Electrocouplings for the resonances of $[70^-]$-multiplet from the Light Front Quark model

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1. Introduction

Last decade has been marked by a significant progress in the experimental study of low-lying baryon resonances (radial/orbital nucleon excitations with $J^P = \frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}$).

Many theoretical approaches to the transition form factors , which start from the first principles, were developed, but some rough estimates may be made on the basis of a light-front quark model.

It implies the construction of a good basis of light-front quark configurations possessing a definite value of the orbital (L) and total ($J = L + S$) angular moments.
The basis should satisfy the Pauli exclusion principle.

Light-front quark wave functions were successfully used by many authors for description of nucleon form factors and transition amplitudes as before appearance of new (polarized electron) data, as well after these. As a rule, the data up to $Q^2 \lesssim 5$ - 6 GeV^2 are discussed.

Now CLAS12 plans measurements up to 12 GeV^2 that initiates the theoretical study of baryon resonance electroproduction at more high Q^2 in terms of relativistic approaches.

Our approach is to fit parameters of light-front quark configurations to the elastic nucleon form factors and use these for excited ($L = 0,1$) nucleon states to calculate the transition form factors at large Q^2 up to 12 GeV^2.

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In our recent work PRD 89, 014032 2014 we have generalized our earlier non-relativistic quark model calculations of Roper resonance electroproduction at $Q^2 < 4\,\mathrm{GeV}^2$ PRD 84, 014004 (2011) by going to more high Q^2 in terms of light-front quark configurations.

By solving this problem we have run into another difficulty: the contribution of excited relativistic quark configurations (at least for $L = 0, 1$)

overestimates the transition amplitudes $N + \gamma^* \to N^*$,

while the elastic $N + \gamma^* \to N$ form factors calculated at the same basis are in a good agreement with the data.

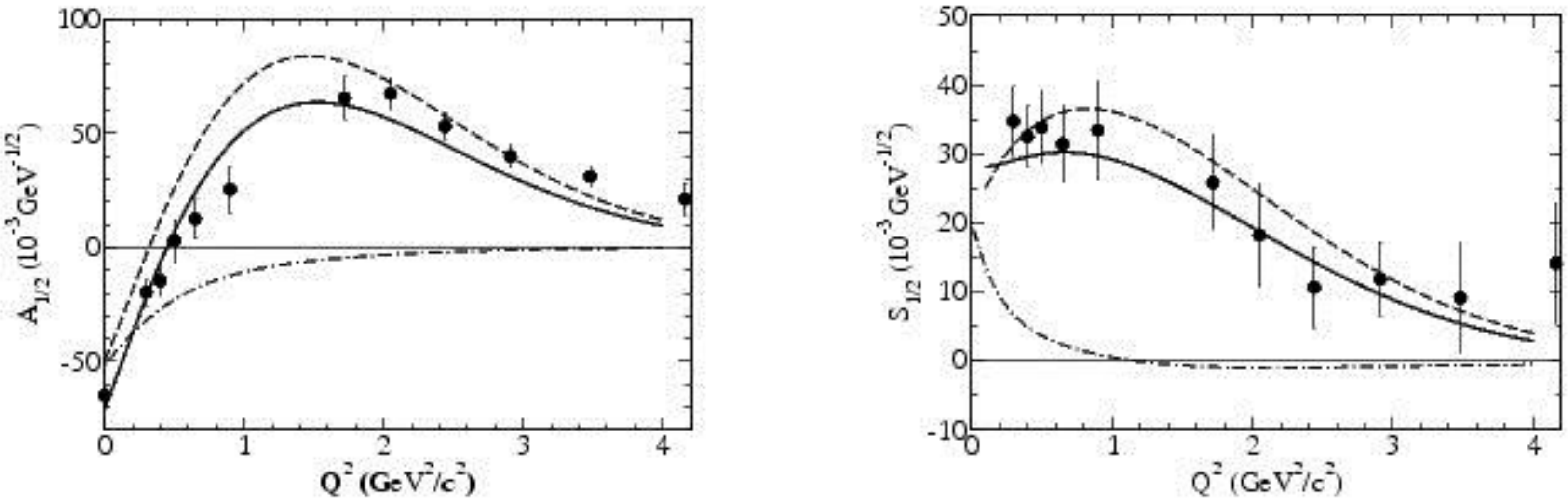
It follows that, along with the quark core, other (more soft) degrees of freedom should be taken into consideration. At high Q^2 the contribution of soft components (the higher Fock states) to the amplitude is quickly dying out, and only the the quark core contribution survives.

The correction for meson cloud was considered in detail by E. Santopinto et al. (J.P.G 24, 753) in a nonrelativistic quark model and by I.G. Aznauryan and V. Burkert (PRC85, 055292) in a light front quark model.

In the case of the Roper resonance it is almost evident that its inner structure cannot be adequately described in terms of only the constituent quark degrees of freedom. Starting from this we have considered the Roper resonance $R = N_{1/2^+}$ as a mixed state of the radially excited quark configuration $3q^* = sp^2[3]_X$ and the "hadron molecule" $(N\sigma)_{mol} = |N + \sigma\rangle$

$$R = \cos\theta\,|3q^*\rangle + \sin\theta\,|N + \sigma\rangle$$

Mixing angle θ was adjusted to optimize the description of the transvers helicity amplitude $A_{1/2}$ only. We found that at the value of $\cos\theta = 0.7$ - 0.8 this model correlates well with the recent CLAS data on the both $A_{1/2}$ and $S_{1/2}$ helicity amplitudes



- - - - - bar quark configuration $sp^2[3]_X$

- · - · - · hadron molecule $N\sigma$

——— $cos\theta 3q^* + sin\theta N\sigma$, $cos\theta = 0.8$

However we used non-relativistic h.o. quark configurations for baryons (Gaussians), and thus such calculations would be senseless at high Q^2.

2. High Q^2. Nucleon and the Roper resonance at Light Front

At high $Q^2 \gtrsim$ 3-4 Gev^2 the contribution of soft components of the baryon (the meson cloud, "molecular" admixtures, etc.) to transition form factors falls off in comparison with the "quark core" contribution. Hence, the quark core should only be considered at high Q^2.
Unfortunately the Gaussian which usually are used as the quark core wave function also predicts form factors which are quickly dying out at $Q^2 \gtrsim$ 3-4 GeV^2.

It is evident that a Gaussian cannot at the same time reproduce the characteristic hadron size $\approx 0.5\, Fm$ and the effect of hard gluon exchange between quarks, which really defines the high momentum part of w.f.
Possible alternatives to the Gaussian wave function are:
- a superposition of many Gaussians (S Capstick, 2007);
- a pole-like w.f.,
- a model with the running quark mass (Aznauryan and Burkert, PRC 85, 055202), following the QCD predictions; etc.

We have chosen a pole-like form of the w.f.

Pole-like form of the nucleon ground state wave function Φ_{0S}

$$\Phi_{0S}(\xi, \eta, k_\perp, K_\perp) = \frac{\mathcal{N}_{0S}}{(1 + \mathcal{M}_0^2/\beta^2)^\gamma}$$

$$\mathcal{M}_0^2 = \frac{M^2 + k_\perp^2}{\eta\xi(1-\xi)} + \frac{\eta M^2 + K_\perp^2}{\eta(1-\eta)}$$

was firstly fitted to the elastic nucleon form factors by Schlumpf (PRD47, 4114) with γ =3.5 and $\beta \approx$2M.

Here k, K are relativistic relative moments in pairs "1-2" and "(12)-3" respectively, the light-front variables are $x_1 = \xi\eta$, $x_2 = (1-\xi)\eta$, $x_3 = 1-\eta$, and $\mathcal{M}_0$ is the mass of the free $3q$ system, M is the constituent quark mass.

Such form is as yet unjustified, but it should be noted that at least in the meson sector the pole-like form of the pion $\bar{q}q$ w.f.

$$f_\pi \varphi_\pi(x, k_\perp^2) = \frac{9}{4\pi^2} \frac{1}{\left(1 + \frac{k_\perp^2}{4M^2 x(1-x)}\right)^\kappa}, \quad \kappa = 1$$

was recently reconstructed starting from the Bethe-Salpeter wave function (C.D. Roberts, arXiv:1509.02925) projected onto the light front (L.Chang *et al*, PRL110, 132001). There are also approximations with κ =1 - 2.

The nucleon pole-like w.f. $\Phi_{0S}(\xi, \eta, k_\perp, K_\perp)$ looks like a generalization of the pion $\bar{q}q$ w.f. for the case of the $3q$ system.

Starting from the "+" component of quark current on the light front

$$I^{(i)+} = e^{(i)} \left(I f_1 + i\,\hat{n}_z \cdot [\sigma^{(i)} \times q_\perp] f_2 \right),$$

(without form factors, $f_1 = 1$, $f_2 = \frac{\varkappa_q}{2M}$, but with small anomalous magnetic moment $\varkappa_q$), we have fitted the free parameters of model to the nucleon data (preserving the characteristic values, find by Schlumpf: $\gamma \approx 3.5$ and $\beta \approx 2$M).

We used data within a full measured range $0 \leq Q^2 \leq 32$ GeV^2 including the electron polarization data on the ratio G_E/G_M at $Q^2 \lesssim 6$-8 GeV^2.

With the values $\gamma = 3.51$, $M = 0.251$ GeV, $\varkappa_u = -0.0028$, $\varkappa_u = 0.0224$, $\beta_u = 0.579(0.59)$ GeV, $\beta_d = 0.5(0.48)$ GeV we have obtained a not so bad description of the data.

We also compared the obtained fit with results of another relativistic approach, in which an effective light-front wave function was derived from the matching of soft-wall AdS/QCD and light-front QCD.

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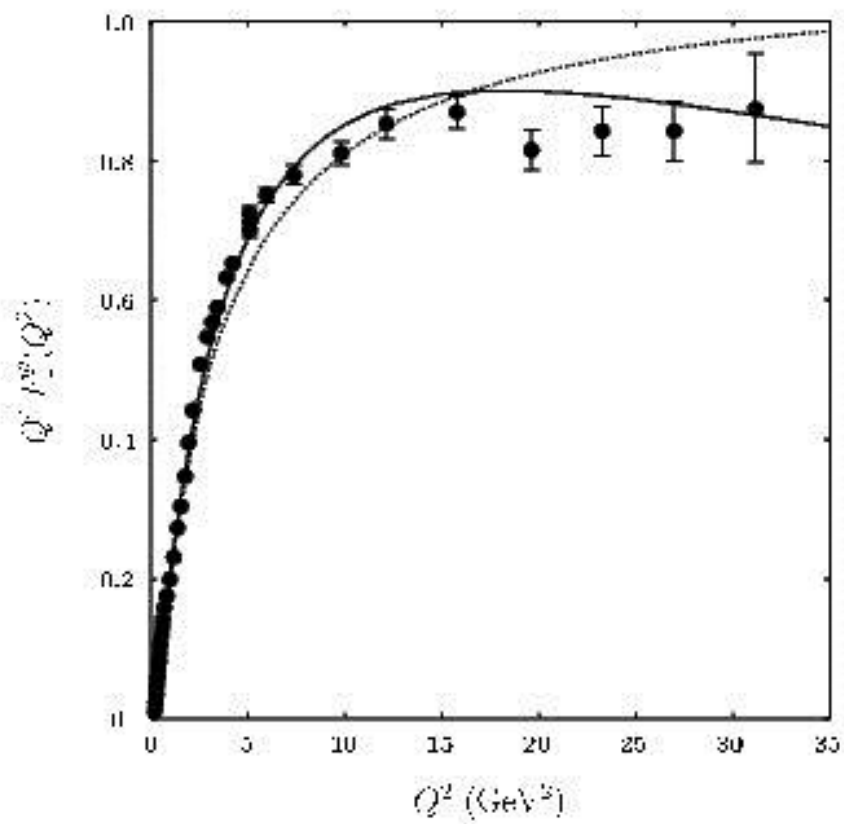
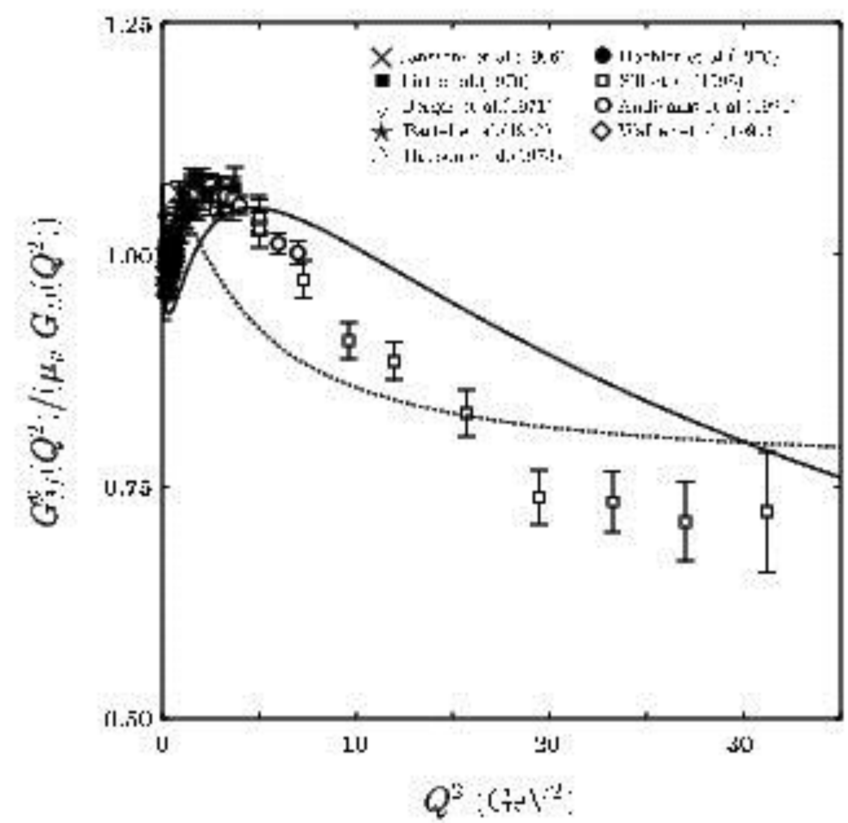
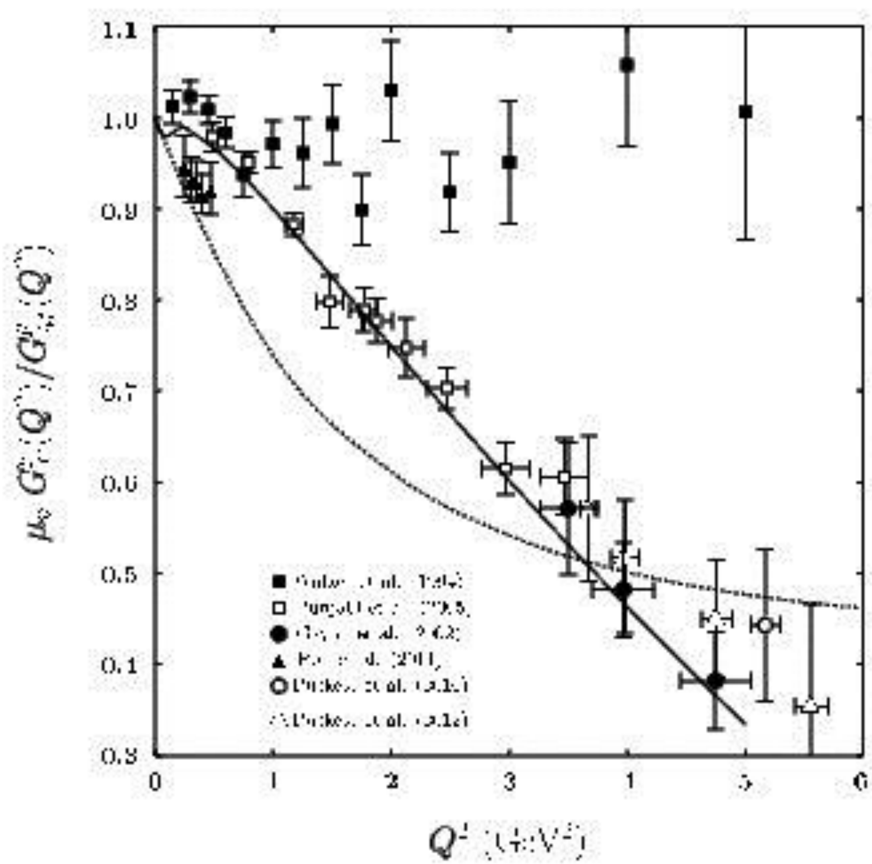
$Q^4 F_1^p$ $\qquad\qquad\qquad\qquad$ $G_M^p/(\mu_p G_D)$



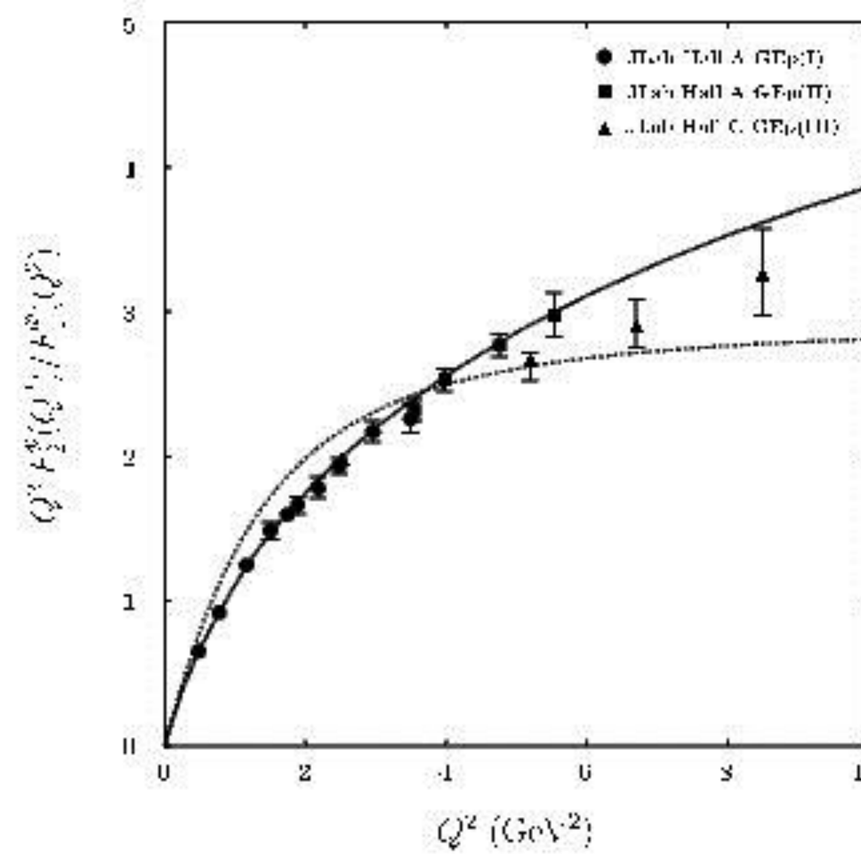


—— **LFQM** $\qquad$ .... **AdS/QCD**

$\mu_p G_E^p / G_M^p$



$Q^2 F_2^p / F_1^p$



—— **LFQM** .... **AdS/QCD**

The pole-like w.f. is also good for the data at $Q^2 \to 0$:

Таблица 1: Electromagnetic properties of nucleons in LF quark models

Quantity	LFQM	AdS/QCD	Data
μ_p (in n.m.)	2.820 (2.820)	2.793	2.793
μ_n (in n.m.)	-1.920 (-1.920)	-1.913	-1.913
μ_u (in n.m.)	3.720 (3.720)	3.673	1.673
μ_d (in n.m.)	-1.020 (-1.020)	-1.033	-2.033
r_E^p (fm)	0.871 (0.872)	0.789	0.8921 ± 0.0073
$\langle r_E^2 \rangle^n$ (fm^2)	-0.014 (-0.022)	-0.108	-0.1161 ± 0.0022
r_M^p (fm)	0.883 (0.872)	0.757	$0.777 \pm 0.013 \pm 0.010$
r_M^n (fm)	0.898 (0.893)	0.773	$0.862^{+0.009}_{-0.008}$
r_E^u (fm)	0.867 (0.866)	0.754	0.8589 ± 0.0107
r_E^d (fm)	0.855 (0.846)	0.638	0.7507 ± 0.0094
r_M^u (fm)	0.875 (0.832)	0.749	0.7288 ± 0.0151
r_M^d (fm)	0.938 (0.949)	0.815	1.0582 ± 0.0434

3. High Q^2. Quark configurations and Melosh transformations.

A good basis of relativistic quark configurations satisfying the Pauli exclusion principle is needed for baryons with $J^P = 1/2^{\pm}, 3/2^{\pm}, 5/2^{\pm}$.

We start from non-relativistic shell-model configurations and change the h.o. wave functions for light-front w.f.'s dependent on the relativistic relative moments $\boldsymbol{k}$, $\boldsymbol{K}$ and expressed in light-front invariants $\boldsymbol{\xi}, \boldsymbol{\eta}$, $\boldsymbol{\lambda}_\perp = (1-\xi)\boldsymbol{p}_{1\perp} - \xi\boldsymbol{p}_{2\perp}$, $\boldsymbol{\Lambda}_\perp = \eta\boldsymbol{p}_{3\perp} - (1-\eta)(\boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp})$.

At this stage, as usual, arrives problems with boosts (in the instant form of dynamics) or rotations (at the light front). In both cases generators of transformations depend on the dynamics, and thus such shell-model basis is only useful for representation of elements of kinematic subgroups of the full Lorentz group.

Nevertheless some difficulties can be resolved by going to the rest frame of each two-body subsystem and using Clebsch-Gordon coefficients of the Lorentz group (B.D. Keister, W.N. Polyzou, Adv. N.P. 20, 225) for definition of states with total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$.

Recall that the spin $\vec{s}_i$ of the particle is uniquely determined in its own rest frame, where $p_i = \overset{\circ}{p}_i \equiv \{M, 0, 0, 0\}$. In the moving frame the canonical spin state is defined by a rotationless Lorentz boost $\lambda(p_i \leftarrow \overset{\circ}{p}_i)$

$$|p_i; s_i\mu_i\rangle_c = U\left(\lambda(p_i \leftarrow \overset{\circ}{p}_i)\right)|\,\overset{\circ}{p}_i; s_i\mu_i\rangle\ ,$$

while the light-front spin state is defined by another type of Lorentz transformation, which leads to the same momentum p_i, but would be represented as a two-step process (M.V. Terent'ev et al. N.P. B158, 497):

$$l(p_i \leftarrow \overset{\circ}{p}_i) = \lambda(p_i \leftarrow p_\infty)\lambda(p_\infty \leftarrow \overset{\circ}{p}_i),$$

where p_∞ is the quark momentum in the infinite momentum frame.

$$|p_i; s_i\mu_i\rangle_f = U\left(l(p_i \leftarrow \overset{\circ}{p}_i)\right)|\,\overset{\circ}{p}_i; s_i\mu_i\rangle\ ,$$

The full manifold of l's forms a subgroup of the Lorentz (Poincare) group. It is a kinematic subgroup in the light-front dynamics — an analog of the rotational subgroup used in the instant form of dynamics.

As a result, canonical and front states are related by a specific rotation named the Melosh transformation.

$$|\boldsymbol{p}_i, s_i\mu_i\rangle_c = \sum_{\mu_i'} |\boldsymbol{p}_i, s_i\mu_i'\rangle_f \, D^{s_i}_{\mu_i'\mu_i}[R(\boldsymbol{p}_i, M_i)],$$

$$R(\boldsymbol{p}_i, M_i) = \underbrace{\lambda(\mathring{p}_i \leftarrow \boldsymbol{p}_i)\lambda(\boldsymbol{p}_i \leftarrow \boldsymbol{p}_\infty)\lambda(\boldsymbol{p}_\infty \leftarrow \mathring{p}_i)}_{Melosh\ rotation}$$

For quarks ($\boldsymbol{s_i = 1/2}$) $\quad D^{\frac{1}{2}}_{\mu_i'\mu_i}[R(\boldsymbol{p}_i, M_i)] = \left(\frac{M + p_i^+ + i\hat{\boldsymbol{n}}_z \cdot [\boldsymbol{\sigma}_i \times \boldsymbol{p}_{i\perp}]}{\sqrt{(M + p_i^+)^2 + p_{i\perp}^2}} \right)_{\mu_i'\mu_i}$

The Lorentz boost at light front $\boldsymbol{l}(\boldsymbol{p}_i' \leftarrow \boldsymbol{p}_i)$ does not rotate the spin

$$|\boldsymbol{p}_i; \boldsymbol{s}_i\boldsymbol{\mu}_i\rangle_{\boldsymbol{f}} \to \boldsymbol{U}(\boldsymbol{l}(\boldsymbol{p}_i' \leftarrow \boldsymbol{p}_i))|\boldsymbol{p}_i; \boldsymbol{s}_i\boldsymbol{\mu}_i\rangle_{\boldsymbol{f}} = |\boldsymbol{p}_i'; \boldsymbol{s}_i\boldsymbol{\mu}_i\rangle_{\boldsymbol{f}}$$

that is very convenient for description of states in moving reference frames.

4. Orbital momentum. Wave functions and matrix elements.

The front spin is not convenient for construction of the total angular momentum $\boldsymbol{J}=\boldsymbol{L}+\boldsymbol{S}$. However for a quark pair such state can be constructed in the pair rest frame (cm). $(p_1+p_2)_{cm}=\{\mathcal{M}_{0(12)},0,0,0\}=\mathring{\mathcal{P}}_{12}$

$$|\mathring{\mathcal{P}}_{12},j_{12}j_{12}^z(ls_{12})\rangle=\sum_{\mu_1\mu_2\mu_{12}m}(s_1\mu_1s_2\mu_2|s_{12}\mu_{12})(lms_{12}\mu_{12}|j_{12}j_{12}^z)$$

$$\times\int d\hat{k}_{cm}Y_{lm}(\hat{k}_{cm})|p_{1cm}\mu_1\rangle_c|p_{2cm}\mu_2\rangle_c,\quad j_{12}=l+s_{12}$$

Under Lorentz boosts this state transforms as an elementary particle with the spin j_{12} and the mass $\mathcal{M}_{0(12)}=\sqrt{\frac{M^2+k_\perp^2}{\xi(1-\xi)}}$ (Yu. Shirokov, 1958). So we should go to the center-of-mass of $3q$ system (CM) and use the same formula for the total angular momentum $J=L+j_{12}+s_3$

$$|\,\mathring{\mathcal{P}}\,JJ^z(LS(j_{12}S_3))\rangle=\sum_{j_{12}^z\mu_3\mu M}(j_{12}j_{12}^zs_3\mu_3|S\mu)(LMS\mu|JJ^z)$$

$$\times\int d\hat{K}Y_{LM}(\hat{K})|p_3\mu_3\rangle_c|\mathcal{P}_{12},j_{12}(ls_{12})\rangle_c,\quad p_3=K,\ \mathcal{P}_{12}=-K$$

with substitution $|p_{1cm}\mu_1\rangle_c\to|p_3\mu_3\rangle_c$ $|p_{2cm}\mu_2\rangle_c\to|\mathcal{P}_{12},j_{12}(ls_{12})\rangle_c$.

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Using the light-front boosts for the transition from reference frame $cm(12)$ to reference frame CM(12-3) (and further from the CM frame to the Breit frame) implles several Melosh rotations that transform the wave functions of initial and final states to a complicated form.

Fortunately the Melosh rotation acting on all spins of the system does not violate the Pauli exclusion principle for the full $3q$ wave function, though the calculation technique (fraction parentage coefficients, etc.) becomes more cumbersome, especially in the case of nonzero orbital moments L and l in the subsystems 12-3 and 12.

Eventually the spin-orbital parts of searched wave functions with orbital moments l and L are:

1) in the case of $l = 0$ (in pair 1-2), $L \neq 0$ (in pair 12-3):

$$\Psi^{(1)}_{J(LS(s_{12}))\mu_J}(\mathcal{P}, K, k; \mu_1, \mu_2, \mu_3) \equiv {}_f\langle p_1\mu_1, p_2\mu_2, p_3\mu_3|\mathcal{P}, J(LS(s_{12}))\mu_J\rangle_f$$
$$= \phi_L(K)\Phi_0(\mathcal{M}_0)\sum_{\mu_S\mu_L} Y_{L\mu_L}(\hat{K})(L\mu_L S\mu_S|J\mu_J)\sum_{\{\bar{\mu}\}}(s_1\bar{\mu}_1 s_2\bar{\mu}_2|s_{12}\bar{\mu}_{12})(s_{12}\bar{\mu}_{12}s_3\bar{\mu}_3)$$
$$\times D^{s_{12}}_{\mu_{12}\bar{\mu}_{12}}[R^{-1}(-K, \mathcal{M}_{0(12)})]D^{s_1}_{\mu_1\bar{\mu}_1}[\tilde{R}^{-1}(k, M_1)]D^{s_2}_{\mu_2\bar{\mu}_2}[\tilde{R}^{-1}(-k, M_2)]D^{s_3}_{\mu_3\bar{\mu}_1}[R^{-1}(K, M_3)],$$

2) a similar expression in the case of $l \neq 0$, i$L = 0$:

$$\Psi^{(2)}_{J(j_{12}(ls_{12})s_3)\mu_J}(\mathcal{P}, K, k; \mu_1, \mu_2, \mu_3) \equiv {}_f\langle p_1\mu_1, p_2\mu_2, p_3\mu_3|\mathcal{P}, J(j_{12}(ls_{12})s_3)\mu_J\rangle_f$$
$$= \phi_l(k)\Phi_0(\mathcal{M}_0)\sum_{\mu_{12}\mu_l} Y_{l\mu_l}(\hat{k})\ldots$$

with a complicated Melosh rotation $\tilde{R}$ in pair 1-2:

$$\tilde{R}^{-1}(k, M_1) = R(p_{1CM}, M_1)R^{-1}(k, M_1)R^{-1}(p_{1CM}, M_1)$$

In the case of $[56^+]$-multiplets (nucleon or Roper), where $L = l = 0$, we denote such w.f. as $\Psi^{(0)}$.

In the case of $[70^-]$-multiplet there are two basis states, $\Psi^{(1)}$ with $L = 1$, $l = 0$ and $\Psi^{(2)}$ with $L = 0$, $l = 1$, which correspond to Yamanouchi symbols $y^{(1)} = 112$ and $y^{(2)} = 121$ respectively (in terms of the irreducible representation of symmetric group S_3 responding to Young scheme [21] in the coordinate space (X)).

Taking into account the spin (S) part of $\Psi^{(n)}$ on can wtite for the $[70^-]$-multiplet

$$\Psi^{(1)}_{s_{12}=1} = |[21]_X\, y^{(1)}_X\rangle\, |[21]_S\, y^{(1)}_S\rangle, \quad \Psi^{(1)}_{s_{12}=0} = |[21]_X\, y^{(1)}_X\rangle\, |[21]_S\, y^{(2)}_S\rangle,$$
$$\Psi^{(2)}_{s_{12}=1} = |[21]_X\, y^{(2)}_X\rangle\, |[21]_S\, y^{(1)}_S\rangle, \quad \Psi^{(2)}_{s_{12}=0} = |[21]_X\, y^{(2)}_X\rangle\, |[21]_S\, y^{(2)}_S\rangle,$$

while for 56^+ one has

$$\Psi^{(0)}_{s_{12}=1} = |[3]_X\rangle\, |[21]_S\, y^{(1)}_S\rangle, \; \Psi^{(0)}_{s_{12}=0} = |[3]_X\rangle\, |[21]_S\, y^{(2)}_S\rangle.$$

Taking into account the isospin part (e.g. for the positive charge)

$$|[21]_T\, y_T^{(1)}\rangle = \sqrt{\frac{2}{3}} uud - \sqrt{\frac{1}{6}}(ud+du)u, \quad |[21]_T\, y_T^{(2)}\rangle = \sqrt{\frac{1}{2}}(ud-du)u$$

one eventually obtains the full w.f. satisfying the Pauli exclusion principle

$$\Psi_N(56^+) = \sqrt{\frac{1}{2}}\left[\Psi^{(0)}_{s_{12}=1}|[21]_T\, y_T^{(1)}\rangle + \Psi^{(0)}_{s_{12}=0}|[21]_T\, y_T^{(2)}\rangle\right],$$

$$\Psi_{N^*}(70^-) = \frac{1}{2}\left[\Psi^{(1)}_{s_{12}=1} - \Psi^{(2)}_{s_{12}=0}\right]|[21]_T\, y_T^{(1)}\rangle + \frac{1}{2}\left[\Psi^{(1)}_{s_{12}=0} + \Psi^{(2)}_{s_{12}=1}\right]|[21]_T\, y_T^{(2)}\rangle.$$

Polynomial parts of w. f.s $\Psi^{(\kappa)}$, $\kappa = 0, 1, 2$, are analogous to h.o. ones:

$\phi_{n=1L=1} = |\boldsymbol{K}|/\beta$ ($\phi_{n=1l=1} = |\boldsymbol{k}|/\beta$) for the 70^- multiplet,
$\phi_{n=2L=0} = [1 - c_R \mathcal{M}_0^2(k, K)/\beta^2]$ for the rad. excited 56^+ (Roper.)

5. Electroproduction of lightest $[56^+]$- and $[70^-]$-resonances.

We calculated the transition amplitudes for the electroproduction of lightest nucleon resonances with $L = 0,1$. The radial parts of w.f.s $\Psi^{(\kappa)}$, $\kappa = 0,1,2$, were choosen in the form

$$\Phi^{(0)}_{2S}(k,K) = \phi_{20}(k,K)\,\Phi_{0S}[\mathcal{M}_0(k,K)], \quad \Phi^{(1)}_{1P} = \phi_{11}(K)\,\Phi_{0S}, \quad \Phi^{(2)}_{1P} = \phi_{11}(k)\,\Phi_{0S}$$

The spin-orbital part of current matrix element for transition $N \to N^*_{1/2^-}$ are defined by functions $\Psi^{(\kappa)}$.
E.g., in the Breit frame, $q^\nu_B = \{0, \boldsymbol{q}_\perp, 0\}$, $Q^2 = \boldsymbol{q}^2_\perp$, the contribution of $\Psi^{(1)}$ to the 3rd quark current matrix element is

$$\langle \Psi^{(1)}_{J'(L'S'(s'_{12}))\mu'_J} | \hat{I}^+_3 | \Psi^{(0)}_{J(LS(s_{12}))\mu_J} \rangle = \delta(\mathcal{P}'_B - \mathcal{P}_B - q_\perp) \sum_{\{\mu_i\}} e_{q_3} (f_1 \delta_{\mu'_3\mu_3} + f_2 q_\perp (-1)^{1/2-\mu_3} \delta_{-\mu'_3\mu_3})$$

$$\times \int \frac{d^2\lambda_\perp d^2\Lambda_\perp d\xi d\eta}{\xi(1-\xi)\eta(1-\eta)} \Psi^{(1)}_{J'(L'S'(s'_{12}))\mu'_J}(\mathcal{P}'_B, K', k'; \mu_1, \mu_2, \mu'_3)\, \Psi^{(0)}_{J(LS(s_{12}))\mu_J}(\mathcal{P}_B, K, k; \mu_1, \mu_2, \mu_3),$$

where $\mathcal{P}_B = -\alpha \boldsymbol{q}_\perp/2$, $\mathcal{P}'_B = (1-\alpha)\boldsymbol{q}_\perp/2$, $\alpha = \frac{q^2_\perp + M^2_{N^*} - M^2_N}{2q_\perp}$.

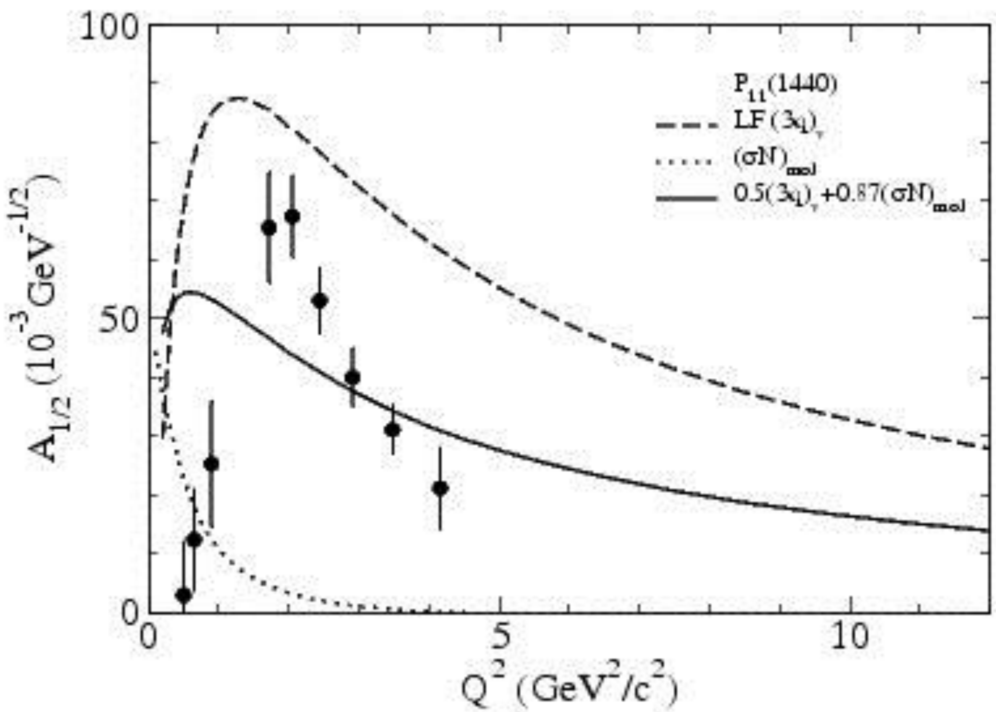
Matrix elements of the I^+ component of quark current allowed us to calculate the transverse ($A_{1/2}$) and longitudinal ($S_{1/2}$) helicity amplitudes for electroproduction of the lightest nucleon resonances $N^*_{1/2^+}(1440)$ and $N^*_{1/2^-}(1535)$ in a large interval Q^2 up to 12 GeV^2.
We used the same technique (fractional parentage coefficients etc.) as in our recent work (PRD 89, 014032) and the same parameters of the light-front quark model. For the $N^*(1535)$ we used a mixed model which is analogous to that of Roper resonance:

$$\boldsymbol{N^*(1535) = \cos\theta^*|3q^*\rangle + \sin\theta^*|\Lambda + K^+\rangle}$$

with $|3q^*\rangle$ constructed from excited quark configurations $\Psi^{(1)}$ and $\Psi^{(2)}$. The value of mixing angle θ^* was adjusted to optimize the description of helicity amplitude $A_{1/2}$, and we have obtained $\cos\theta^* = 0.6$. We also updated the fit of parameter θ for the Roper resonance reducing $\cos\theta = 0.57$ to the new value $\cos\theta = 0.5$ which is more close to CLAS data at $Q^2 \gtrsim 3$ - 4 GeV^2.

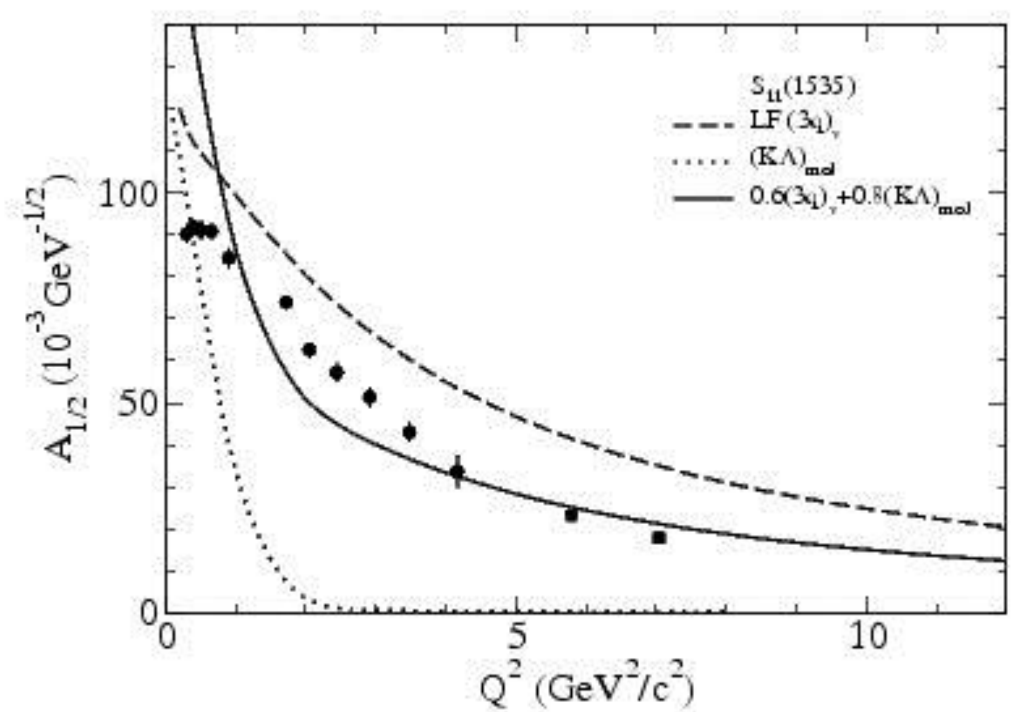
$N \to N_{1/2^+}(1440)$

$\cos\theta = 0.5$



$N \to N_{1/2^-}(1535)$

$\cos\theta^* = 0.6$



Transverse helicity amplitude $A_{1/2}$ for electroproduction of the lightest nucleon resonances.

6. Summary

1. The lightest nucleon resonances are described at light front as mixed states of the $3q$ **cluster possessing a definite value of the inner orbital momentum** $L = \mathbf{0,1}$ **and a hadron molecular state,** $N + \sigma$ **or** $\Lambda + K$.

2. Nucleon elastic form factors are successfully described in a large interval of Q^2 **by the lowest light-front quark configuration without any hadron-molecular admixtures.**

3. But in the case of nucleon resonances the quark core overestimates the transition amplitudes at large Q^2**, and thus the weight of quark core in the resonance should be relatively small (about 1/3 - 1/4) because of excitation of higher Fock states.**

Such situation is also characteristic of other relativistic models recently developed. It could be considered as evidence of a large role of the soft meson cloud in resonances as compared to the ground state of the nucleon.

Suplement

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7. Search of the lightest hybrid baryons.

In the context of projected extensive search of baryon resonances in the mass region 2$\lesssim W \lesssim$3 GeV (now available with the upgraded CEBAF) there is an interest in study of a possible "hybrid" baryons.

We suggest that the possible lightest hybrid baryon $\Lambda^g_{1/2^+}$ with quantum numbers of Λ hyperon could be a component of the molecular state $\Lambda^g + K$ coupled to the negative parity baryon resonance $N^{*g}_{1/2^-}$ which we denote as $S_{11}(2300)$ (a rough estimate of its mass gives $\gtrsim$2.3 GeV):

$$N^{*g}_{1/2^-} \simeq S_{11}(2300) = \cos\theta_g N^{*c}_{1/2^-}[(3q^*)^c g] + \sin\theta_g |\Lambda^g_{1/2^+} + K^+\rangle$$

Its quark part $(3q^*)^c$ (a colored state) should be the same parity as the ΛK ground state, i.e. $J^P = 1/2^-$ (it implies the positive parity of the constituent gluon). This is realized in the $s^2p[21]_X$ quark configuration which has been used for conventional $N^*_{1/2^-}$ resonances (the $\underline{70}^-$ multiplet of SU(6)).

The hybrid hyperon $\Lambda^{g}_{1/2^+}$ is determined as a confined state of the constituent gluon $g(1^+)$ (the gluon lowest eigenmode with $J^P = 1^+$) and the colored $3q^s$ cluster of positive parity (different from $3q^*)^c$)

$$(3q^s)^c \sim s^3[3]_X \circ \underbrace{[21]_C \circ [21]_S}_{[3]_{cs}} \circ [1^3]_F (uds)$$

which stands out as being a state of the full color-spin symmetry $[3]_{cs}$ that implies the maximal color-magnetic (hyperfine) attractive forces in the $3q$ system. But in this case the strange quark s is needed to form the flavor-antisymmetric state $|uds[1^3]_F\rangle$.

The mass of the bound $(3q)^c + g(1^+)$ state $\sim$ 1.8 GeV was evaluated in the MIT Bag model (T. Barns, F.E. Close, Phys. Lett.B 123, 89 (1983)) for the similar quark-gluon configuration which determines a baryon resonance of positive parity $N^{g}_{1/2^+}$ (without the s quark, but with a more slight quark attraction in the state $[21]_{CS}$)

$$(3q)^c \sim s^3[3]_X \circ \underbrace{[21]_C \circ [21]_S}_{[21]_{cs}} \circ [21]_F (uud)$$

So there are two candidates for the lowest hybrid baryon, non-strange and strange, both of positive parity (quark configuration $s^3[3]_X[21_{SF}]$)

$$\underline{70}^+ : N^g_{1/2^+}(\approx 1800) = |(3q)^c g(1^+)\rangle$$
$$\underline{70}^+ : \Lambda^g_{1/2^+}(\approx 1800) = |(3q^s)^c g(1^+)\rangle$$

(Possibly the MIT Bag model underestimates the resonance masses and they might be greater ($>$ 2-2.2 GeV)).

We suggest that the strange hybrid baryon could be seen at the JLab in the K^+ electroproduction at $W \gtrsim\approx 2.3$ GeV as a negative parity molecular state

$$N^{*g}_{1/2^-}(\approx 2300) = \Lambda^g_{1/2^+}(\approx 1800) + K^+, \qquad \underline{70}^+ \otimes \underline{35}^- \to \underline{70}^-$$

which should predominantly decay into the s-wave YK^+ channel.

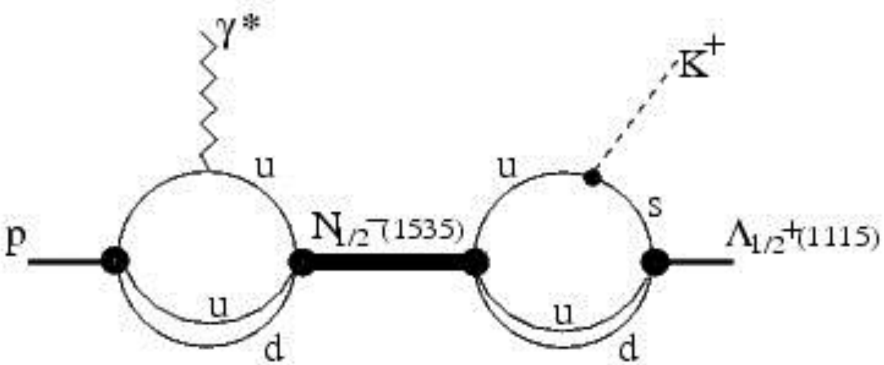
. . . .

<u>**The proposed mechanism**</u>

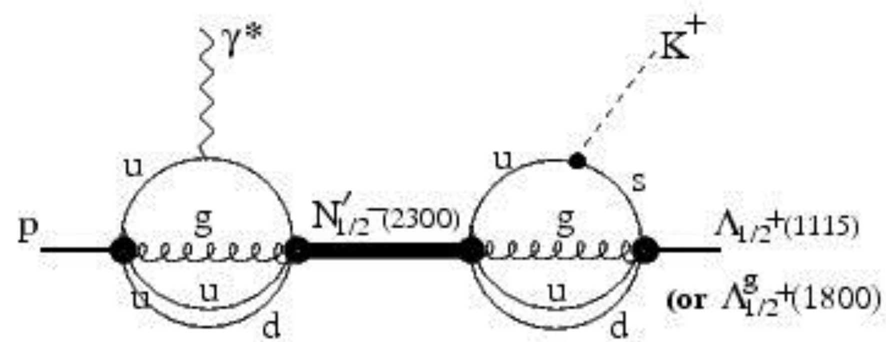
We propose that the electro-excitation of hybrid baryons can occurs due to a small admixture of the constituent gluon component in the proton ($g + uud$) and formation (after γ^* absorption) of the low-lying configuration $g + uds$ followed by emission of K^+ meson

It implies the following mechanism of K^+ electroproduction at two different regions of W:

$W \cong 1.5$ - 1.8 GeV



$W \gtrsim 2.3$ - 2.6 GeV



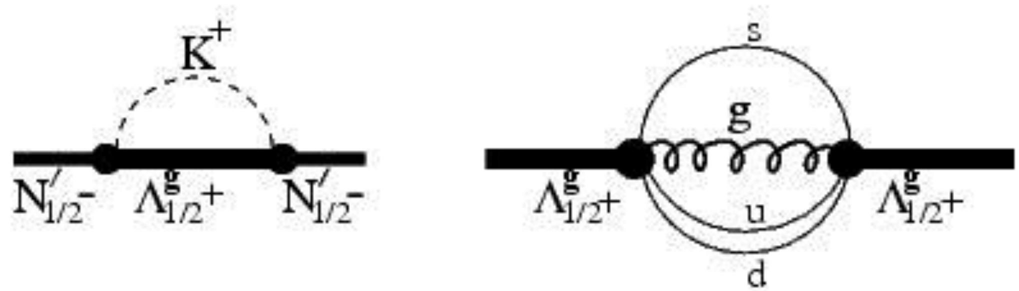
A study of processes with hybrid baryons gives a possibility to test the $3q + g$ Fock component in conventional baryons (like nucleons, etc.)

Appendix 1

Relativistic model

Our consideration will be based on the use of interpolating currents in terms of constituent quarks and gluon. In particular, the interpolating current describing a coupling of three quarks in the color state $[\mathbf{21}]_C$ with the constituent gluon is given by the convolution of quark fields and the stress tensor of gluon field with the color antisymmetric tensor ε^{abc}:

$$J_{ijk} = \varepsilon^{abc} G^{ad}_{\mu\nu} \gamma^{\mu} q^{d}_{i} q^{b}_{j} C \gamma^{\nu} q^{c}_{k}, \qquad C = \gamma^{0}\gamma^{2},$$



The corresponding phenomenological Lagrangian describing the coupling of hybrid baryon B with its interpolating current (including the coupling constant g_B) is given by their product

$$\mathcal{L}_B(x) = g_B\,\bar{B}_{ijk}(x) J_{ijk}(x) + H.c.$$

with a non-local extension of the interpolating current $J_{ijk}(x)$ that implies the integration over coordinates smeared-out with a correlation function $F\Big(\sum\limits_{i<j}(x_i - x_j)^2\Big)$ describing the distribution quarks and constituent gluons in a hybrid baryon.

$$J_{ijk}(x) = \int d^4\{x_1 \ldots x_4\}_x\, F\Big(\sum_{i<j}(x_i - x_j)^2\Big)$$
$$\times \varepsilon^{abc}\, G^{ad}_{\mu\nu}(x_4)\,\gamma^\mu q^d_i(x_1)\, q^b_j(x_2)\, C\gamma^\nu q^c_k(x_3)\,,$$

Such Lagrangian is an extension of relativistic quark model for mesons, baryons and tetraquarks proposed and developed in 90-th and 00-th by a large group of authors:

G. V. Efimov, M. A. Ivanov, J. G. Körner, M. P. Locher, V. E. Lyubovitskij, *et al.*

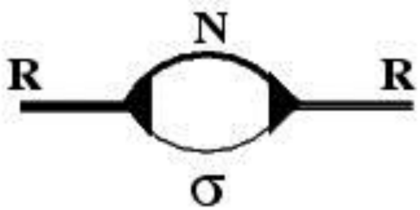
In this approach the coupling constant g_B is calculated from the compositeness condition formulated in 60-th [S. Weinberg 1962, A. Salam 1963, K. Hayashi *et al.* 1967].

It is a condition of equality to zero value of the renormalization factor Z_B for point-like elementary particle B in the case if the particle is fully composite: Z_B is the one minus the derivative Σ'_B of mass operator which is described by the loop diagram

$$Z_B = 1 - \Sigma'_B(m_B) = 0.$$

Here m_B is the mass of hybrid baryon.

Appendix 2

The hadron loop 

$\Sigma_{N\sigma}$ gives a negative contribution to the mass of the Roper resonance, and the $RN\sigma$ coupling constant $g_{RN\sigma}$ is defined by the 'compositeness condition'

$$Z_R \equiv 1 - \frac{d}{d\not{p}}\Sigma_{N\sigma}(\not{p})|_{\not{p}=m_R} = 0,$$

i.e. the elementary particle R has a zero weight in the hadron molecule.

We use effective Lagrangians (Dubna group: G. Efimov, M. Ivanov, V. Lyubovitskij) for description of non-local RNσ and NNσ interactions, e.g.

$$\mathcal{L}_{str}(x) = g_{RN\sigma} \bar{R}(x) \int d^4y \, \Phi_R(y^2) N(x+\alpha y) \sigma(x-(1-\alpha)y), \quad \alpha = \frac{M_\sigma}{m_N + M_\sigma},$$

and h.o. Gaussians as Fourier transforms of $\Phi_N(y^2)$ and $\Phi_R(y^2)$

$$\tilde{\Phi}_N(k_E^2) = exp(-\frac{k_E^2}{\Lambda^2}) \quad \text{and} \quad \tilde{\Phi}_R(k_E^2) = (1-\lambda\frac{k_E^2}{\Lambda^2}) exp(-\frac{k_E^2}{\Lambda^2})$$

with the orthogonality condition $\int \tilde{\Phi}_R(^2 k_E) \tilde{\Phi}_N(k_E^2) d^4 k_E = 0$.

The electromagnetic interaction term for this non-local vertex

$$\mathcal{L}_{em}^{(1)} = g_{RN\sigma}\bar{R}(x)\int dy\,\Phi_R(y^2)e^{-ieI(x+\alpha y,x,P)}N(x+\alpha y)\sigma(x-(1-\alpha)y) + h.c.$$

is generated when the non-local Lagrangian are gauged with a gauge field exponential $e^{-ieI(x+\alpha y,x,P)}$ where

$$I(y,x,P) = \int_x^y dz_\mu A^\mu(z), \quad P \text{ is the path of integration}$$

S.Mandelstam, Ann.Phys. 19, 1 (1963); J.Terning, Ph.Rev. D44, 887 (1991) slide 12a

The full Lagrangian of electromagnetic interaction

$$\mathcal{L}_{em} = \mathcal{L}_{em}^{(1)} + \mathcal{L}_{em}^{(2)}$$

includes also the standard term

$$\mathcal{L}_{em}^{(2)} = e_{B}\bar{B}(x)\not{A}(x)B(x), \quad B = N, R$$

obtained by minimal substitution $\partial^\mu B \rightarrow (\partial^\mu - e_{B}A^\mu)B$

Only the total sum of the first order diagrams (including the contact terms $\mathcal{L}_{em}^{(1)}$) satisfies the gauge invariance