Electroweak Interactions with nuclei:

The Relativistic Mean Field Approach

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INT Workshop on neutrino-nucleus interactions

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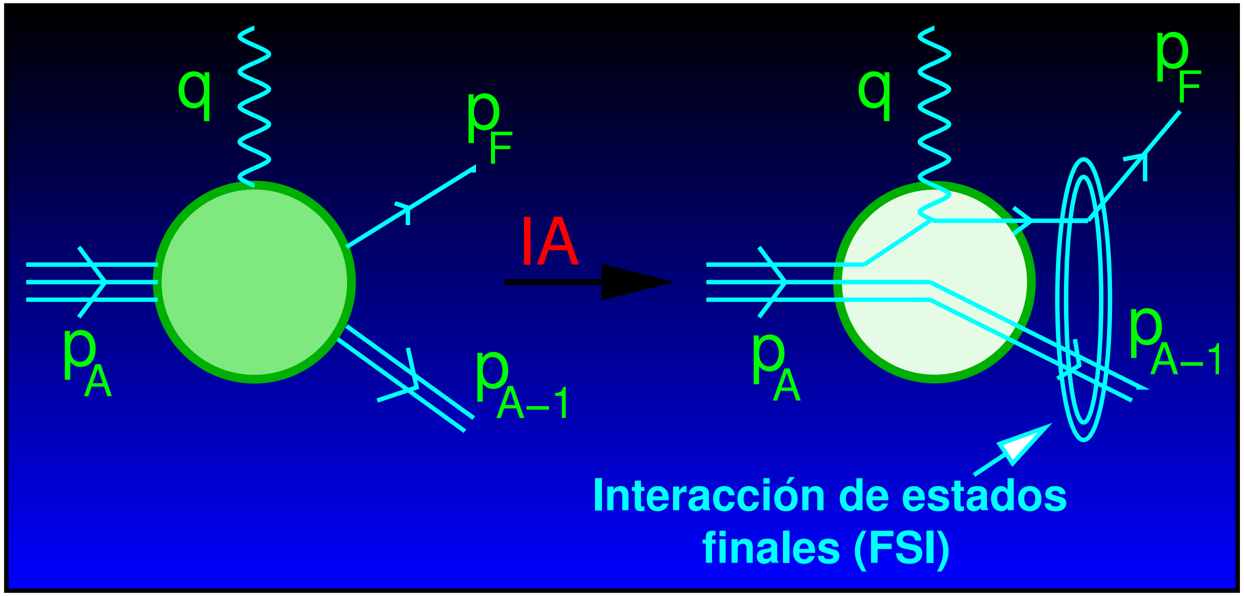
SOME QUESTIONS OF INTEREST

- *Importance of relativistic effects: kinematics & dynamics*
- *Evidence for such effects in electron scattering?*
- *Inclusive & semi-inclusive reactions*
- *Natural outcomes. Transverse enhancement*
- *Model extended to neutrino reactions*
- *Parity-violating electron scattering*
- *MEC effects handled within RMF?*

The model: Relativistic Impulse Approximation (RIA)



Nuclear Current $\Longrightarrow$ One-body operator

$$J_N^\mu(\omega, \vec{q}) = \int d\vec{p}\, \overline{\Psi}_F(\vec{p}+\vec{q})\, \hat{J}_N^\mu\, \Psi_B(\vec{p})$$

Scattering off a nucleus $\Longrightarrow$ incoherent sum of single–nucleon scattering processes

Basic ingredients of RIA

Nucleon Wave Functions $\Longrightarrow$ *Solutions of Dirac equation with phenomenological relativistic potentials (scalar and vector terms):*

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- Ψ_B: **Bound nucleon wave function** $\Longrightarrow$
  Relativistic Mean Field (RMF) Approach

Local potentials obtained from a Lagrangian fitted to properties of nuclear matter, radii and nuclear masses.

The non-relativistic reduction of the RMF formalism leads to a Schrödinger-like equation but with presence of non-local terms.

$$\left[-\frac{\boldsymbol{\nabla}^2}{2m_N} - V_{\rm DEB}\right]\phi_{\rm nr}(\mathbf{r}) = E_{\rm nr}\phi_{\rm nr}(\mathbf{r}); \quad \Psi_{up}(\mathbf{r}) = K(r)\phi_{\rm nr}(\mathbf{r})$$

$K(r) \sim 0.8$ *in the nuclear interior going to unity asymptotically*

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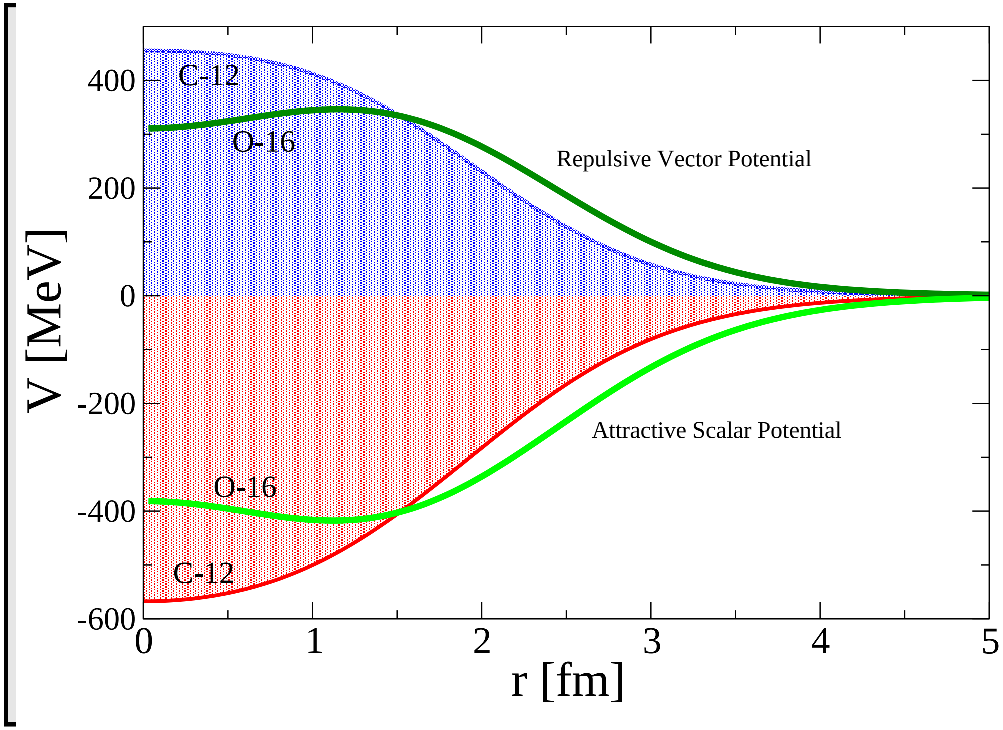
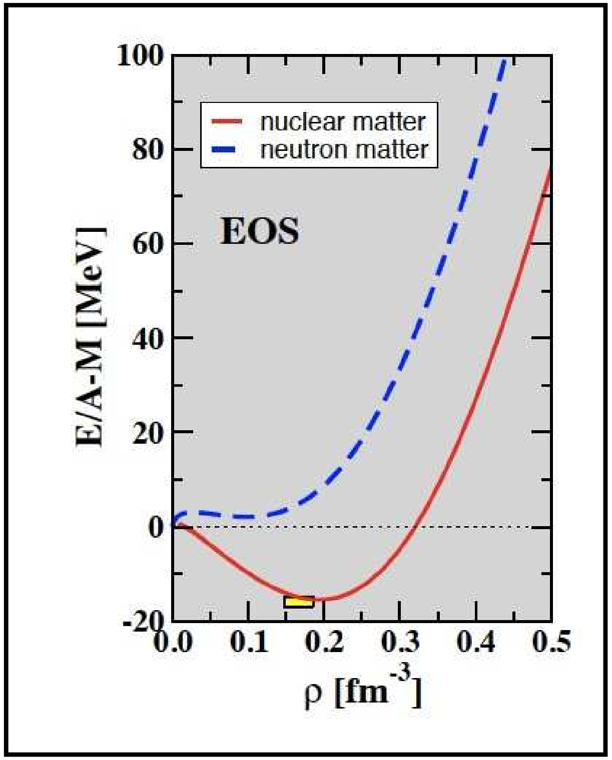
- Ψ_F: **Ejected nucleon wave function** $\Longrightarrow$
  Dependence with final state interactions (FSI)

The Relativistic Mean Field Approach (RMF)

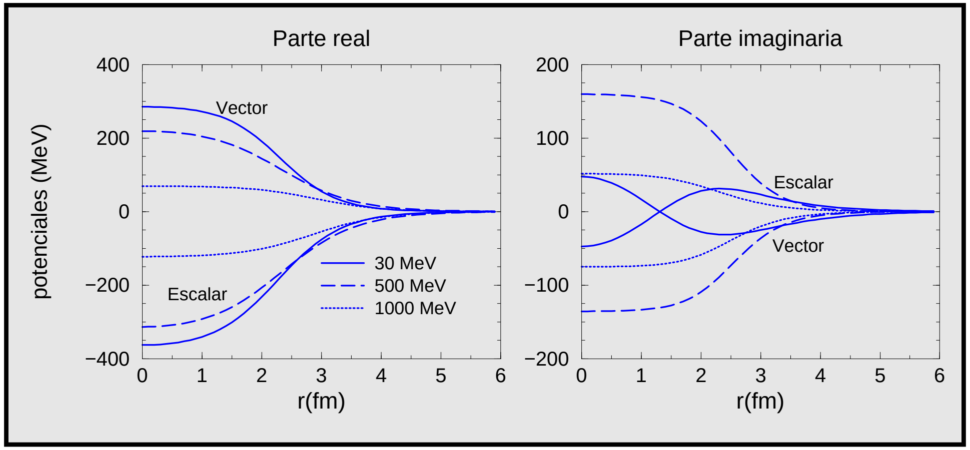
Large scalar (attractive) and vector (repulsive) potentials that lead to saturation. Nonlocalities & correlation effects accounted for by the RMF? Important difference with non-relativistic models

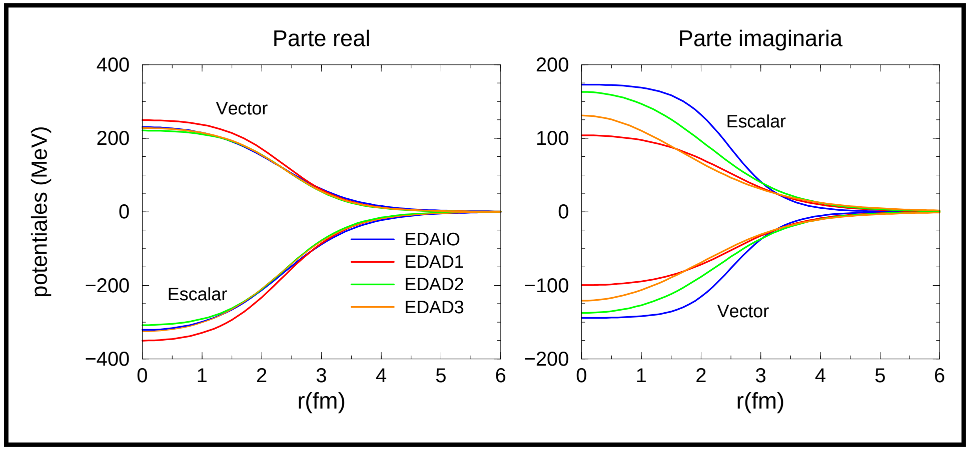




Final State Interactions (FSI)

Final State Interactions between the ejected nucleon and the residual nucleus described by the Optical Model: complex energy-dependent relativistic potentials fitted to elastic nucleon-nucleus scattering data





Nucleon current operator and uncertainties

- **Electromagnetic current:** (e, e')

$$\hat{J}^{\mu}_{cc1} = (F_1 + F_2)\gamma^{\mu} - \frac{F_2}{2m_N}(\overline{P} + P_N)^{\mu}$$

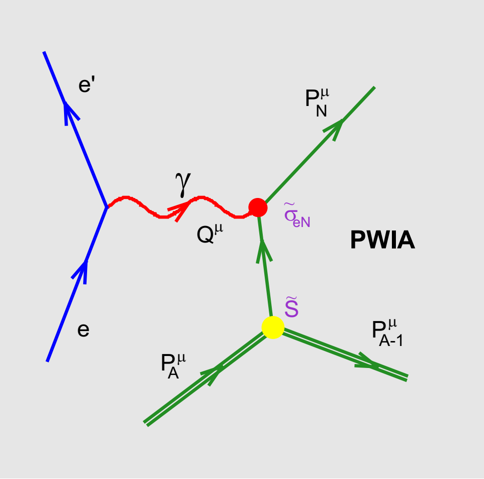
$$\hat{J}^{\mu}_{cc2} = F_1\gamma^{\mu} + \frac{iF_2}{2m_N}\sigma^{\mu\nu}Q_{\nu}$$

- Nucleon (bound and ejected) off-shell $\Longrightarrow$ dependence with the current operator: off-shell effects or Gordon ambiguities
- RIA model $\Longrightarrow$ Gauge invariance violation ($Q_{\mu}J^{\mu} \neq 0$):
  - *Lorentz Gauge:* use of $\hat{J}^0, \quad \hat{J}^3$
  - *Coulomb Gauge:* $\hat{J}^3 \rightarrow \frac{\omega}{q}\hat{J}^0$
  - *Weyl Gauge:* $\hat{J}^0 \rightarrow \frac{q}{\omega}\hat{J}^3$

Important terminology

- **RELATIVISTIC DISTORTED WAVE IMPULSE APPROXIMATION (RDWIA)**
  Relativistic distorted (Dirac) wave functions, Ψ_B, Ψ_F *and the relativistic nucleon current operator* J_p^μ.
- **RELATIVISTIC PLANE WAVE IMPULSE APPROXIMATION (RPWIA)**
  - *Final State Interactions neglected* $\Longrightarrow$ Ψ_F*-relativistic plane wave (u-Dirac spinor)*
  - *Bound Wave Function:* $\Psi_B = \begin{pmatrix} \phi^{up} \\ \phi^{down} \end{pmatrix} = \begin{pmatrix} \phi^{up} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M+S-V}\phi^{up} \end{pmatrix} = \alpha u + \beta v$

  i.e. Ψ_B *includes negative energy components* $\Longrightarrow$ *coupling to Dirac sea*
- **PLANE WAVE IMPULSE APPROXIMATION (PWIA)**
  Negative Energy Components in Ψ_B *are projected out*
  $\Longrightarrow$ *Nuclear dynamics and electron-proton interaction are decoupled. The cross section factorizes:*



$$\frac{d\sigma}{d\Omega_e d\varepsilon_f d\Omega_p} = K f_{rec}^{-1} \sigma^{ep} N(p)$$

with $N(p)$*-single-particle momentum distribution and* σ^{ep}*-single-proton cross section:*

$$\sigma^{ep} \sim \eta_{\mu\nu}\mathcal{W}^{\mu\nu} = \eta_{\mu\nu}\left\{\overline{\sum}_{s_i s_f}\left[\overline{u}(\mathbf{p}_f, s_f)J_p^\mu u(\mathbf{p}_i, s_i)\right]^*\left[\overline{u}(\mathbf{p}_f, s_f)J_p^\nu u(\mathbf{p}_i, s_i)\right]\right\}$$

Non-relativistic approaches

Relativistic vs non-relativistic approaches

- **KINEMATICAL EFFECTS:**

  Relativistic (4x4) current operator vs non-relativistic (2x2) reductions. Non-relativistic reduction in powers of: $\frac{q}{m_N}, \frac{\omega}{m_N}, \frac{p}{m_N}$*?*

- **DYNAMICAL EFFECTS:**

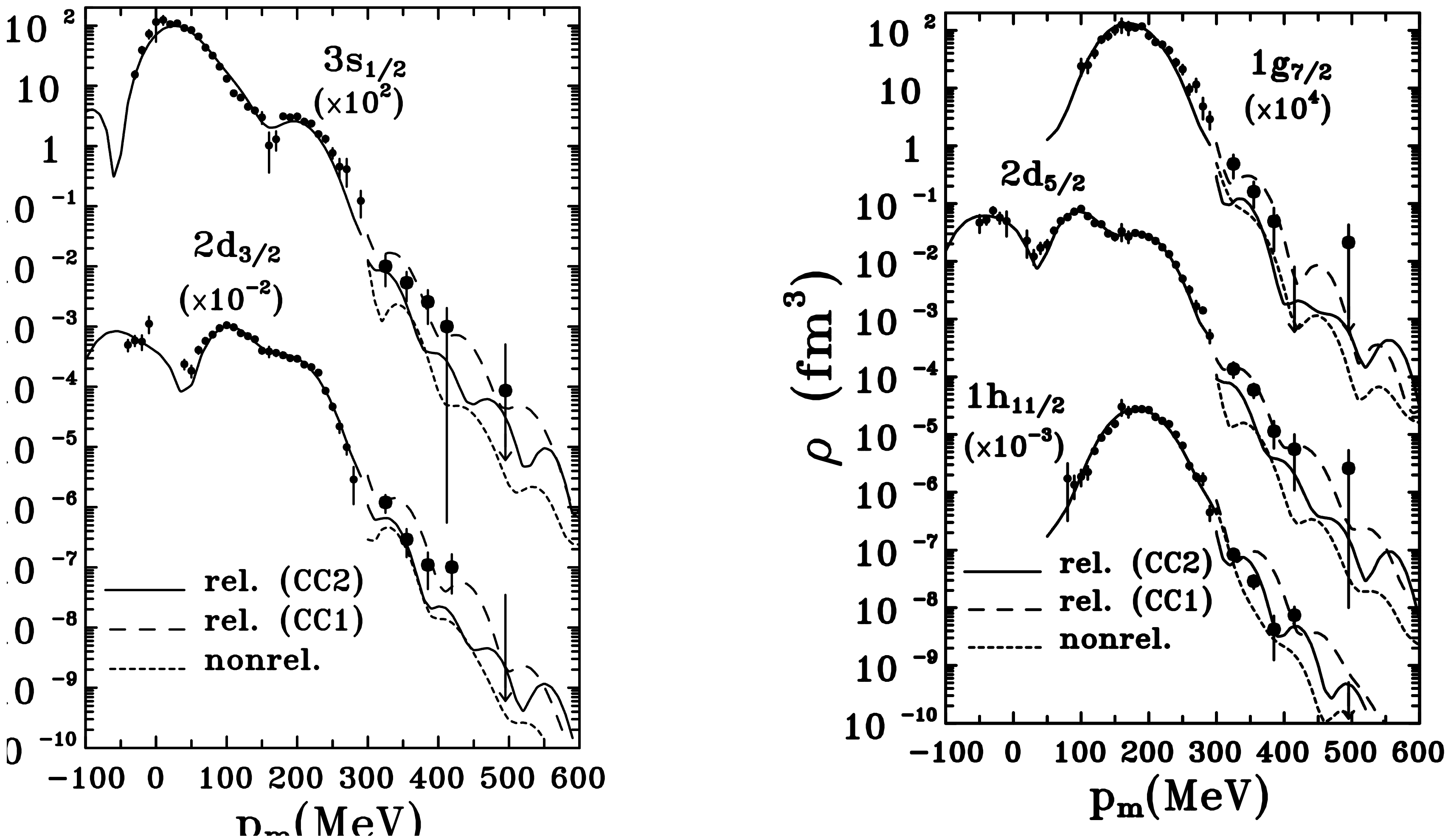
  Relativistic vs non-relativistic wave functions:

  - *Upper components of* Ψ *vs Schrödinger solutions* $\Rightarrow$ *Darwin Factor*
  - *Lower components of* Ψ $\Longrightarrow$ *Dynamical enhancement due to* **(S-V)** *relativistic potentials*

THE RIA MODEL APPLIED TO $(e, e'N)$ REACTIONS

Analysis of reduced cross sections

- $p \leq 300$ *MeV/c*: *Darwin term. Larger spectroscopic factors*
- $p > 300$ *MeV/c*: *Dynamical enhancement of lower components*



Cross Section for ^{16}O at low-$|Q^2|$

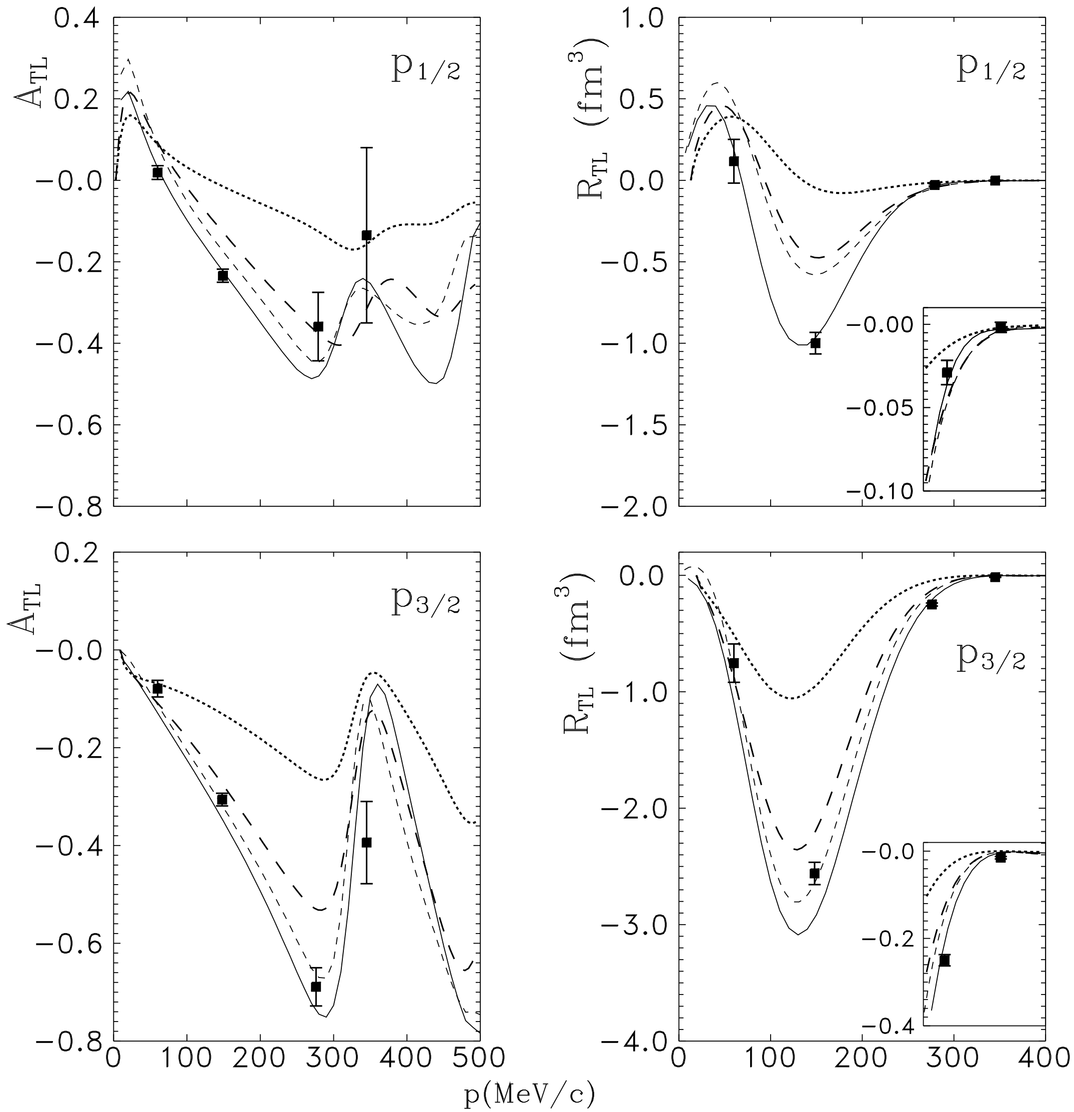
- TL **response &** A_{TL} **asymmetry at high-**$|Q^2|$
  - Kinematics:

    $\varepsilon_i = 2445$ MeV,
    $|\mathbf{q}| = 1$ GeV/c,
    $\omega \approx 439$ MeV,
    $|Q^2| = 0.8$ (GeV/c)^2

  - Observables:

$$A_{TL} = \frac{\sigma(0^o)-\sigma(180^o)}{\sigma(0^o)+\sigma(180^o)}$$

$$R^{TL} = \frac{\sigma(0^o)-\sigma(180^o)}{K v_{TL}}$$



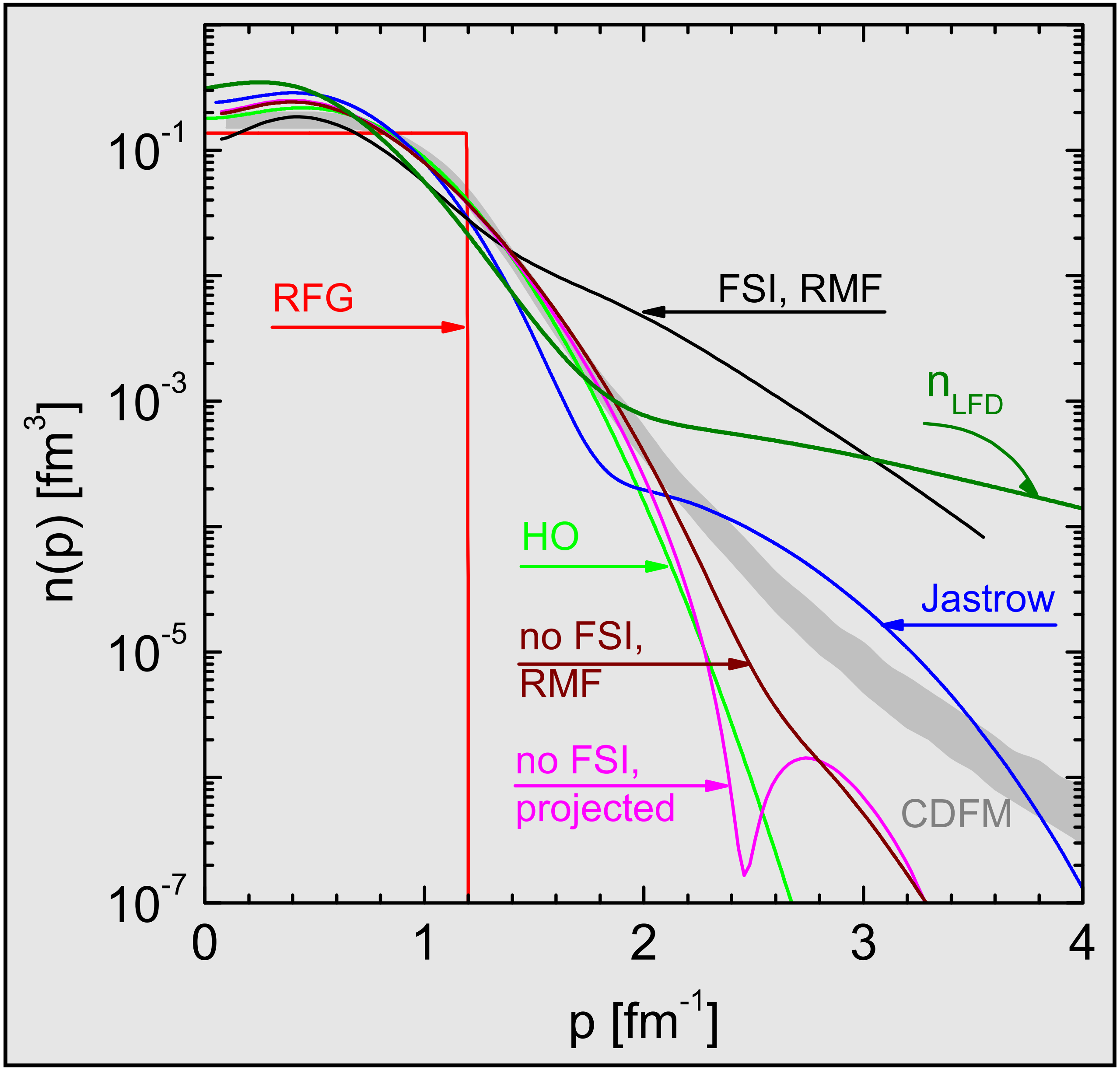
Solid: CC2

Dashed: PCC2

Long-dashed: NR+SO

Dotted: NR (no SO)

Reduced Cross Section and the RMF



Strong scalar & vector potentials: *Significant enhancement of the reduced cross section at high momentum. Role of lower components in the nucleon wave functions (mainly in the final state).*

Explicit correlations?

INCLUSIVE (e, e') PROCESSES & SCALING

The SuperScaling Approach (SuSA)

- *Scaling of the first kind below the QE peak ($\psi \leq 0$)*
- *Excellent scaling of the second kind in the same region*
- *Breaking of scaling above the QE peak ($\psi > 0$) $\Longrightarrow$ Effects beyond the IA (mainly located in the T channel)*
- **LONGITUDINAL RESPONSE SUPERSCALES**

The SuperScaling Approach (SuSA)

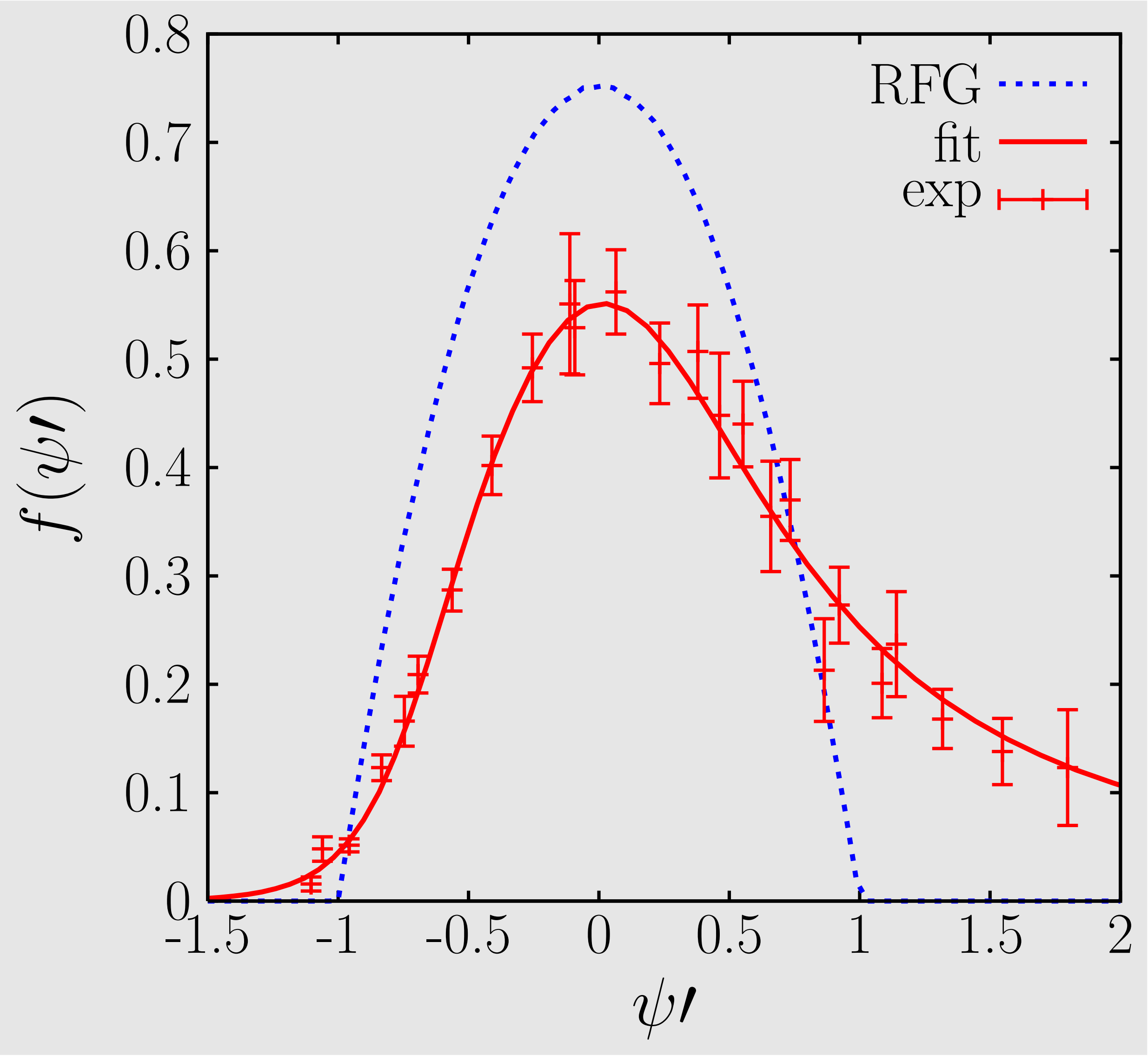
- *Scaling of the first kind be*

PRC60 (1999) 065502

PRL82 (1999) 3212

PRC65 (2002) 025502

- **LONGITUDINAL RESPO**



Experimental superscaling function: asymmetric shape with a long tail extended to positive ψ-values

$\Downarrow$

strong constraints to models

Nucleon wave functions in RIA

Solutions of Dirac equation with phenomenological relativistic potentials

- Ψ_B: **Bound nucleon wave function** $\Longrightarrow$
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Local potentials from a Lagrangian fitted to properties of nuclear matter and nuclei

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  Dependence with final state interactions (FSI)

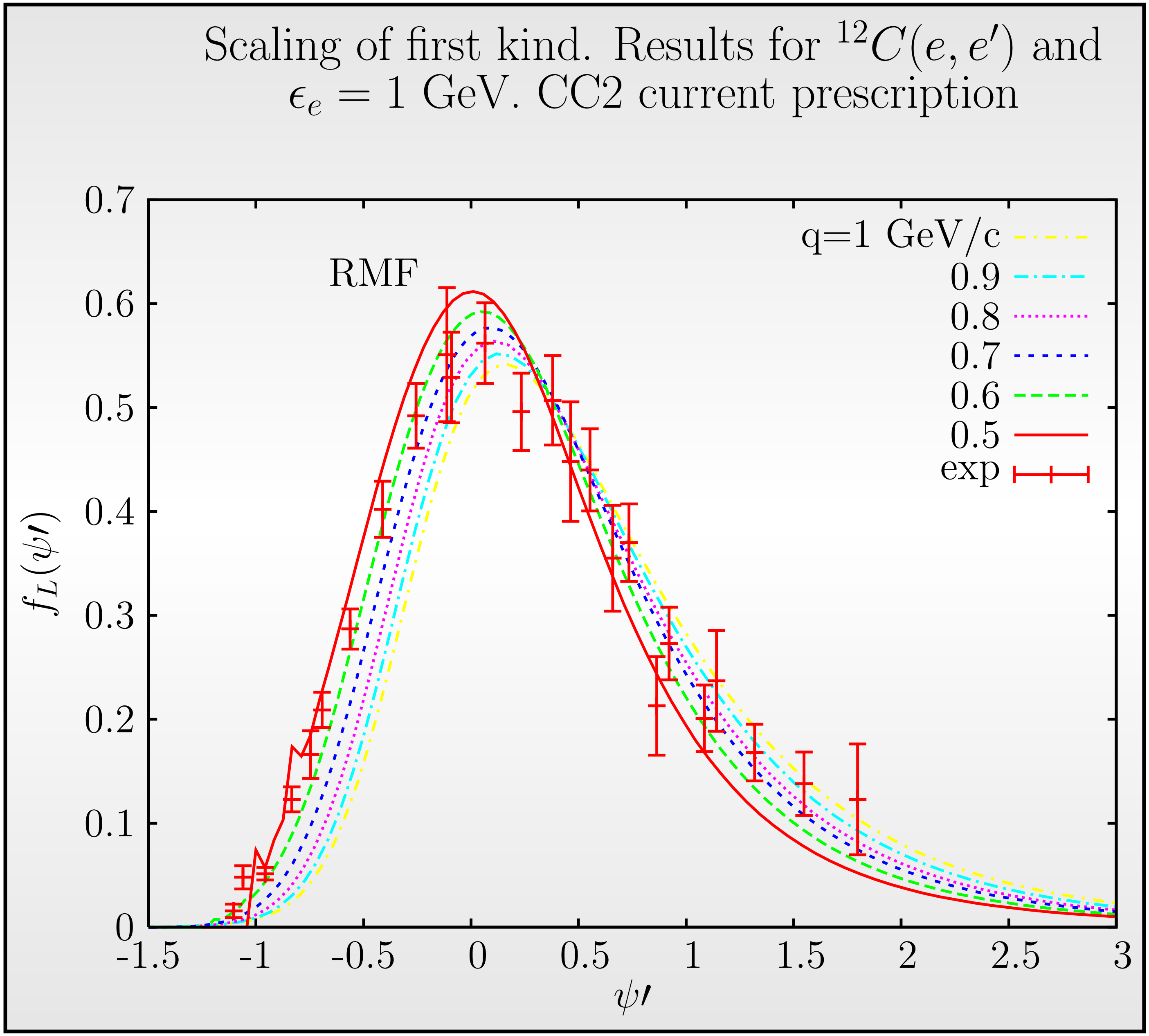
Retain inelastic channels $\Longrightarrow$ *Use of pure real potentials (no absortion)*

RMF $\Leftrightarrow$ **rROP** $\Leftrightarrow$ **RPWIA**

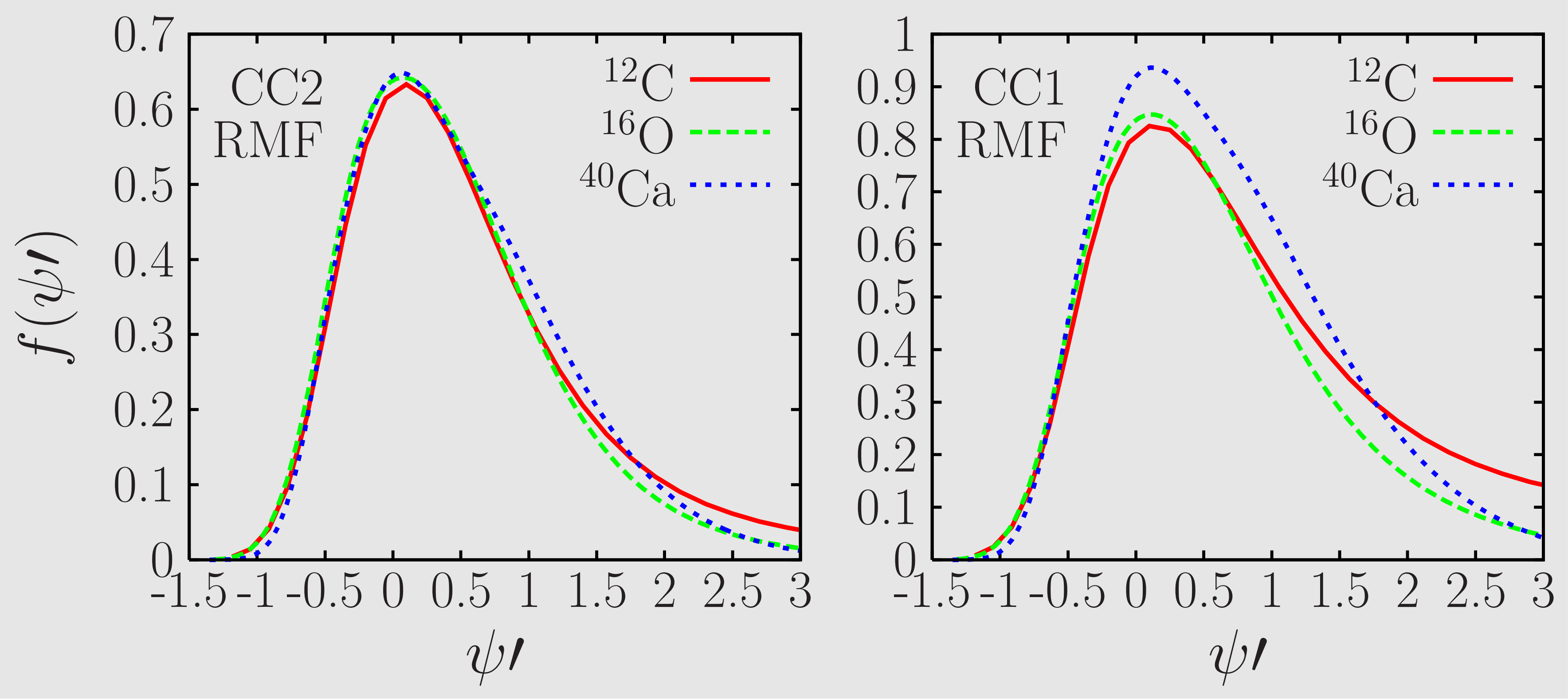
Description of FSI with the RMF approach:

- *Preserves orthogonalization property.*
- *Maintains the continuity equation (aprox.).*

How Scaling of the 1^{er} kind behaves (RMF)

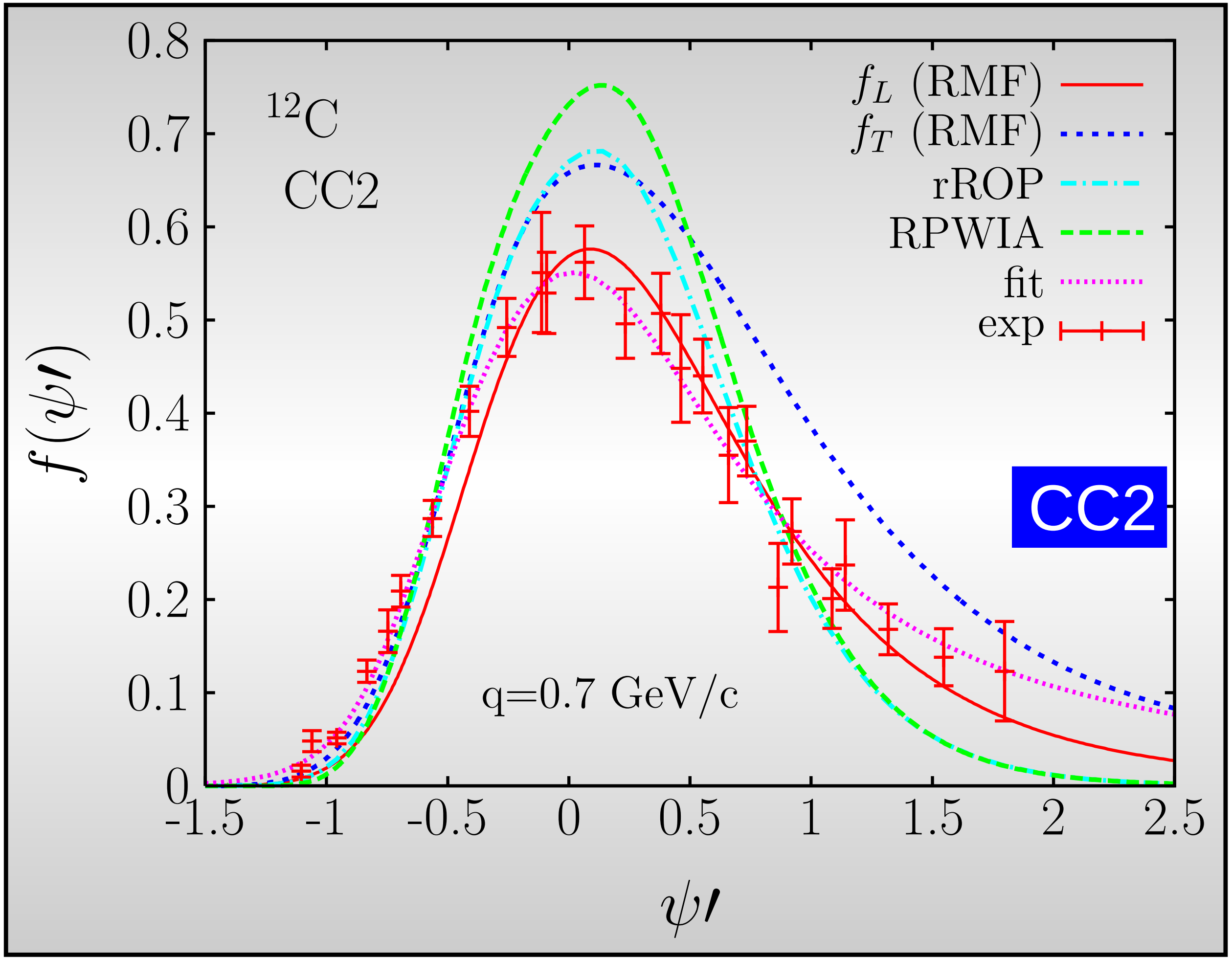


Scaling of the second kind in RIA



Scaling of 2^a kind: excellent with the CC2 current operator

RMF: Comparison with (e, e') data



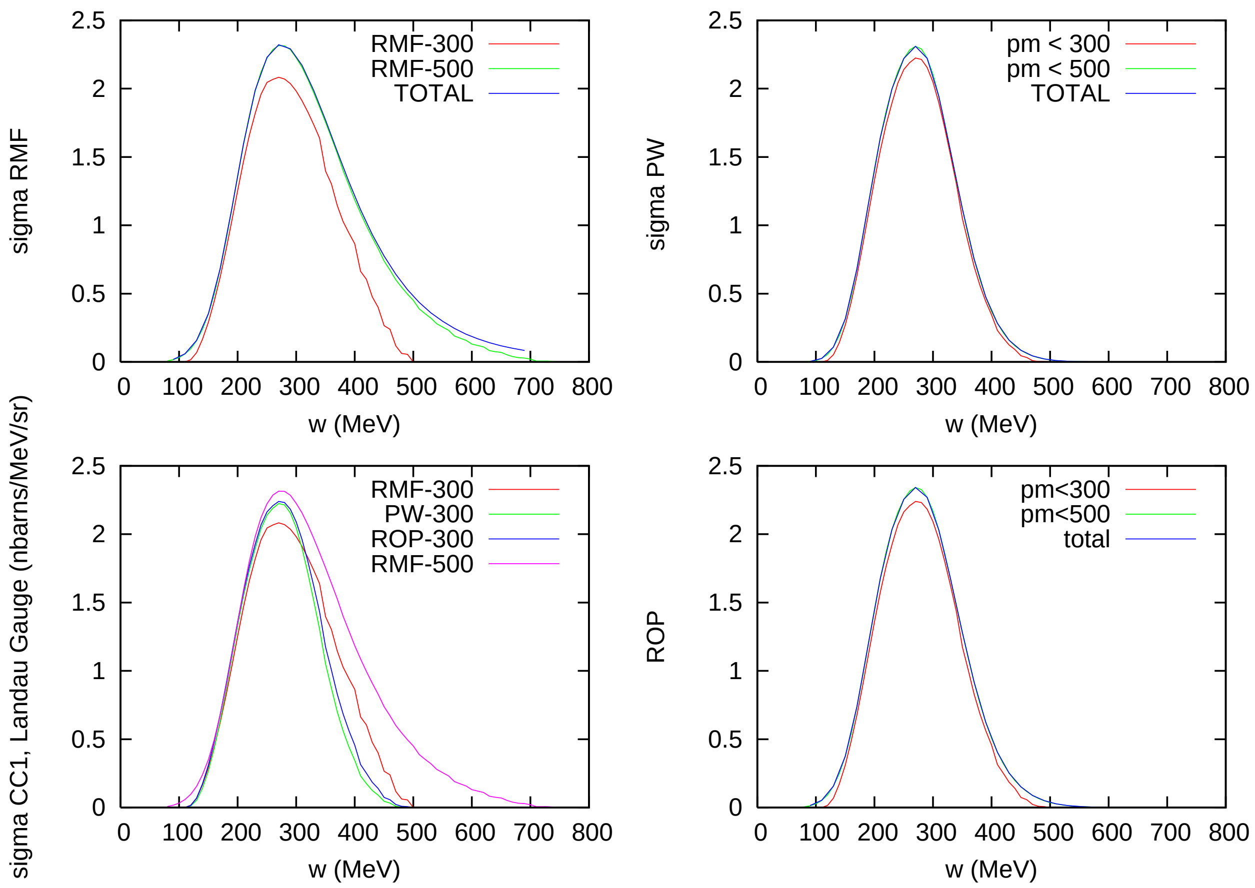
Only the description of FSI provided by RMF leads to an asymmetric function $f(\psi')$ in accordance with the behavior shown by data. Moreover, $f_T > f_L$

Asymmetry in the RMF approach

The RMF approach produces a shift of strength to higher missing momenta.

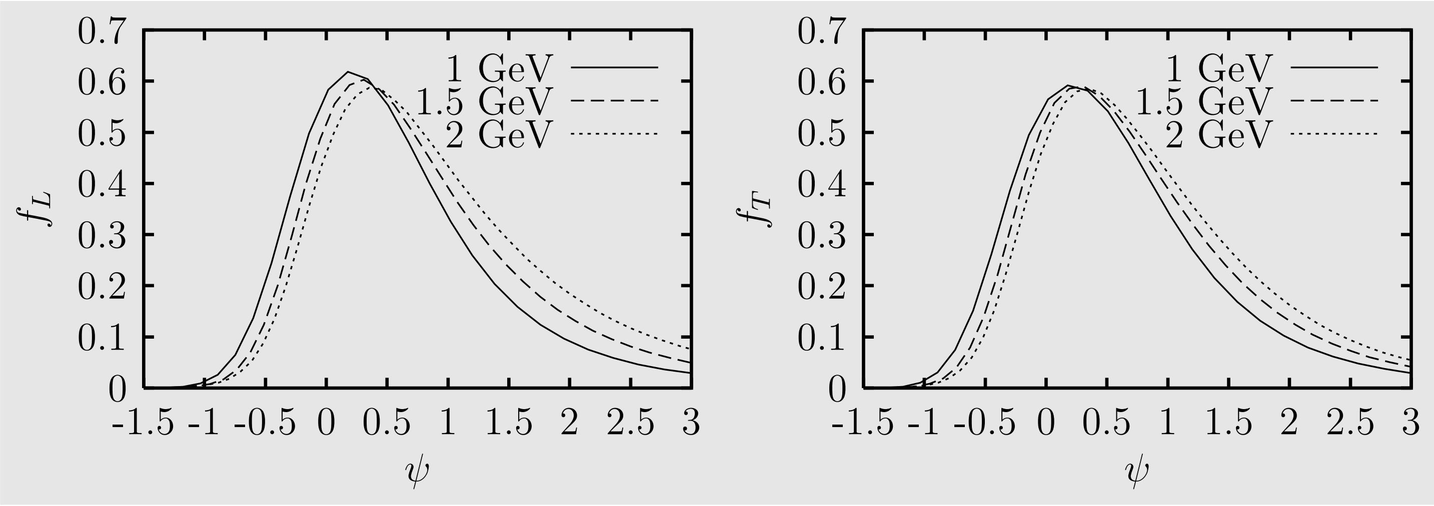
$$E_{eff} = E - V, \quad M_{eff} = M - S; \quad p^2_{eff} = p^2 + (V^2 - S^2) + 2M(S - V) - 2T_N V$$

When increasing T_N, the value of p_{eff} decreases due to the strong relativistic potentials



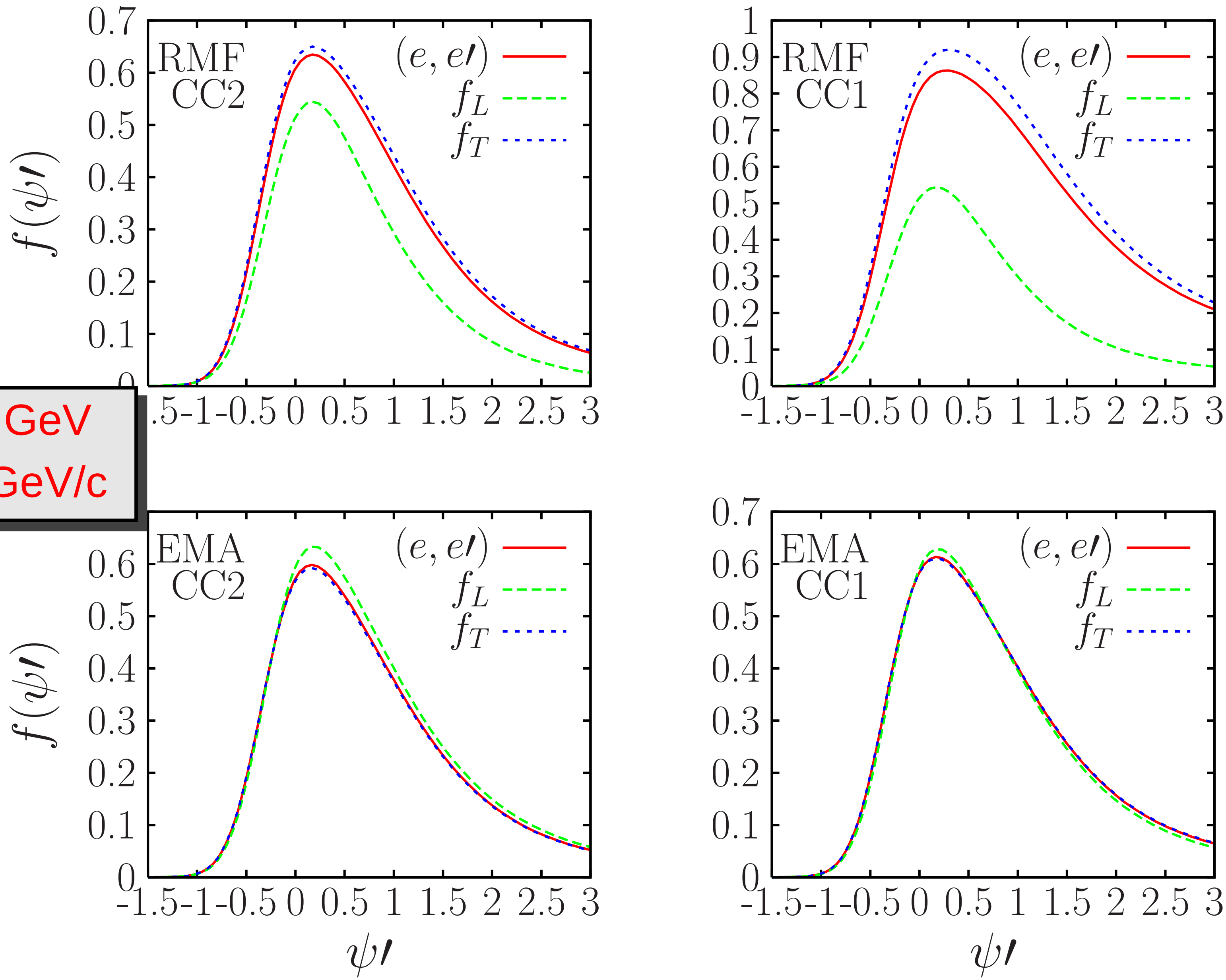
Other models: 0^{th} kind scaling & asymmetry

- RFG: by construction $f_L(\psi) = f_T(\psi) = f(\psi)$
- RPWIA: $f_L(\psi) = f_T(\psi) = f(\psi)$–symmetric
- Semi-relativistic (SR) and/or NR approaches with FSI:
  - *Woods-Saxon potential: symmetric scaling functions.*
  - *Dirac Equation-Based (DEB) potential: leads to an asymmetric function $f(\psi)$.*



In all cases: $f_L(\psi) = f_T(\psi) = f(\psi)$

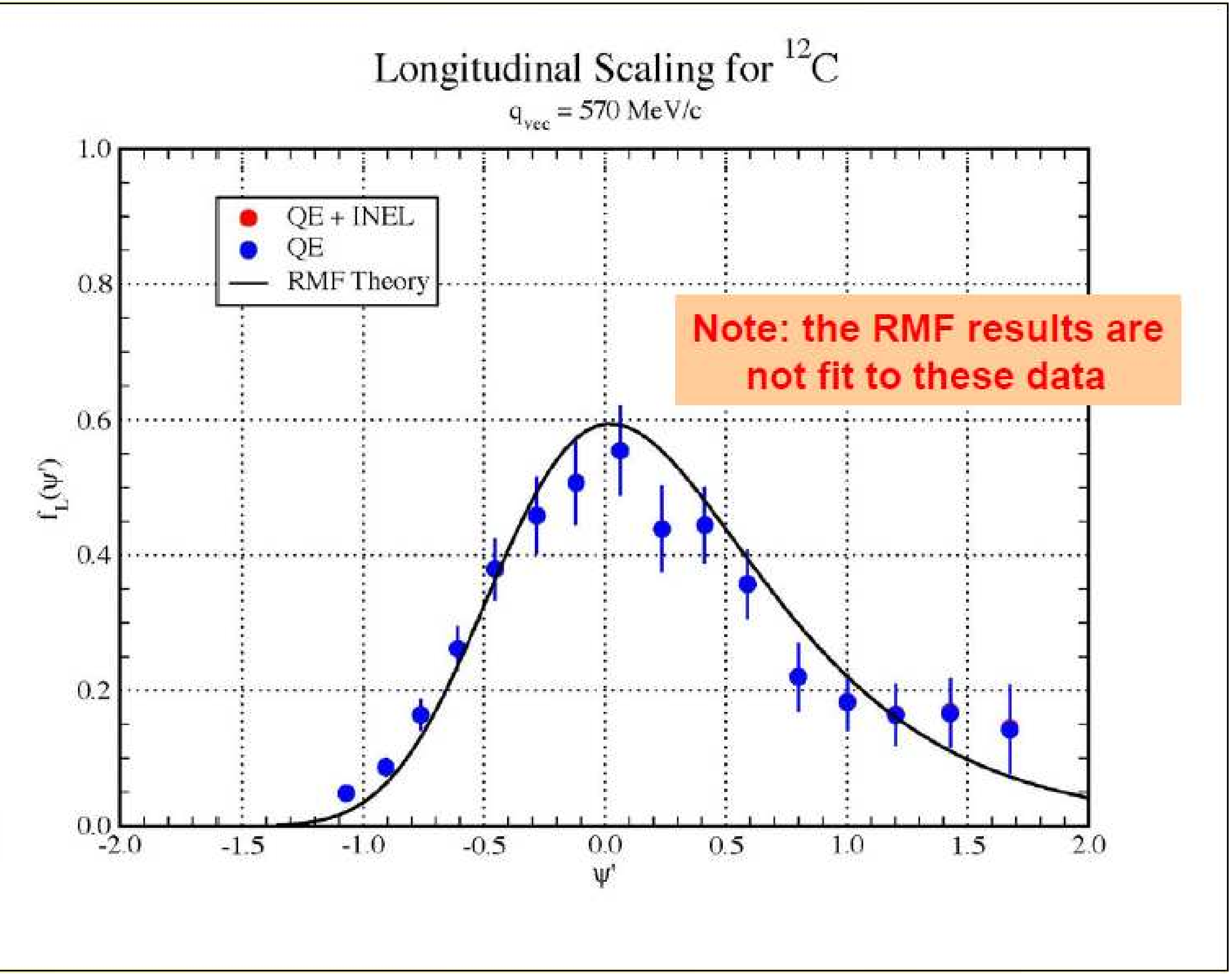
Scaling of the 0^{th} kind in RMF: T enhancement



$\varepsilon_e = 1$ GeV

$q = 1$ GeV/c

Scaling in QE L/T-channels



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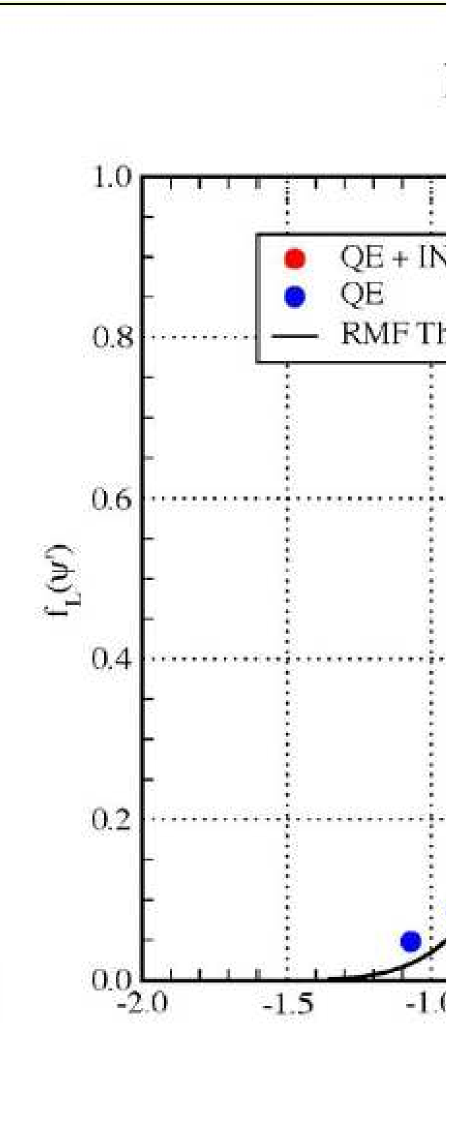
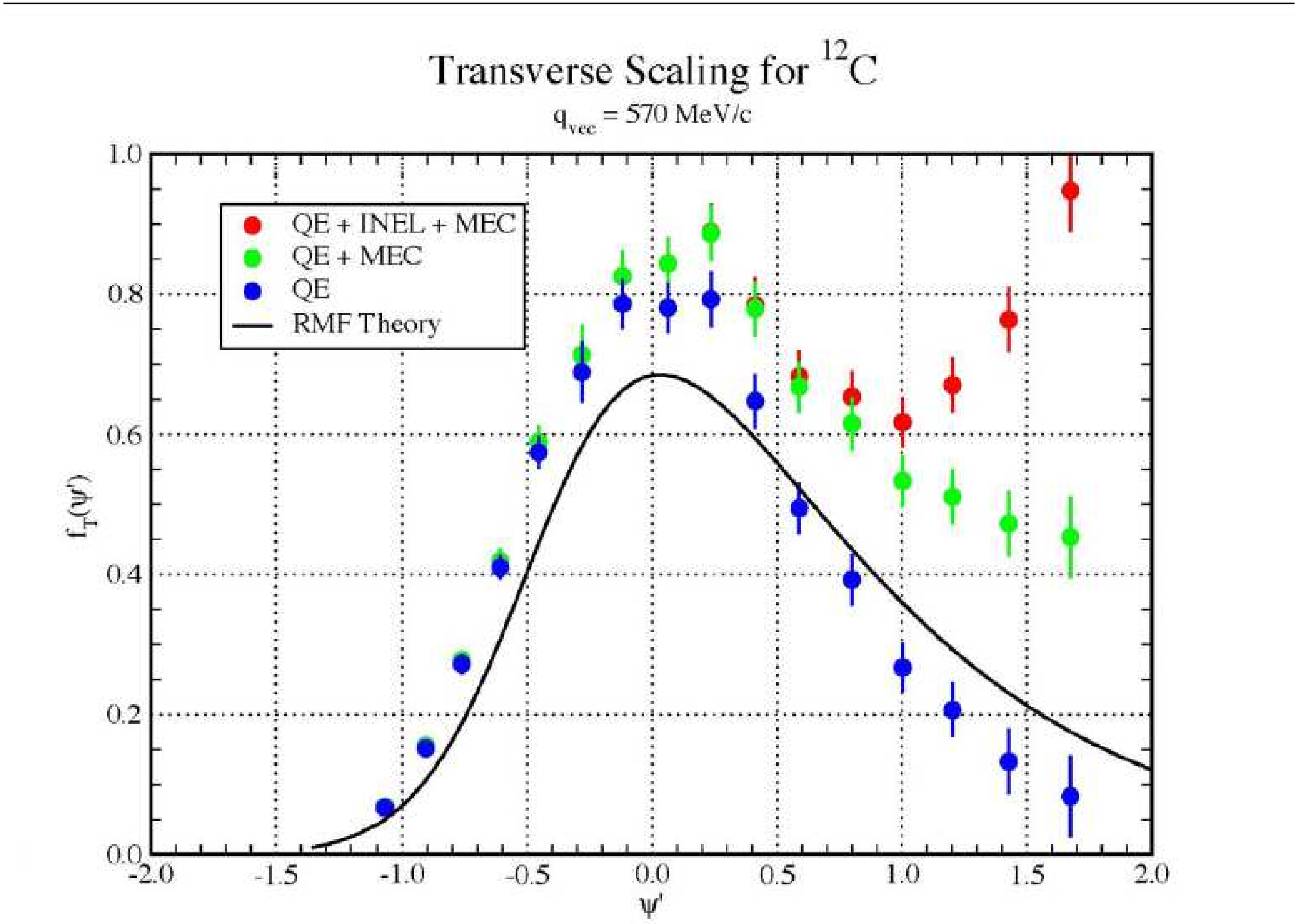
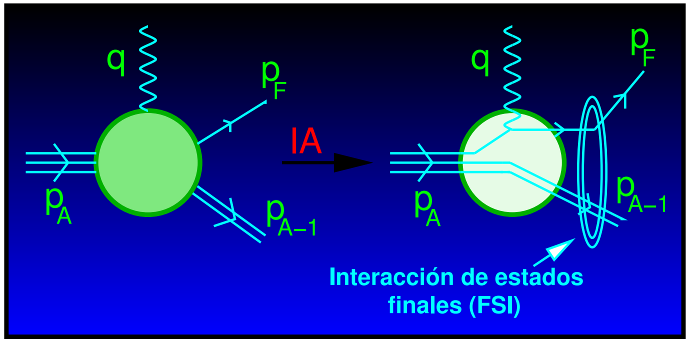




INTERACTION OF NEUTRINOS with NUCLEI

RELATIVISTIC IMPULSE APPROXIMATION



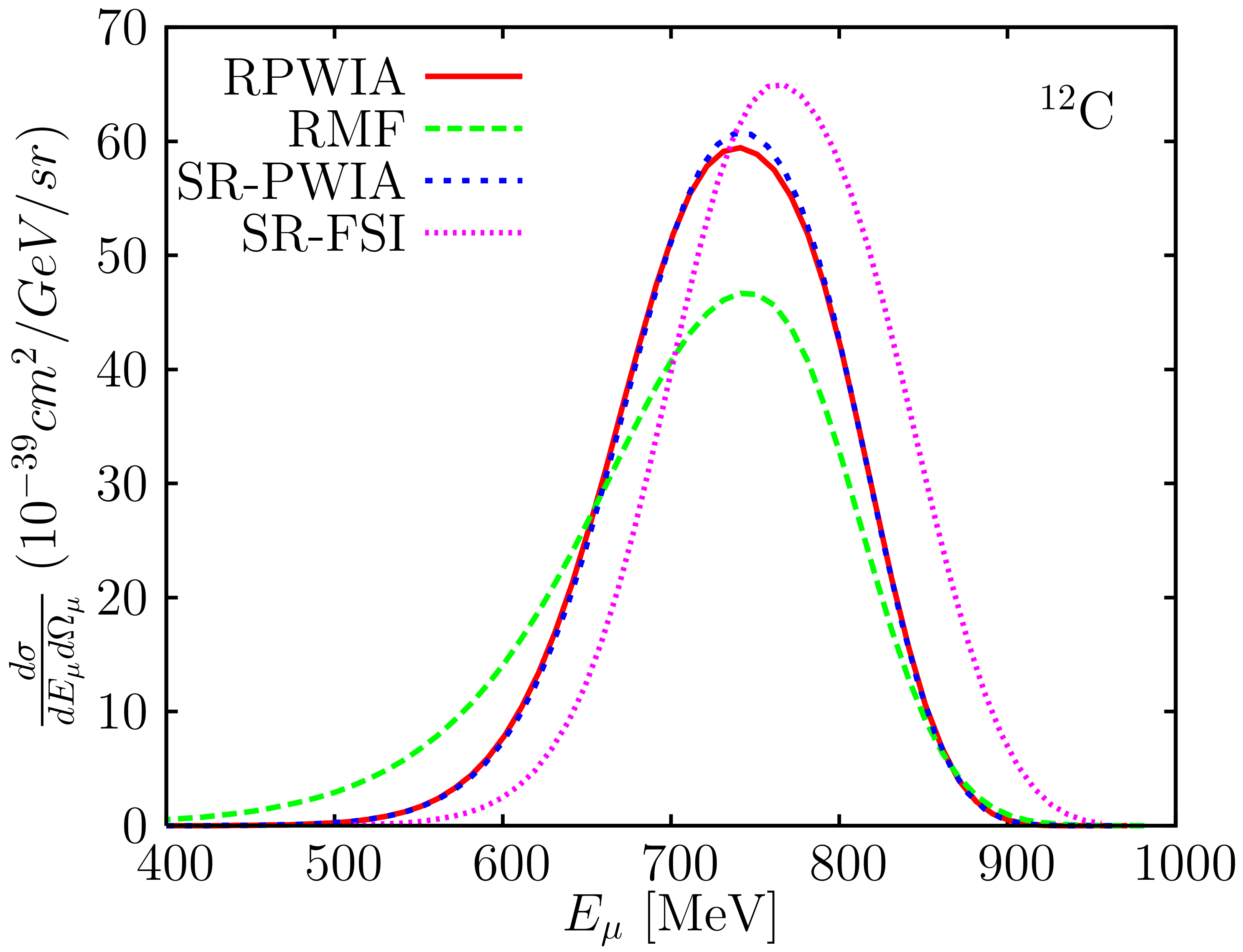
- *Incident neutrino interacts with only one nucleon which is then emitted, while the $(A-1)$ remaining nucleons are simply spectators.*
- *The weak nuclear current is the sum of single nucleon currents.*
- *Target and residual nuclei can be adequately described within an independent particle model.*

Weak single-nucleon current operator

$$\hat{J}^{\mu}_{wsn} = \hat{J}^{\mu}_{V} - \hat{J}^{\mu}_{A} = \tilde{F}_1\gamma^{\mu} + \frac{i\tilde{F}_2}{2m_N}\sigma^{\mu\nu}Q_{\nu} + G_A\gamma^{\mu}\gamma^5 + \frac{G_P}{2m_N}Q^{\mu}\gamma^5$$

- **Neutral Currents**: *Strangeness content in $\tilde{G}_{E,M,A}$ and dependence with Weinberg angle (no G_P)*
- **Charge-changing Currents:** *Pure isovector form factors $\tilde{F}_i^V = (F_i^p - F_i^n)$*

RIA & SR approximations. FSI effects



RMF applied to (ν, μ) & Scaling

PROCEDURE

- *Evaluate the inclusive* (ν, μ) *cross section with a specific RIA model and divide it by the corresponding single-nucleon cross section [weighted by the appropriate proton* (Z) *and neutron* (N) *numbers]* $\Longrightarrow$ **THEORETICAL SCALING FUNCTION**
- *Does the theoretical RIA scaling function satisfy scaling properties?*
  - *Scaling of the first kind:* $f(q,\psi) \stackrel{q\to\infty}{\longrightarrow} f(\psi)$
  - *Scaling of the second kind:* $f(\psi)$ – *independent on the nucleus*

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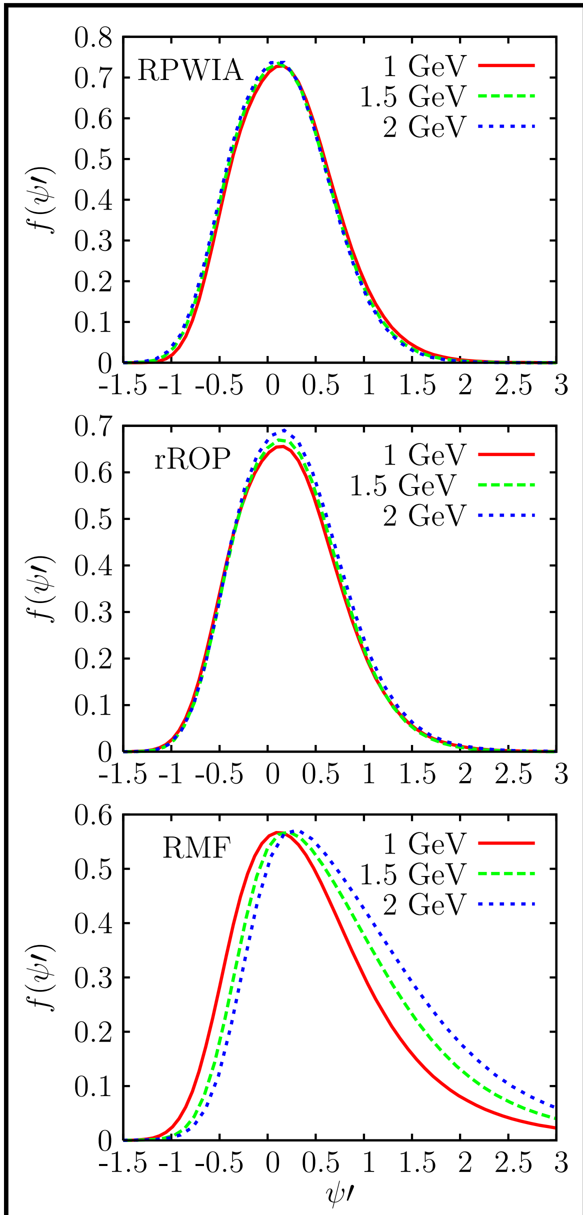

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$\nu, \mu)$ *cross section with a specific* ... *he*

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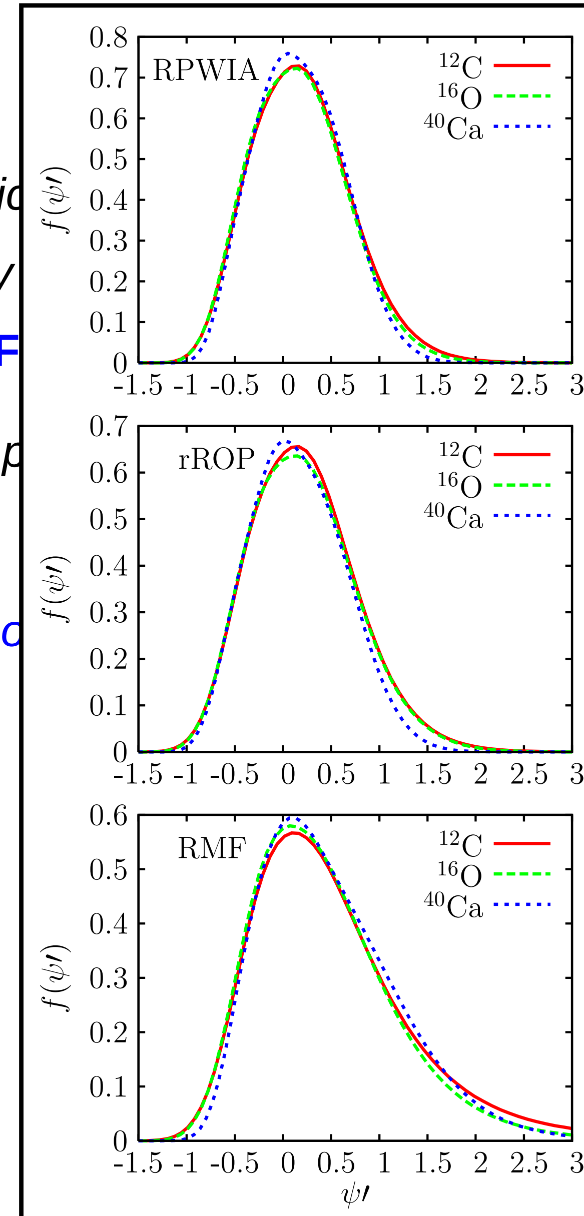

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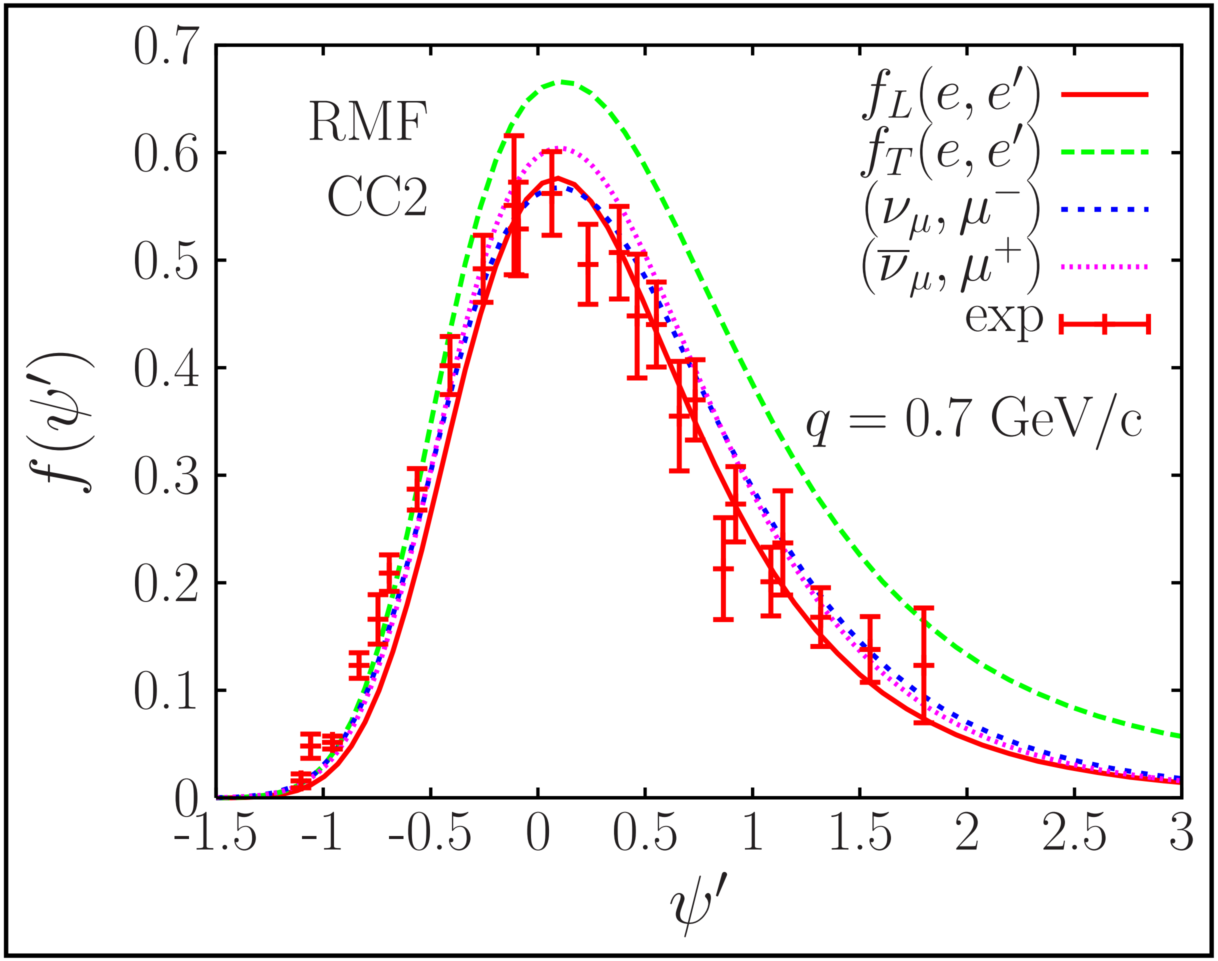
  - *Scaling of the first kind:* $f(q, \psi) \stackrel{q \to \infty}{\longrightarrow} f(\psi)$
  - *Scaling of the second kind:* $f(\psi)$ – *independent on the nucleus*
- *Is the function $f(\psi)$ obtained from (ν, μ) cross sections evaluated within RIA consistent with the function $f(\psi)$ obtained from (e, e') calculations (with the same model)?, and with $f_{exp}(\psi)$?*

Similar scaling function $f(\psi)$ for (e, e') and (ν, μ) processes?

(e, e') vs (ν, μ). SuSA vs RMF



Basic result: *the function $f(\psi)$ evaluated for (ν, μ) processes agrees better with the contribution $f_L(\psi)$ [corresponding to (e, e')] than with $f_T(\psi)$.*

BASIC CONCLUSIONS: (e, e') & (ν_μ, μ)

ELECTRON SCATTERING

- *Superscaling shows up in RIA, even with FSI: rROP & RMF.*
- *RMF-FSI description leads to an asymmetric superscaling function which fits data.*
- *Some first-kind scaling violation with FSI switched on.*

Contrary to most non-relativistic models, $f_L(\psi) \neq f_T(\psi)$ within RMF

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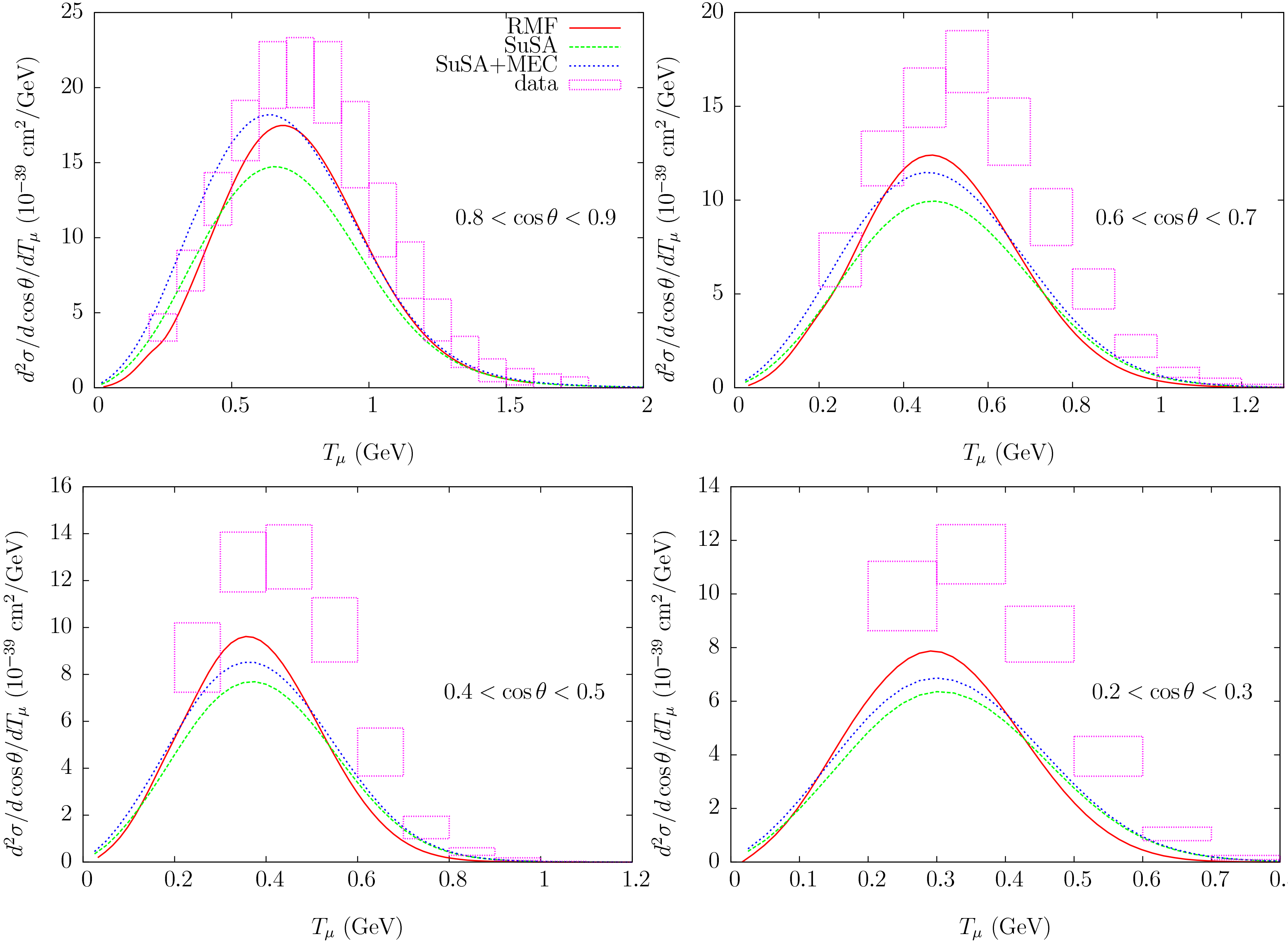
NEUTRINO SCATTERING

- *Superscaling fulfilled by RIA calculations.*
- *Scaling functions from QE (e, e') and (ν, μ) cross sections follow similar trends.*
- *Differences between (e, e') and (ν, μ) RIA+RMF results are consistent with the isoscalar/isovector nucleon f.f. contributions in the two processes.*

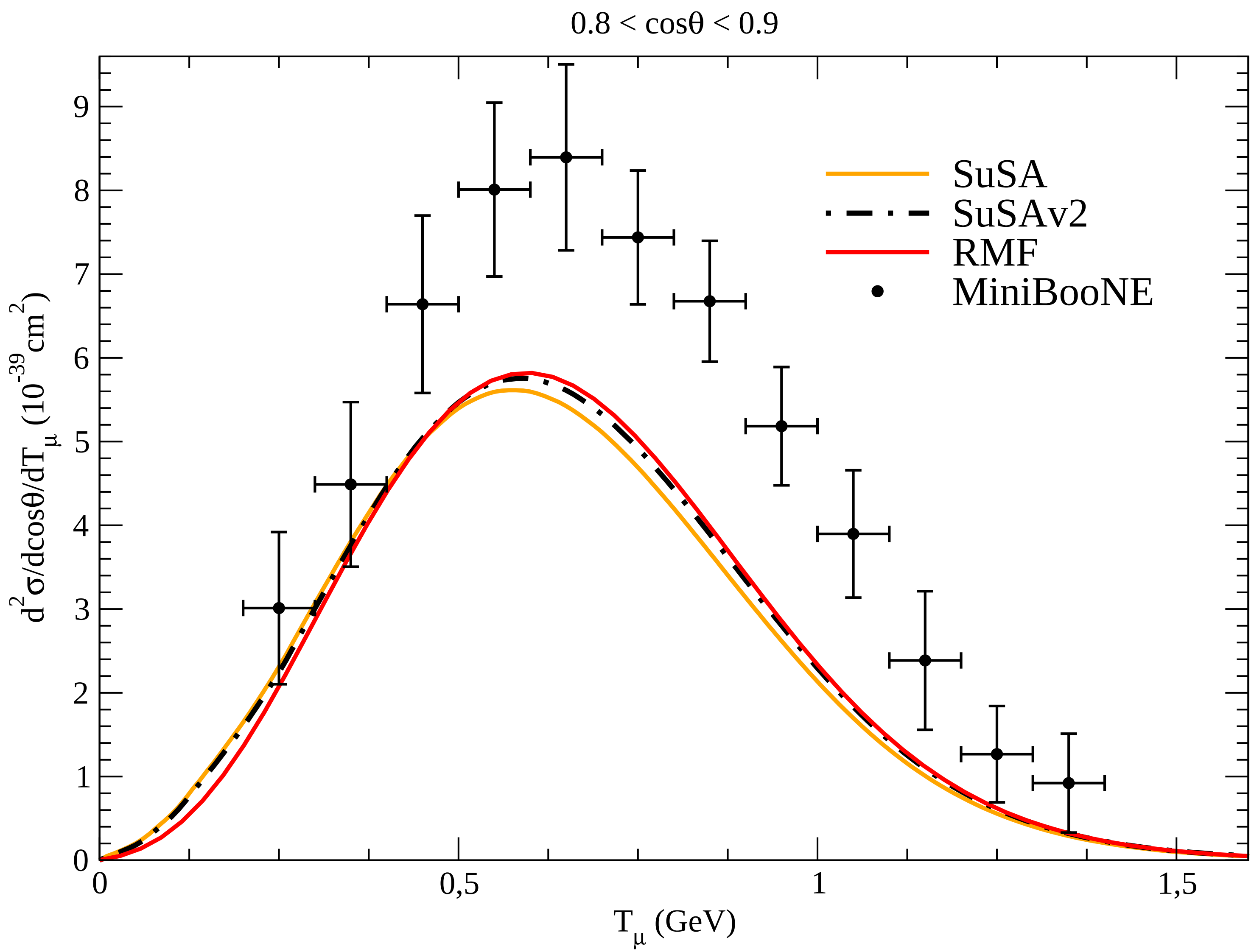
COMPARISON WITH DATA:

MiniBooNE, MinerνA, NOMAD & T2K

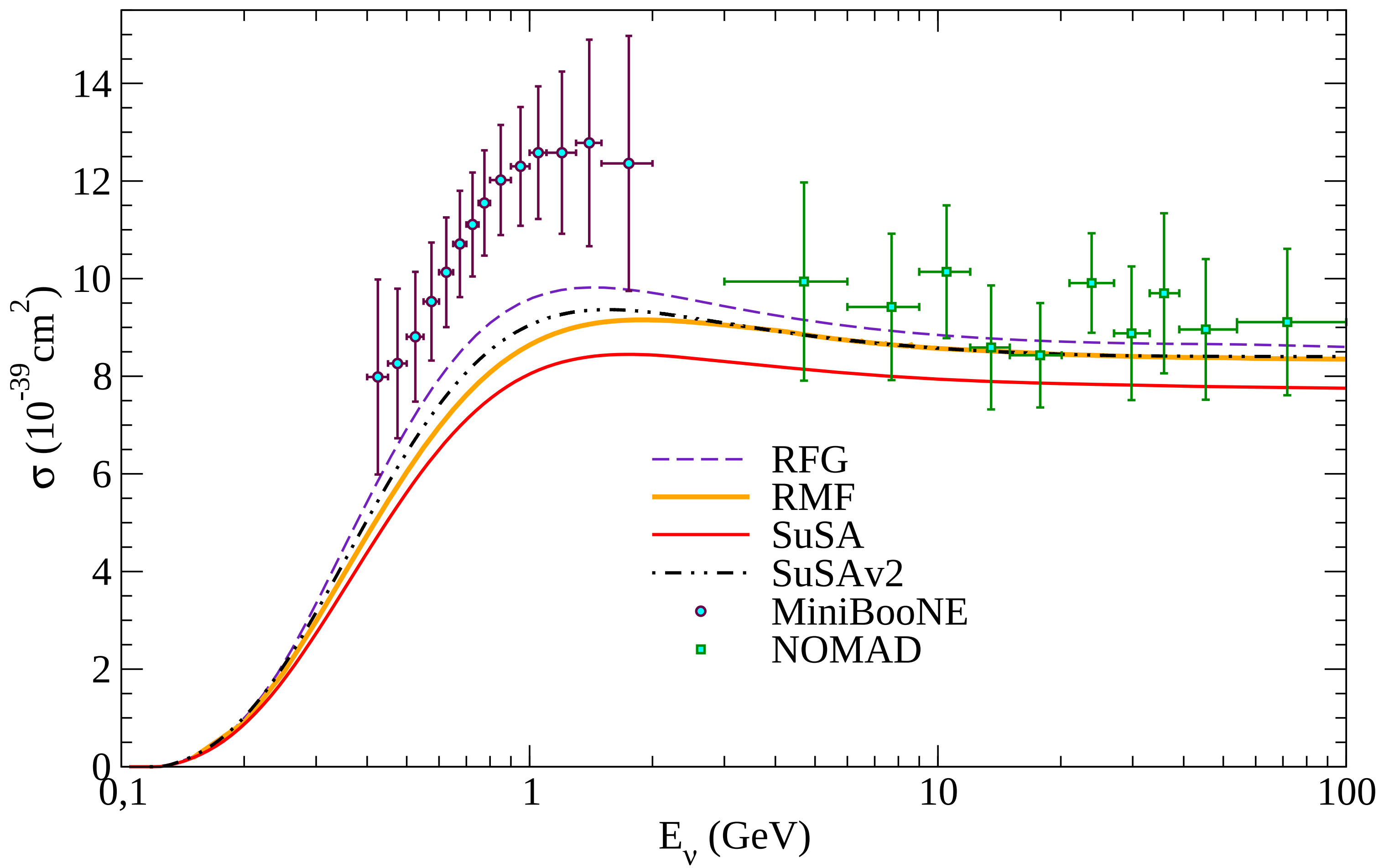
Flux-averaged double-differential CCQE: SuSA & RMF



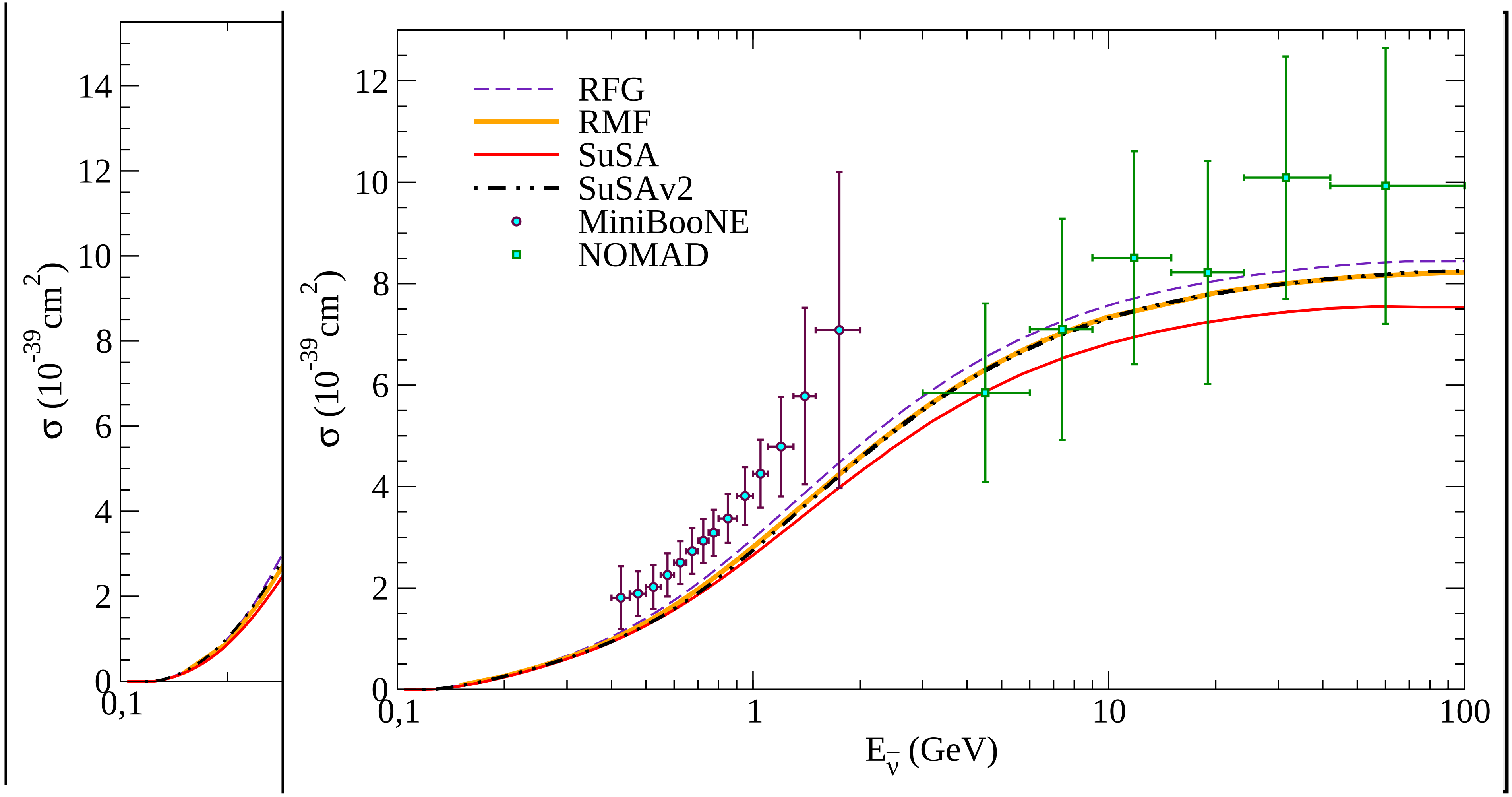
Antineutrinos: strong enhancement of 2p2h effects?



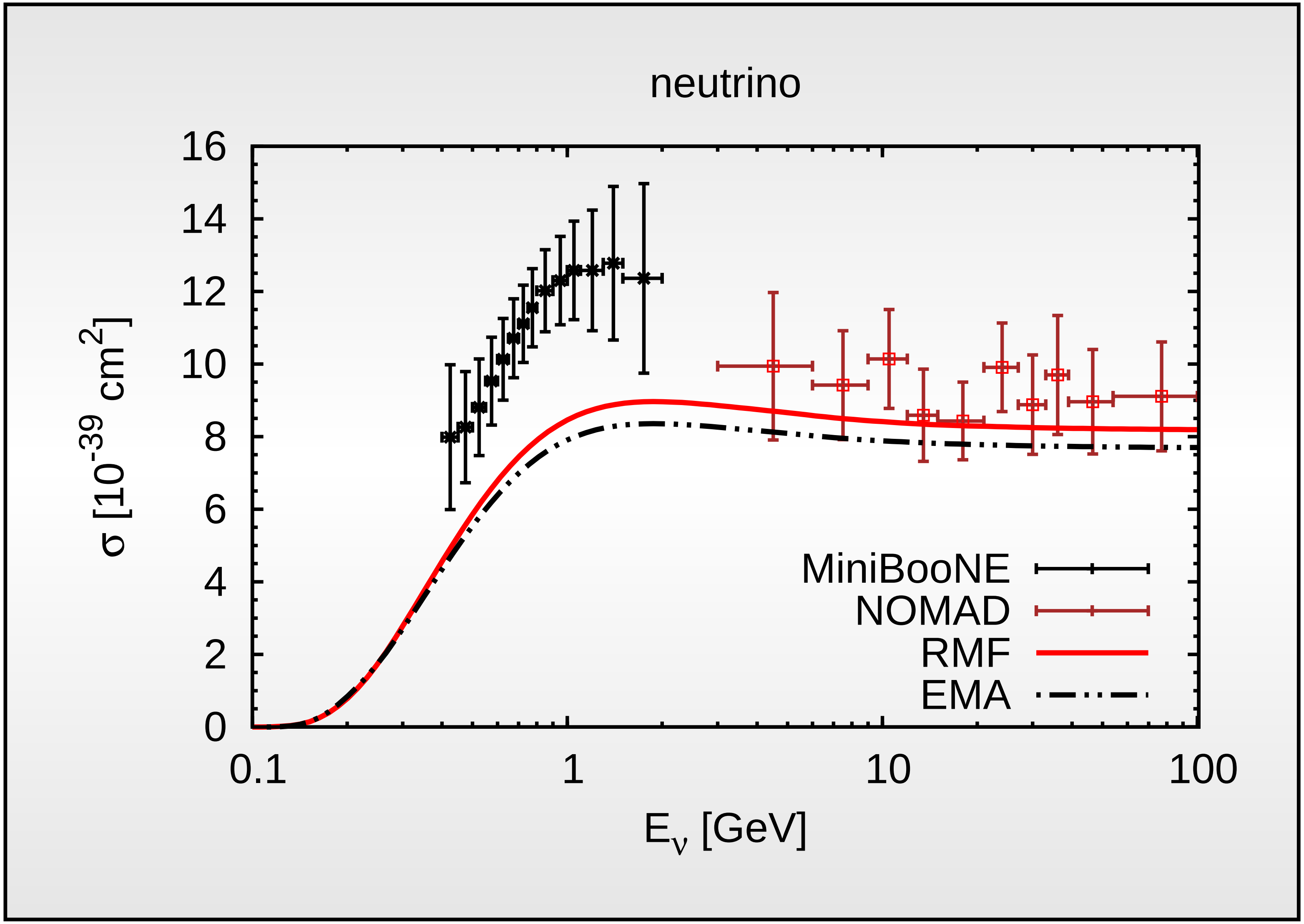
MiniBooNE & NOMAD: SuSA *vs* RMF



MiniBooNE & NOMAD: SuSA *vs* RMF



NOMAD: spinor distortion in RMF



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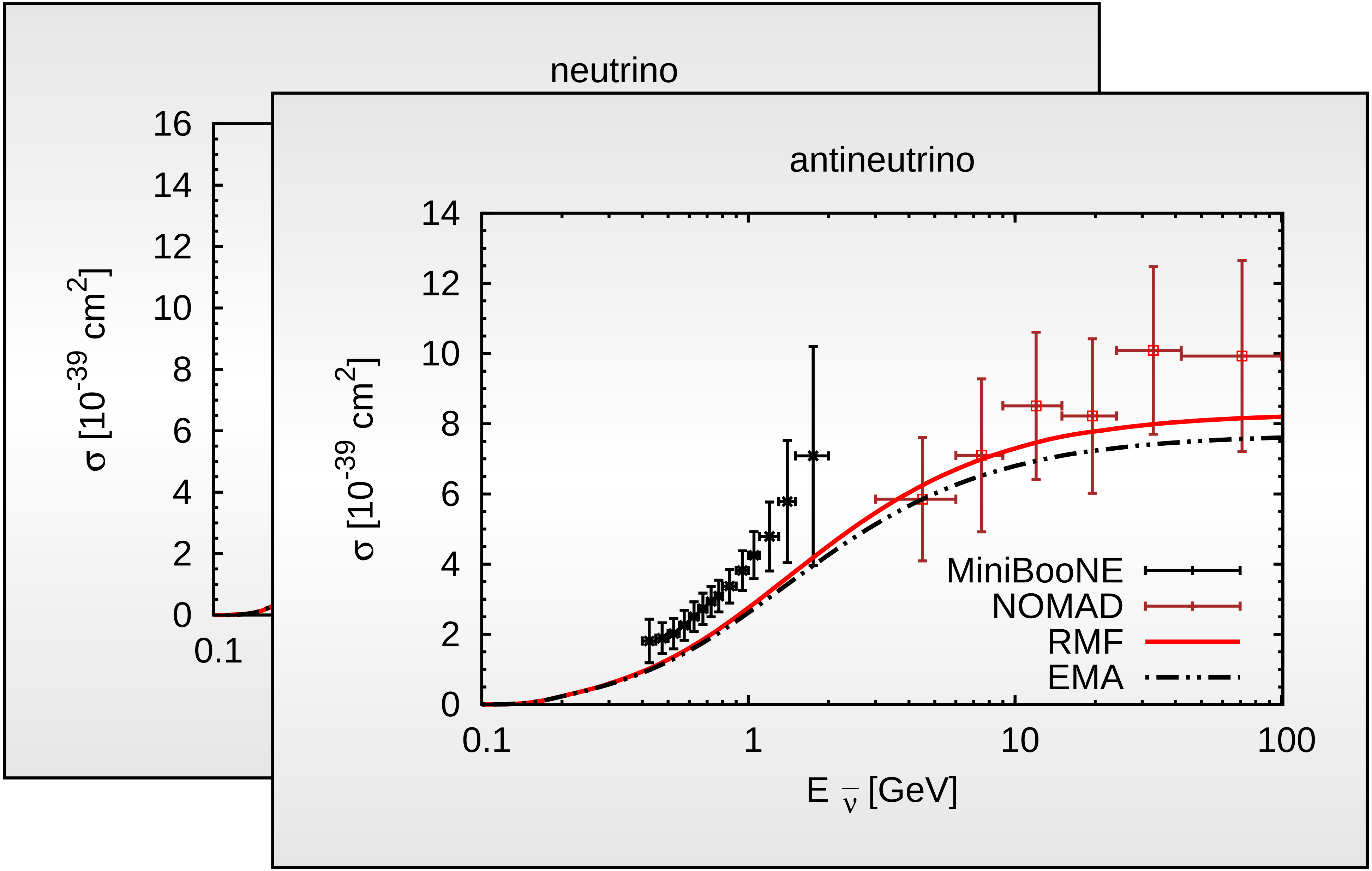
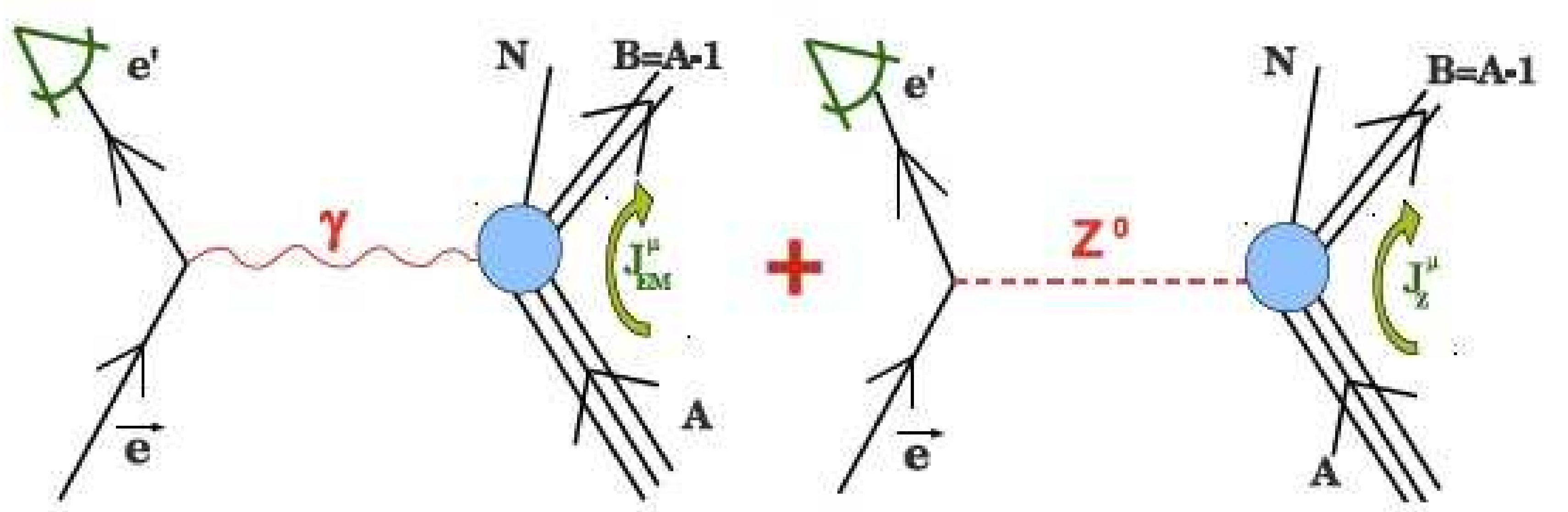


Feynman diagrams in first Born approximation:



Scaling in PV (e, e') responses

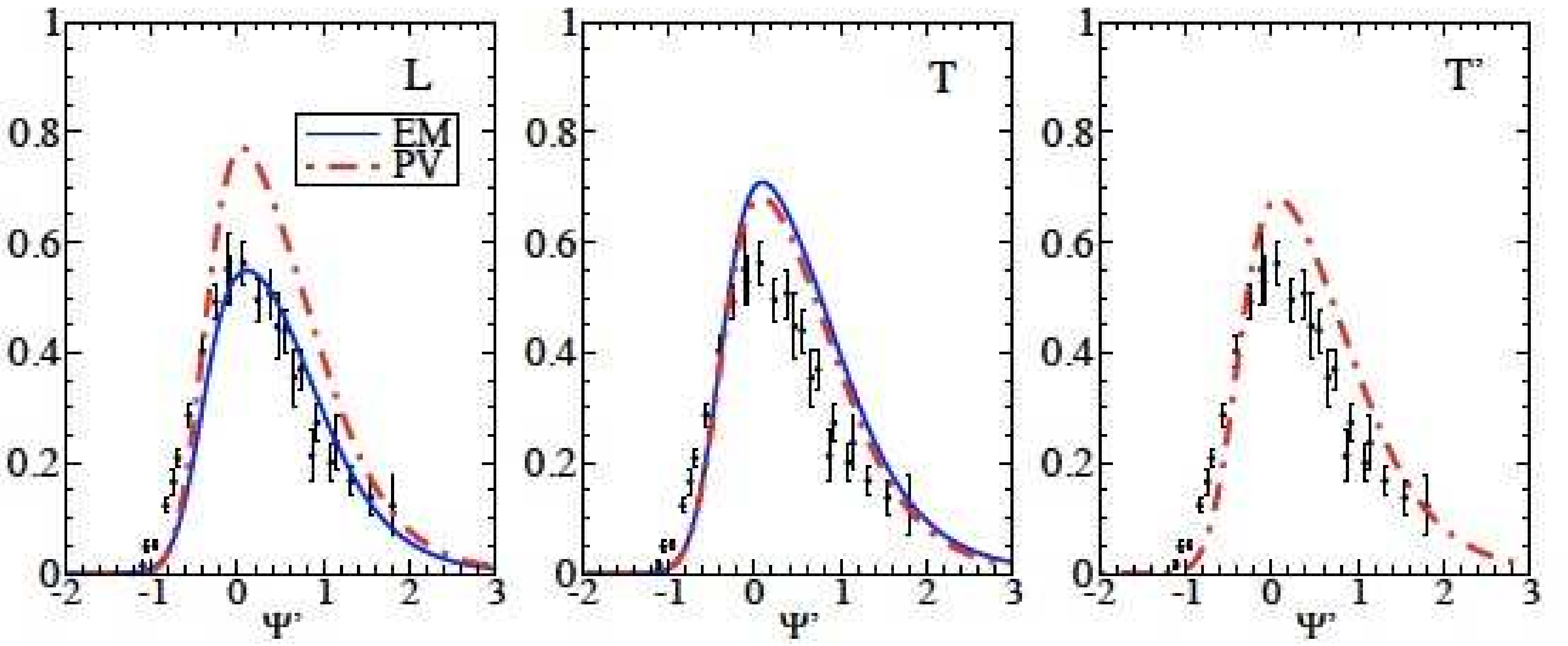


Figure: Experimental longitudinal scaling function compared with RMF-FSI predictions.

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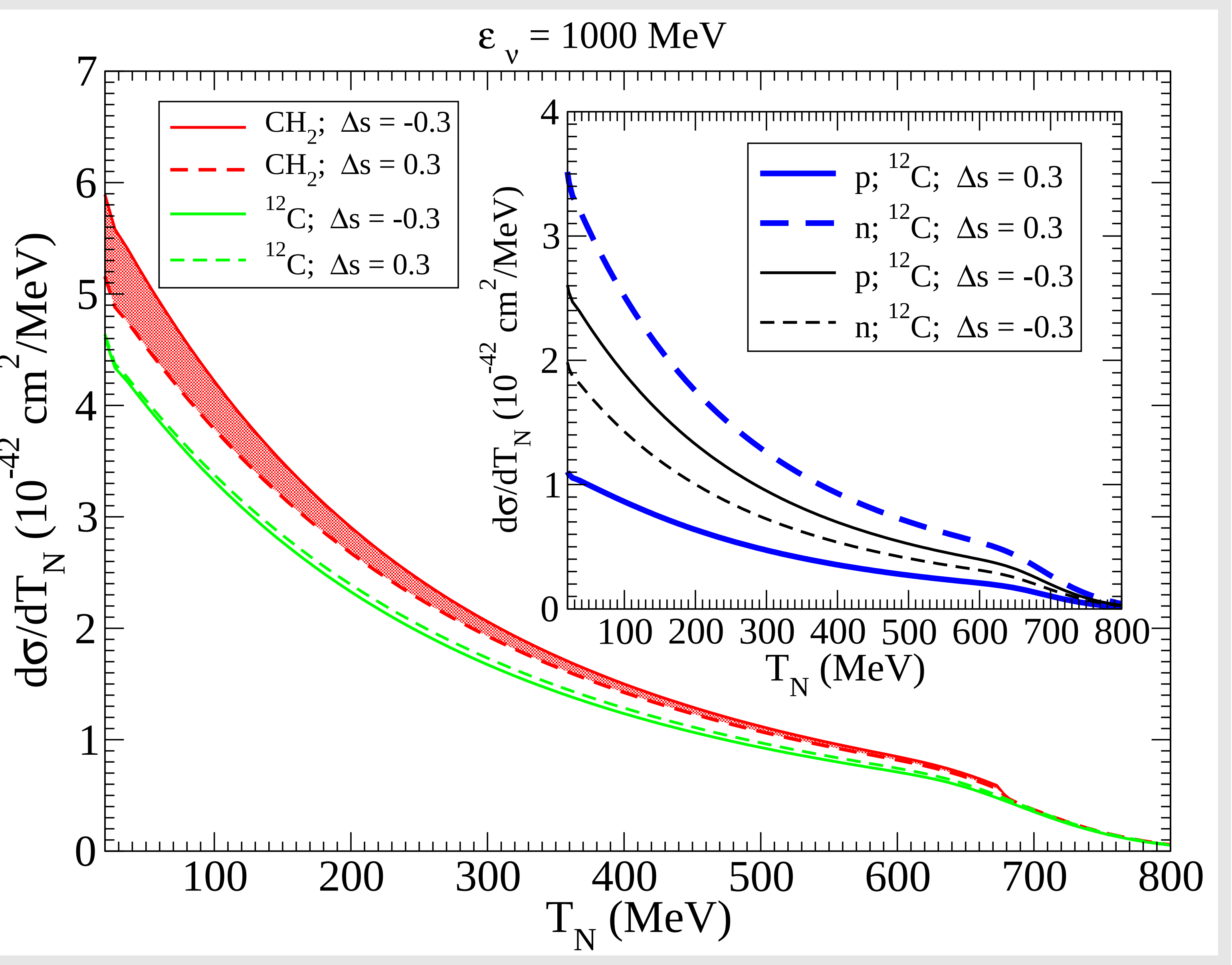
APPLICATION TO NEUTRAL CURRENT NEUTRINO PROCESSES: $(\nu, N)\nu'$

NCQE & axial strangeness



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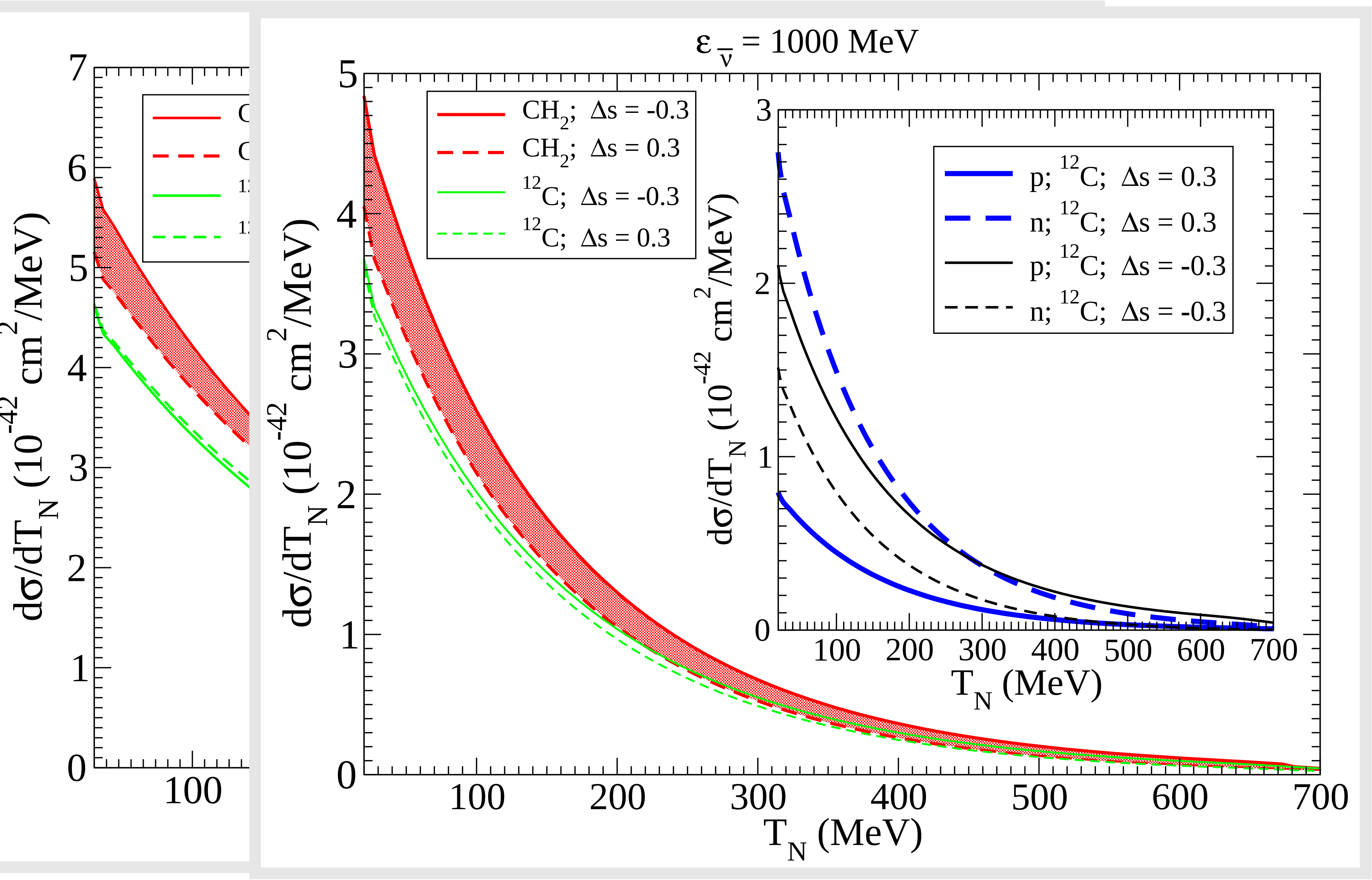
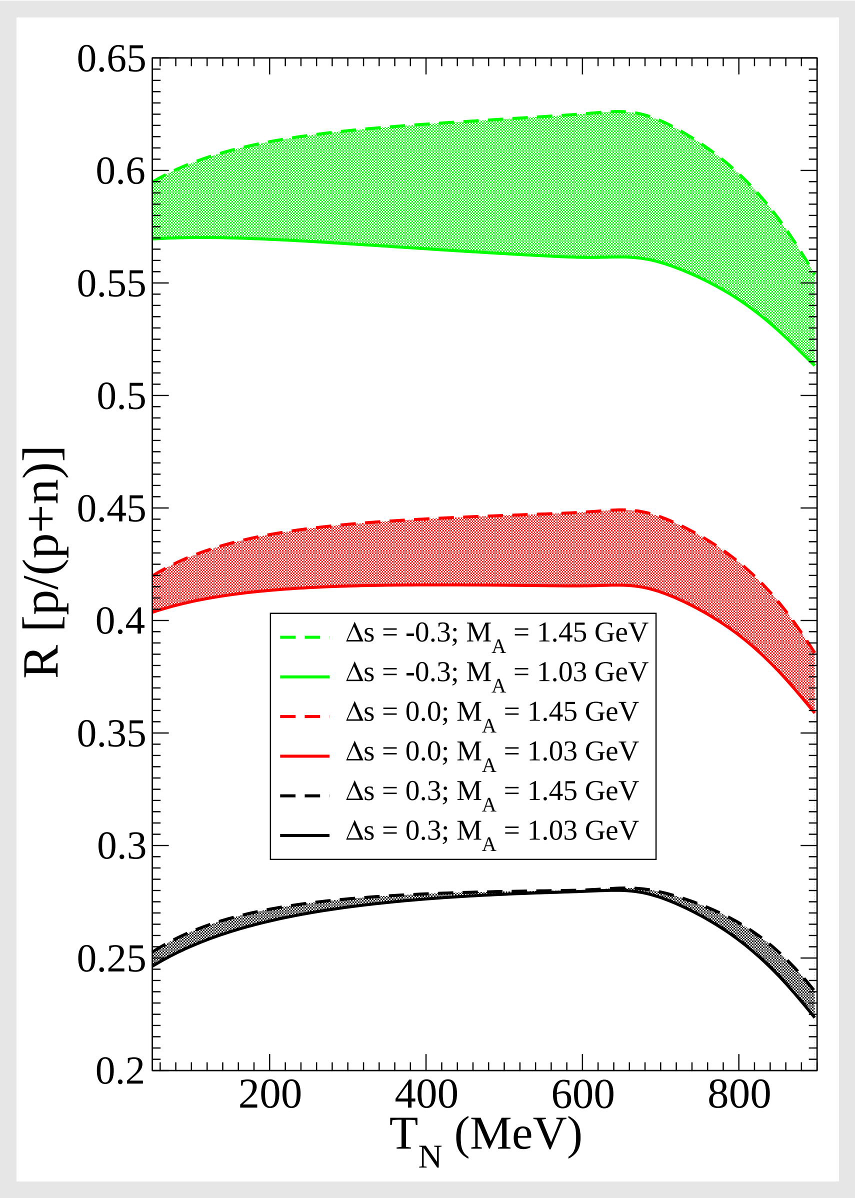
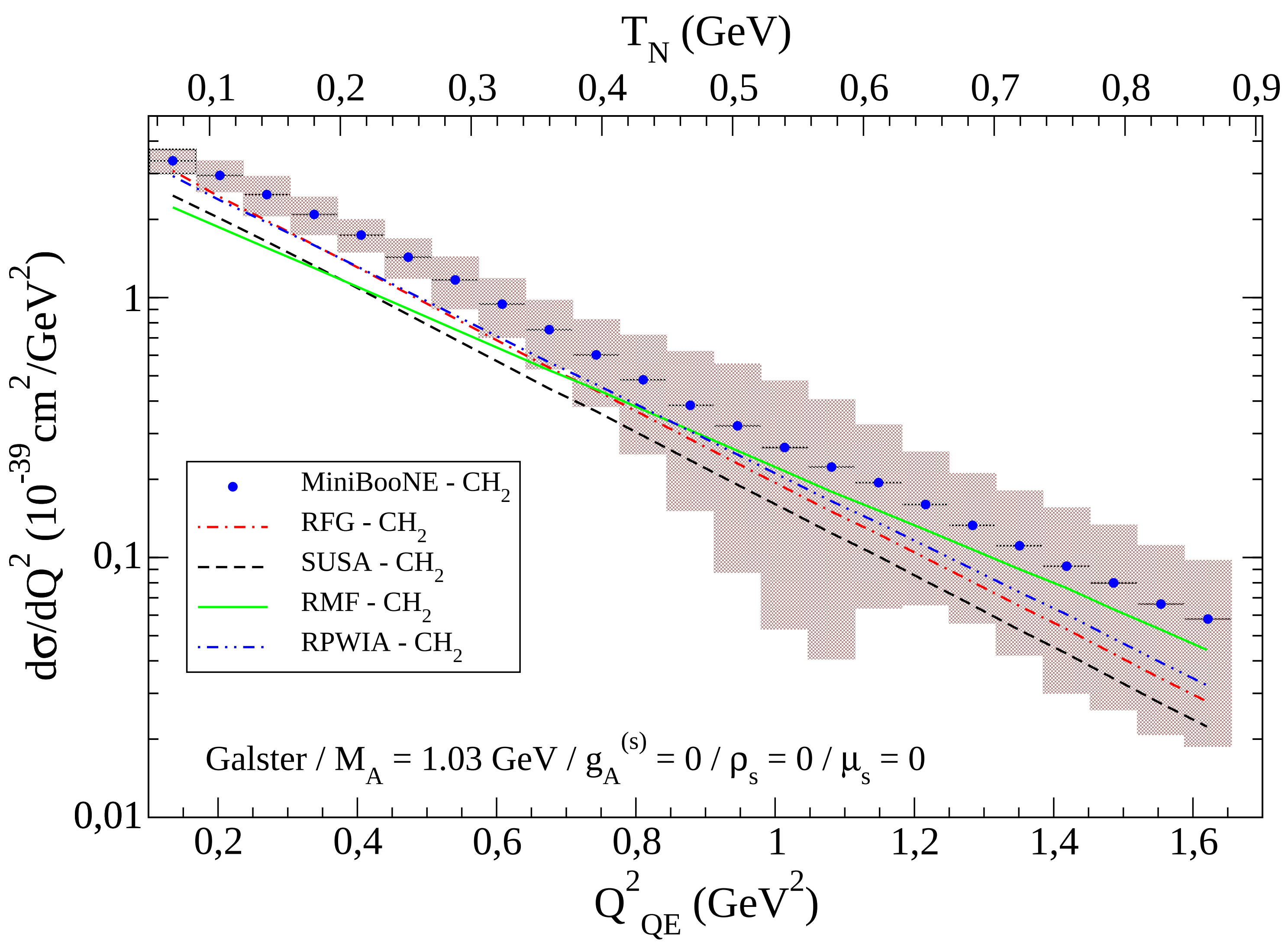


P/N RATIO: axial strangeness *vs* axial mass

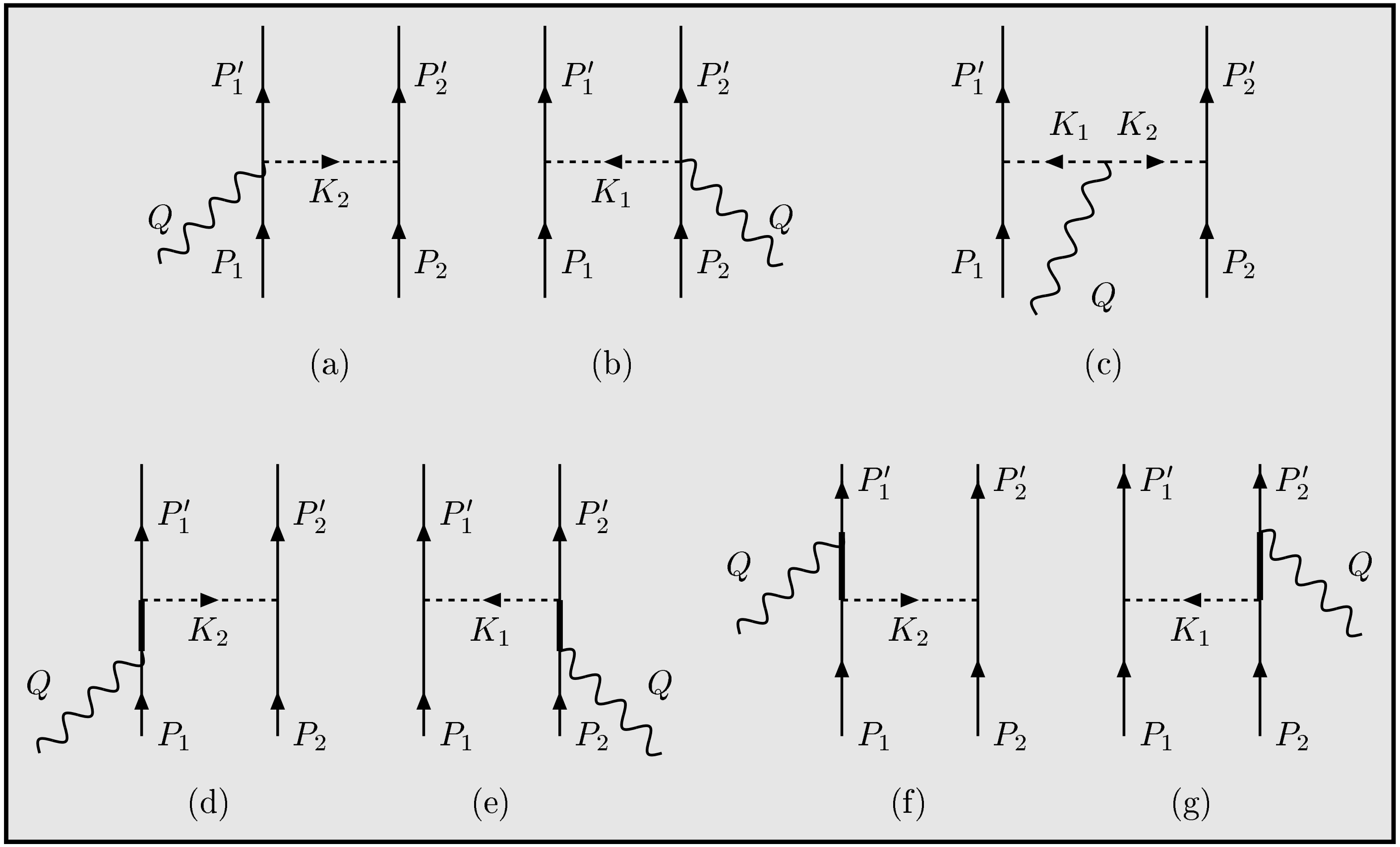


Correlation between axial strangeness & axial mass: for Δs-positive (grey band) NCQE is almost insenstive to M_A, whereas for Δs-negative (green) uncertainty due to M_A is $\sim 10-12\%$.

Comparison with MiniBooNE data



SuSAv2-MEC: QE + 2p-2h MEC + Inelastic

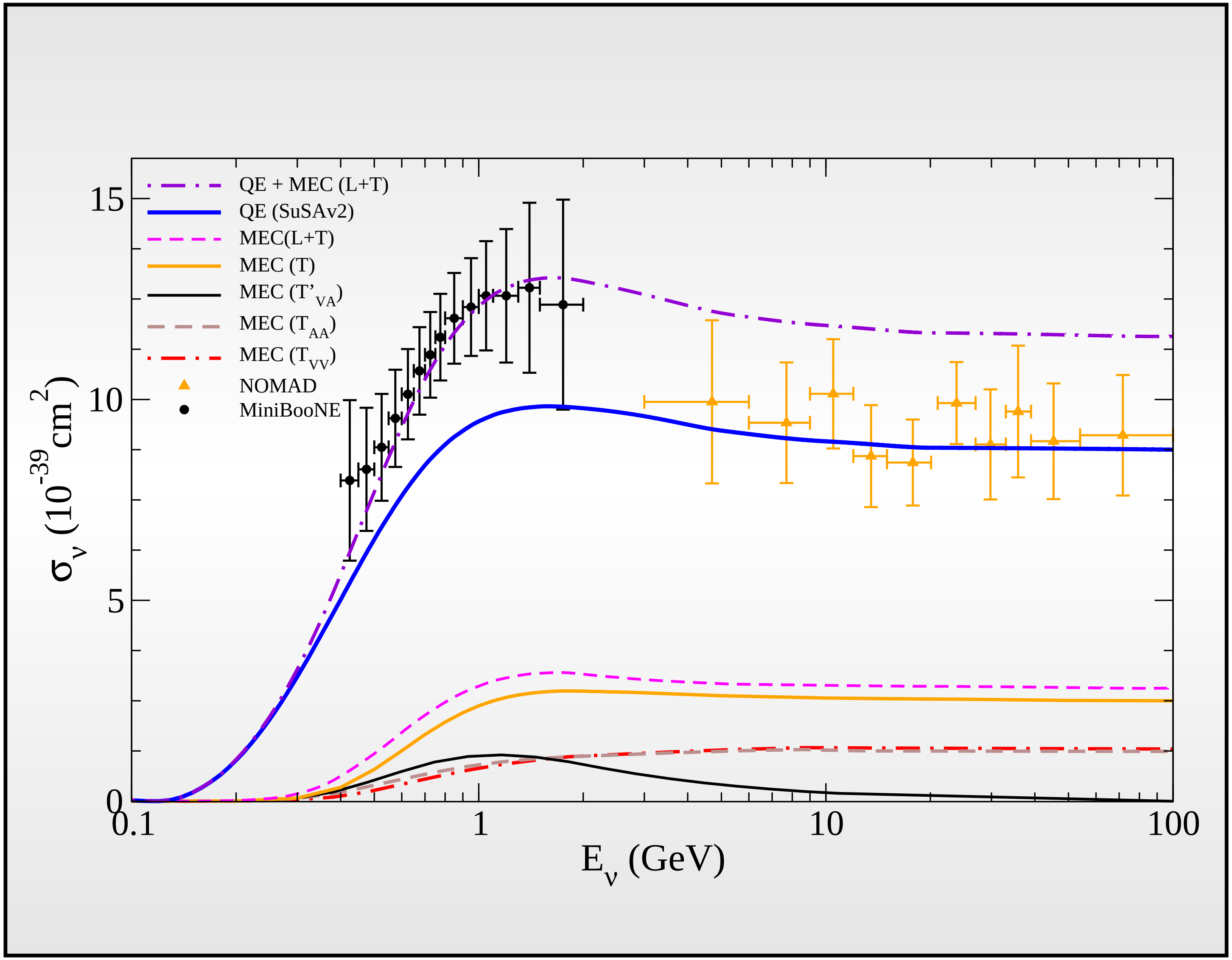


"MEC"

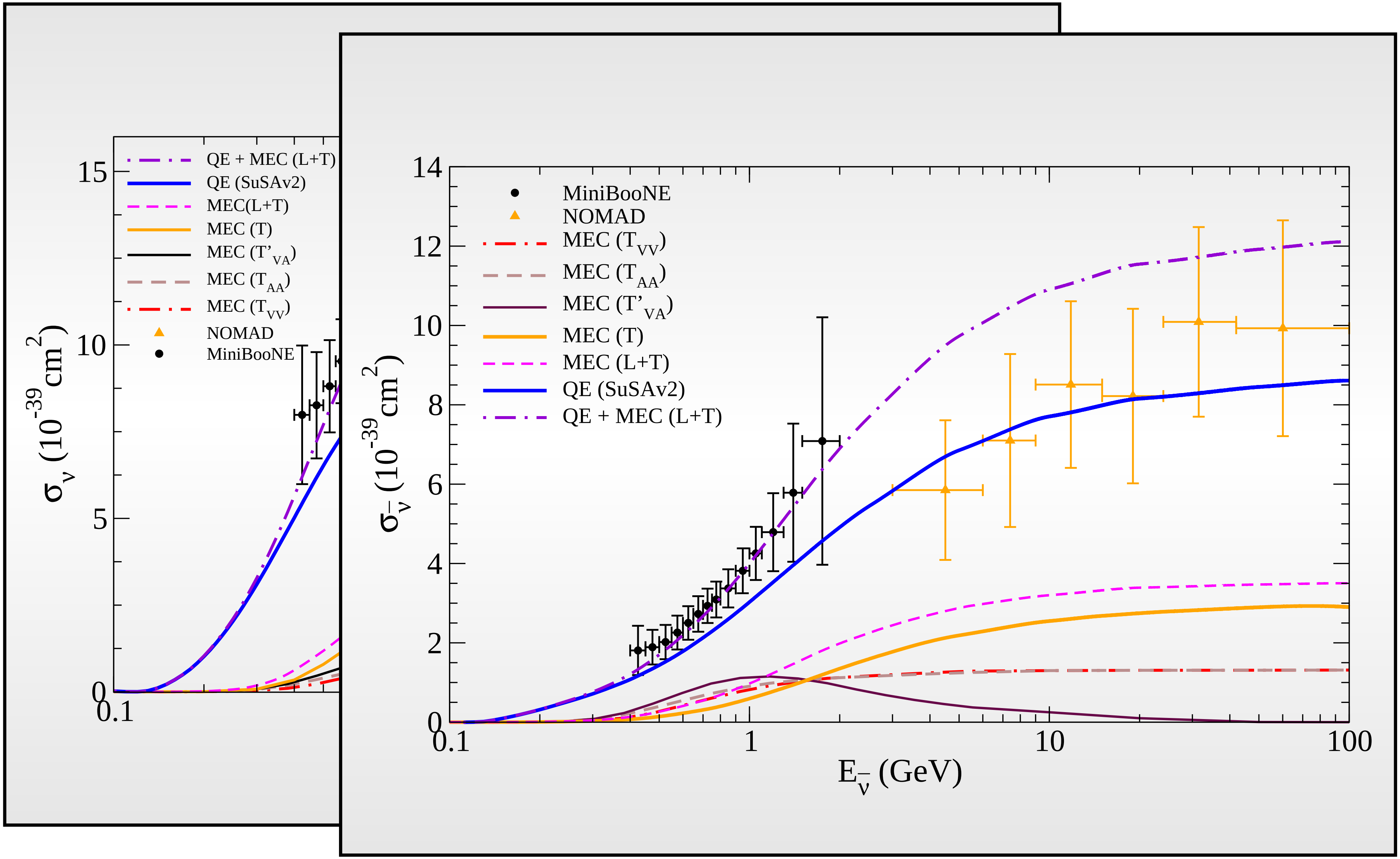
"correlations" and "Δ-MEC"

Application to CC neutrino scattering processes

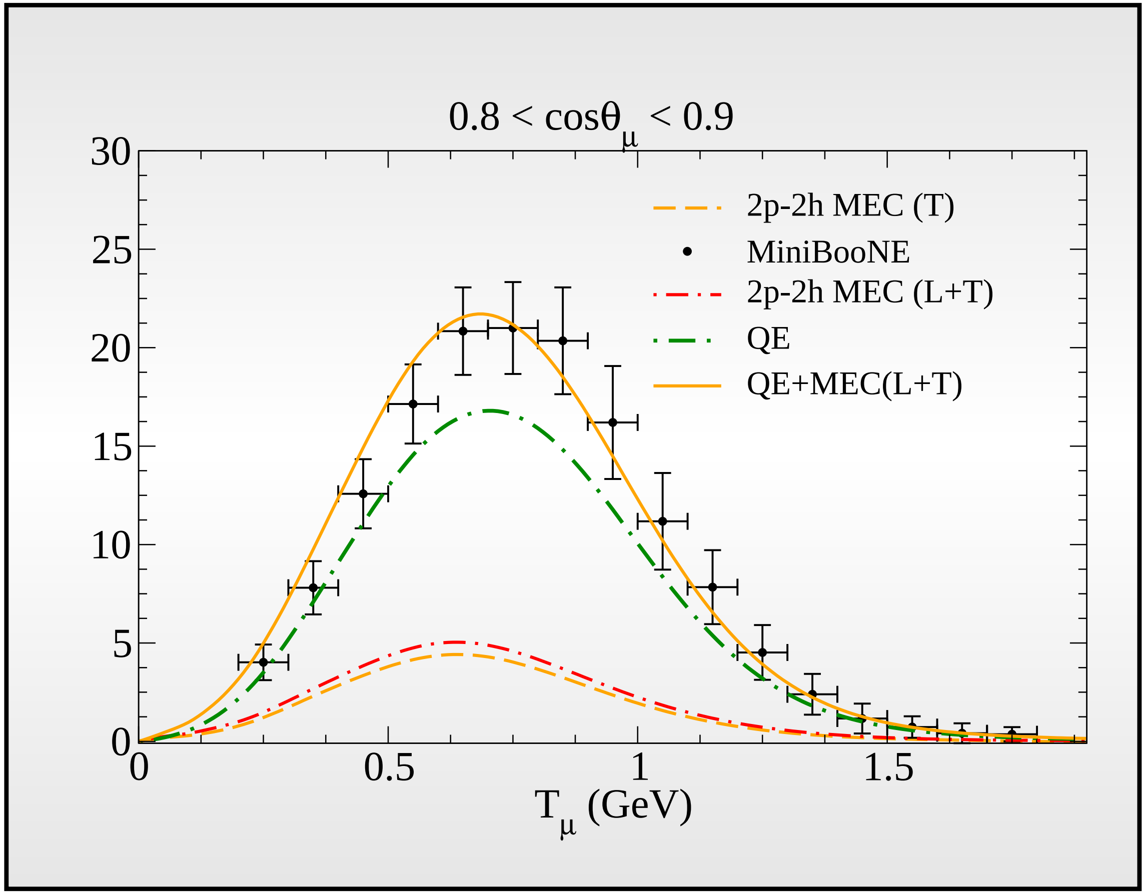
Integrated cross section: 2p-2h effects



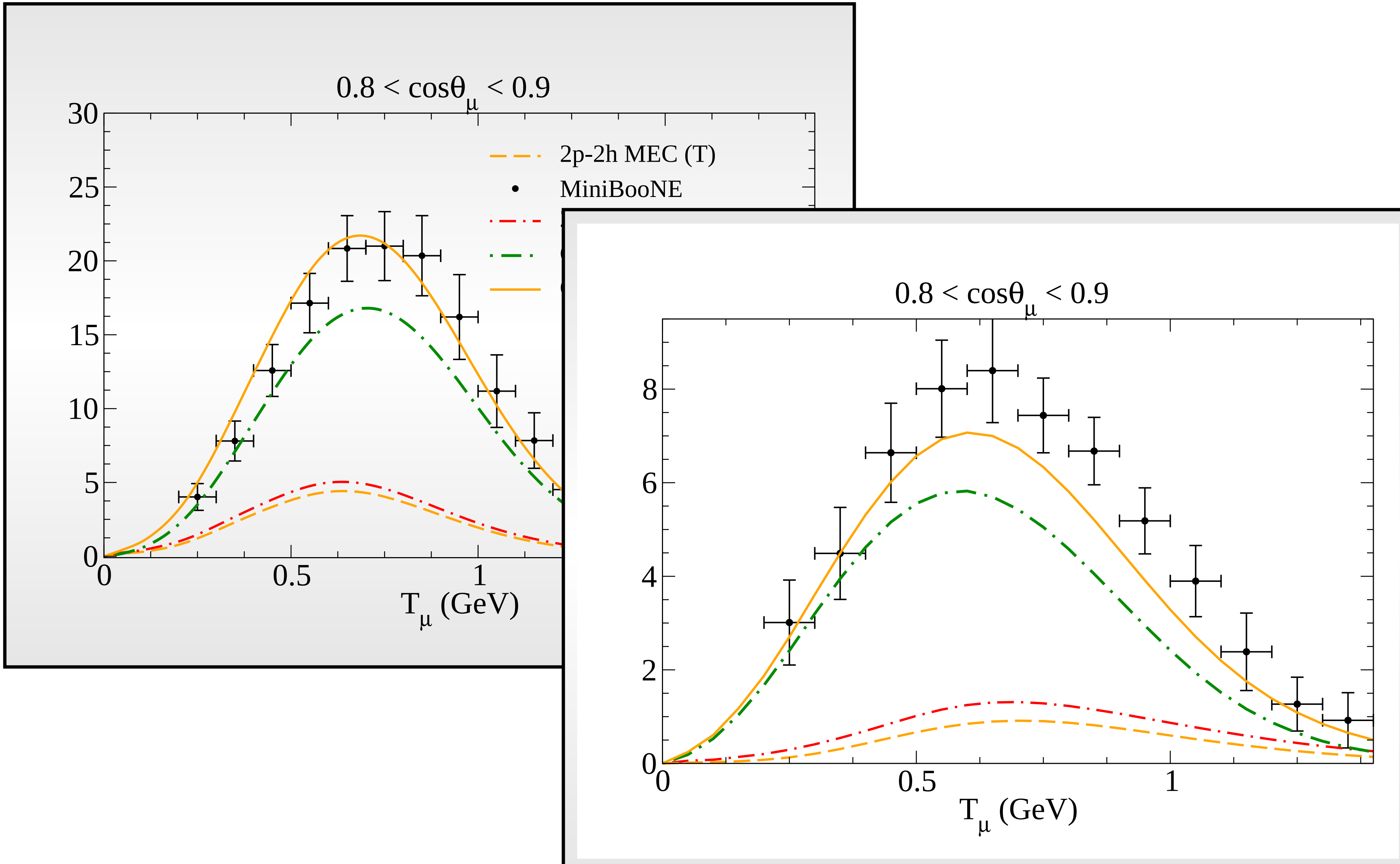
Integrated cross section: 2p-2h effects



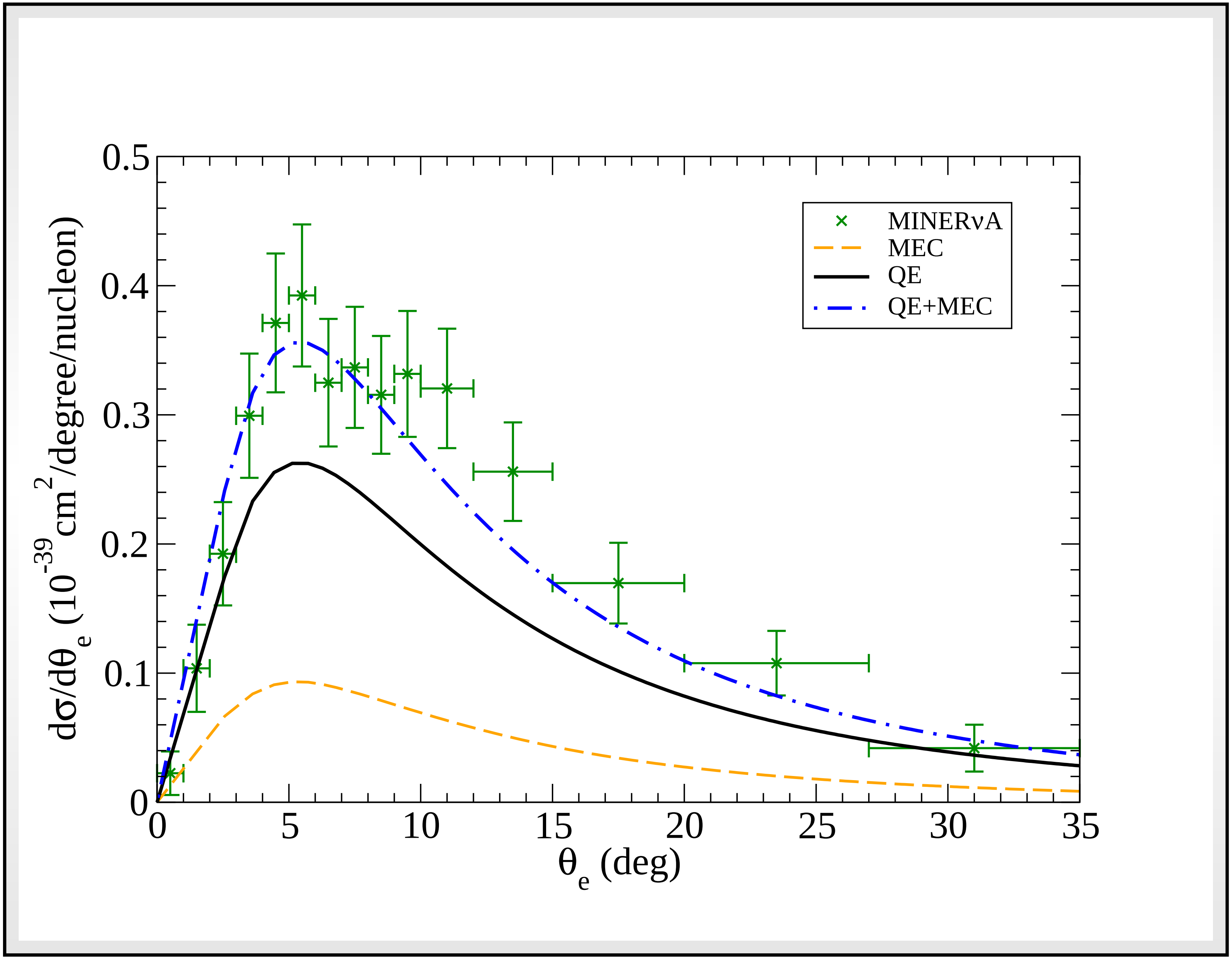
Flux-averaged double-differential CCQE-ν_μ & $\overline{\nu}_\mu$



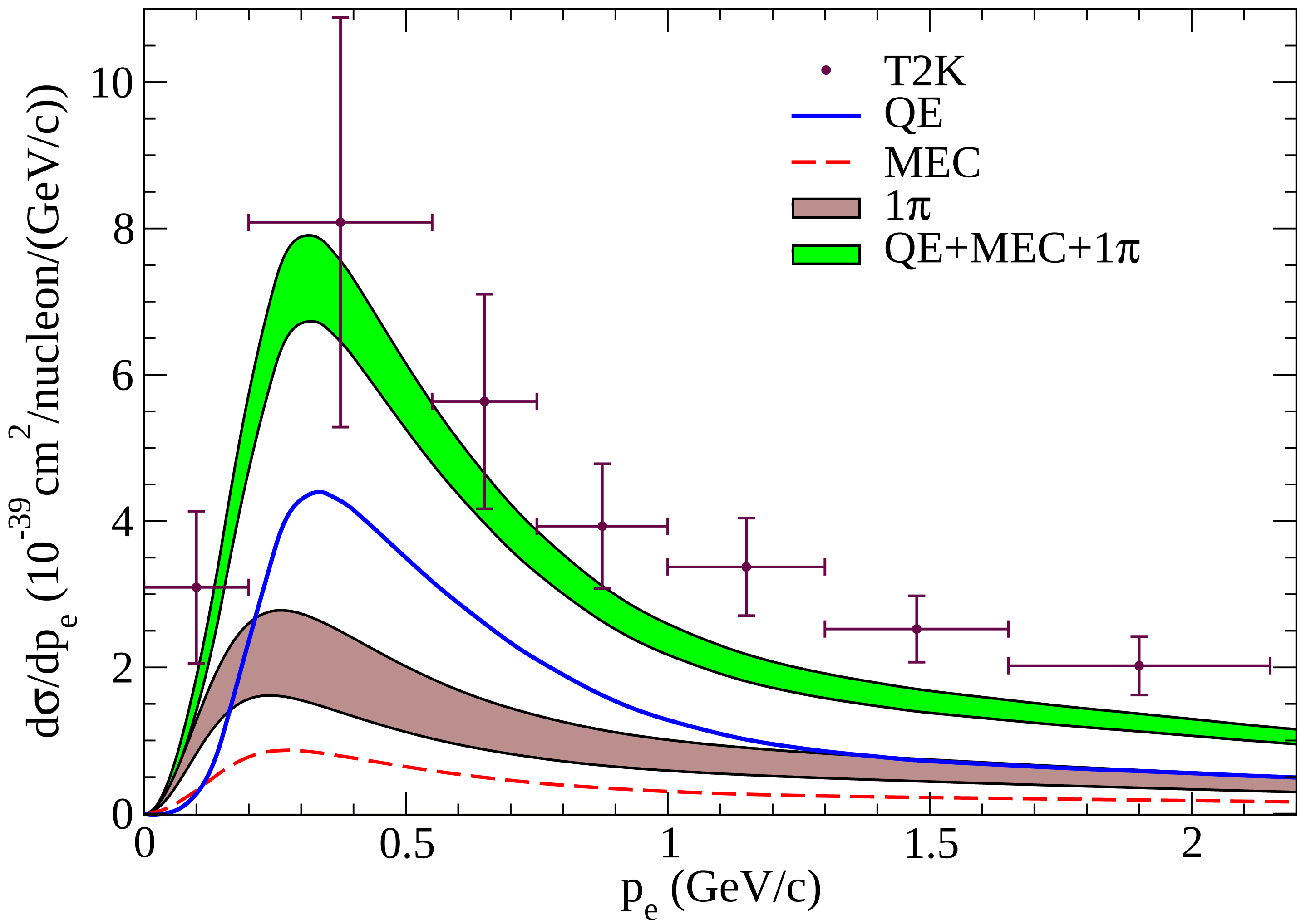
Flux-averaged double-differential CCQE-ν_μ & $\overline{\nu}_\mu$



MINERνA: case of ν_e



T2K experiment: Δ-contribution



SUMMARY

- *The RIA/RMF describes in a reasonable way QE (e, e') data, satisfying scaling behavior and providing an asymmetric superscaling L function in accordance with data.*
- *Contrary to most NR/SR models (likewise RFG), RMF violates scaling of zeroth order, i.e., $f_T > f_L$. This seems to be consistent with (e, e') data analysis.*
- *RMF applied to neutrino scattering also satisfies scaling/superscaling properties.*
- *RMF provides results in excellent agreement with SuSA/SuSAv2 approaches.*
- *Significant discrepancy with MiniBooNE data: Important enhancement of 2p-2h effects.*
- *RMF results (likewise SuSA/SuSAv2) in accordance with NOMAD data.*
- *Correlation between axial strangeness and axial mass in NC processes. Some discrepancy with MiniBooNE data: role of 2p-2h?, strangeness? ...*
- *RMF leads to scaling for the PV (e,e') responses. Similar to the EM ones.*
- *SuSAv2-MEC applied to neutrino reactions describes properly MiniBooNE, MinerVa and T2K data. Significant enhancement due to 2p2h-MEC.*

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Juan Antonio Caballero (06/12/16)