Spectrum of Octet & Decuplet Baryons

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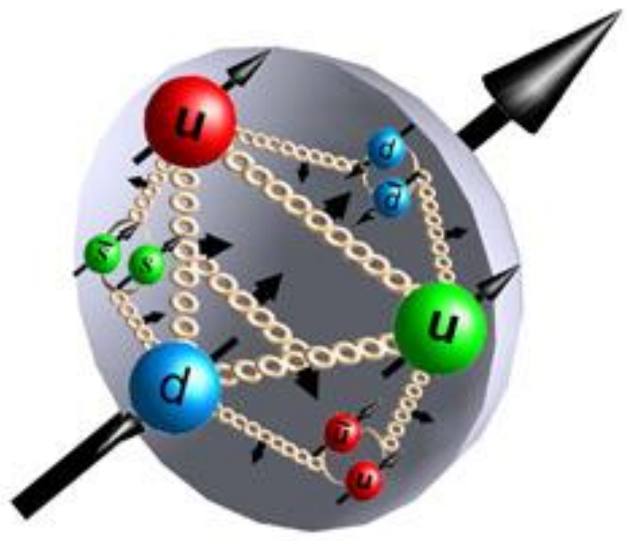


Universal Truths & DSEs

- **Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.**
- **Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.**

- **Dyson-Schwinger equations**
  - ✓ A Nonperturbative symmetry-preserving tool for the study of Continuum-QCD
  - ✓ Well suited to Relativistic Quantum Field Theory
  - ✓ A method connects observables with long-range behaviour of the running coupling
  - ✓ Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling

Two Additional Questions

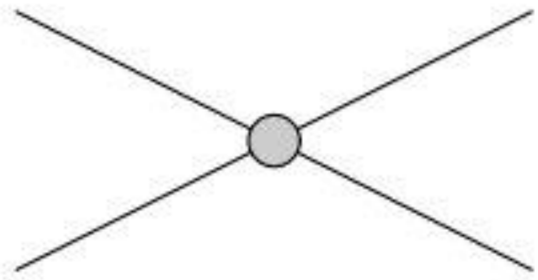
1. How hard must one work in order to compute the spectrum of low-lying baryons?
2. To which features of the interaction is the spectrum sensitive?

Baryons

Models employed

- Two models:
  - NJL-like interaction: contact model
  - QCD -kindred interaction: realistic model

Baryons



Contact Model

Vector-vector contact interaction

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu} \frac{4\pi\alpha_{\rm IR}}{m_G^2}$$

2 parameters:

m_G = 0.8 GeV & *αIR = 0.93π*

Gap Equation:

$$M = m + \frac{M}{3\pi^2 m_G^2} \int_0^\infty ds\, s\, \frac{1}{s+M^2}$$

m_u	m_s
0.007	0.17

M_0	M_u	M_s
0.36	0.37	0.53

Meson/Diquark Spectrum

	m_π	m_K	m_ρ	m_{K^*}	m_ϕ	m_σ	m_κ	m_{a_1}	m_{K_1}	m_{f_1}
n=0 DSE	0.14	0.50	0.93	1.03	1.13	1.29	1.40	1.38	1.48	1.59
expt.	0.14	0.50	0.78	0.89	1.02	1.0 - 1.2		1.23	1.34	1.42
n=1 DSE	$1.33_{\pm 0.06}$	$1.33_{\pm 0.07}$	$1.29_{\pm 0.05}$	$1.40_{\pm 0.05}$	$1.51_{\pm 0.05}$	$1.42_{\pm 0.02}$	$1.53_{\pm 0.02}$	$1.47_{\pm 0.02}$	$1.57_{\pm 0.01}$	$1.67_{\pm 0.02}$
expt.	$1.3_{\pm 0.1}$	1.46*	$1.46_{\pm 0.03}$	1.68*	$1.68_{\pm 0.02}$			$1.65_{\pm 0.02}$		

	$[u,d]_{0^+}$	$[s,u]_{0^+}$	$\{u,u\}_{1^+}$	$\{s,u\}_{1^+}$	$\{s,s\}_{1^+}$	$[u,d]_{0^-}$	$[s,u]_{0^-}$	$\{u,u\}_{1^-}$	$\{s,u\}_{1^-}$	$\{s,s\}_{1^-}$
n=0 qq	0.78	0.93	1.06	1.16	1.26	1.37	1.47	1.45	1.55	1.65
$q\bar{q}$	0.14	0.50	0.93	1.03	1.13	1.29	1.40	1.38	1.48	1.59
n=1 qq	$1.34_{\pm 0.05}$	$1.35_{\pm 0.05}$	$1.32_{\pm 0.04}$	$1.42_{\pm 0.04}$	$1.53_{\pm 0.04}$	$1.48_{\pm 0.03}$	$1.57_{\pm 0.02}$	$1.52_{\pm 0.01}$	$1.62_{\pm 0.02}$	$1.71_{\pm 0.01}$
$q\bar{q}$	$1.33_{\pm 0.06}$	$1.33_{\pm 0.07}$	$1.29_{\pm 0.05}$	$1.40_{\pm 0.05}$	$1.51_{\pm 0.05}$	$1.42_{\pm 0.02}$	$1.53_{\pm 0.02}$	$1.47_{\pm 0.02}$	$1.57_{\pm 0.01}$	$1.67_{\pm 0.02}$

Faddeev Amplitudes

- Faddeev amplitudes: $\Psi_{\alpha\beta\gamma}$
- Separable *Ansatz:*

$$\Psi_{\alpha\beta\gamma} = \sum_a \Gamma^a_{\beta\gamma} D^a \Phi^a_\alpha(p, P)$$

Φ : the quark-diquark vertex.

a = scalar, axial-vector, pesudoscalar, vector .

- Octet baryons: *I = 1/2* → *a = scalar, axial-vector.*
  (parity partners: *a = pseudoscalar, vector*)
- Decuplet baryons: *I = 3/2* → *a = axial-vector.*

Faddeev Amplitudes

- Constraints:
  - *Positive energy:*

$$\Phi\Lambda^{+} = \Phi$$

  - *Positive/negative parity:*

$$\hat{\mathbf{P}}\Psi = \pm\Psi$$

- Independent components:
  - *Nucleon: scalar – 2; axial-vector – 6.*

$$\mathcal{S}(\ell;P) = s_1(\ell;P)\,I_{\rm D} + \left(i\gamma\cdot\hat{\ell} - \hat{\ell}\cdot\hat{P}\,I_{\rm D}\right)\,s_2(\ell;P)$$

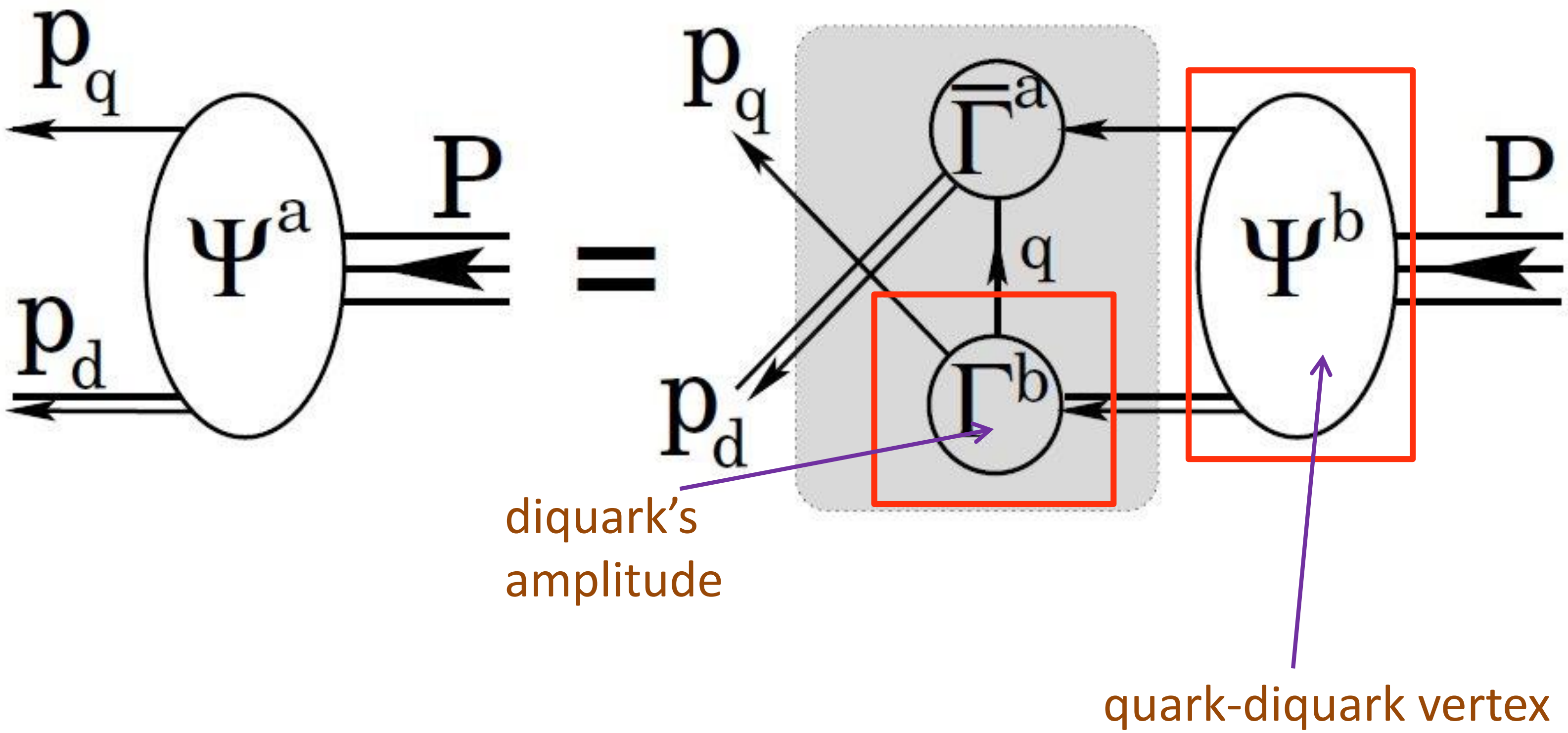
$$\mathcal{A}^i_\nu(\ell;P) = \sum_{n=1}^{6} p^i_n(\ell;P)\,\gamma_5\,A^n_\nu(\ell;P)\,,\ i = +,0,-$$

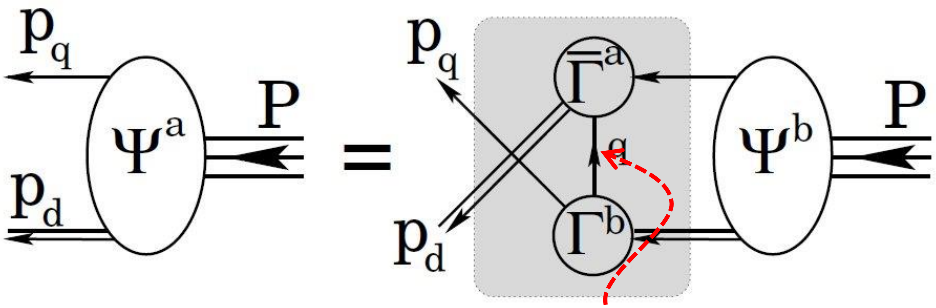
$$A^1_\nu = \gamma\cdot\hat{\ell}^\perp\,\hat{P}_\nu\,,\ A^2_\nu = -i\hat{P}_\nu\,,\qquad A^3_\nu = \gamma\cdot\hat{\ell}^\perp\,\hat{\ell}^\perp_\nu\,,$$
$$A^4_\nu = i\,\hat{\ell}^\perp_\mu\,,\qquad A^5_\nu = \gamma^\perp_\nu - A^3_\nu\,,\ A^6_\nu = i\gamma^\perp_\nu\gamma\cdot\hat{\ell}^\perp - A^4_\nu$$

  - *Delta: axial-vector – 8.*

$$\mathcal{D}_{\nu\rho}(\ell;P) = \mathcal{S}^\Delta(\ell;P)\delta_{\nu\rho} + \gamma_5\mathcal{A}^{i\,\Delta}_\nu(\ell;P)\ell^\perp_\rho$$

Faddeev Equation



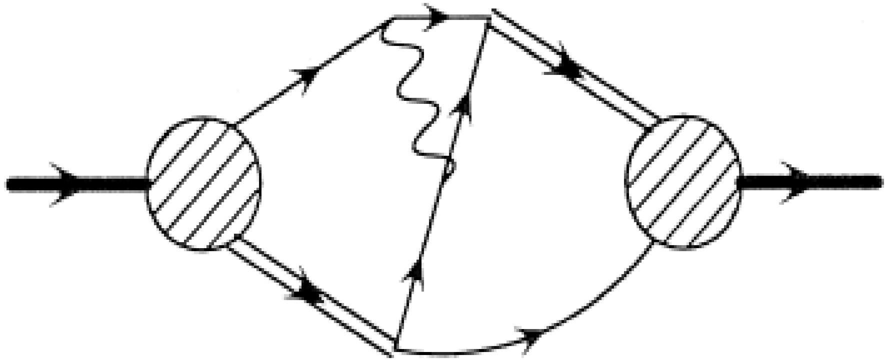


$$\frac{1}{i\gamma \cdot p + M} \to \frac{g_B^2}{M}$$

- Static “approximation”
  - Implements analogue of contact interaction in Faddeev-equation
- In combination with contact-interaction diquark-correlations, generates Faddeev equation kernels which themselves are momentum-independent
- The merit of this truncation is the *dramatic simplifications* which it produces
- Used widely in hadron physics phenomenology

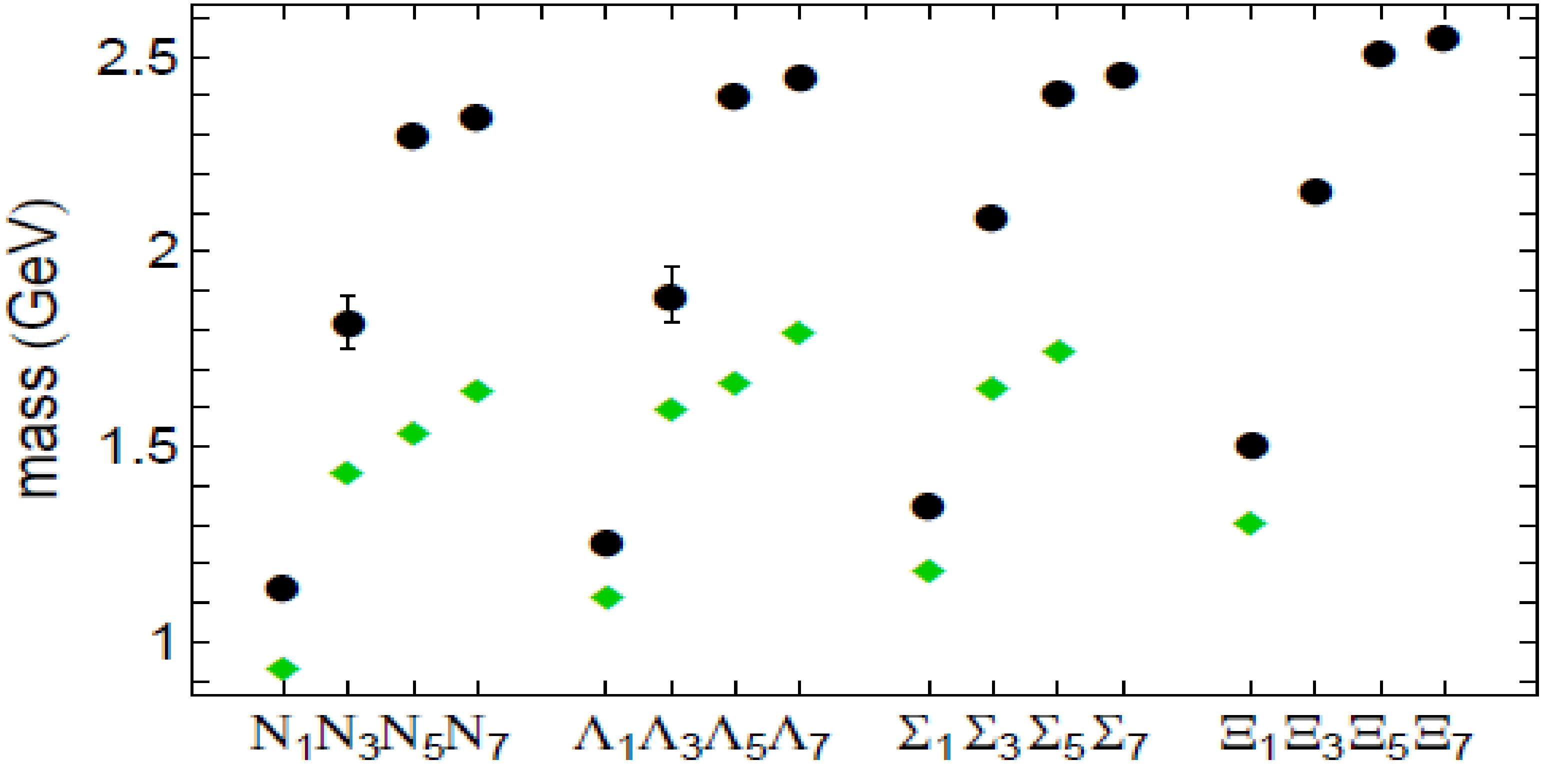
Pion-Loops & Baryons Masses

- Pion-loops: meson exchange between two distinct dressed-quarks within the bound states.
- For example:

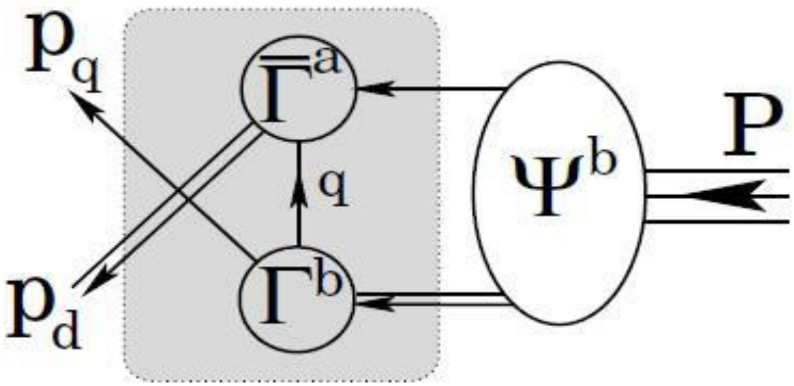


- Typically produce sizable reductions of the masses.

Spectrum of baryons



Spectrum of hadrons with strangeness
Chen Chen, Lei Chang, Craig D. Roberts, Shaolong Wan and David J. Wilson
arXiv:1204.2553 [nucl-th], Few Body Syst. **53** (2012) pp. 293-326



Realistic Model: Selected Papers

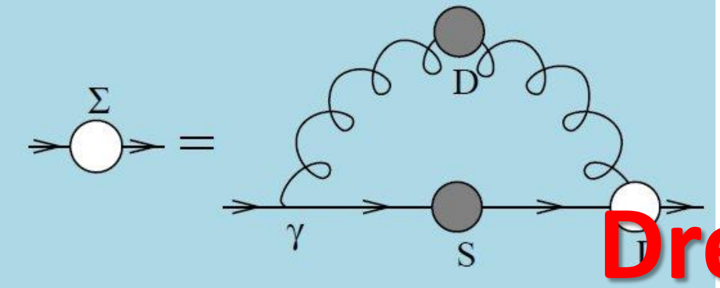
Survey of nucleon electromagnetic form factors
I.C. Cloët, G. Eichmann, B. El-Bennich, T. Klähn and C.D. Roberts
arXiv:0812.0416 [nucl-th], Few Body Syst. **46** (2009) pp. 1-36

Nucleon and Δ elastic and transition form factors
Jorge Segovia, Ian C. Cloët, Craig D. Roberts and Sebastian M. Schmidt
arXiv:1408.2919 [nucl-th], Few Body Syst. **55** (2014) pp. 1185-1222 [on-line]

50 PRL *Completing the picture of the Roper resonance*
Jorge Segovia, Bruno El-Bennich, Eduardo Rojas, Ian C. Cloët, Craig D. Roberts, Shu-Sheng Xu and Hong-Shi Zong
arXiv:1504.04386 [nucl-th], Phys. Rev. Lett. **115** (2015) 171801

Understanding the nucleon as a Borromean bound-state
Jorge Segovia, Craig D. Roberts and Sebastian M. Schmidt
arXiv:1506.05112 [nucl-th], Phys. Lett. B **750** (2015) pp. 100-106

RAPID *Dissecting nucleon transition electromagnetic form factors*
Jorge Segovia and Craig D. Roberts
arXiv:1607.04405 [nucl-th], Phys. Rev. C **94** (2016) 042201(R)/1-6



Realistic Model: Dressed Quark Propagators

$$\boxed{S(p) = -i\gamma\cdot p\,\sigma_V(p^2) + \sigma_S(p^2)}$$

$$\bar\sigma_S(x) = 2\,\bar m\,\mathcal{F}(2(x+\bar m^2)) + \mathcal{F}(b_1 x)\,\mathcal{F}(b_3 x)\,\left[b_0 + b_2\mathcal{F}(\epsilon x)\right]\,,$$

$$\bar\sigma_V(x) = \frac{1}{x+\bar m^2}\,\left[1-\mathcal{F}(2(x+\bar m^2))\right]\,,$$

with $x = p^2/\lambda^2$, $\bar m = m/\lambda$,

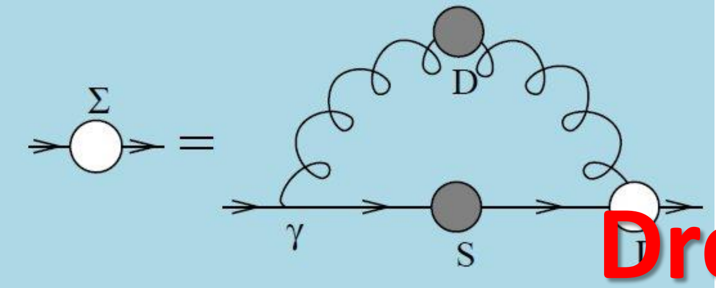
$$\mathcal{F}(x) = \frac{1-\mathrm{e}^{-x}}{x}\,,$$

$\bar\sigma_S(x) = \lambda\,\sigma_S(p^2)$ and $\bar\sigma_V(x) = \lambda^2\,\sigma_V(p^2)$.

The mass-scale, $\lambda = 0.566$ GeV,

$\bar m$	b_0	b_1	b_2	b_3
0.00897	0.131	2.90	0.603	0.185

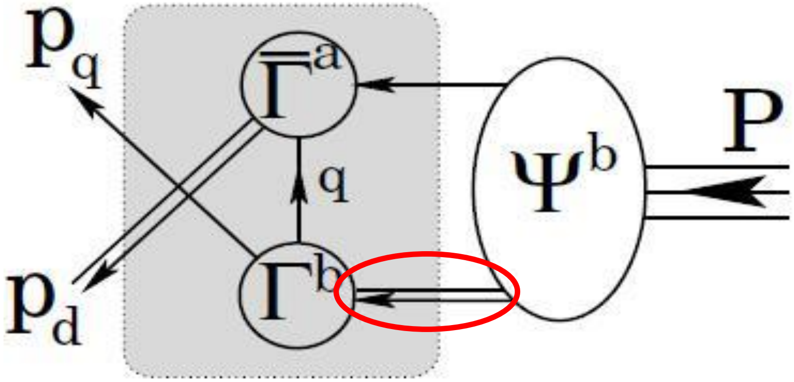
were fixed in a least-squares fit to light-meson observables



Realistic Model: Dressed Quark Propagators

$$S(p) = -i\gamma \cdot p\,\sigma_V(p^2) + \sigma_S(p^2)$$

- Based on solutions to the gap equation that were obtained with a dressed gluon-quark vertex.
- Mass function has a real-world value at *k^2 = 0*, not the highly inflated value typical of RL truncation.
- Propagators are entire functions, consistent with sufficient condition for confinement and completely unlike known results from RL truncation.
- Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.

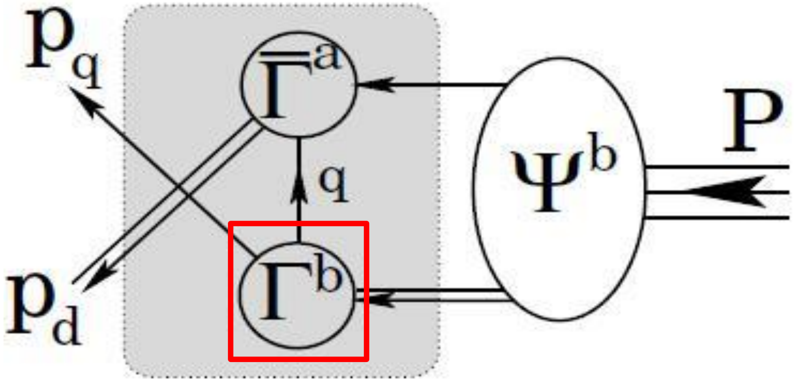


Realistic Model: Diquark Propagators

$$\Delta^{0^+}(K) = \frac{1}{m_{0^+}^2}\mathcal{F}(K^2/\omega_{0^+}^2)\,,$$

$$\Delta^{1^+}_{\mu\nu}(K) = \left(\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^+}^2}\right)\frac{1}{m_{1^+}^2}\mathcal{F}(K^2/\omega_{1^+}^2)$$

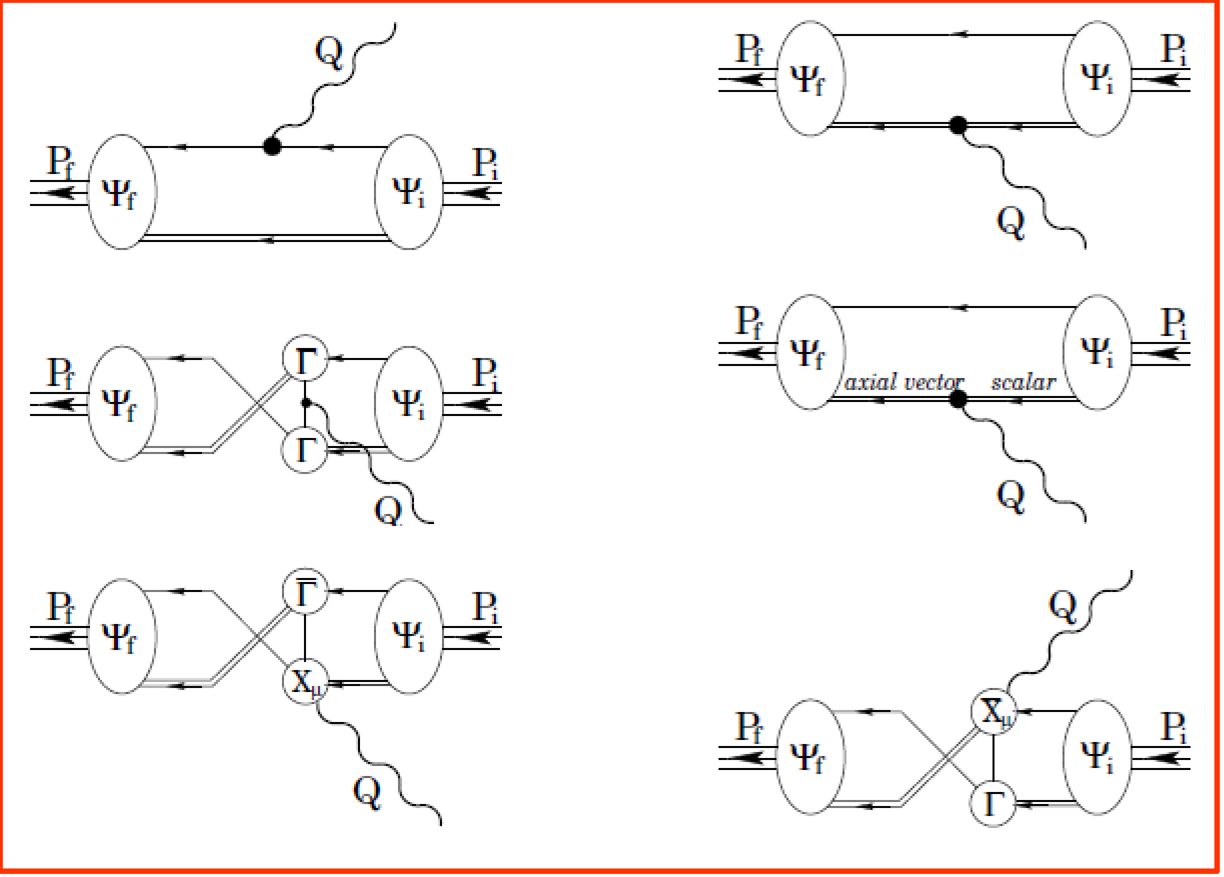
- The *four* parameters *m_J^P* are diquark pseudoparticle masses & *ω_J^P* are widths characterising the diquark amplitudes. *ω_J^P* can be obtained by *m_J^P* via normalisation conditions.
- The *F-functions*: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and *1/q^2* evolution (UV) of meson propagators.
- Notably, our diquarks are confined. That is NOT true of RL studies. It is this that enables us to reach arbitrarily high values of momentum transfer.



Realistic Model: Diquark Amplitudes

$$
\begin{aligned}
\chi^{sc}(k;K) &= g_{sc}(\gamma_5 C)\mathcal{F}(k^2/\omega_{0^+}^2)\frac{\tau_2}{\sqrt{2}}\frac{\epsilon_{ABD}}{\sqrt{2}},\\
\chi_\mu^{ax}(k;K) &= g_{ax}(\gamma_\mu C)\mathcal{F}(k^2/\omega_{1^+}^2)\frac{\mathrm{t}^i}{\sqrt{2}}\frac{\epsilon_{ABD}}{\sqrt{2}},\\
\chi^{ps}(k;K) &= g_{ps}(C)\mathcal{F}(k^2/\omega_{0^-}^2)\frac{\tau_2}{\sqrt{2}}\frac{\epsilon_{ABD}}{\sqrt{2}},\\
\chi_\mu^{vc}(k;K) &= g_{vc}(\gamma_\mu\gamma_5 C)\mathcal{F}(k^2/\omega_{1^-}^2)\frac{\tau_2}{\sqrt{2}}\frac{\epsilon_{ABD}}{\sqrt{2}},
\end{aligned}
$$

- **The *Ansatze* of diquark amplitudes retain only that single Dirac-amplitude which would represent a point particle with the given quantum numbers in a local Lagrangian density. They are usually the *dominant* amplitudes in a solution of the rainbow-ladder BSE for the lowest mass *J^P* and mesons.**
- **Simple form. Just one parameter, which connects mass with width.**
- **Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.**



Interaction current

- The minimal construction which ensures that with our computed baryon Faddeev amplitudes, the Ward-Green-Takahashi identities are satisfied. It is not a RL current because we have the seagull-terms. The seagull terms are an implicit correction for the non-triangle-diagram pieces that should be in our current because our inputs are not RL-based.

Realistic Model:
Why do we do it this way?

- ✓RL truncation imposes severe limitations on the domain of physics that can be accessed. We use non-RL insights to build a QCD -kindred framework.
- ✓Algebraic parametrisations are invaluable because computations require repeated evaluation of multidimensional integrals with integrands sampling a large subdomain of the complex plane. This is made much simpler when using realistic, algebraic parametrisations of the elements in the integrands. This is precisely what we are using and it enables us to reach ***large-Q^2*** when computing form factors.
- ✓These algebraic parametrisations are the precursors to our modern Nakanishi-like representations.

Realistic Model:
Spectrum with strangeness

- Parameters: diquark masses

$$
\begin{aligned}
m_{[ud]} &= 0.73 \text{ GeV}, \\
m_{[us]} = m_{[ds]} &= 0.89 \text{ GeV}, \\
m_{\{uu\}} = m_{\{ud\}} = m_{\{dd\}} &= 0.925 \text{ GeV}, \\
m_{\{us\}} = m_{\{ds\}} &= 1.025 \text{ GeV}, \\
m_{\{ss\}} &= 1.115 \text{ GeV},
\end{aligned}
$$

- *u/d-* & *s-* quark propagators:

f	$\bar{m}_f$	b_0^f	b_1^f	b_2^f	b_3^f
$u=d$	0.00897	0.131	2.90	0.603	0.185
s	0.210	0.105	3.18	0.858	0.185

Realistic Model:
Spectrum with strangeness

Table 1: The masses of octet and decuplet baryons with strangeness. Where m^{real} are the values of the realistic model, m^{CI} are the values of the contact model, and $m^{\mathrm{expt.}}$ are the experimental values. All dimensioned quantities are listed in GeV.

	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω
m^{real}	1.14	1.28	1.35	1.46	1.39	1.51	1.64	1.76
m^{CI}	1.14	1.26	1.35	1.43	1.39	1.51	1.63	1.76
$m^{\mathrm{expt.}}$	0.94	1.12	1.19	1.31	1.23	1.39	1.53	1.67
$m^{\mathrm{real}} - m^{\mathrm{expt.}}$	0.2	0.16	0.16	0.15	0.16	0.12	0.11	0.09
$m^{\mathrm{real}} - m^{\mathrm{CI}}$	0	0.02	0	0.03	0	0	0.01	0

- Contact model is good enough to calculate the spectrum.
- However, the Faddeev amplitudes from the contact model are constants, so it is hard to give quantitative predictions of *large-q^2* form factors.

Nucleon & *N*(1535)*: All Diquarks Included

- Nucleon:

$$\Gamma^{N}_{\alpha\beta\gamma} = (\chi^{sc})_{\beta\gamma} D(\Phi^{sc}_{+} u)_\alpha + (\chi^{ax}_{\mu})_{\beta\gamma} D_{\mu\nu}(\Phi^{ax}_{+\nu} u)_\alpha + (\chi^{ps})_{\beta\gamma} D(\Phi^{ps}_{+} u)_\alpha + (\chi^{vc}_{\mu})_{\beta\gamma} D_{\mu\nu}(\Phi^{vc}_{+\nu} u)_\alpha$$

- *N*(1535)*:

$$\Upsilon^{N^*}_{\alpha\beta\gamma} = (\chi^{sc})_{\beta\gamma} D(\Phi^{sc}_{-} u)_\alpha + (\chi^{ax}_{\mu})_{\beta\gamma} D_{\mu\nu}(\Phi^{ax}_{-\nu} u)_\alpha + (\chi^{ps})_{\beta\gamma} D(\Phi^{ps}_{-} u)_\alpha + (\chi^{vc}_{\mu})_{\beta\gamma} D_{\mu\nu}(\Phi^{vc}_{-\nu} u)_\alpha$$

- Diquark masses (*inferred from DSE study*):

$$\begin{aligned} m_{\rm sc} &= 0.8 \text{ GeV} \\ m_{\rm ax} &= 0.9 \text{ GeV} \\ m_{\rm ps} &= 1.3 \text{ GeV} \\ m_{\rm vc} &= 1.4 \text{ GeV} \end{aligned}$$

Realistic Model Results:

	$N_{n=0}$	$N_{n=1}$	$N^*_{n=0}$	$N^*_{n=1}$
m^{DSE}	1.19	1.73	1.83	1.94
$m^{\mathrm{expt.}}$	0.94	1.44	1.54	1.65
$m^{\mathrm{DSE}} - m^{\mathrm{expt.}}$	0.25	0.29	0.29	0.29

✓Nucleon: the *unflavoured* diquark components (*ps*, *vc*) are almost ***ZERO***.

✓*N*(1535):* ***ALL*** the diquark components are important!

✓For nucleon & *N*(1535)*, the mass splittings between the computed and experimental values are very close!

✓ m_N*(1535) > m_Roper

Two Additional Questions

1. How hard must one work in order to compute the spectrum of low-lying baryons?
2. To which features of the interaction is the spectrum sensitive?

Baryons

Conclusions & Outlook

- ✓ One doesn't need to work very hard to compute the spectrum of low-lying baryons.
- ✓ A very sensible picture is obtained using just a contact interaction
- ✓ Demonstration that for all but nucleon and Delta ground-states, one should include all possible diquark correlatoins
- ✓ Finally, however, if one wants reliable wave functions, with which to compute transition form factors, for example, then only a realistic interaction is acceptable

- ➢ Future plan: masses of other excited nucleons; form factors; PDAs; PDFs; GPDs; TMDs…

Thank you!