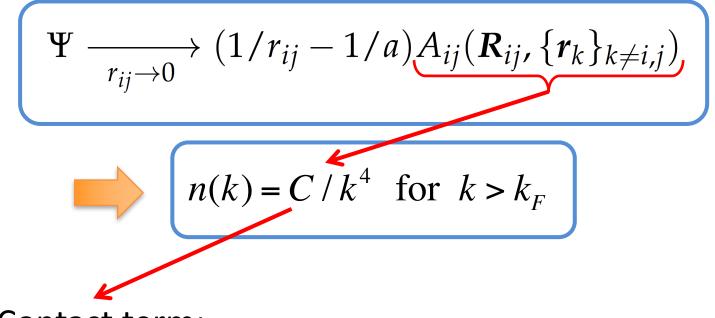




 $\Psi \xrightarrow[r_{ij} \to 0]{} (1/r_{ij} - 1/a) A_{ij}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j}),$ $n(k) = C / k^4$ for $k > k_F$







Tan's Contact term:

- 1. Measures the number of SRC different fermion pairs.
- 2. Determines the thermodynamics through a series of universal relations.

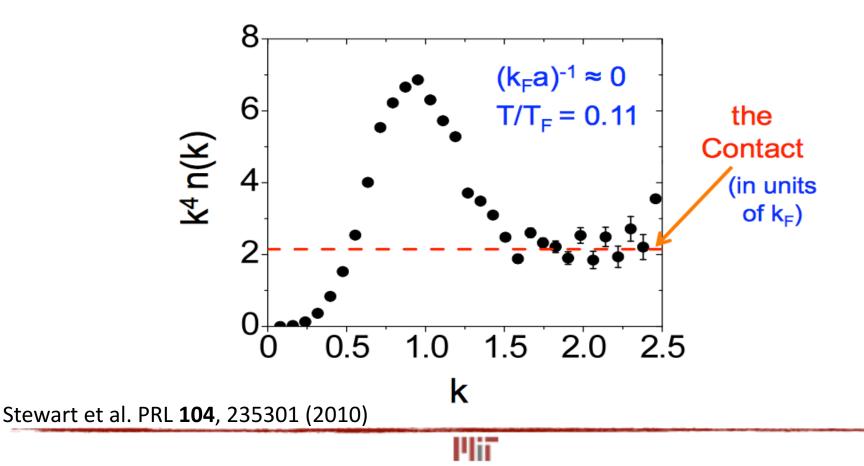






Two spin-state mixtures of ultra-cold ⁴⁰K and ⁶Li atomic gas systems.

=> extracted the contact and verified the universal relations



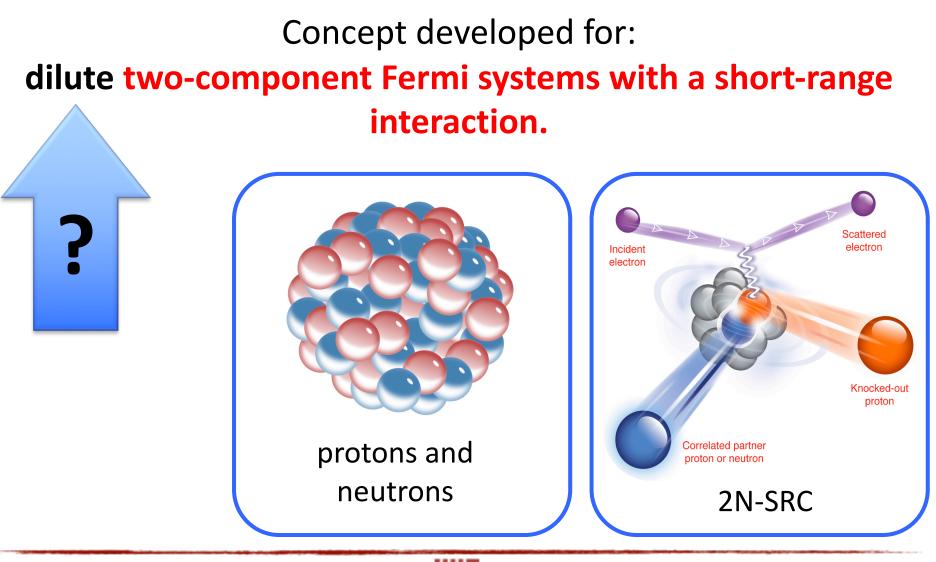




What About a *Nuclear* Contact ?

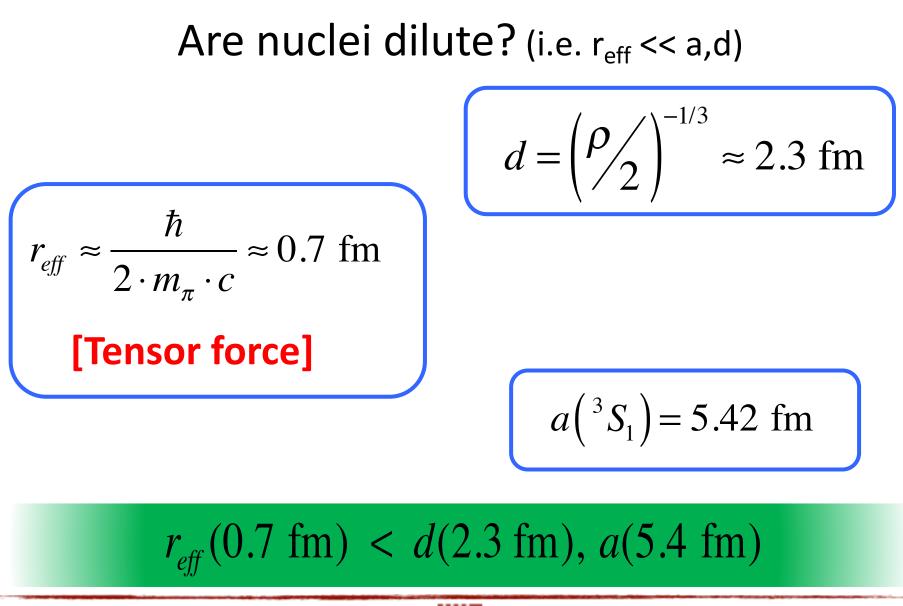






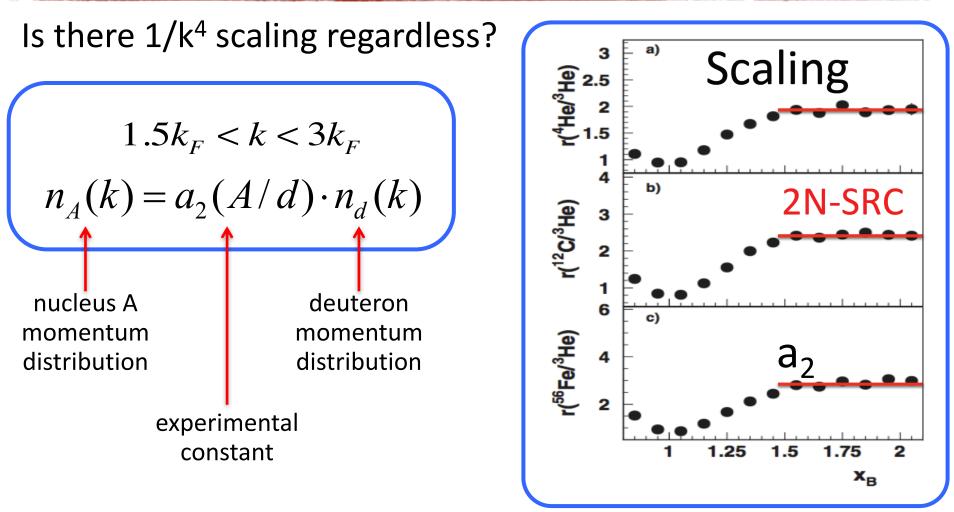




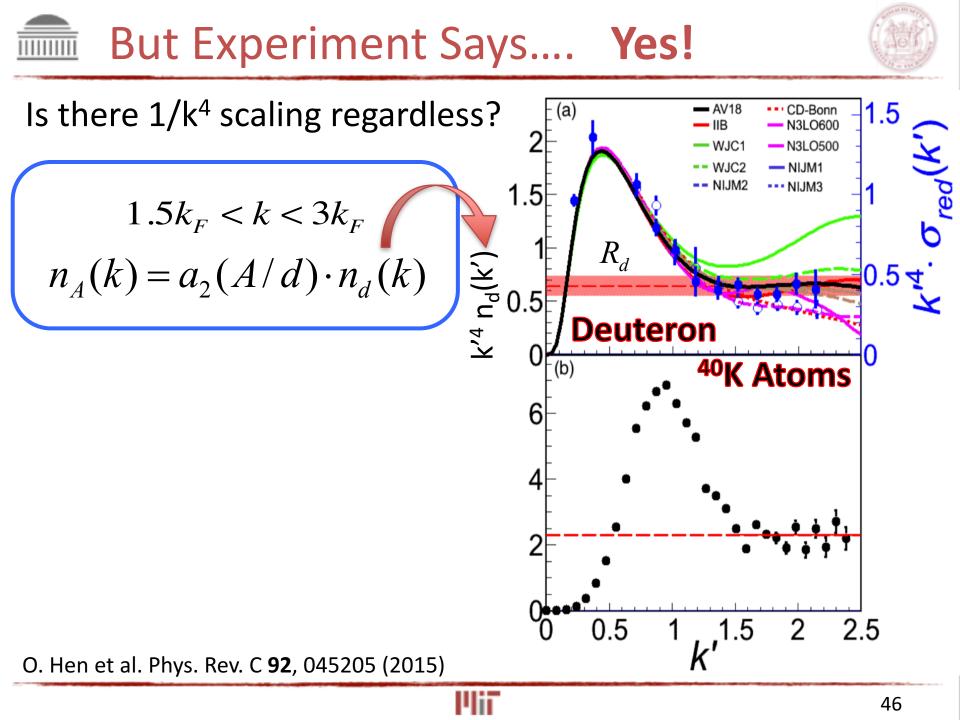


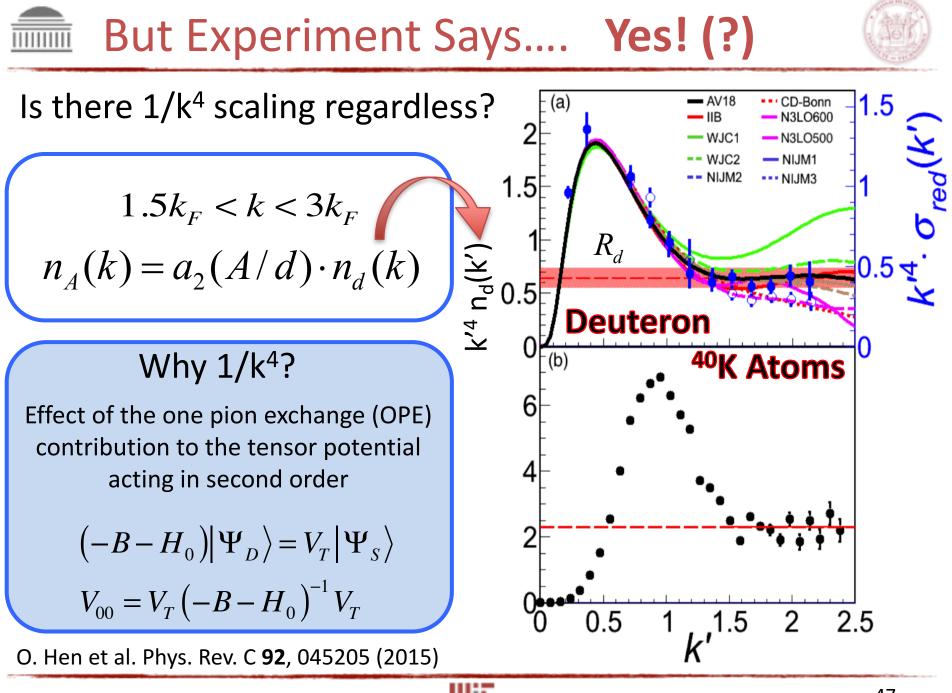
But Experiment Says....

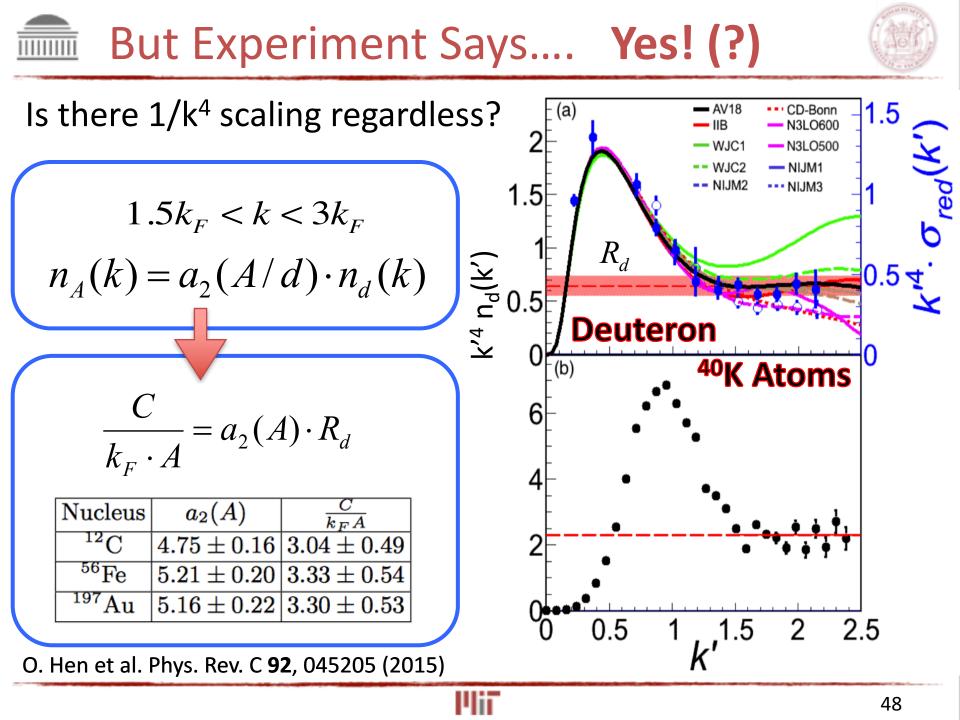




The momentum distribution of nucleons in medium to heavy nuclei is proportional to that of deuteron at high momenta.



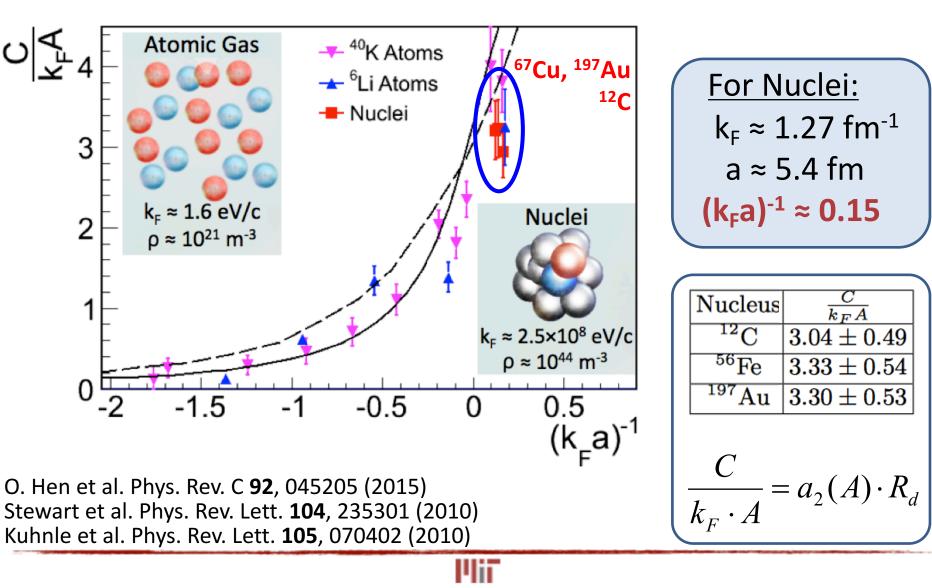




Comparing with atomic systems



Equal contacts for equal interactions strength!





How can we reconcile the experimental observation with theory expectation?

i.e. is there a region in which the nuclear wave function fully factorizes?





Going Back to the Theory...

- 1. Generalize the contact formalism to nuclear systems.
- 2. Use it to make specific predictions of nuclear properties.
- 3. Check using experimental data and full many-body calculations.



Issue: Scale separation does not necessarily work in nuclear systems.

Solution: assume a more general form for the 2-body wavefunction.

Atomic System:

$$\Psi \longrightarrow (1/r_{ij} - 1/a) A_{ij}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j})$$

Nuclear System:

$$\Psi \longrightarrow (\boldsymbol{\varphi}(\boldsymbol{r})_{ij}) A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j})$$





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Nuclear System:

$$\Psi \longrightarrow (\boldsymbol{\varphi}(\boldsymbol{r})_{ij}) A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j})$$

known solution for the two-body (nuclear) problem





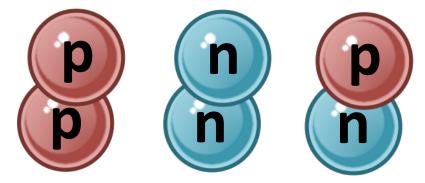
Consider the factorized wave function:

$$\Psi \longrightarrow_{r_{ij}\to 0} \sum_{\alpha} \varphi_{\alpha}(\mathbf{r}_{ij}) A^{\alpha}_{ij}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k\neq i,j})$$

In nuclear physics we have 3 possible types of pairs:



For each pair we have different channels





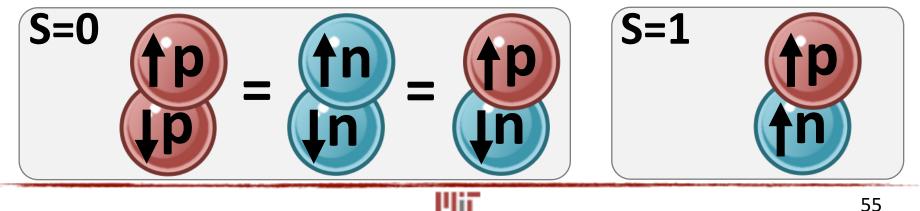


Consider the factorized wave function:

$$\Psi \longrightarrow_{r_{ij}\to 0} \sum_{\alpha} \varphi_{\alpha}(\mathbf{r}_{ij}) A_{ij}^{\alpha}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k\neq i,j})$$

In nuclear physics we have 3 possible types of pairs: ij = {pp, nn, pn} For each pair we have different channels α = (s,l)jm

Reduced to 2 contacts by imposing L=0 and symmetry considerations

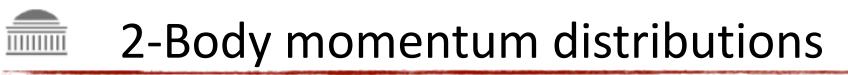


Relating to Momentum Space

$$\begin{aligned}
\Psi \longrightarrow \sum_{\alpha} \varphi_{\alpha}(\mathbf{r}_{ij}) A_{ij}^{\alpha}(\mathbf{R}_{ij}, \{\mathbf{r}_{k}\}_{k \neq i,j}) \\
\Psi \longrightarrow \sum_{\alpha} \varphi_{\alpha}(\mathbf{r}_{ij}) A_{ij}^{\alpha}(\mathbf{R}_{ij}, \{\mathbf{r}_{k}\}_{k \neq i,j}) \\
\end{aligned}$$
One Body:

$$\begin{aligned}
n_{p}(\mathbf{k}) &= \sum_{\alpha} \left| \widetilde{\varphi}_{pp}^{\alpha}(\mathbf{k}) \right|^{2} 2C_{pp}^{\alpha} + \sum_{\alpha} \left| \widetilde{\varphi}_{pn}^{\alpha}(\mathbf{k}) \right|^{2} C_{pn}^{\alpha}
\end{aligned}$$

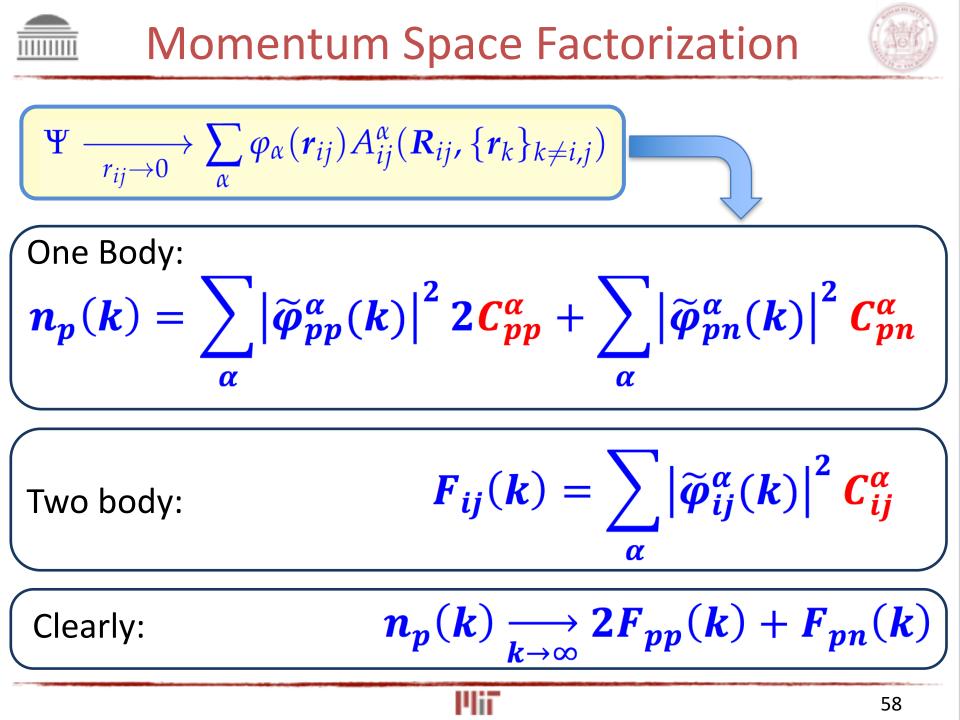
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- <u>One Body momentum distribution [n_N(k)]:</u>
 Probability to find a nucleon, N, in the nucleus with momentum k.

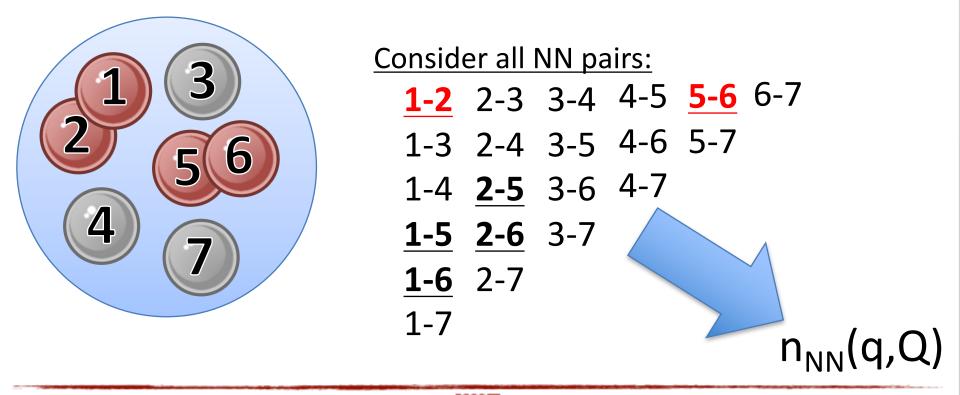
<u>Two Body momentum distribution [n_{NN}(q,Q)]:</u>
 Probability to find a NN pair in the nucleus with relative (c.m.) momentum q (Q).

n_{NN}(q,Q) – computational Frontier!



Two-Body Momentum Distributions

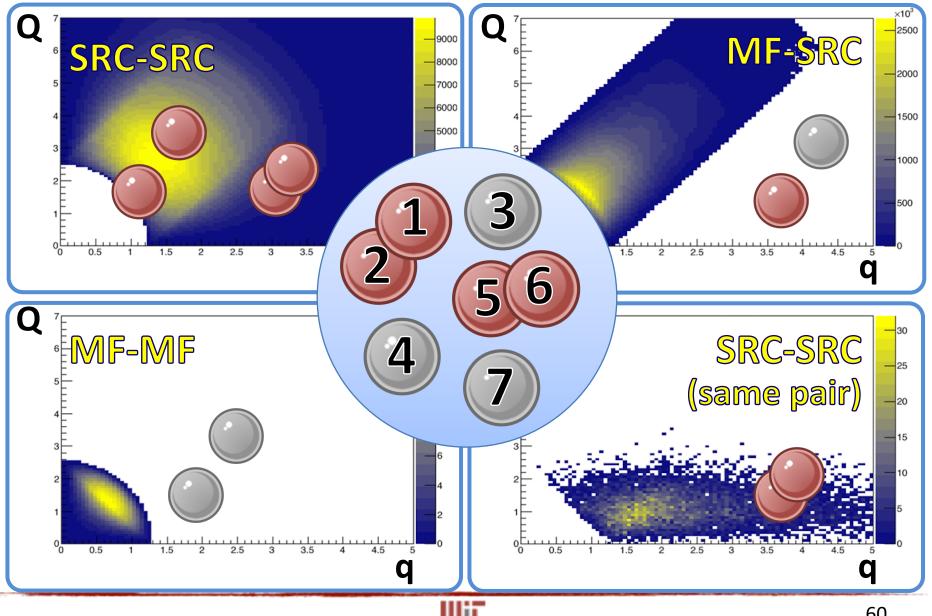
 n_{NN}(q,Q) – Mathematical object that counts all possible NN pairs, regardless of their state:





Toy model to the rescue

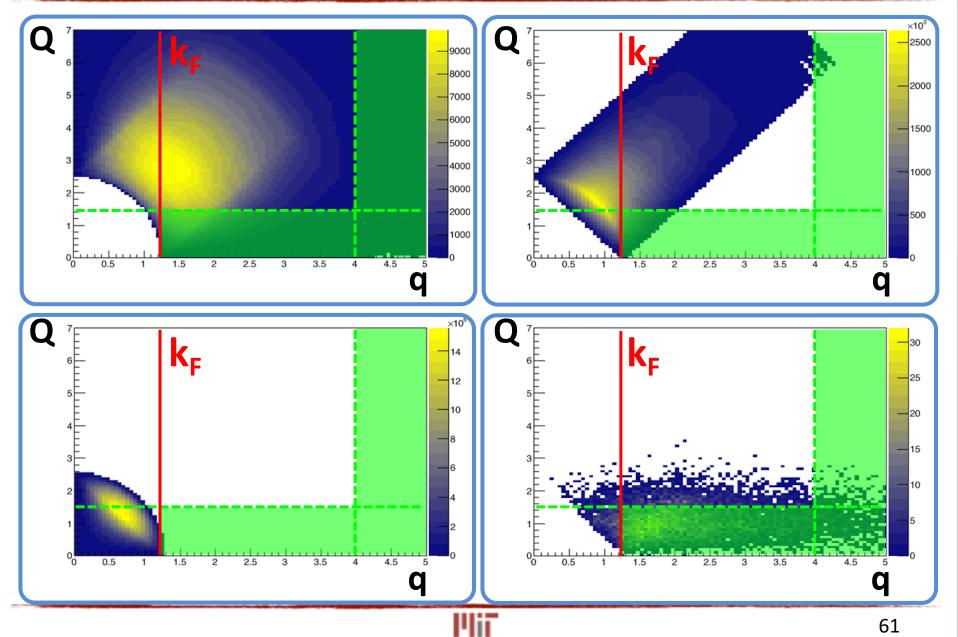






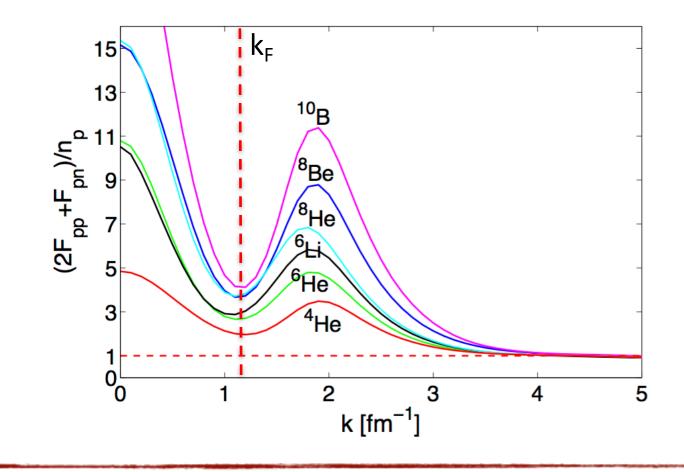
Toy model to the rescue







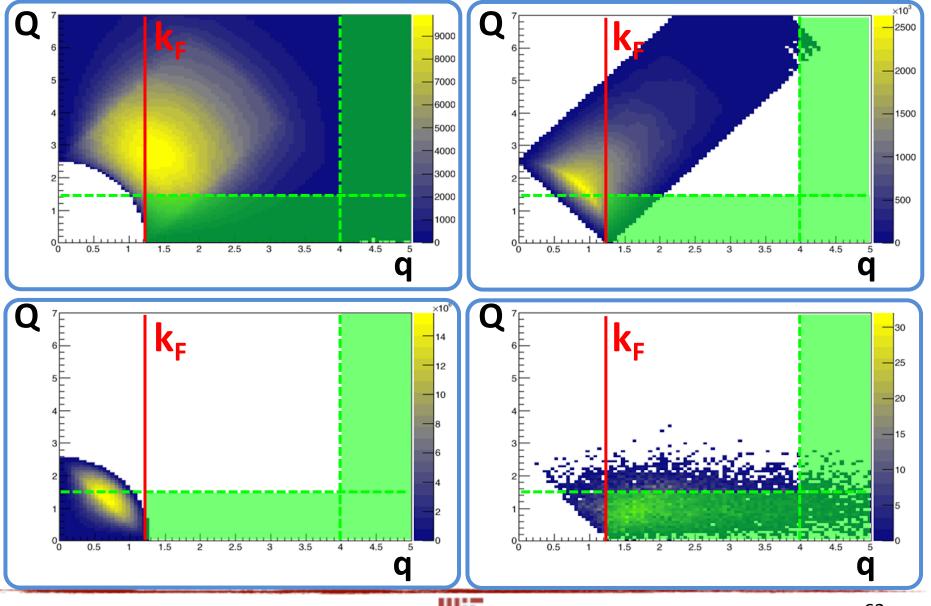
 Weiss and Barnea (PRC 2015): contact interactions dominate when n_{pn}(q)+2n_{pp}(q) = n_p(k)





Toy model to the rescue

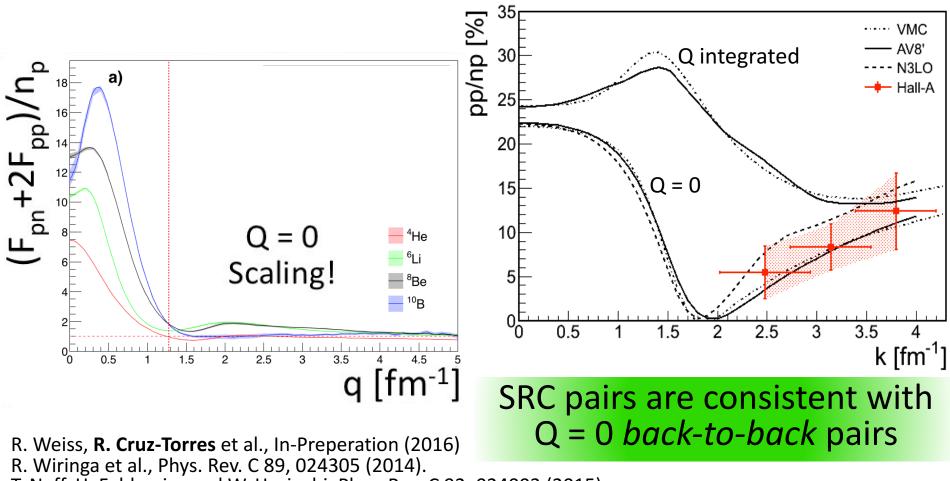




Two-Body Scaling for Q=0



Restricting Q=0 restores scaling starting from $k > k_F$ AND gives consistent results with experimental data!

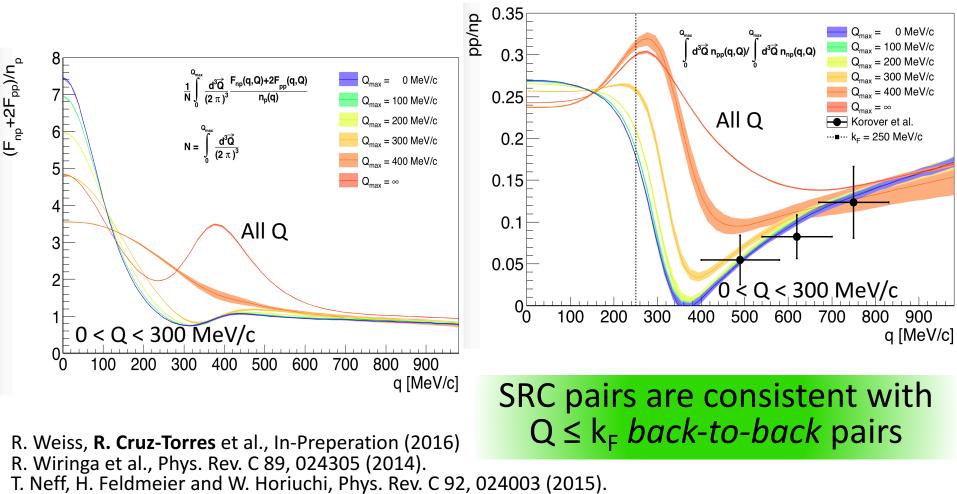


T. Neff, H. Feldmeier and W. Horiuchi, Phys. Rev. C 92, 024003 (2015). I. Korover, N. Muangma, and O. Hen et al., Phys. Rev. Lett 113, 022501 (2014).

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Two-Body Scaling for Low Q

- Restricting Q=0 restores scaling starting from $k > k_F$ AND
- gives consistent results with experimental data!



I. Korover, N. Muangma, and O. Hen et al., Phys. Rev. Lett 113, 022501 (2014).

Extracting the nuclear contact(s)

3:44 PM Contacts John Appleseed Kate Bell NUCLEAR Linda Napp David Taylor Hank M. Zakroff n



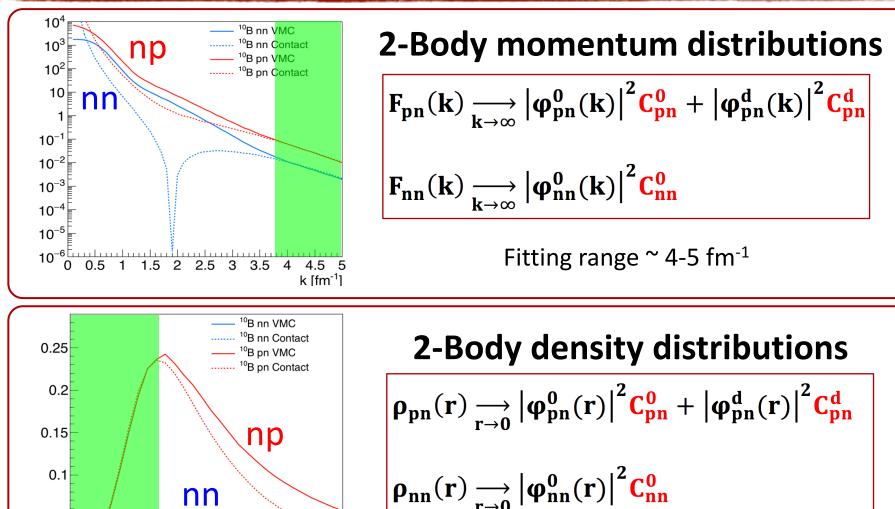
0.05

0⊾ 0

0.5

Extracting the Contacts





Fitting range ~ 0.25-1 fm

R. B. Wiringa et al., PRC 89, no. 2, 024305 (2014).

1.5

2

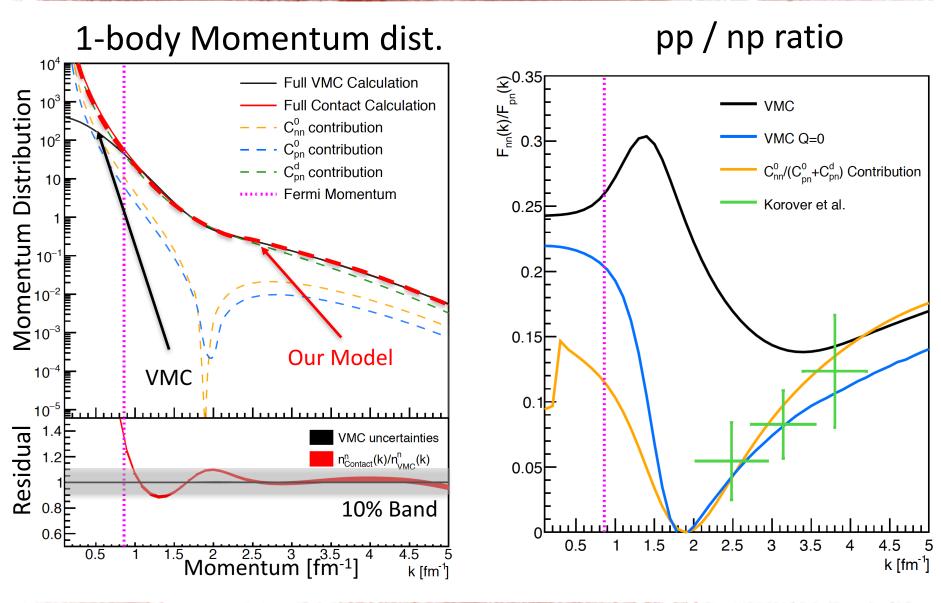
2.5

3 r [fm]



⁴He Results

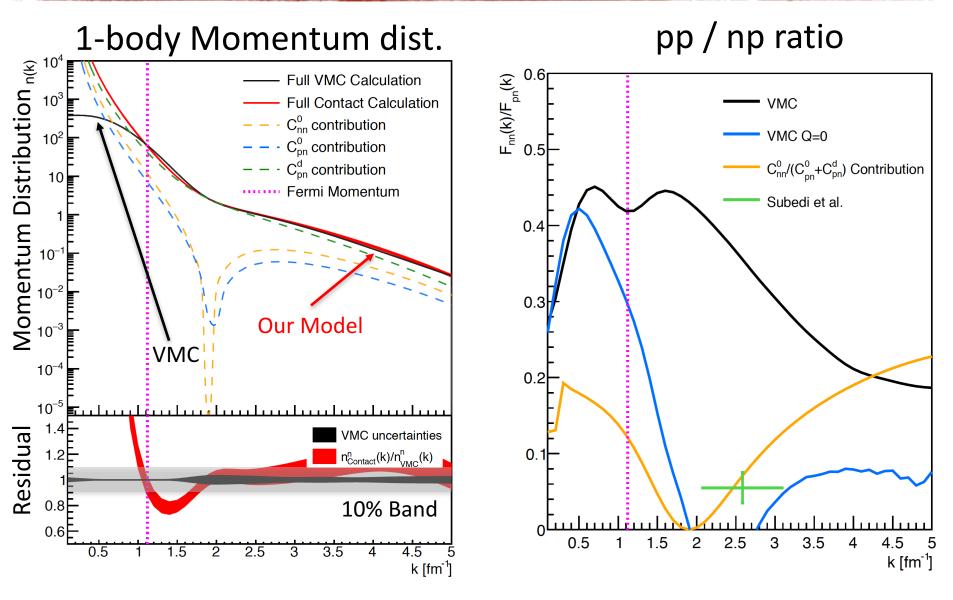






¹²C Results









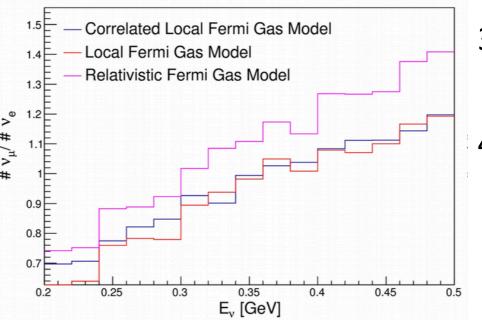
Consistent nuclear contacts extracted from:

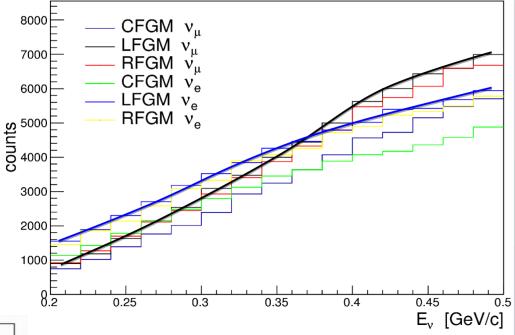
- (1) Experimental data,
- (2) many-body *momentum* space calculations,
- (3) many-body *coordinate* space calculations.

Α	k-space				r-space			
	$C_{pn}^{s=1}$	$C_{pn}^{s=0}$	$C_{nn}^{s=0}$	$C^{s=0}_{pp}$	$C_{pn}^{s=1}$	$C_{pn}^{s=0}$	$C_{nn}^{s=0}$	$C_{pp}^{s=0}$
4 He	$12.3{\pm}0.1$	$0.69{\pm}0.03$	$0.65{\pm}0.03$		$11.61{\pm}0.03$	$0.567{\pm}0.004$		
	$14.9{\pm}0.7~(\exp)$	$0.8{\pm}0.2~({ m exp})$			11.01±0.00	0.001±0.004		
⁶ Li	$10.5{\pm}0.1$	$0.53{\pm}0.05$	$0.49{\pm}0.03$		$10.14{\pm}0.04$	$0.415{\pm}0.004$		
⁷ Li	10.6 ± 0.1	0.71 ± 0.06	0.78 ± 0.04	0.44 ± 0.03	9.0 ± 2.0	0.6 ± 0.4	0.647 ± 0.004	0.350 ± 0.004
⁸ Be	$13.2{\pm}0.2$	$0.86{\pm}0.09$	$0.79{\pm}0.07$		$12.0{\pm}0.1$	$0.603{\pm}0.003$		
⁹ Be	$12.3{\pm}0.2$	$0.90{\pm}0.10$	$0.84{\pm}0.07$	$0.69{\pm}0.06$	$10.0{\pm}3.0$	$0.7{\pm}0.7$	$0.65{\pm}0.02$	$0.524{\pm}0.005$
$^{10}\mathbf{B}$	$11.7{\pm}0.2$	$0.89{\pm}0.09$	$0.79 {\pm} 0.06$		$10.7{\pm}0.2$	$0.57{\pm}0.02$		
12C	$16.8{\pm}0.8$	$1.4{\pm}0.2$	$1.3{\pm}0.2$		$14.9{\pm}0.1$	$0.83{\pm}0.01$		
	$18\pm2 \text{ (exp)}$		$1.5\pm0.5~(\exp)$)	14.0±0.1	0.00±0.01		

Relevance To Neutrino Scattering?

- Initial implementation of SRCs in GENIE's Local Fermi-Gas Model.
- Examination of model dependence of 1-lepton 1proton CCQE predictions





- 3. Correlation effects reduced for v_{μ} / v_e ratios!
- 4. Next step: contact formalism implementation and then using spectral and decay functions.

Relevance To Neutrino Scattering?

8000

7000

6000

\$1000 stunos

3000

2000

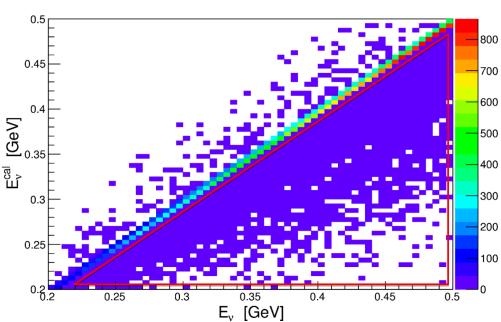
1000

CFGM

0.25

0.3

- Initial implementation of SRCs in GENIE's Local Fermi-Gas Model.
- Examination of model dependence of 1-lepton 1proton CCQE predictions



Calorimetric energy reconstruction works well for parts of the phase-space.

0.35

0.4

0.45

E_v [GeV/c]

Should we focus on these parts? uB data forthcoming!



The Correlations group



MIT (Or Hen):





Reynier Torres



Efrain Segarra



Afroditi Papadopoulou



Axel Schmidt



George Laskaris



Maria Patsyuk



Taofeng Wang (*visiting scientist)

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Theory Collaborators (lots!)

Contact works: G. Miller, N. Barnea and R. Weiss



Questions?

Thank You!