Impact parameter analysis in e + N → e + pi+ N

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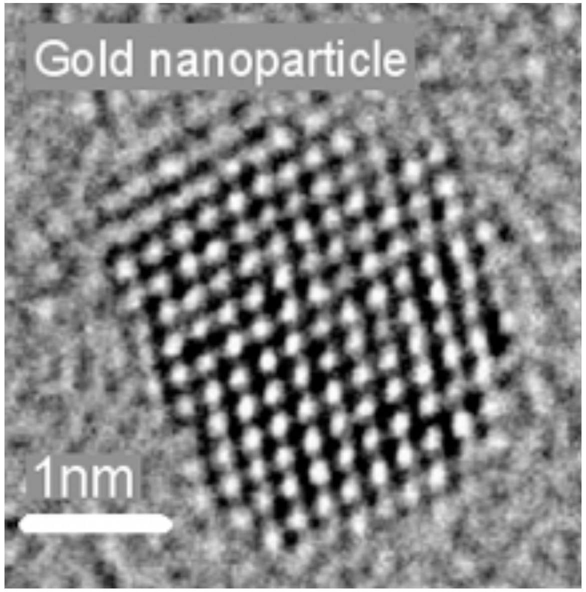
INT Workshop November 14 - 18, 2016

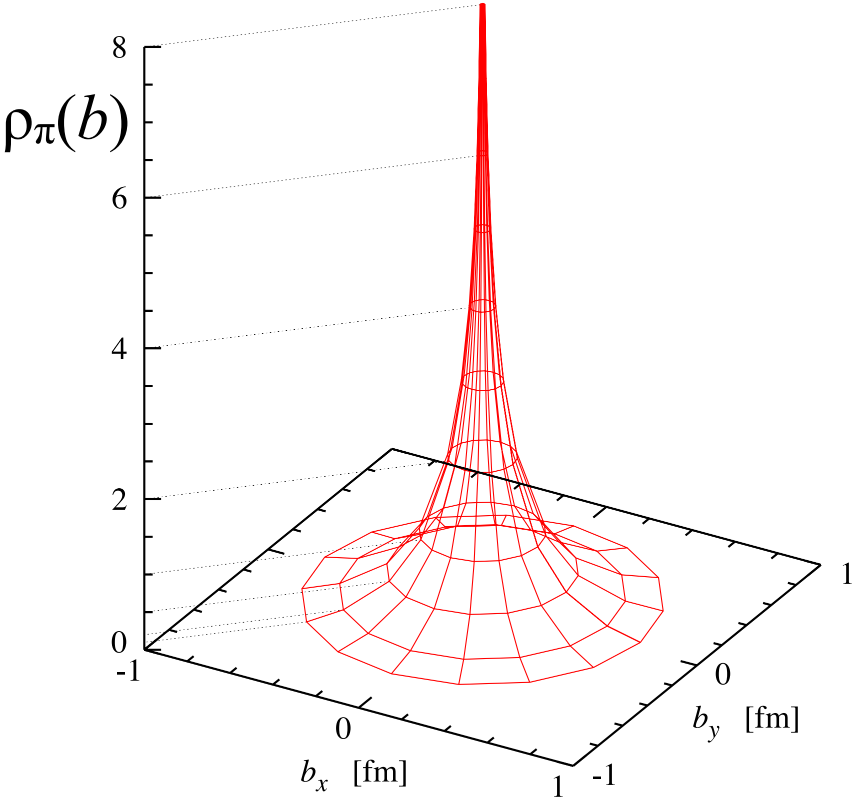
Relativistic electron microscopy of hadron dynamics

Gold atoms: 3D



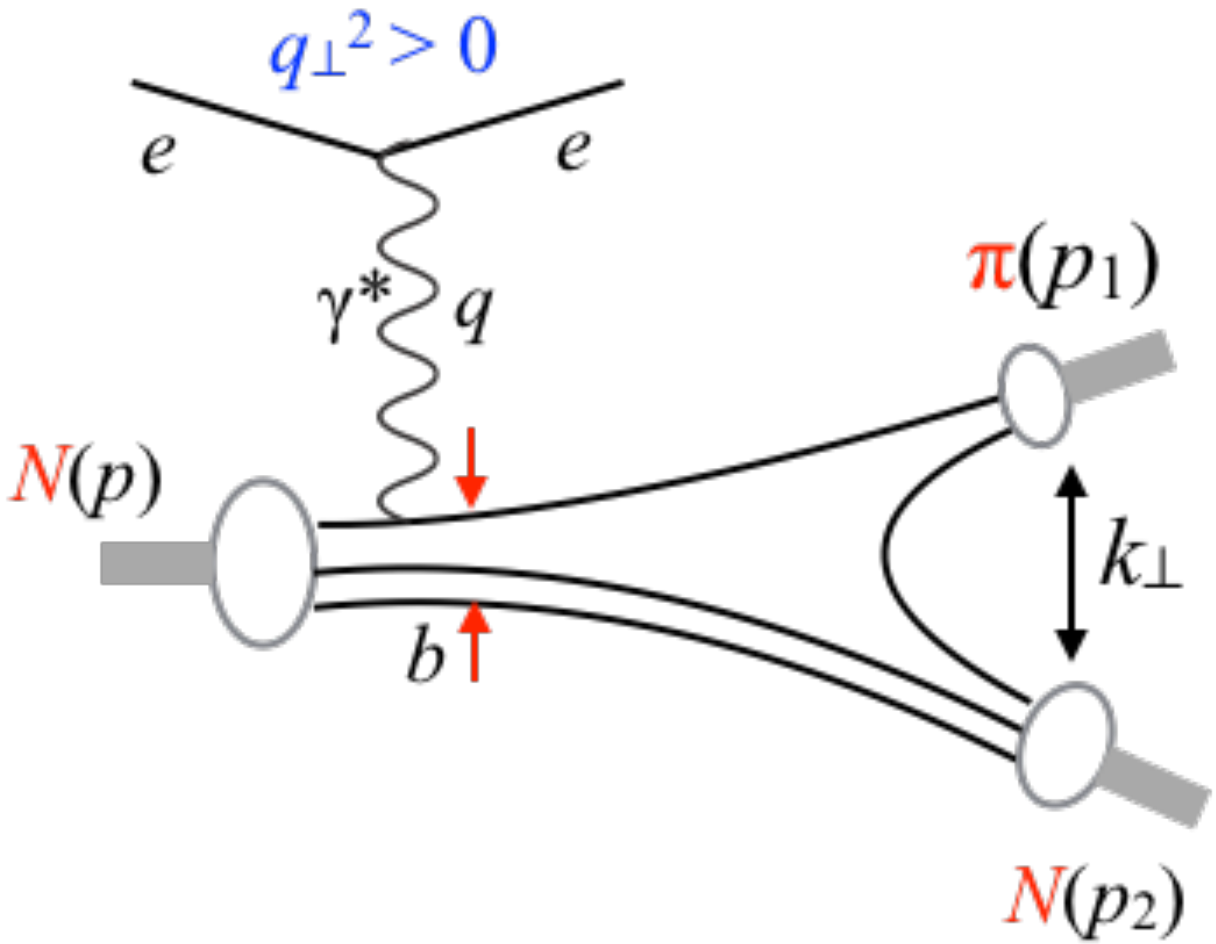
www.york.ac.uk/nanocentre/facilities/fetem/

The pion: 2D



Miller, Strikman, Weiss, arXiv 1011.1472

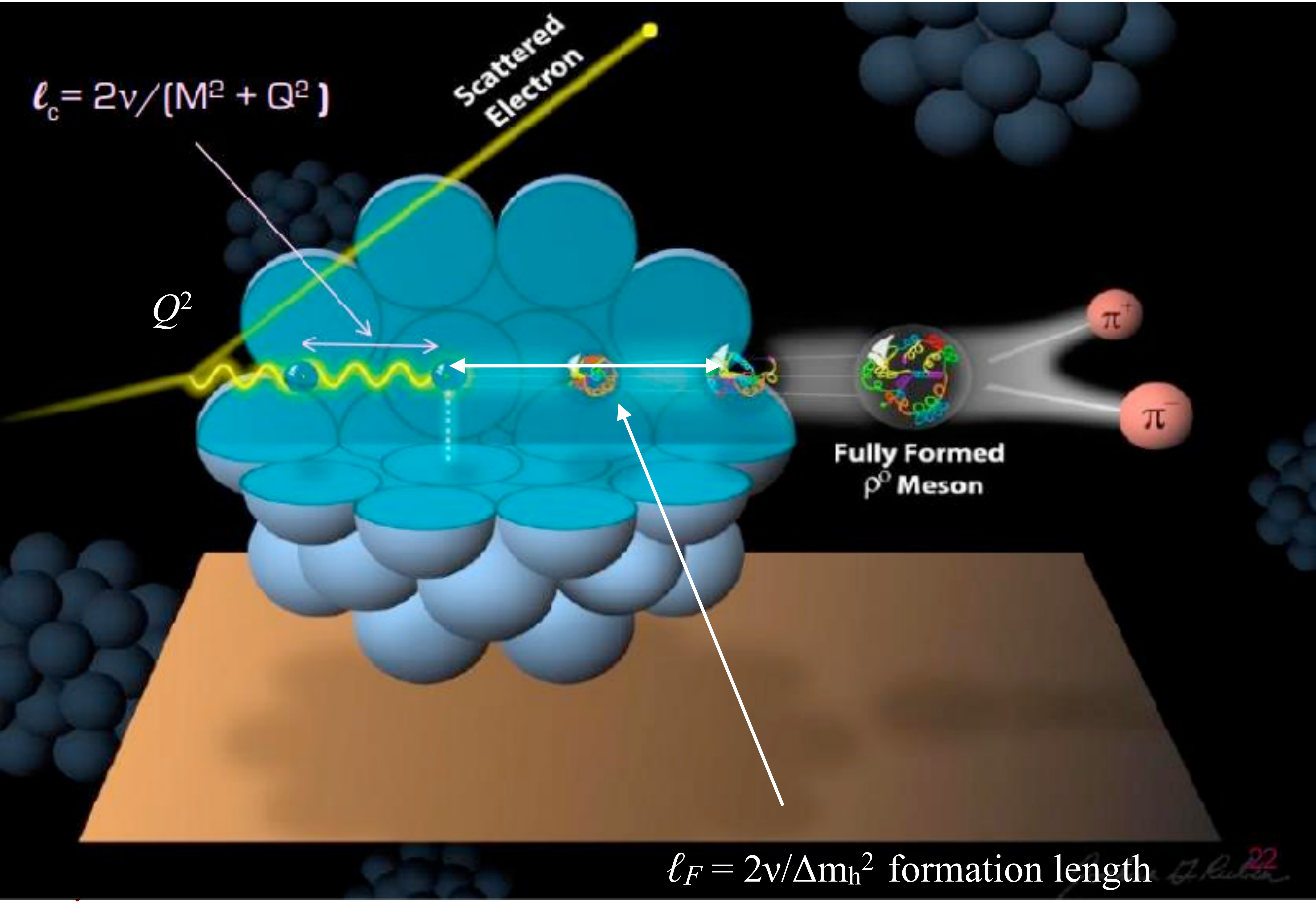
Process dependence of transverse size



How does the target charge distribution depend on the properties of the final state?

PH and S. Kurki
arXiv:1101.4810

Compare: Color Transparency in eA → e ρ X



Electron microscopy of relativistic charges

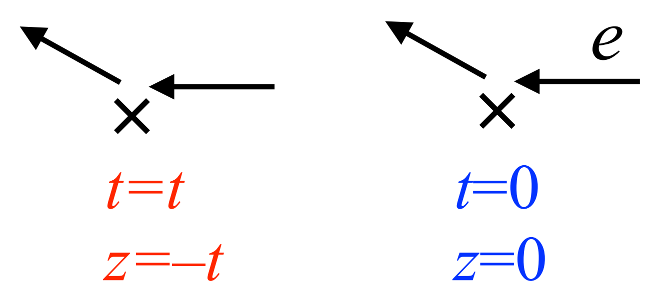
Light quarks in hadrons move with v $\approx c = 1$: How can we get a sharp picture using probes moving with the same speed?

In a frame where $p_h^z \to +\infty$ (IMF or LF), the quark energy $E_q = x\, E_h \to \infty$, hence

$$v_{q\perp} = \frac{p_{q\perp}}{x E_h} \to 0$$

The electron can resolve the transverse positions of the quarks with arbitrary accuracy in hard collisions $(Q \to \infty)$

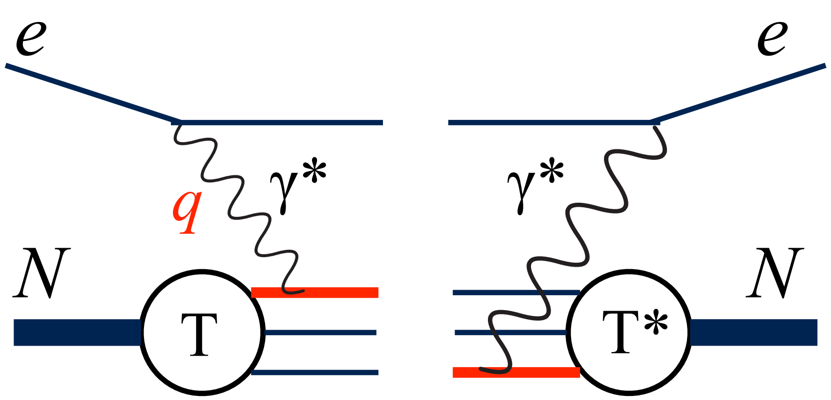
In $eh \to eX$ where the electron $p_e^z \to -\infty$ it scatters from all target quarks at equal Light-Front (LF) time $x^+ = t+z$



The transverse structure of h can be measured at equal Light-Front time x^+

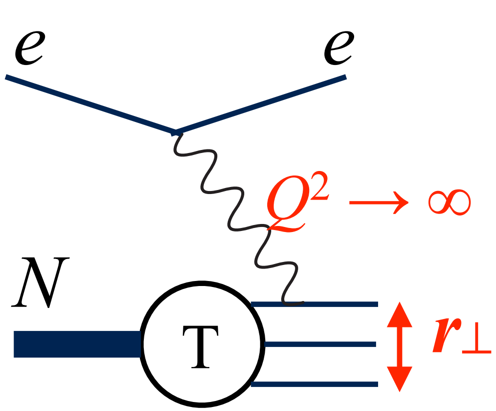
DIS: large Q^2 resolves single quarks

At low Q^2 the γ^* may interact with different quarks in T and T^*:



Such contributions do not reflect the properties of a single quark.

Bj limit: $q^0 = \nu \to \infty$ and $Q^2 \to \infty$, with $x_{Bj} = \dfrac{Q^2}{2m_h\nu}$ fixed



At large Q^2 the γ^* is coherent on a single quark

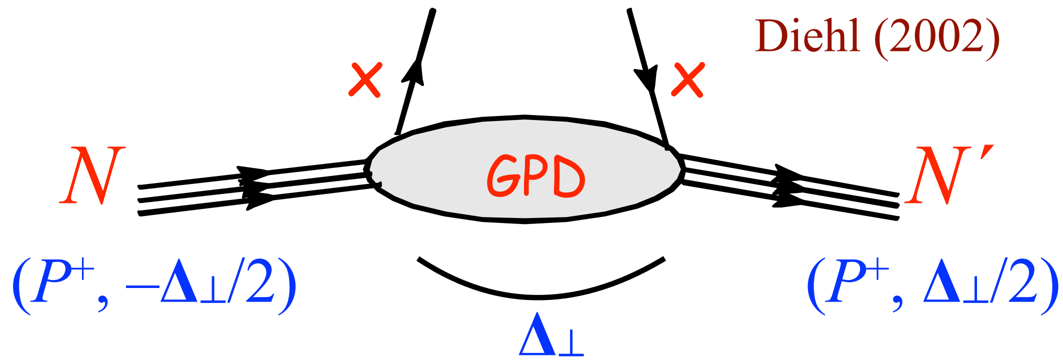
Verified by scaling in Q^2 (up to log's)

The quark can be at any $r_\perp$ in the target.

Impact parameter distributions via the GPD's

Soper (1977)
Burkardt (2000)
Diehl (2002)

$$\int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{b}\cdot\boldsymbol{\Delta}_\perp}\, \mathrm{GPD}(\boldsymbol{\Delta}_\perp)$$



determines the transverse position $\boldsymbol{b}$ of the struck quark

$$f_{q/N}(x,\boldsymbol{b}) = \sum_{n,\lambda_i,k}\prod_{i=1}^{n}\left[\int dx_i \int 4\pi d^2\boldsymbol{b}_i\right]\delta\Big(1-\sum_i x_i\Big)\frac{1}{4\pi}\delta^2\Big(\sum_i x_i\boldsymbol{b}_i\Big)$$

$$\times \quad \delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_k)\delta(x-x_k)\,|\psi_n^\lambda(x_i,\boldsymbol{b}_i,\lambda_i)|^2$$

"Center of momentum" at the origin

LF wave function

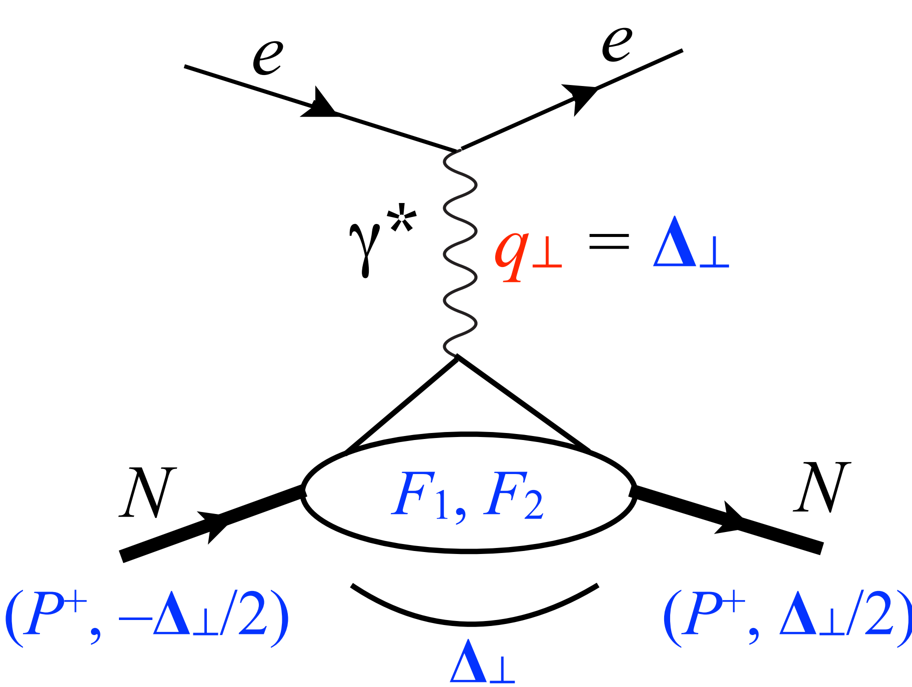
Note: * $\boldsymbol{b}$ is conjugate to the (finite) nucleon momentum transfer $\boldsymbol{\Delta}_\perp$, *not* to the momentum transfer ($Q \to \infty$) in the hard collision $eq \to eq$

* The 2-dimensional FT must be done in a frame where $\Delta^+ = 0$

Nucleon Form Factors

In $eN \to eN$ the electron and nucleon momentum transfers are the same:

$$q_\perp = \mathbf{\Delta}_\perp$$



The γ* couples to a single quark in the form factor (amplitude, not σ !)

A 2-dim. FT over $\boldsymbol{q}_\perp$ will give the distribution of the struck quark in $\boldsymbol{b}$

Note: * The $\gamma^*(\boldsymbol{q}_\perp)$ scatters coherently over quarks within $\Delta\boldsymbol{b} \sim 1/\boldsymbol{q}_\perp$, and thus measures charge with this resolution.

* A FT over $-\infty < \boldsymbol{q}_\perp < \infty$ gives the $\boldsymbol{b}$-distribution with δ-function accuracy.

Nucleon Charge Distribution from eN → eN

$$\rho_0(\boldsymbol{b}) = \frac{1}{2p^+}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\, \langle p^+, \tfrac{1}{2}\boldsymbol{q}, \lambda |\, J^+(0)\, | p^+, -\tfrac{1}{2}\boldsymbol{q}, \lambda \rangle$$

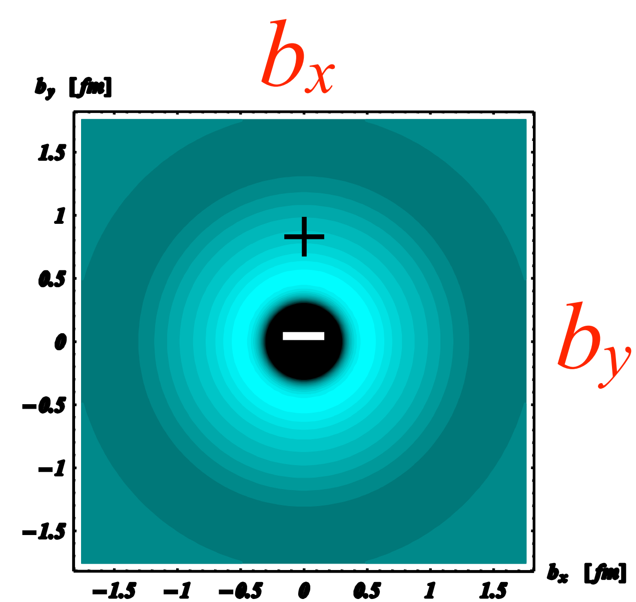
$$= \int_0^\infty \frac{dQ}{2\pi}\, Q\, J_0(b\,Q) F_1(Q^2)$$

$$= \sum_{n,\lambda_i,k} e_k \Big[\prod_{i=1}^{n} \int dx_i \int 4\pi d^2\boldsymbol{b}_i\Big]\, \delta(1-\sum_i x_i)\, \frac{1}{4\pi}\delta^{(2)}(\sum_i x_i \boldsymbol{b}_i)$$

$$\times\ \delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_k)\, |\psi_n^\lambda(x_i, \boldsymbol{b}_i, \lambda_i)|^2$$

Complementary to pdf's, but no factorization, hence no universality.

Neutron charge distribution vs. ***b***



Miller (2007)
Carlson and Vanderhaeghen (2008)

Beyond elastic form factors

The expression of $\rho_0(\boldsymbol{b})$ in terms of LC wave functions,

$$\rho_0(\boldsymbol{b}) = \frac{1}{2p^+}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\, \langle p^+, \tfrac{1}{2}\boldsymbol{q}, \lambda \big| J^+(0) \big| p^+, -\tfrac{1}{2}\boldsymbol{q}, \lambda\rangle$$

is based on the Fock expansion of the initial and final states:

$$|P^+, \boldsymbol{P}_\perp, \lambda\rangle_{x^+=0} = \sum_{n,\lambda_i} \prod_{i=1}^{n} \Big[\int_0^1 \frac{dx_i}{\sqrt{x_i}} \int \frac{d^2\boldsymbol{k}_i}{16\pi^3}\Big] 16\pi^3 \delta(1-\sum_i x_i)\,\delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

$$\times \psi_n(x_i, \boldsymbol{k}_i, \lambda_i)\, |n;\, x_i P^+, x_i\boldsymbol{P}_\perp + \boldsymbol{k}_i, \lambda_i\rangle_{x^+=0}$$

Any state f is defined by its LF wave functions $\psi_n^f(x_i, \boldsymbol{k}_i, \lambda_i)$

The $\boldsymbol{b}$-space analysis applies similarly for any final state f: $\langle f|\, J^+(0)\, |N\rangle$

By comparing the $\boldsymbol{b}$-distributions for various states f one learns about the reaction dynamics.

Beyond elastic form factors

The $\boldsymbol{b}$-distribution of the struck quark in $eN \to e\,f$ is given by:

$$\mathcal{A}_{fN}(\boldsymbol{b}) = \frac{1}{4\pi}\sum_n \Big[\prod_{i=1}^{n}\int_0^1 dx_i \int 4\pi d^2\boldsymbol{b}_i\Big]\delta(1-\sum_i x_i)\delta^2(\sum_i x_i\boldsymbol{b}_i)$$

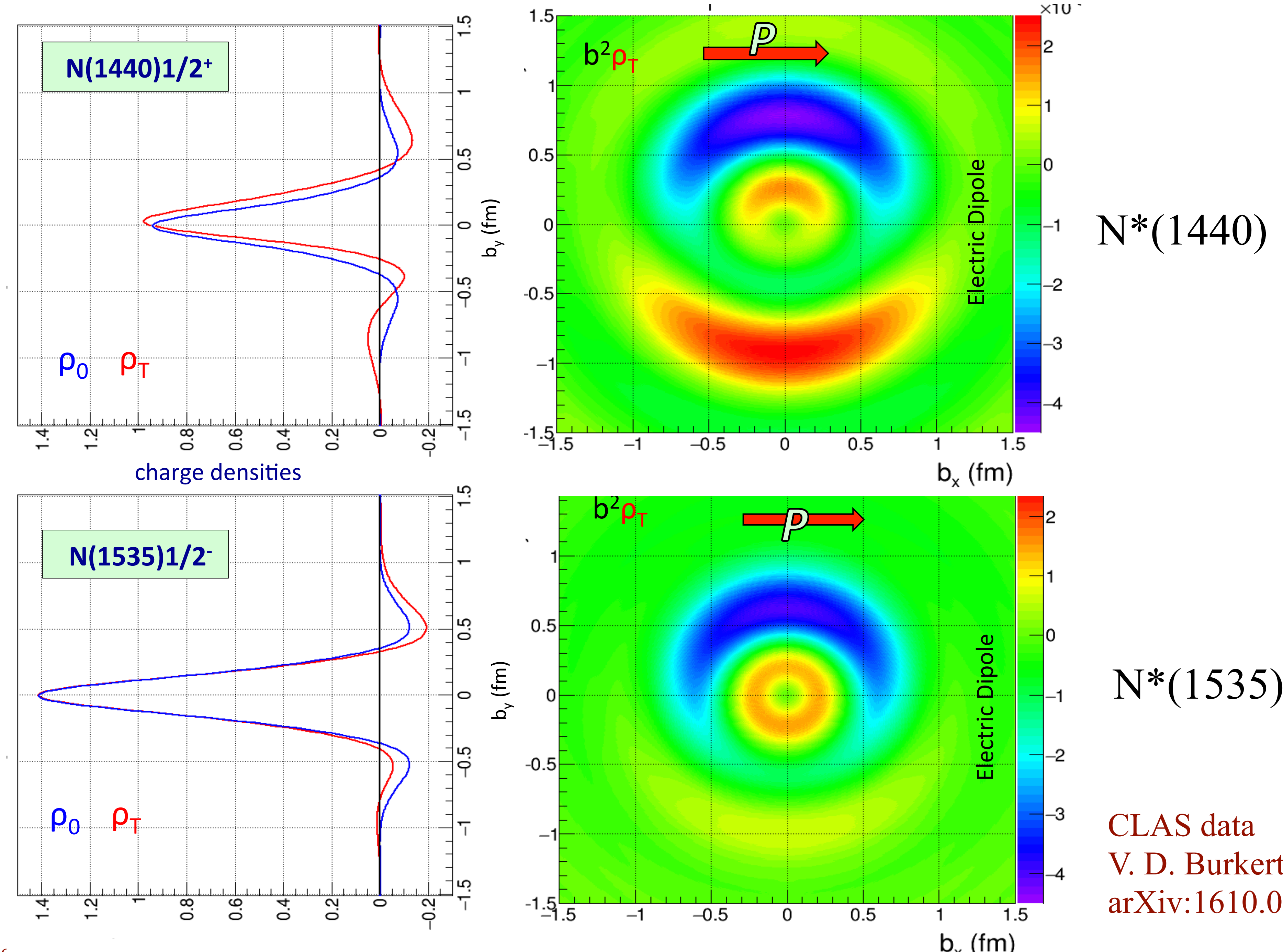
$$\times\ \psi_n^{f\,*}(x_i,\boldsymbol{b}_i)\psi_n^{N}(x_i,\boldsymbol{b}_i)\sum_k e_k\delta^2(\boldsymbol{b}_k-\boldsymbol{b})$$

The expression is diagonal in the Fock states n, and $\propto\ \psi_n^{f\,*}\psi_n^{N}$

The γ^* both causes the transition $N \to f$
and measures the contributing Fock states

$f = N^*$:   Transition form factors: $eN \to eN^*$

Comparison of N* transition form factors



CLAS data
V. D. Burkert
arXiv:1610.00400

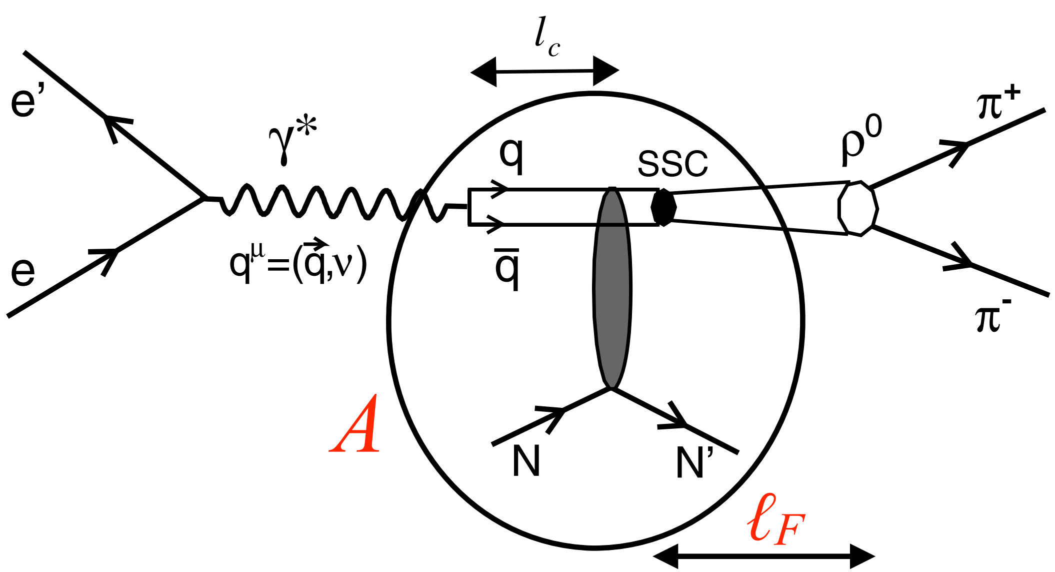
Connection to Color Transparency

Brodsky, and Mueller (1988)

Hard processes are expected to involve transversally compact Fock states
Measure their size via rescattering in nuclei

Example: $e\,A \to e\,\rho\,X \to e\,\pi\,\pi\,X$

CLAS Collaboration
arXiv:1201.2735



Expect the transverse size of the $q\bar{q}$ created by the $\gamma^*(Q^2)$ to decrease as Q^2 grows.

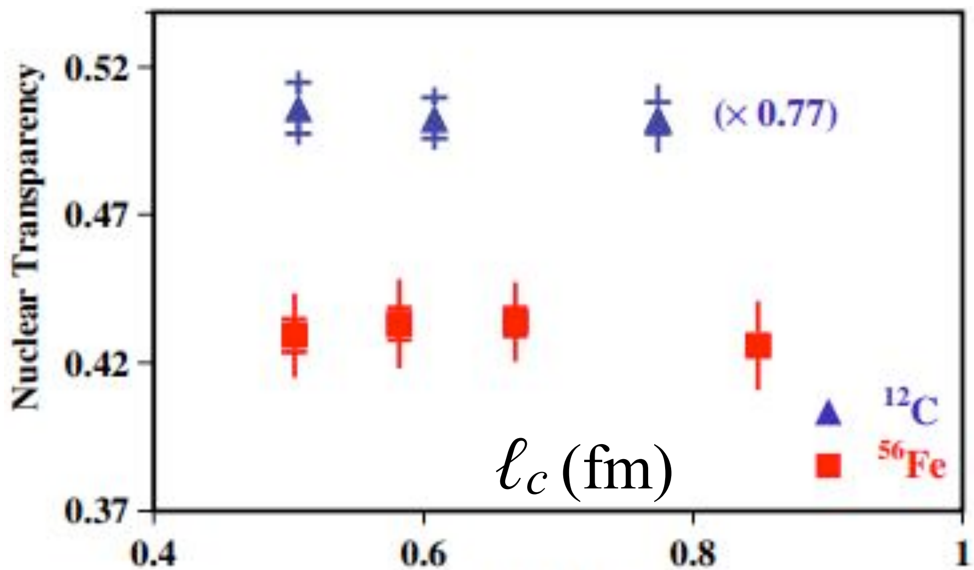
Test by measuring the absorption of the the $q\bar{q}$ in the target nucleus A.

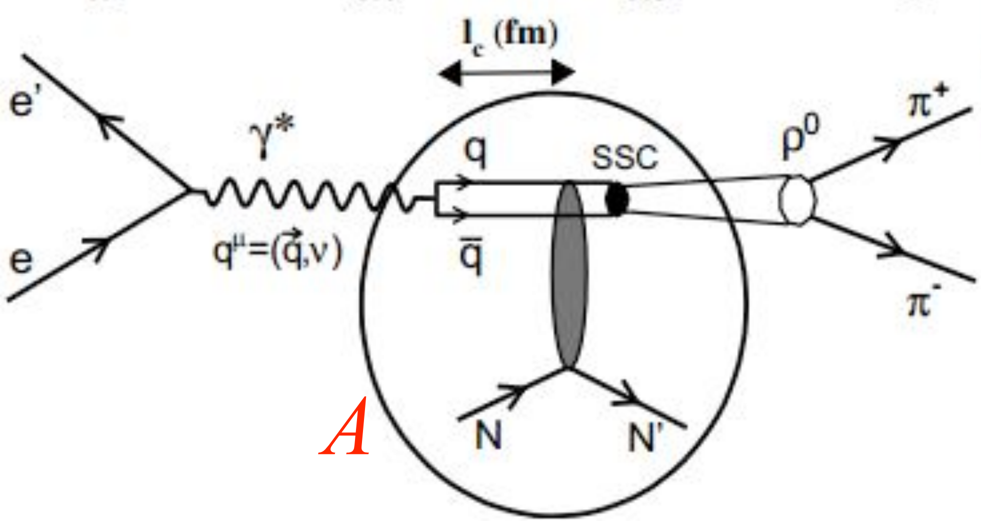
Choose kinematics:

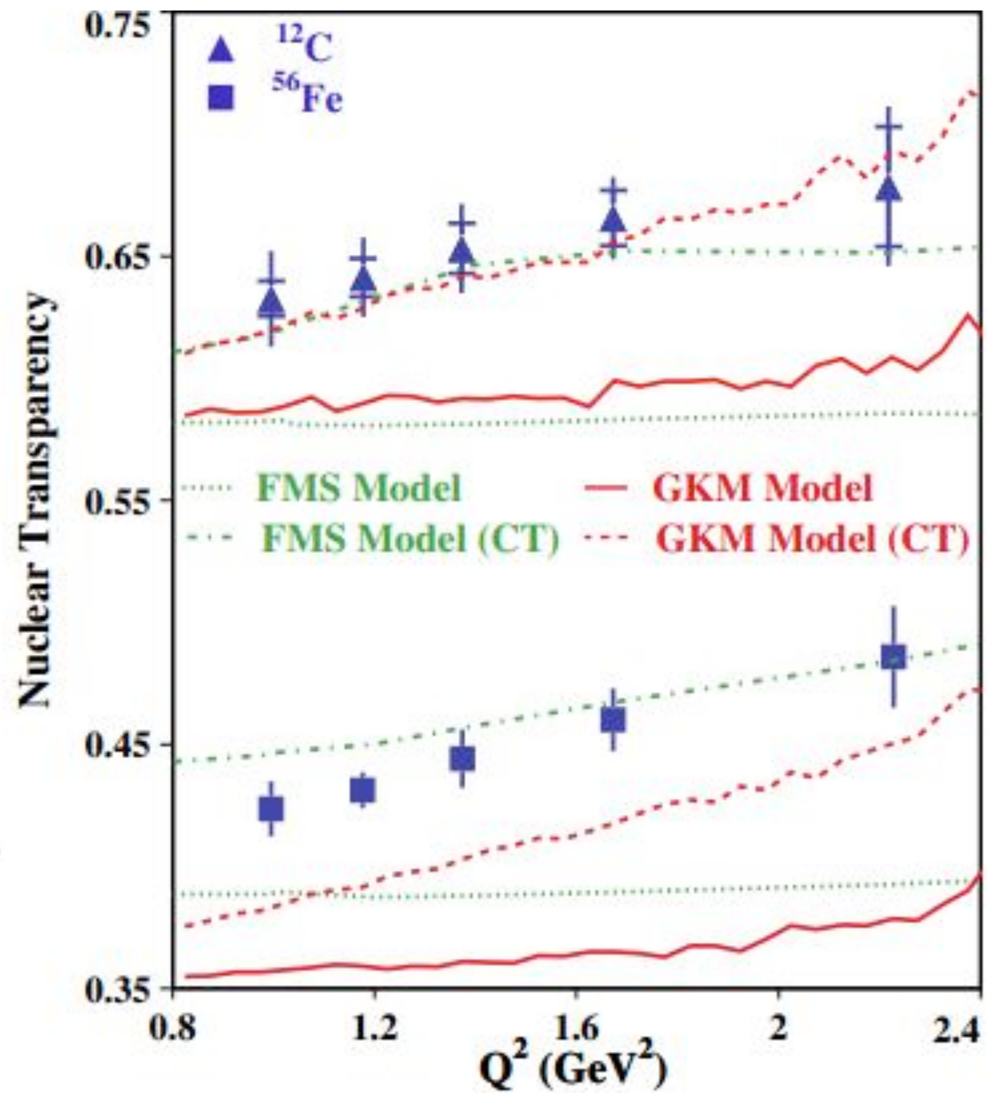
– The γ^* coherence length ℓ_c is short (< 1 fm)

– The ρ formation length ℓ_F is long (> 1fm)

Evidence for color Transparency







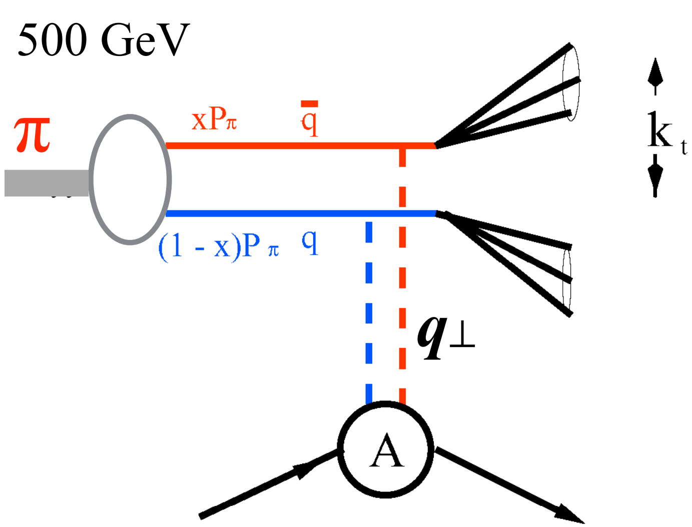
CLAS Collaboration
arXiv:1201.2735

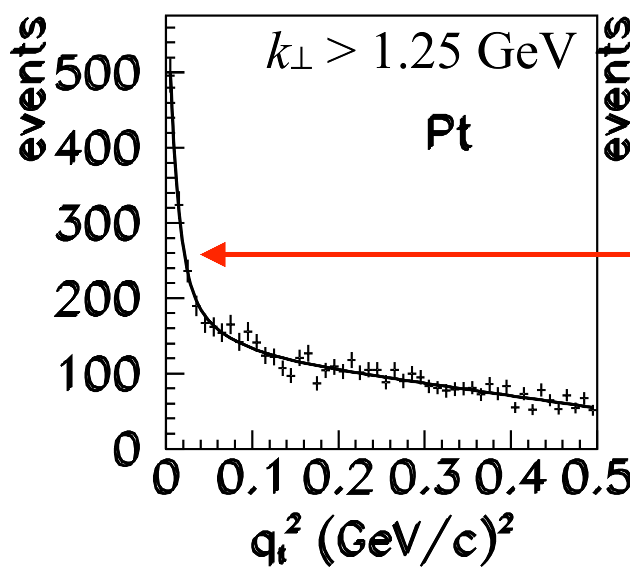
Hard scale from the final state

In the CLAS experiment, the virtuality Q^2 of the γ^* gave the hard scale, and $\sigma(A)$ reflected the size of the $q\bar{q}$ which (later) formed into the ρ.

We may also consider $e\,A \rightarrow e\,\pi\,\pi\,X$ at low Q^2, where the relative $\boldsymbol{k}_\perp$ of the pions provides the hard scale.

This is analogous to the E791 measurement of "exclusive" dijet production $\pi\,A \rightarrow$ jet jet A : The relative $\boldsymbol{k}_\perp$ of the jets provides the hard scale.





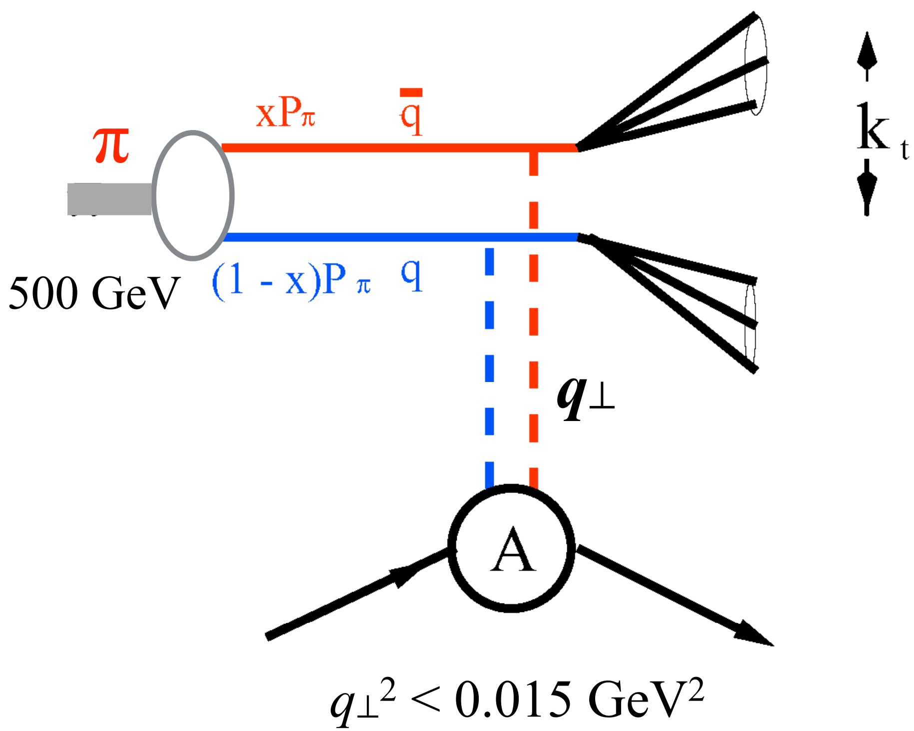
At low $\boldsymbol{q}_\perp$ the scattering is coherent on the nuclear target A =*Pt*.

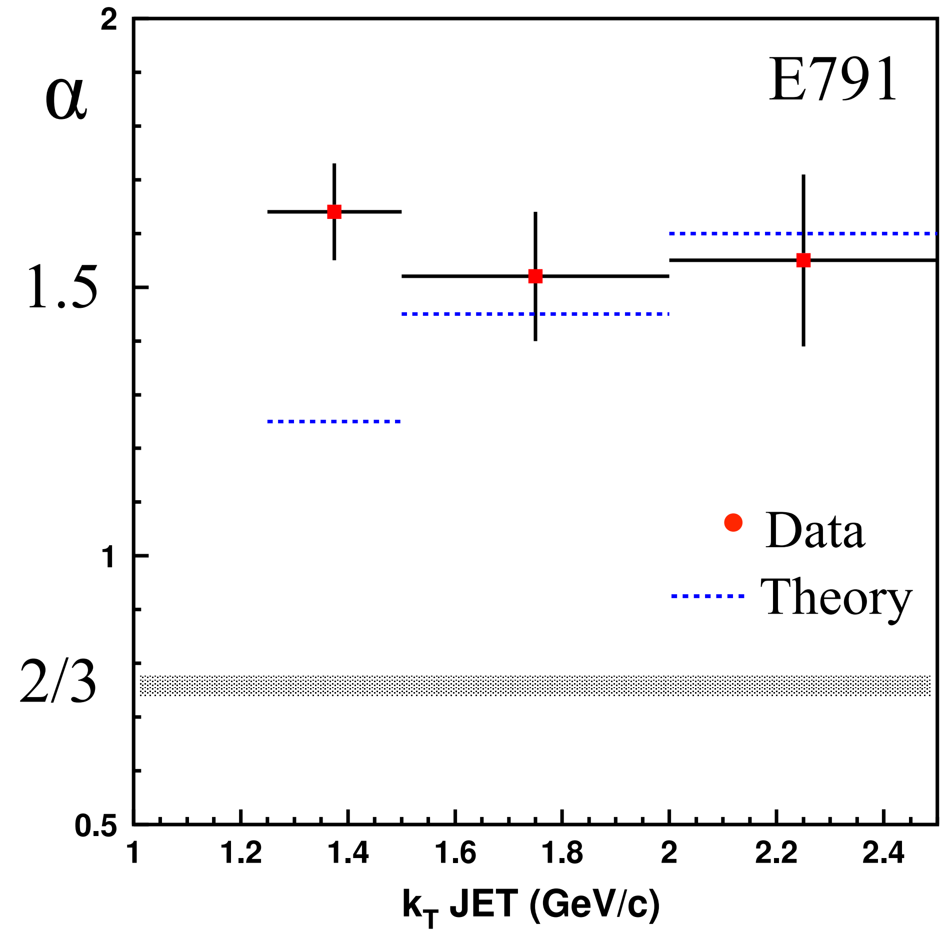
E791 Collaboration
hep-ex/0010044

Use $\sigma(A)$ to measure the size of the $q\bar{q}$ which creates the jets.

$\sigma(\pi A \to \text{jet jet } A) \propto A^{\alpha}$

E791 Collaboration
hep-ex/0010044

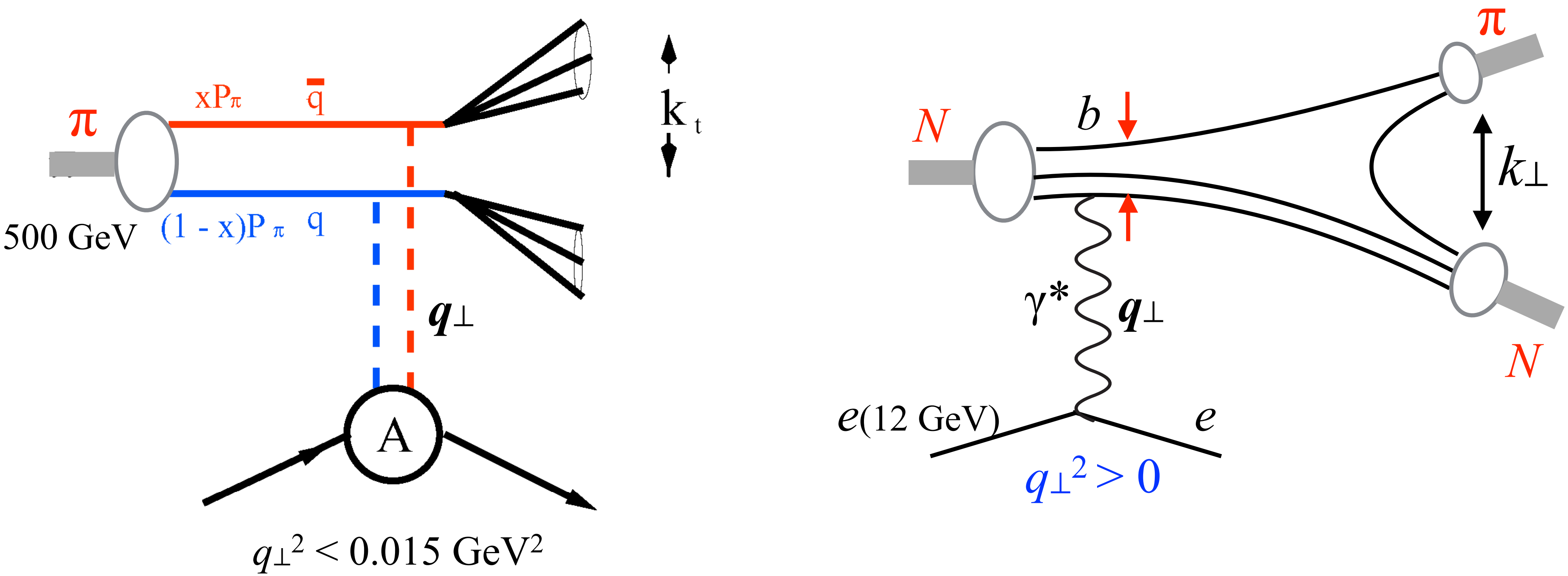




$\alpha \simeq 1.5$ indicates that the nucleus is transparent to the compact $q\bar{q}$ Fock states of the pion, selected by $k_\perp > 1.25$ GeV.

$\sigma(e\,N \to e\,\pi\,N)$

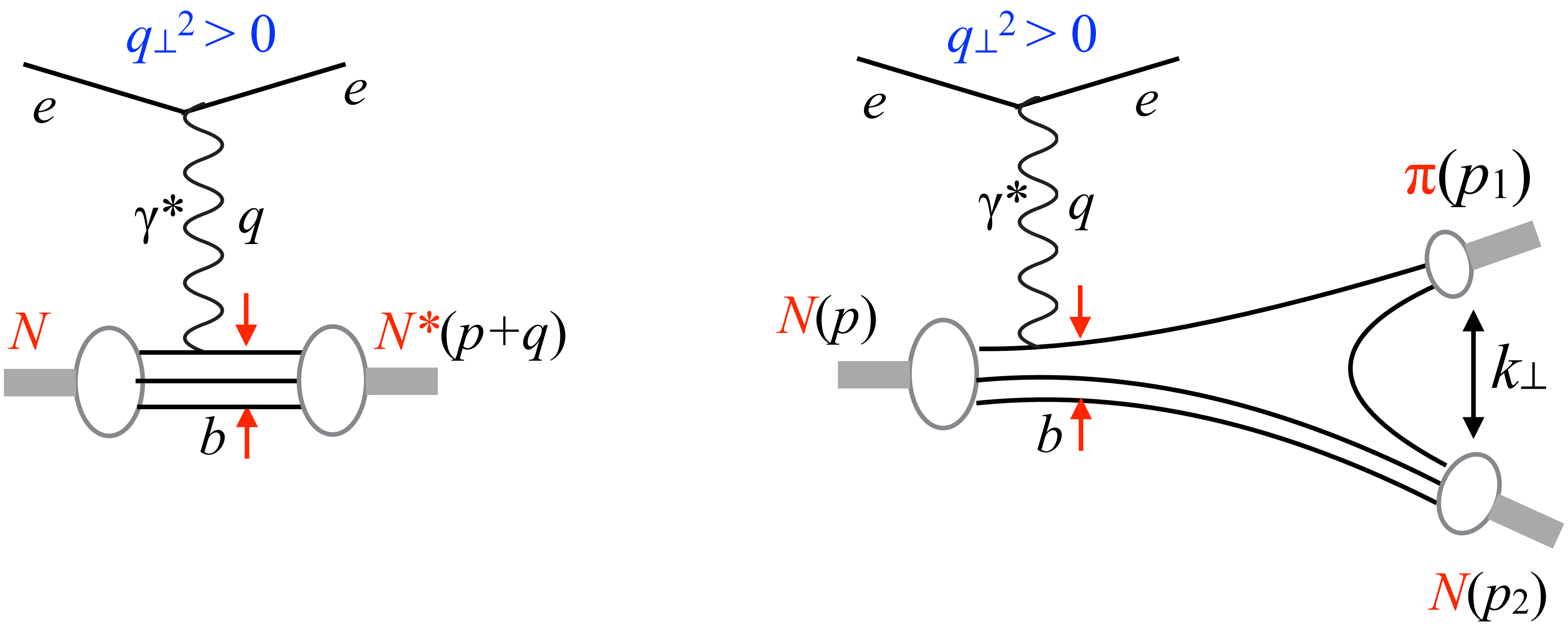
Replace A → *e*: Let the photon measure the transverse size, $b(k_\perp)$



Note: * The photon measures the size of the nucleon Fock states at $x^+ = 0$
The asymptotic πN state emerges from these Fock states as $x^+ \to \infty$

* There is no issue of coherence or formation lengths.

Comparison with the transition form factor



- $p_1 + p_2 = p + q$ is not sufficient to fix $p_1(q)$ and $p_2(q)$ separately

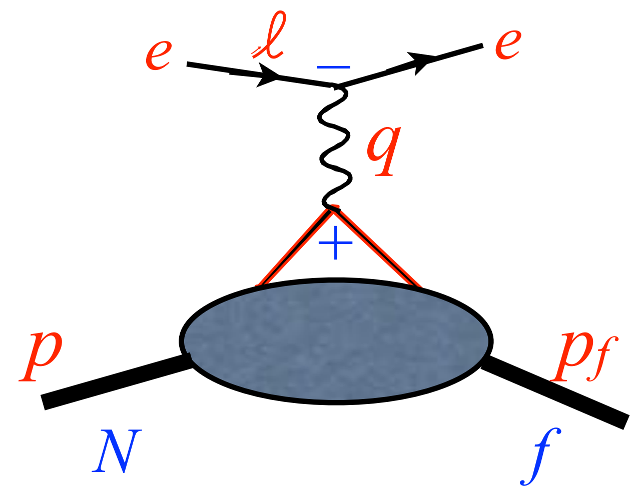
- The N and N^* wave functions are independent of LF time x^+
  The π N state develops from the Fock states measured at $x^+ = 0$

- The $eN \rightarrow eN^*$ amplitude is real, whereas $eN \rightarrow e\,\pi\,N$ has dynamical phases due to the πN interactions in the final state.

FT of γ* matrix element in momentum space

In the frame:

$$p = (p^+, p^-, -\tfrac{1}{2}\boldsymbol{q})$$
$$q = (0^+, q^-, \boldsymbol{q})$$
$$p_f = (p^+, p^- + q^-, \tfrac{1}{2}\boldsymbol{q})$$



we have

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle = \mathcal{A}_{fN}(\boldsymbol{b})$$

$$\mathcal{A}_{fN}(\boldsymbol{b}) = \frac{1}{4\pi}\sum_n \Big[\prod_{i=1}^n \int_0^1 dx_i \int 4\pi d^2\boldsymbol{b}_i\Big]\delta(1-\sum_i x_i)\delta^2(\sum_i x_i\boldsymbol{b}_i)$$

$$\psi_n^{f\,*}(x_i,\boldsymbol{b}_i)\psi_n^N(x_i,\boldsymbol{b}_i)\sum_k e_k\delta^2(\boldsymbol{b}_k-\boldsymbol{b})$$

Example: f = π (p_1) N(p_2)

In order to conform with the Lorentz covariance of LF states, at any p_f :

$$|\pi N(p_f^+, \boldsymbol{p}_f; \Psi^f)\rangle \equiv \int_0^1 \frac{dx}{\sqrt{x(1-x)}} \int \frac{d^2\boldsymbol{k}}{16\pi^3} \Psi^f(x, \boldsymbol{k}) |\pi(p_1) N(p_2)\rangle$$

where $\Psi^f(x,\boldsymbol{k})$ defines the final state in terms of the relative variables x, $\boldsymbol{k}$:

$$p_1^+ = x p_f^+ \qquad \boldsymbol{p}_1 = x\boldsymbol{p}_f + \boldsymbol{k}$$

$$p_2^+ = (1-x) p_f^+ \qquad \boldsymbol{p}_2 = (1-x)\boldsymbol{p}_f - \boldsymbol{k}$$

With x, $\boldsymbol{k}$ being independent of p_f, our choice of $\Psi^f(x,\boldsymbol{k})$ defines the pion and nucleon momenta p_1, p_2 at all photon momenta q.

QED illustration: $e + \gamma^* \to e + \mu^+ \mu^-$

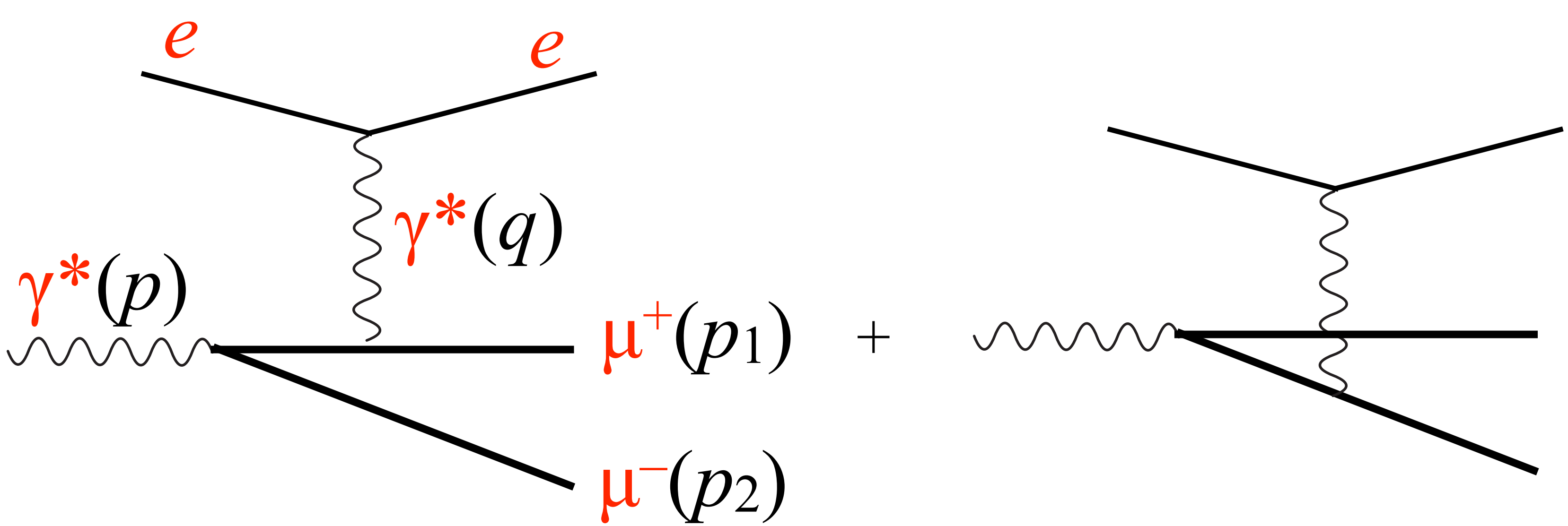
The "target" is a virtual photon $\gamma^*(p)$. The Fourier transform of

$$\mathcal{A}^{\mu\mu,\lambda}_{\lambda_1,\lambda_2} = \frac{1}{2p^+} \langle \mu^-(p_1,\lambda_1)\mu^+(p_2,\lambda_2)|\, J^+(0)\, |\gamma^*(p,\lambda)\rangle$$

gives, denoting $m = m_\mu$ and $M^2 = m^2 - x(1\text{-}x)\, p^2$,

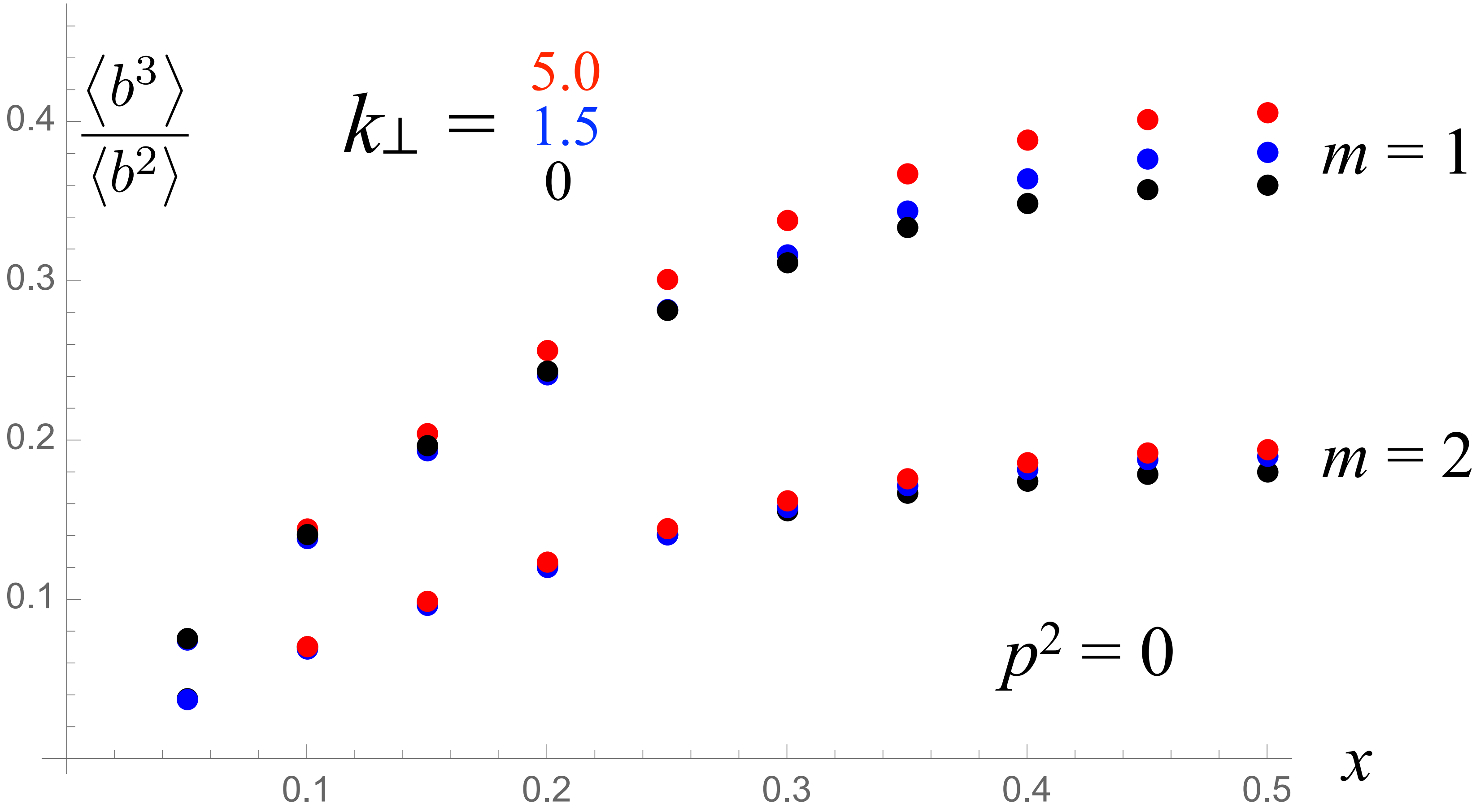
$$\mathcal{A}^{\mu\mu,+1}_{+\frac{1}{2}+\frac{1}{2}}(\boldsymbol{b}; x, \boldsymbol{k}) = \frac{em}{\sqrt{2}\pi}\sqrt{x(1-x)}\left[\frac{K_0\left(\frac{M\,b}{1-x}\right)}{(1-x)^2}\exp\left(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\right) - \frac{K_0\left(\frac{M\,b}{x}\right)}{x^2}\exp\left(+i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{x}\right)\right]$$

This agrees with the general expression in terms of the LF wave functions.



$e + \gamma^* \rightarrow e + \mu^+ \mu^-$ example (1)

Average Impact parameter vs. x: m and $k_\perp$ dependence



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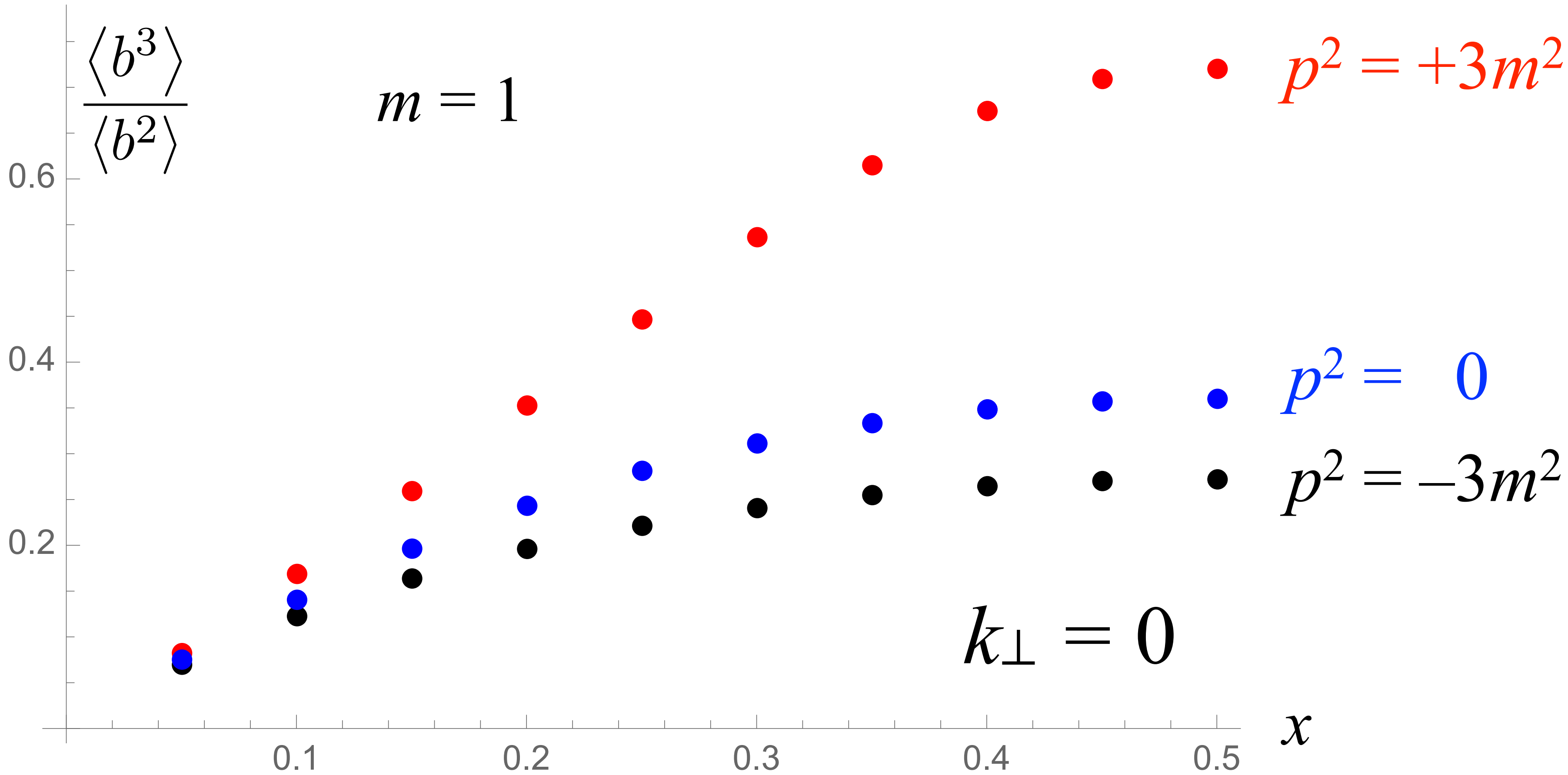
$e + \gamma^* \to e + \mu^+ \mu^-$ example (2)

Average Impact parameter vs. x: p^2 dependence



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Fourier transform of the cross section

The $\gamma^*+N \to f$ amplitudes have dynamical phases (resonances,...).

$\Rightarrow$ Calculating their Fourier transforms requires an amplitude analysis.

One can also Fourier transform the measured cross section itself.

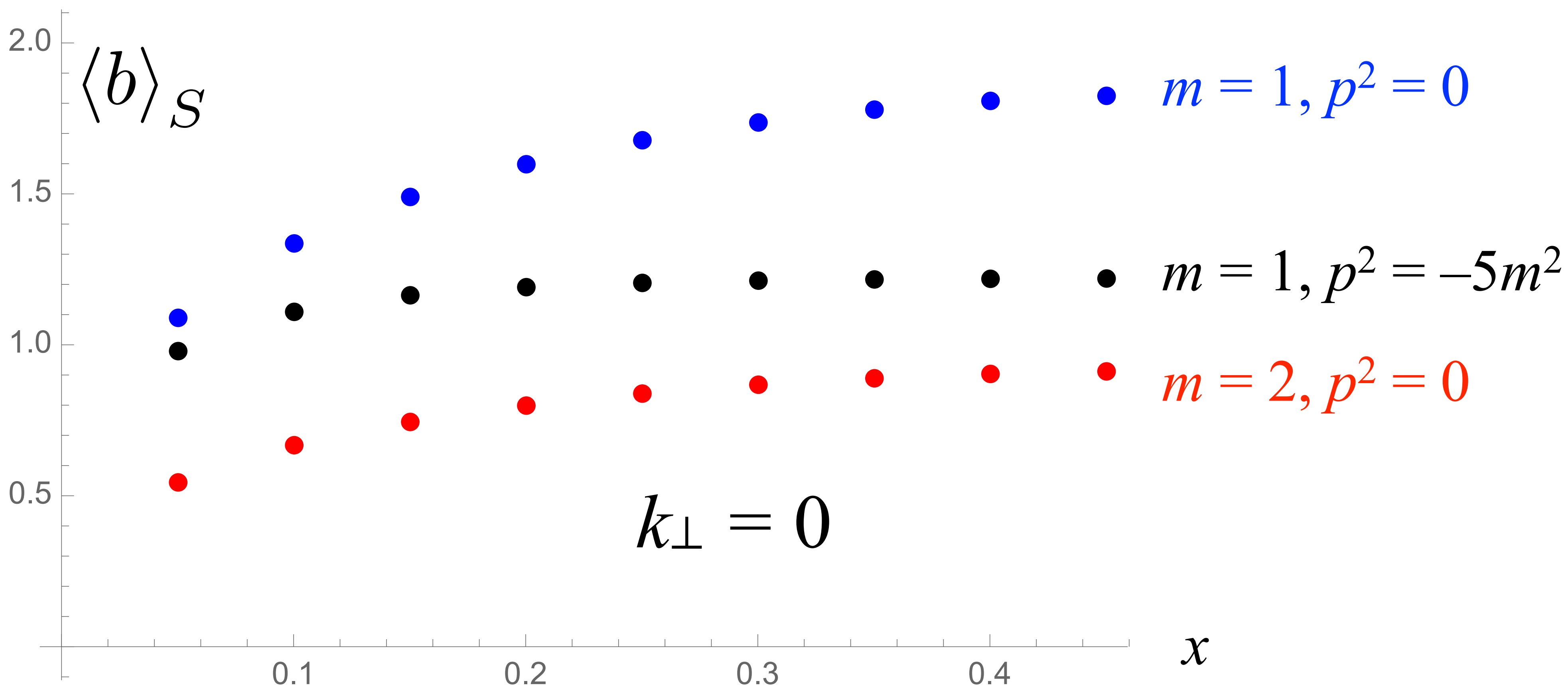
Then the $\boldsymbol{b}$-distribution reflects the difference between the impact parameters of the photon vertex in the amplitude and its complex conjugate:

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \left| \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle \right|^2 = \int d^2\boldsymbol{b}_q\, \mathcal{A}_{fN}(\boldsymbol{b}_q)\, \mathcal{A}^*_{fN}(\boldsymbol{b}_q - \boldsymbol{b})$$

A narrowing of $A_{fN}(\boldsymbol{b}_q)$ as a function of the final state f will be reflected in the convolution.

$e + \gamma^* \rightarrow e + \mu^+ \mu^-$ example (3)

Average Impact parameter from cross section



Non-flip amplitude, b^4 moment, normalized to S(b=0), to power 1/6.

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Example: $\gamma^{(*)}$ + D → p + n at 90°

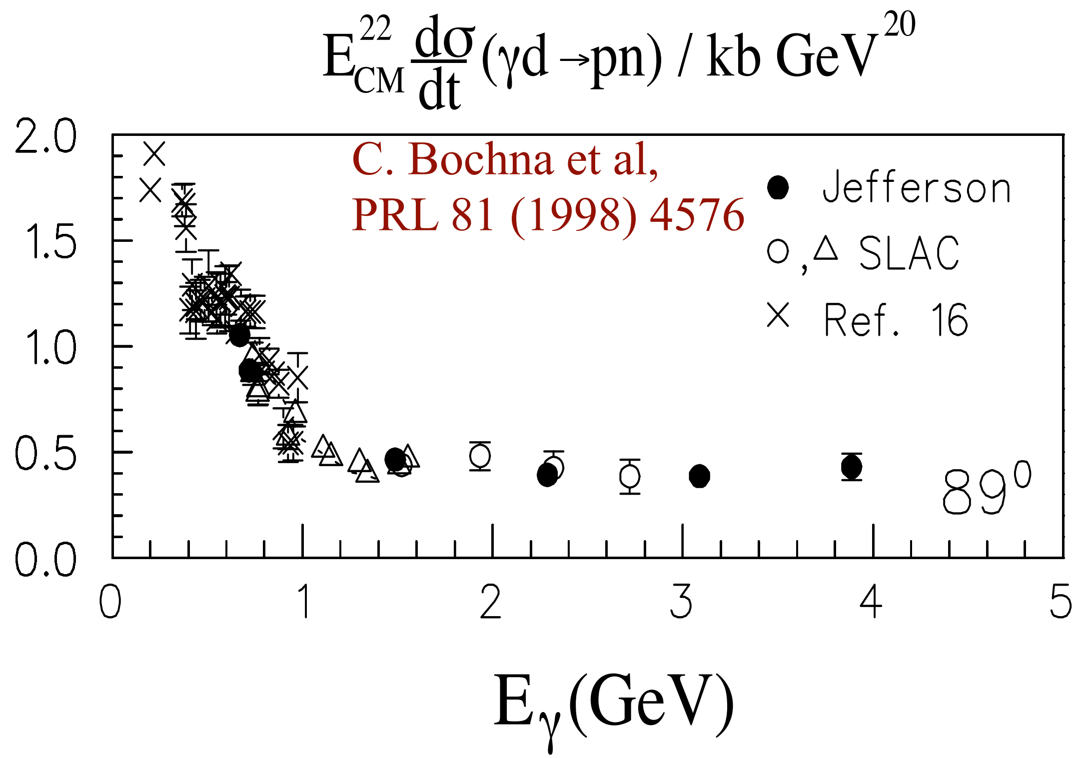
The 90° break-up cross section at $q^2=0$ agrees with dimensional scaling for $E_\gamma > 1$ GeV.

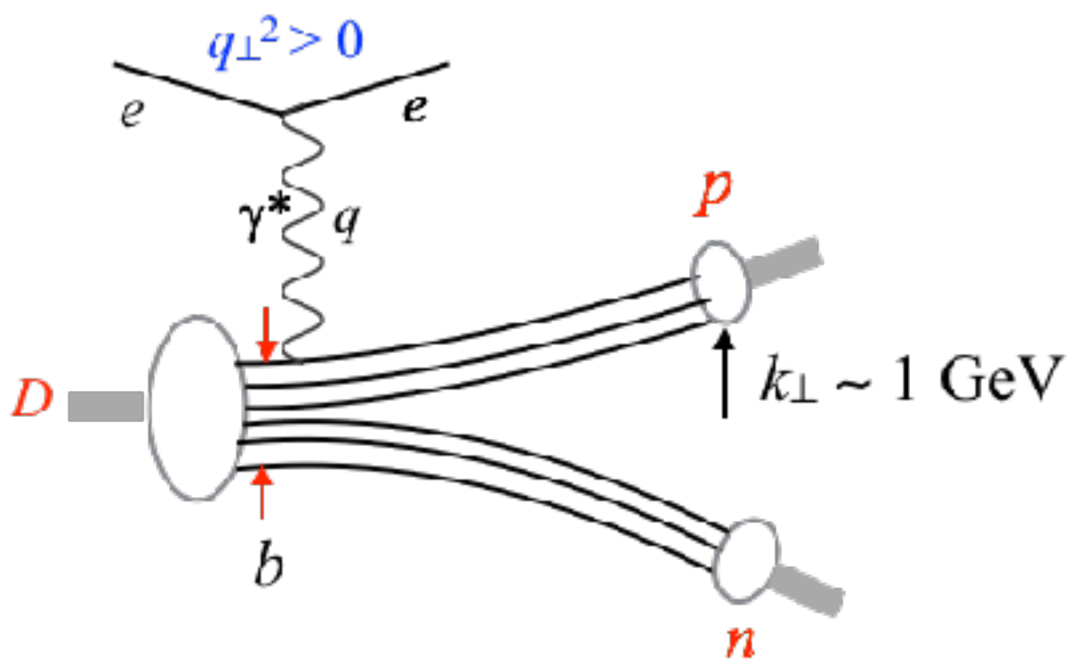
$$\sigma(\gamma D \to pn) \propto E^{-22}$$

Does this mean that only compact configurations of the deuteron, with $R < 0.2$ fm, contribute to this process?

If so, expect no q^2-dependence for $q^2 < 1\ \mathrm{GeV}^2$.

With electroproduction data R could be measured:





Many other processes within reach

Multiparticle final states

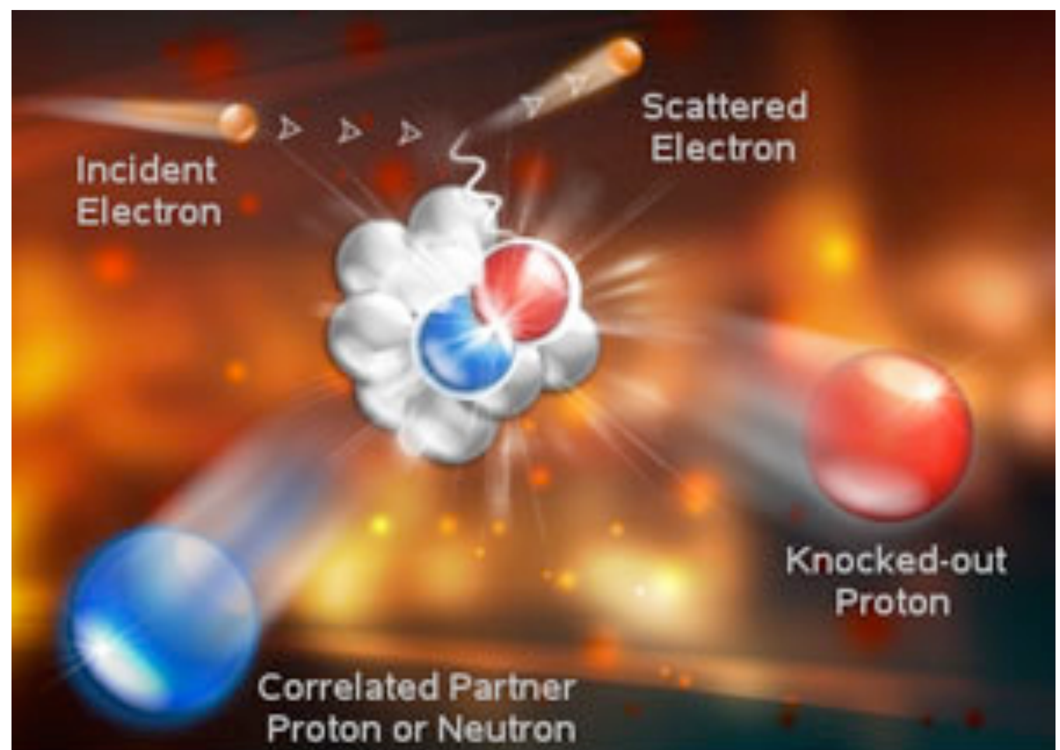
$$\gamma^* N \to \pi\pi N, \ldots$$

Heavier flavors

$$\gamma^* N \to K\Lambda, K^*\Lambda \ldots$$

$$\gamma^* N \to D\Lambda_c, \ldots$$

Nuclear targets



Summary

The $q_\perp$ -dependence of the virtual photon measures the charge distribution in transverse space.

The charge density is measured at an instant of Light-Front time $x^+ = t + z$

Unlike pdf's, no "leading twist" limit is implied: All Q^2 are useful

The density can be determined for any initial and final state: $\gamma^* A \to f$

Comparisons of ***b***-distributions in different processes give insights into the scattering dynamics in transverse space.

Model independent analysis

Ready to be tried out with Jlab data!