Finite Energy Sum Rules in Hadro- and Photoproduction Reactions

Vincent MATHIEU

Indiana University

-

Joint Physics Analysis Center

INT-16-62W

November 2016

INDIANA UNIVERSITY

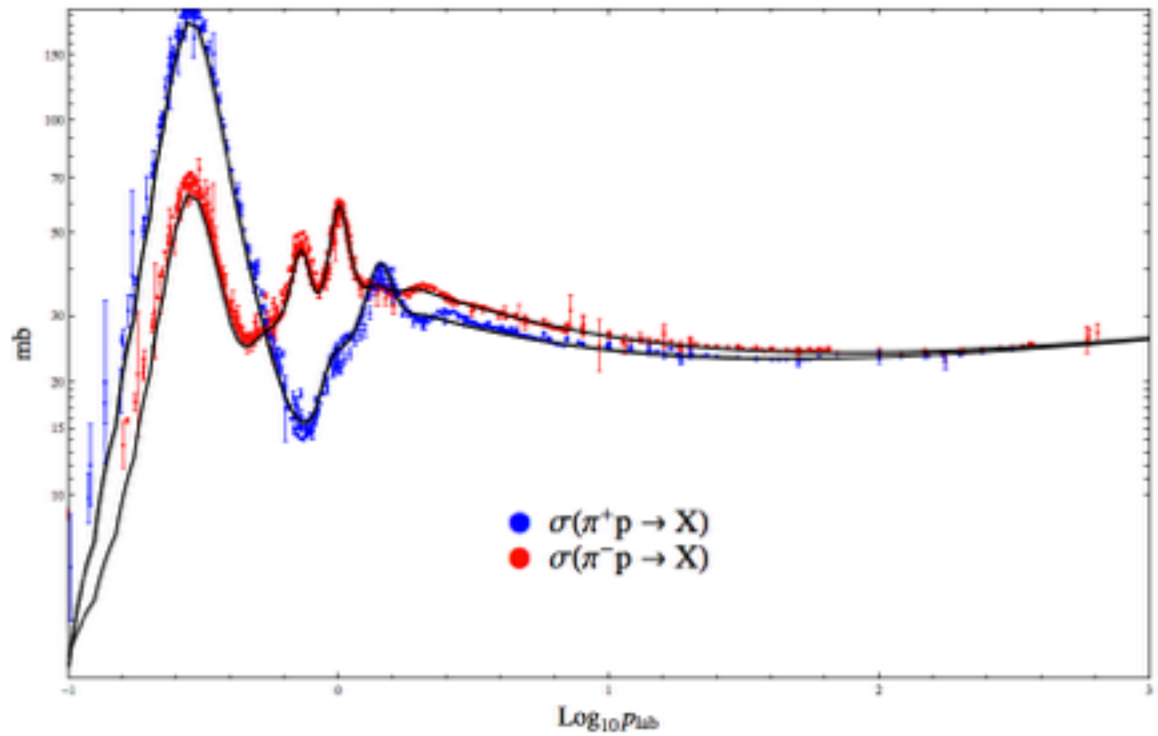
Jefferson Lab

Thomas Jefferson National Accelerator Facility

Outline and Motivations

- **about extracting resonance properties**

  $g(Q^2), M, \Gamma$ **from exp./lattice data**



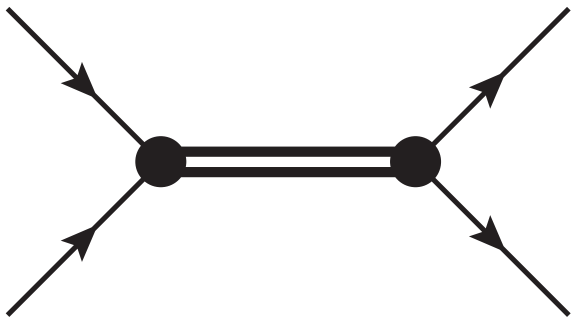
- **Derivation of Finite Energy Sum Rules (FESR)**
- **Illustrations of FESR with**

  $\pi p \to \pi p$ $\gamma p \to \pi p$ $\gamma p \to \eta p$

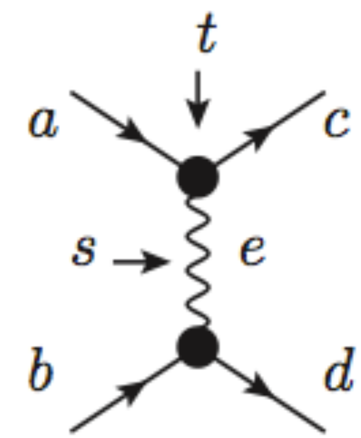
- **Methodology**
- **Conclusion**

$\pi^- p \to \pi^0 n$

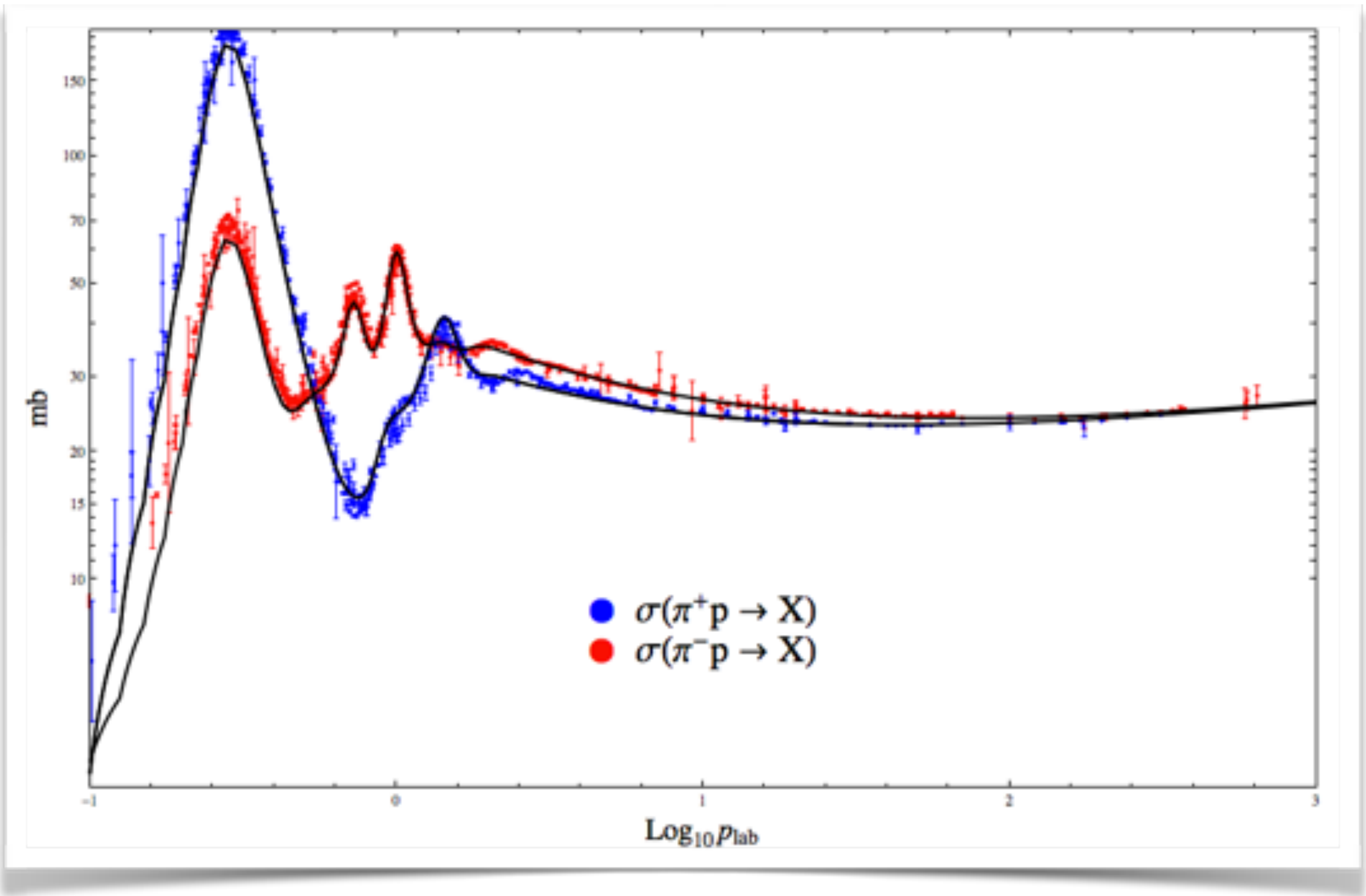
Low energy: baryon resonances



High energy: Regge exchange

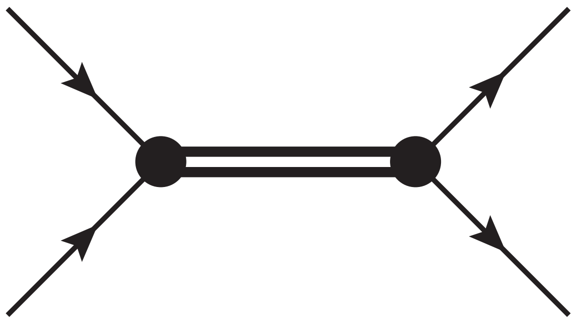


Total cross section

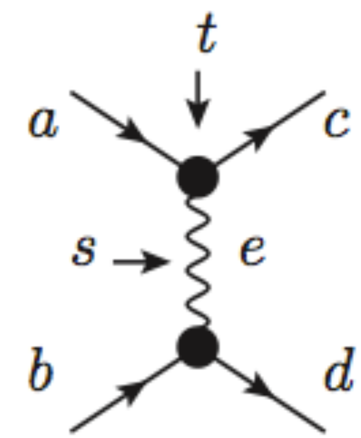


$\pi^- p \to \pi^0 n$

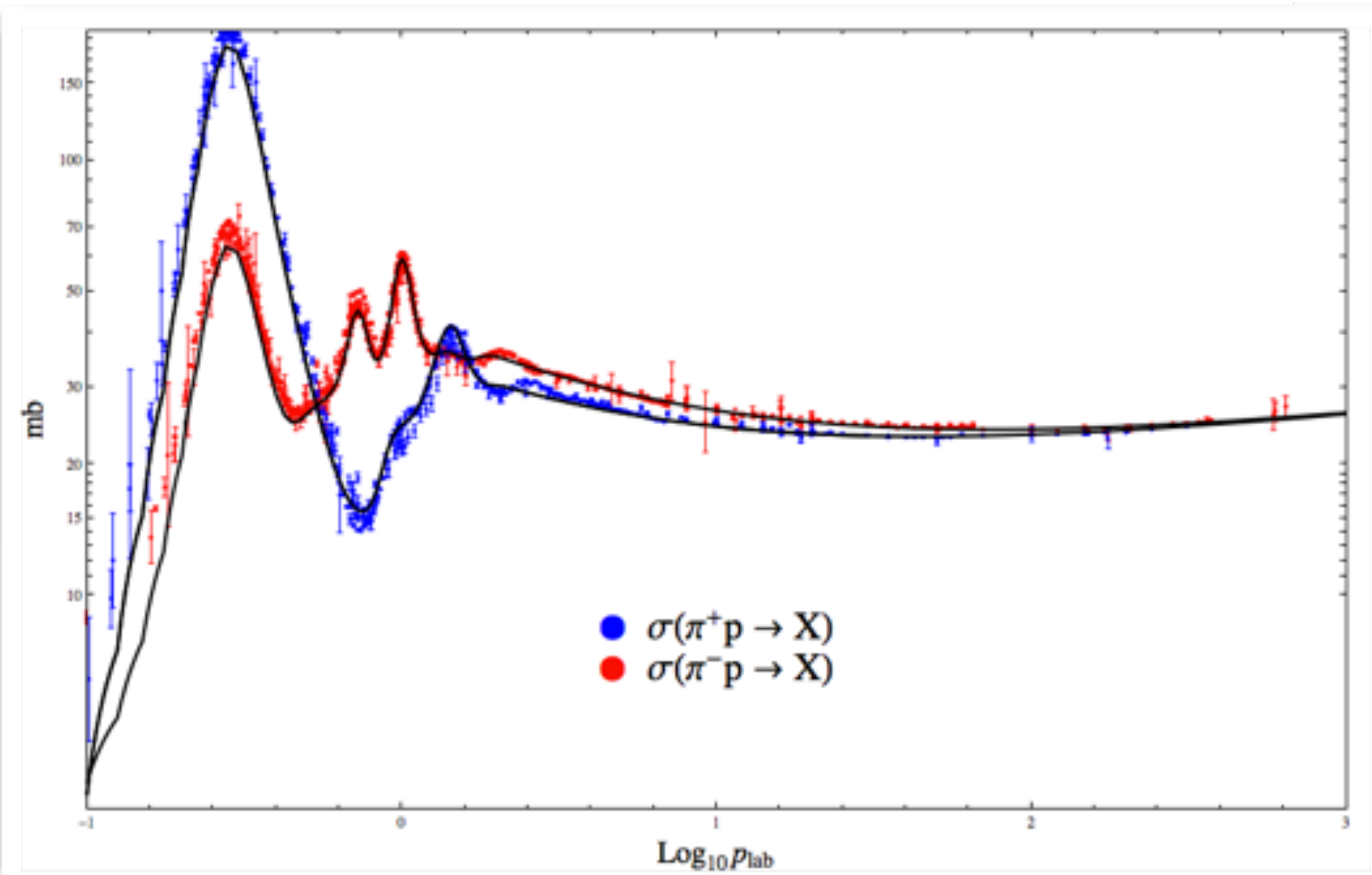
Low energy: baryon resonances



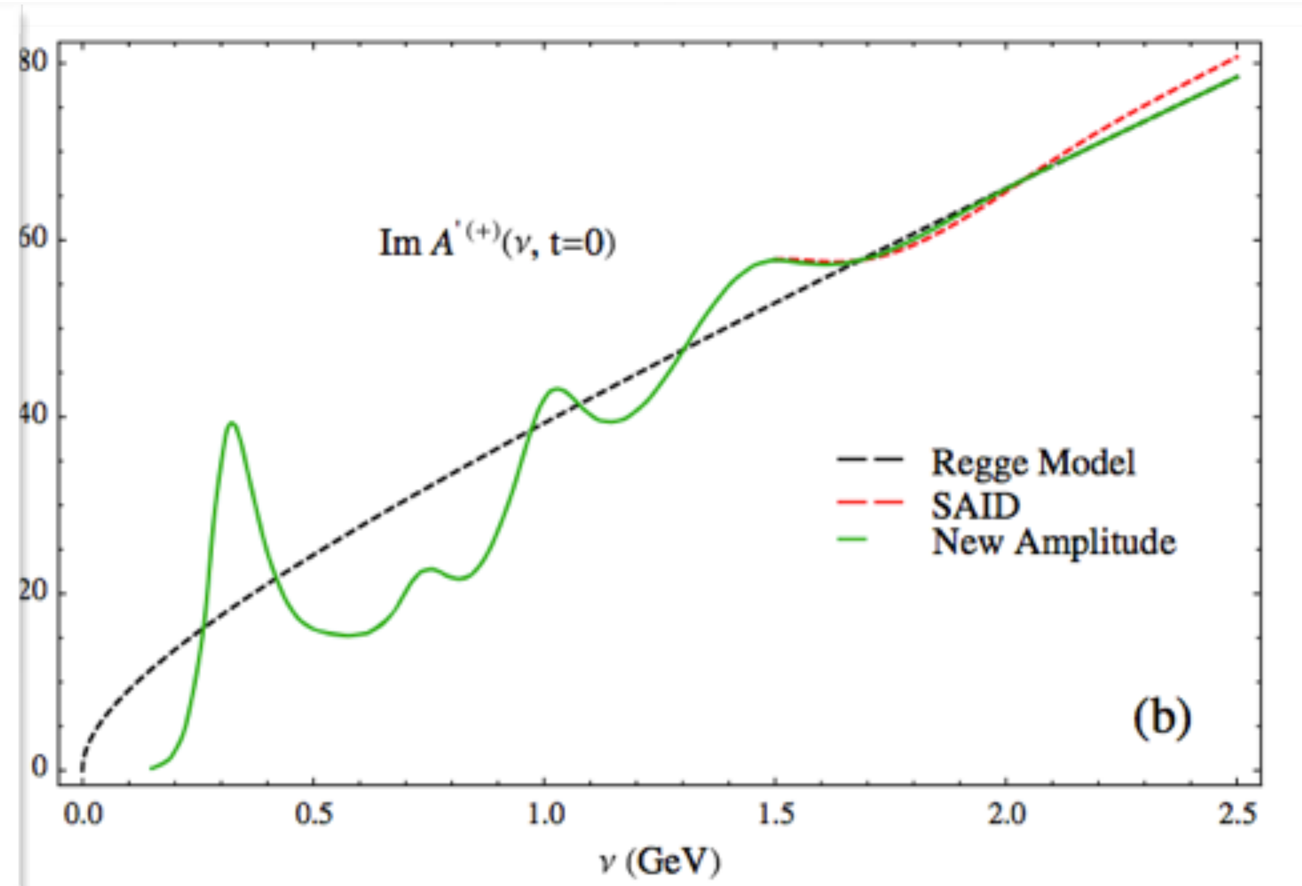
High energy: Regge exchange



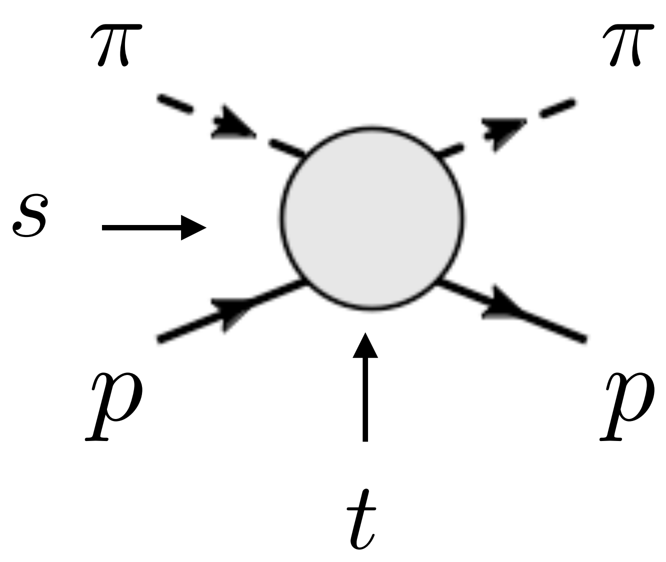
Total cross section



Amplitude



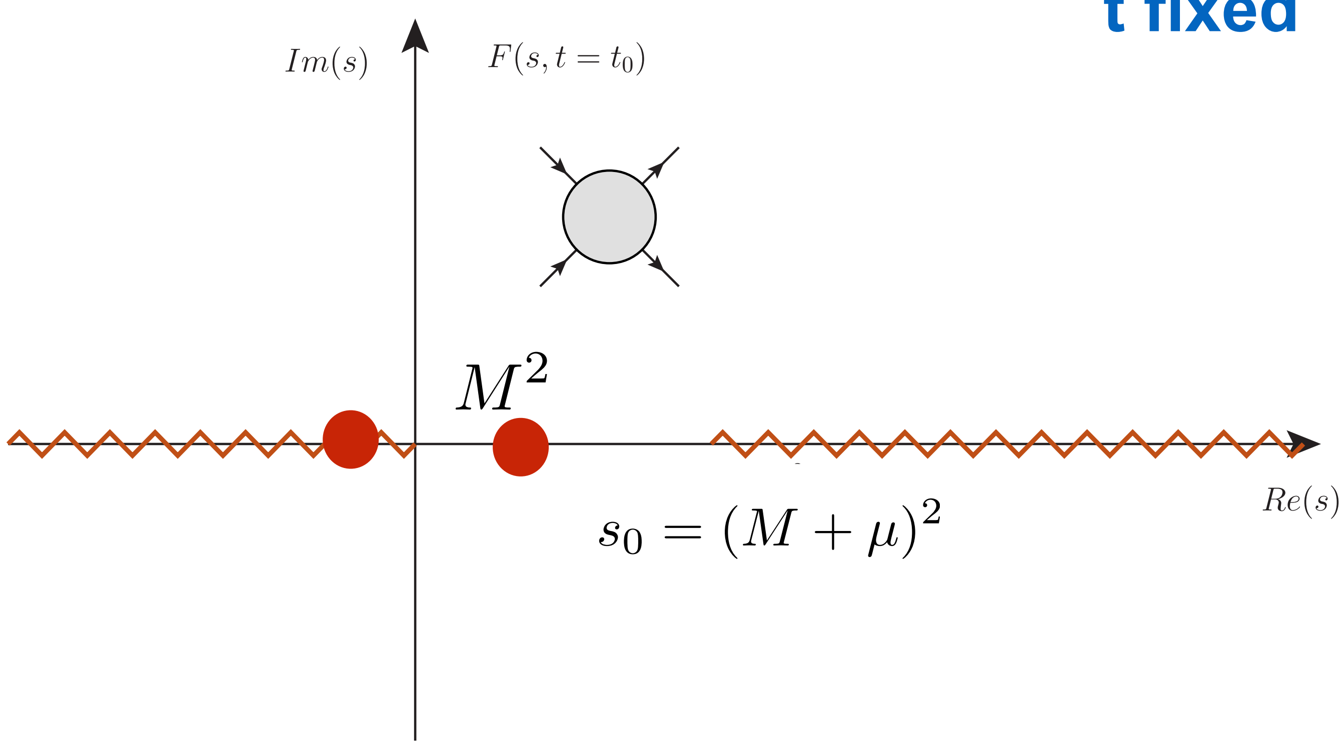
Dispersion Relation



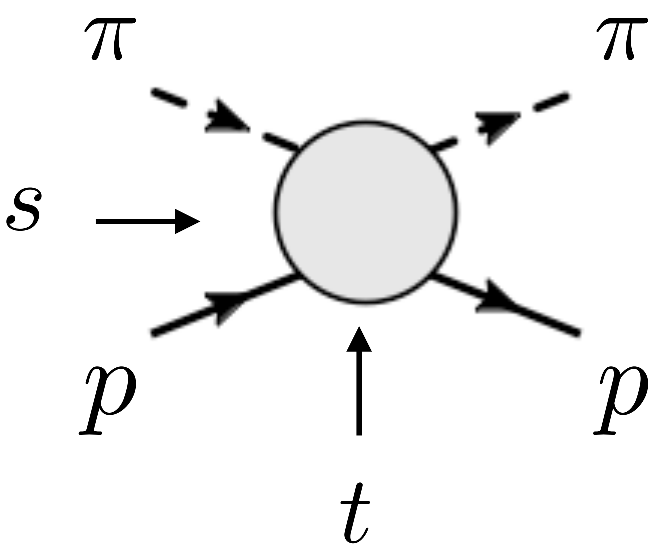
Decompose into scalar function

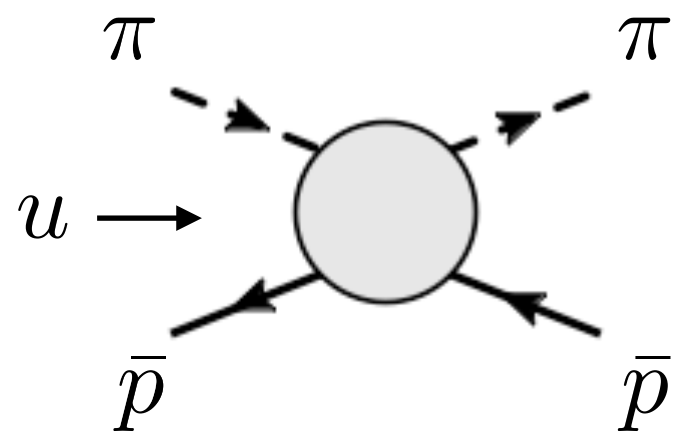
t fixed

$$T = \bar{u}(p_4, \lambda_4) \left(A + \frac{1}{2} (\not{p}_1 + \not{p}_3) B \right) u(p_2, \lambda_2)$$



Dispersion Relation





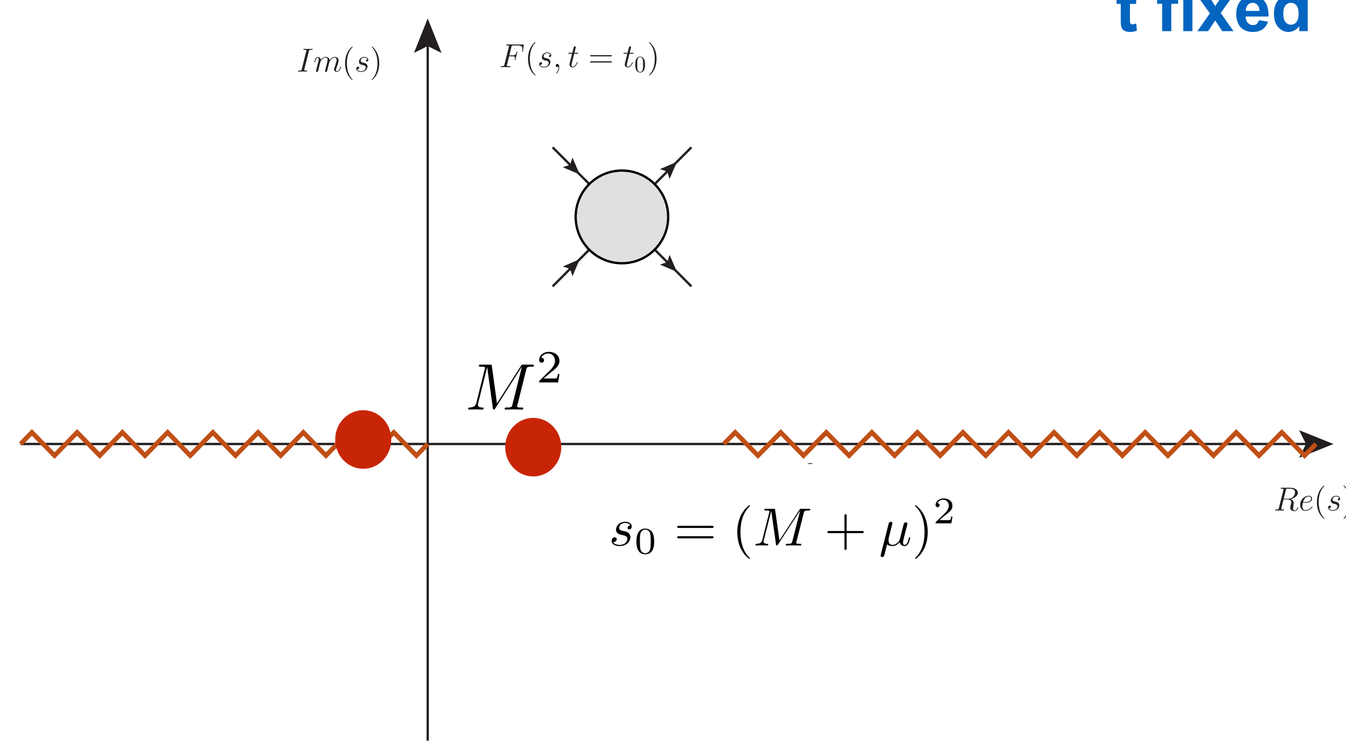
Conjugation charge relate

πp **and** $\pi \bar{p}$ **scatterings**

Decompose into scalar function

$$T = \bar{u}(p_4, \lambda_4) \left(A + \frac{1}{2} (\not{p}_1 + \not{p}_3) \, B \right) u(p_2, \lambda_2)$$

t fixed



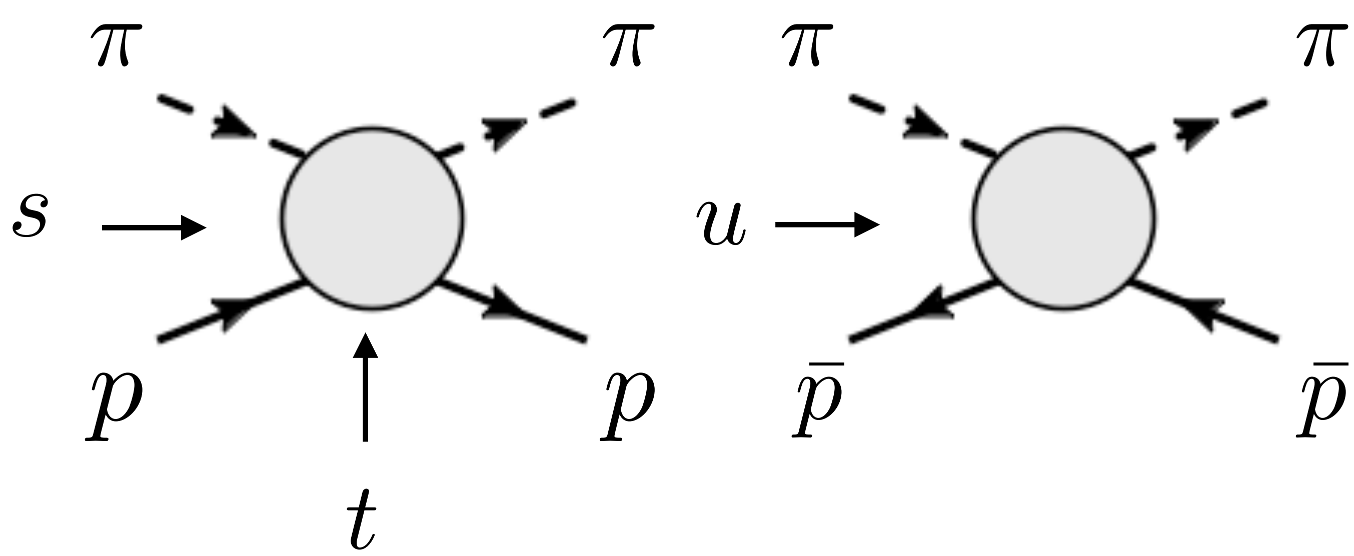
Introduce the crossing variable

$$\nu = \frac{s - u}{2}$$

$$u(s,t) = -s - t + 2M^2 + 2\mu^2$$

and symmetrize the two cuts

Dispersion Relation



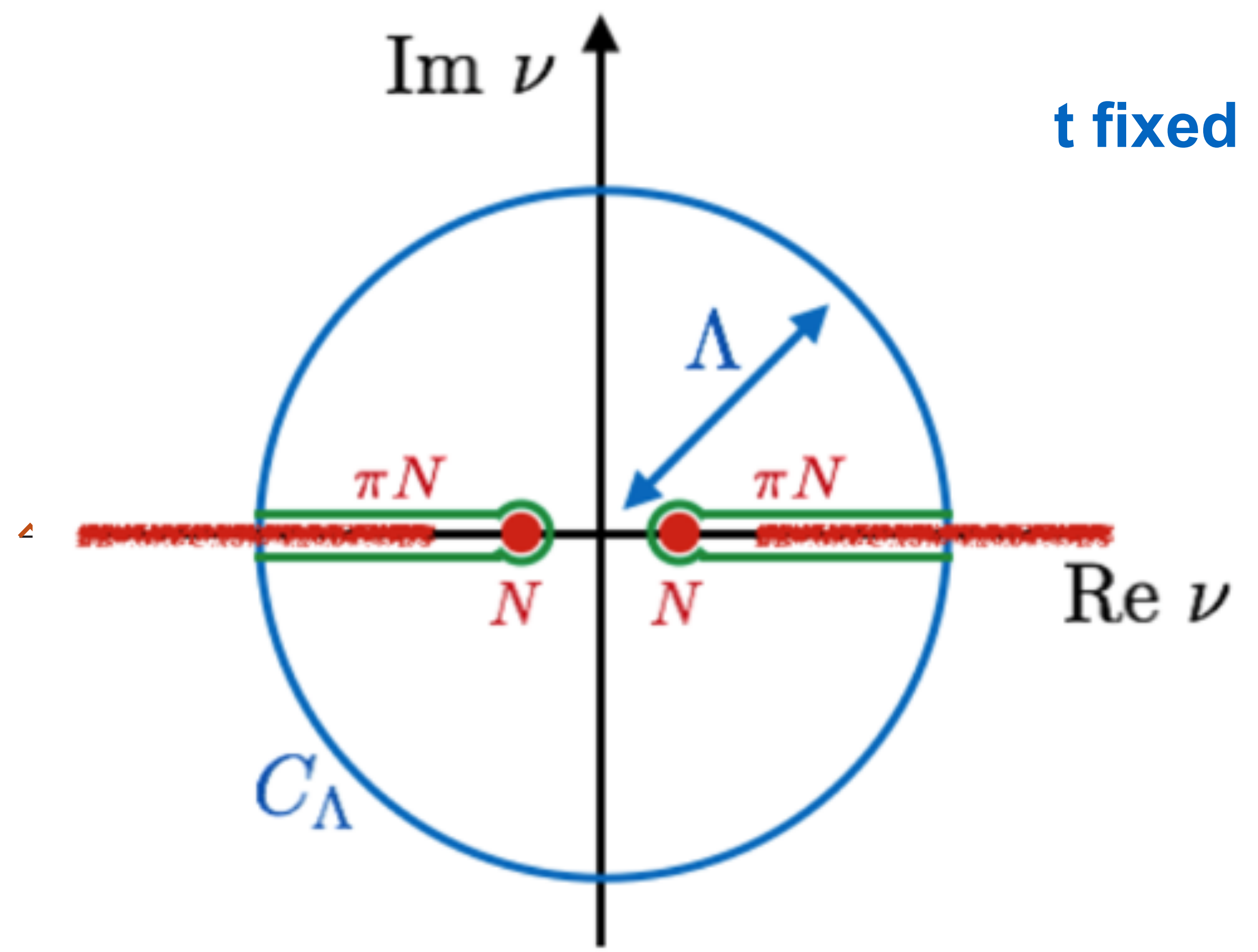
Conjugation charge relate

πp **and** $\pi \bar{p}$ **scatterings**

Decompose into scalar function

$$T = \bar{u}(p_4, \lambda_4)\left(A + \frac{1}{2}\,(\not{p}_1 + \not{p}_3)\,B\right)u(p_2, \lambda_2)$$

t fixed



Introduce the crossing variable

$$\nu = \frac{s-u}{2}$$

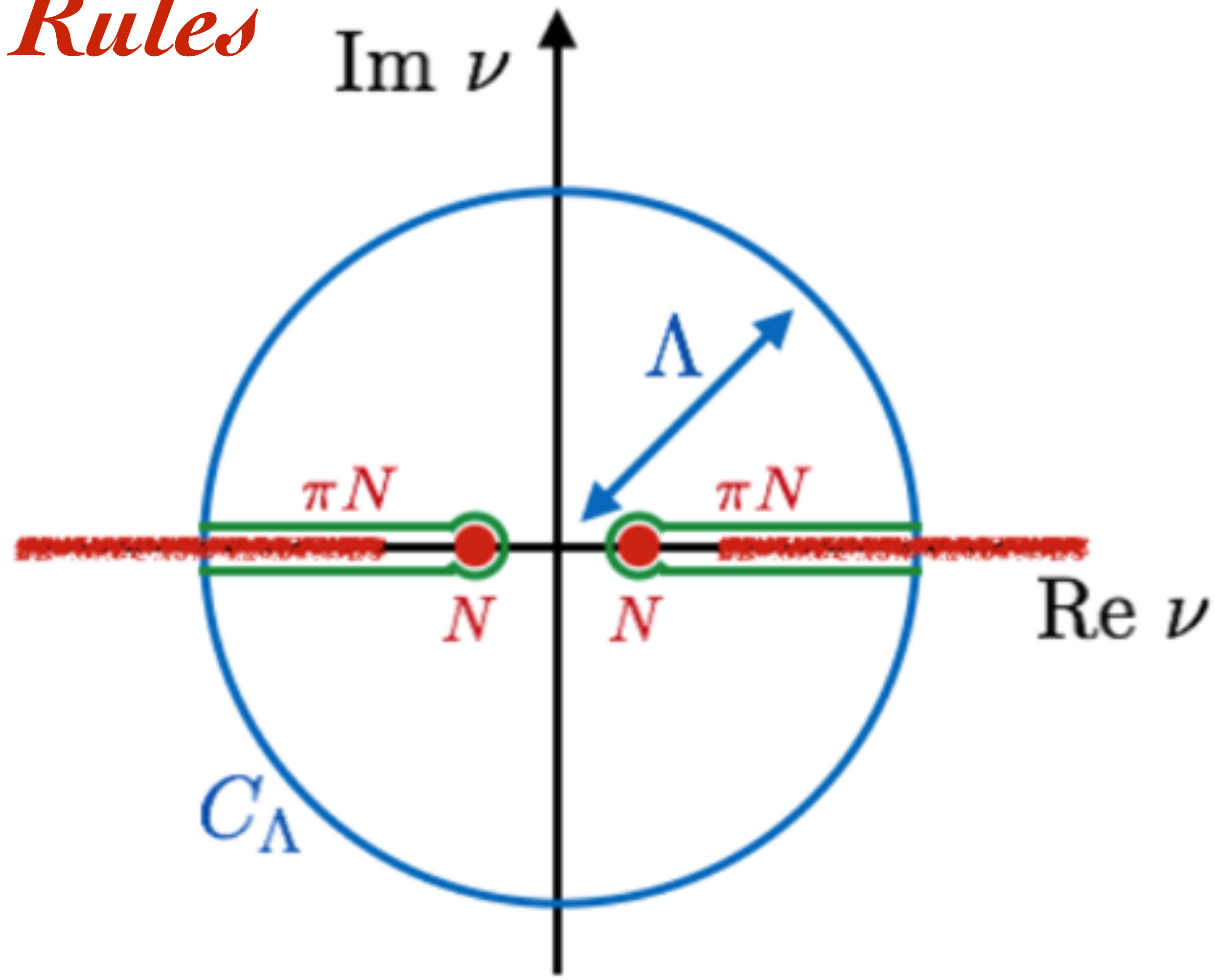
$$u(s,t) = -s - t + 2M^2 + 2\mu^2$$

and symmetrize the two cuts

Finite Energy Sum Rules

Cauchy contour

$$\oint_C A(\nu, t) d\nu = 0$$

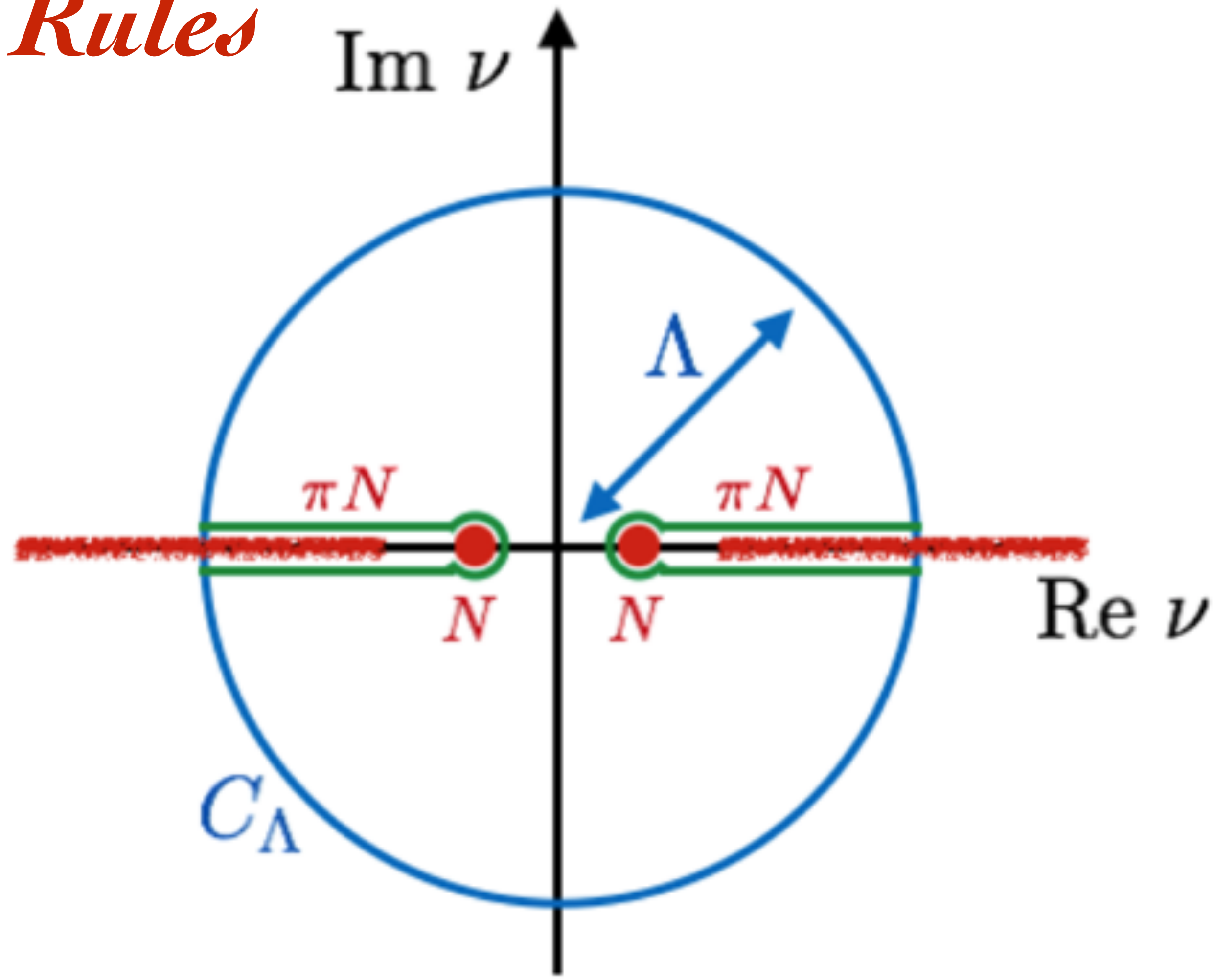


Finite Energy Sum Rules

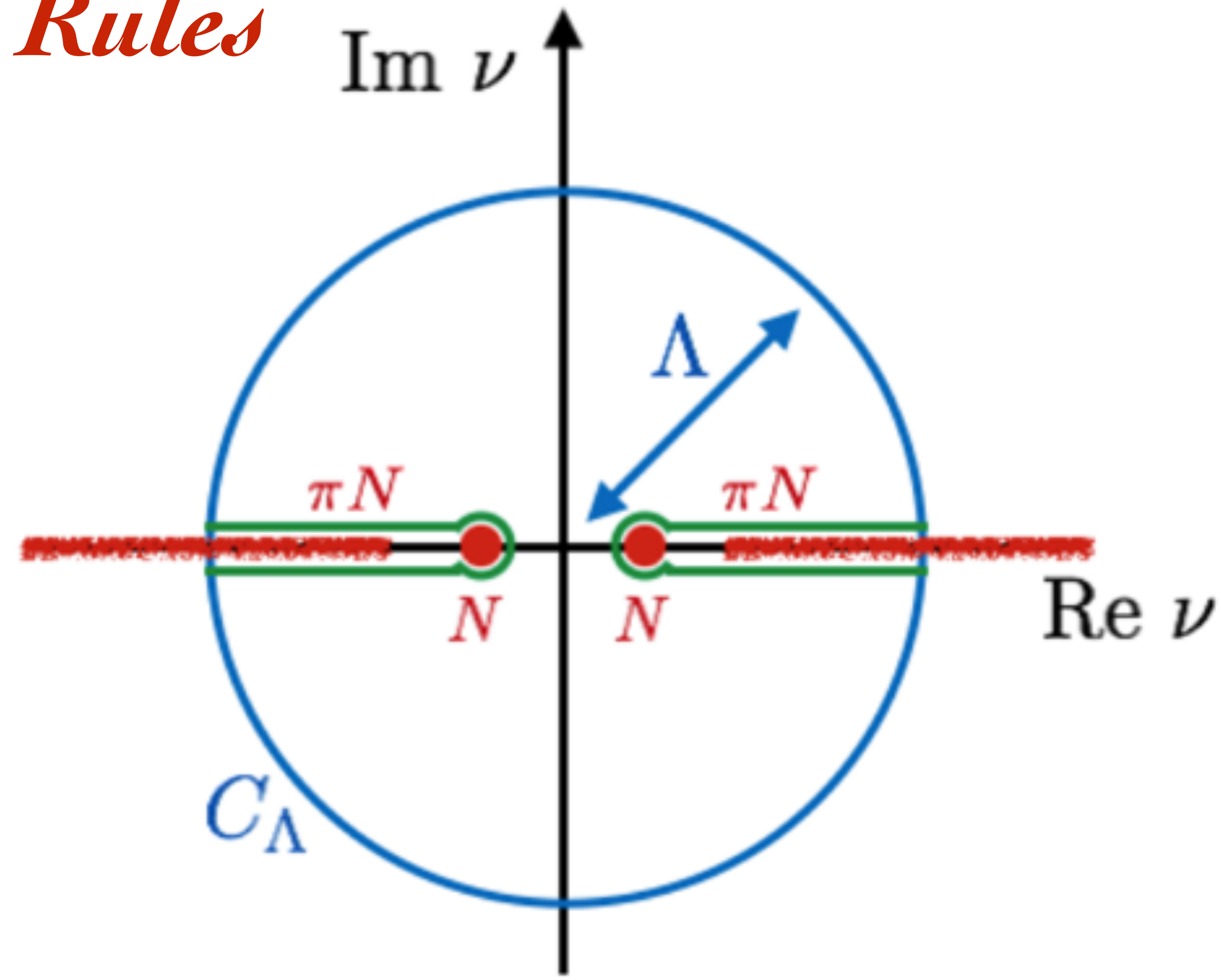
Cauchy contour

$$\oint_C A(\nu,t)d\nu = 0$$

$$2i\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)d\nu = -\oint_{C_\Lambda} A(\nu,t)d\nu$$



Finite Energy Sum Rules



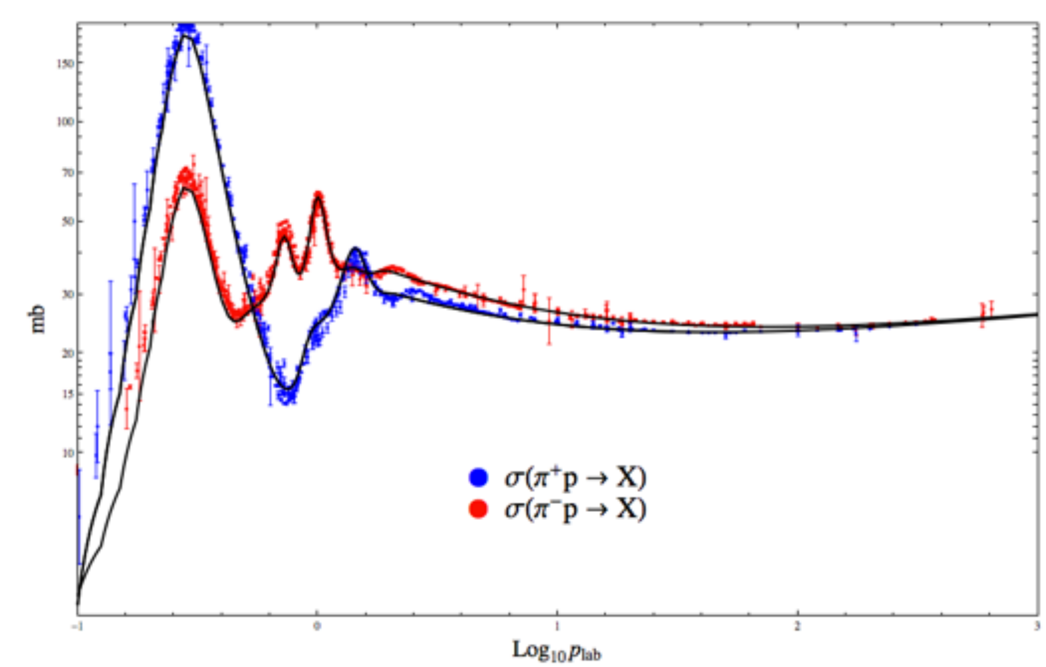
Cauchy contour

$$\oint_C A(\nu,t)d\nu = 0$$

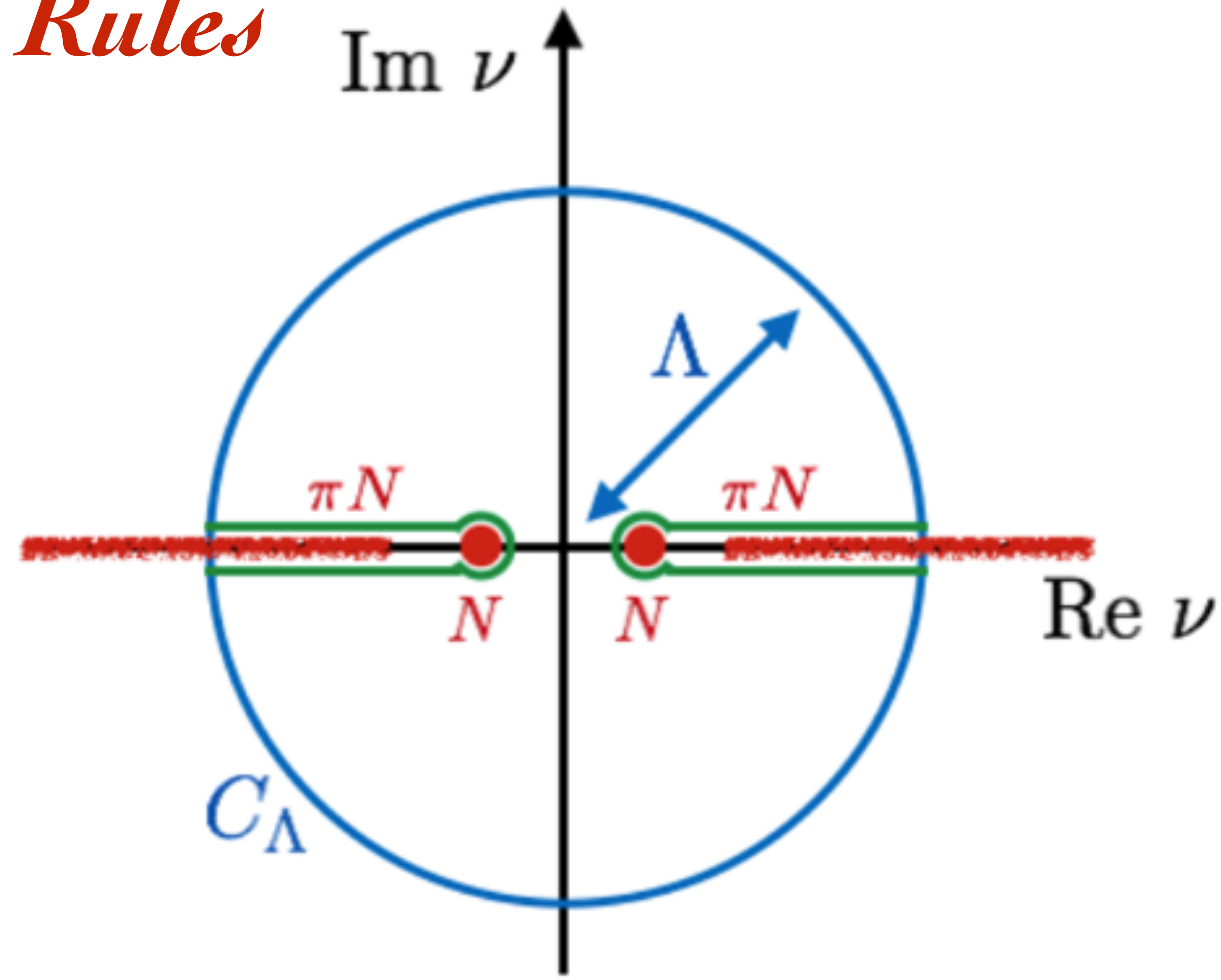
$$2i\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)d\nu = -\oint_{C_\Lambda} A(\nu,t)d\nu$$

Assume Regge form at $\nu = \Lambda$ **:**

$$A(\nu,t) = \beta(t)\frac{\pm 1 - e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)}\nu^{\alpha(t)}$$



Finite Energy Sum Rules



Cauchy contour

$$\oint_C A(\nu,t)d\nu = 0$$

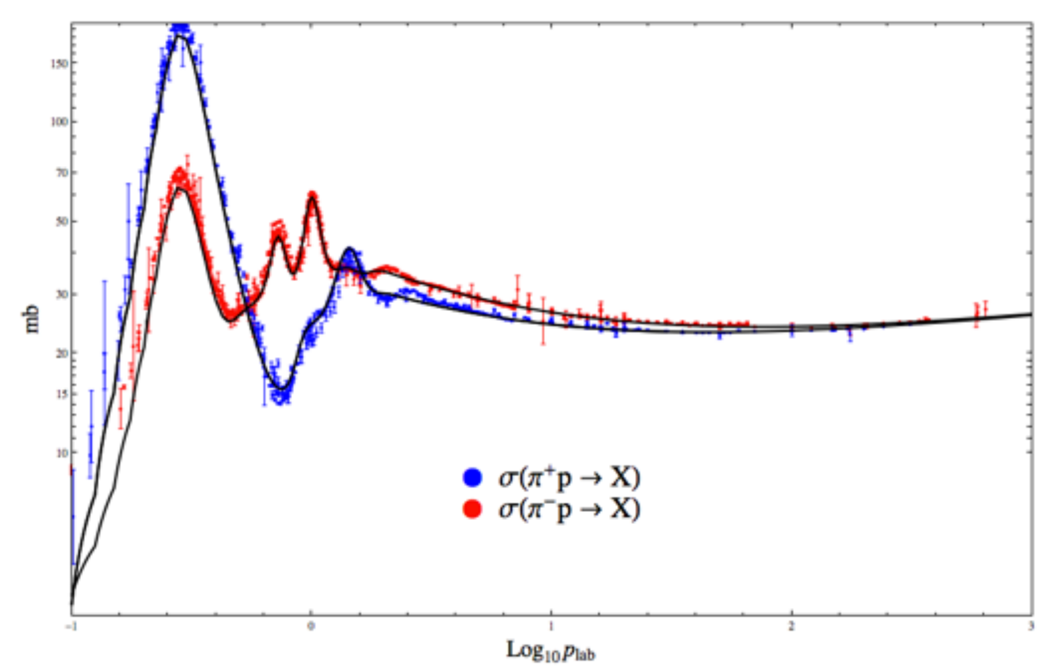
$$2i\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)d\nu = -\oint_{C_\Lambda} A(\nu,t)d\nu$$

Assume Regge form at $\nu = \Lambda$ **:**

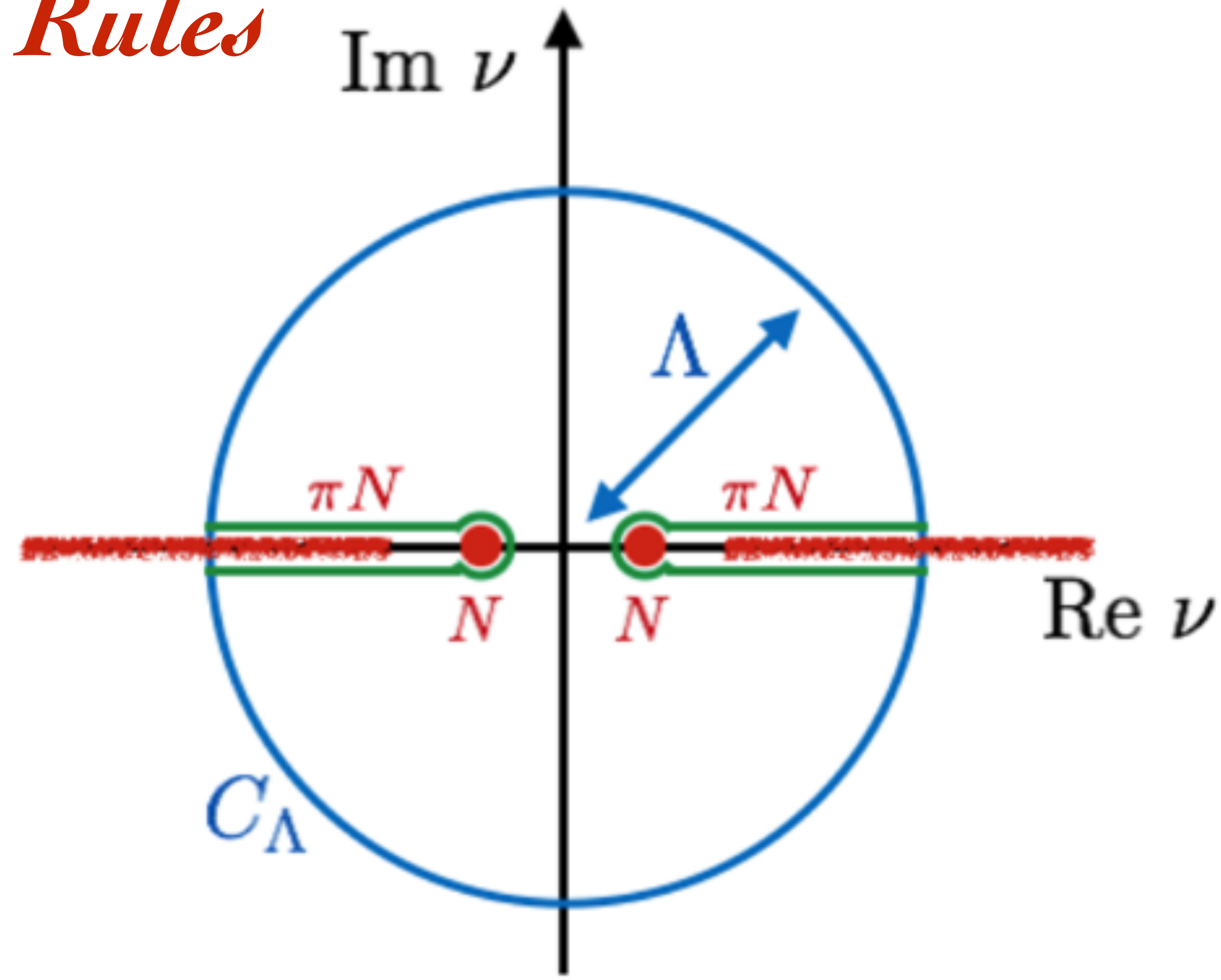
$$A(\nu,t) = \beta(t)\frac{\pm 1 - e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\nu^{\alpha(t)}$$

$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)d\nu = \beta(t)\frac{\Lambda^{\alpha(t)+1}}{\alpha(t)+1}$$

t fixed



Finite Energy Sum Rules



Cauchy contour

$$\oint_C A(\nu,t)\nu^k d\nu = 0$$

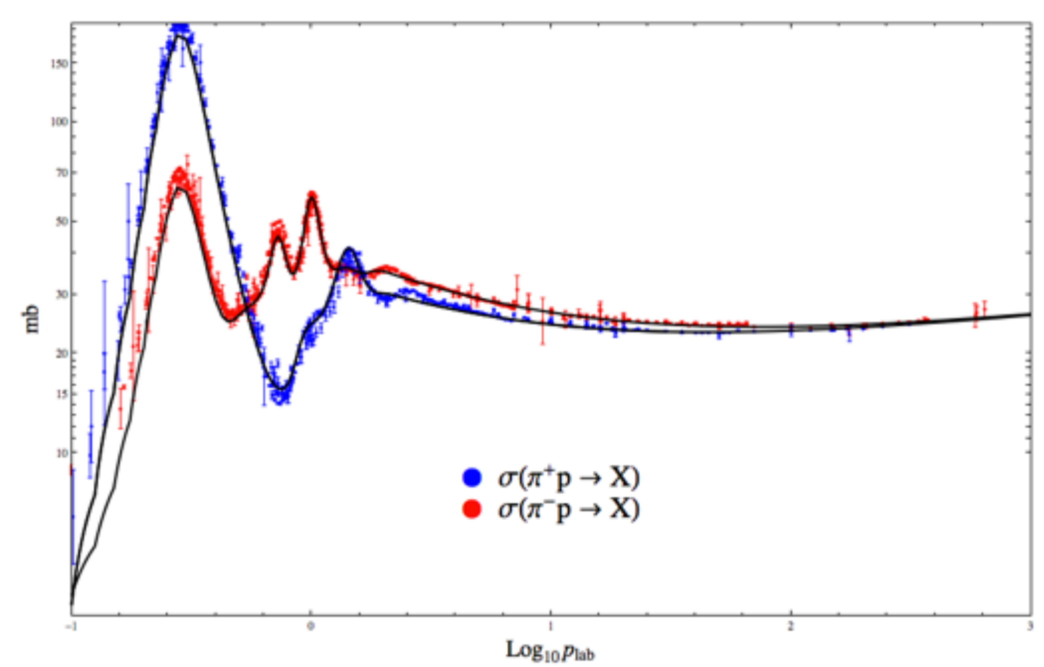
$$2i\int_{\nu_0}^{\Lambda} \text{Im } A(\nu,t)d\nu = -\oint_{C_\Lambda} A(\nu,t)d\nu$$

Assume Regge form at $\nu = \Lambda$ **:**

$$A(\nu,t) = \beta(t)\frac{\pm 1 - e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\nu^{\alpha(t)}$$

$$\int_{\nu_0}^{\Lambda} \text{Im } A(\nu,t)d\nu = \beta(t)\frac{\Lambda^{\alpha(t)+1}}{\alpha(t)+1}$$

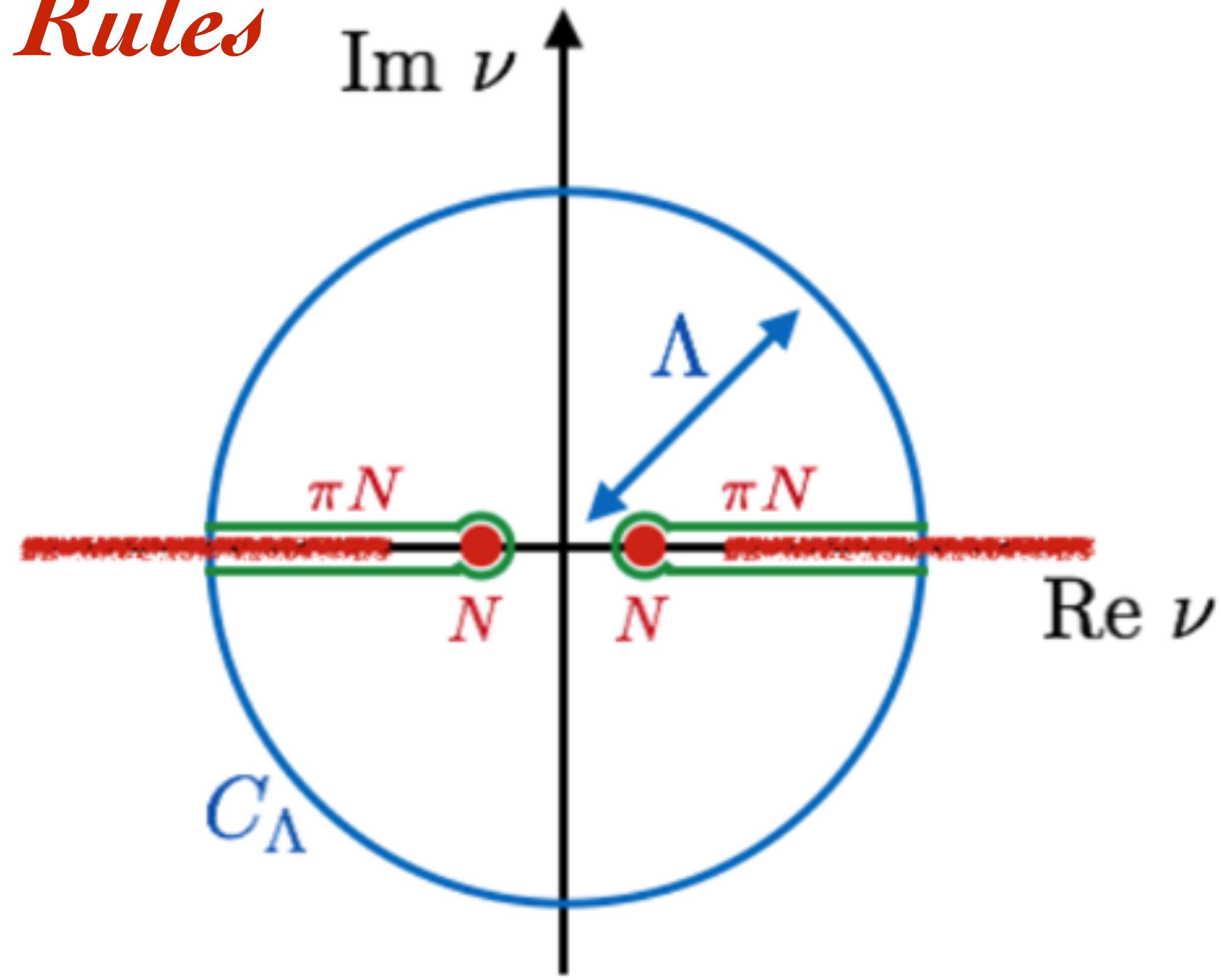
t fixed



Finite Energy Sum Rules

Cauchy contour

$$\oint_C A(\nu,t)\nu^k d\nu = 0$$

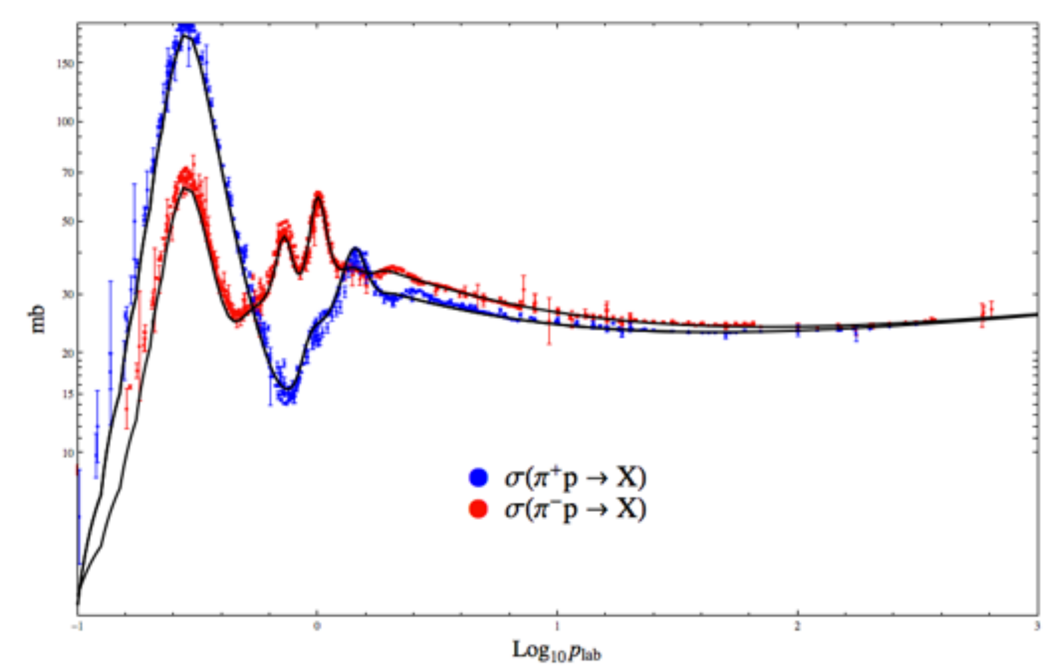


$$2i\int_{\nu_0}^{\Lambda} \mathrm{Im}\; A(\nu,t)d\nu = -\oint_{C_\Lambda} A(\nu,t)d\nu$$

Assume Regge form at $\nu = \Lambda$ **:**

$$A(\nu,t) = \beta(t)\frac{\pm 1 - e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\nu^{\alpha(t)}$$

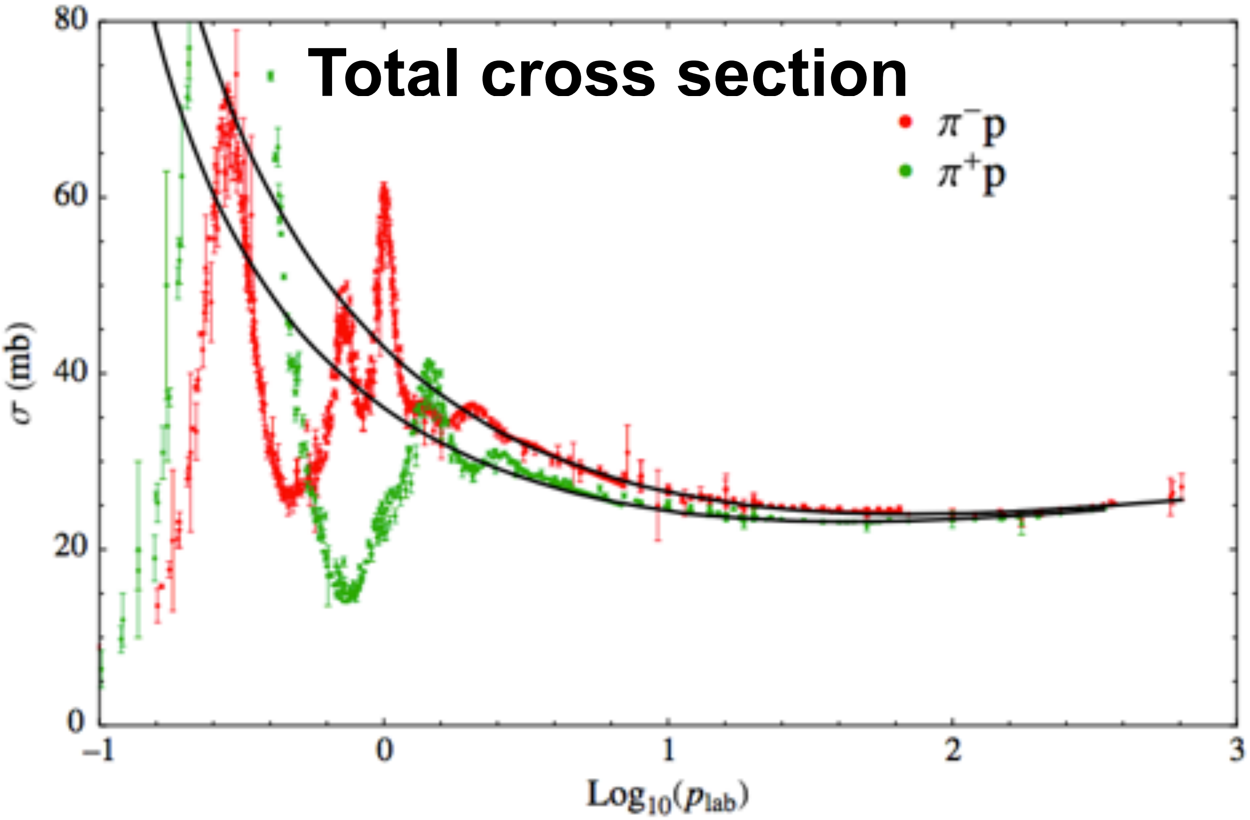
$$\boxed{\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\; A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}}$$



t fixed

Application to πN : High Energy Fit

VM et al (JPAC) PRD92
arXiv:1506.01764



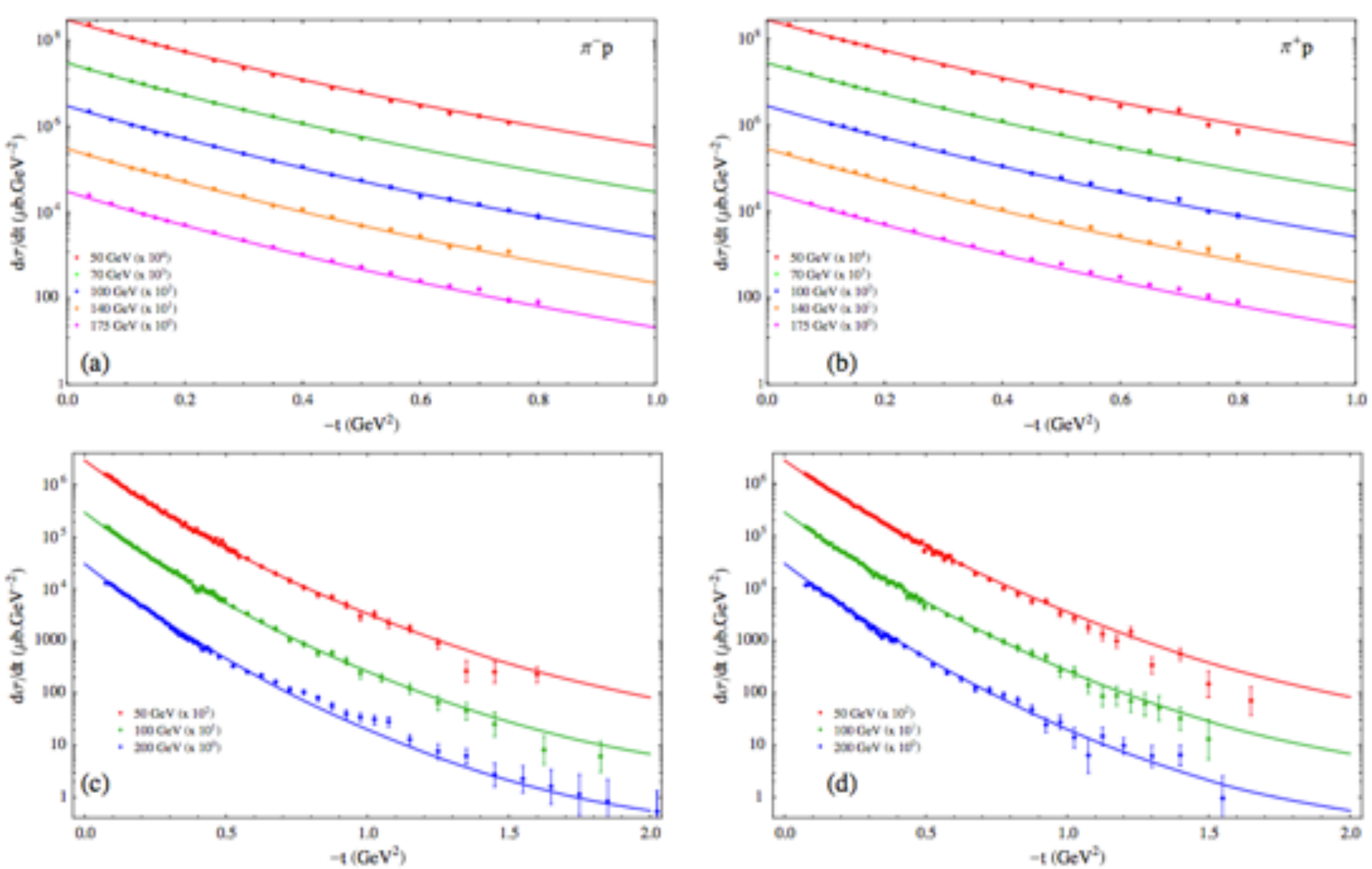
Fit to the world data on

$$\pi^{\pm} p \to \pi^{\pm} p$$

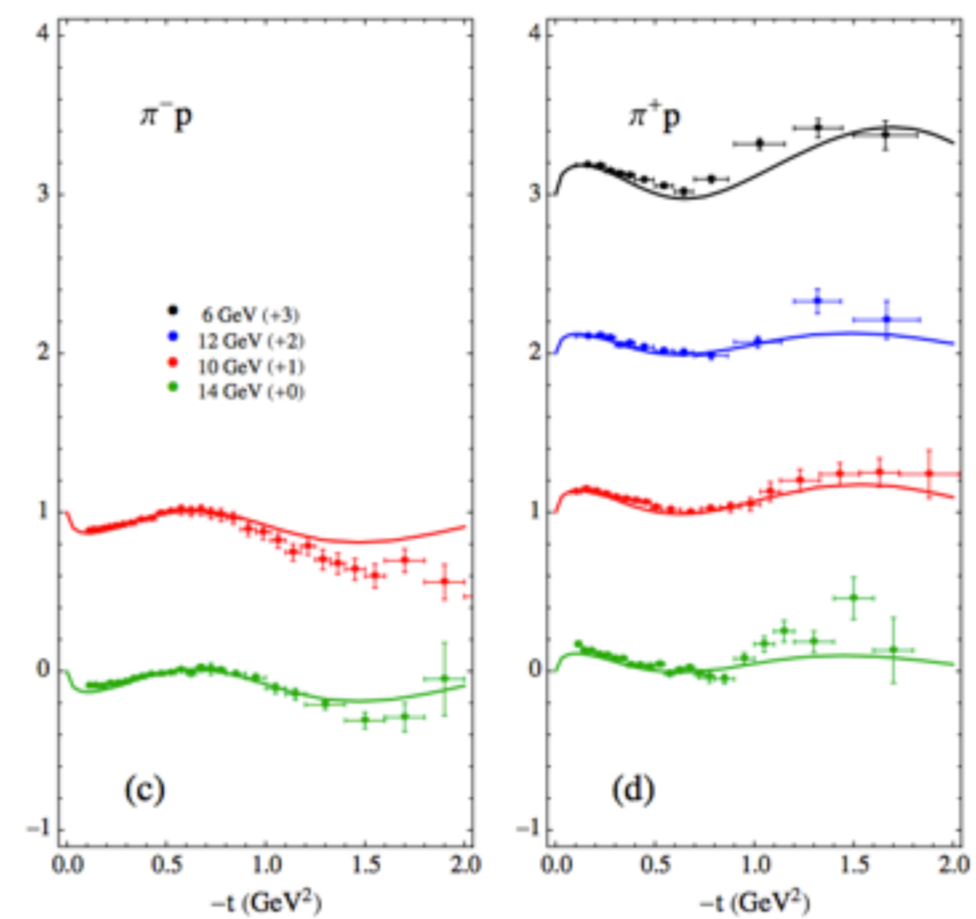
$$\pi^- p \to \pi^0 n$$

for beam energy > 2 GeV

Differential cross section



Polarization observable

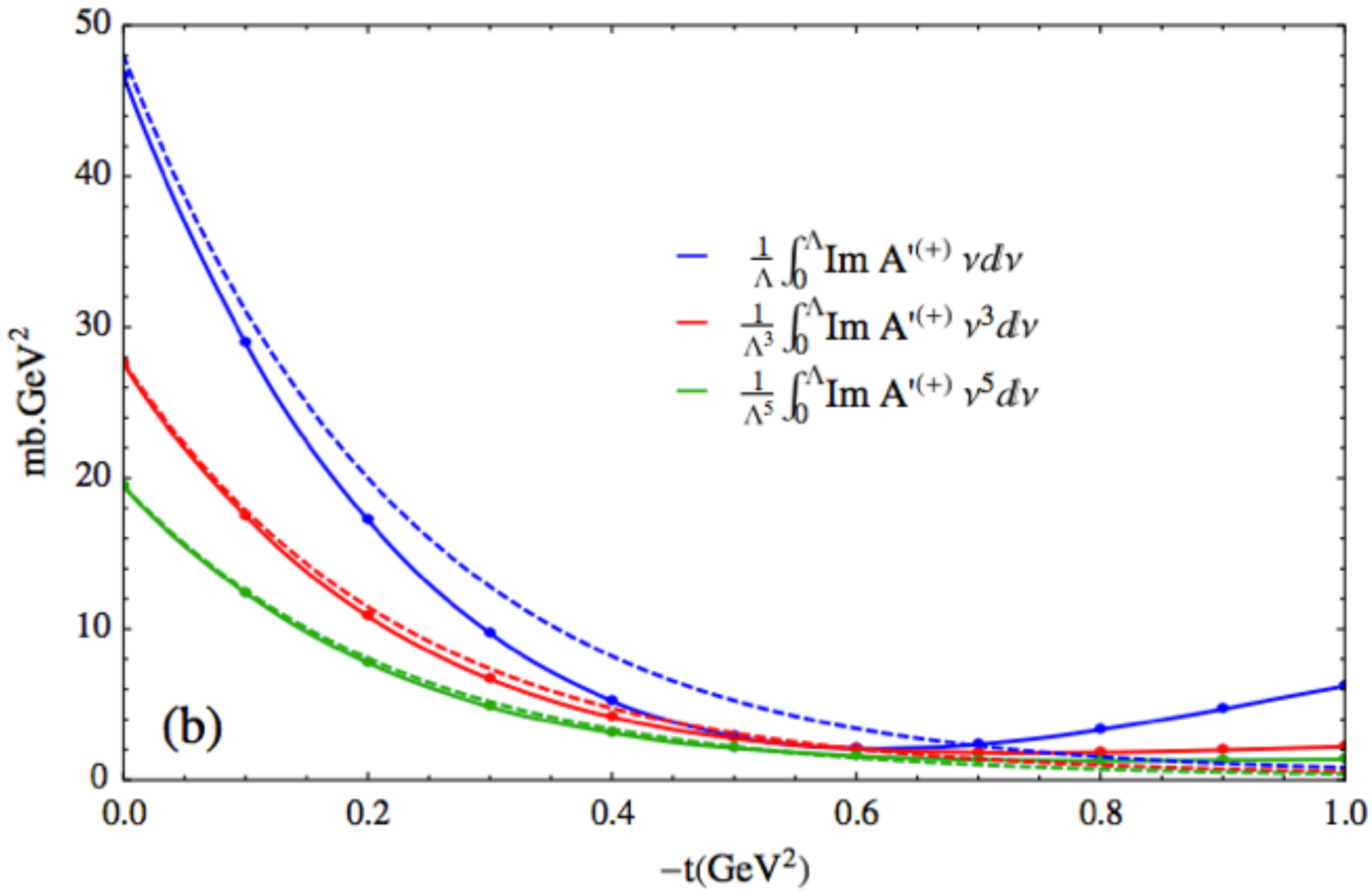


Let's compare both side of the sum rule

Solid line: SAID

Dashed line: Regge

excellent match !

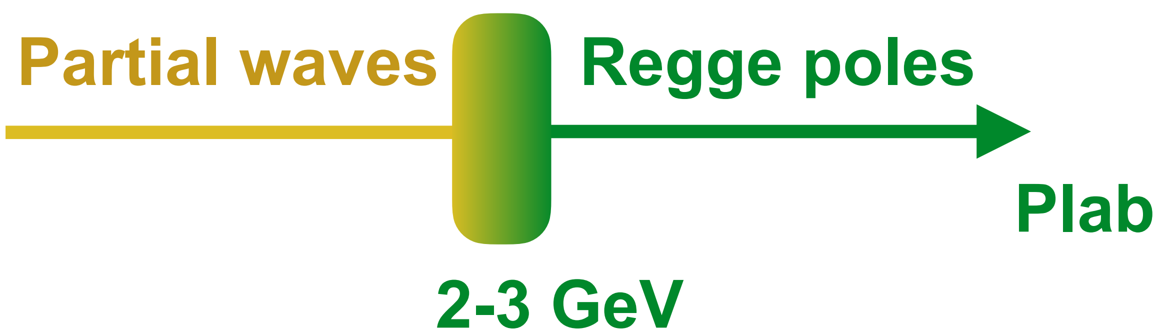


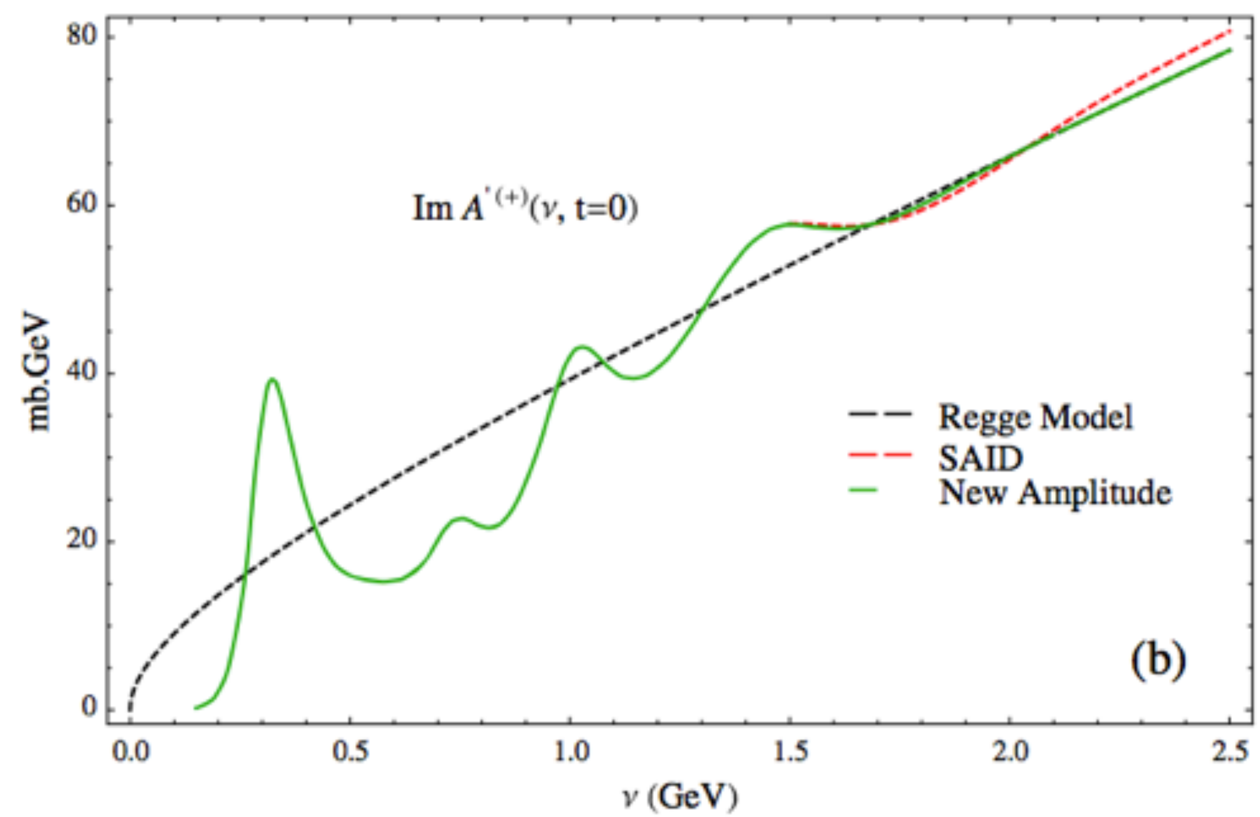
$$\boxed{\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}}$$

VM et al (JPAC) PRD92
arXiv:1506.01764

Checking Analyticity

Match low energy (PW)
and high energy (Regge)
imaginary parts





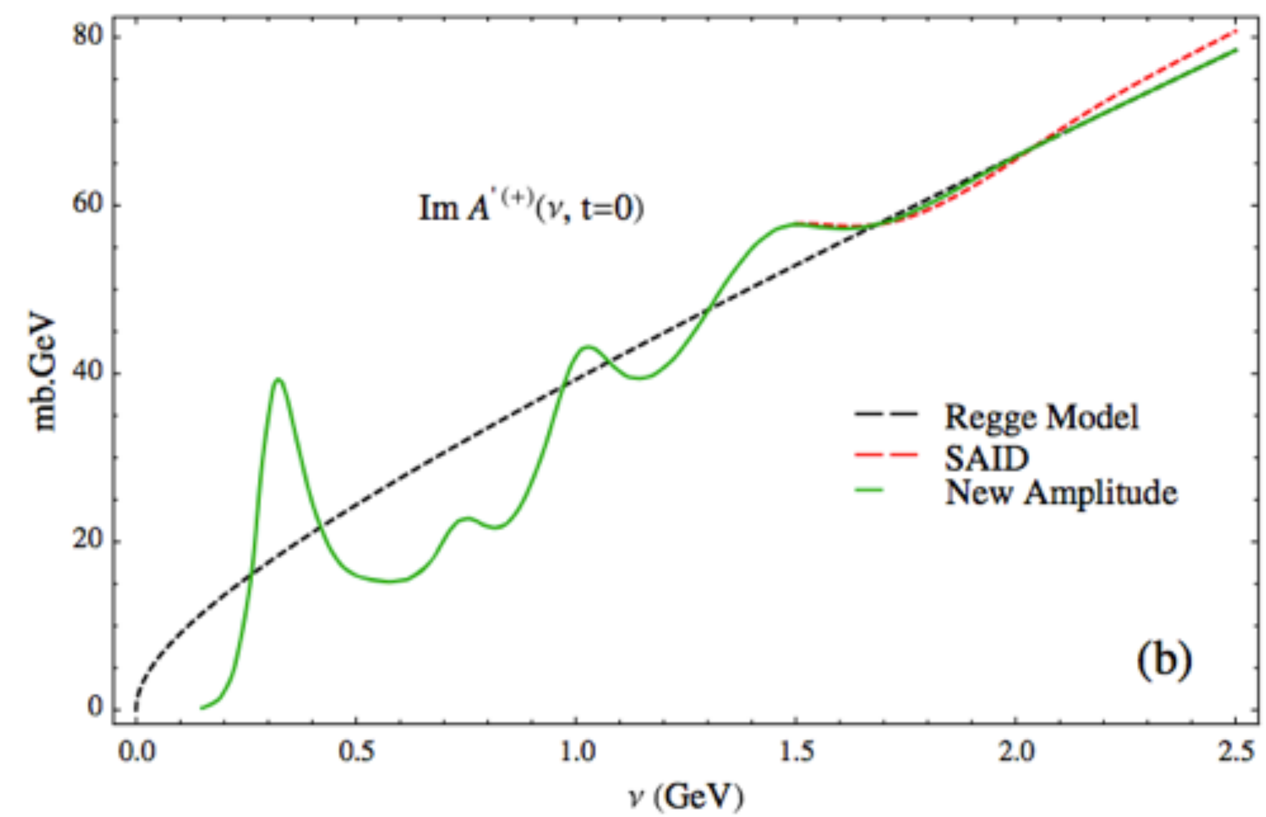
Reconstruct the real part
from the dispersion relation

$$A(\nu, t) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \frac{\mathrm{Im}\, A(\nu', t)}{\nu'^2 - \nu^2} \nu' d\nu'$$

VM et al (JPAC) PRD92 arXiv:1506.01764

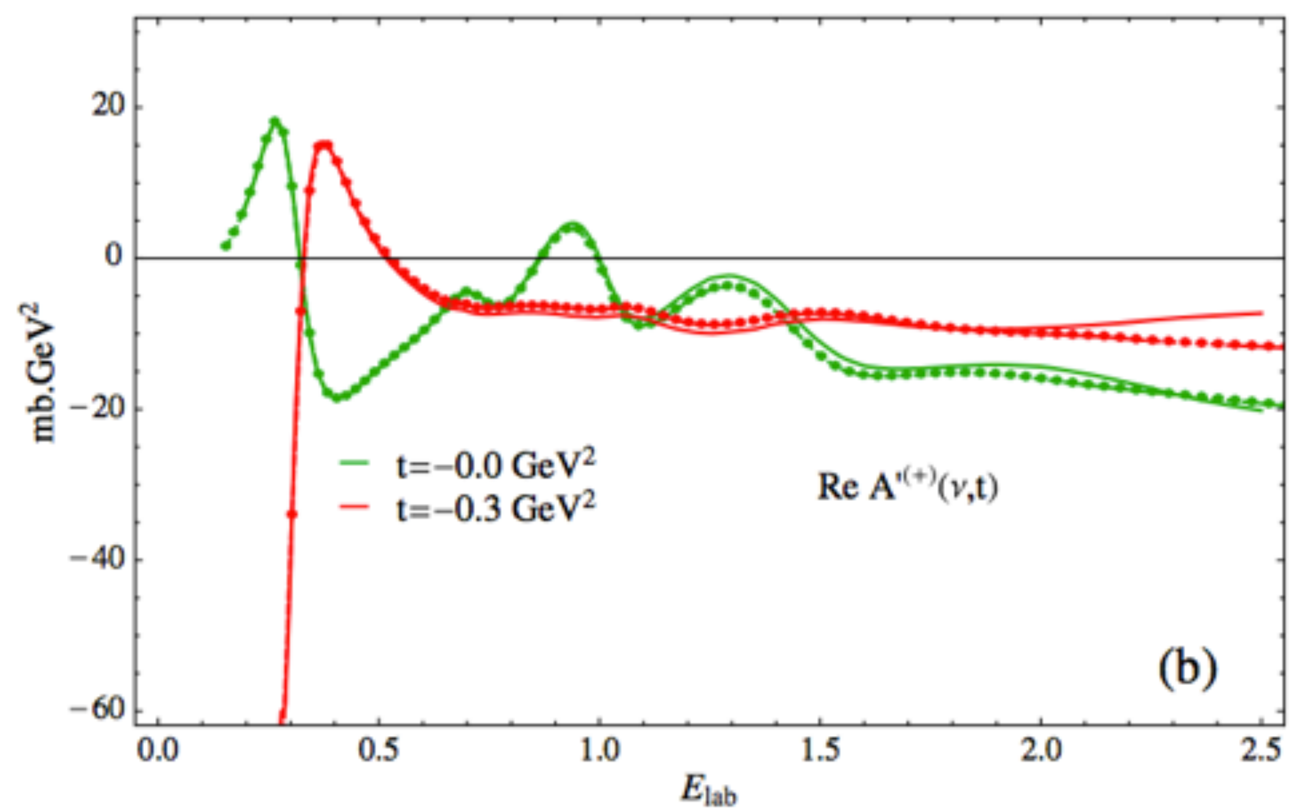
Checking Analyticity

Match low energy (PW) and high energy (Regge) imaginary parts



Reconstruct the real part from the dispersion relation

$$A(\nu,t) = \frac{2}{\pi}\int_{\nu_0}^{\infty} \frac{\operatorname{Im} A(\nu',t)}{\nu'^2-\nu^2}\nu' d\nu'$$

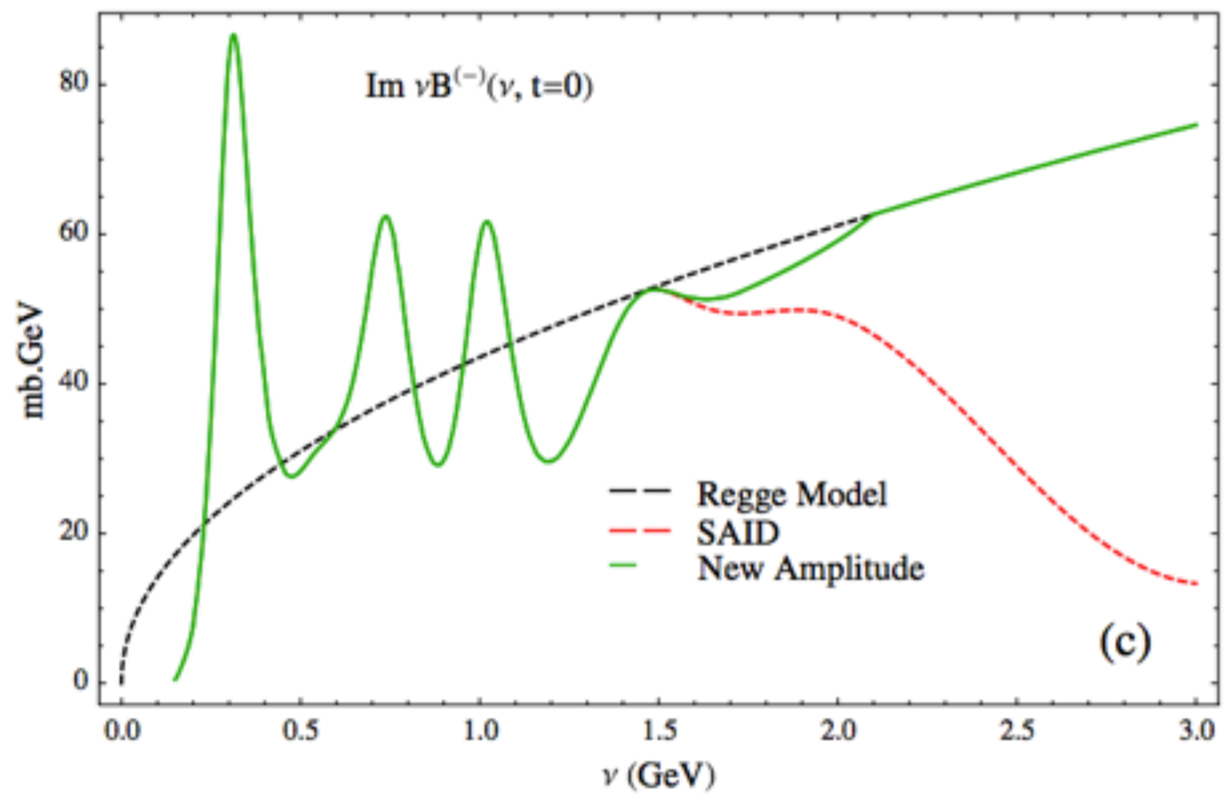


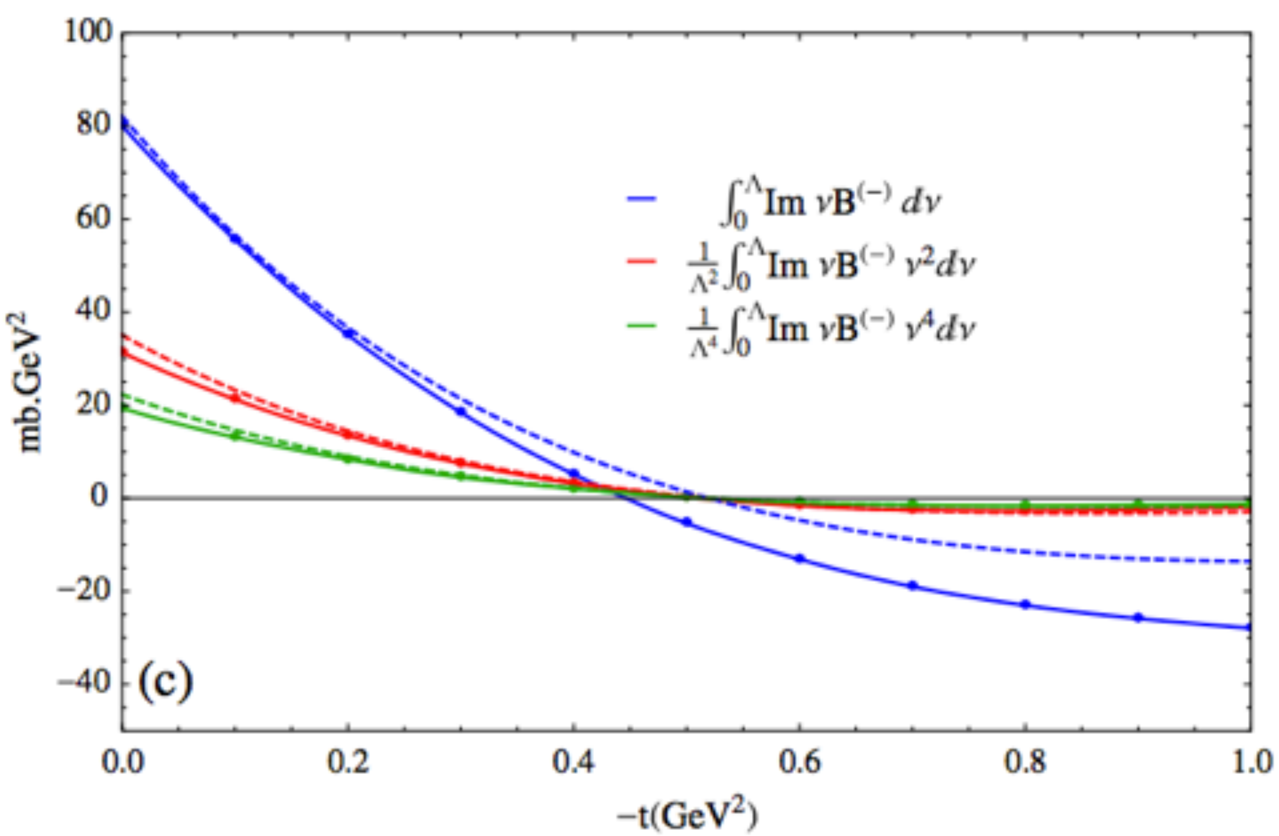
VM et al (JPAC) PRD92 arXiv:1506.01764

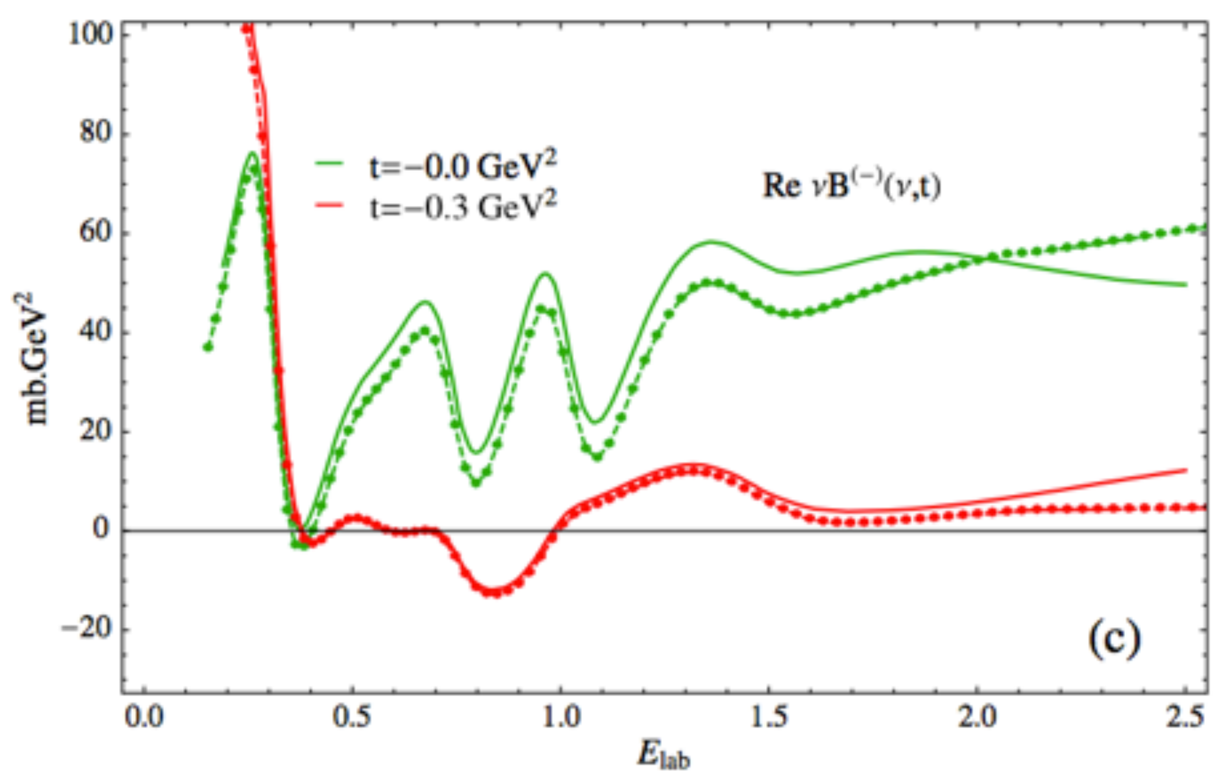
VM et al (JPAC) PRD92
arXiv:1506.01764

Similar results for the other amplitude

$$T = \bar{u}(p_4, \lambda_4)\left(A + \frac{1}{2}(\not{p}_1 + \not{p}_3)\, B\right) u(p_2, \lambda_2)$$







Application to Pseudoscalar Photoproduction

$$\gamma N \to \pi N$$

$$(\pm 1)\left(\pm\frac{1}{2}\right) \longrightarrow 0\left(\pm\frac{1}{2}\right)$$

8 helicity configurations related by pair via parity

4 indep. helicity configurations

use CGLN basis $A_1, \ldots, A_4$

Isospin symmetry:
every amplitude has an isospin index (+,-,0)

$$A^a_{ji} = A^{(+)}\delta^{a3}\delta_{ji} + A^{(-)}\frac{1}{2}[\tau^a, \tau^3]_{ji} + A^{(0)}\tau^a_{ji}$$

12 indep. helicity/isospin configurations

$$\gamma p \to \pi^+ n : \sqrt{2}\left(A^{(0)} + A^{(-)}\right)$$

$$\gamma n \to \pi^- p : \sqrt{2}\left(A^{(0)} - A^{(-)}\right)$$

$$\gamma p \to \pi^0 p : \quad A^{(+)} + A^{(0)}$$

$$\gamma n \to \pi^0 n : \quad A^{(+)} - A^{(0)}$$

Application to Pseudoscalar Photoproduction

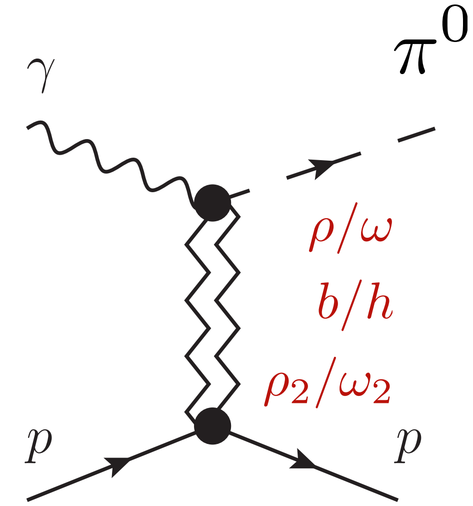
$$\gamma N \to \pi N$$

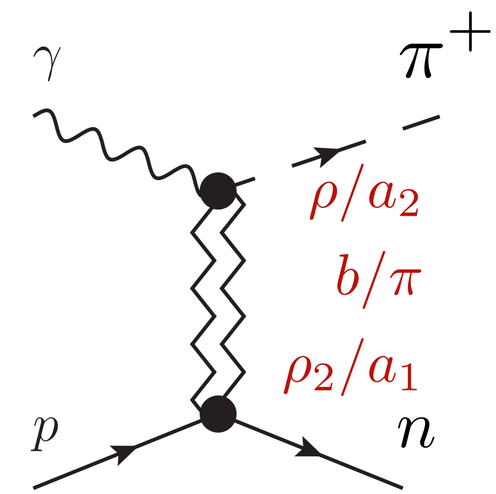
$$(\pm 1)\left(\pm\frac{1}{2}\right) \longrightarrow 0\left(\pm\frac{1}{2}\right)$$

8 helicity configurations related by pair via parity

4 indep. helicity configurations

use CGLN basis $A_1, \ldots, A_4$





Isospin symmetry:
every amplitude has an isospin index (+,-,0)

$$A^a_{ji} = A^{(+)}\delta^{a3}\delta_{ji} + A^{(-)}\frac{1}{2}[\tau^a, \tau^3]_{ji} + A^{(0)}\tau^a_{ji}$$

12 indep. helicity/isospin configurations

$$\gamma p \to \pi^+ n : \sqrt{2}\left(A^{(0)} + A^{(-)}\right)$$

$$\gamma n \to \pi^- p : \sqrt{2}\left(A^{(0)} - A^{(-)}\right)$$

$$\gamma p \to \pi^0 p : \quad A^{(+)} + A^{(0)}$$

$$\gamma n \to \pi^0 n : \quad A^{(+)} - A^{(0)}$$

A_i	(0)	$(+)$	$(-)$
A_1	ρ	ω	a_2
A_2	b	h	π
A_3	ρ_2	ω_2	a_1
A_4	ρ	ω	a_2

VM et al (JPAC), in preparation

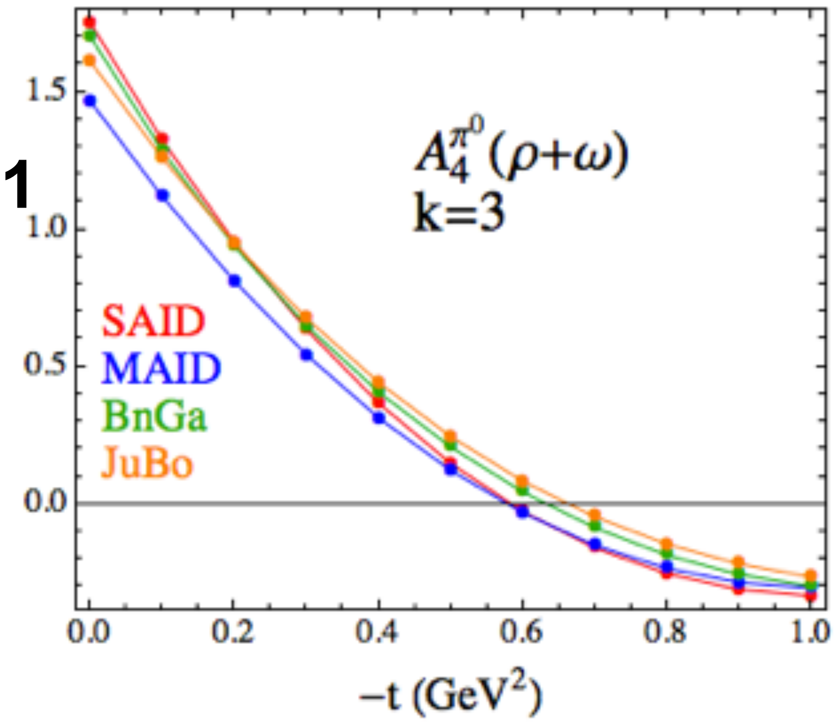
Use CGLN basis $A_{1,2,3,4}$

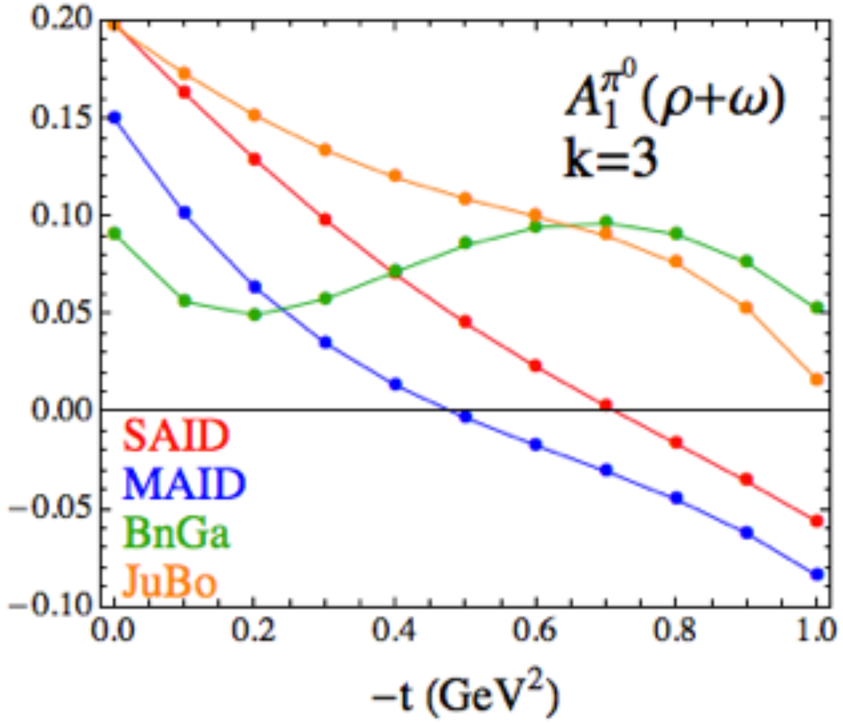
$\gamma p \to \pi^0 p$

Agreed on dominant A4

Variation on subdominant A1

$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$





VM et al (JPAC), in preparation

Use CGLN basis $A_{1,2,3,4}$

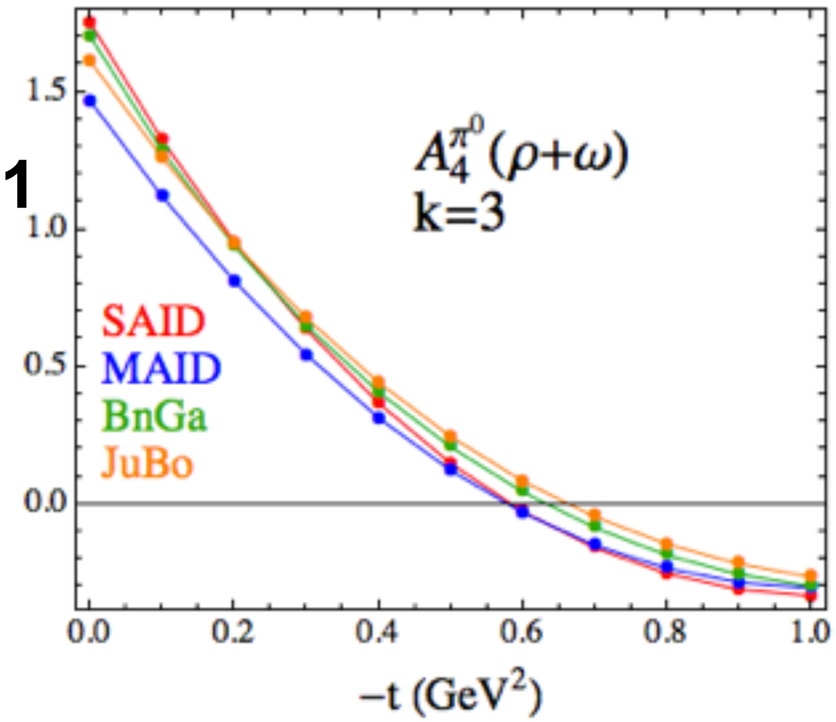
$\gamma p \to \pi^0 p$

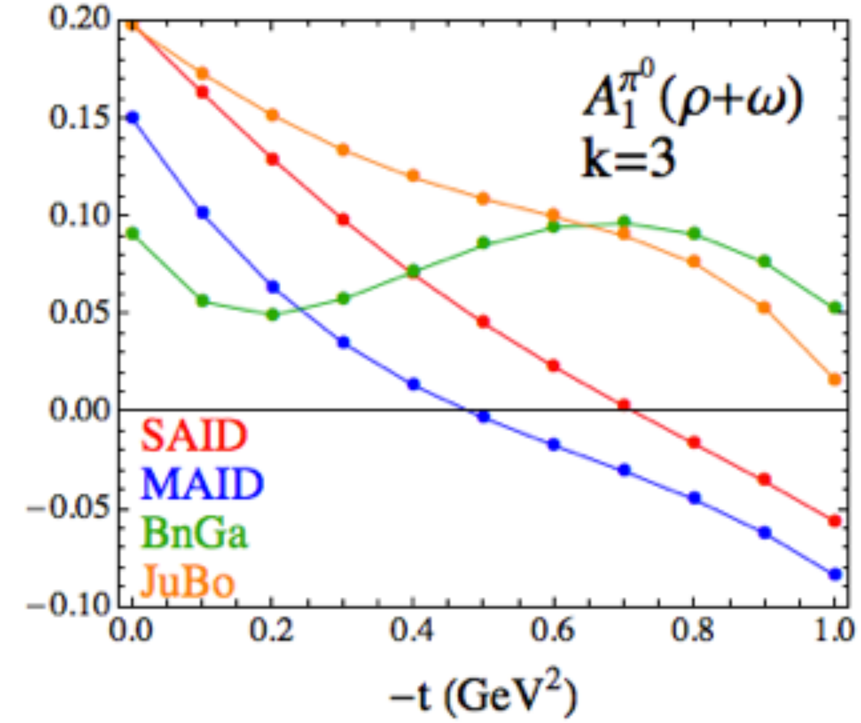
Agreed on dominant A4

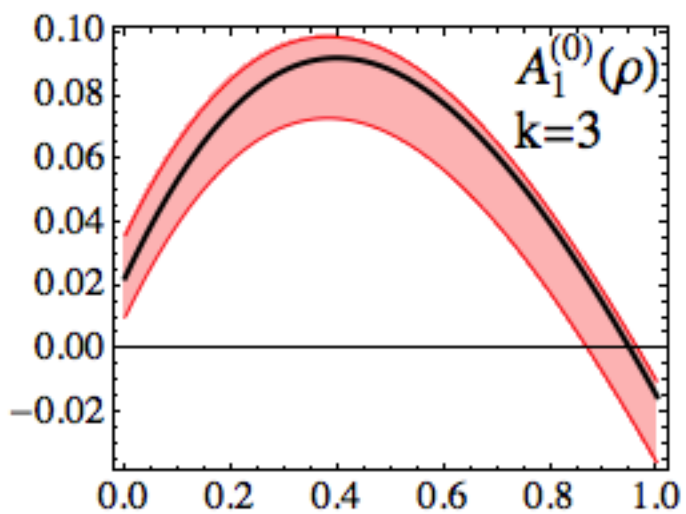
Variation on subdominant A1

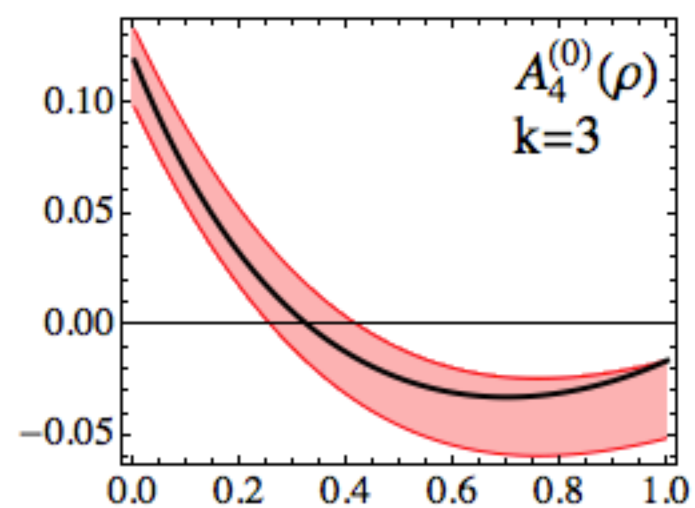
$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$

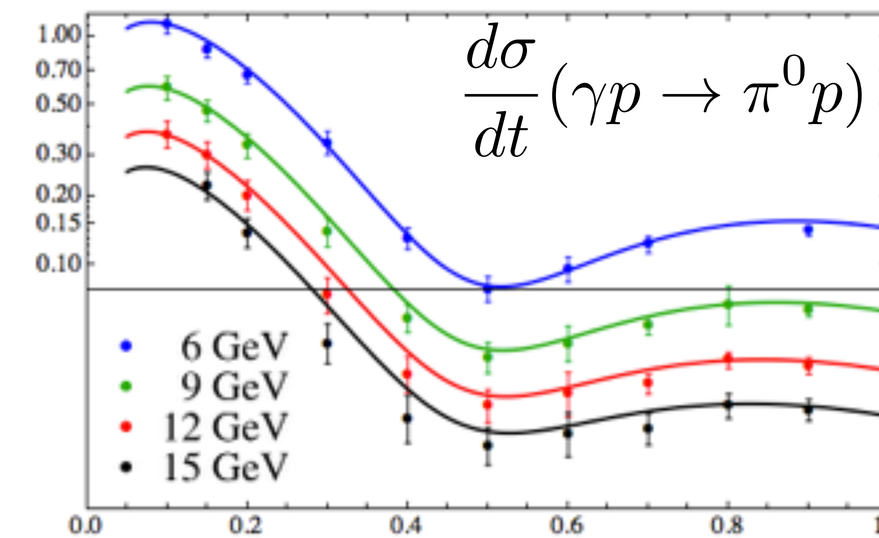
Use SAID to fit
FESR and observable

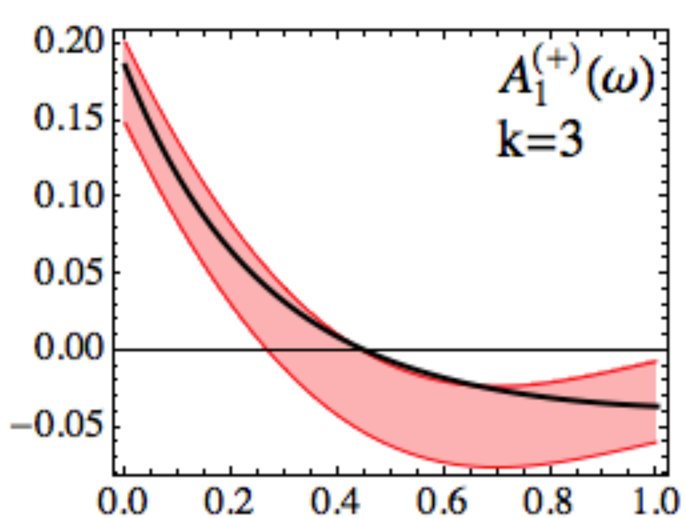


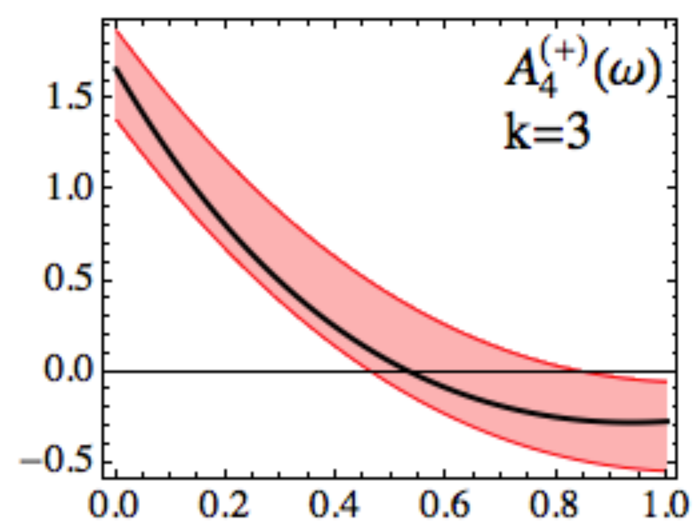


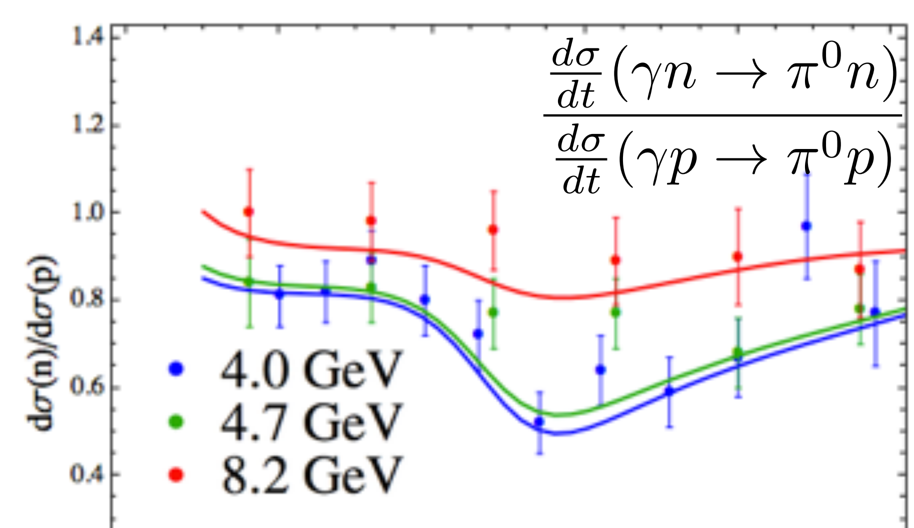












Application to Pseudoscalar Photoproduction

$$\gamma N \to \eta N$$

$$(\pm 1)\left(\pm\frac{1}{2}\right) \longrightarrow 0\left(\pm\frac{1}{2}\right)$$

8 helicity configurations related by pair via parity

4 indep. helicity configurations

use CGLN basis $A_1, \ldots, A_4$

Isospin symmetry:
every amplitude has an isospin index (s,v)

$$A_{ji} = A^s \delta_{ji} + A^v \tau^3_{ji}$$

8 indep. helicity/isospin configurations

$$\gamma p \to \eta p : A^s + A^v$$

$$\gamma n \to \eta n : A^s - A^v$$

Application to Pseudoscalar Photoproduction

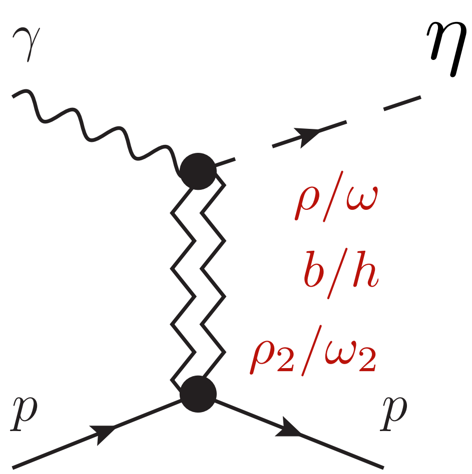
$$\gamma N \to \eta N$$

$$(\pm 1)\left(\pm\frac{1}{2}\right) \longrightarrow 0\left(\pm\frac{1}{2}\right)$$

8 helicity configurations related by pair via parity

4 indep. helicity configurations

use CGLN basis $A_1, \ldots, A_4$



Isospin symmetry:
every amplitude has an isospin index (s,v)

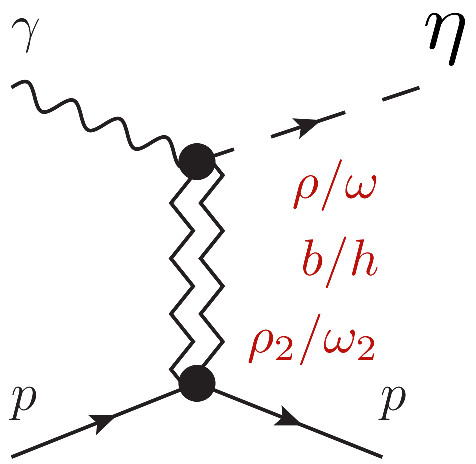
$$A_{ji} = A^s \delta_{ji} + A^v \tau^3_{ji}$$

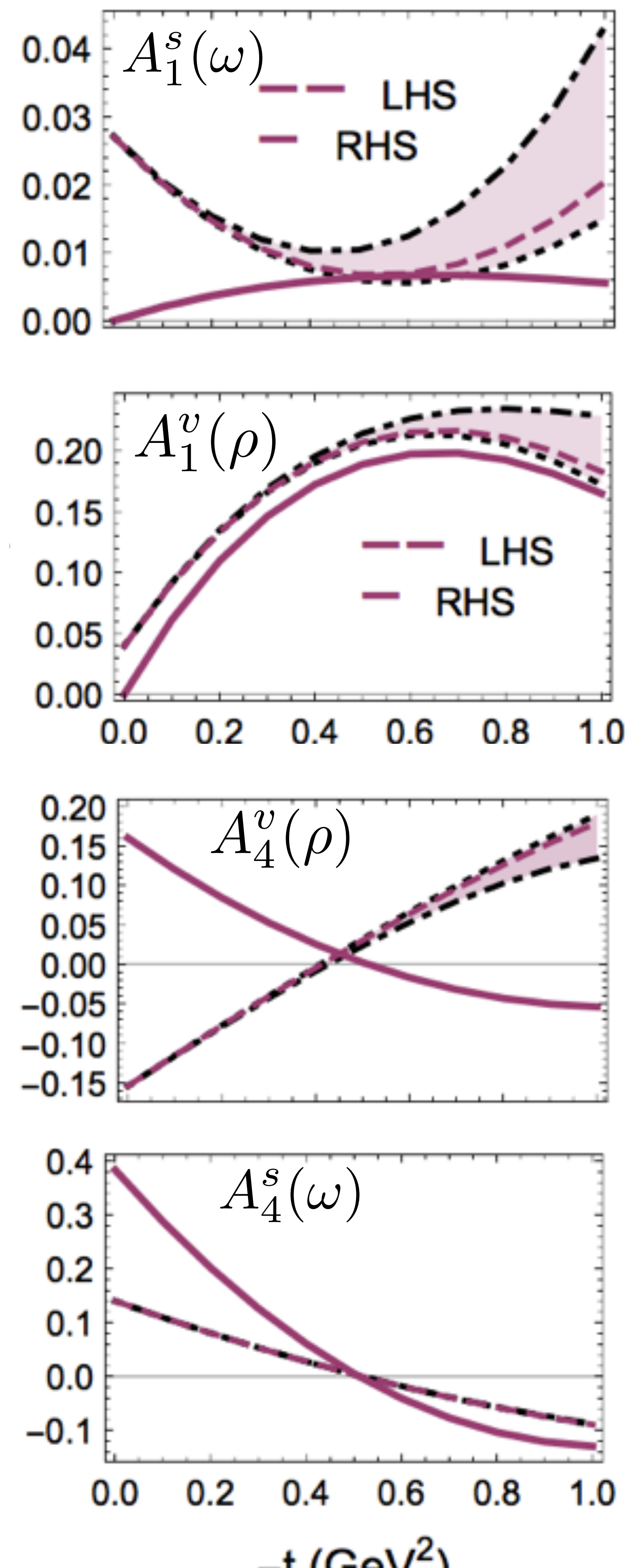
8 indep. helicity/isospin configurations

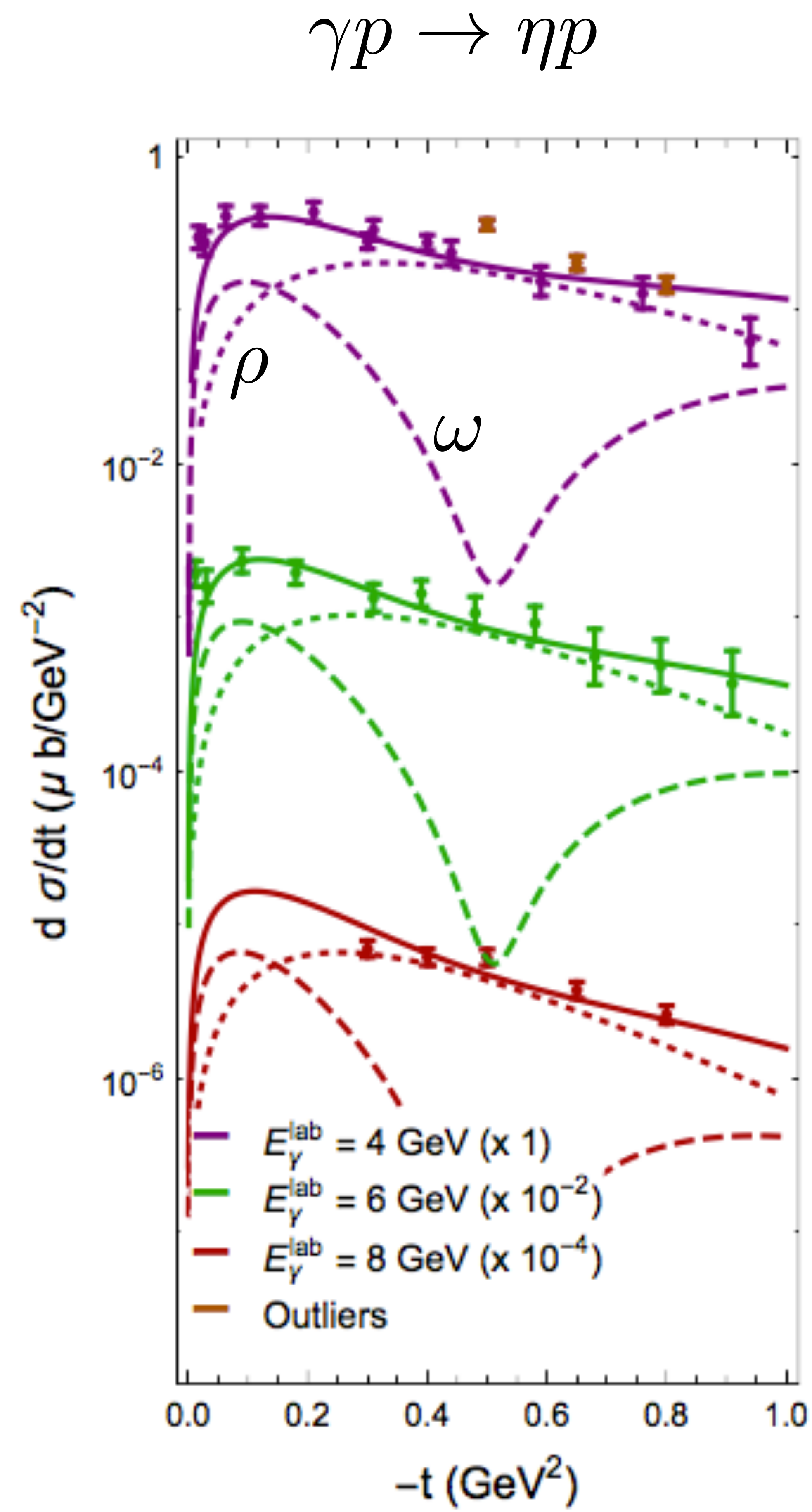
$$\gamma p \to \eta p : A^s + A^v$$

$$\gamma n \to \eta n : A^s - A^v$$

A_i	v	s
A_1	ρ	ω
A_2	b	h
A_3	ρ_2	ω_2
A_4	ρ	ω







Dashed lines: MAID 2001

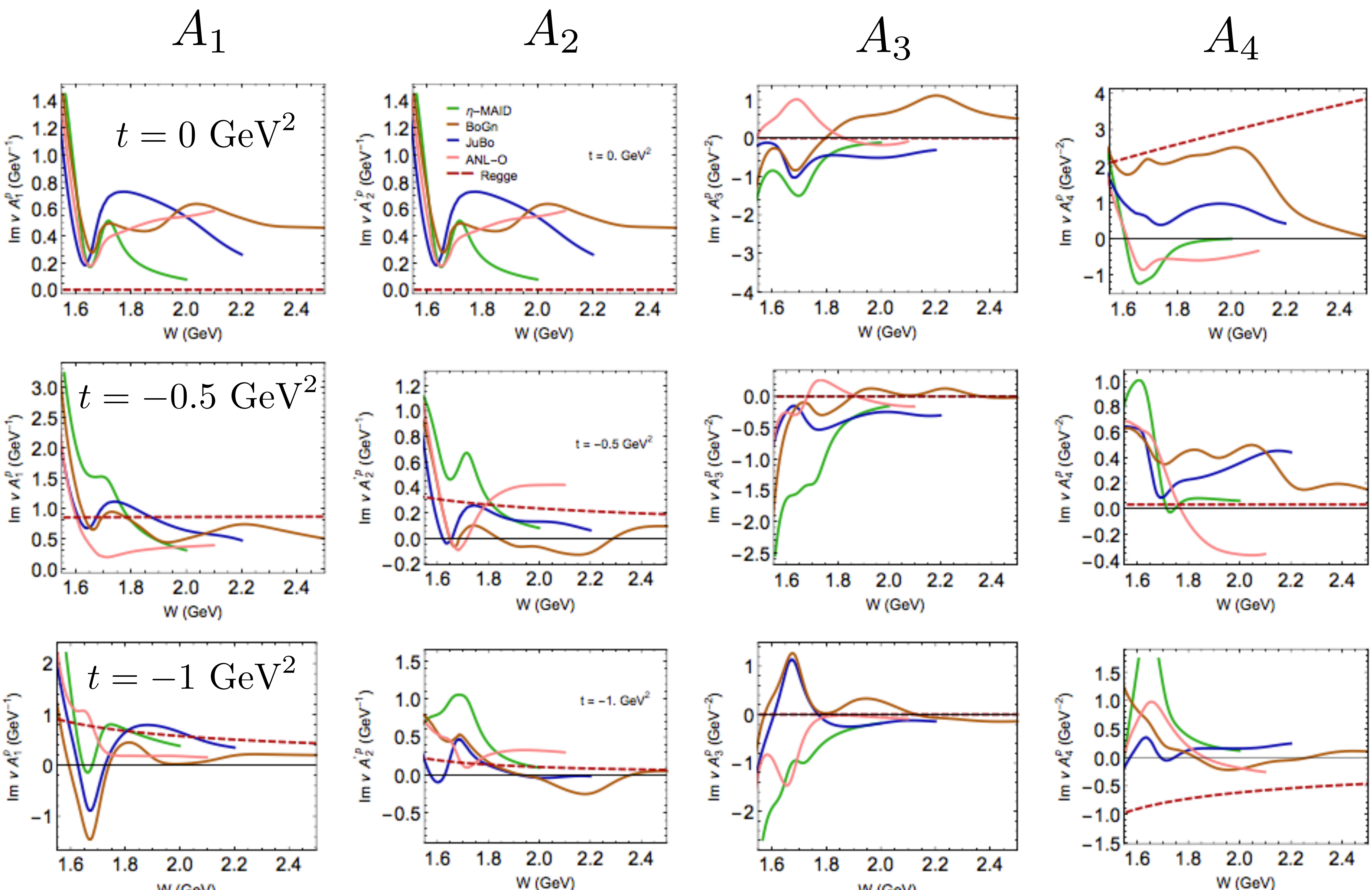
Solid lines: Regge

Not perfect agreement

But similar features

J. Nys et al (JPAC)
arXiv:1611.04658

Amplitude Comparison for $\gamma p \to \eta p$



INDIANA UNIVERSITY BLOOMINGTON

Jefferson Lab
Thomas Jefferson National Accelerator Facility

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

Joint Physics Analysis Center

HOME PROJECTS PUBLICATIONS LINKS

U.S. DEPARTMENT OF ENERGY

National Science Foundation

JPAC acknowledges support from DOE and NSF

INDIANA UNIVERSITY

Interactive webpage: http://www.indiana.edu/~jpac/index.html

Indiana University

- **Adam Szczepaniak** Professor
- **Geoffrey Fox** Professor
- **Emilie Passemar** Professor
- **Tim Londergan** Professor
- **Vincent Mathieu** Postdoctoral researcher
- **Ina Lorenz** Postdoctoral researcher
- **Andrew Jackura** PhD student

Jefferson Lab

- **Michael R. Pennington** Professor
- **Viktor Mokeev** Professor
- **Vladiszlav Pauk** Postdoctoral researcher
- **Alessandro Pilloni** Postdoctoral researcher

George Washington University

- **Ron Workman** Professor
- **Michael Doring** Professor

Universidad Nacional Autonoma de Mexico

- **Cesar Fernandez-Ramirez** Professor

Johannes Gutenberg University, Mainz

- **Igor Danilkin** Postdoctoral researcher

Bonn University

- **Misha Mikhasenko** PhD student

University of Valencia

- **Astrid Hiller Blin** PhD student

Ghent University

- **Jannes Nys** PhD student

Interactive webpage: http://www.indiana.edu/~jpac/index.html

November 2016:

- The $\gamma p \to \eta p$ page is online.

June 2016:

- The $\gamma p \to J/\psi p$ page is online.
- The πN page is online.

October 2015:

- The $\bar{K}N$ page is online.

May 2015:

- The website is launched.
- The $\gamma p \to \pi^0 p$ page is online.
- The $\omega, \phi \to 3\pi$ page is online.
- The $\eta \to 3\pi$ page is online.

INDIANA UNIVERSITY

Interactive webpage: http://www.indiana.edu/~jpac/index.html

November 2016:

- The $\gamma p \to \eta p$ page is online.

June 2016:

- The $\gamma p \to J/\psi p$ page is online.
- The πN page is online.

October 2015:

- The $\bar{K}N$ page is online.

May 2015:

- The website is launched.
- The $\gamma p \to \pi^0 p$ page is online.
- The $\omega, \phi \to 3\pi$ page is online.
- The $\eta \to 3\pi$ page is online.

Resources

- **Publication:** [Nys16]
- **C/C++ observables:** C-code main, Input file, C-code source, C-code header, Eta-MAID 2001 multipoles
- **C/C++ minimal script to calculate the amplitudes:** C-code zip
- **Data:** Dewire , Braunschweig
- **Contact person:** Jannes Nys
- **Last update:** November 2016

Interactive webpage: http://www.indiana.edu/~jpac/index.html

November 2016:

- The $\gamma p \to \eta p$ page is online.

June 2016:

- The $\gamma p \to J/\psi p$ page is online.
- The πN page is online.

October 2015:

- The $\bar{K}N$ page is online.

May 2015:

- The website is launched.
- The $\gamma p \to \pi^0 p$ page is online.
- The $\omega, \phi \to 3\pi$ page is online.
- The $\eta \to 3\pi$ page is online.

Resources

- **Publication:** [Nys16]
- **C/C++ observables:** C-code main, Input file, C-code source, C-code header, Eta-MAID 2001 multipoles
- **C/C++ minimal script to calculate the amplitudes:** C-code zip
- **Data:** Dewire , Braunschweig
- **Contact person:** Jannes Nys
- **Last update:** November 2016

Run the code

E_γ in GeV 9

t cos

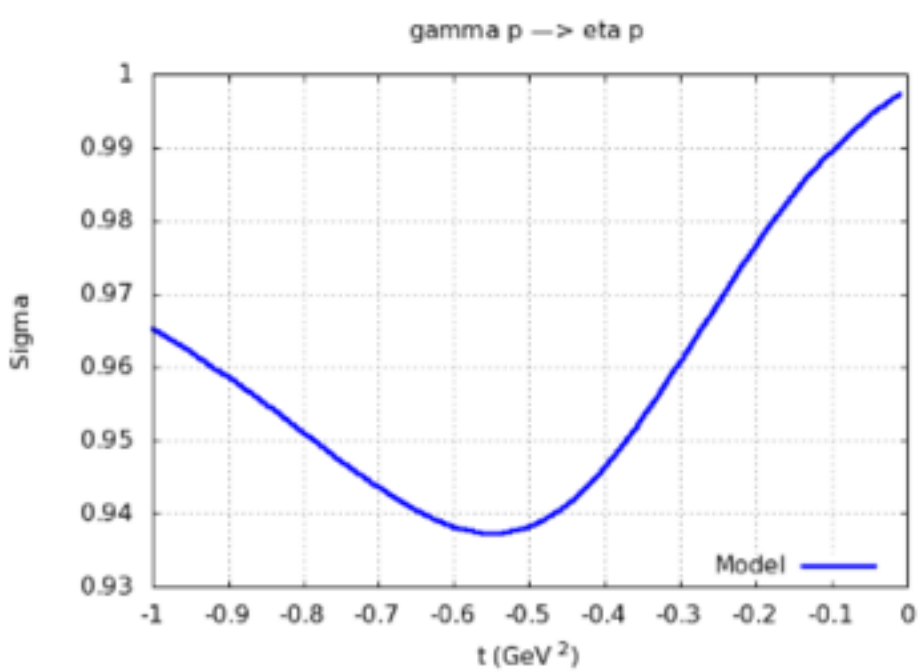
t in GeV² (min max step) -1 0 0.01

cos θ (min max step) 0.85 1 0.01

Start reset

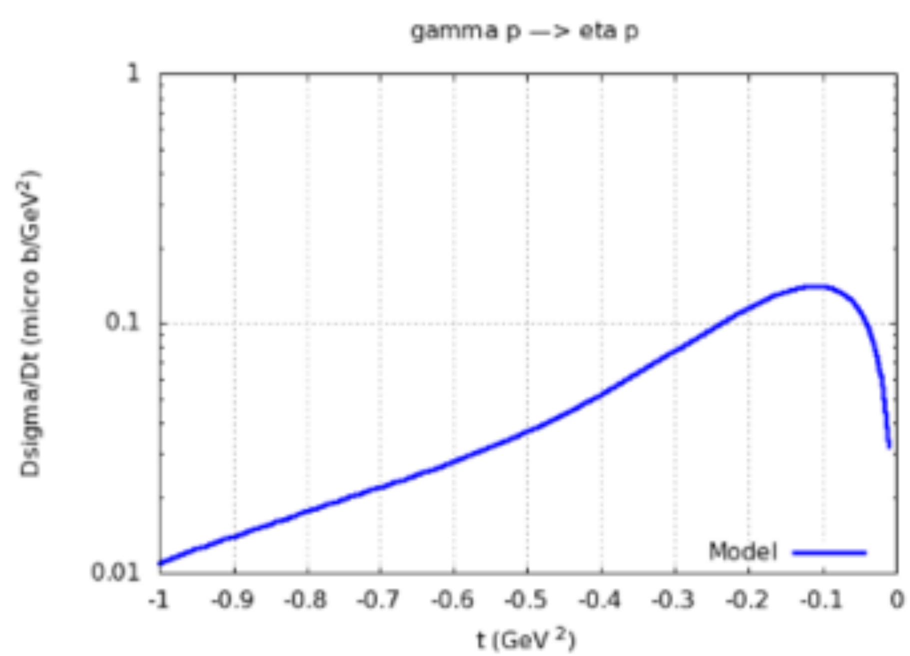
Observable: photon beam asymmetry

Download the the plot with Ox=t , the plot with Ox=cos .



Observable: differential cross section

Download the the plot with Ox=t , the plot with Ox=cos .



Methodology: A

Choose one single channel: $\gamma p \to \pi^0 p$

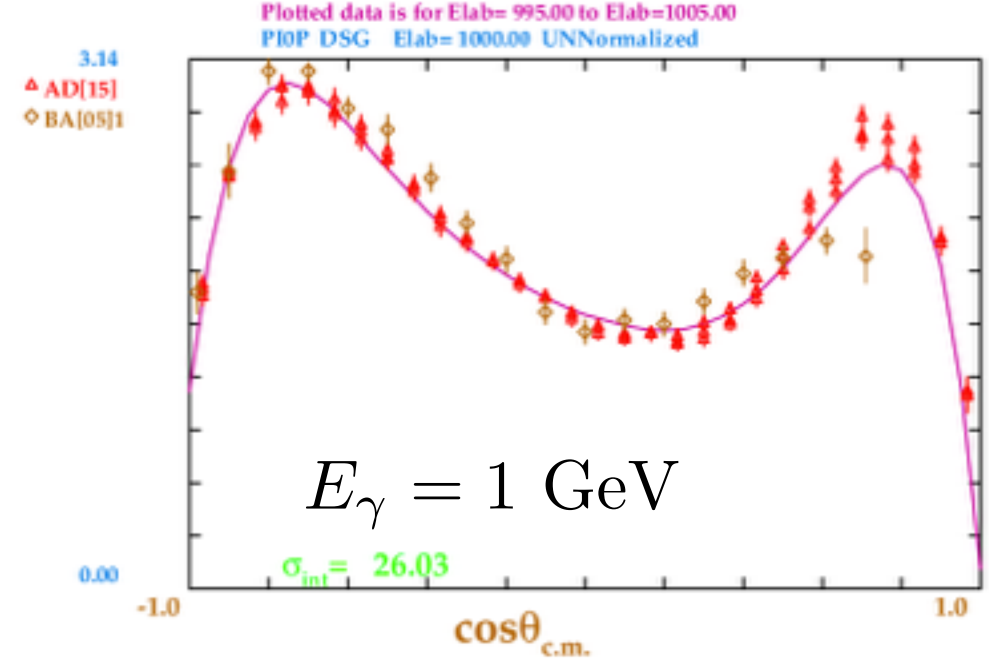
Fit independently low and high energy data

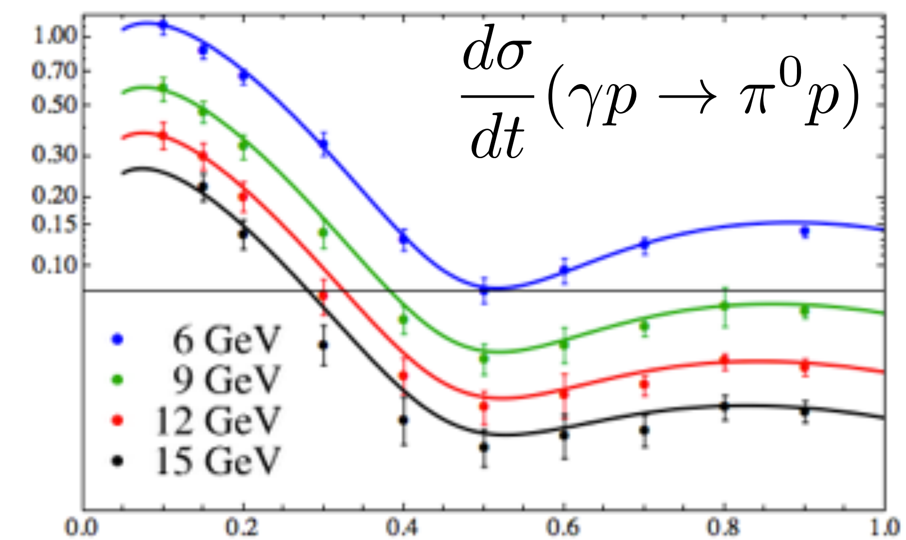
resonances Regge

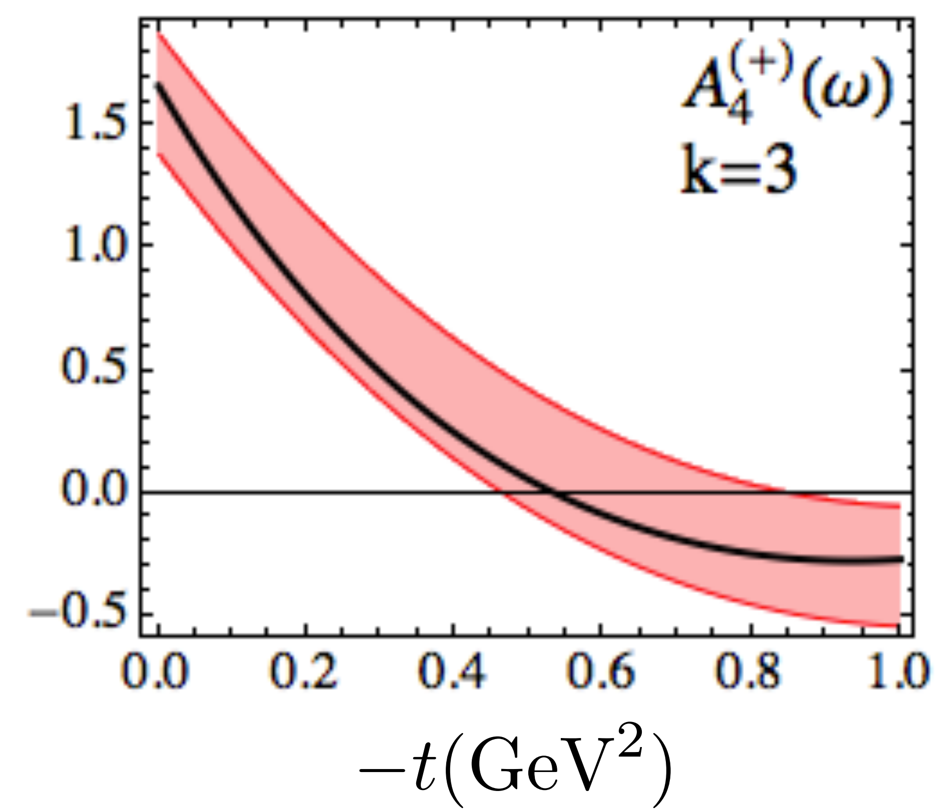
Compare amplitudes

Impose FESR as penalty function in fit

$$\boxed{\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}}$$

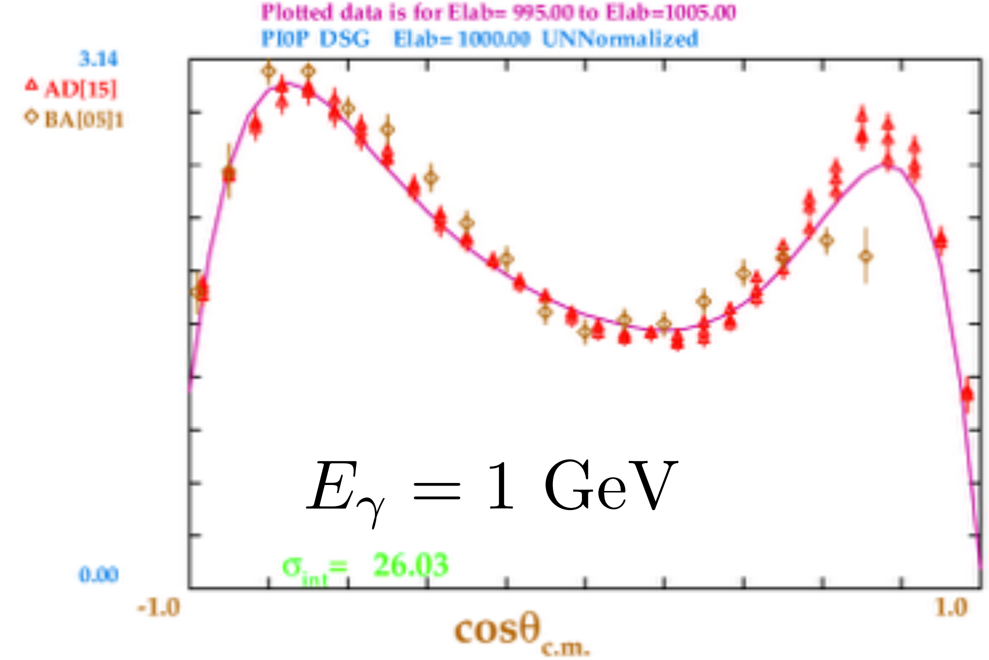






Methodology: B

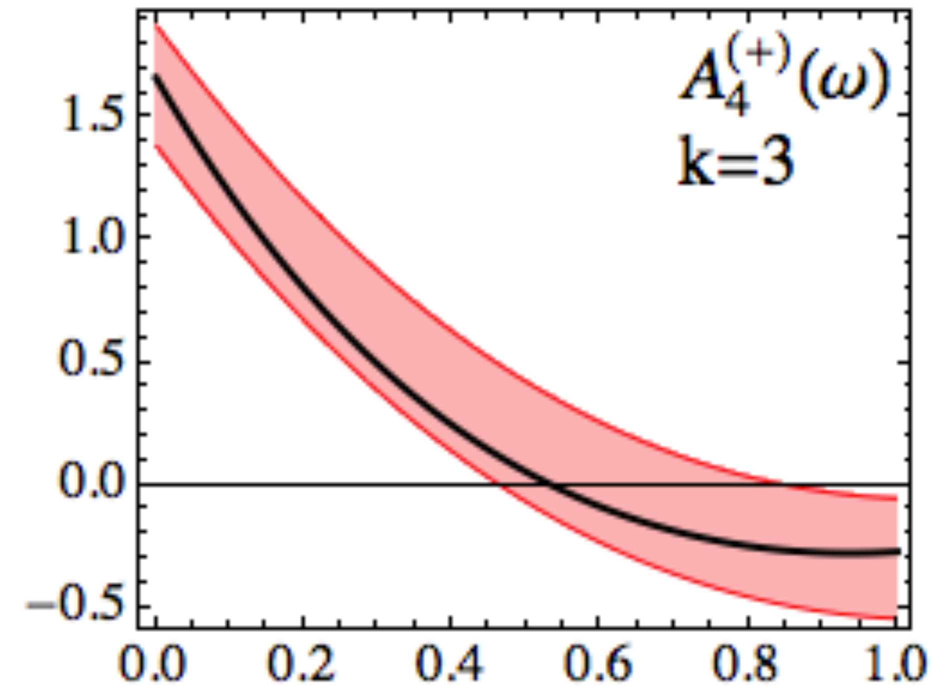
Choose one single channel: $\gamma p \to \pi^0 p$



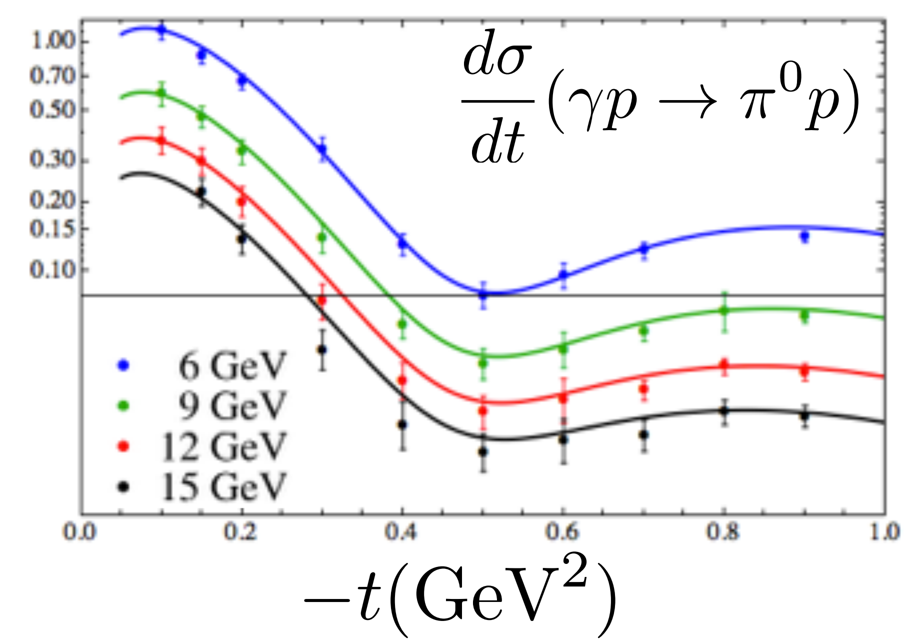
Propose imaginary part (only real parameters)

Reconstruct real part from dispersion relation:

$$\boxed{A(\nu, t) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \frac{\mathrm{Im}\, A(\nu', t)}{\nu'^2 - \nu^2} \nu' d\nu'}$$

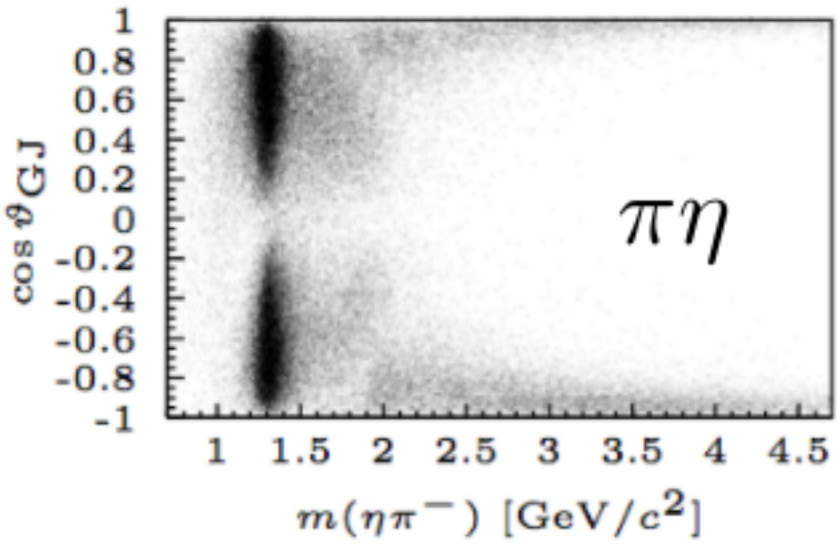


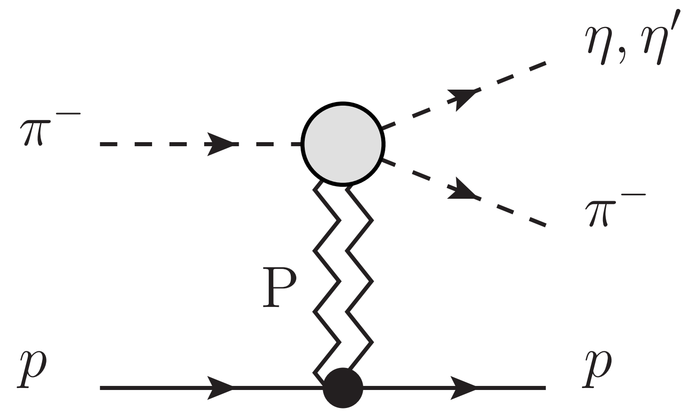
Fit all data and iterate



Going Further: Eta-Pi @COMPASS

COMPASS PLB740
ArXiv:1408.4286

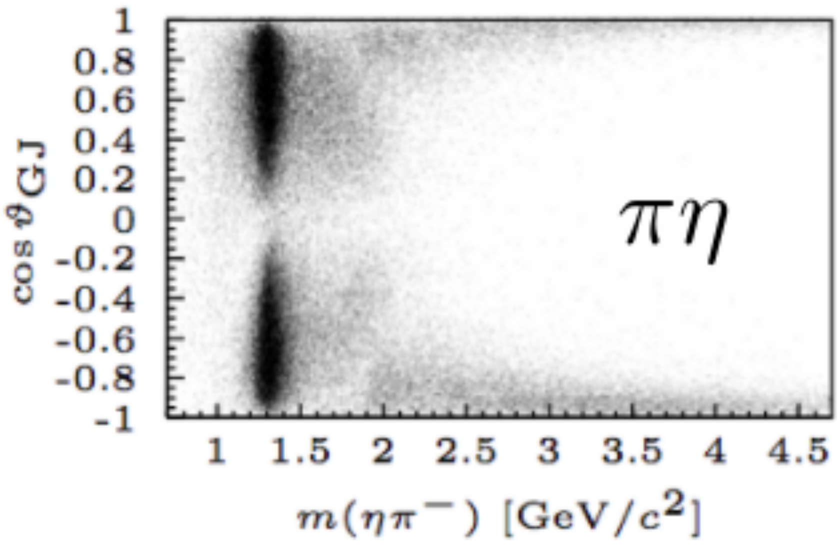


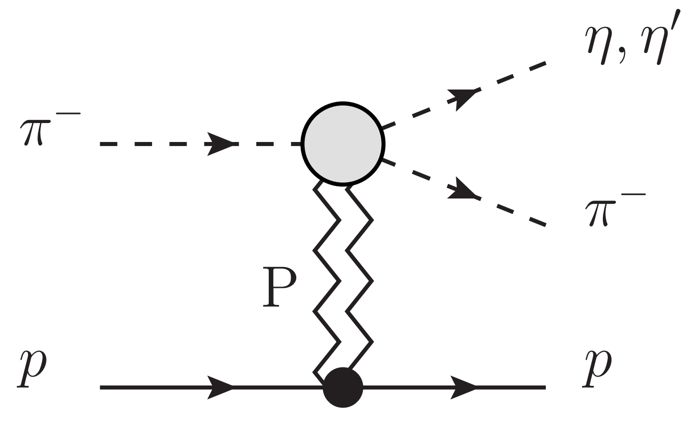


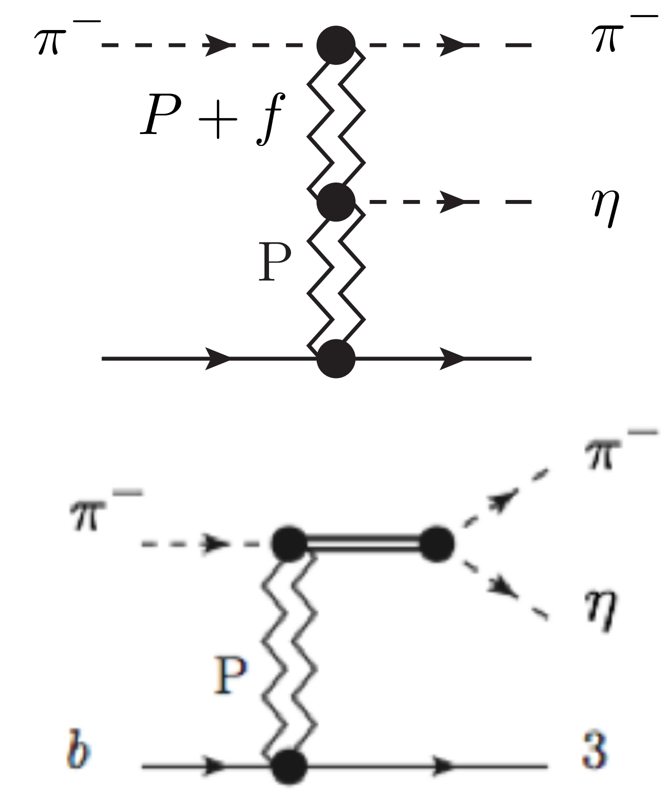
A. Jackura et al (JPAC), in preparation

Going Further: Eta-Pi @COMPASS

COMPASS PLB740
ArXiv:1408.4286



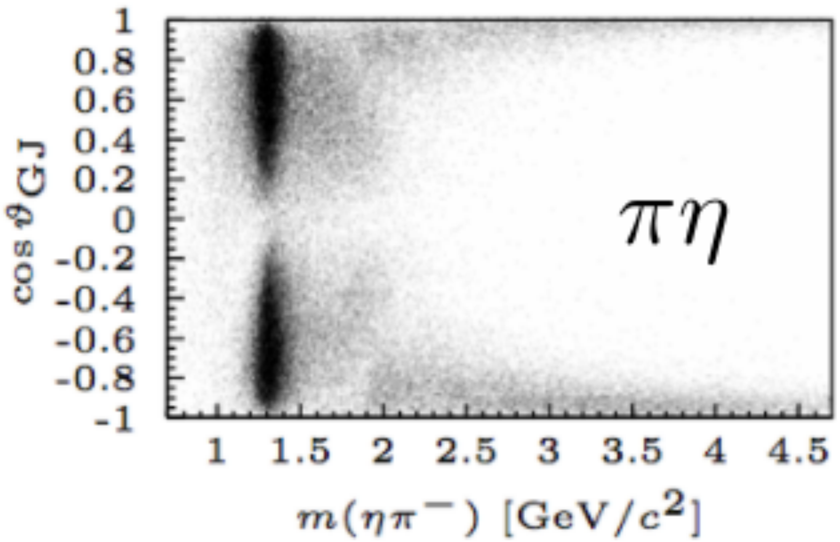


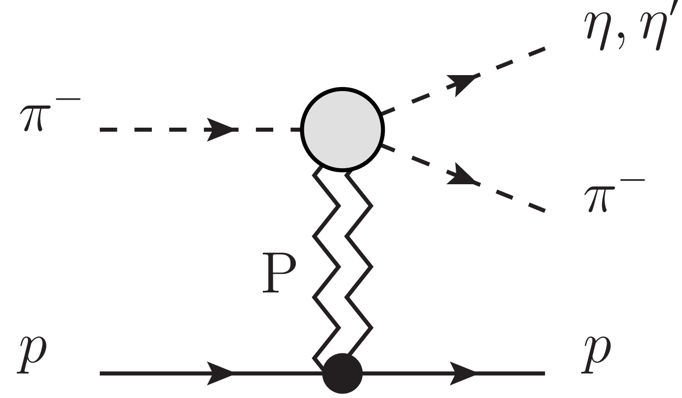


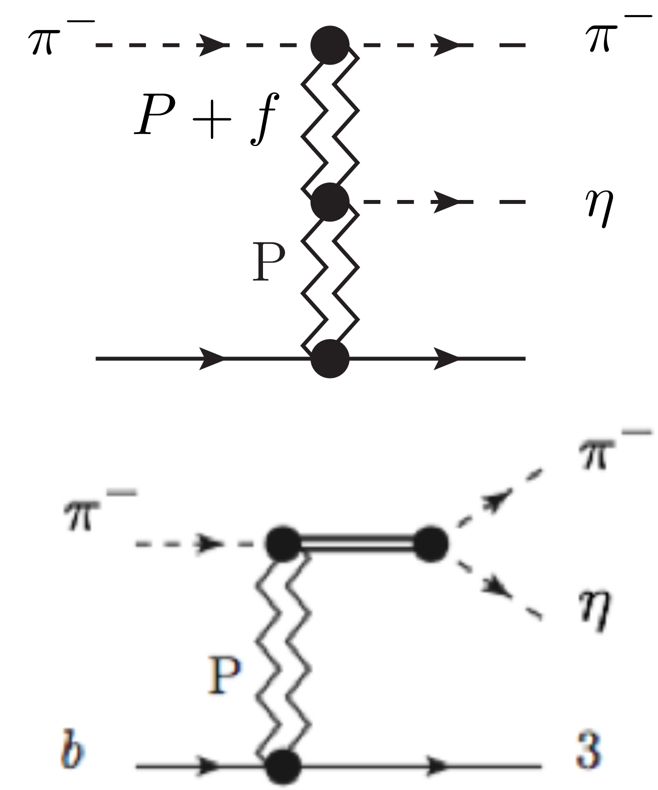
A. Jackura et al (JPAC), in preparation

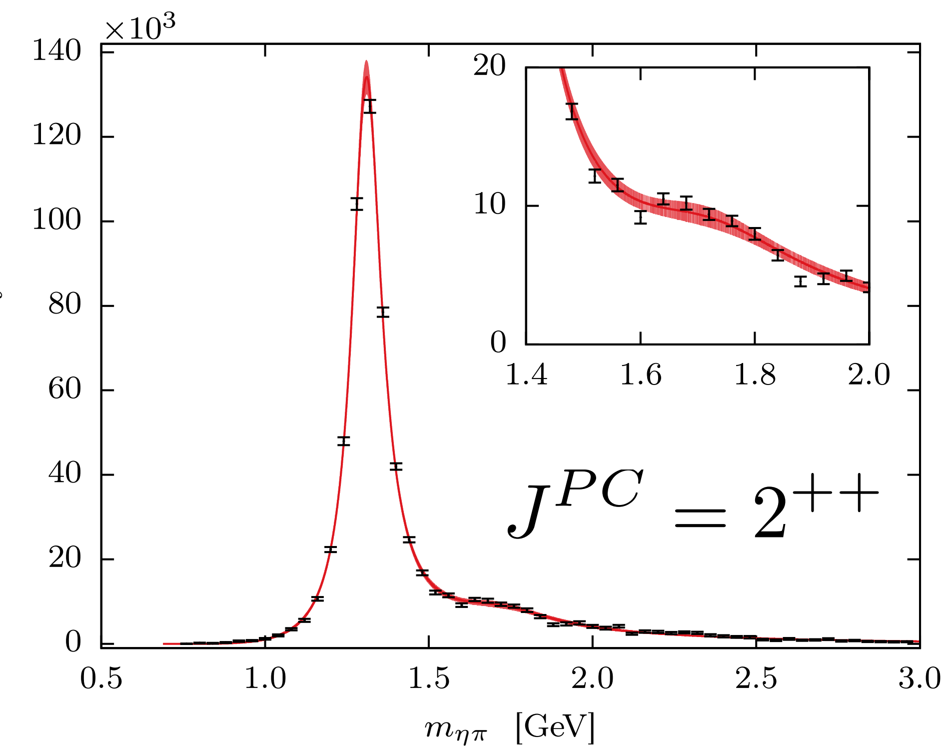
Going Further: Eta-Pi @COMPASS

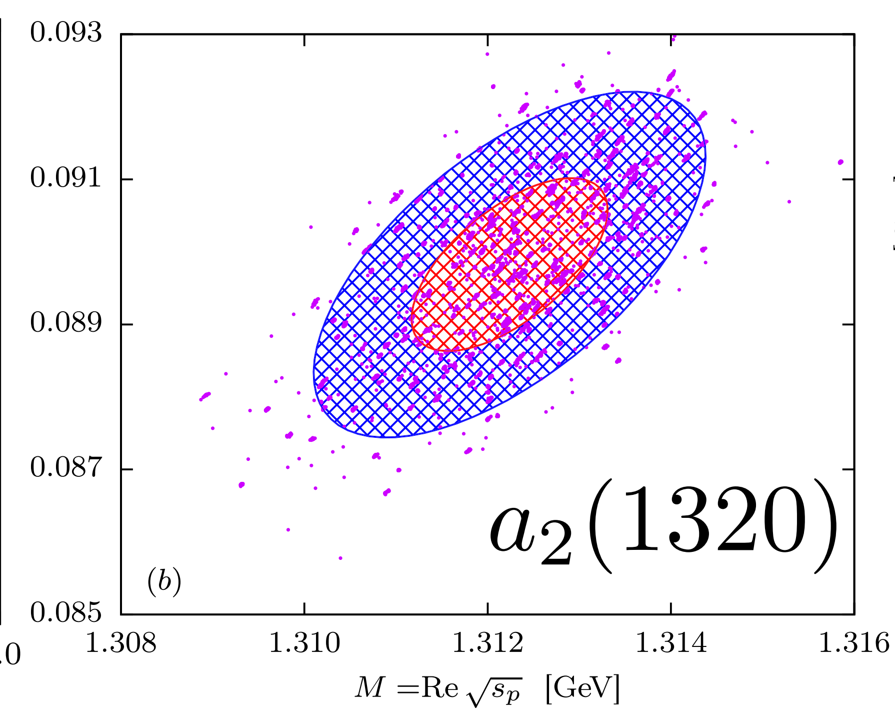
COMPASS PLB740
ArXiv:1408.4286

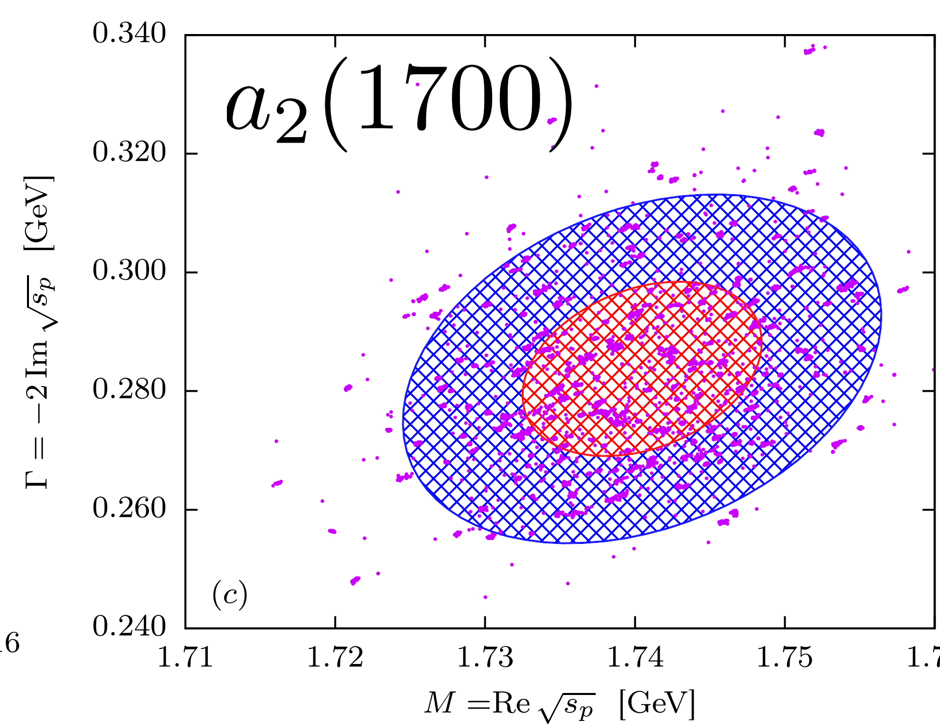








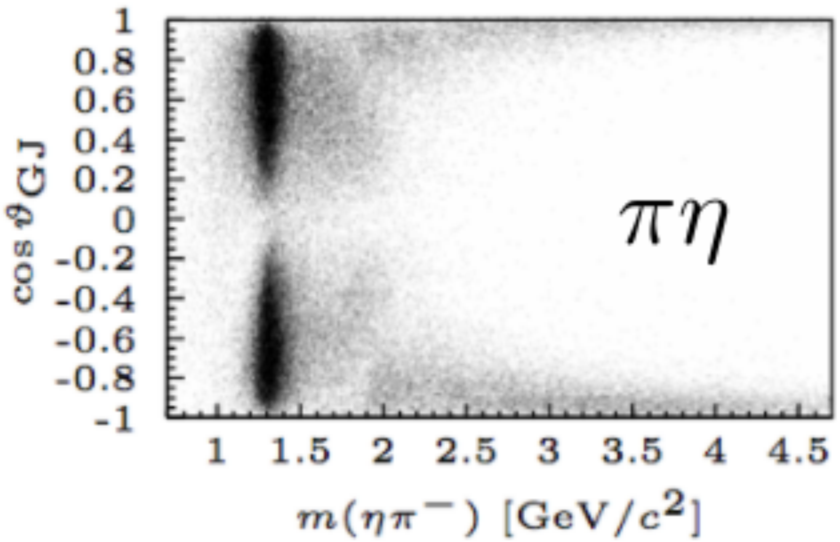


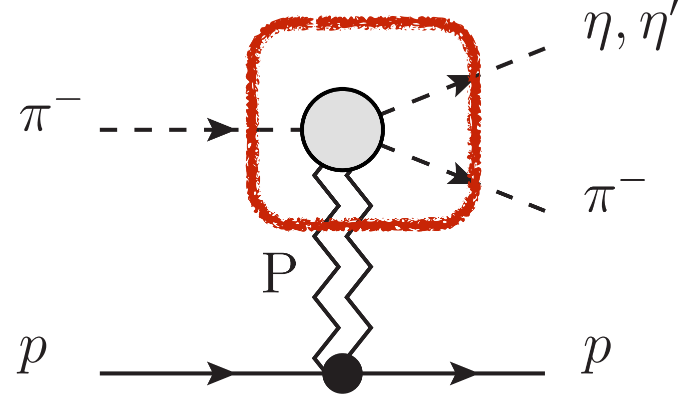


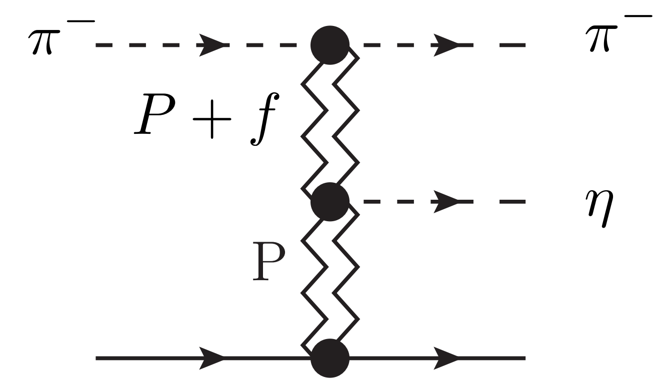
A. Jackura et al (JPAC), in preparation

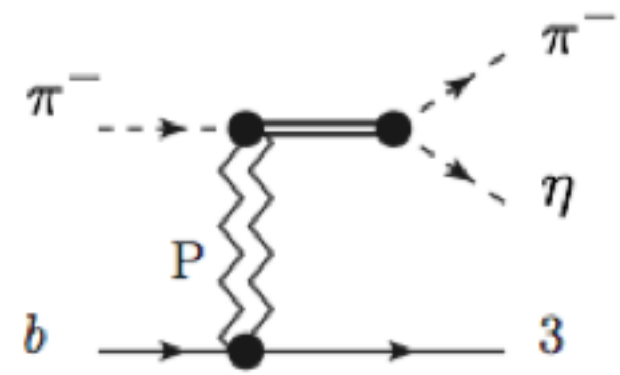
Going Further: Eta-Pi @COMPASS

COMPASS PLB740
ArXiv:1408.4286

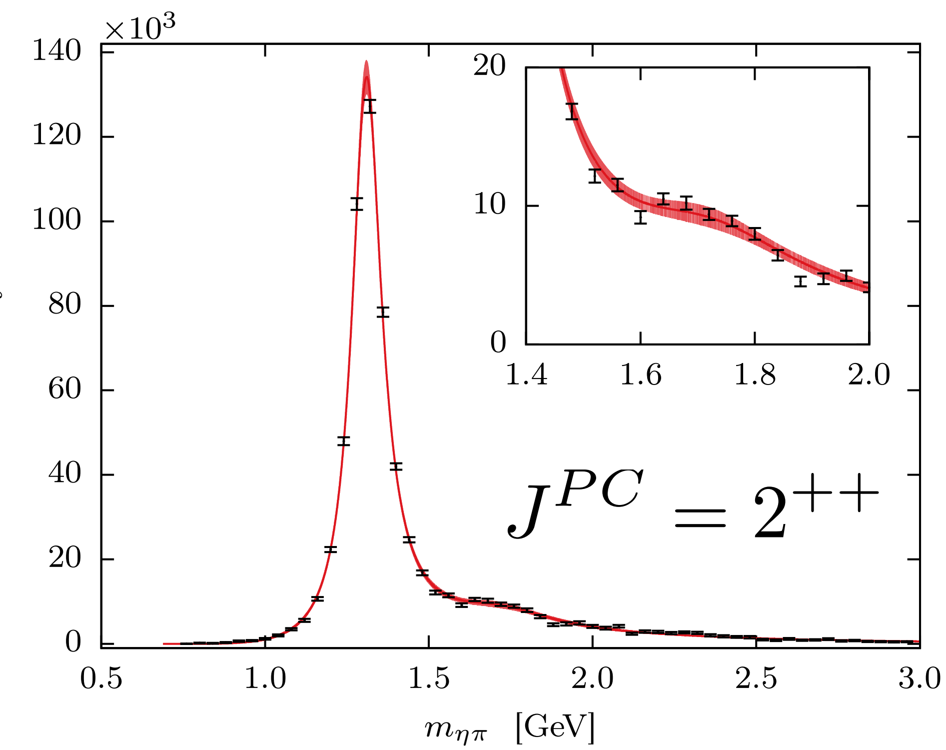


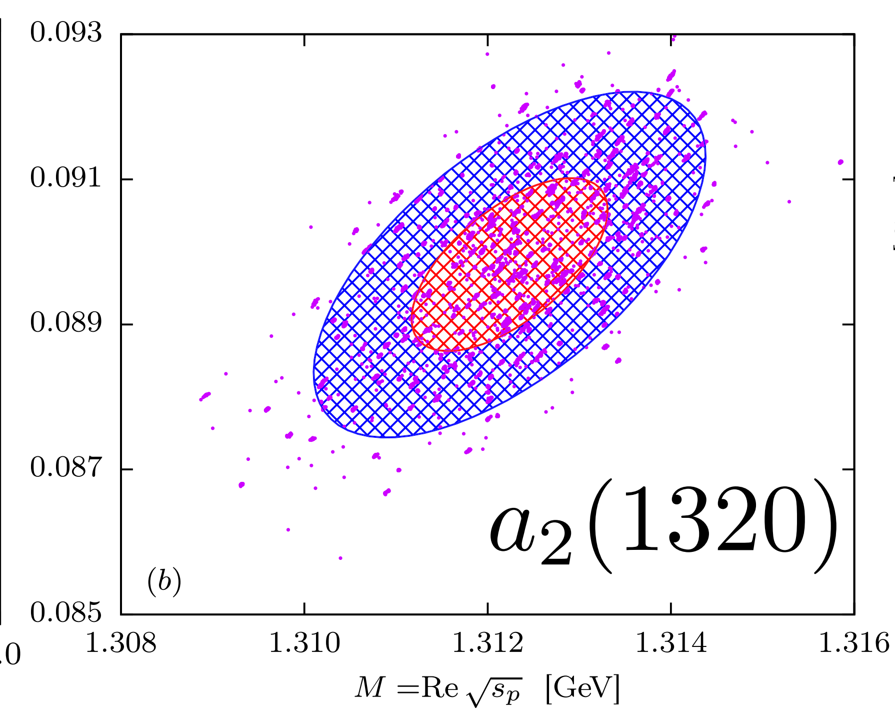


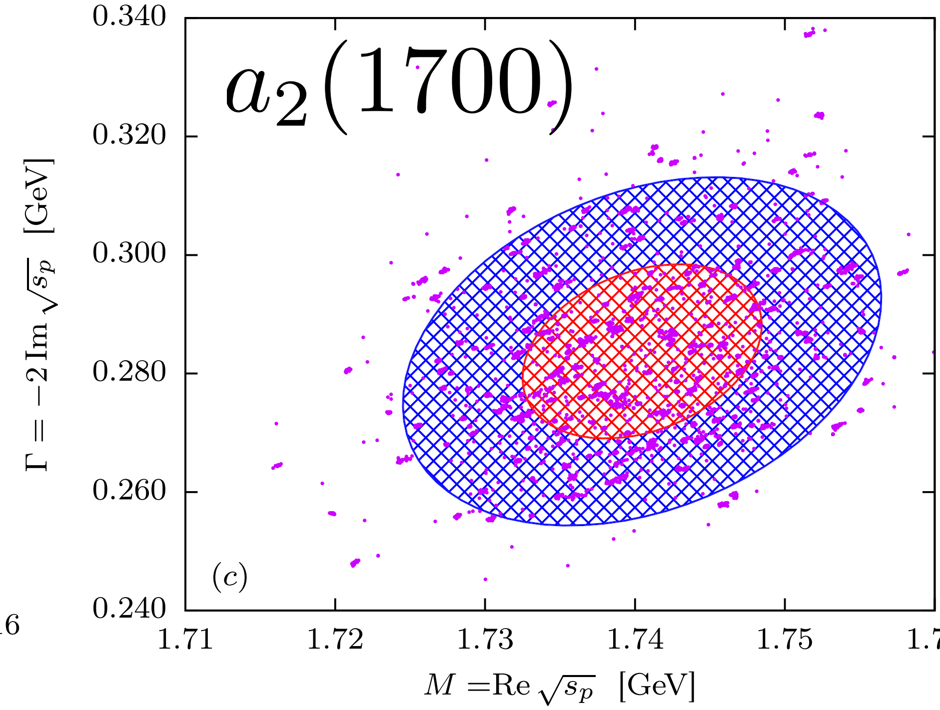




FESR with Reggeons could reduce uncertainties on pole parameters



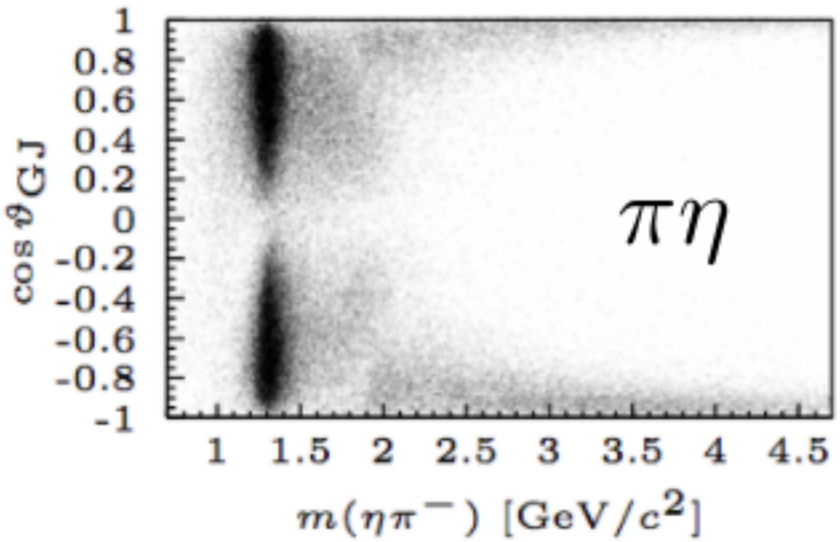


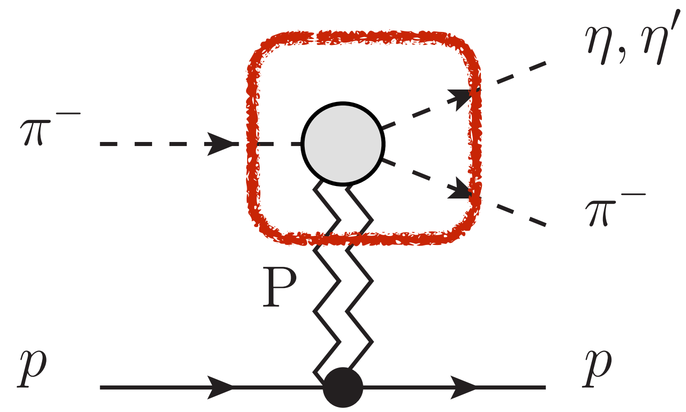


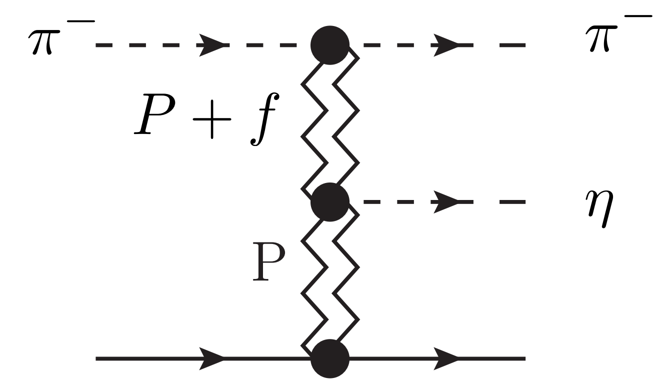
A. Jackura et al (JPAC), in preparation

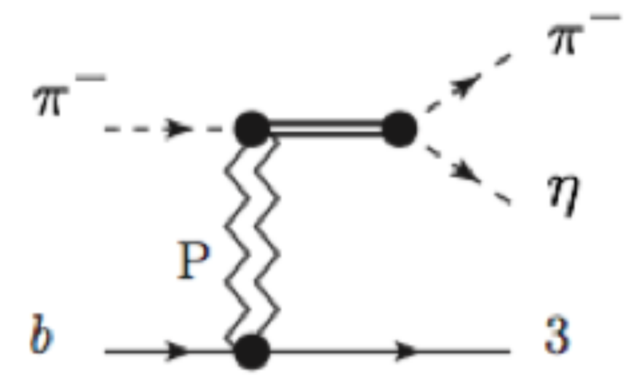
Going Further: Eta-Pi @COMPASS

COMPASS PLB740
ArXiv:1408.4286

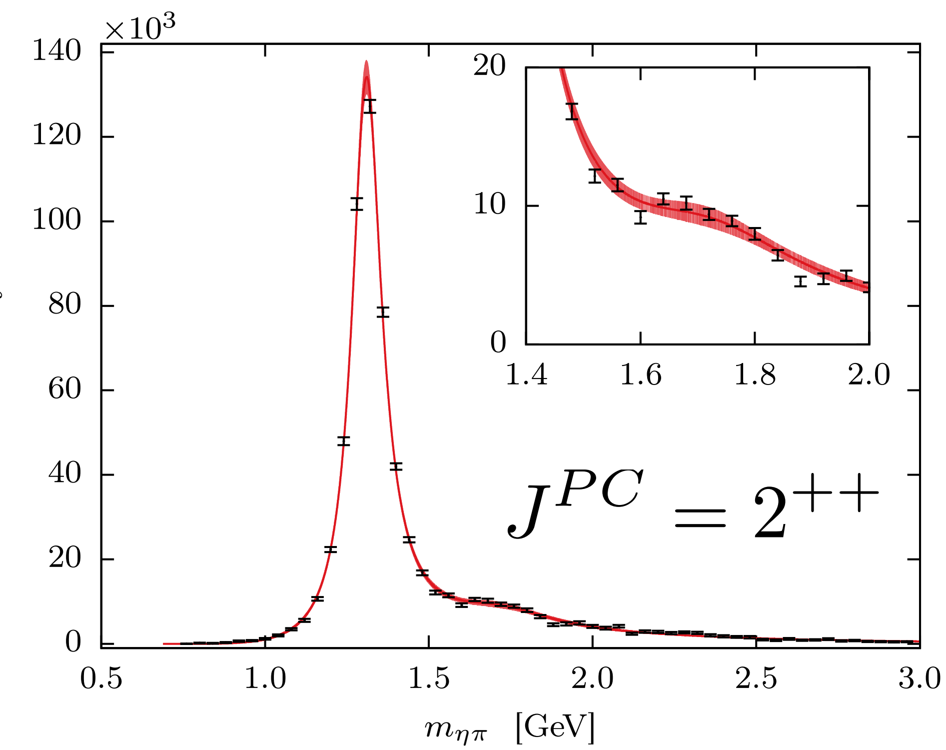


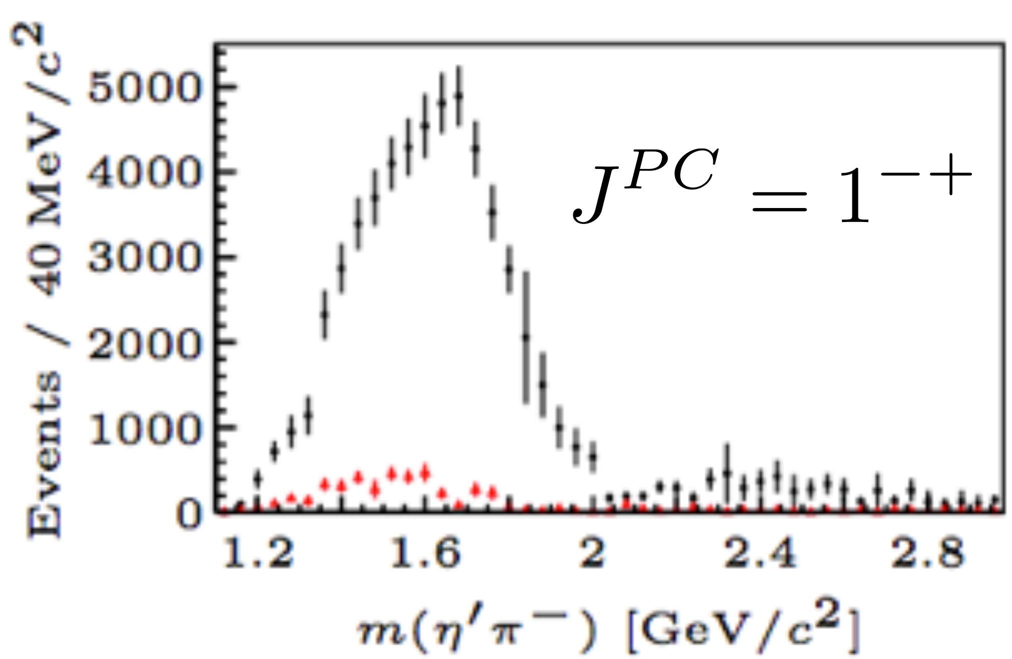






FESR with Reggeons could reduce uncertainties on pole parameters constraint exotic production



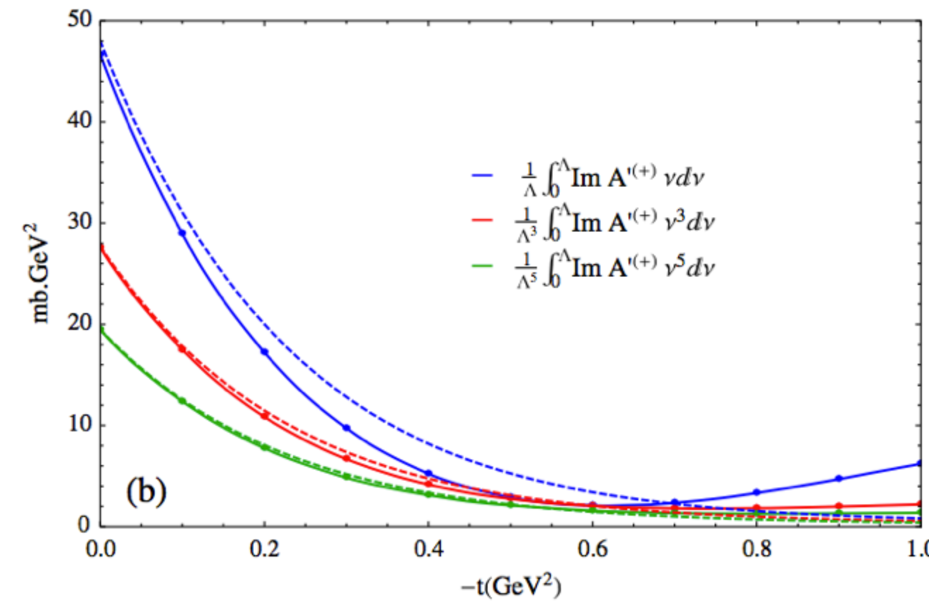


A. Jackura et al (JPAC), in preparation

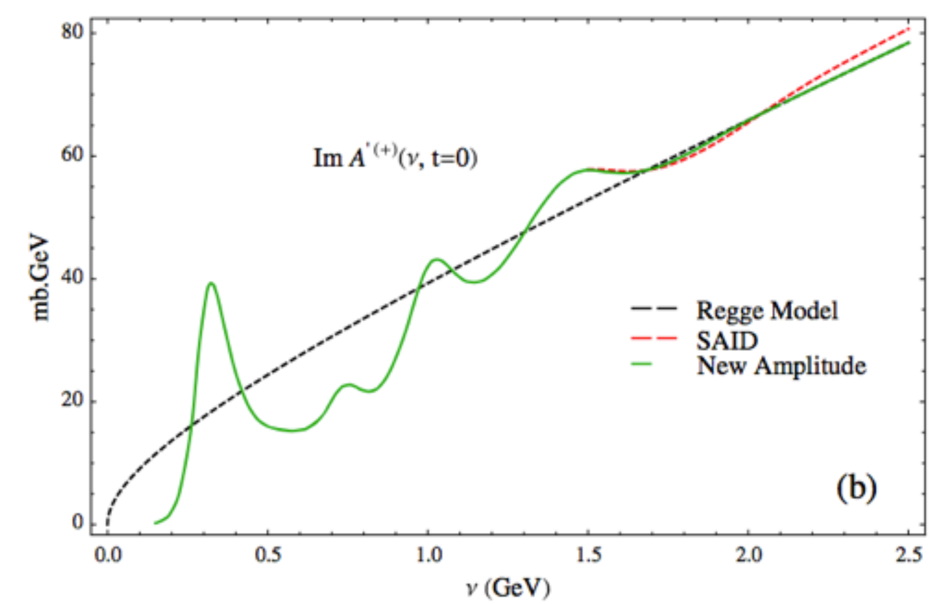
Summary: Methodology

Use constraints from analyticity: FESR

$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$



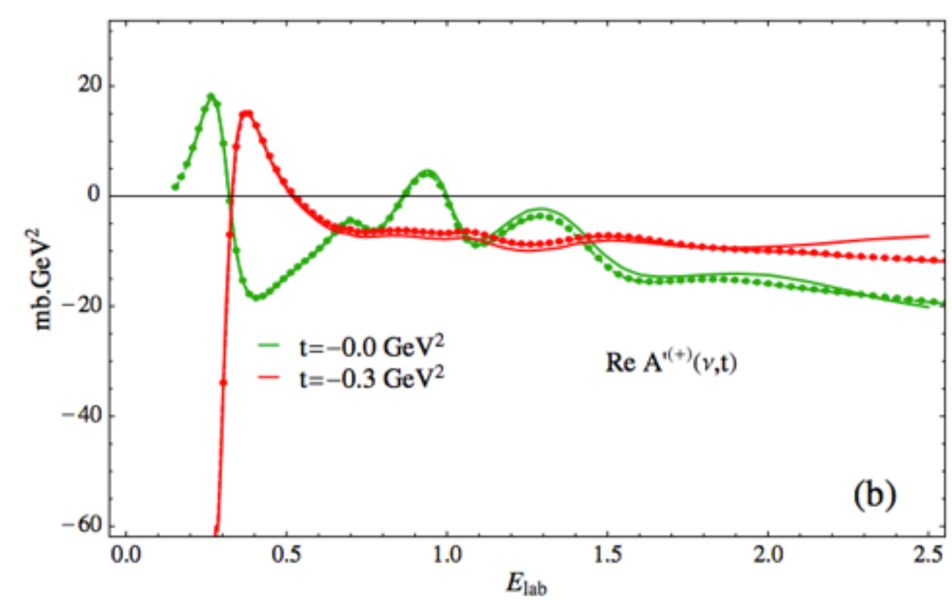
Great agreement for $\pi p \to \pi p$



Possibly useful for $\gamma p \to \pi p$ $\quad \gamma p \to \eta p$

to resolve ambiguities

Work in progress for $KN \to KN$

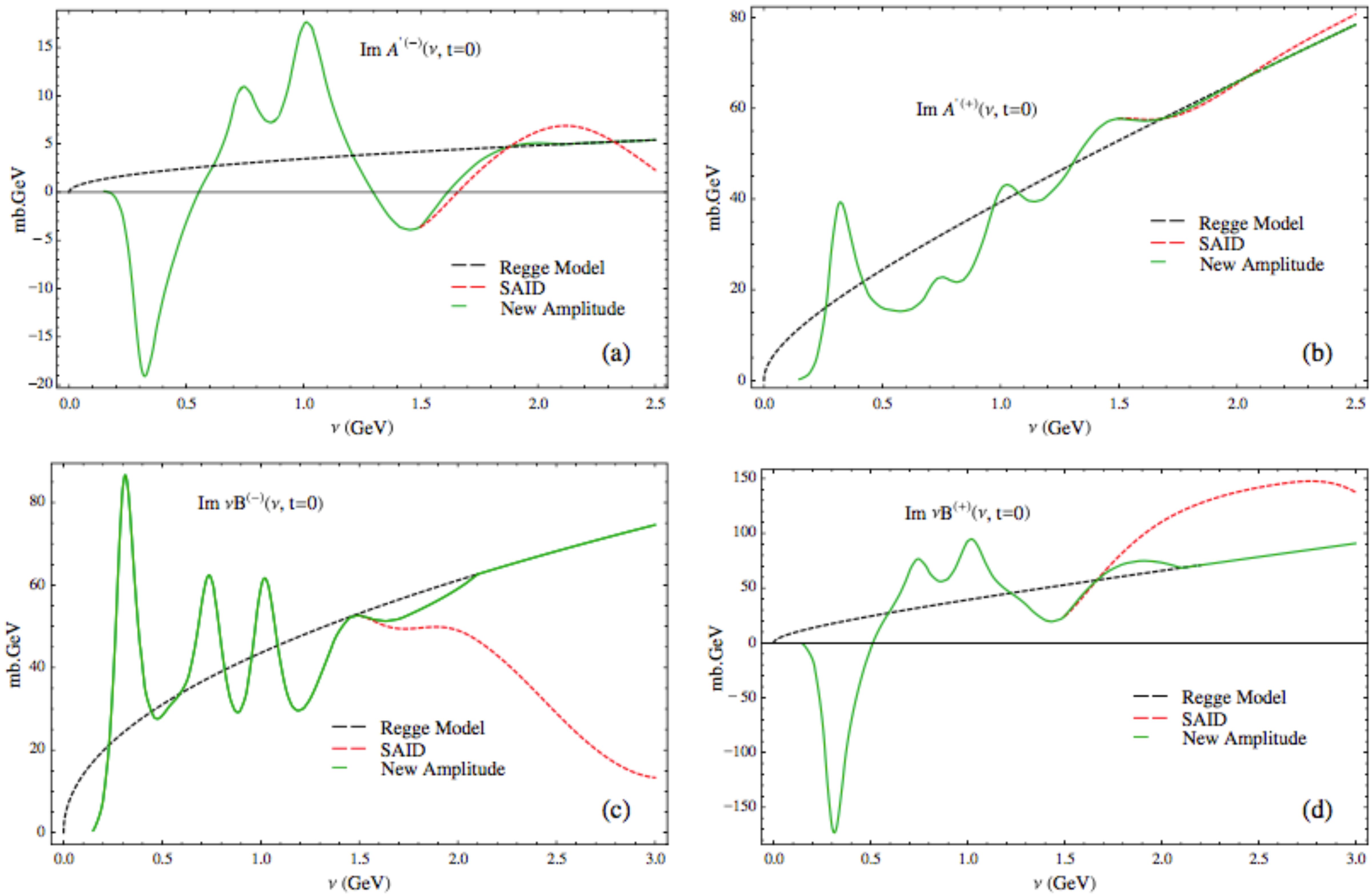


VM et al (JPAC) PRD92
arXiv:1506.01764

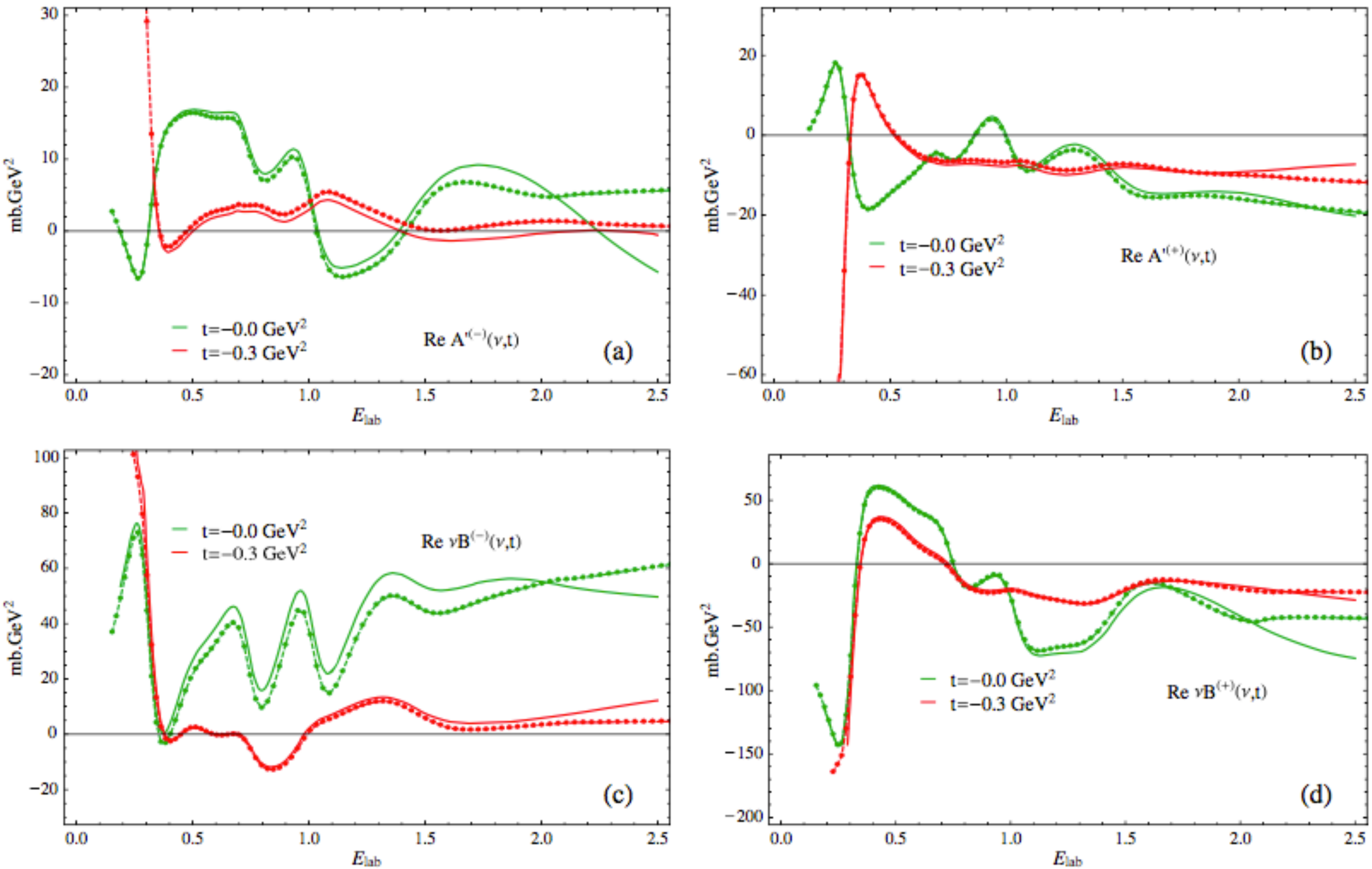
J. Nys et al (JPAC)
arXiv:1611.04658

Backup Slides

$$\pi N \rightarrow \pi N$$

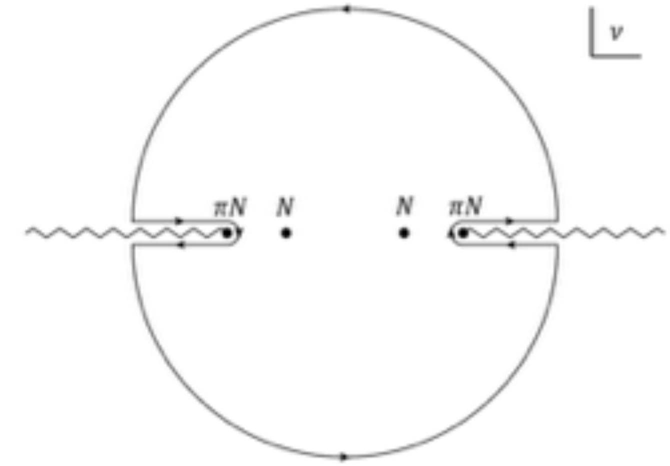


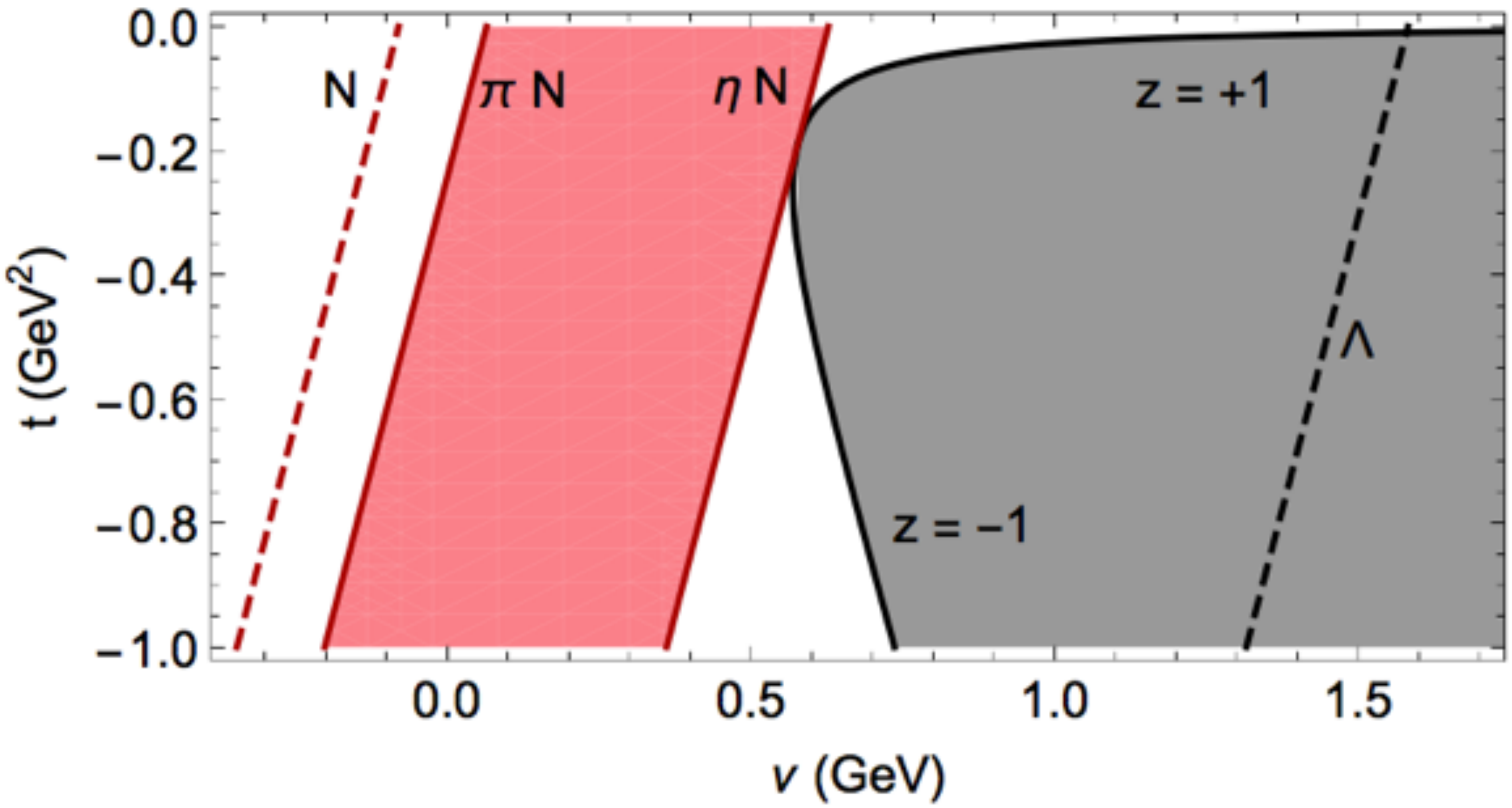
$\pi N \to \pi N$



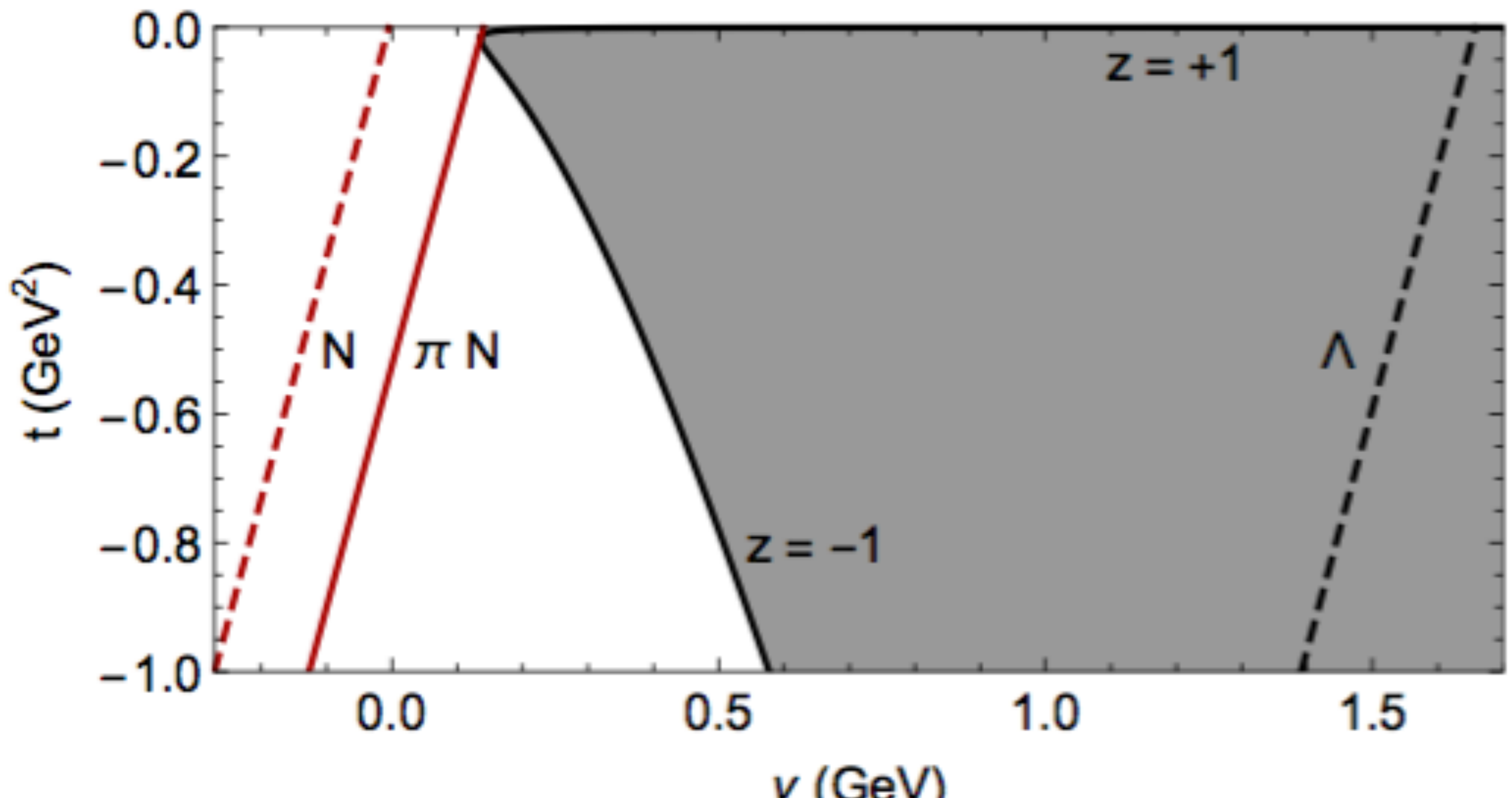
t=−0.0 GeV²
t=−0.3 GeV²
Re A′(−)(ν,t)
(a)
Re A′(+)(ν,t)
(b)
Re νB(−)(ν,t)
(c)
Re νB(+)(ν,t)
(d)
mb.GeV²
E_lab

Integration Region

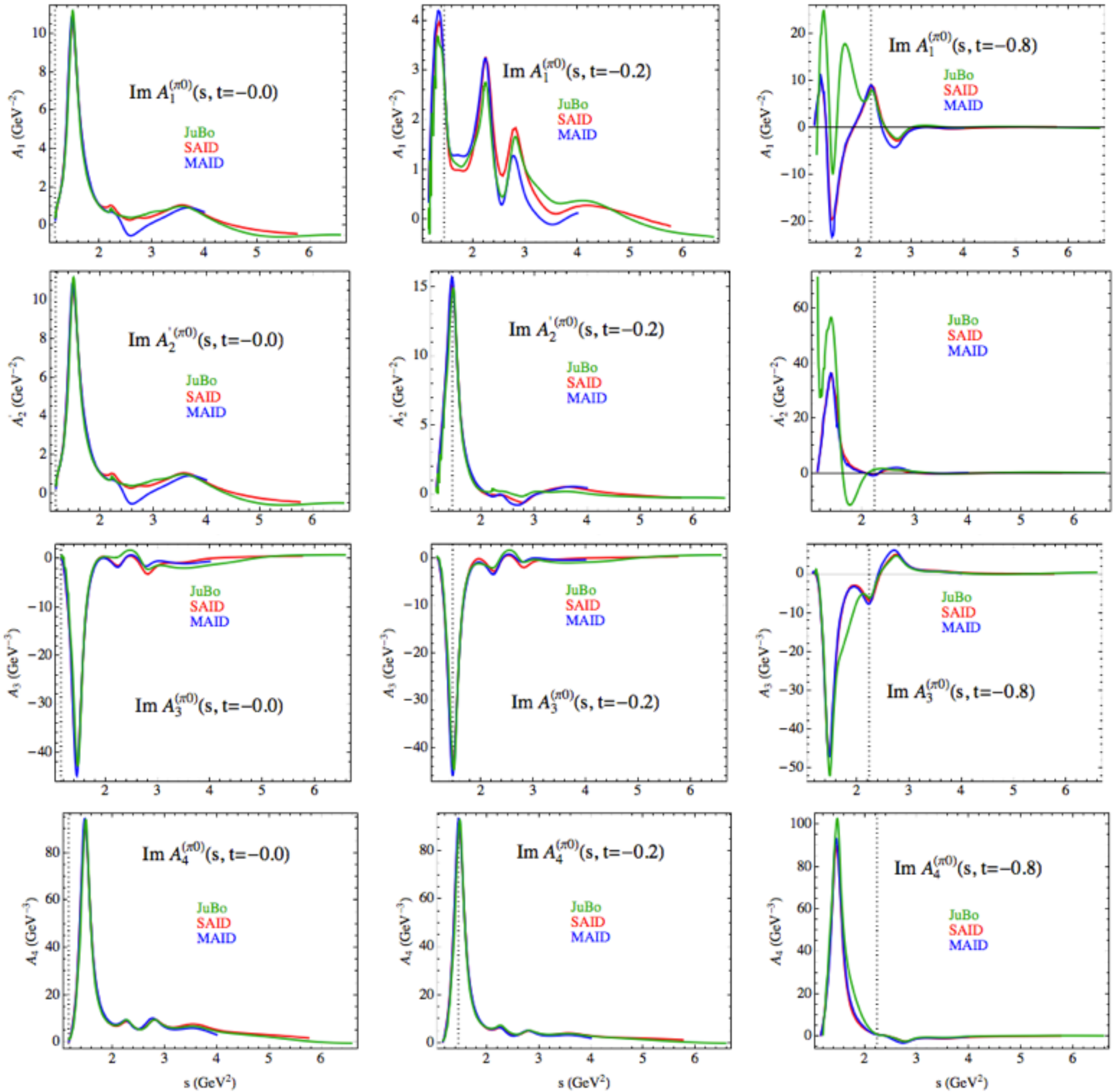




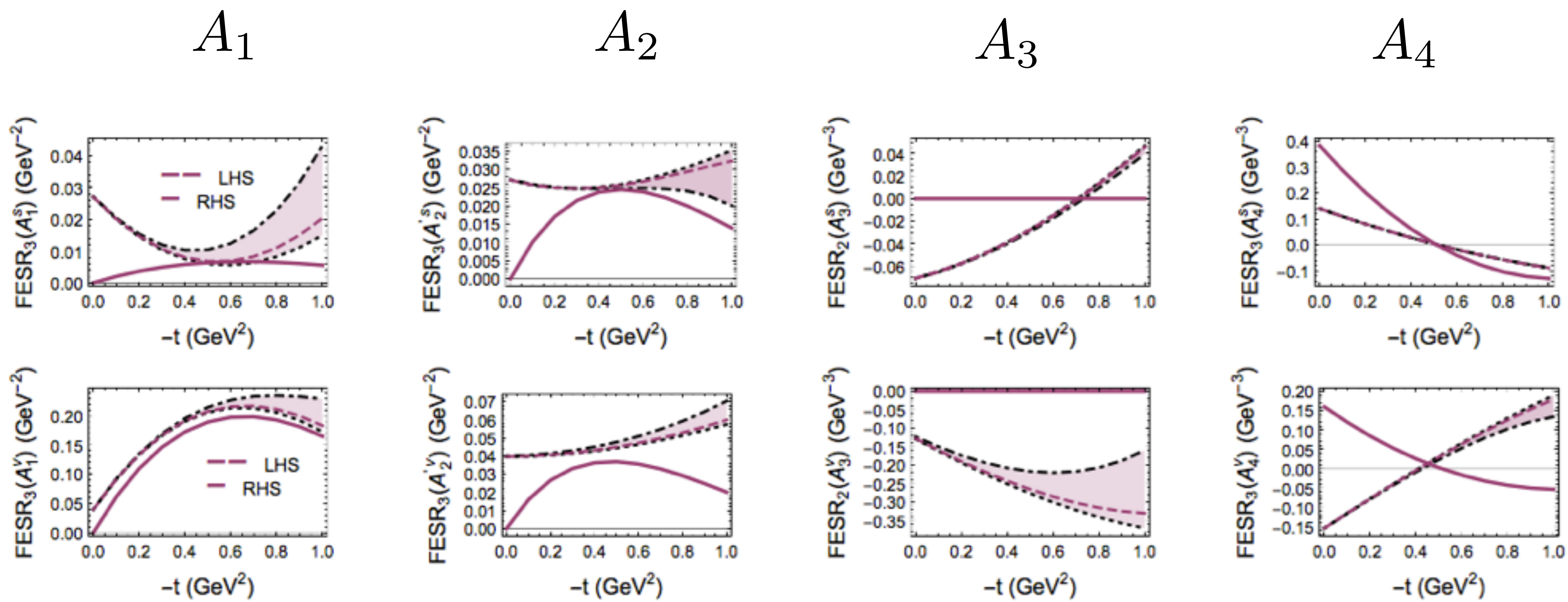
$$\gamma N \to \eta N$$



$$\gamma N \to \pi N$$

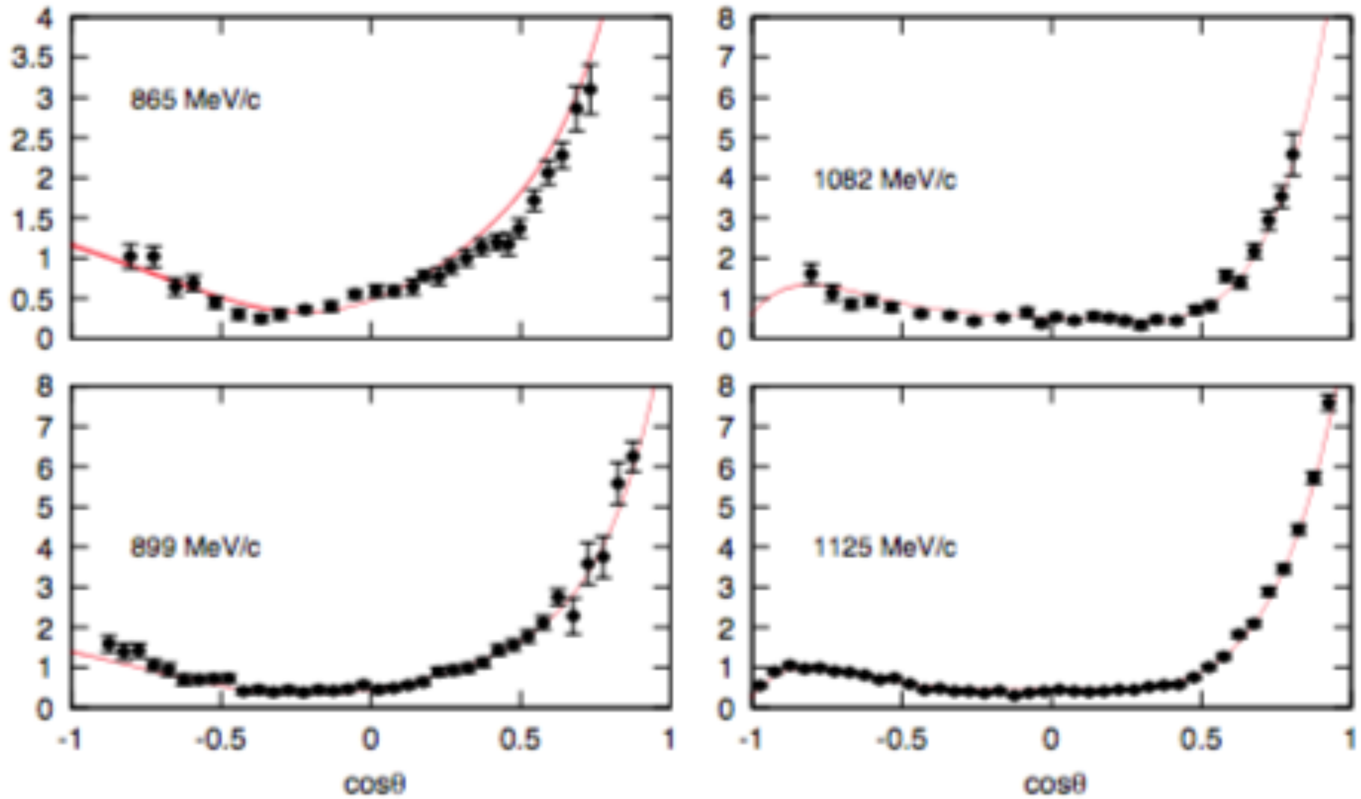

Im $A_1^{(\pi 0)}$(s, t=−0.0)
Im $A_1^{(\pi 0)}$(s, t=−0.2)
Im $A_1^{(\pi 0)}$(s, t=−0.8)
Im $A_2^{'(\pi 0)}$(s, t=−0.0)
Im $A_2^{'(\pi 0)}$(s, t=−0.2)
Im $A_3^{(\pi 0)}$(s, t=−0.0)
Im $A_3^{(\pi 0)}$(s, t=−0.2)
Im $A_3^{(\pi 0)}$(s, t=−0.8)
Im $A_4^{(\pi 0)}$(s, t=−0.0)
Im $A_4^{(\pi 0)}$(s, t=−0.2)
Im $A_4^{(\pi 0)}$(s, t=−0.8)
JuBo
SAID
MAID
A_1 (GeV^{-2})
$A_2^{'}$ (GeV^{-2})
A_3 (GeV^{-3})
A_4 (GeV^{-3})
s (GeV^2)

$\gamma N \to \eta N$

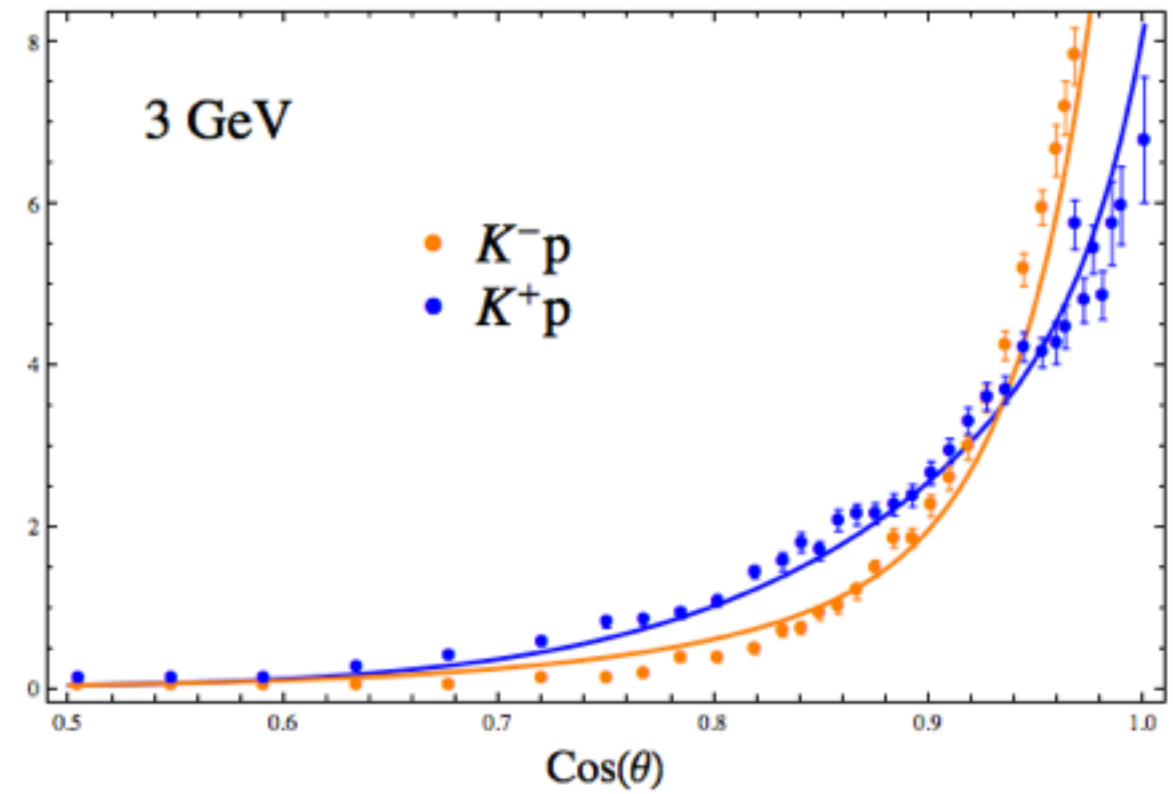


$K^- p \to K^- p$ *Energy Evolution*

C. Fernandez-Ramirez et al. (JPAC) ArXiv:1510:07065

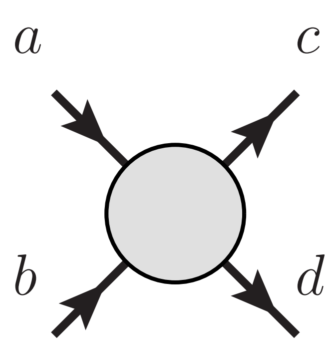


VM (unpublished)

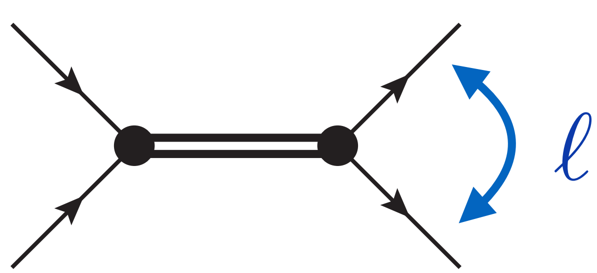


Partial wave expansion

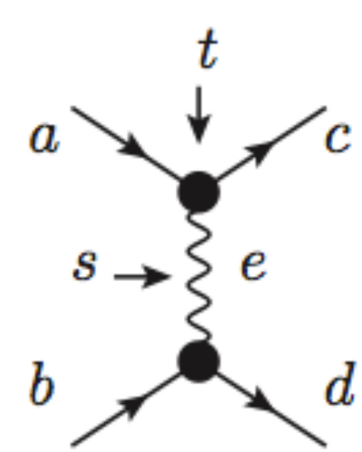
Regge pole expansion



$$= \sum_{\ell=0}^{\cancel{\infty}\ L_{max}}$$

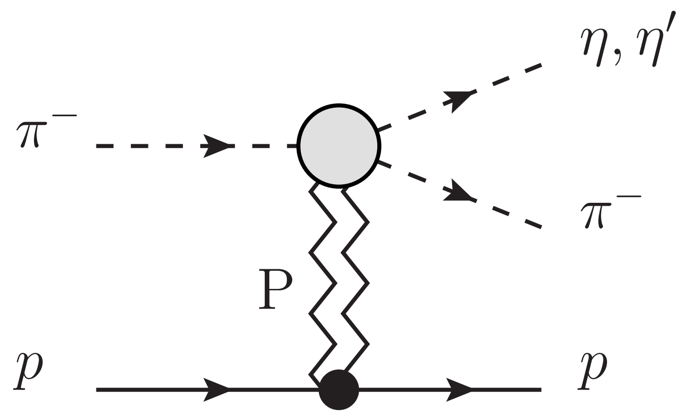


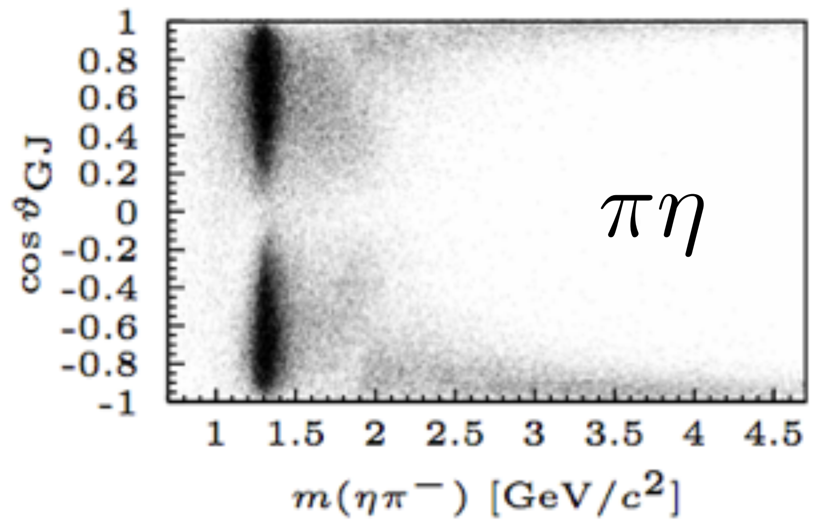
$$\ell \quad =$$

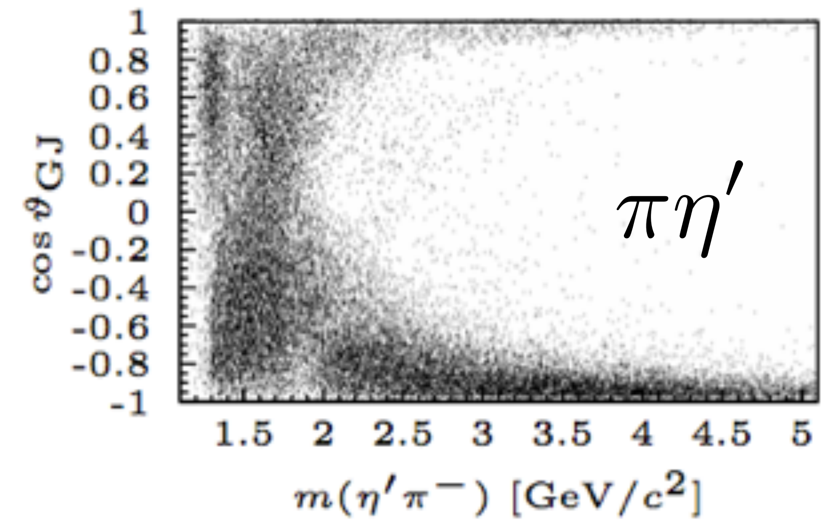


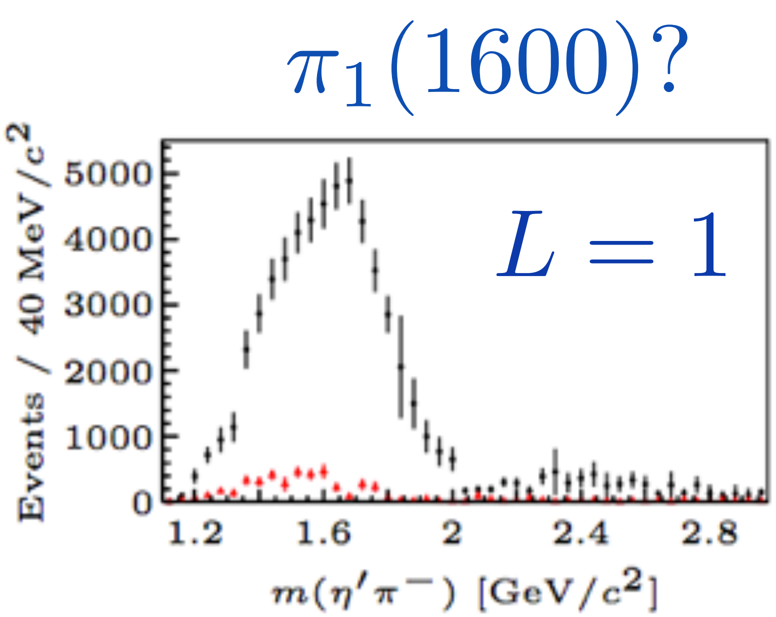
$$+\cancel{\mathcal{O}\left(\frac{1}{\sqrt{s}}\right)}$$

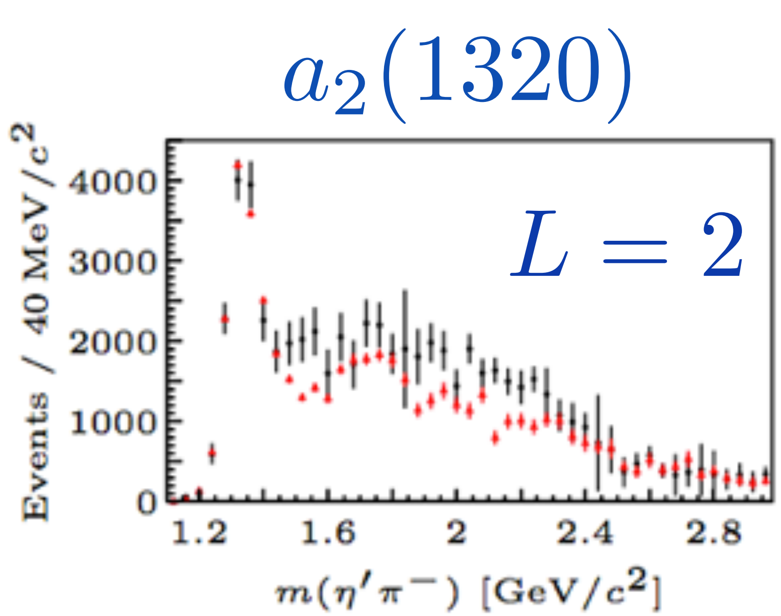
Discovering (?) New Resonances: Eta(')-Pi @COMPASS

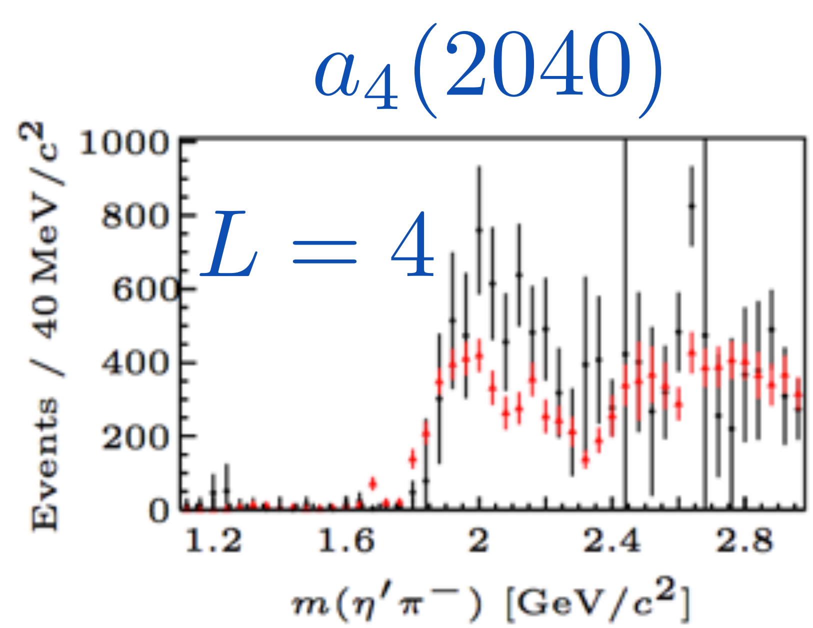


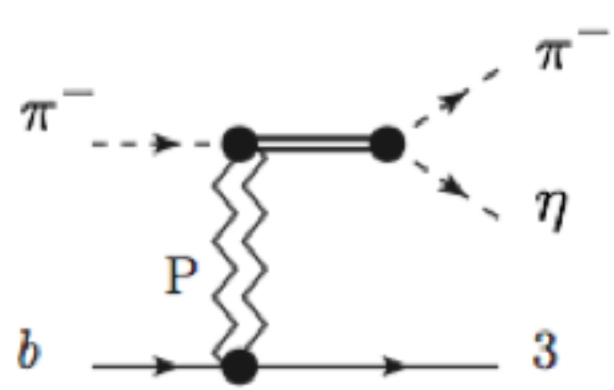












black: $\pi\eta'$

red: $\pi\eta$ **(scaled)**

Resonance in angular mom. L = 1 ?

COMPASS ArXiv:1408.4286 PLB740 303

Interactive webpage: http://www.indiana.edu/~jpac/index.html

$\pi N \to \pi N$	VM et al (JPAC)	arXiv:1506.01764	PRD92 7 074004
$\gamma p \to \pi^0 p$	VM et al	arXiv:1505.02321	PRD92 7 074013
$\eta \to \pi^+\pi^-\pi^0$	P. Guo et al (JPAC)	arXiv:1505.01715	PRD92 5 054016
$\omega, \phi \to \pi^+\pi^-\pi^0$ $\to \gamma^*\pi^0$	I. Danilkin et al (JPAC)	arXiv:1409.7708	PRD91 9 094029
$\gamma p \to K^+K^-p$	M. Shi et al (JPAC)	arXiv:1411.6237	PRD91 3 034007
$KN \to KN$	C. Fernandez-Ramirez et al (JPAC)	arXiv:1510.07065	