Progress in Dyson-Schwinger Studies of Hadron Properties: From Spectrum to Structure

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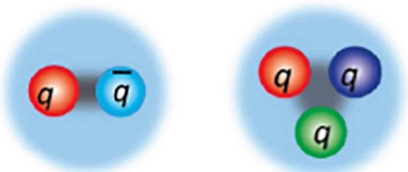
Fundamental Forces versus Bound States:

QCD

Non-perturbative

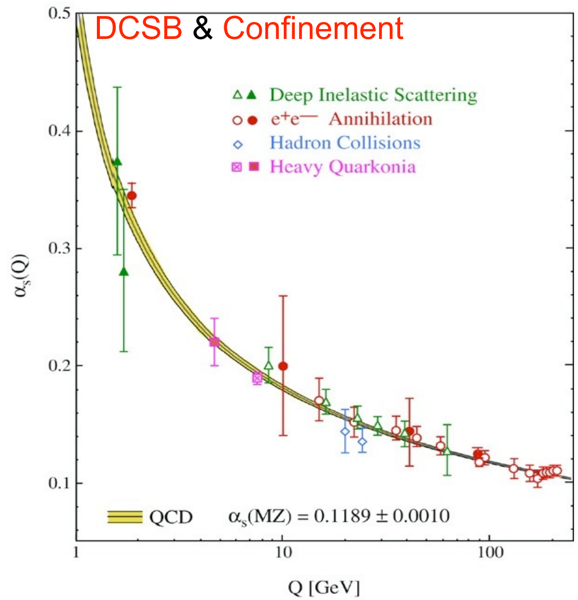


meson

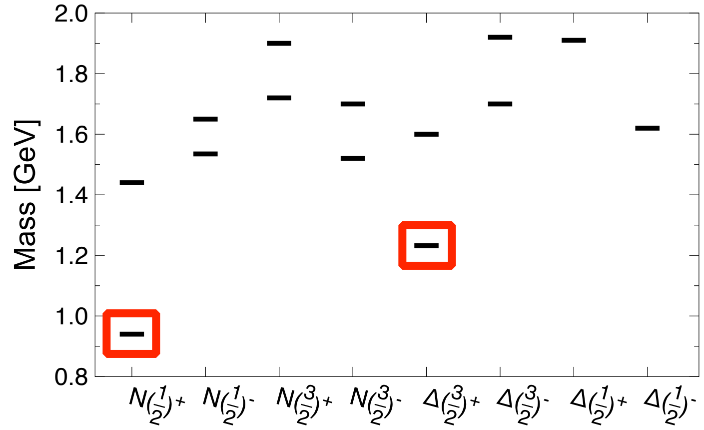
baryon

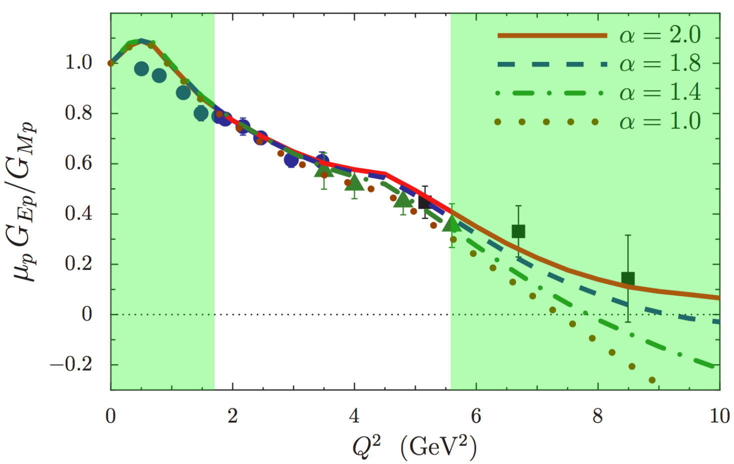
Theory

Exp.



QCD running coupling constant





Dyson-Schwinger Equations: EoM of Green functions

Classical Mechanics

Quantum Field Theory

Principle of Least Action

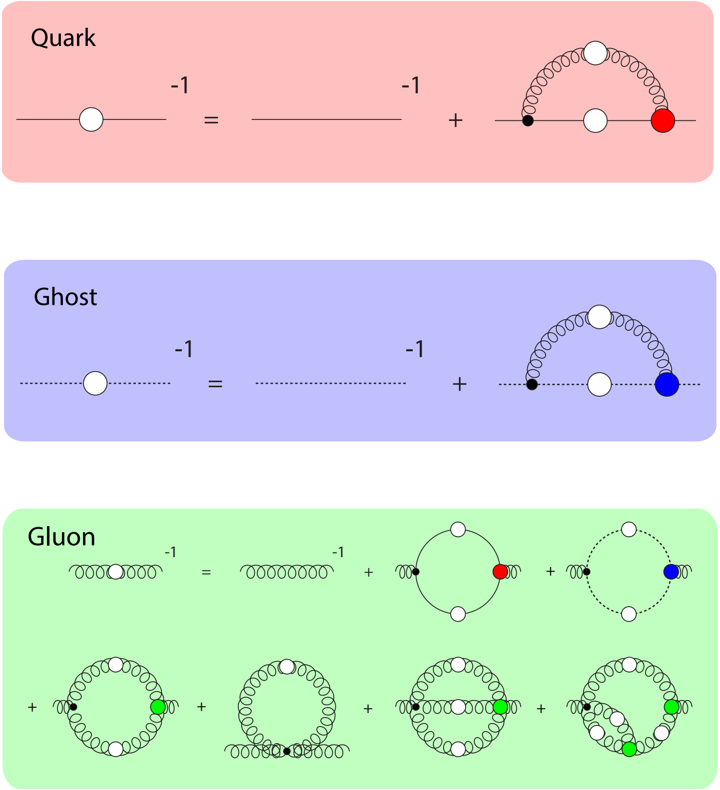
$$\frac{\delta S[q]}{\delta q}$$

$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle$$

Equations of Motion (EoM)

Euler-Lagrange Equation

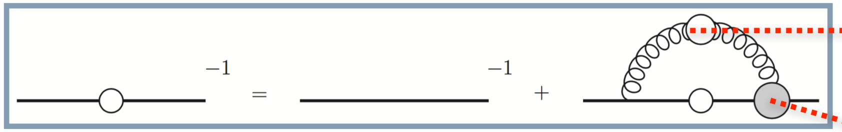
Dyson-Schwinger Equation



- Complicated integral equations
- Coupled tower of all equations

Dyson-Schwinger Equations: Equations for hadron properties

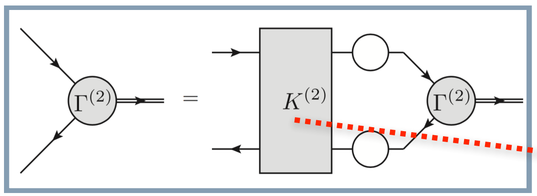
1. One-body gap equation:



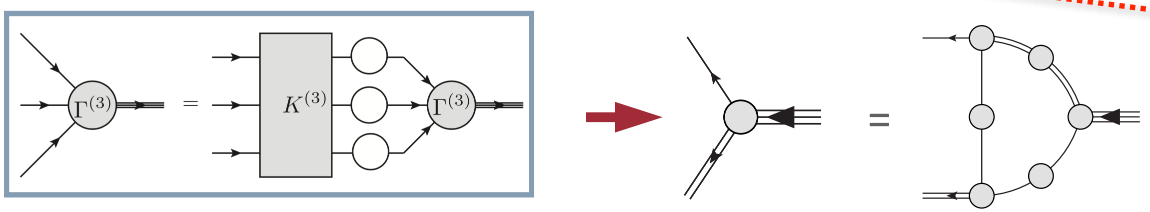
Gluon propagator

2. Bound-state equations:

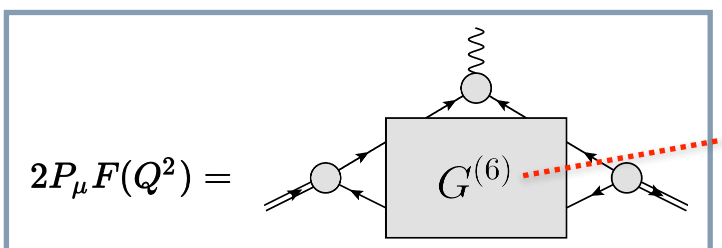
Quark-gluon vertex



4-point scattering kernel



3. Form factor equations:

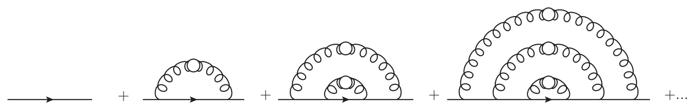


6-point Green function

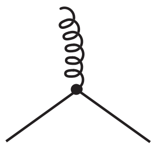
Dyson-Schwinger Equations: The simplest approximation

Rainbow diagrams of quark propagator:

$S(p) =$



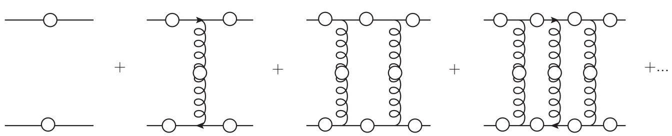
$$\Gamma^a_\mu(k,p) = \frac{\lambda^a}{2}\gamma_\mu$$

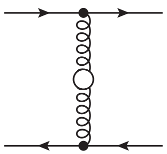


$$\mathcal{K}^{ab}_{\mu\nu}(k,q,P) = g^2 D^{ab}_{\mu\nu}(k)\left[\frac{\lambda^a}{2}\gamma_\mu\right]\left[\frac{\lambda^b}{2}\gamma_\nu\right]$$

Ladder diagrams of 4-point Green function:

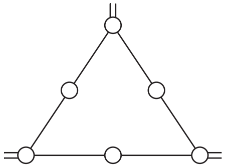
$G^4(k,q,P) =$

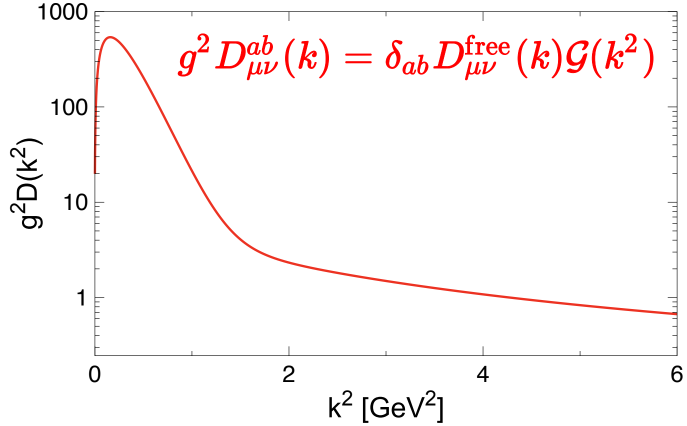




Triangle diagrams (Impulsion) of form factor:

$\Lambda_\mu(P,Q) = 2P_\mu F(Q^2) =$



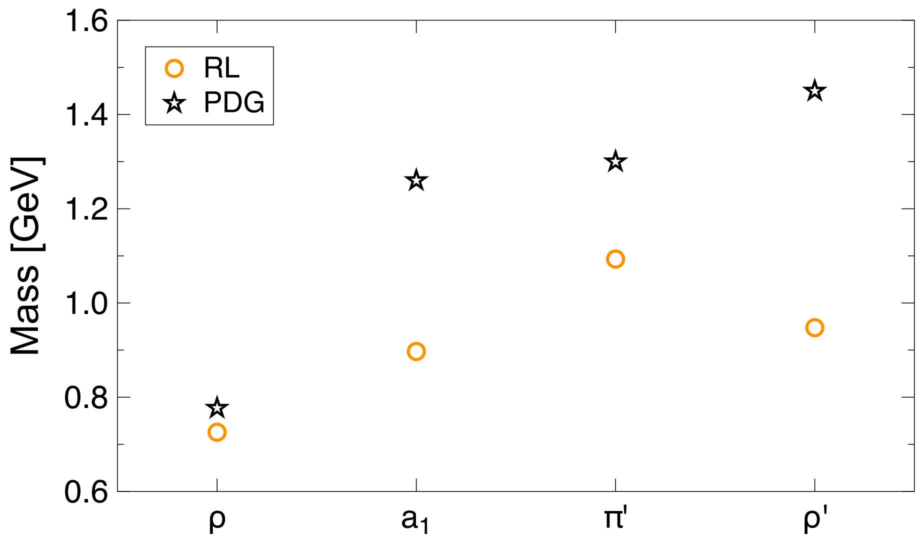


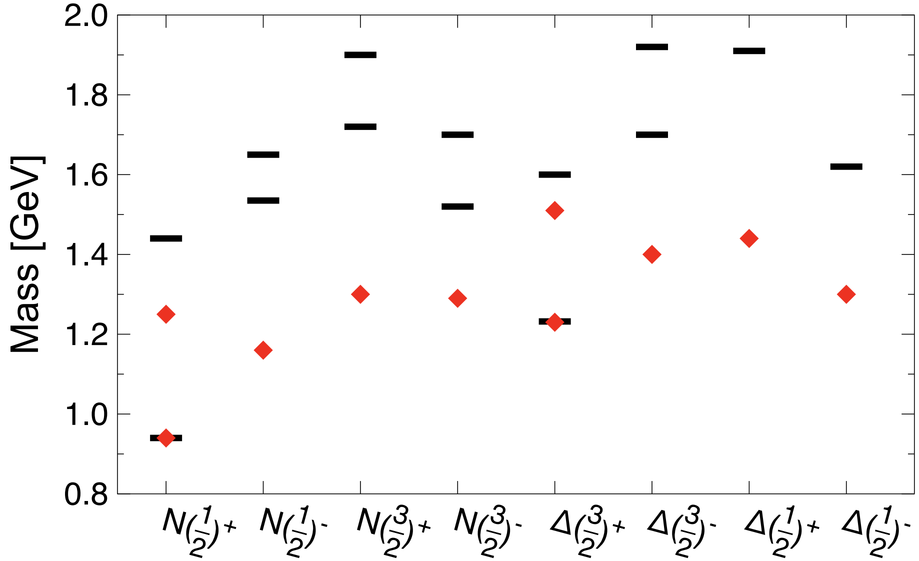
Dyson-Schwinger Equations: Failures of the simplest approximation

✦ Heavy ground states: light, e.g., rho-a_1 mass splitting;

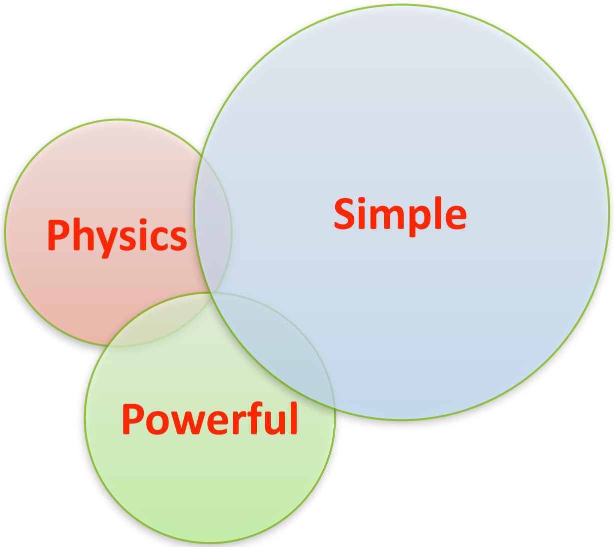
✦ Radial excitation states: light, e.g., pion’, rho’, excited baryons;

✦ Hadron spectrum: systematically wrong ordering and magnitudes.

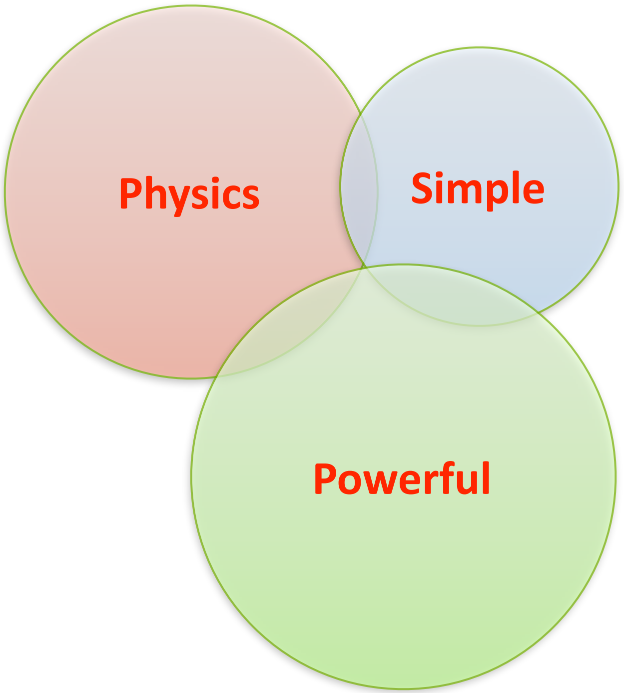




Rainbow-Ladder (Impulsion)



Beyond

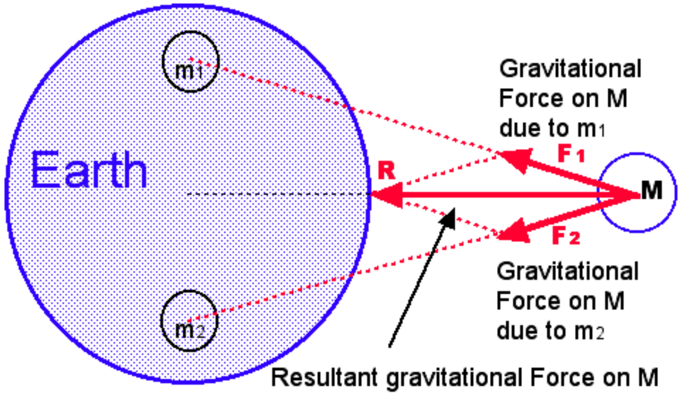


I. DCSB in quark-gluon vertex

II. Symmetries of the kernels

III. Current conservation in FF

I. DCSB in quark-gluon vertex: Ward-Green-Takahashi Identities



$$[\Gamma_\mu(p,q)]_{\alpha\beta} = \{\gamma_\mu, p_\mu, q_\mu\} \times \{\mathbf{1},\ \gamma\cdot p,\ \gamma\cdot q,\ \sigma_{p,q}\}$$

- The WGTIs express the curls and divergences of the vertices.
- The WGTIs of the vertices in different channels couple together.
- The WGTIs involve contributions from high-order Green functions.

❑ **Gauge symmetry:** vector WGTI

$$iq_\mu \Gamma_\mu(k,q) = S^{-1}(k) - S^{-1}(p)$$

$\nabla \cdot \Phi$

❑ **Chiral symmetry:** axial-vector WGTI

$$q_\mu \Gamma^A_\mu(k,q) = S^{-1}(k) i\gamma_5 + i\gamma_5 S^{-1}(p) - 2im\Gamma_5(k,p)$$

❑ **Lorentz symmetry + :** transverse WGTIs

$\nabla \times \Phi$

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ & + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ & + A^V_{\mu\nu}(k,p)\,, \end{aligned}$$

$$\begin{aligned} q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) = {} & S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) \\ & + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) \\ & + V^A_{\mu\nu}(k,p)\,, \qquad \sigma^5_{\mu\nu} = \sigma_{\mu\nu}\gamma_5 \end{aligned}$$

He, PRD, 80, 016004 (2009)

I. DCSB in quark-gluon vertex: Solution of WGTIs

Qin et. al., PLB 722, 384 (2013)

- Defining proper projection tensors and contract them with the transverse WGTIs, one can decouple the WGTIs and obtain a group of equations for the vector vertex:

$$T^1_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta} t_\alpha q_\beta \mathbf{I}_\mathrm{D}, \qquad T^2_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta} \gamma_\alpha q_\beta .$$

$$q_\mu i\Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p),$$

$$q\cdot tt\cdot\Gamma(k,p) = T^1_{\mu\nu}[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)] + t^2 q\cdot\Gamma(k,p) + T^1_{\mu\nu}V^A_{\mu\nu}(k,p),$$

$$q\cdot t\gamma\cdot\Gamma(k,p) = T^2_{\mu\nu}[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)] + \gamma\cdot tq\cdot\Gamma(k,p) + T^2_{\mu\nu}V^A_{\mu\nu}(k,p).$$

- It is a group of full-determinant linear equations and a unique solution:

$$\Gamma^{\mathrm{Full}}_\mu(k,p) = \Gamma^{\mathrm{BC}}_\mu(k,p) + \Gamma^{\mathrm{T}}_\mu(k,p) + \Gamma^{\mathrm{FP}}_\mu(k,p)$$

- The quark propagator contributes to the longitudinal and transverse parts. The **DCSB** terms are highlighted.

$$\Gamma^{\mathrm{BC}}_\mu(k,p) = \gamma_\mu\Sigma_A + t_\mu \not{t}\frac{\Delta_A}{2} - it_\mu\Delta_B,$$

$$\Gamma^{\mathrm{T}}_\mu(k,p) = -\sigma_{\mu\nu}q_\nu\Delta_B + \gamma^T_\mu q^2\frac{\Delta_A}{2} - \left(\gamma^T_\mu[\not{q},\not{t}] - 2t^T_\mu\not{q}\right)\frac{\Delta_A}{4}.$$

$$S(p) = \frac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$$

$$\Sigma_\phi(x,y) = \frac{1}{2}[\phi(x) + \phi(y)],$$

$$\Delta_\phi(x,y) = \frac{\phi(x) - \phi(y)}{x-y}.$$

$$X^T_\mu = X_\mu - \frac{q\cdot X q_\mu}{q^2}$$

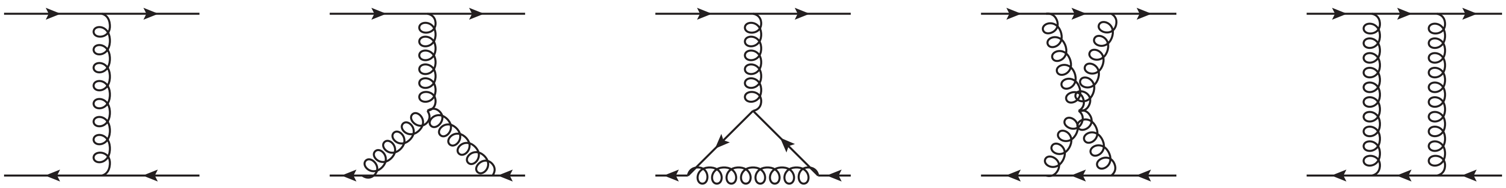
- The unknown high-order terms contribute to the transverse part, i.e., the longitudinal part has been completely determined by the quark propagator.

I. DCSB in quark-gluon vertex: Summary

◆ The Lagrangian symmetries are able to constrain structures of the fermion—gauge-boson vertex, and even determine some structures uniquely.

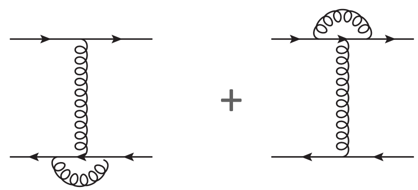
◆ DCSB reshapes the appearance of the vertex, dramatically. This must result in remarkable consequences in observables.

II. Symmetries of the kernels: Discrete symmetries

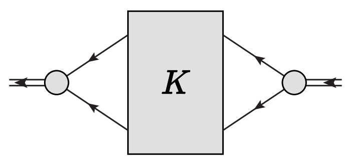


+ etc.

✦ Permutation: $\mathscr{P}\,\mathcal{K}(q_\pm,k_\pm) = \mathcal{K}^*(q_\pm,k_\pm) = K^\mu_R(k_\mp,q_\mp)\otimes K^\mu_L(k_\mp,q_\mp)$

$\frac{1}{2!}$  + etc.

✦ Charge-conjugation: C $\mathcal{K}(q_\pm,k_\pm) = \overline{\mathcal{K}}(q_\pm,k_\pm) = C\,K^\mu_L(-k_\pm,-q_\pm)^T\,C^{-1}\otimes C\,K^\mu_R(-k_\pm,-q_\pm)^T\,C^{-1}$

$\langle\chi_i|K|\chi_j\rangle =$ $\bar{\chi}_i$  χ_j $= \delta_{ij}$

✦ P and T symmetries: P $\mathcal{K}(q_\pm,k_\pm) = \widehat{\mathcal{K}}(q_\pm,k_\pm) = P\,K^\mu_L(q_\pm,k_\pm)\,P^{-1}\otimes P\,K^\mu_R(q_\pm,k_\pm)\,P^{-1}$

$$K = \mathbf{1}\otimes\mathbf{1} + \gamma_5\otimes\gamma_5 + \cancel{\mathbf{1}\otimes\gamma_5 + \gamma_5\otimes\mathbf{1}}$$

Lorentz covariance guarantees CPT-symmetry; T-symmetry is obtained for free.

II. Symmetries of the kernels: Continuous symmetries

In the chiral limit, the color-singlet axial-vector WGTI (**chiral symmetry**) is written as

$$P_\mu \Gamma_{5\mu}(k,P) = S^{-1}\left(k+\frac{P}{2}\right) i\gamma_5 + i\gamma_5 S^{-1}\left(k-\frac{P}{2}\right)$$

Assuming **DCSB**, i.e., the mass function is generated, we have the following identity

$$\lim_{P\to 0} P_\mu \Gamma_{5\mu}(k,P) = 2i\gamma_5 B(k^2) \neq 0$$

The axial-vector vertex must involve a pseudo scalar pole (Goldstone theorem)

$$\Gamma_{5\mu}(k,0) \sim \frac{2i\gamma_5 f_\pi E_\pi(k^2) P_\mu}{P^2} \propto \frac{P_\mu}{P^2} \qquad f_\pi E_\pi(k^2) = B(k^2)$$

Assuming there is a radially excited pion, its decay constant vanishes

$$\lim_{P^2\to M_{\pi_n}^2} \Gamma_{5\mu}(k,P) \sim \frac{2i\gamma_5 f_{\pi_n} E_{\pi_n}(k,P) P_\mu}{P^2 + M_{\pi_n}^2} < \infty \qquad f_{\pi_n} = 0$$

DCSB means **much more** than massless pseudo-scalar meson.

II. Symmetries of the kernels: Continuous symmetries

The Bethe-Salpeter equation and the quark gap equation are written as

$$\boxed{\Gamma^H_{\alpha\beta}(k,P)} = \gamma^H_{\alpha\beta} + \int_q \mathcal{K}(k_\pm, q_\pm)_{\alpha\alpha',\beta'\beta}[S(q_+)\Gamma^H(q,P)S(q_-)]_{\alpha'\beta'},$$

$$\boxed{S^{-1}(k)} = S_0^{-1}(k) + \int_q D_{\mu\nu}(k-q)\gamma_\mu S(q)\Gamma_\nu(q,k),$$

The color-singlet axial-vector and vector WGTIs are written as

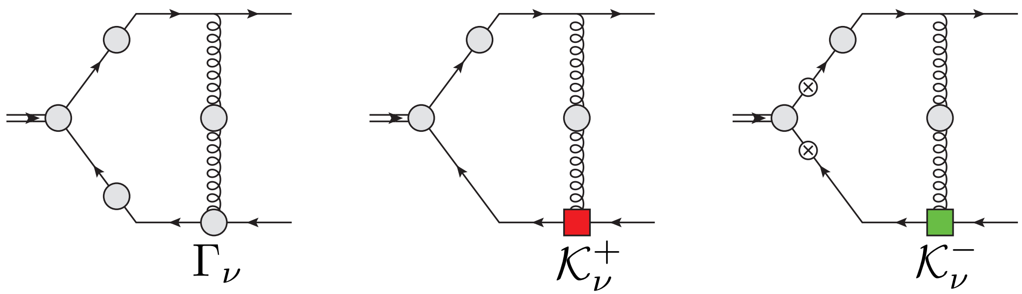
$$\begin{aligned} P_\mu\Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) &= S^{-1}(k_+)i\gamma_5 + i\gamma_5 S^{-1}(k_-),\\ iP_\mu\Gamma_\mu(k,P) &= S^{-1}(k_+) - S^{-1}(k_-). \end{aligned}$$

The kernel satisfies the following WGTIs: quark propagator + quark-gluon vertex

$$\boxed{\begin{aligned} \int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+) - S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} &= \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+) - S(q_-)\Gamma_\nu(q_-,k_-)],\\ \int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+)\gamma_5 + \gamma_5 S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} &= \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+)\gamma_5 - \gamma_5 S(q_-)\Gamma_\nu(q_-,k_-)]. \end{aligned}}$$

II. Symmetries of the kernels: Continuous symmetries

Assuming the scattering kernel has the following structure:



Ladder-like term

Symmetry-rescuing term

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

$$\Gamma_\nu^\Sigma = \Gamma_\nu^+ + \gamma_5\Gamma_\nu^+\gamma_5 \qquad \Gamma_\nu^\Delta = \Gamma_\nu^+ - \Gamma_\nu^-$$

$$B_\Sigma = 2B_+ \qquad B_\Delta = B_+ - B_-$$

$$A_\Delta = i(\gamma\cdot q_+)A_+ - i(\gamma\cdot q_-)A_-$$

Inserting the ansatz for the kernel into its WGTIs, we have

$$\int_q D_{\mu\nu}\gamma_\mu S_+\boxed{(\Gamma_\nu^+ - \Gamma_\nu^-)} = \int_q D_{\mu\nu}\gamma_\mu S_+\boxed{(S_+^{-1} - S_-^{-1})\mathcal{K}_\nu^+} + \int_q D_{\mu\nu}\gamma_\mu S_+\boxed{\gamma_5(S_+^{-1} - S_-^{-1})\gamma_5\mathcal{K}_\nu^-},$$

$$\int_q D_{\mu\nu}\gamma_\mu S_+\boxed{(\Gamma_\nu^+\gamma_5 + \gamma_5\Gamma_\nu^-)} = \int_q D_{\mu\nu}\gamma_\mu S_+\boxed{(S_+^{-1}\gamma_5 + \gamma_5 S_-^{-1})\mathcal{K}_\nu^+} + \int_q D_{\mu\nu}\gamma_\mu S_+\boxed{(\gamma_5 S_+^{-1} + S_-^{-1}\gamma_5)\mathcal{K}_\nu^-}.$$

Eventually, the solution is straightforward:

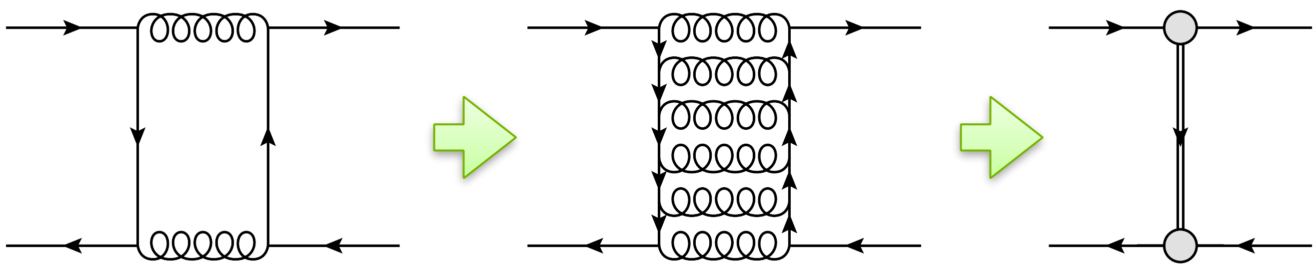
$$\boxed{\mathcal{K}_\nu^\pm = (2B_\Sigma A_\Delta)^{-1}[(A_\Delta \mp B_\Delta)\Gamma_\nu^\Sigma \pm B_\Sigma\Gamma_\nu^\Delta].}$$

- ✦ The form of scattering kernel is simple.
- ✦ The kernel has no kinetic singularities.
- ✦ All channels share the same kernel.

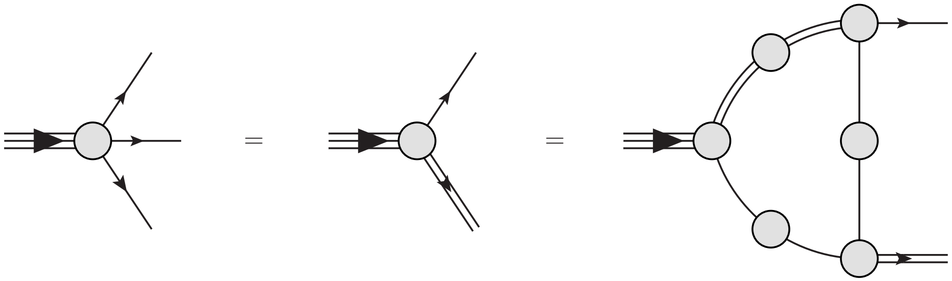
II. Symmetries of the kernels: Meson cloud and diquark

In Quantum Field theory (infinitely many degrees of freedom), high-order Green functions **cannot** completely truncated by low-order ones (unclosed).

For example, meson cloud, e.g., pion cloud, goes into the scattering kernel:



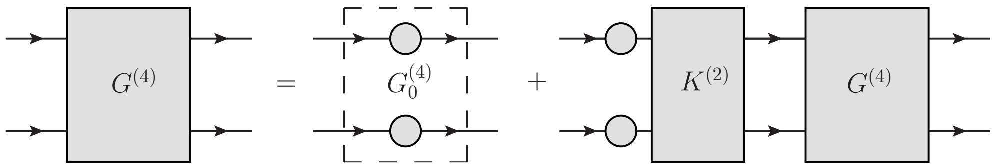
In baryons, two quarks tend to bind together to form a particle-like soft object:



- ✦ What is the **off-shell** meson and diquark?
- ✦ How to make the system **self-consistent**?

II. Symmetries of the kernels: Meson cloud and diquark

In QFT, Meson cloud and diquark are encoded in the four-point Green function:

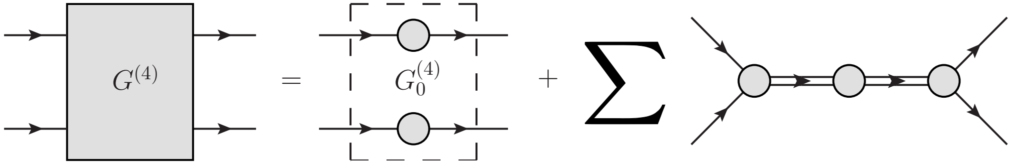


$$G^{(4)} = G_0^{(4)} + G_0^{(4)} \cdot K^{(2)} \cdot G^{(4)}$$

The kernel can be decomposed by its orthogonal eigenbasis:

$$G_0^{(4)} |\Gamma_i\rangle = \lambda_i \, G_0^{(4)} \cdot K^{(2)} \cdot G_0^{(4)} |\Gamma_i\rangle \qquad \langle \Gamma_i | G_0^{(4)} | \Gamma_j \rangle = \delta_{ij} \qquad K^{(2)} = \sum_i \lambda_i \, |\Gamma_i\rangle \langle \Gamma_i|$$

Accordingly, the four-point Green function can be decomposed:



$$G^{(4)} = G_0^{(4)} + \sum_i |\chi_i\rangle \frac{\lambda_i(P^2)}{1 - \lambda_i(P^2)} \langle \chi_i|$$

- ✦ The basis is classified by J^P quantum number, and radial quantum number n_r.
- ✦ Meson cloud and diquark correspond to components with quantum numbers.

II. Symmetries of the kernels: Meson cloud and diquark

The start point is the Bethe-Salpeter equation with meson cloud

$$\boxed{\Gamma^H_{\alpha\beta}(k,P)} = \gamma^H_{\alpha\beta} + \int_q \mathcal{K}(k_\pm, q_\pm)_{\alpha\alpha',\beta'\beta} [\boxed{S(q_+)}\Gamma^H(q,P)\boxed{S(q_-)}]_{\alpha'\beta'}.$$

The color-singlet axial-vector and vector WGTIs (|P| = 0) are written as

$$\begin{aligned} i\hat{P}_\mu \Gamma_\mu(k,0) &= \hat{P}_\mu \frac{\partial S^{-1}(k)}{\partial k_\mu}, \\ 2m\Gamma_5(k,0) &= S^{-1}(k)\gamma_5 + \gamma_5 S^{-1}(k), \end{aligned}$$

The Bethe-Salpeter kernel can modify the quark propagator as

$$\left[\hat{P}_\mu \frac{\partial S^{-1}(k)}{\partial k_\mu}\right]_{\alpha\beta} = [i\hat{\not{P}}]_{\alpha\beta} - \int_q \mathcal{K}(k,q)_{\alpha\alpha',\beta'\beta} \left[\hat{P}_\mu \frac{\partial S(q)}{\partial q_\mu}\right]_{\alpha'\beta'},$$

$$\left[S^{-1}(k)\gamma_5 + \gamma_5 S^{-1}(k)\right]_{\alpha\beta} = [2m\gamma_5]_{\alpha\beta} + \int_q \mathcal{K}(k,q)_{\alpha\alpha',\beta'\beta} \left[S(q)\gamma_5 + \gamma_5 S(q)\right]_{\alpha'\beta'},$$

Using the quark dress functions, the new quark gap equation reads

$$\boxed{\begin{cases} \dfrac{\partial |k| A(k^2)}{\partial |k|} = 1 + \dfrac{1}{4}\displaystyle\int_q \left[k^{\parallel}_\mu\right]_{\beta\alpha} \mathcal{K}_{\alpha\alpha',\beta'\beta} \left[\frac{\partial S(q)}{\partial q_\mu}\right]_{\alpha'\beta'}, \\ B(k^2) = m + \dfrac{1}{4}\displaystyle\int_q \left[\gamma_5\right]_{\beta\alpha} \mathcal{K}_{\alpha\alpha',\beta'\beta} \left[\gamma_5 \sigma_B(q^2)\right]_{\alpha'\beta'}, \end{cases}}$$

II. Symmetries of the kernels: Summary

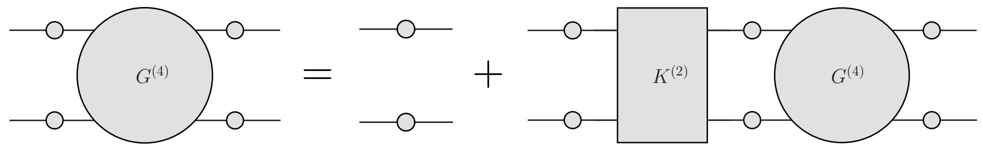
- The quark—anti-quark scattering kernel can be constrained by discrete and continuous symmetries, i.e., CPT and vector and axial-vector WGTIs.

- The meson cloud and diquark can be expressed as components of four-point Green function with corresponding quantum numbers.

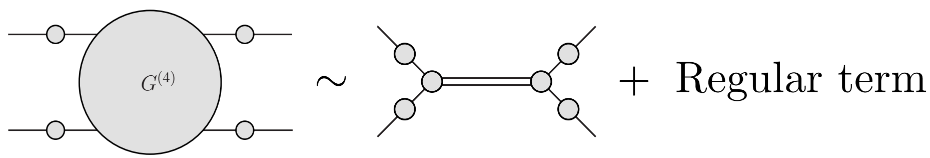
- The self-consistency can be guaranteed by WGTIs. The quark self-energy and BS kernel can be expressed as the core part plus the meson cloud part.

III. Current conservation in Form Factor: Normalization of wave-fn

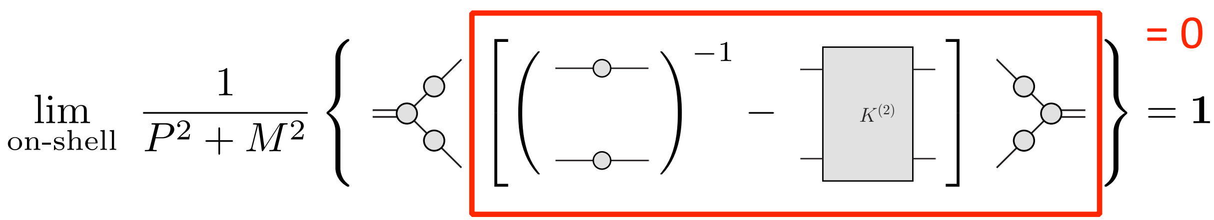
The Dyson-Schwinger equation of the **four-point Green function** is written as



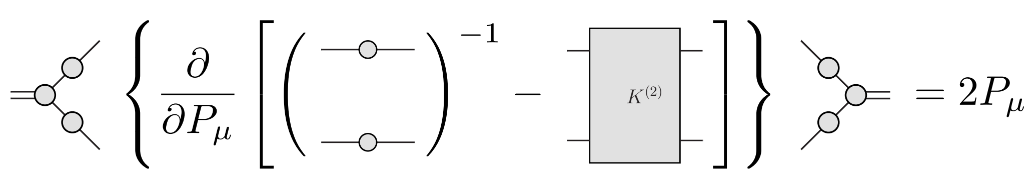
Assuming that there is a **bound state**



the **wave function** of the bound state has to satisfy the following condition

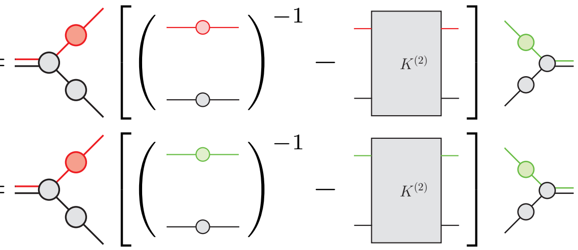


The **differential form** of current conservation is obtained as ($F(Q^2=0)=1$)



III. Current conservation in Form Factor: Beyond triangle diagram

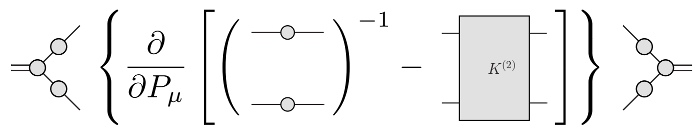
Introduce a function depending on ***(P, Q)***, i.e., $\mathcal{G}(P,Q) \equiv \mathcal{G}_+(P,Q) - \mathcal{G}_-(P,Q)$

$\mathcal{G}_+(P,Q) =$ 

$\mathcal{G}_-(P,Q) =$

$$\boxed{q_+ + \frac{Q}{2}} \qquad \boxed{q_+ - \frac{Q}{2}}$$

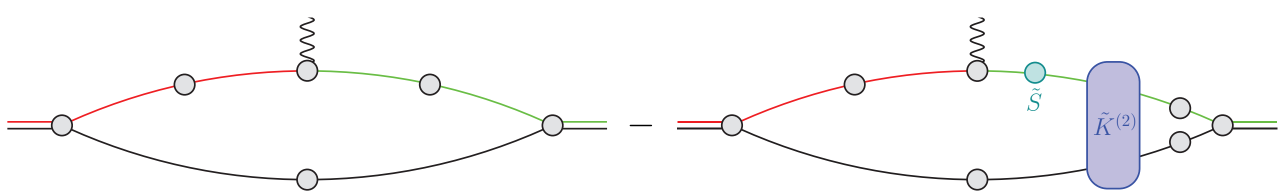
Then the function can reproduce the normalization condition as

$\lim_{Q\to 0} \frac{\mathcal{G}(P,Q)}{Q_\mu} =$  $= 2P_\mu$

Inserting the **color-singlet vector Ward identity** into the function,

$$Q_\mu \Gamma_\mu\left(q_+ + \frac{Q}{2}, q_+ - \frac{Q}{2}\right) = S^{-1}\left(q_+ + \frac{Q}{2}\right) - S^{-1}\left(q_+ - \frac{Q}{2}\right) \qquad \mathcal{G}(P,Q) = Q_\mu \Lambda_\mu(P,Q)$$

Eventually, the **form factor** can be defined as $\Lambda_\mu(P,Q) = 2P_\mu F(Q^2)$ with $F(Q^2 = 0) = 1$

$\Lambda_\mu(P,Q) =$ 

$$\tilde{S}\left(q_+ \pm \frac{Q}{2}\right) = \left[S^{-1}\left(q_+ + \frac{Q}{2}\right) - S^{-1}\left(q_+ - \frac{Q}{2}\right)\right]^{-1} \qquad \tilde{K}\left(q_+ \pm \frac{Q}{2}, q_-, q'_+ \pm \frac{Q}{2}, q'_-\right) = K\left(q_+ + \frac{Q}{2}, q_-, q'_+ + \frac{Q}{2}, q'_-\right) - K\left(q_+ - \frac{Q}{2}, q_-, q'_+ - \frac{Q}{2}, q'_-\right)$$

III. Current conservation in Form Factor: Summary

- Hadrons can be considered as either elementary particles or composite states of quarks and gluons. This imposes a normalization condition on wave functions of bound-states.

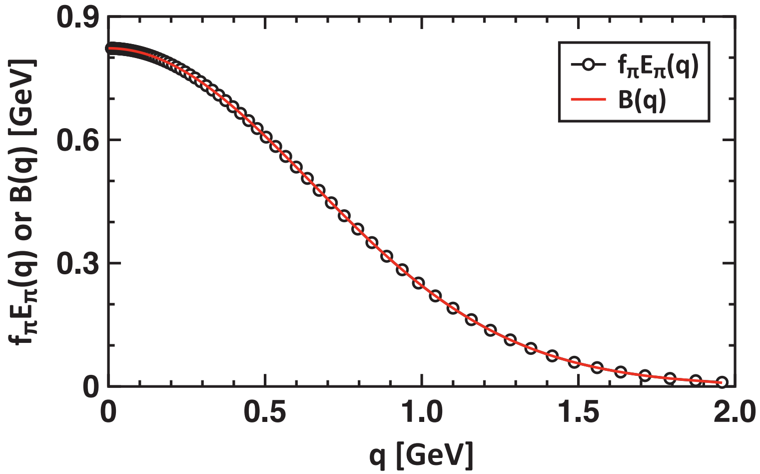
- Combing the normalization with the vector WGTI, the form factor beyond simplest impulsion approximation (triangle diagram) can be constructed.

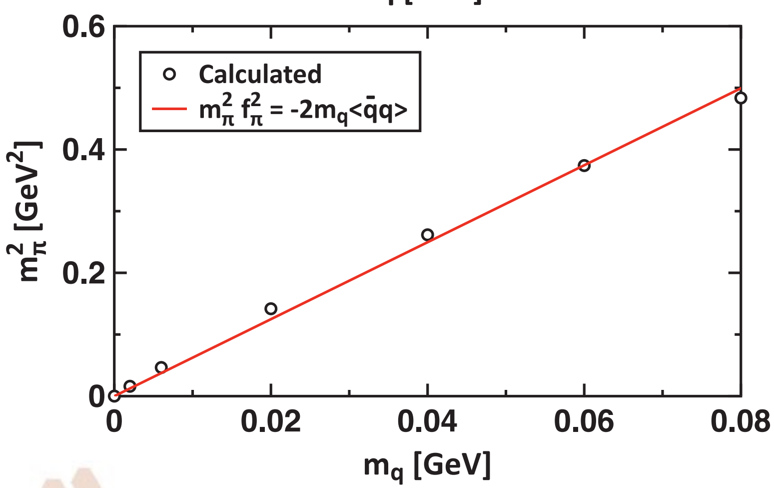
IV. Numerical implementation: The first step

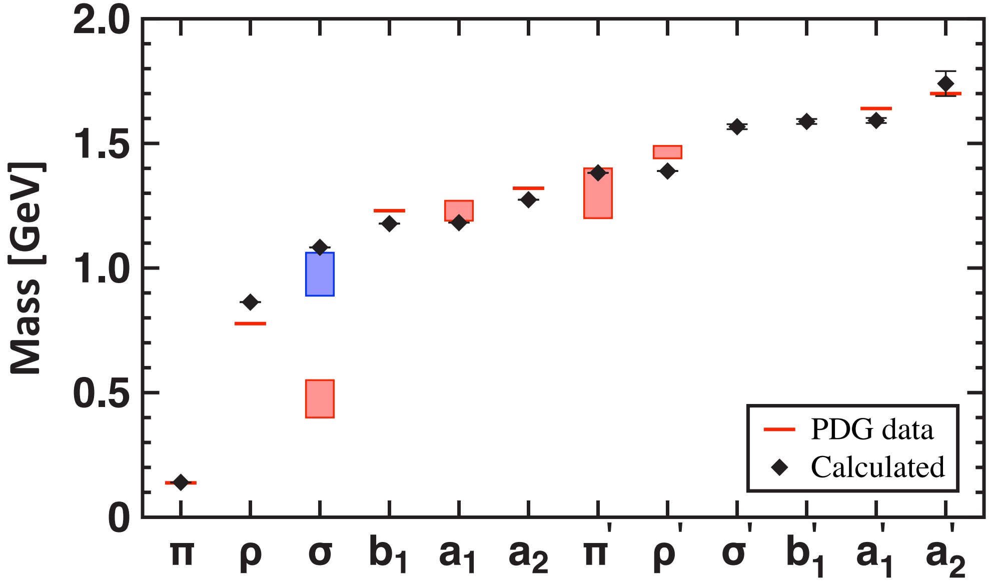
Let the quark-gluon vertex includes both longitudinal and transverse parts:

$$\Gamma_\mu(p,q) = \Gamma_\mu^{\mathrm{BC}}(p,q) + \eta\,\Gamma_\mu^{\mathrm{T}}(p,q) \qquad \Gamma_\mu^{\mathrm{T}}(p,q) = \Delta_B \tau_\mu^8 + \Delta_A \tau_\mu^4$$

$$\boxed{\begin{aligned}\tau_\mu^4 &= 4 l_\mu^{\mathrm{T}} \gamma\cdot k + 4 i \gamma_\mu^{\mathrm{T}} \sigma_{\nu\rho} l_\nu k_\rho,\\ \tau_\mu^8 &= 3\, l_\mu^{\mathrm{T}} \sigma_{\nu\rho} l_\nu k_\rho / (l^{\mathrm{T}} \cdot l^{\mathrm{T}}).\end{aligned}}$$







	$-\langle\bar{q}q\rangle_0^{1/3}$	$\rho_\pi^{1/2}$	f_π	m_π	m_ρ	m_σ	m_{b_1}	m_{a_1}	m_{a_2}	$m_{\pi'}$	$m_{\rho'}$	$m_{\sigma'}$	$m_{b_1'}$	$m_{a_1'}$	$m_{a_2'}$
this work	0.291	0.526	0.089	0.14	0.86	1.08	1.17	1.18	1.27	1.38	1.39	1.56 ± 0.01	1.57 ± 0.01	1.58 ± 0.01	1.74 ± 0.05
PDG	-	-	0.092	0.14	0.78	0.50	1.24	1.26	1.32	1.30	1.45	-	-	1.64	1.70

TABLE I: The meson spectrum (Full vertex, $(D\omega)^{1/3} = 0.64$ GeV, $\omega = 0.60$ GeV, $\eta = 0.95$ and $m_q = 2.5$ MeV).

Summary

✦ Based on WGTIs resulting from **symmetries**, a **systematic** and **self-consistent** method to construct **the quark-gluon vertex, the scattering kernels**, and **the form factors** beyond the simplest approximation, is proposed;

✦ The **numerical implementation** is in progress. The ***first*** step has showed that the light meson spectrum, including **ground** and **radially excited mesons**, can be well described.

Outlook

✦ The ***second*** step: To calculate diquark spectrum with the sophisticated kernel, and baryon spectrum, accordingly.

✦ The ***third*** step: To calculate form factors of mesons and diquarks beyond triangle diagram, and those of baryons, accordingly.

✦ The ***forth*** step: To calculate contributions of meson cloud, quantitatively, and identify their importance.