

RELATIVISTIC GREEN'S FUNCTION MODEL

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OPTICAL POTENTIAL

The OP provides a suitable framework to describe elastic nucleon-nucleus scattering

Its use can be extended to calculate the cross section of a wide variety of nuclear reactions, for instance, to describe FSI in the exclusive QE ($e, e'p$) knockout reaction

RGF: the OP is used to describe FSI in the inclusive QE (e, e') reaction. Different but consistent treatment of FSI in the exclusive and inclusive electron scattering

OPTICAL POTENTIAL

In elastic nucleon-nucleus scattering the imaginary part of the OP accounts for the flux lost in the elastically scattering beam toward open inelastic channels

The RGF formalism can translate the flux lost toward inelastic channels (imaginary part of the OP) into the strength observed in inclusive reactions.

OP powerful tool to include, in a relatively simple and somewhat model-independent way, important contributions not included in other FSI models based on the impulse approximation.

In principle the OP would require the solution of the full many-body nuclear problem ... but the availability of phenomenological OP's makes our calculations feasible and allows us to include inelastic contributions in a simple phenomenological way

RGF successful in the description of data..... BUT

OPTICAL POTENTIAL

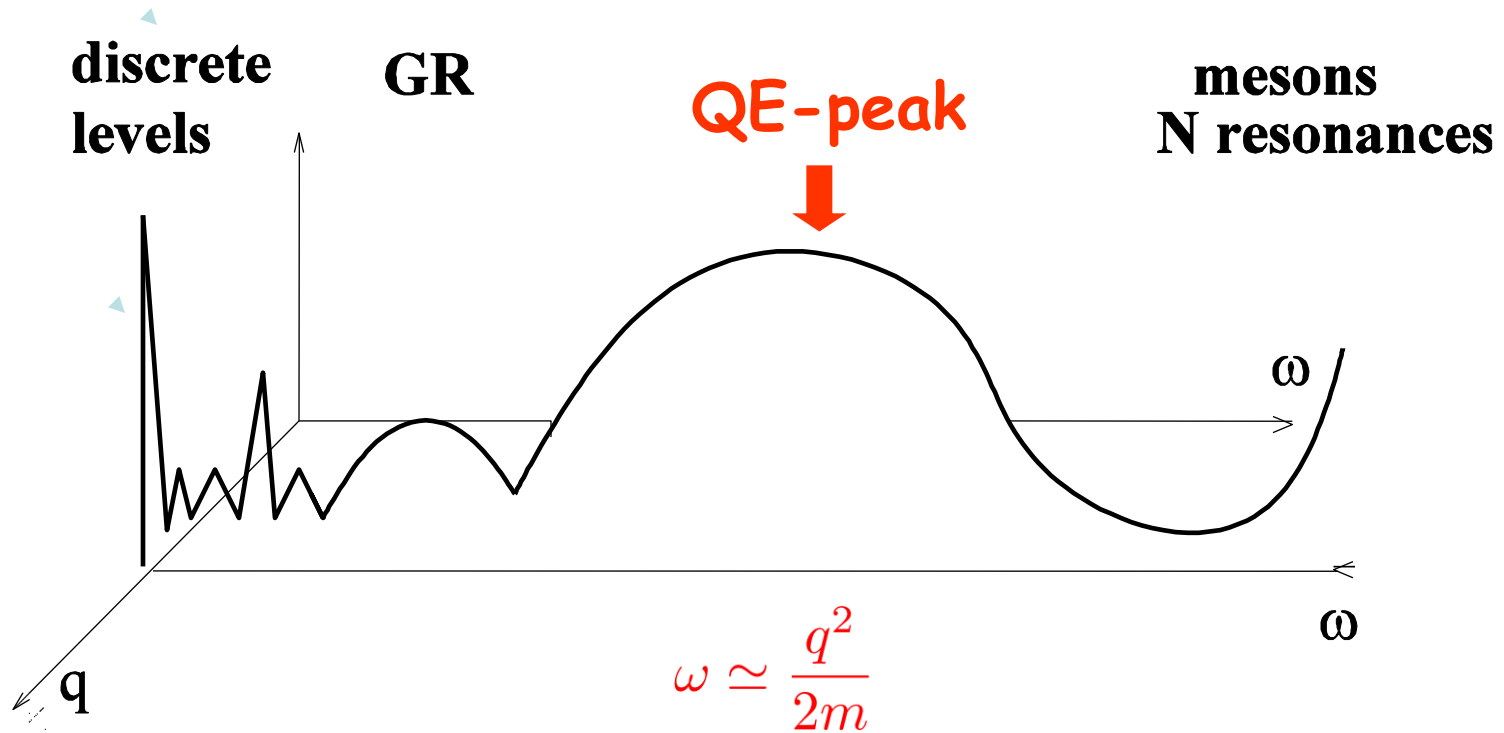
BUT there are some caveats

The use of a phenomenological OP does not allow us to disentangle and evaluate the role of a specific inelastic contribution

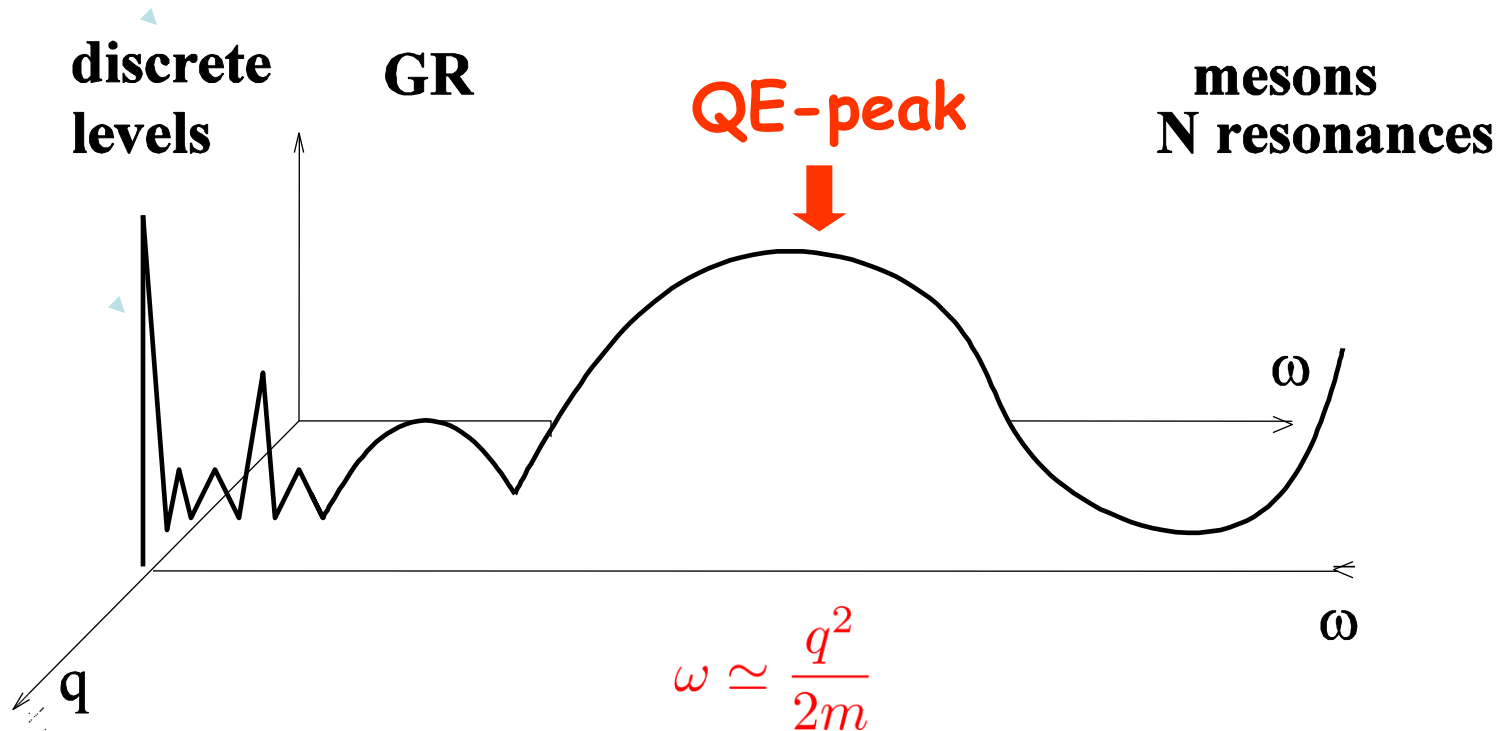
Available proton-nucleus scattering data do not completely constrain the shape and size of the OP

Different OP's are available, equivalent in the description of proton-nucleus scattering data, but with different imaginary parts, give different inelastic contributions in RGF calculations and produce theoretical uncertainties on the predictions of the RGF model

nuclear response to the electroweak probe

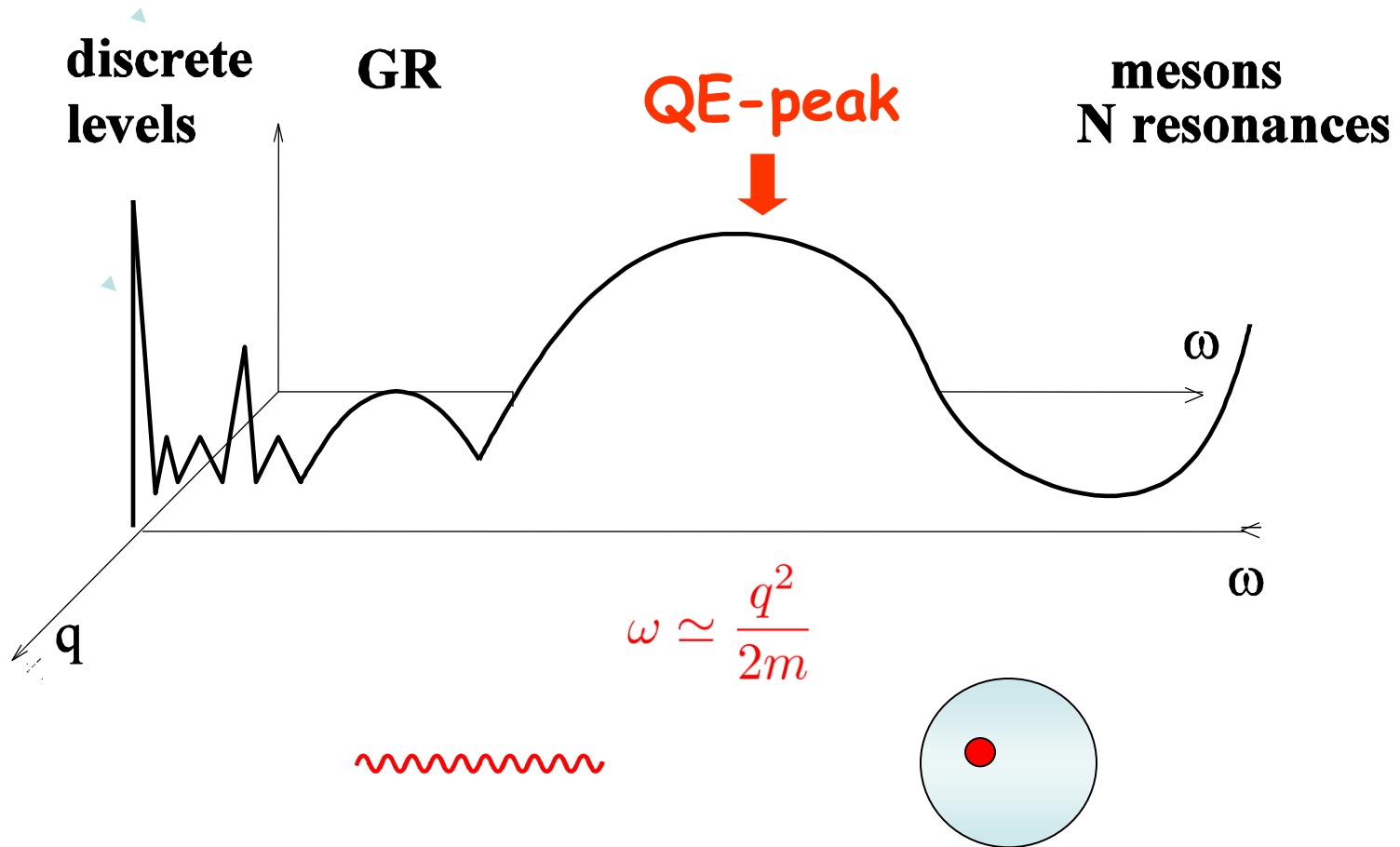


nuclear response to the electroweak probe



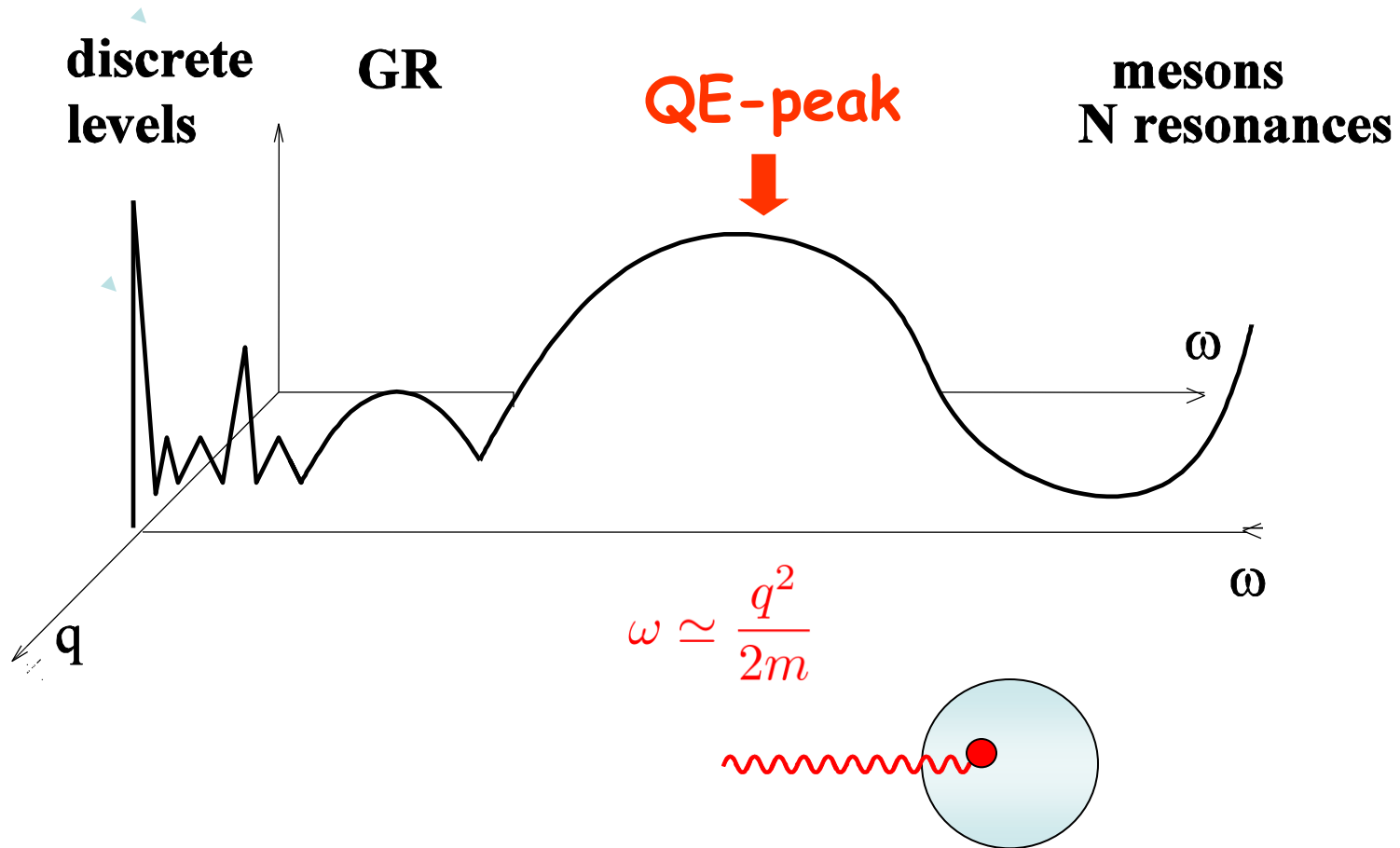
QE-peak dominated by one-nucleon knockout

nuclear response to the electroweak probe



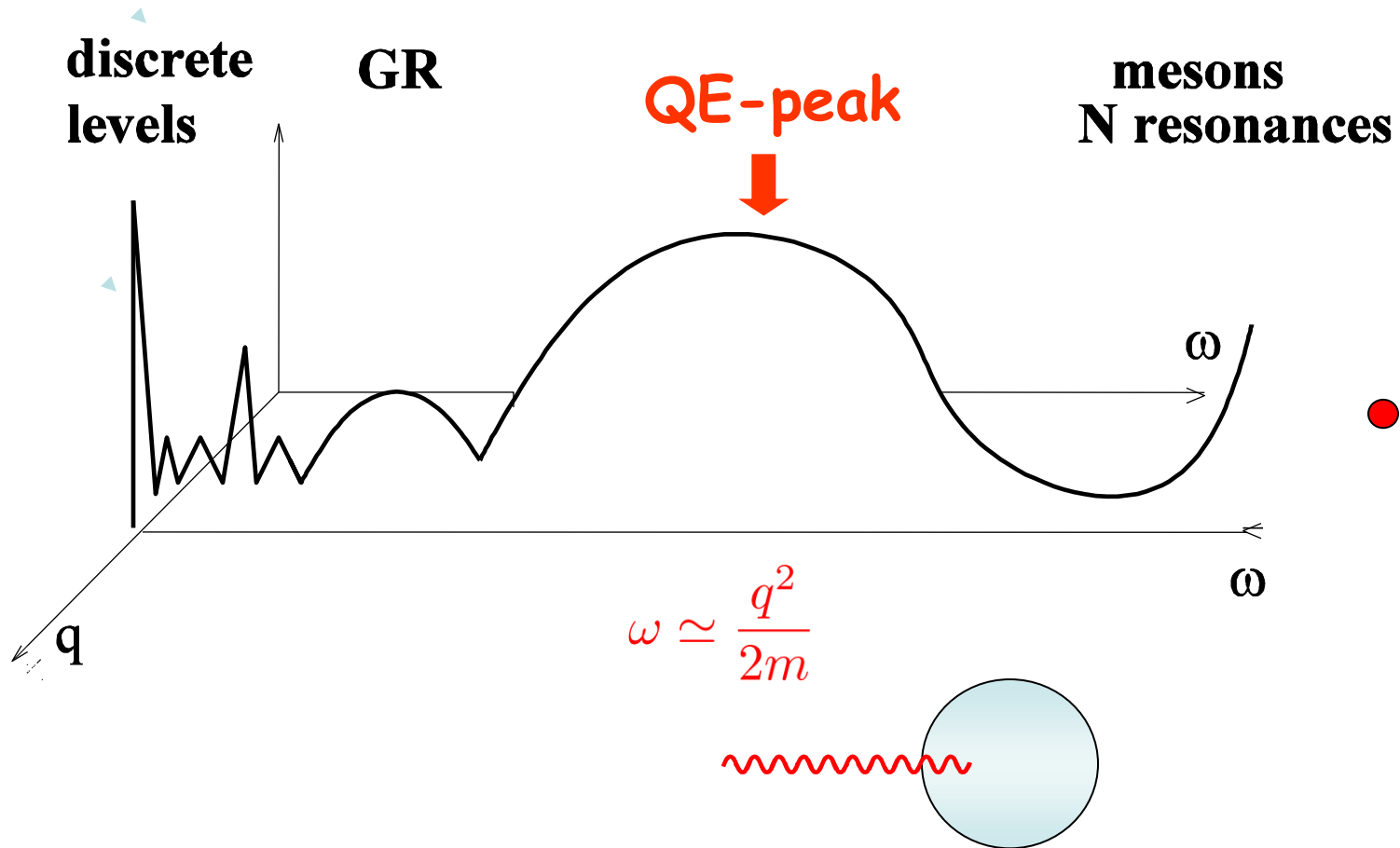
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nuclear response to the electroweak probe



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QE e-nucleus scattering

$$e + A \Rightarrow e' + N + (A - 1)$$

- both e' and N detected $(A-1)$ discrete eigenstate n exclusive $(e,e'p)$

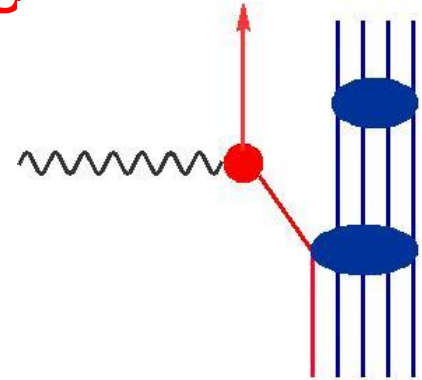
QE e-nucleus scattering

$$e + A \Rightarrow e' + N + (A - 1)$$

- both e' and N detected ($A-1$) discrete eigenstate n **exclusive** ($e,e'p$)
- only e' detected, all final nuclear states included **inclusive** (e,e')

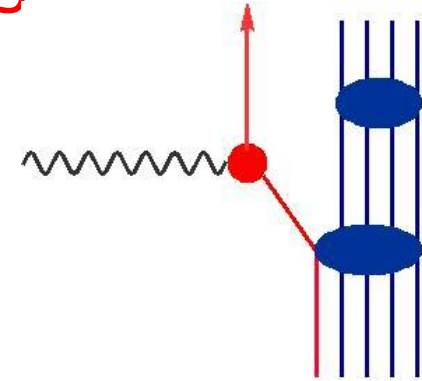
IMPULSE APPROXIMATION

- ✱ EXCLUSIVE SCATTERING: interaction through a 1-body current on a quasi-free nucleon, direct 1NKO

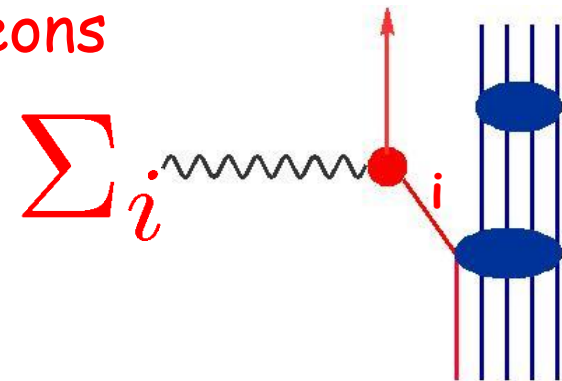


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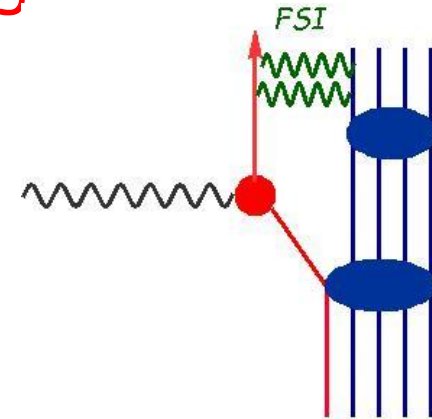


- INCLUSIVE SCATTERING: c.s given by the sum of integrated direct 1NKO over all the nucleons

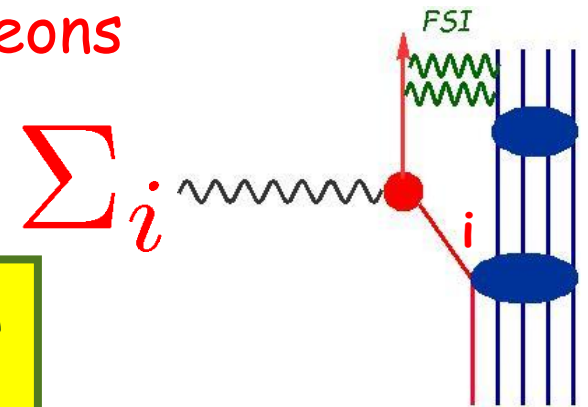


IMPULSE APPROXIMATION

- EXCLUSIVE SCATTERING: interaction through a 1-body current on a quasi-free nucleon, direct 1NKO



- INCLUSIVE SCATTERING: c.s given by the sum of integrated direct 1NKO over all the nucleons



FINAL-STATE INTERACTION between the emitted nucleon and the residual nucleus

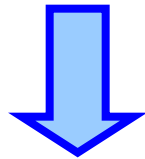
EXCLUSIVE SCATTERING: FSI

RDWIA

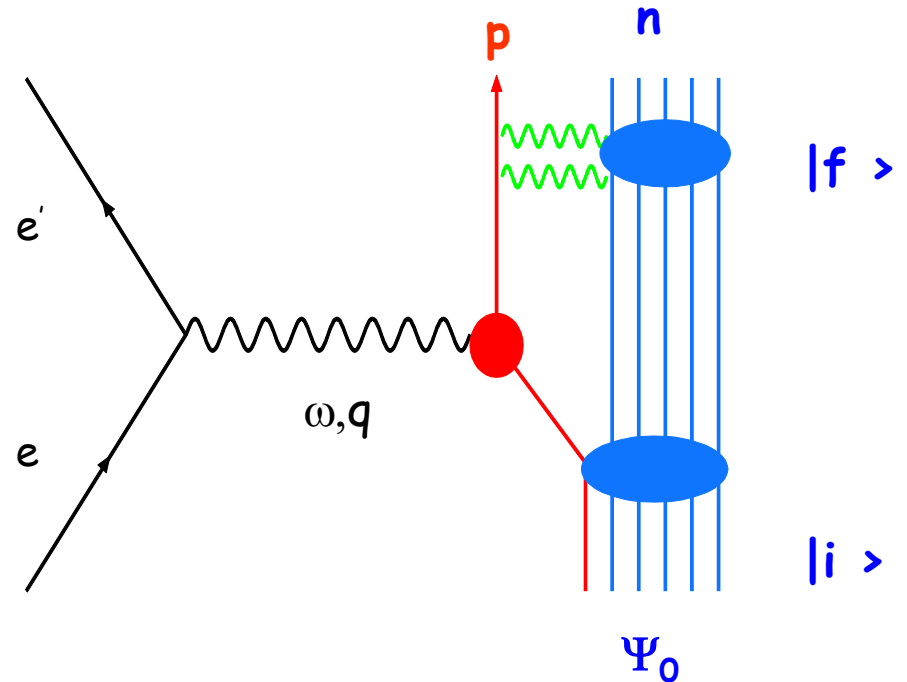
FSI described by a complex OP with an imaginary absorptive part. The imaginary part gives a reduction of the calculated c.s. which is essential to reproduce data

DWIA ($e, e'p$)

- ☀ exclusive reaction: n
- ☀ DKO mechanism: the probe interacts through a one-body current with one nucleon which is then emitted the remaining nucleons are spectators



$$\langle f | J^\mu(\mathbf{q}) | i \rangle \longrightarrow \lambda_n^{1/2} \langle \chi_{\mathbf{p}}^{(-)} | j^\mu(\mathbf{q}) | \phi_n \rangle$$



Direct knockout DWIA (e,e'p)

$$\lambda_n^{1/2} \langle \chi^{(-)} | j^\mu | \phi_n \rangle$$

- j^μ one-body nuclear current
- ϕ_n s.p. bound state overlap function
- λ_n spectroscopic factor
- $\chi^{(-)}$ s.p. scattering w.f. eigenfunction of an OP

Direct knockout DWIA (e,e'p)

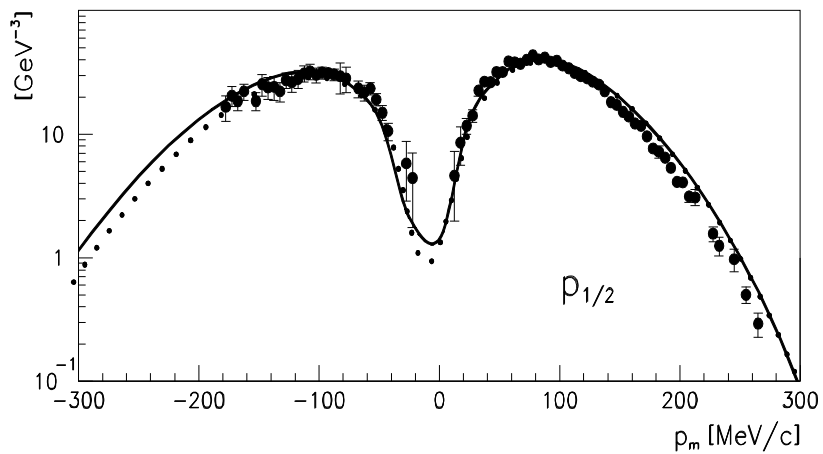
$$\lambda_n^{1/2} \langle \chi^{(-)} | j^\mu | \phi_n \rangle$$

- j^μ one-body nuclear current
- ϕ_n s.p. bound state overlap function
- λ_n spectroscopic factor
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DWIA and RDWIA: excellent description of (e,e'p) data

DWIA - RDWIA

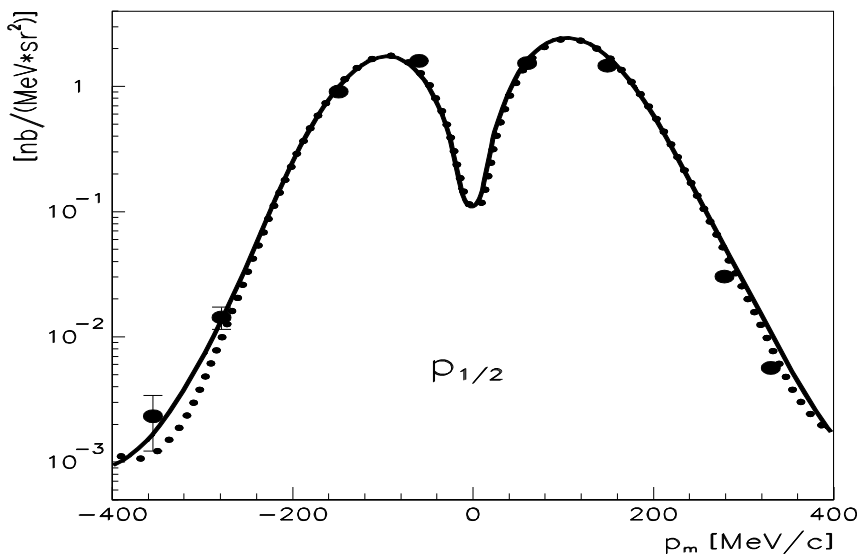
$^{16}\text{O}(e,e'p)$



NIKHEF parallel kin $E_0 = 520$ MeV $T_p = 90$ MeV

— rel RDWIA $\lambda_n = 0.7$
••••• nonrel DWIA $\lambda_n = 0.65$

JLab (ω, q) const kin $E_0 = 2445$ MeV $\omega = 439$ MeV $T_p = 435$ MeV



— RDWIA diff opt.pot. $\lambda_n = 0.7$
•••••

INCLUSIVE SCATTERING: FSI

RDWIA

sum of $1NKO$ where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

INCLUSIVE SCATTERING: FSI

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REAL POTENTIAL

INCLUSIVE SCATTERING: FSI

RDWIA

sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

REAL POTENTIAL

RMF RELATIVISTIC MEAN FIELD: same strong real energy-independent potential of bound states

Fits well the scaling function obtained from QE (e, e') data, it can be considered to represent the nucleonic $1p1h$ contribution to the inclusive response at intermediate energy. As the energy increases, however, the energy-independent RMF is too strong and unable to predict the data

INCLUSIVE SCATTERING: FSI

RDWIA

sum of 1NKO where FSI are described by a complex OP with an imaginary absorptive part does not conserve the flux

REAL POTENTIAL

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RGF

complex energy dependent OP: conserves the flux, consistent description of FSI in exclusive and inclusive QE electron scattering

FSI for the inclusive scattering : Green's Function Model

- with suitable approximations (basically related to the IA) the components of the inclusive response can be written in terms of the s.p. optical model Green's function
- the explicit calculation of the s.p. GF can be avoided by its spectral representation which is based on a biorthogonal expansion in terms of the eigenfunctions of the non Herm optical potential V and V^+
- matrix elements similar to RDWIA
- scattering states eigenfunctions of V and V^+ (absorption and gain of flux): the imaginary part redistributes the flux and the total flux is conserved

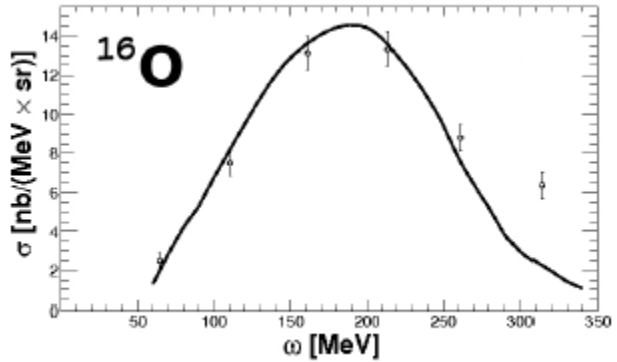
Relativistic Green's Function Model

- the imaginary part of the OP includes inelastic channels, contributions not included in other models based on the IA
- energy dependence of the OP reflects the different contribution of the different inelastic channels open at different energies, results sensitive to the kinematic conditions

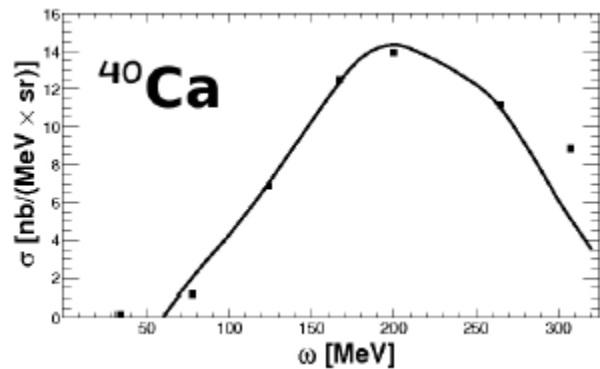
RGF: comparison with QE (e, e') data

(e, e')

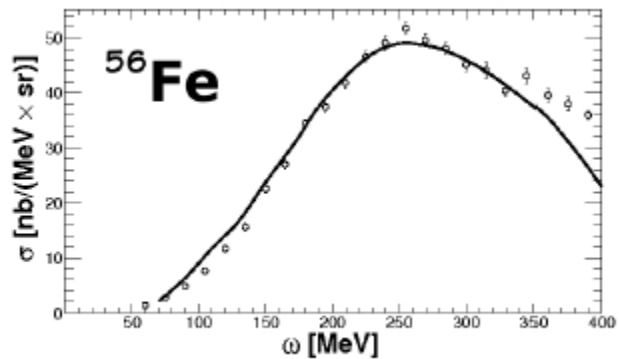
RGF



$$E_0 = 1080 \text{ MeV} \quad \vartheta = 32^\circ$$



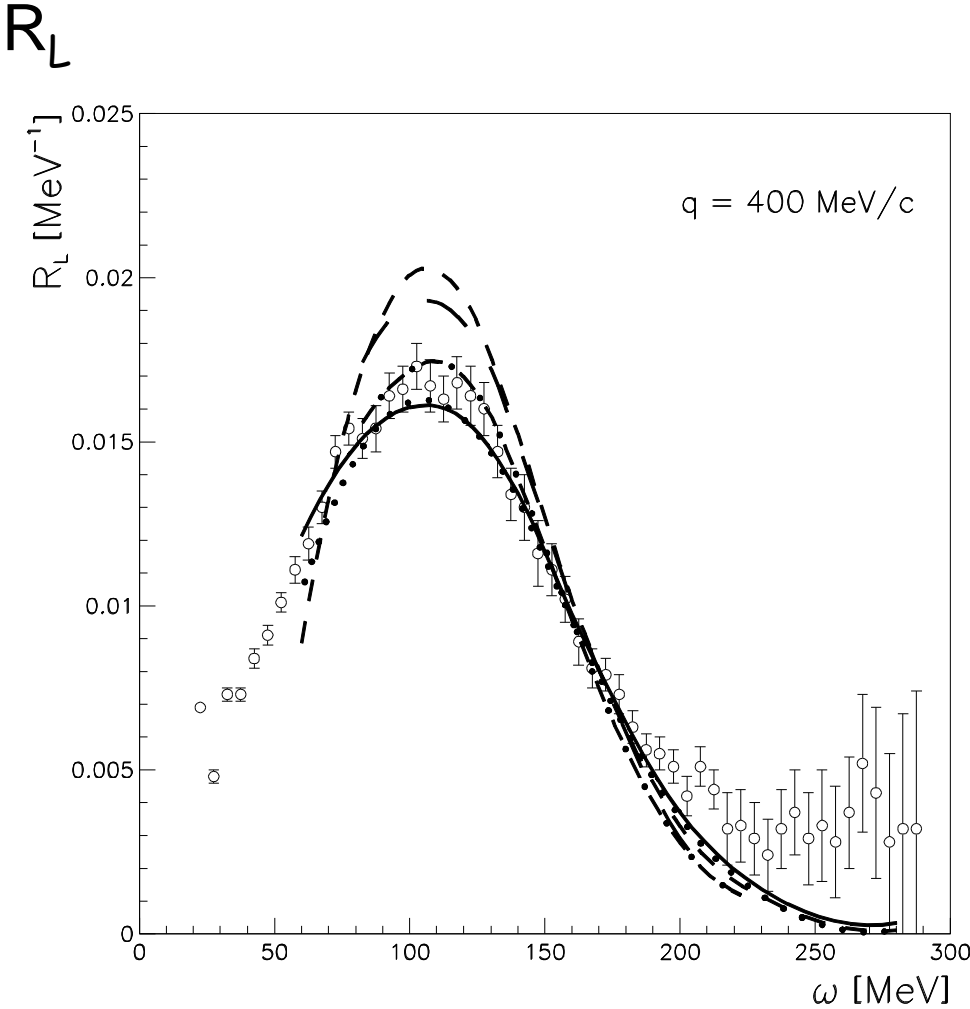
$$E_0 = 841 \text{ MeV} \quad \vartheta = 45.5^\circ$$



$$E_0 = 2020 \text{ MeV} \quad \vartheta = 20^\circ$$

RGF - GF

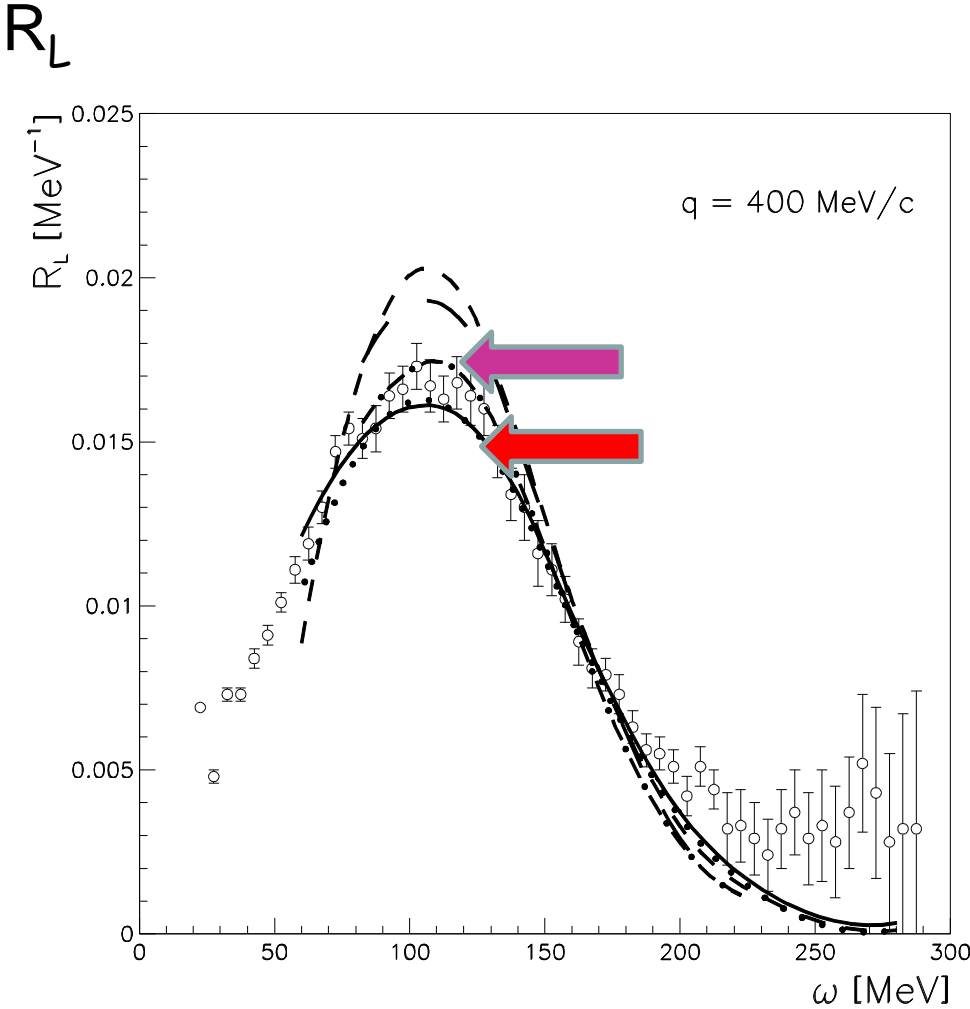
$^{12}\text{C}(e, e')$



- RGF
- - - RGF without dispersive integral
- ⋯ RGF without $\sqrt{1 - v'(E)}$
- · - · GF
- - - GF without $\sqrt{1 - v'(E)}$

RGF - GF

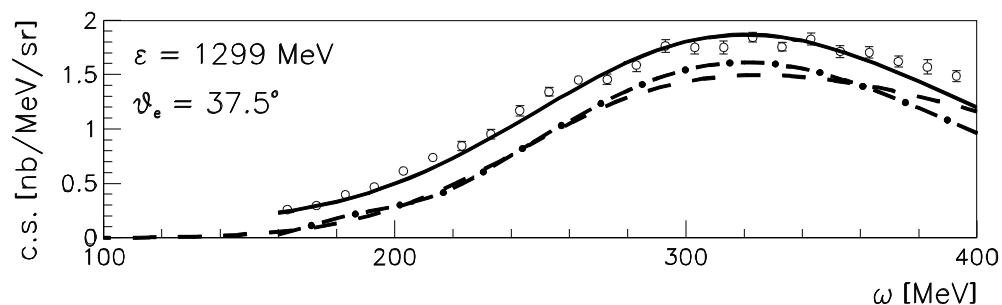
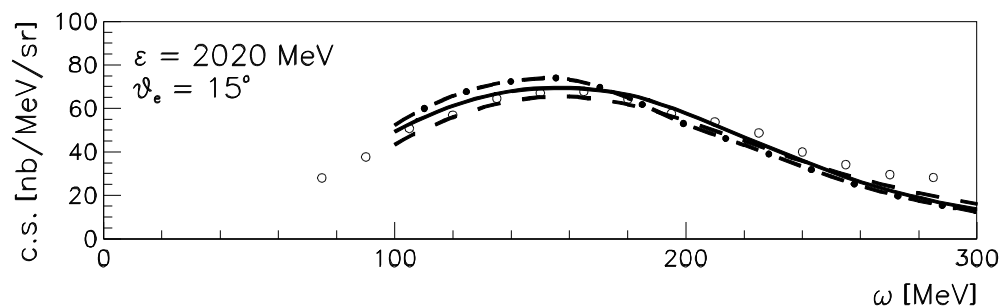
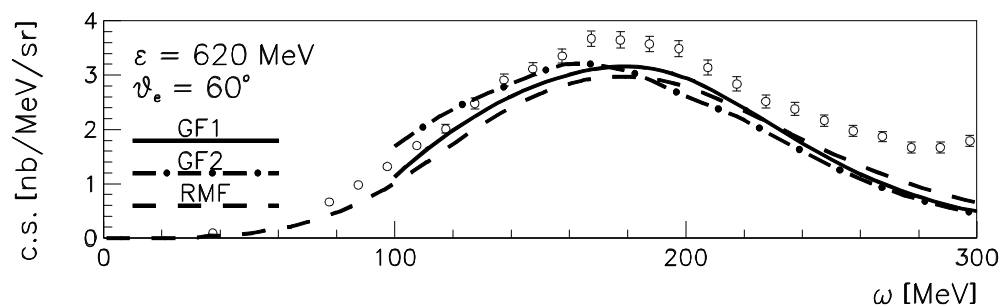
$^{12}\text{C}(e, e')$



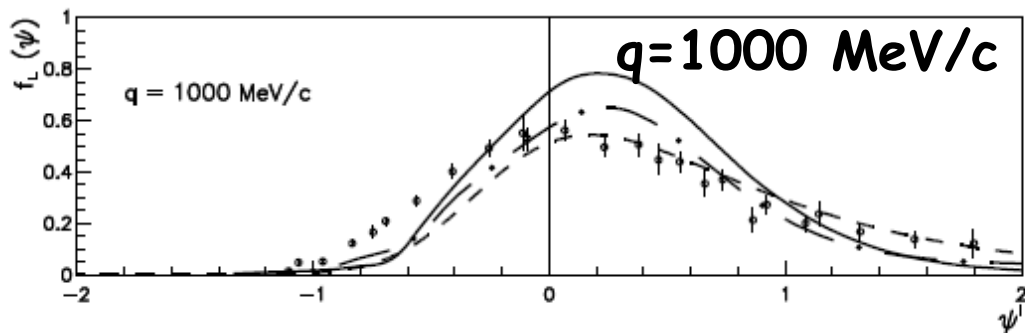
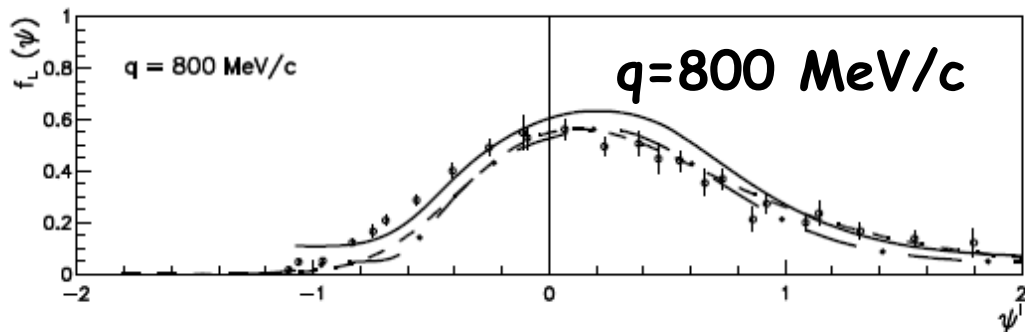
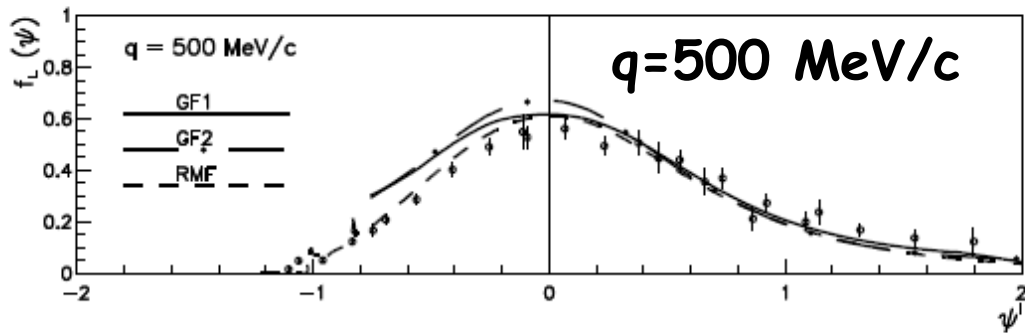
- RGF ←
- - - RGF without dispersive integral
- ⋯ RGF without $\sqrt{1 - v'(E)}$
- · - · GF ←
- - - GF without $\sqrt{1 - v'(E)}$

$^{12}\text{C}(e, e')$

relativistic models



QE SCALING FUNCTION: RGF, RMF



Differences between Electron and Neutrino Scattering

- **electron scattering :**

beam energy known, cross section as a function of ω

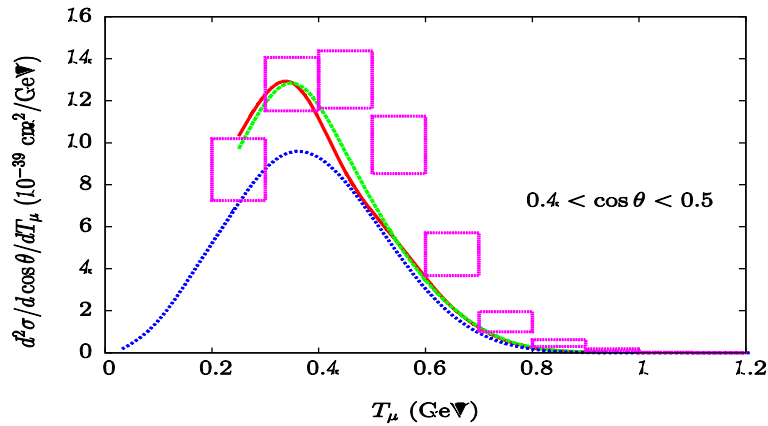
- **neutrino scattering:**

beam energy and ω not known

calculations over the energy range relevant for the neutrino flux

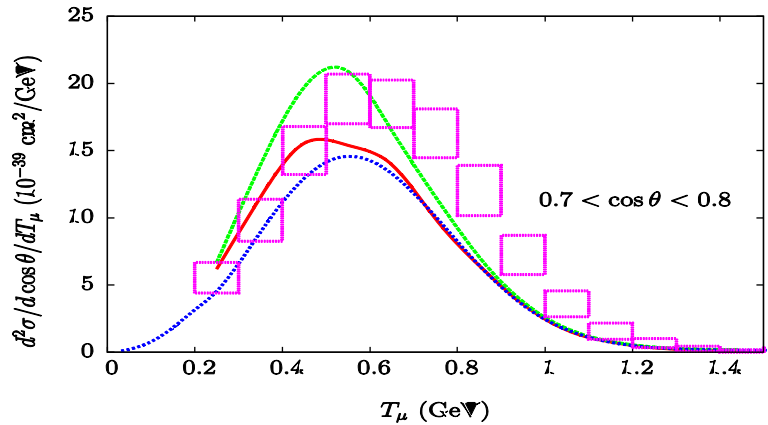
the flux-average procedure can include contributions from different kinematic regions where the neutrino flux has significant strength, contributions other than direct 1-nucleon emission

Comparison with MiniBooNe CCQE data



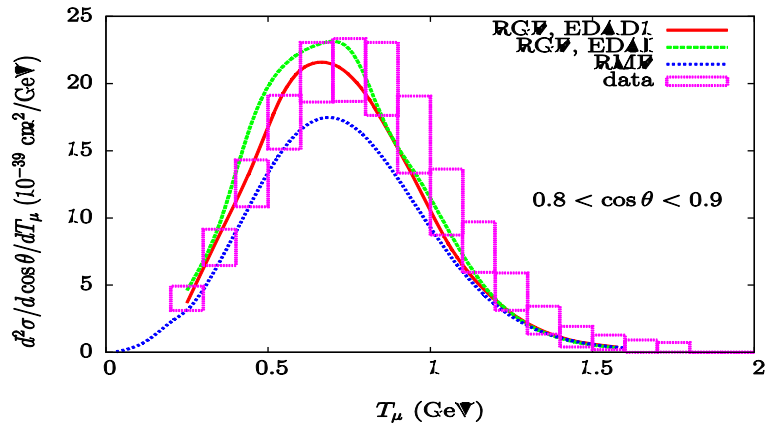
$0.4 < \cos\theta_\mu < 0.5$

$^{12}\text{C}(\nu_\mu, \mu^-)$



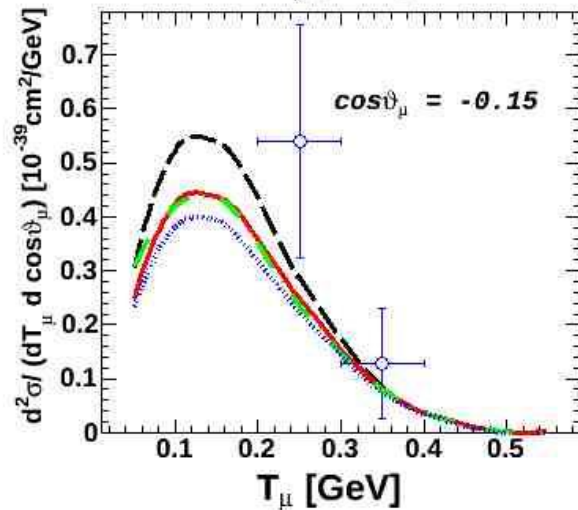
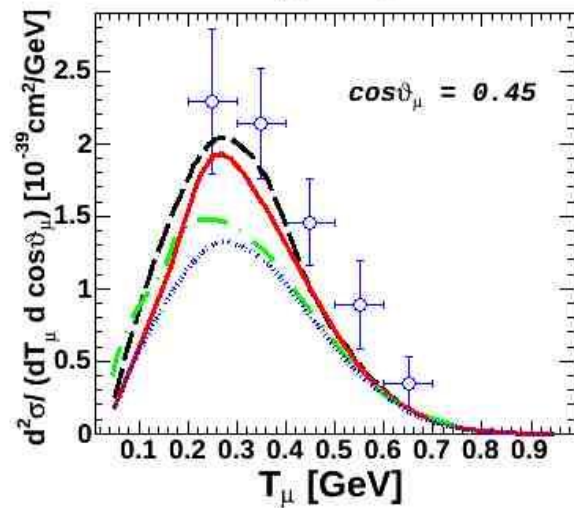
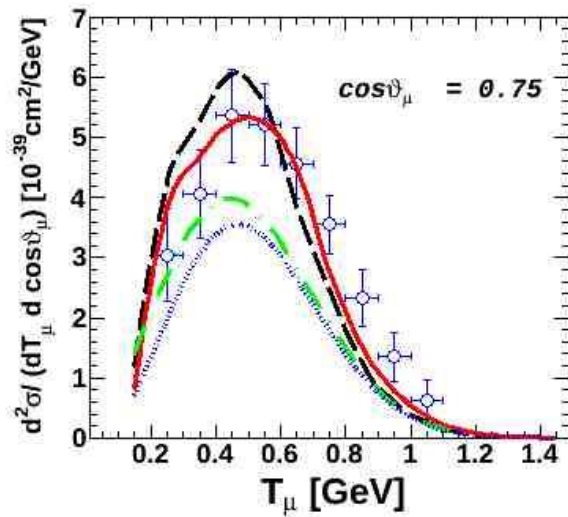
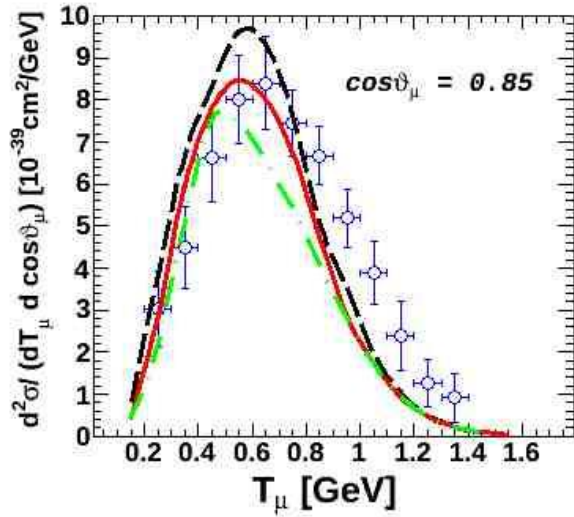
$0.7 < \cos\theta_\mu < 0.8$

- RGF-EDAI
- RGF-EDAD1
- RMF



$0.8 < \cos\theta_\mu < 0.9$

Comparison with MiniBooNe CCQE data

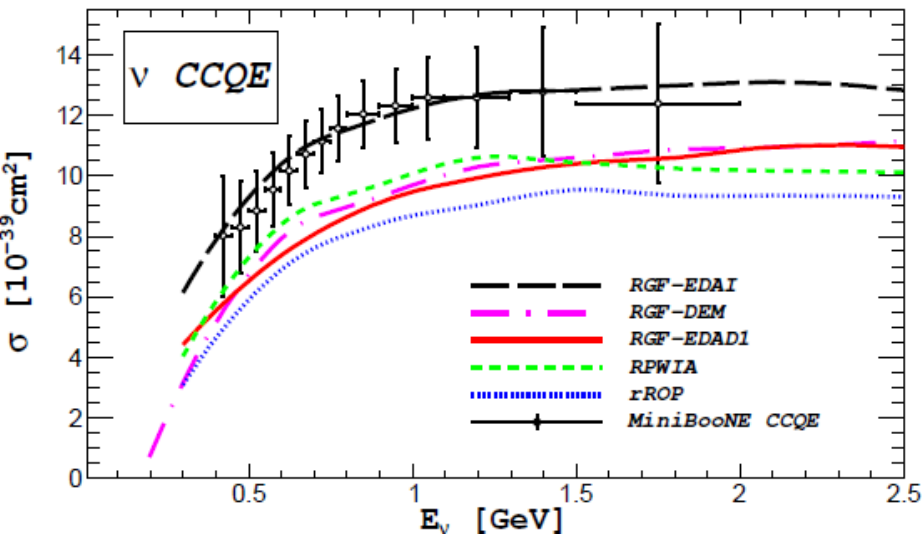


$$^{12}\text{C}(\bar{\nu}_\mu, \mu^+)$$

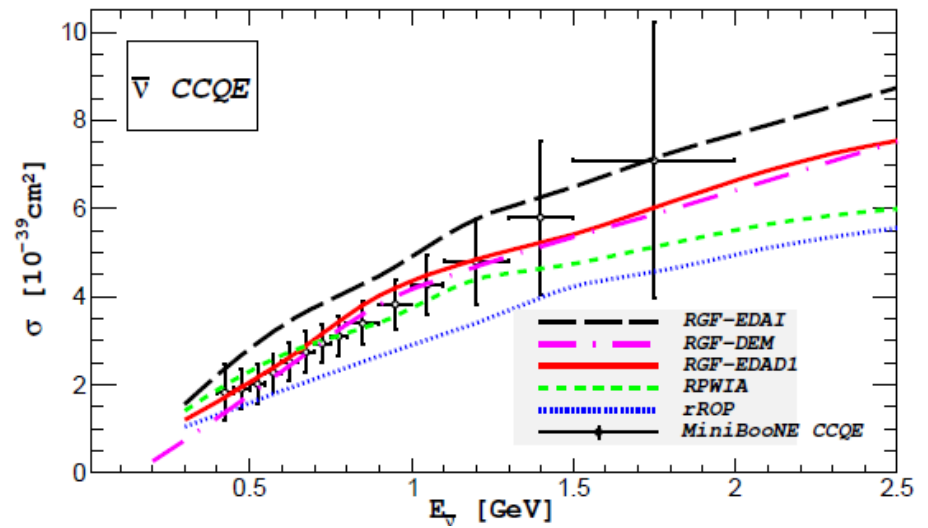
- - - RPWIA
- - - rROP
- - - RGF EDAI
- - - RGF-EDAD1

Comparison MiniBooNE CCQE neutrino-antineutrino scattering

$^{12}\text{C}(\nu_\mu, \mu^-)$

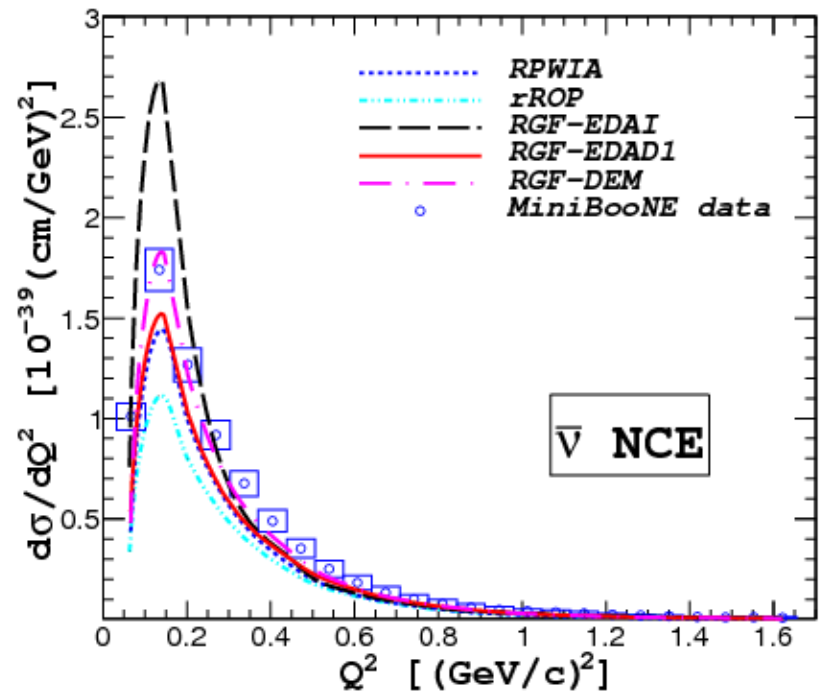
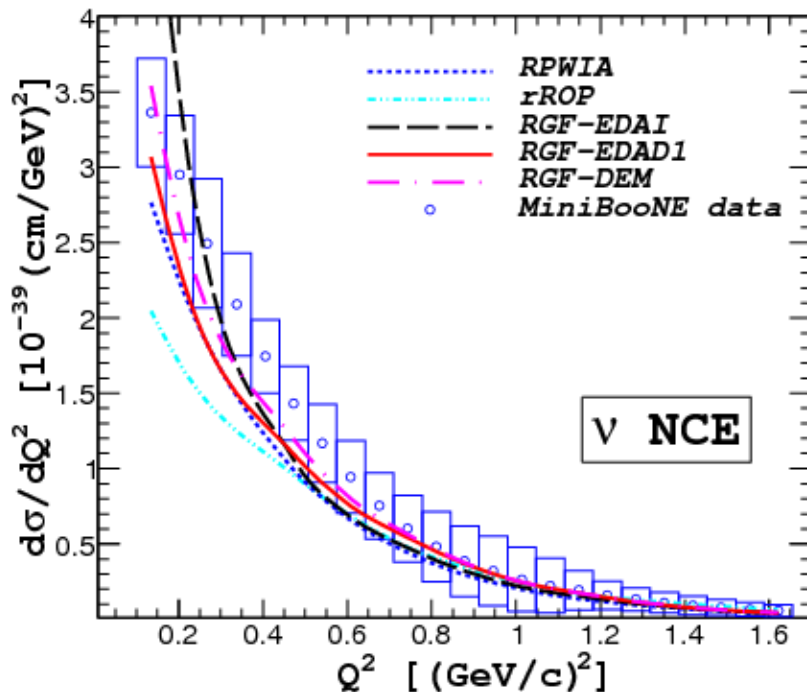


$^{12}\text{C}(\bar{\nu}_\mu, \mu^+)$



- RGF EDAI
- RGF-EDAD1
- · - RGF-DEM
- rROP
- RPWIA

Comparison with MiniBooNE NCE data



RGF

successful in comparison with data: (e,e') , CCQE and NCE MiniBooNE data, MINERvA CCQE data

the imaginary part of the ROP includes the overall effect of inelastic channels (rescattering, non-nucleonic, multi-nucleon....)

a phenomenological ROP does not allow us to disentangle and evaluate the role of specific inelastic processes

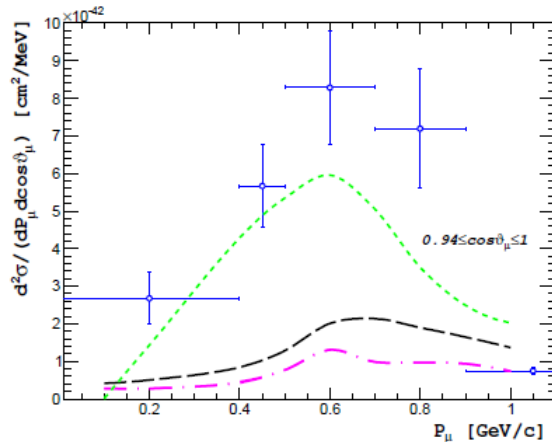
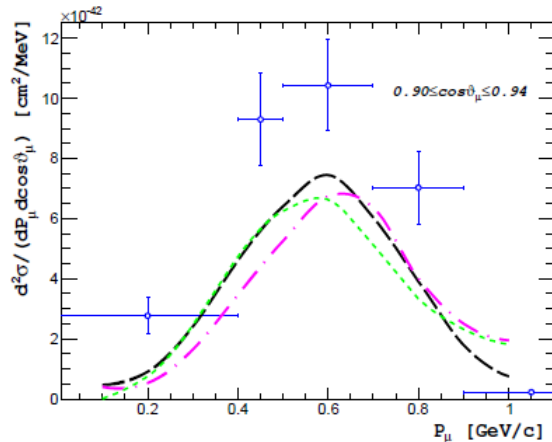
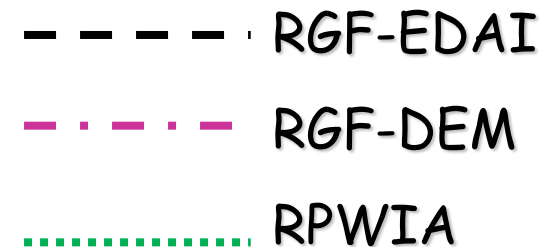
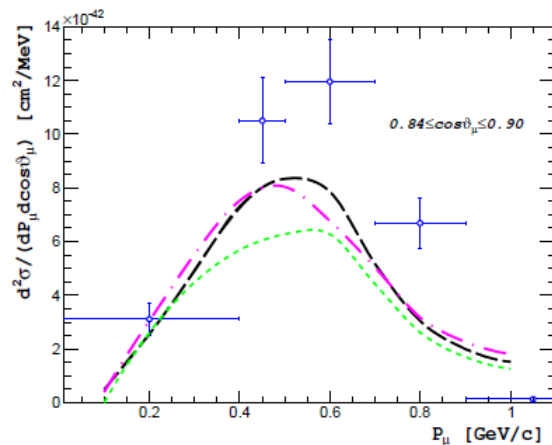
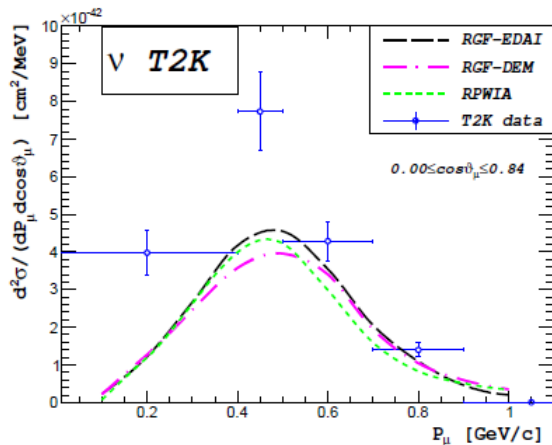
the agreement of the RGF results with data should be interpreted with care

MEC are not included

does the model include also pion production channels?

comparison with T2K CC-inclusive data....

Comparison with T2K CC inclusive data



RGF underestimates CC inclusive data !

RGF

To reduce theoretical uncertainties due to different OPs a less phenomenological optical potential has been obtained for ^{12}C within **RIA**:

GLOBAL spanning a wide range of nucleon energies (20-1040 MeV)

RELATIVISTIC

FOLDING the relativistic Horowitz-Love-Franey t-matrix for the NN scattering amplitudes with relativistic mean-field nuclear densities via the t_ρ approximation

OPTICAL

POENTIAL

GRFOP

GRFOP AND RGF

Global relativistic folding optical potential and the relativistic Green's Function model:

M.V. Ivanov, J.M. Vignote, R. Alvarez-Rodriguez, A. Meucci, C.Giusti, J.M. Udias PRC 94 014608 (2016)

GRFOP

shape dictated by the shape of nuclear densities

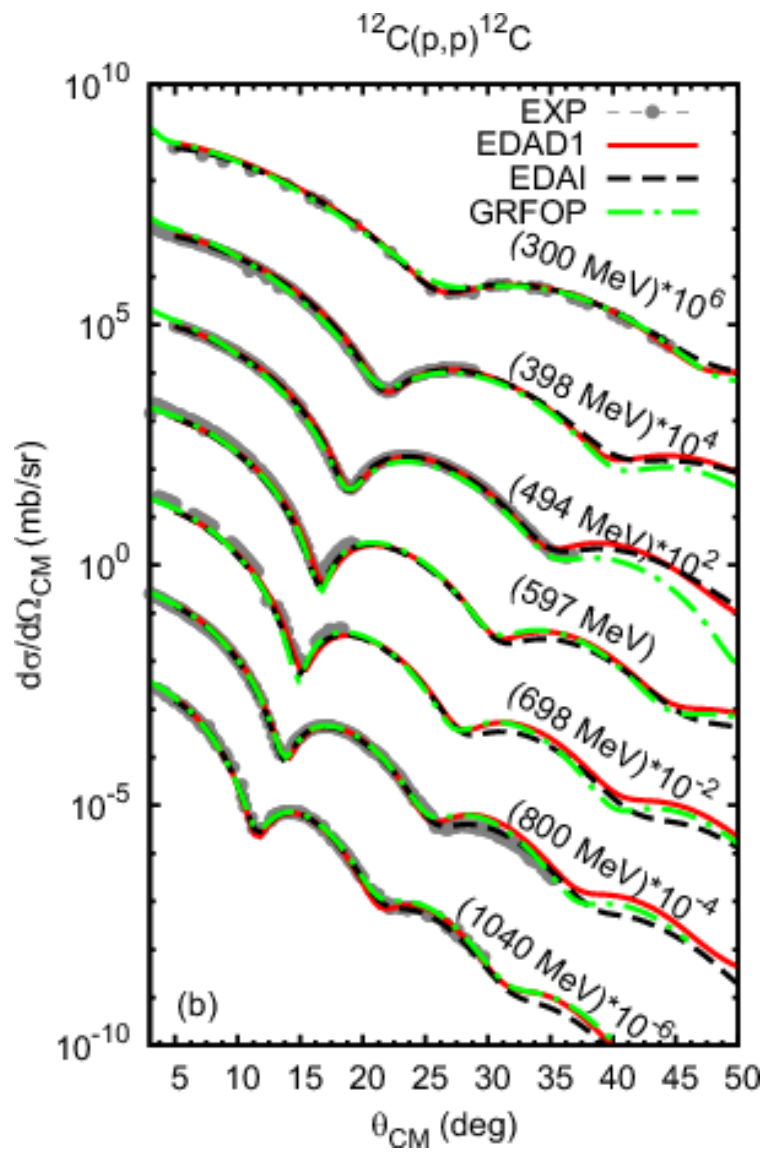
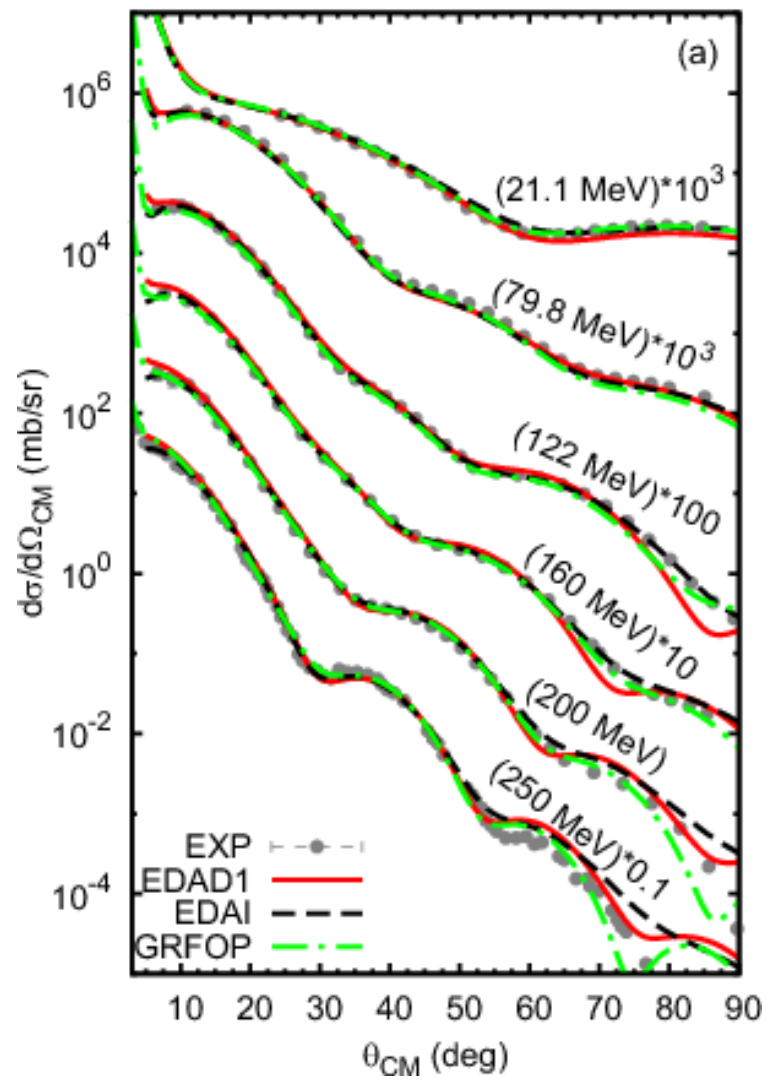
strength dictated by effective parametrizations of the NN scattering amplitudes

GRFOP

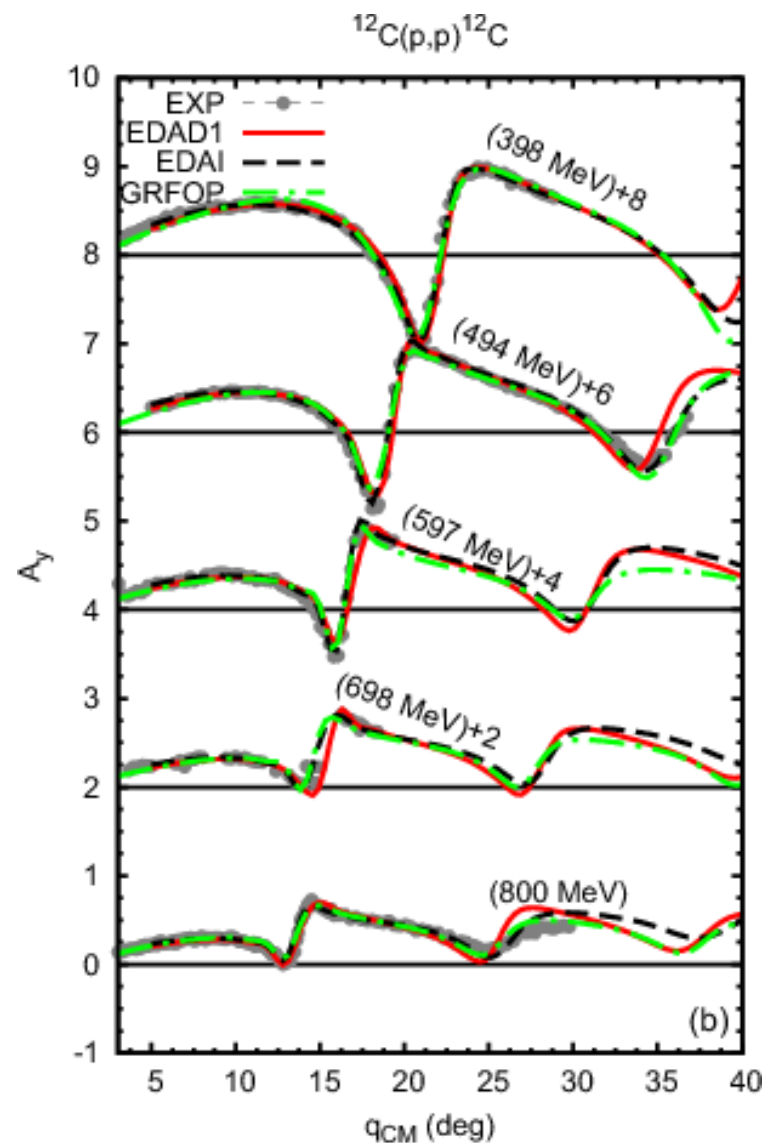
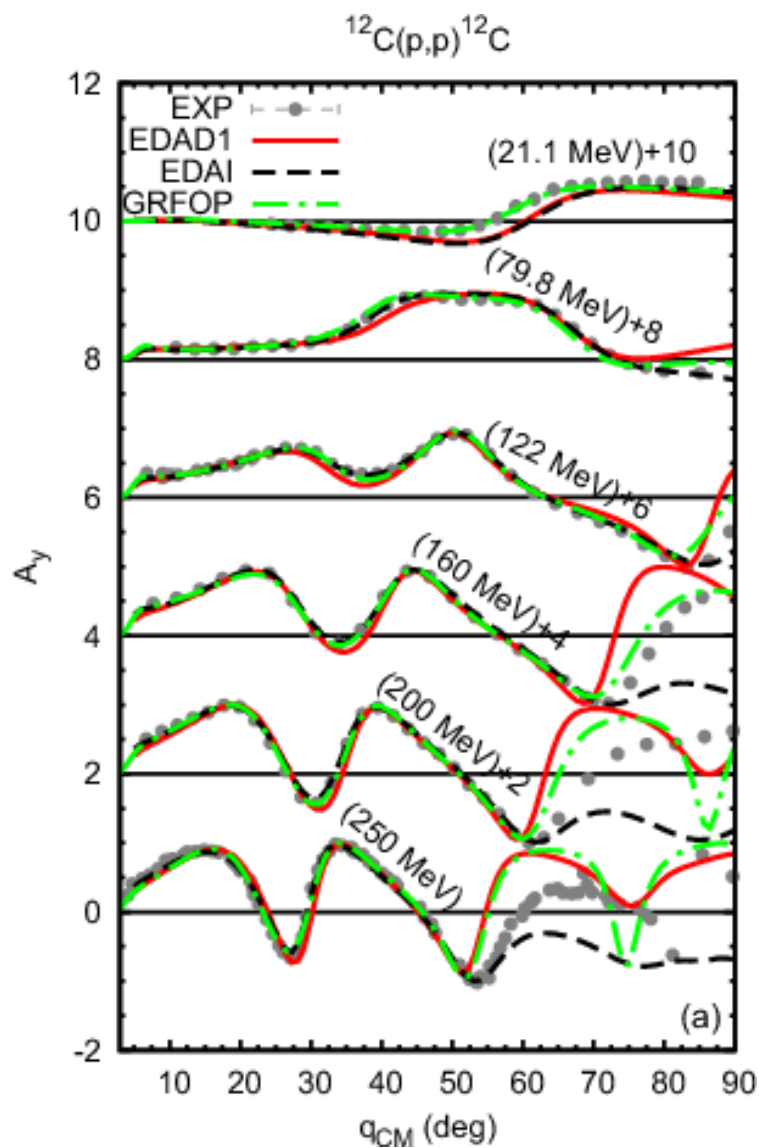
- derived from all available elastic proton- ^{12}C scattering data
- folding approach with proton density taken from electron scattering data and neutron density fitted to data
- imaginary part built from the effective NN interaction

$^{12}\text{C}(p,p)^{12}\text{C}$ cross section

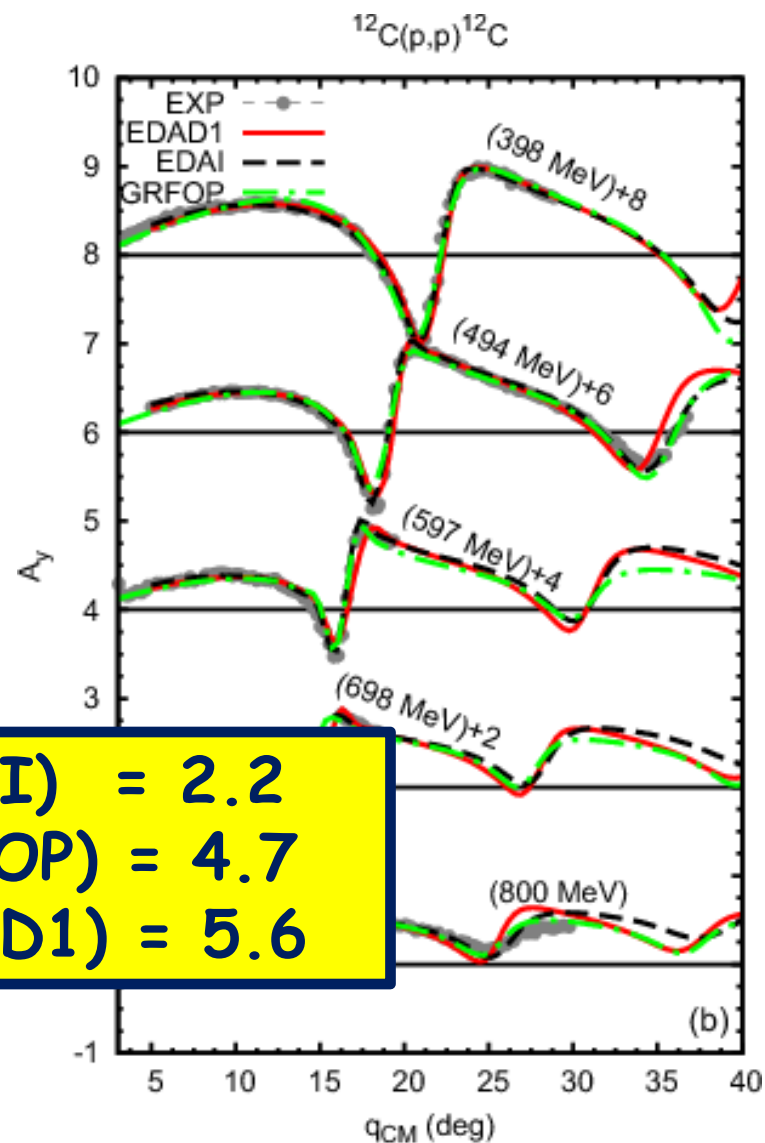
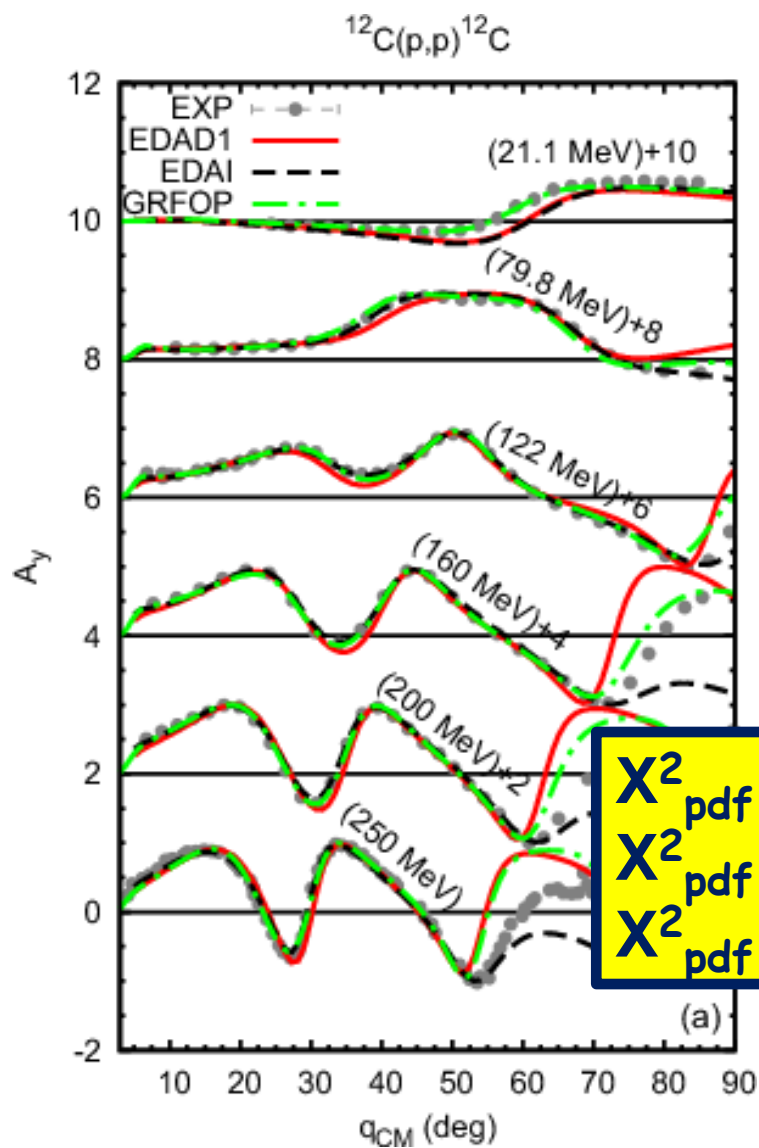
$^{12}\text{C}(p,p)^{12}\text{C}$



$^{12}\text{C}(p,p)^{12}\text{C}$ analyzing power



$^{12}\text{C}(p,p)^{12}\text{C}$ analyzing power



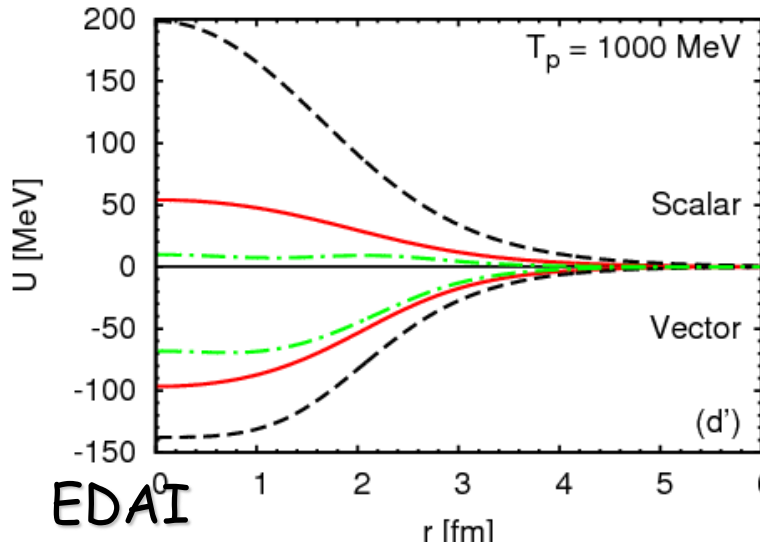
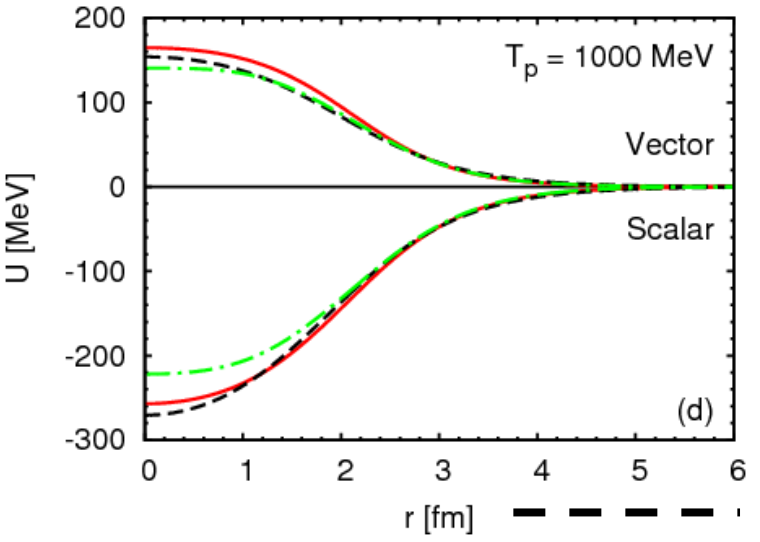
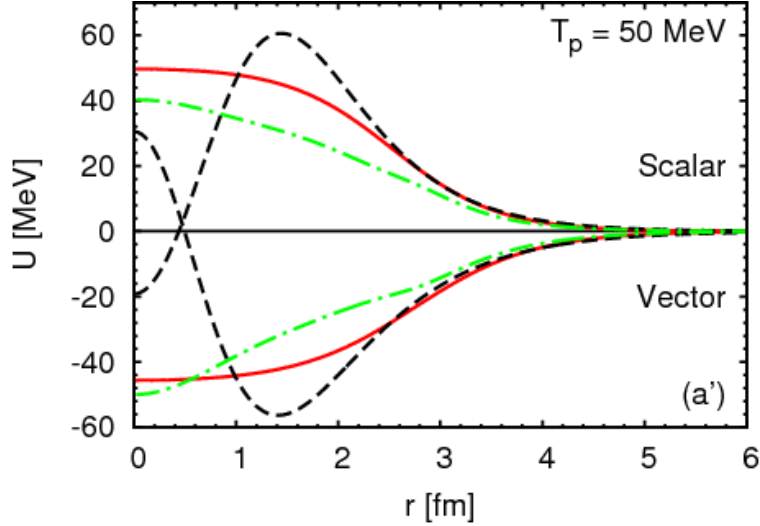
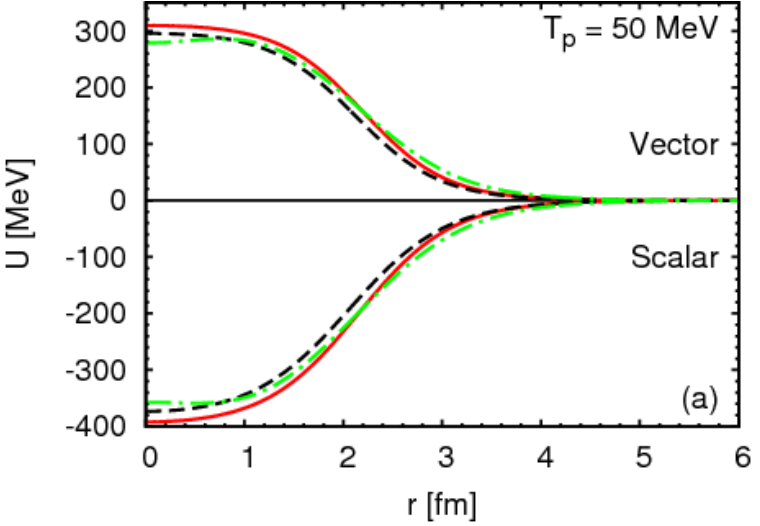
X^2_{pdf} (EDAI) = 2.2
 X^2_{pdf} (GRFOP) = 4.7
 X^2_{pdf} (EDAD1) = 5.6

Re OP

Im OP

50 MeV

1000 MeV



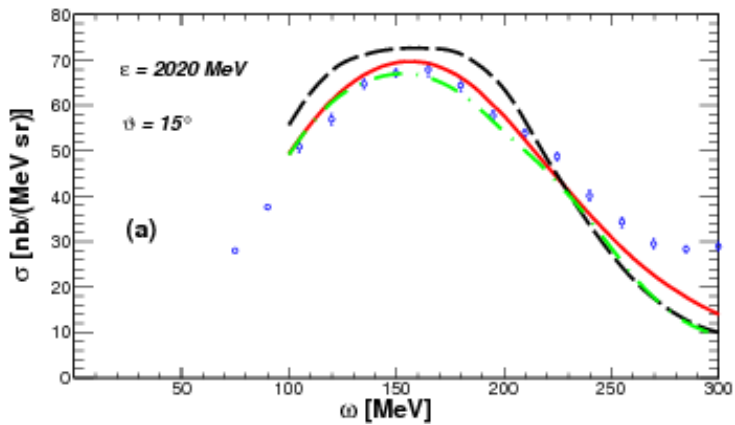
EDAD0
EDAD1
GRFOP

EDAD0

EDAD1

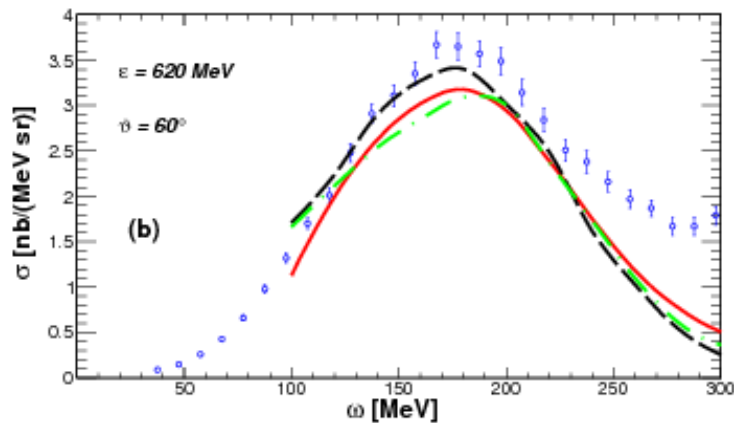
GRFOP

$^{12}\text{C}(e, e')$



$\epsilon = 2020 \text{ MeV}$

$\theta = 15^\circ$



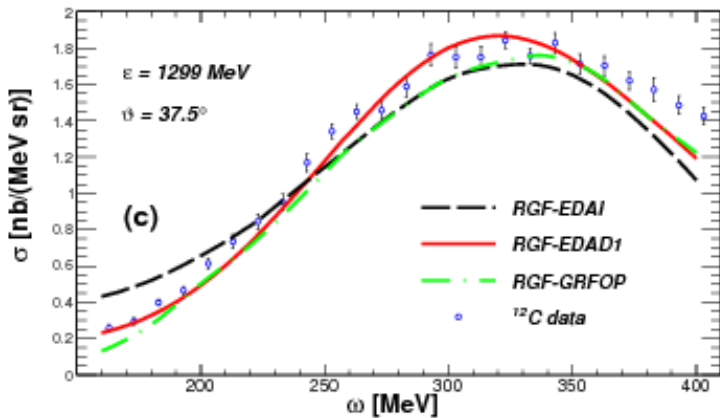
$\epsilon = 620 \text{ MeV}$

$\theta = 60^\circ$

--- RGF-EDAI

— RGF-EDAD1

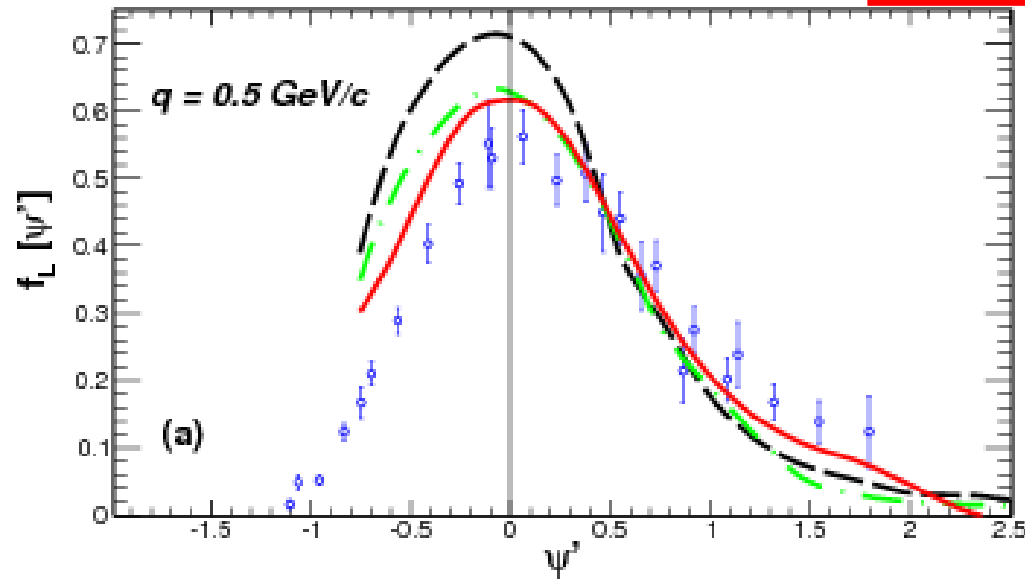
- · - · - RGF-GRFOP



$\epsilon = 1299 \text{ MeV}$

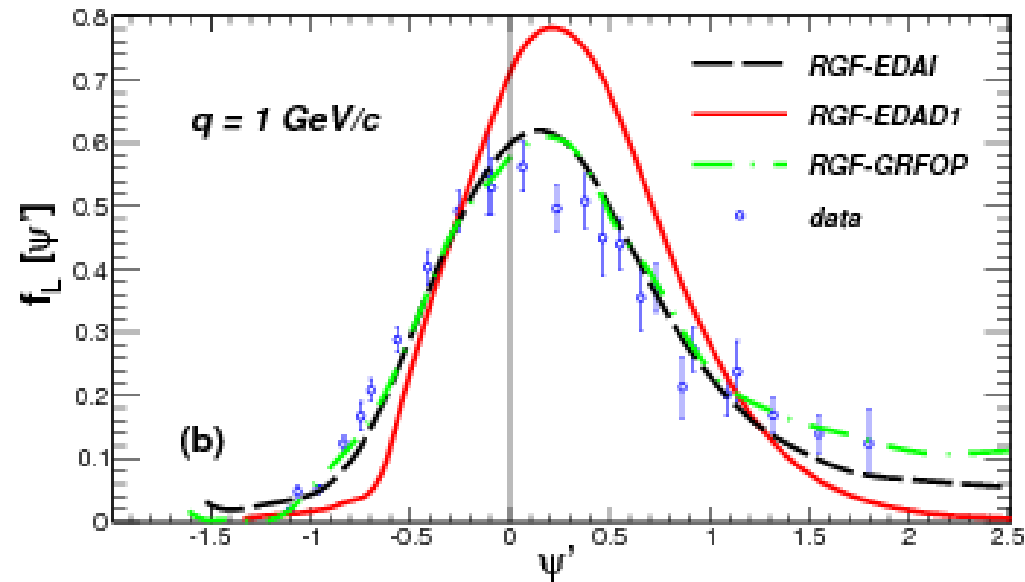
$\theta = 37.5^\circ$

QE SCALING FUNCTION



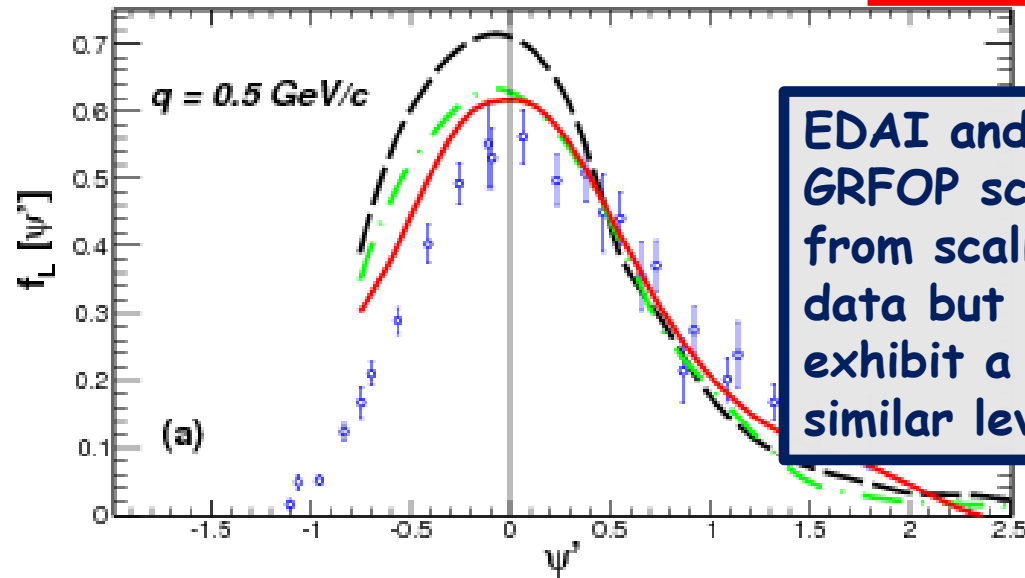
$q = 500 \text{ MeV}/c$

--- RGF-EDAI
— RGF-EDAD1
- . - . RGF-GRFOP



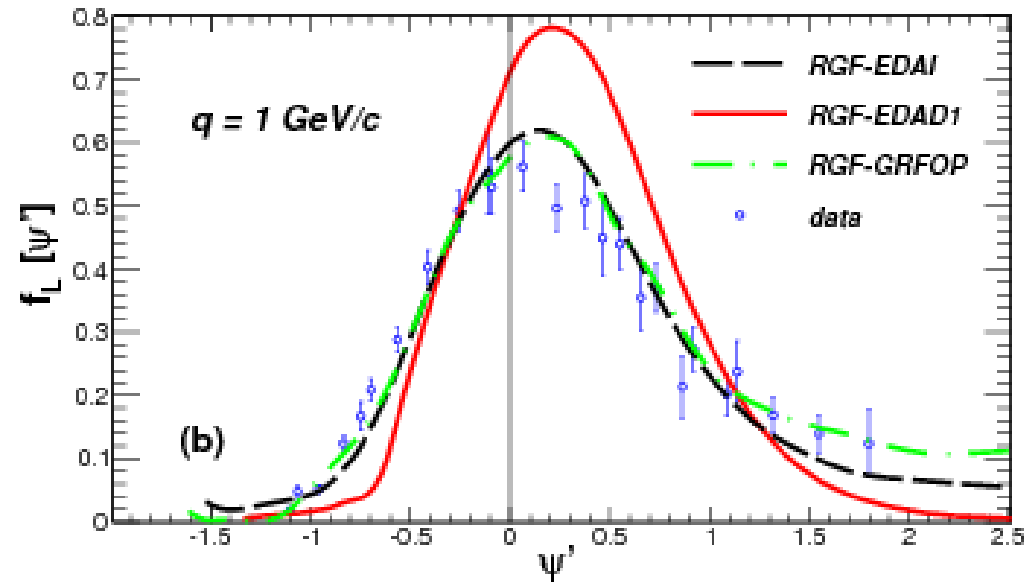
$q = 1 \text{ GeV}/c$

QE SCALING FUNCTION

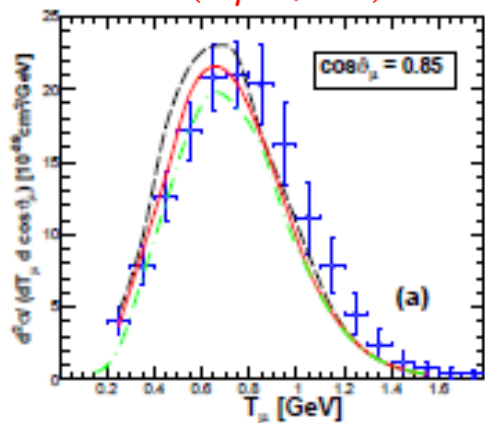
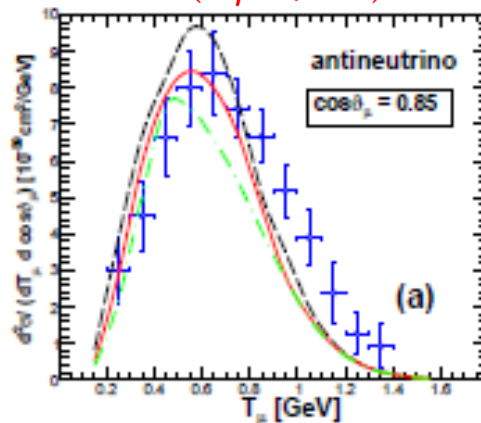


EDAI and EDAD1 do not scale enough
GRFOP scales better, its mild departure
from scaling is not only compatible with the
data but it is also favored as the data
exhibit a slight departure from scaling at a
similar level as the one of RGF-GRFOP

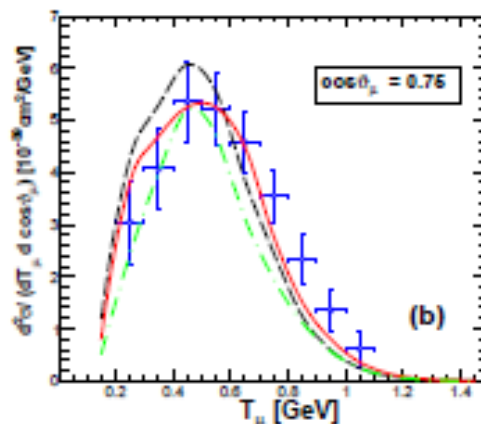
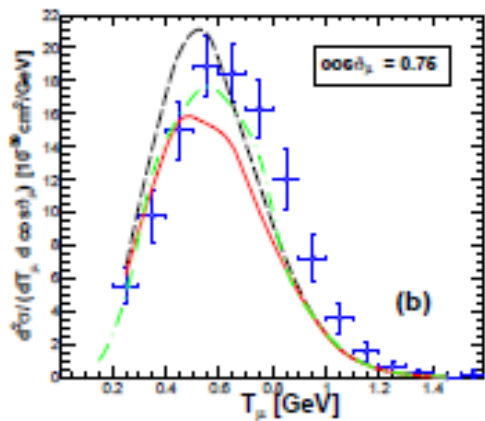
--- RGF-EDAI
— RGF-EDAD1
-.- RGF-GRFOP



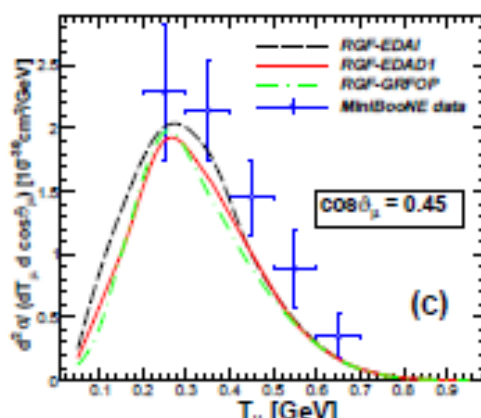
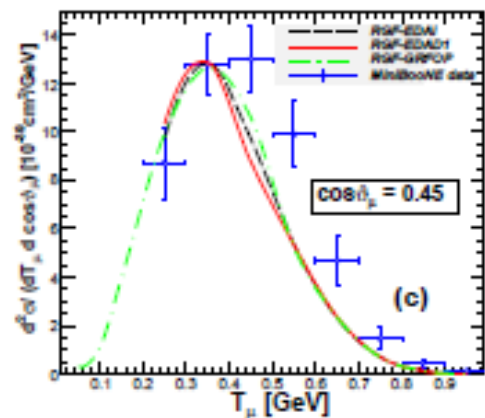
$q = 1 \text{ GeV}/c$

$^{12}C(\nu_\mu, \mu^-)$  $^{12}C(\bar{\nu}_\mu, \mu^+)$ 

MiniBooNe CCQE data



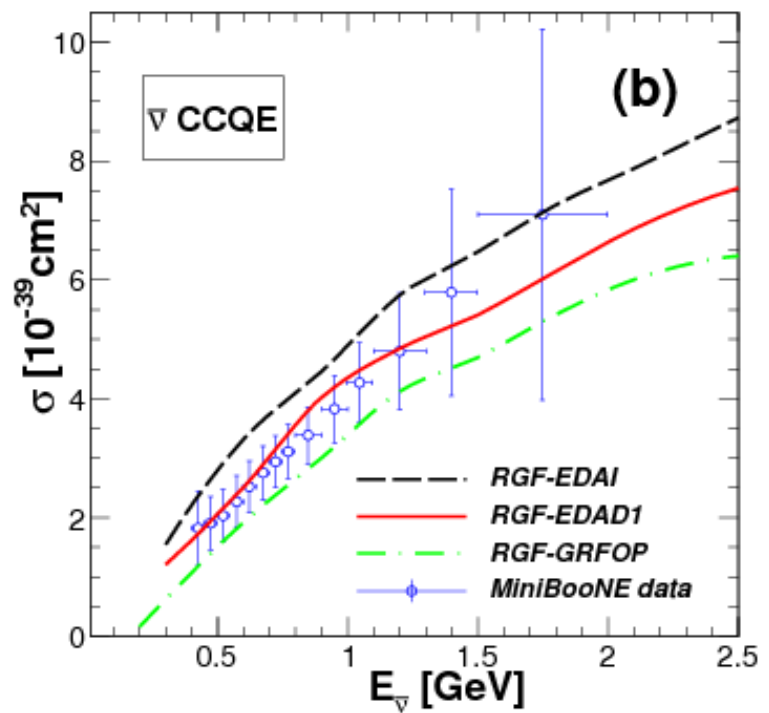
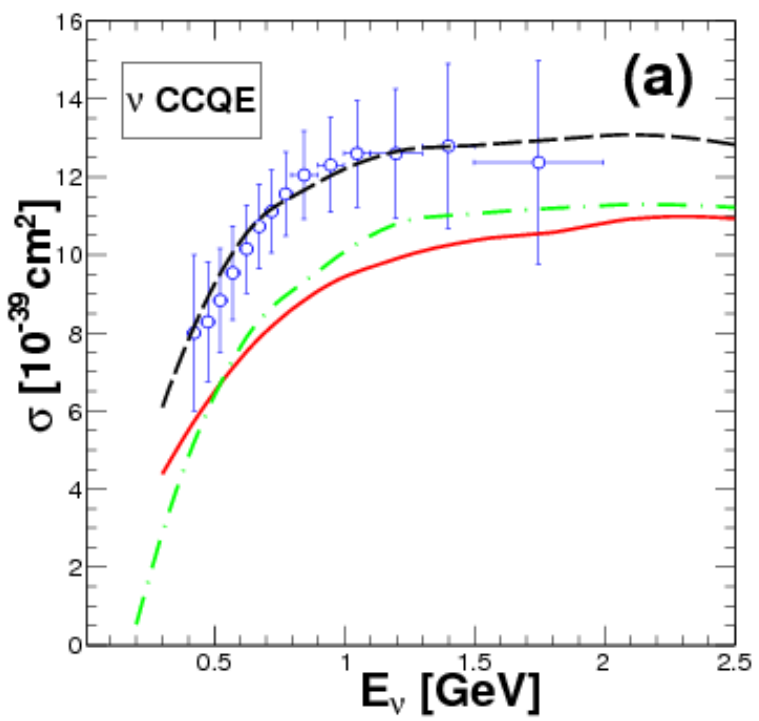
--- RGF-EDAI
 — RGF-EDAD1
 - · - · RGF-GRFOP



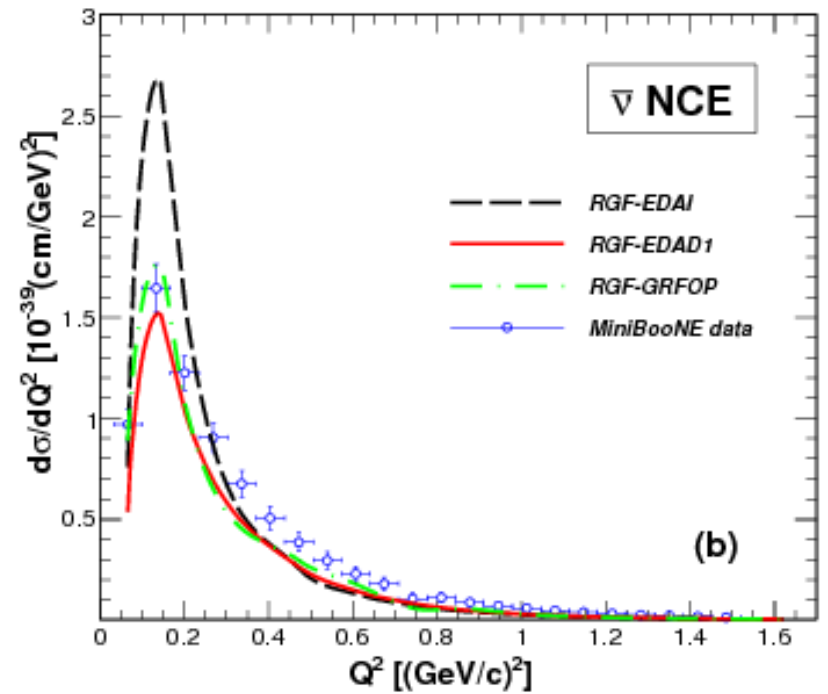
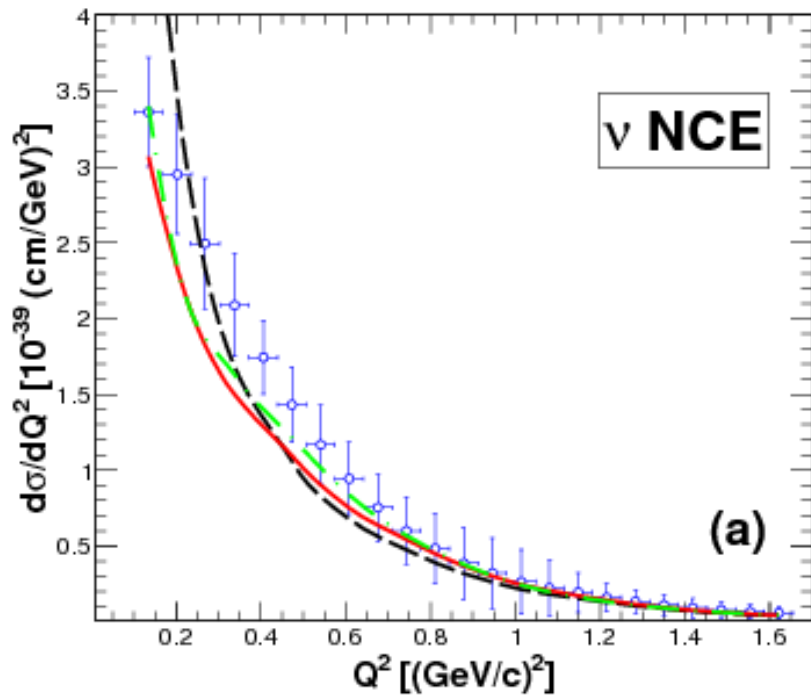
MiniBooNe CCQE data

$$^{12}\text{C}(\nu_{\mu}, \mu^{-})$$

$$^{12}\text{C}(\bar{\nu}_{\mu}, \mu^{+})$$



MiniBooNE NCE data



RGF-GRFOP

- generally between RGF-EDAI and RGF-EDAD1 results
- in many cases in better agreement with data
- good agreement with (e,e') data
- good agreement with the experimental scaling function
- reasonable agreement with CCQE and NCE data
- use of GRFOP reduces the theoretical uncertainties in RGF predictions and confirms previous findings in comparison with data
- RIA can provide successful Dirac optical potentials able to fit elastic nucleon-nucleus scattering data and useful alternatives to phenomenological OP
- GRFOP can be improved extending the range of validity of the parametrization or including an A -dependence

RGF: prospects....

- **OPTICAL POTENTIAL** calculations of more theoretical OP 's would improve the theoretic content of the model
- **MEC** require a new **consistent** model
 - suitable** approximations are required to make calculations feasible and give **reliable and consistent** results
 - relativistic vs. nonrelativistic...
 - correlations.....
 - exclusive semi-inclusive inclusive processes...