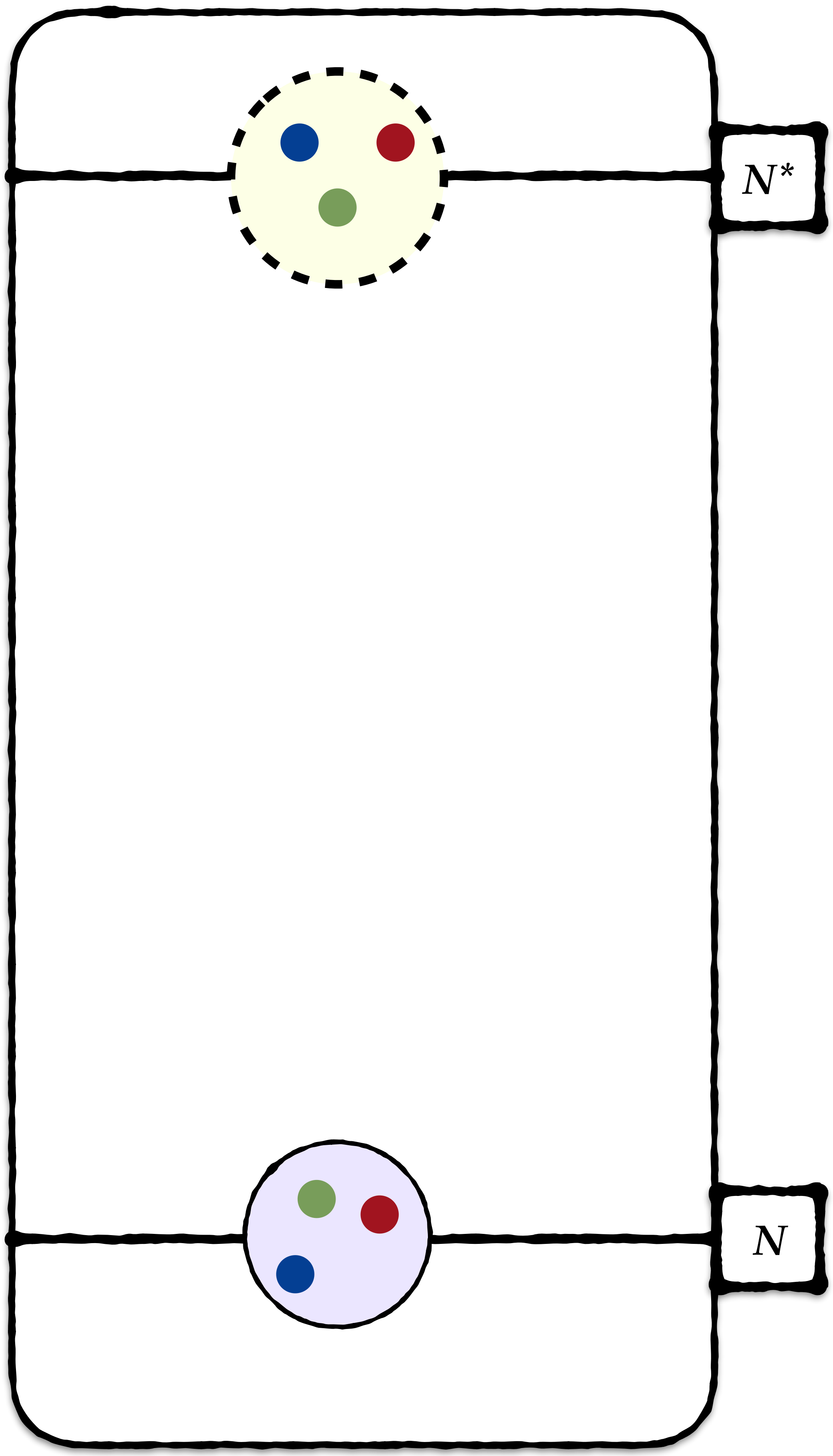
Towards systematic studies of resonances from lattice QCD

Raúl Briceño

rbriceno@jlab.org

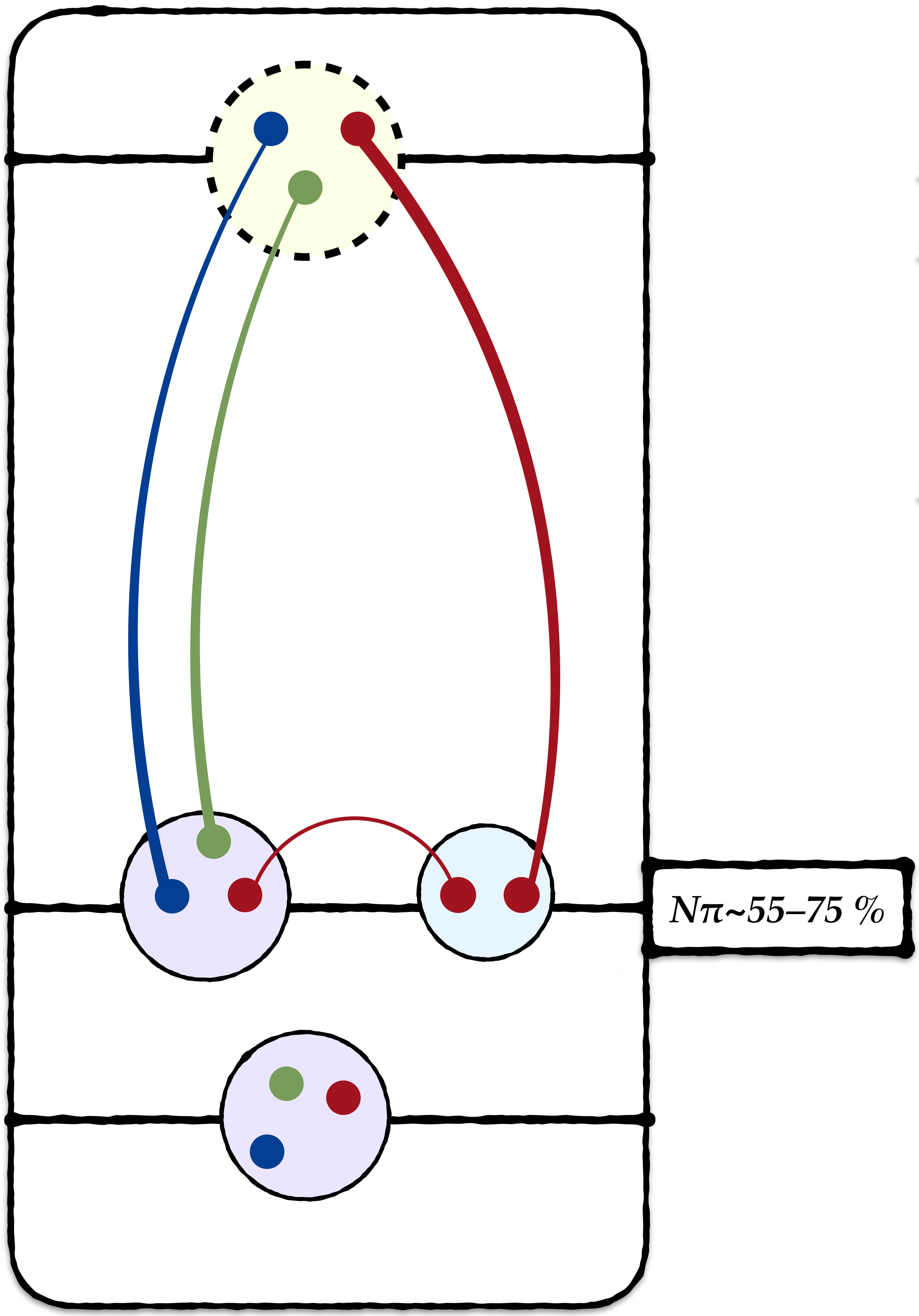
Jefferson Lab

INT, N* workshop
Nov, 2016



the Roper

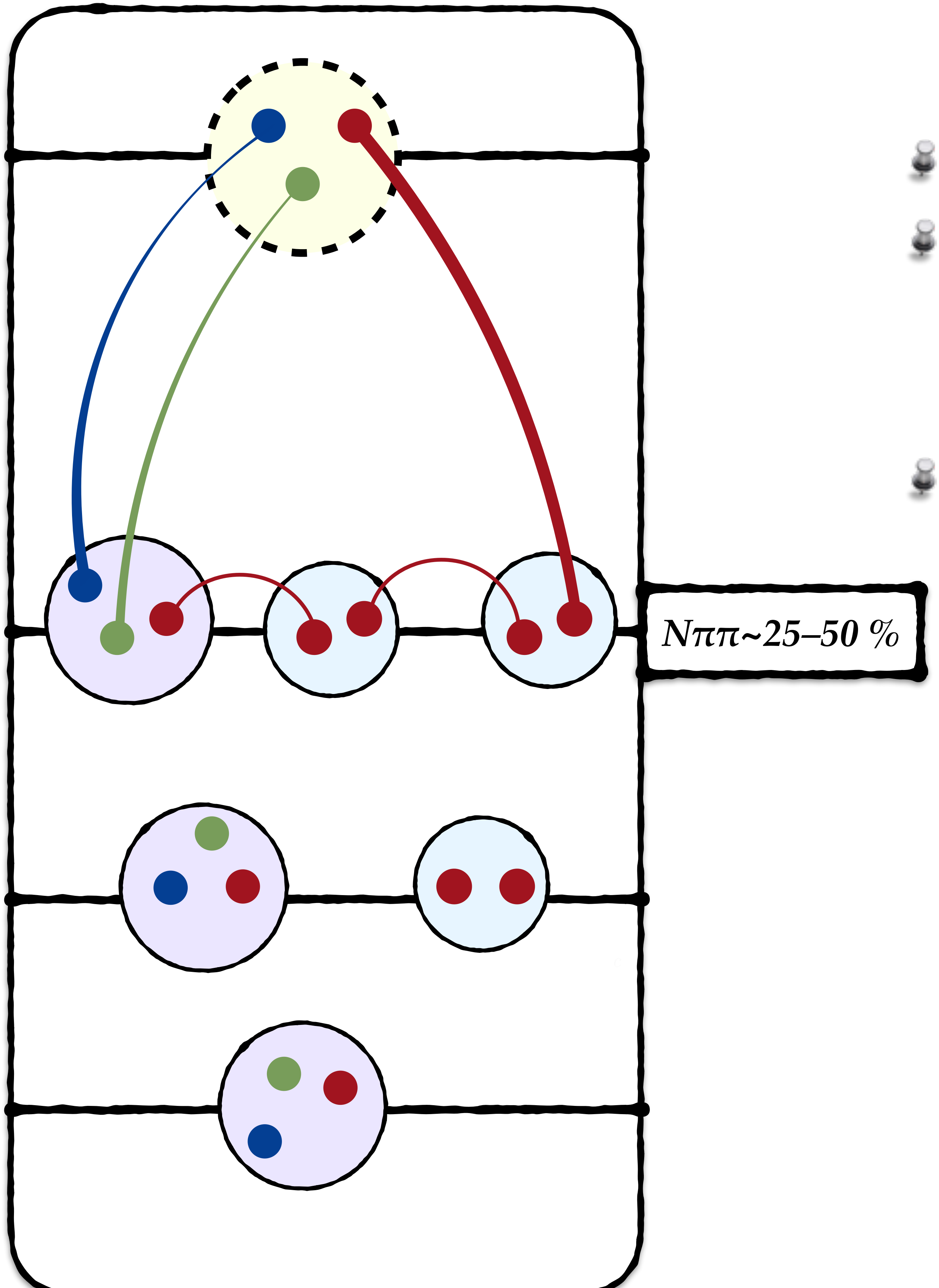
- Excited state of the nucleon



the Roper

- Excited state of the nucleon
- Dynamical enhancement in amps.
  - Complex pole in unphys. sheet
  - Fairly broad
- Strongly coupled to:
  - $N\pi$

the Roper



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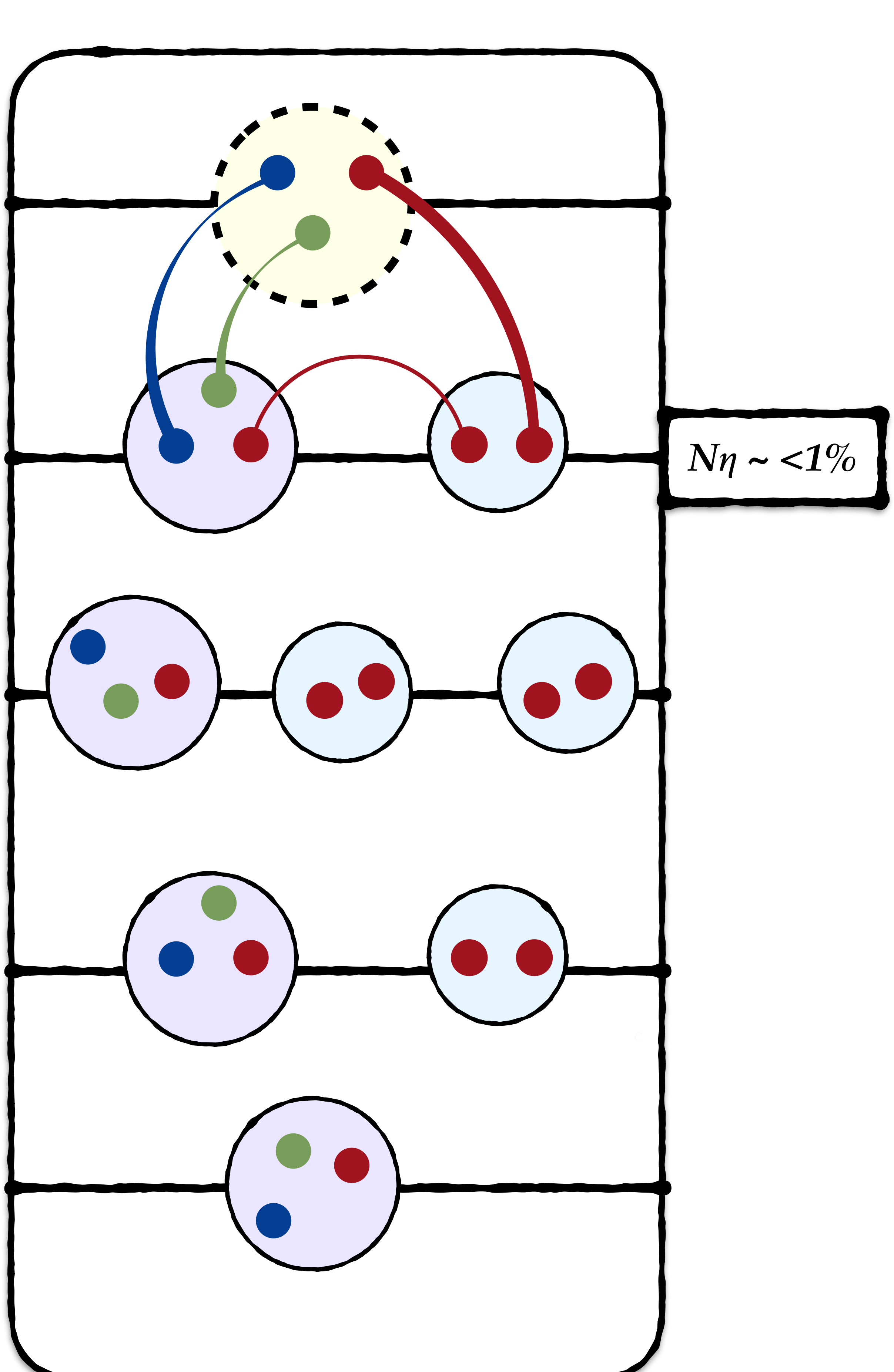

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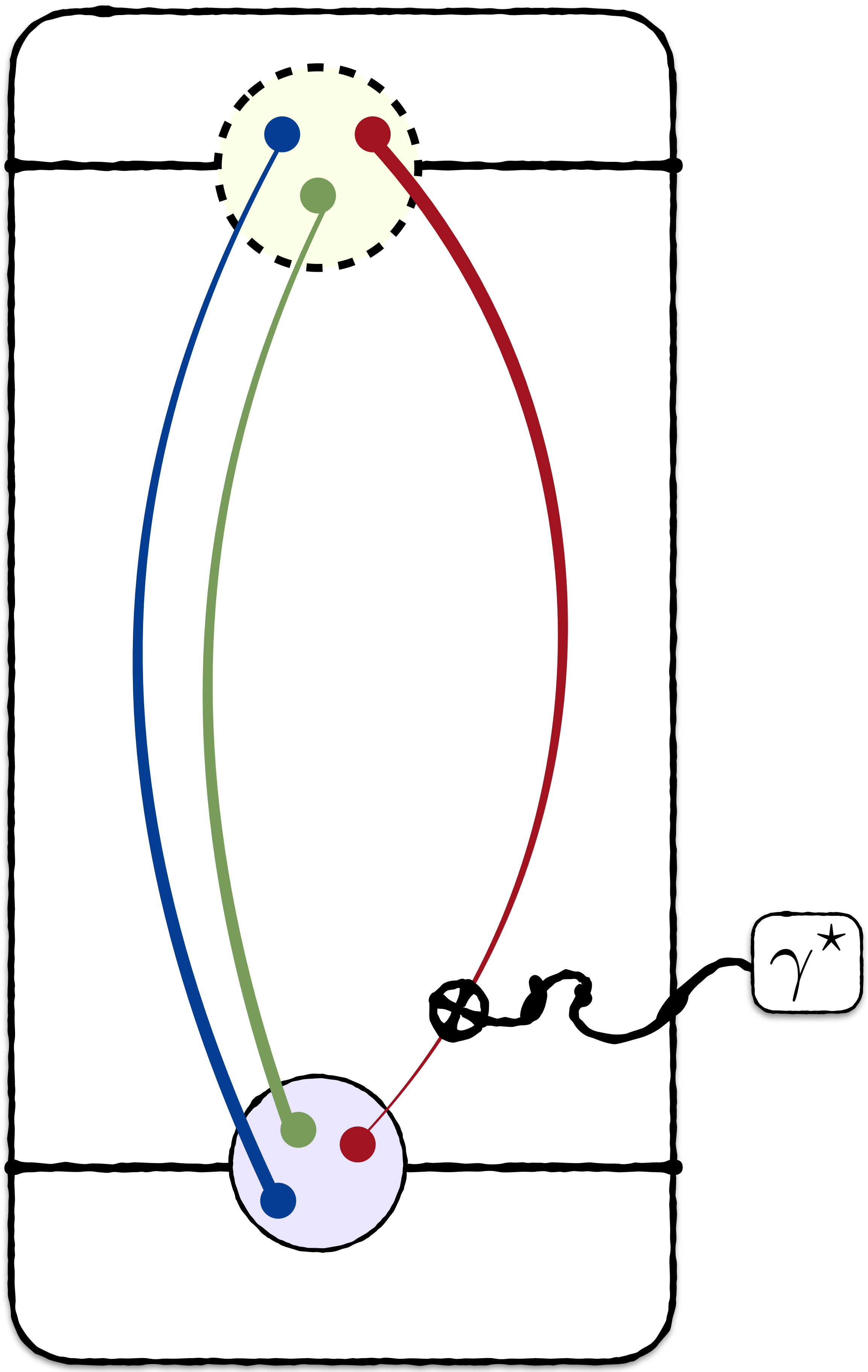
- Vanishing couple to $N\eta$?



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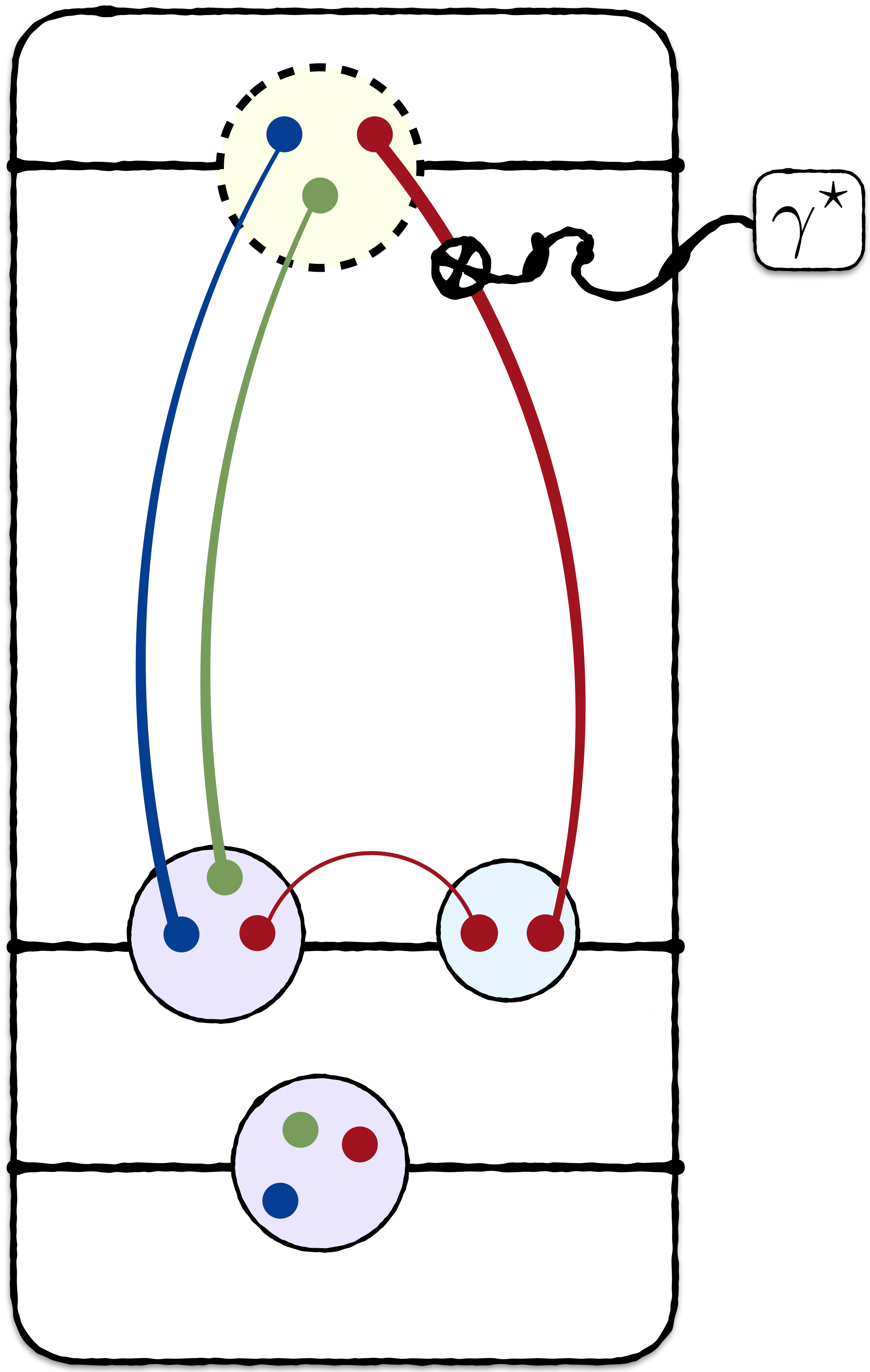

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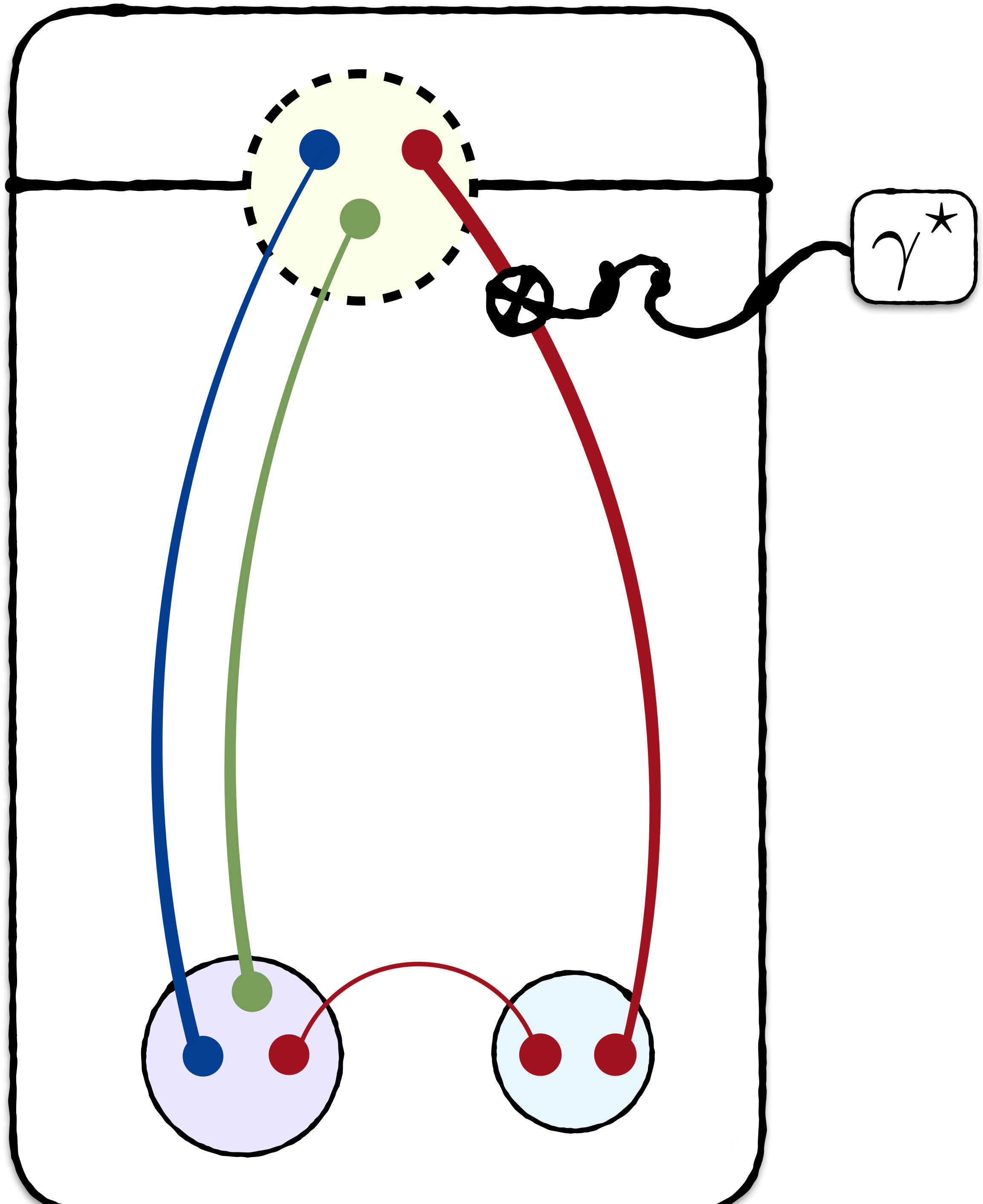
- Photo-,electro-produced
- Elastic form factors?



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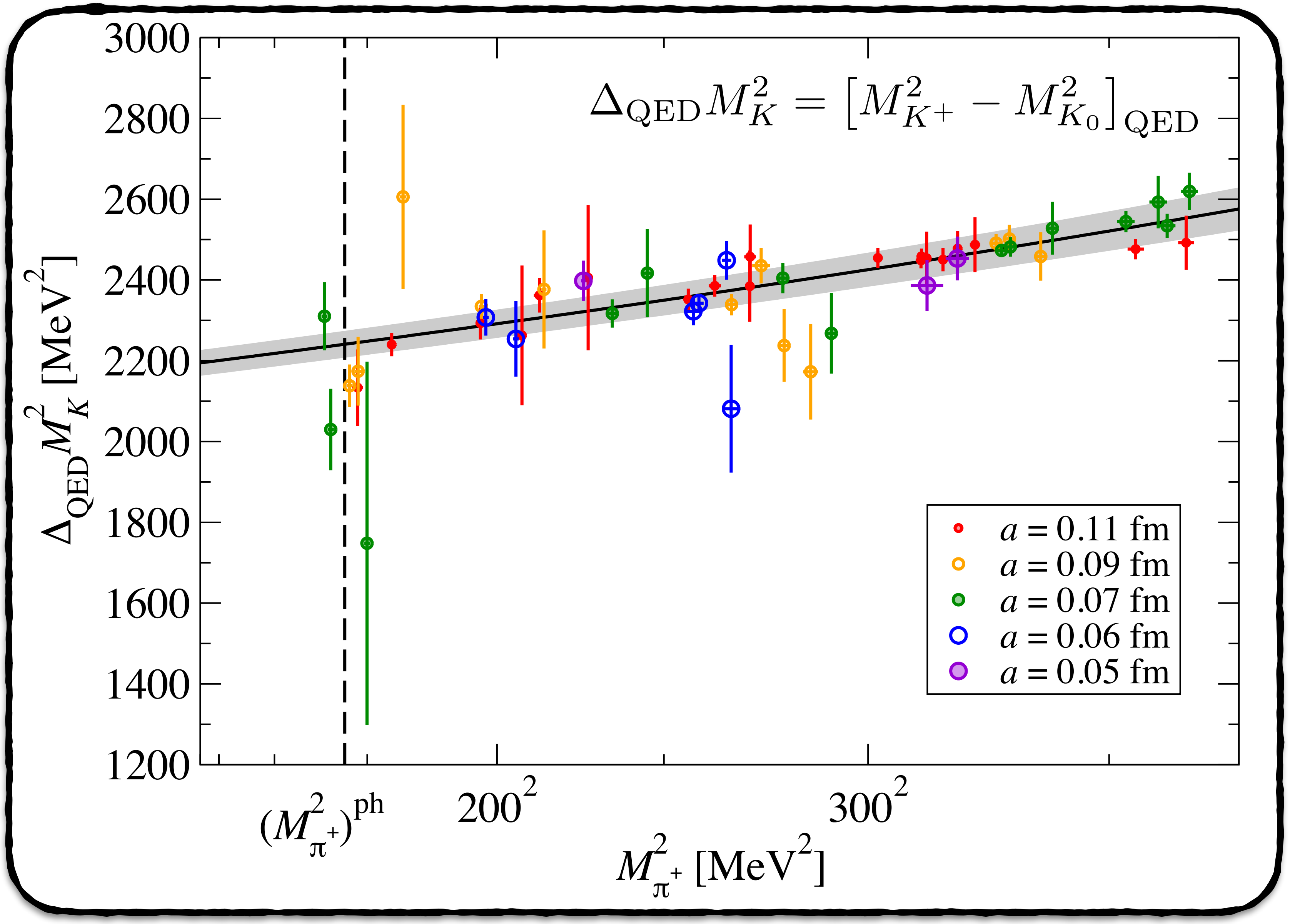
demand for lattice:

- Stable states generated *"exactly"*
- Resonant/non-resonant amplitudes are generated *"exactly"*
- QED/weak can be introduced perturb. or non-perturb.

Spectroscopy in LQCD

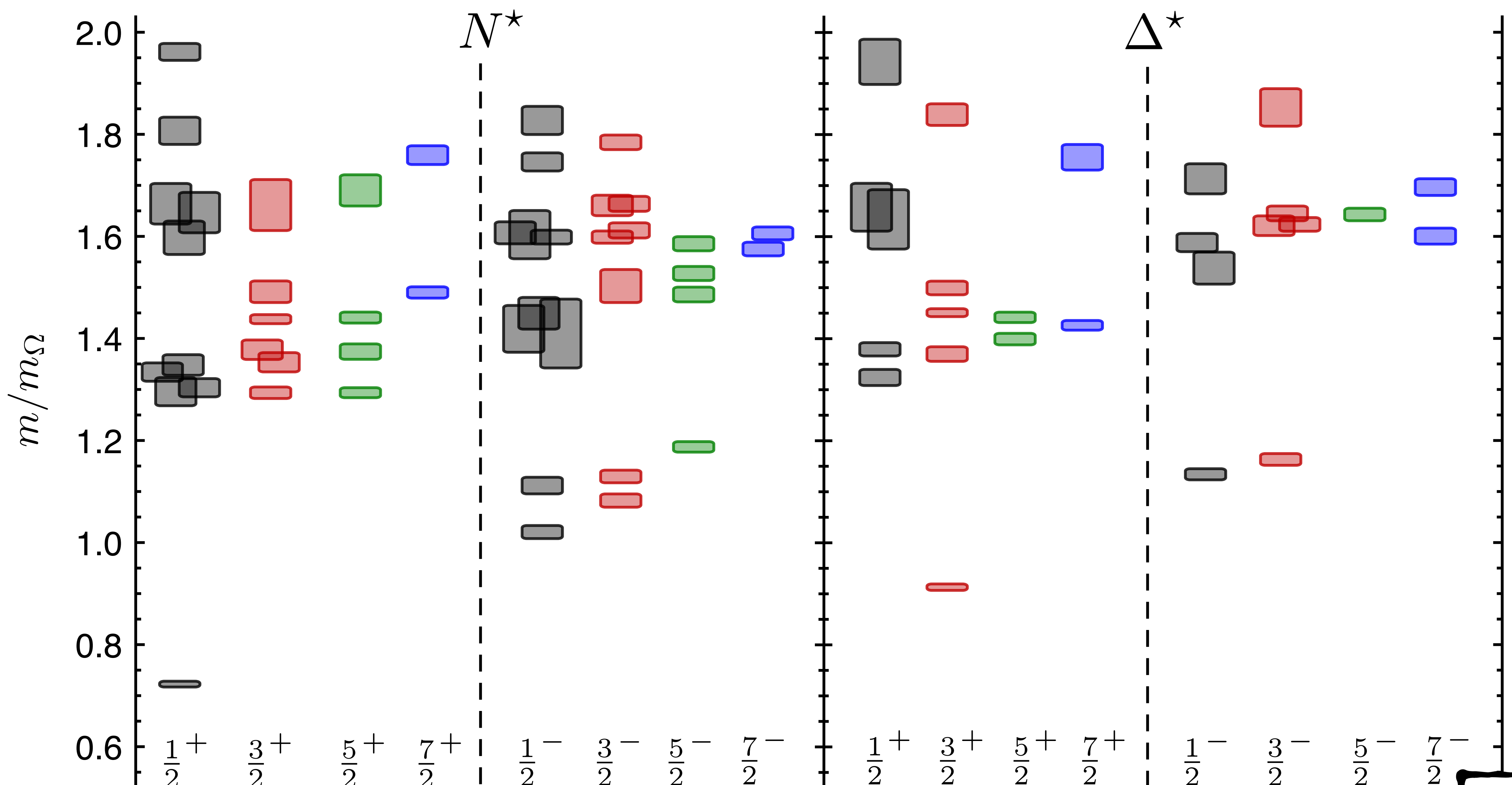
- *Vanilla* spectroscopy - QCD stable states [non-composite states]
  - Physical or lighter quark masses [down to m_π~120 MeV] ✓
  - Non-degenerate light-quark masses: N_f=1+1+1+1 ✓
  - Dynamical QED ✓



Fodor et al. [BMWc] (2016)

Spectroscopy in LQCD

- *Vanilla* spectroscopy - QCD stable states [non-composite states]
- the frontier of spectroscopy - hadronic resonances [composite states]



m_π=391 MeV

Edwards, Dudek, Richards, Wallace [Hadspec Collab.] (2011)

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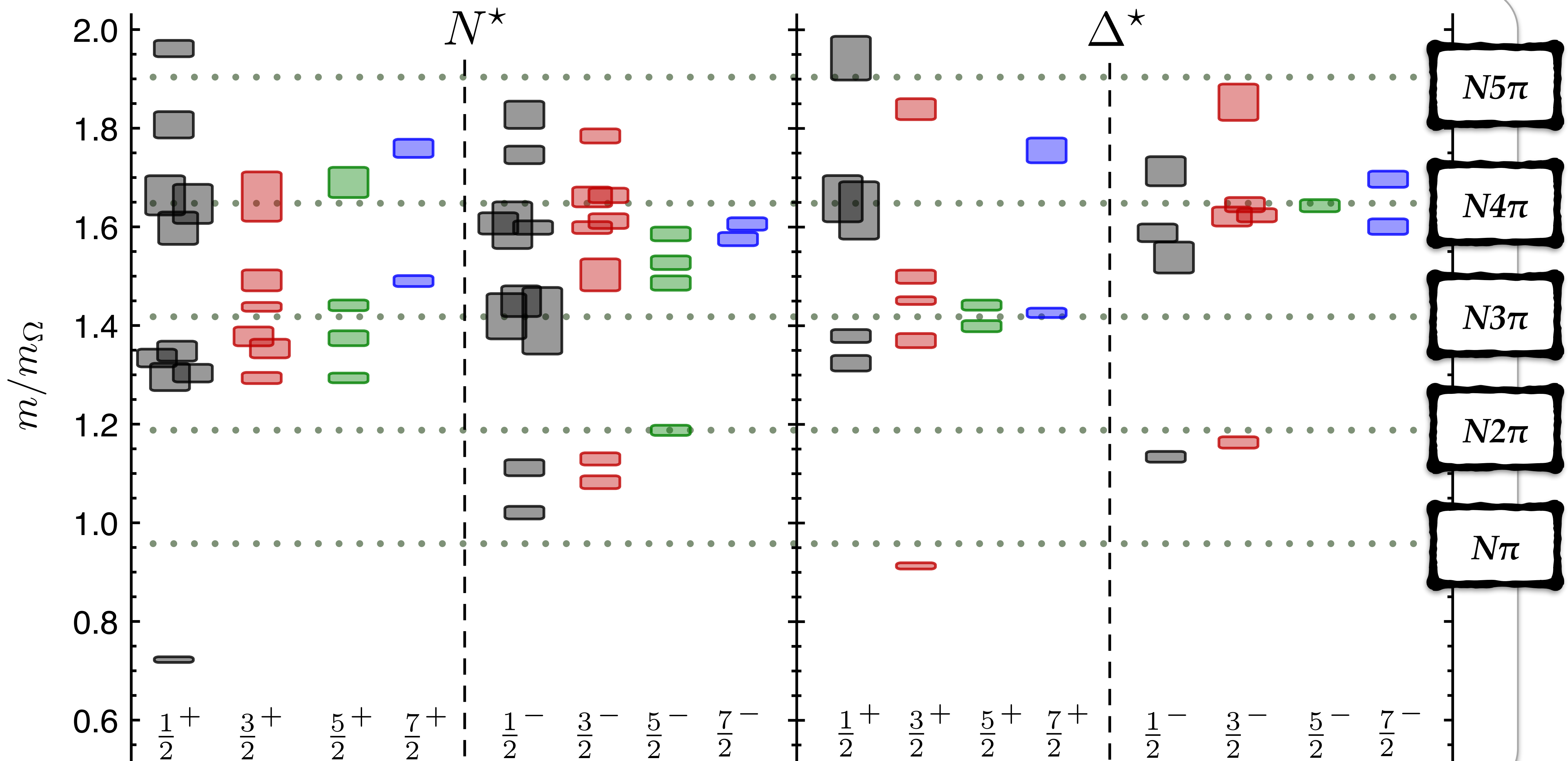


Edwards, Dudek, Richards, Wallace [Hadspec Collab.] (2011)

not all thresholds shown
not all threshold are expected to matter

Broad goals

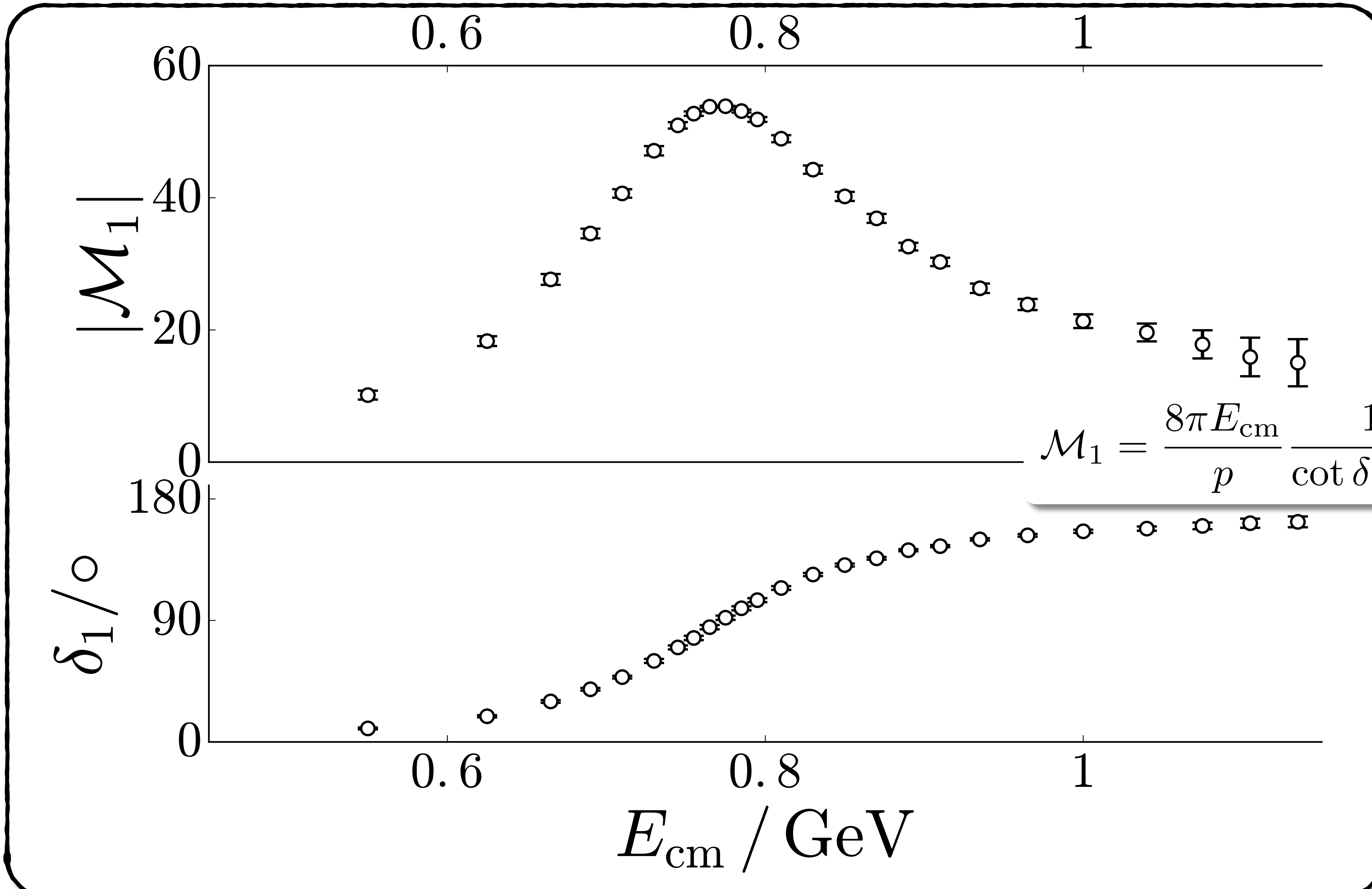
- Strongly coupled **2**-body
- Strongly coupled **2, 3**-body
- Spin-dependent amps.
- Narrow resonances
- Broad resonances
- Photo-, electro-production
- Transition form factors
- Elastic form factors

Broad goals

	formalism	numerical
Strongly coupled **2**-body	☐	☐
Strongly coupled **2, 3**-body	☐	☐
Spin-dependent amps.	☐	☐
Narrow resonances	☐	☐
Broad resonances	☐	☐
Photo-, electro-production	☐	☐
Transition form factors	☐	☐
Elastic form factors	☐	☐

A pseudo-quantitative definition

(bump in cross sections/amplitude - e.g., ππ scattering in ϱ-channel)

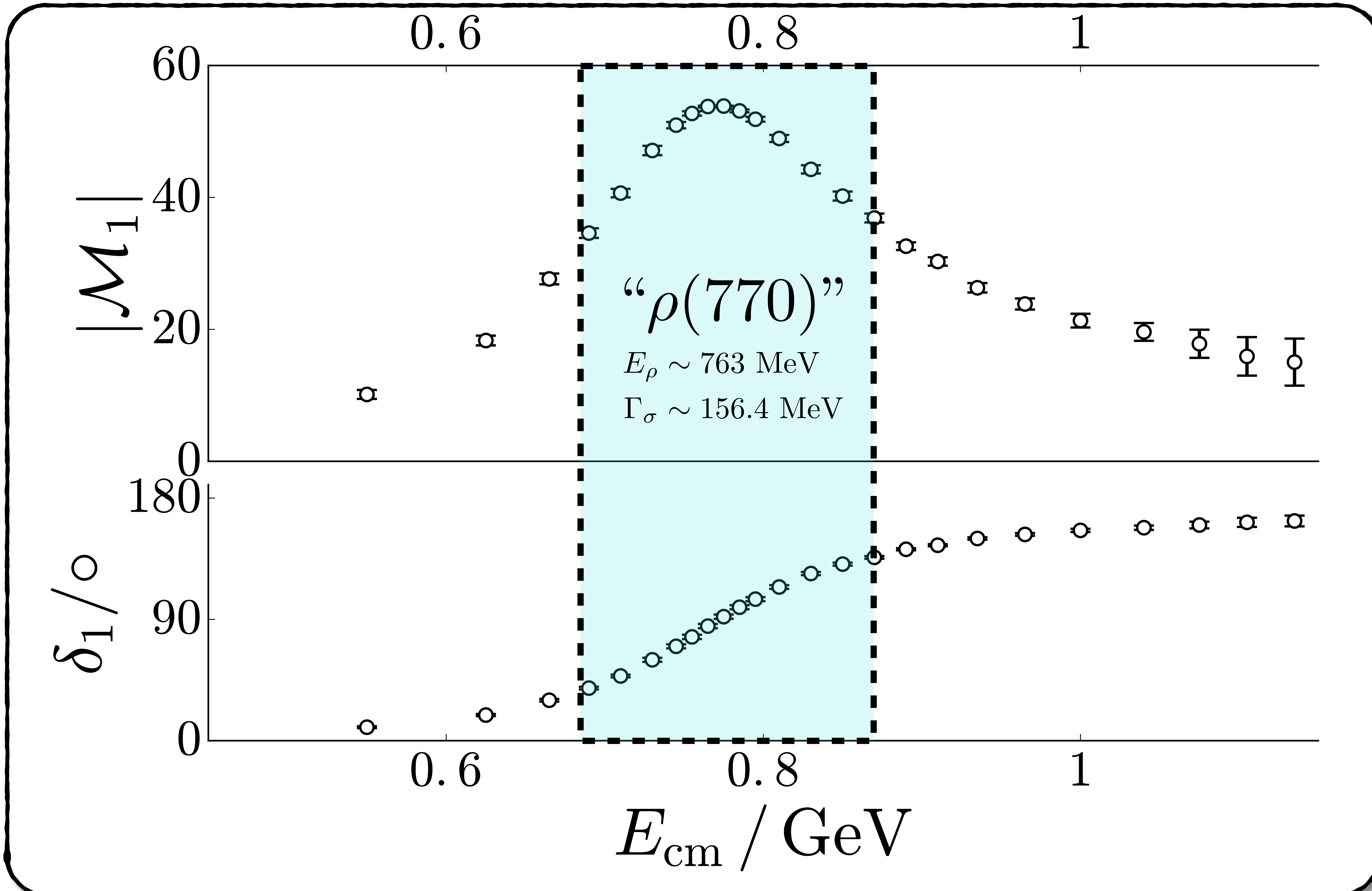


$$\mathcal{M}_1 = \frac{8\pi E_{\mathrm{cm}}}{p}\frac{1}{\cot\delta_1 - i}$$

Protopopescu et al. (1972)

A pseudo-quantitative definition

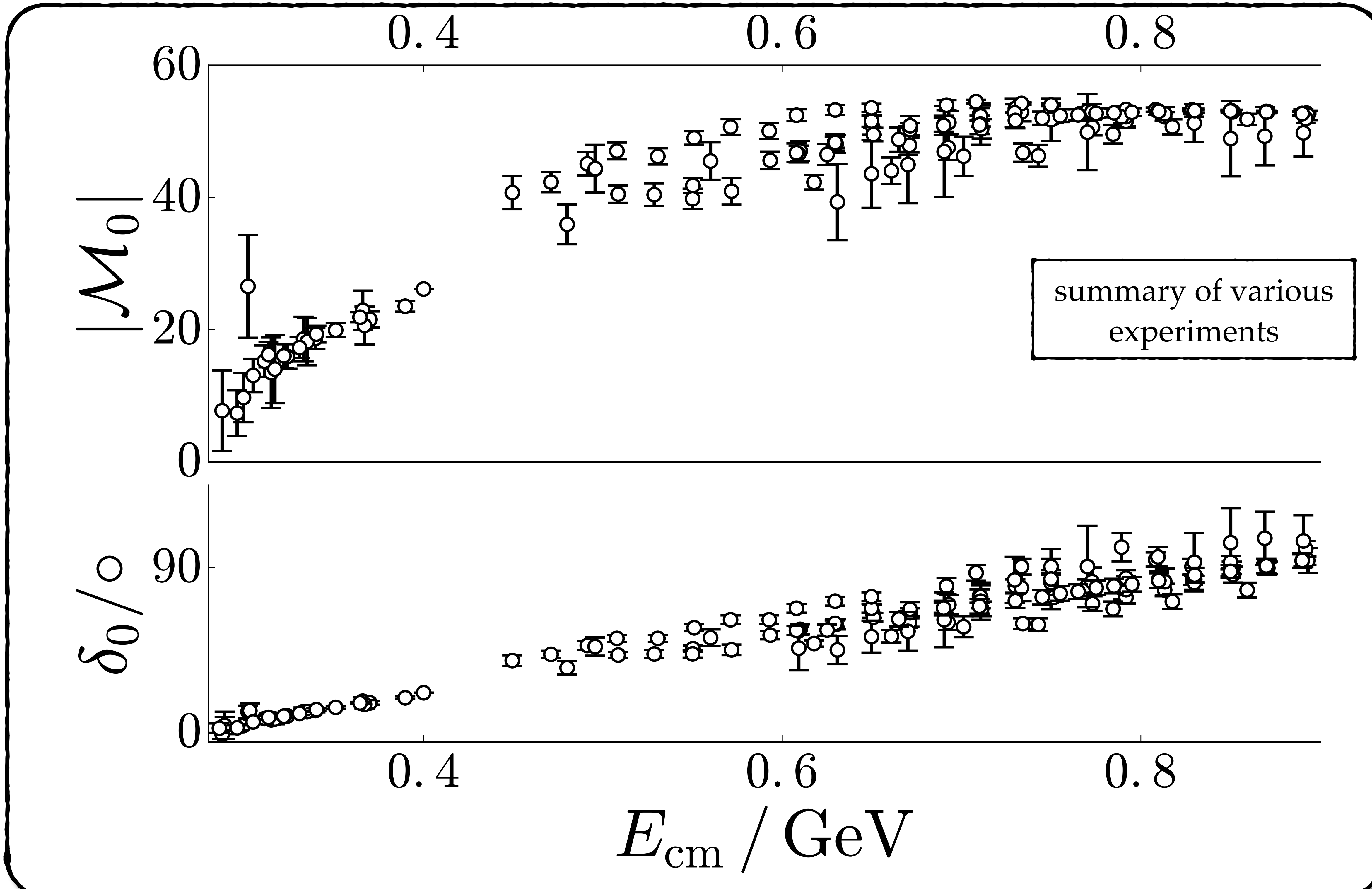
(bump in cross sections/amplitude - e.g., ππ scattering in ϱ-channel)



Protopopescu et al. (1972)

A counter example

(Isoscalar, scalar $\pi\pi$ scattering)



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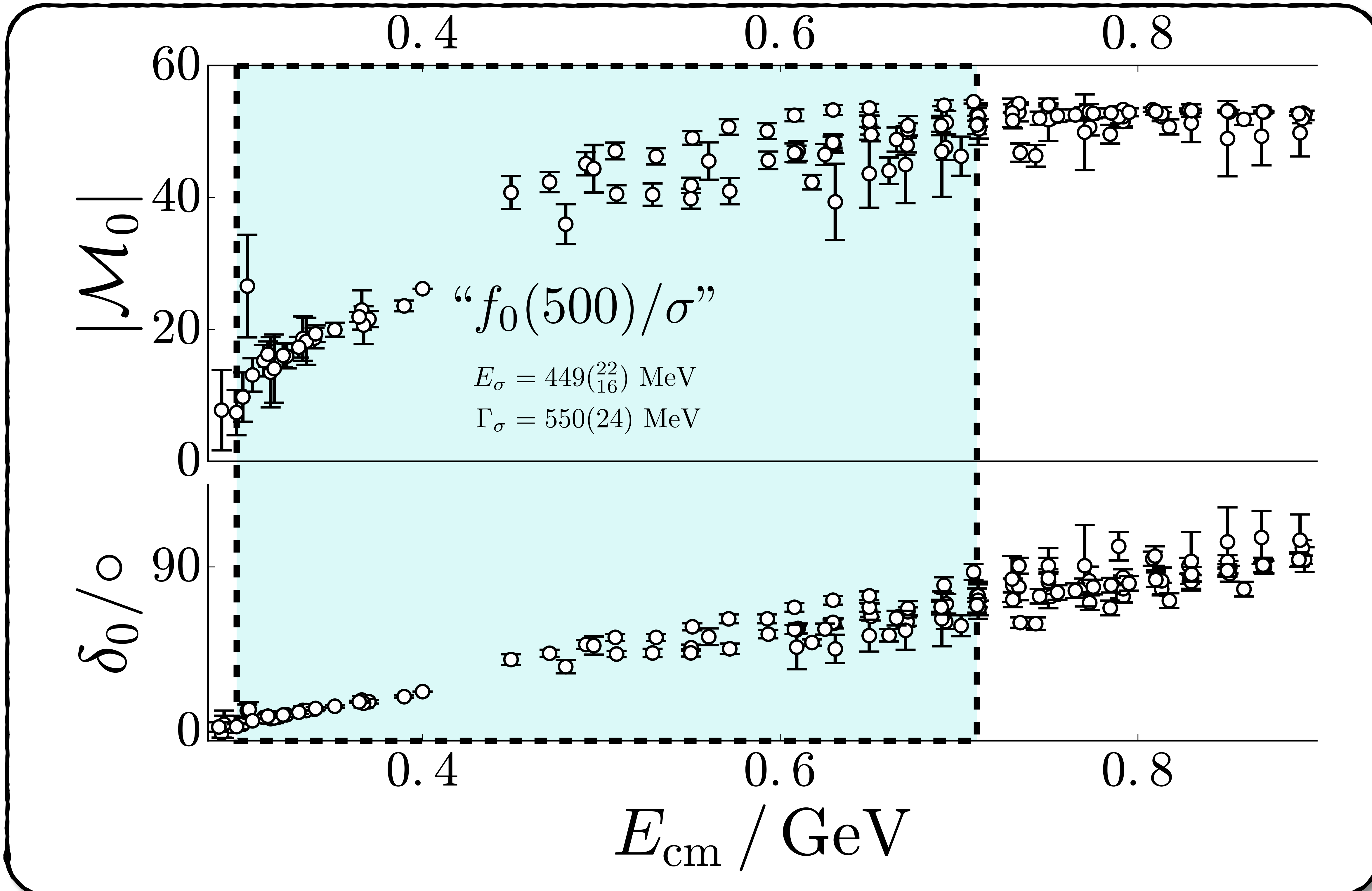
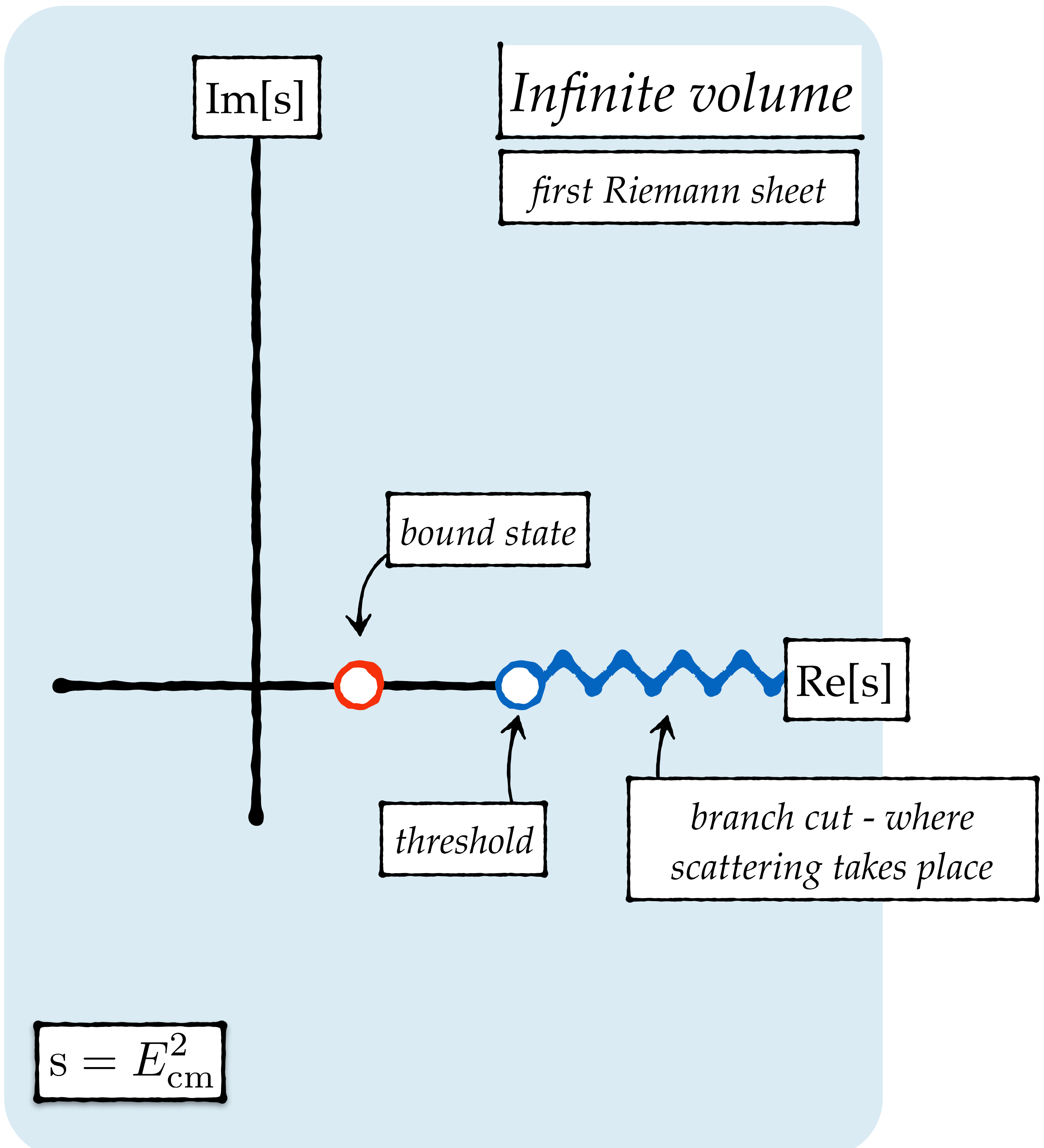
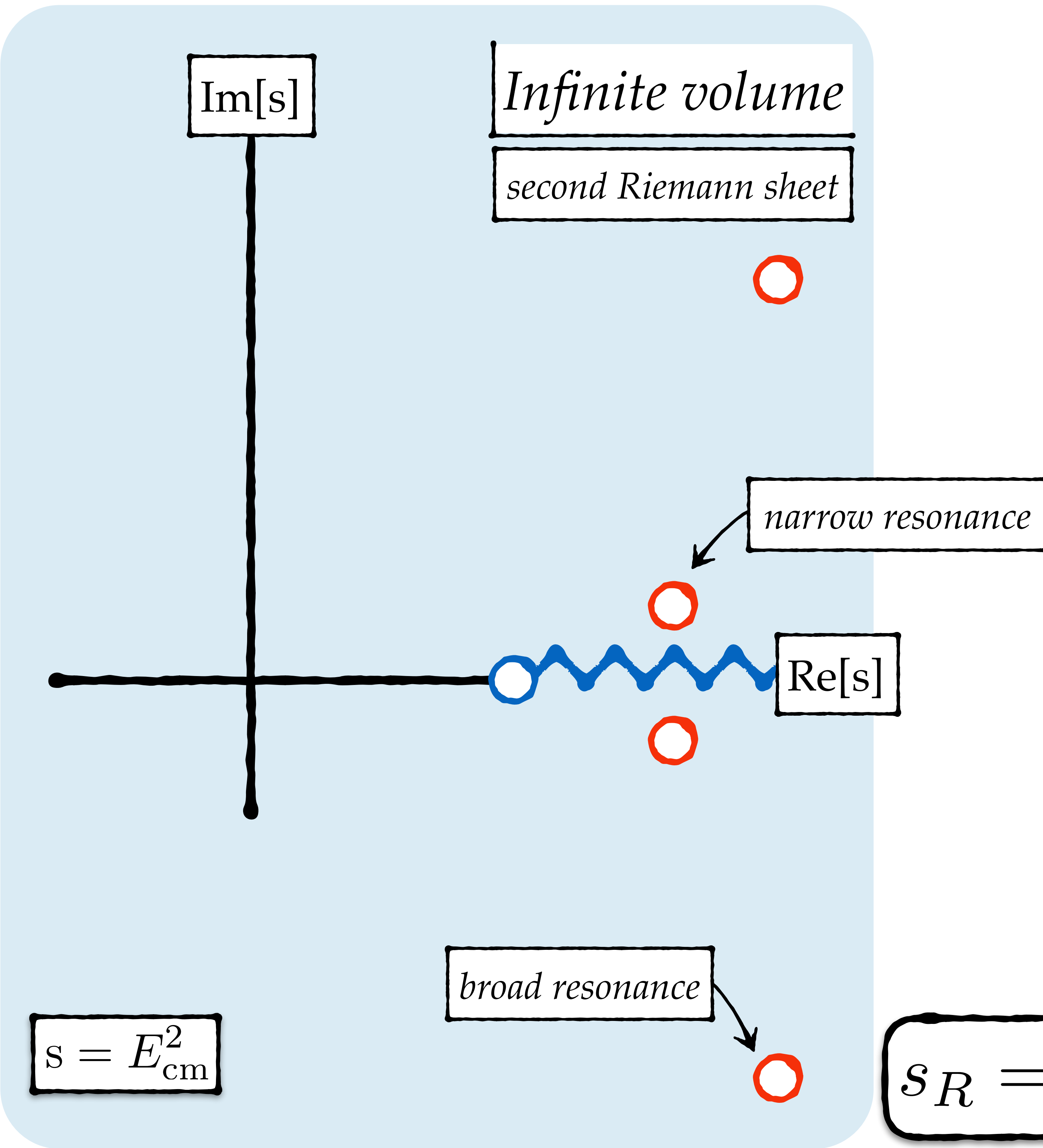


Spectroscopy recap

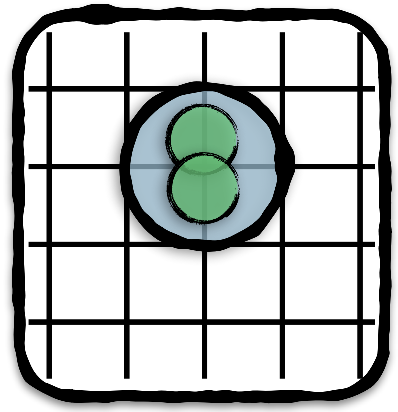


Spectroscopy recap

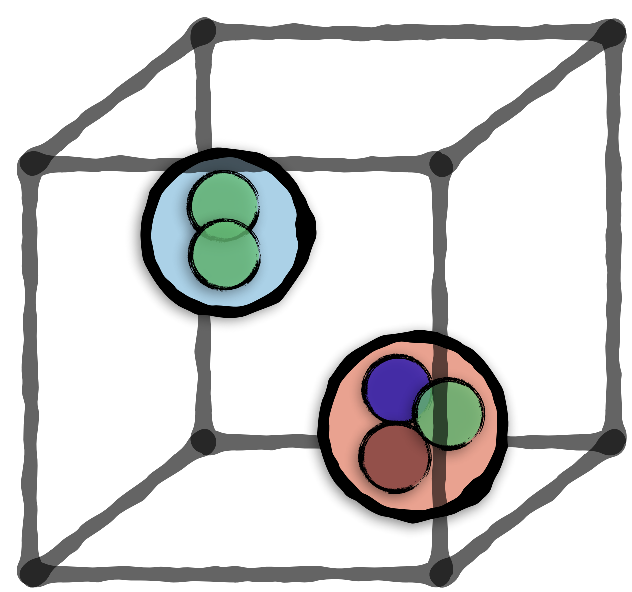


$$s_R = (E_R - \tfrac{i}{2}\Gamma_R)^2$$

Lattice QCD

- Lattice spacing:  $\} a \sim 0.03 - 0.1 \text{ fm}$

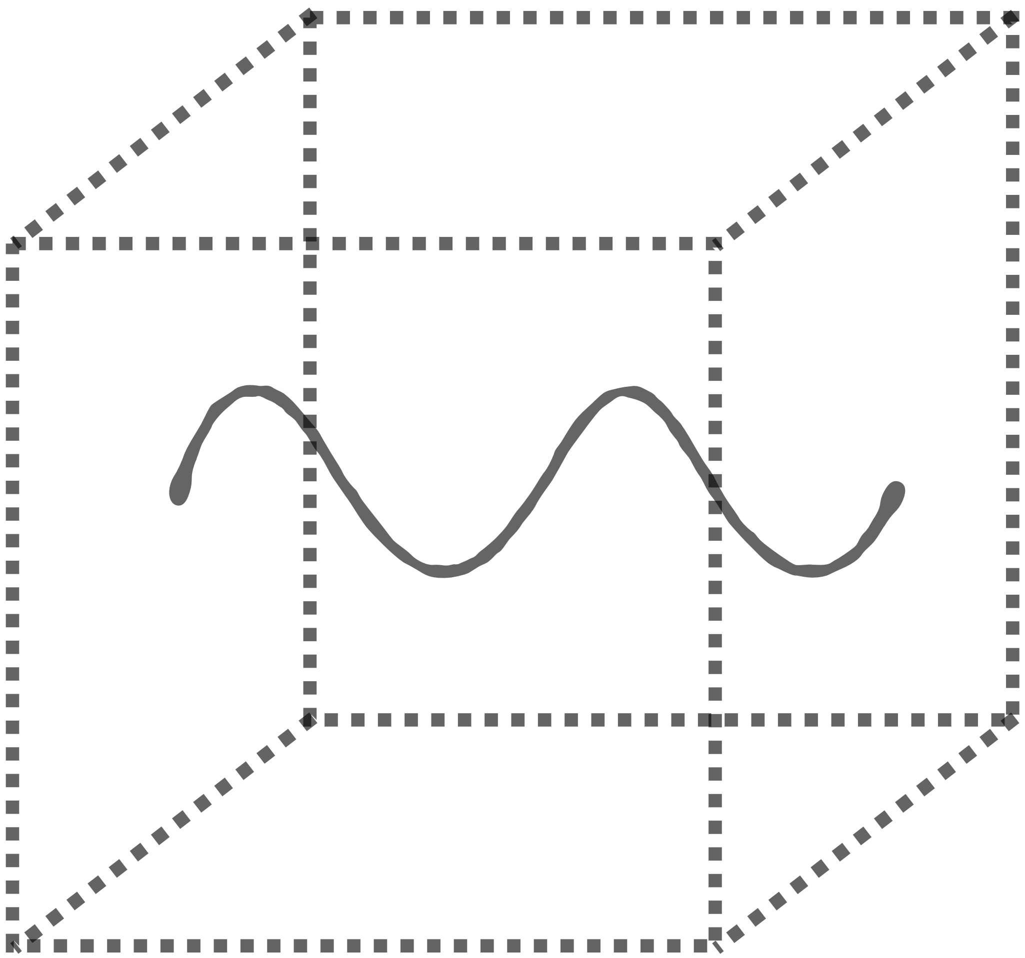
- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$

- Finite volume: 

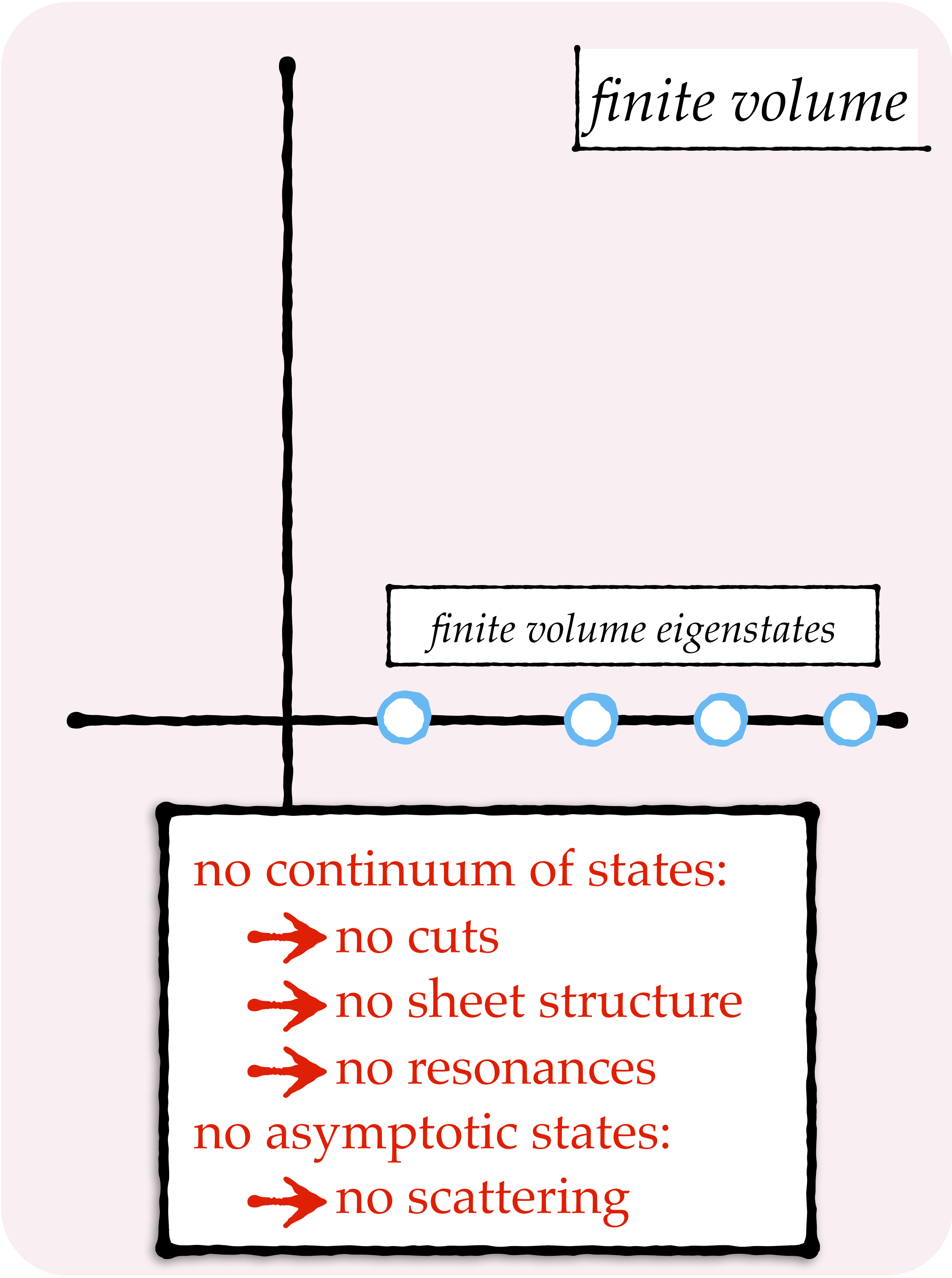
- Quark masses: $m_q \to m_q^{\text{phys.}}$

Have we '*mangled*' QCD too much?

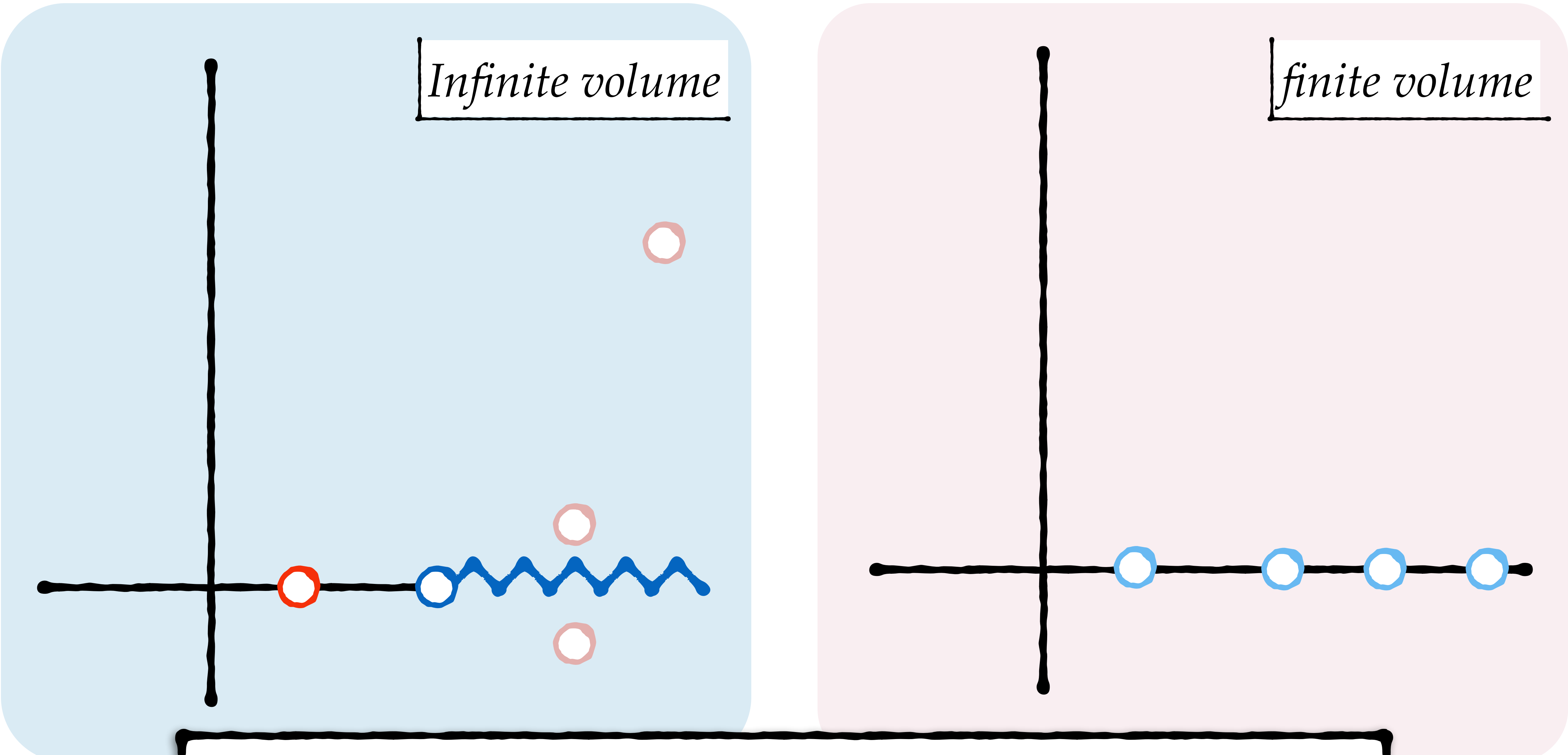
Finite volume spectrum



"only a discrete number of modes can exist in a finite volume"



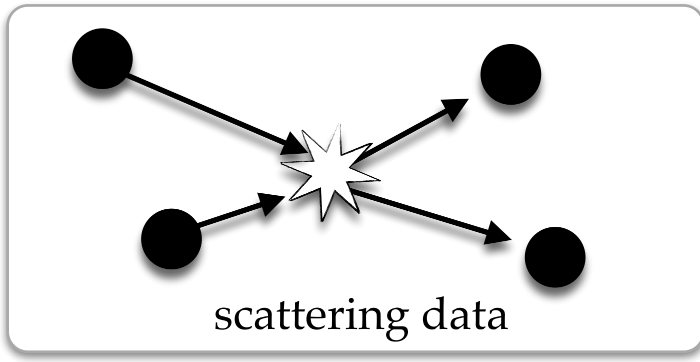
Finite vs. infinite volume spectrum



both pictures *are* QCD

the connection is perhaps not obvious since we have historically been "confined" to thinking about infinite volume physics

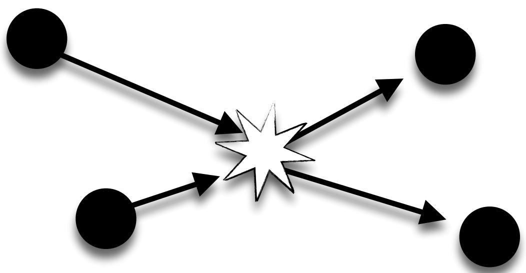
Experiment



amplitude analysis →

partial wave amplitudes

Experiment



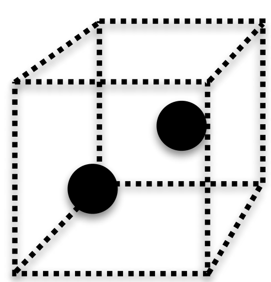
scattering data

amplitude analysis →

partial wave amplitudes

Lattice QCD

FV spectrum



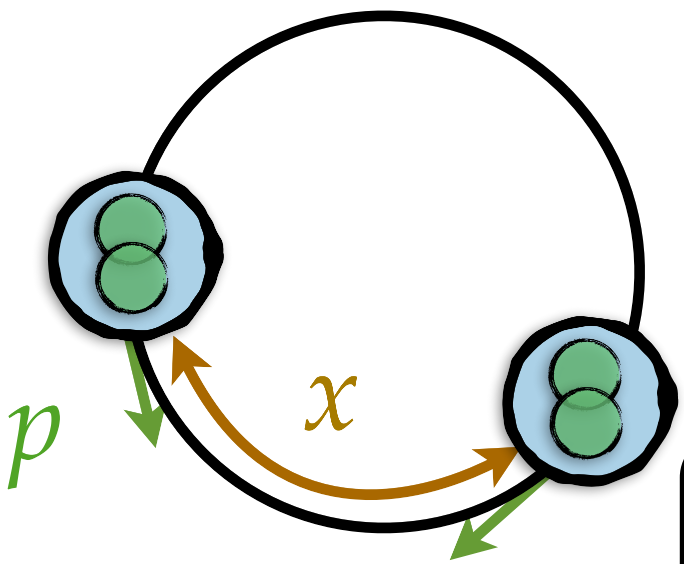
Lüscher formalism →

partial wave amplitudes

Physics in a 1D-box

Two identical particles:

infinite volume scattering phase shift



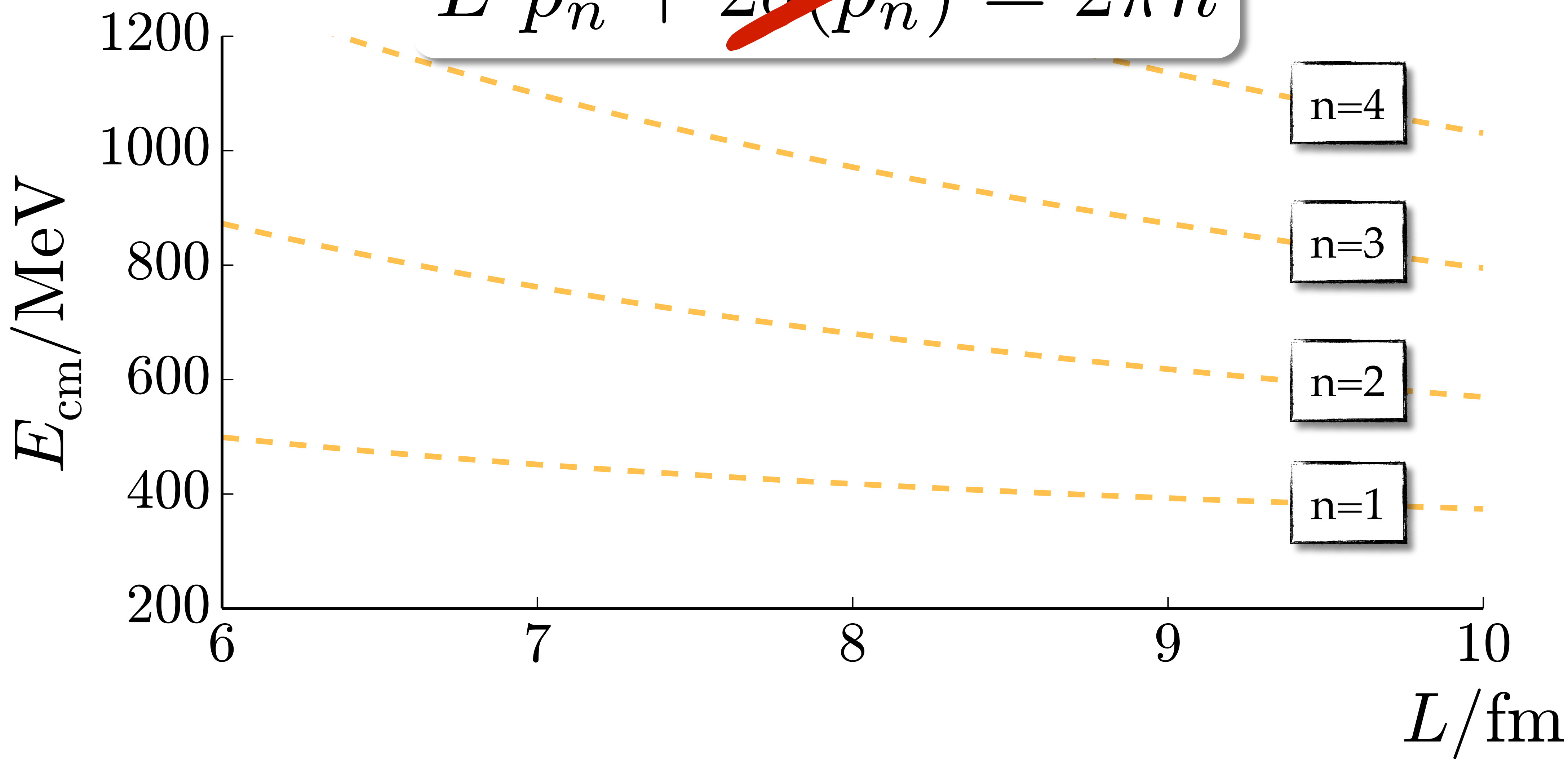
$$\psi(x) \sim \cos(p|x| + \delta(p))$$

Asymptotic wavefunction

Periodicity: $L\ p_n + 2\delta(p_n) = 2\pi n$

Physics in a 1D-box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$



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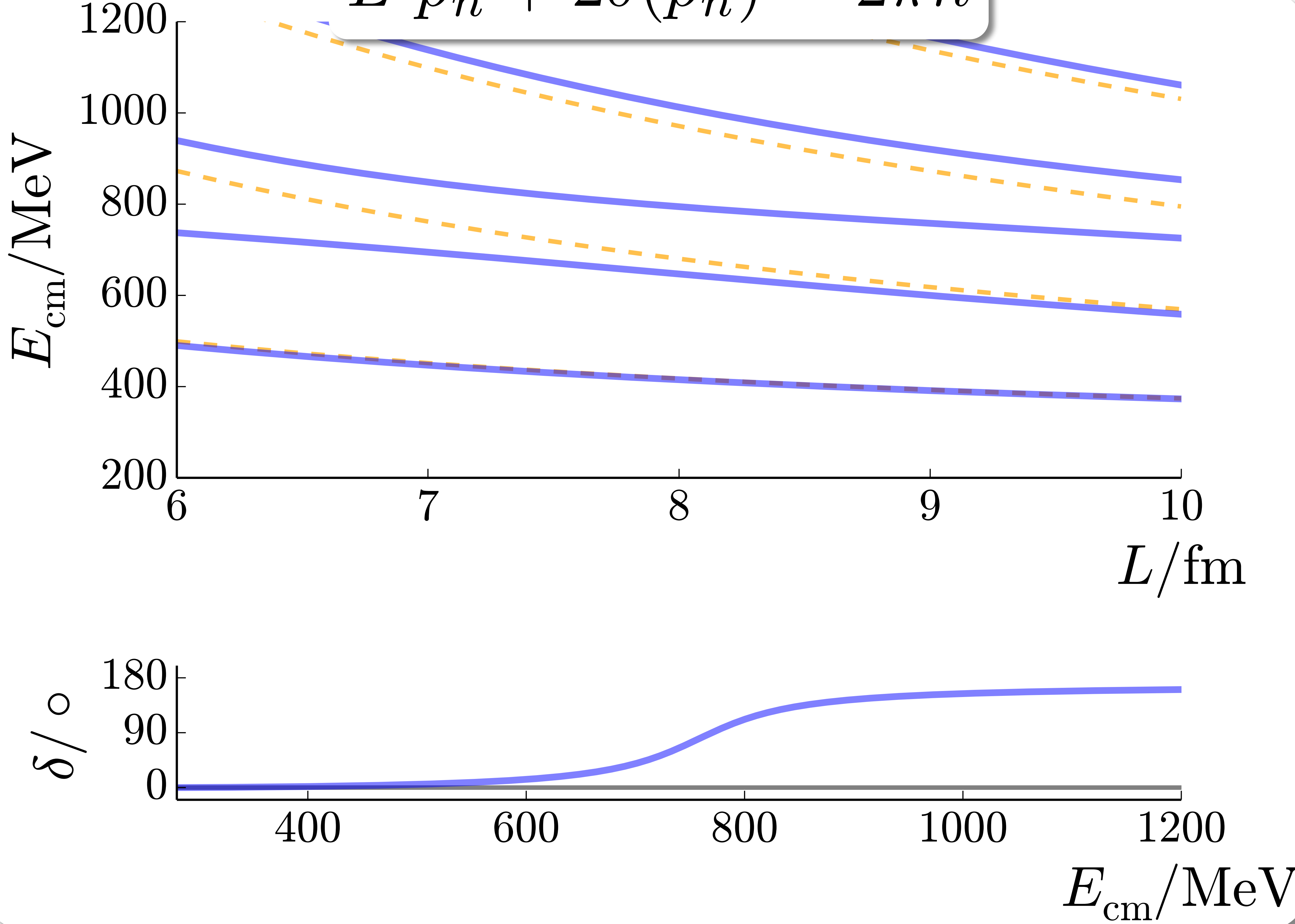

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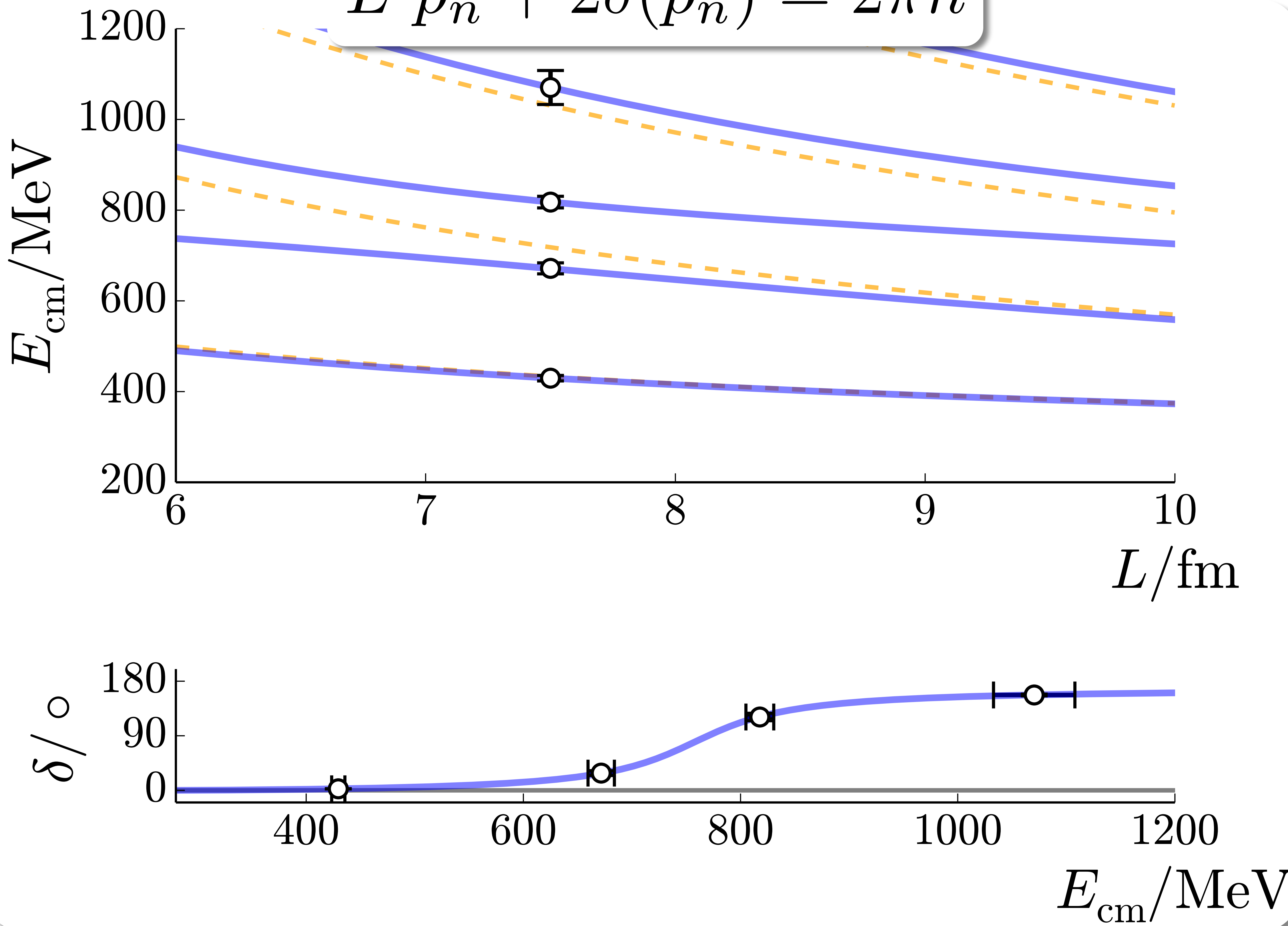

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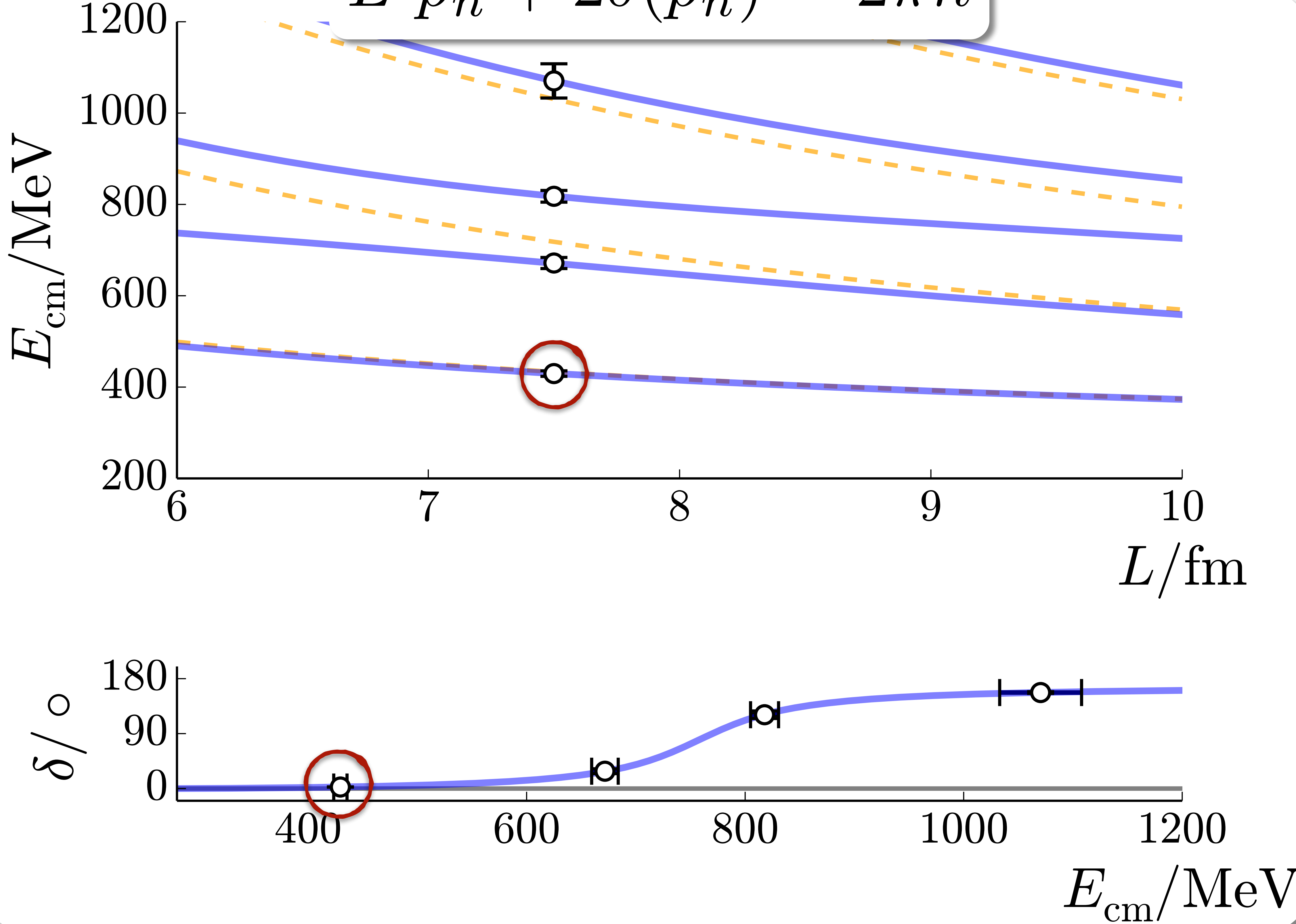

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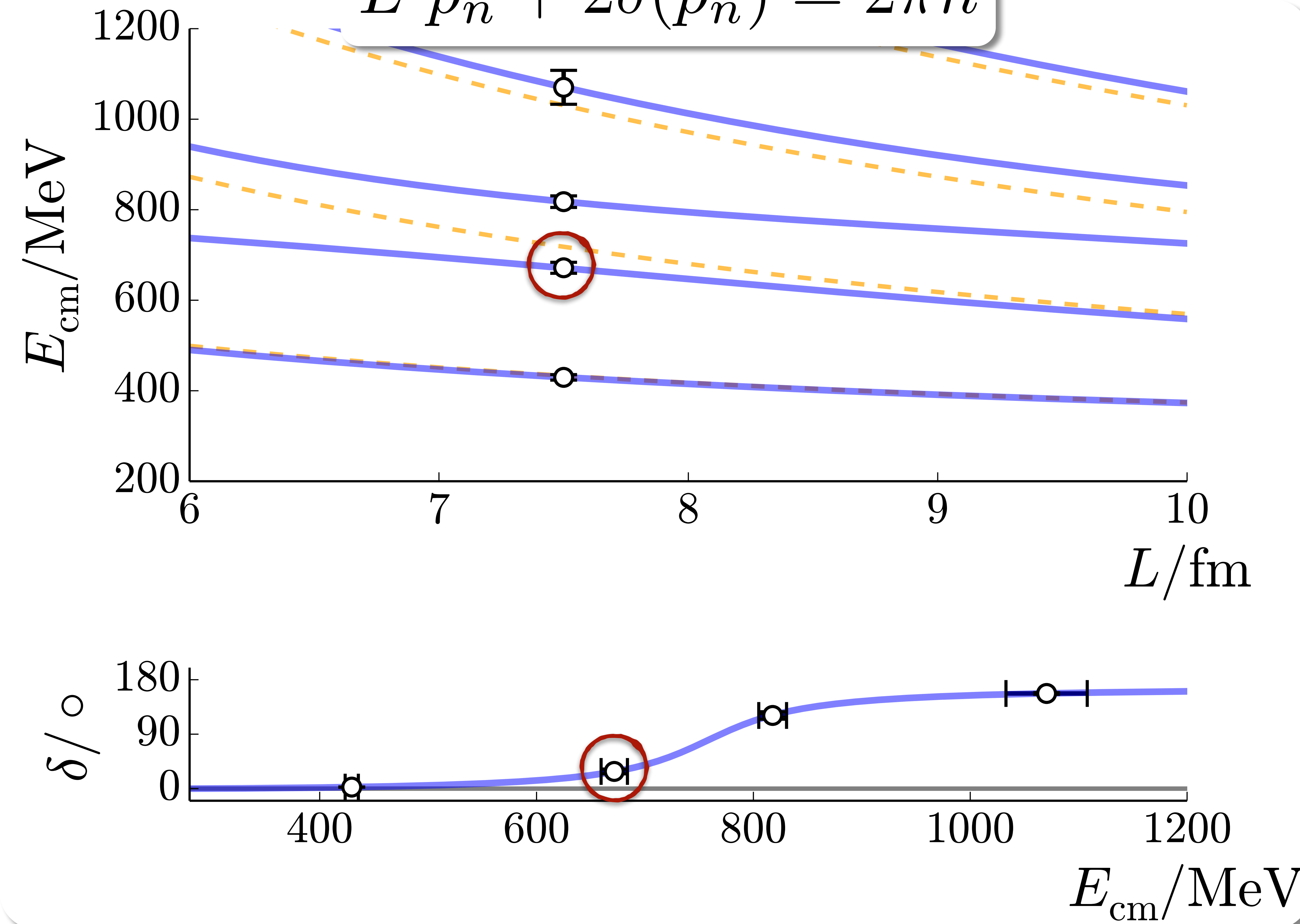

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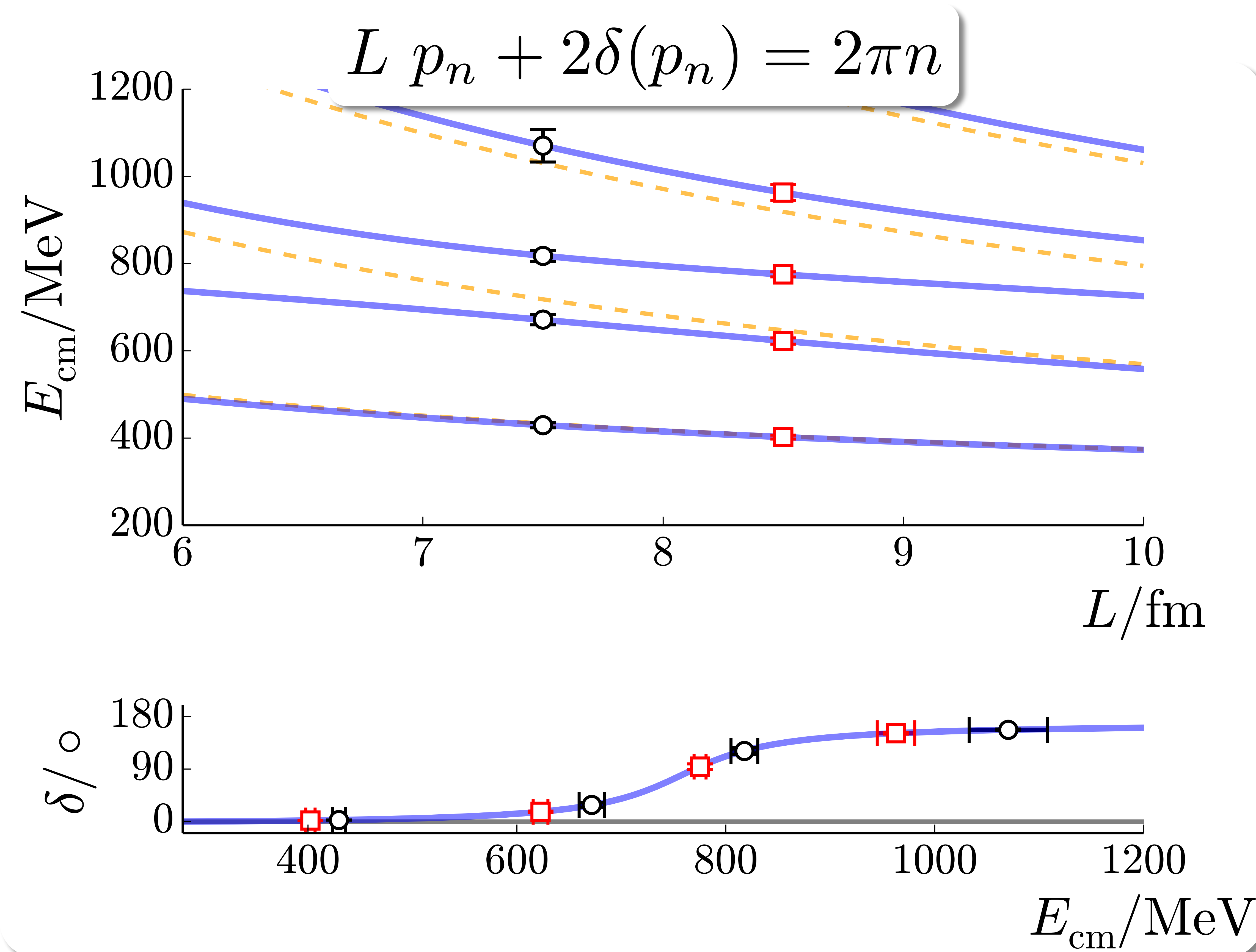
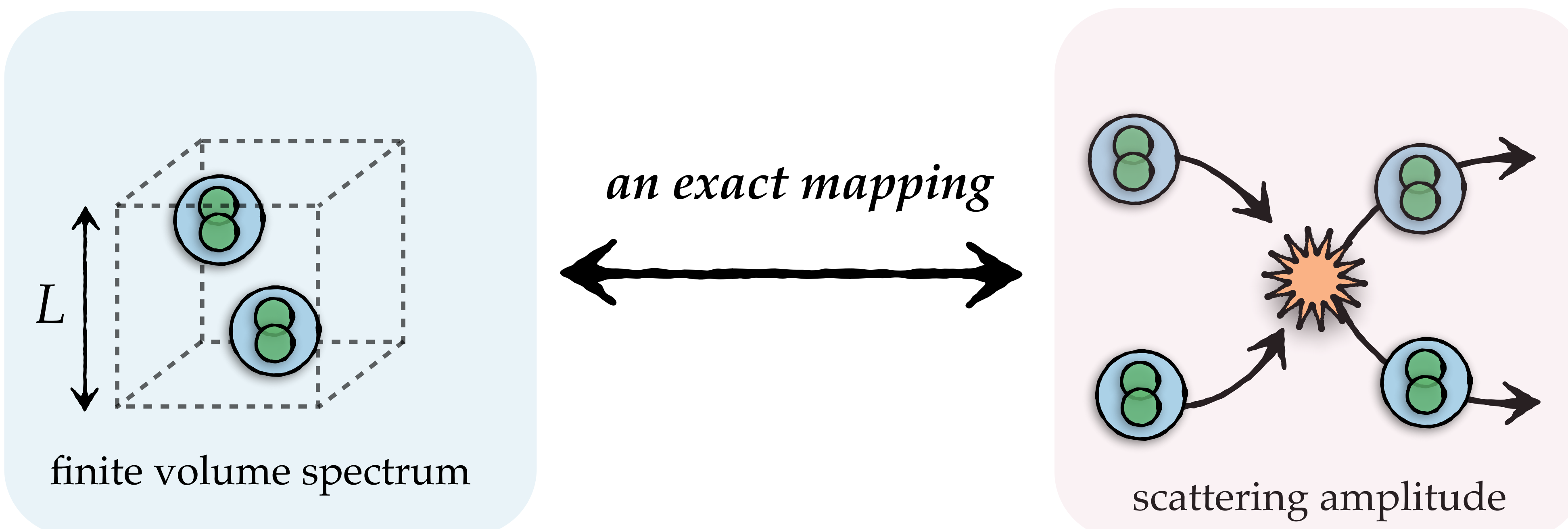


Lüscher formalism

spectrum satisfy: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$



E_L = finite volume spectrum

L = finite volume

F = known function

$\mathcal{M}$ = scattering amplitude

$$\mathcal{M} = \frac{8\pi E_{\rm cm}}{p}\frac{1}{\cot\delta - i}$$

Lüscher formalism

spectrum satisfy: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Bernard, Lage, Meißner & Rusetsky (2008) [Nπ systems]
- Gockeler, Horsley, Lage, Meißner, Rakow, Rusetsky, Schierholz, & Zanotti (2012) [Nπ systems]
- RB, Davoudi, Luu & Savage (2013) [generic spinning systems]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- RB (2014) / RB & Hansen (2015) [moving inelastic spinning particles]

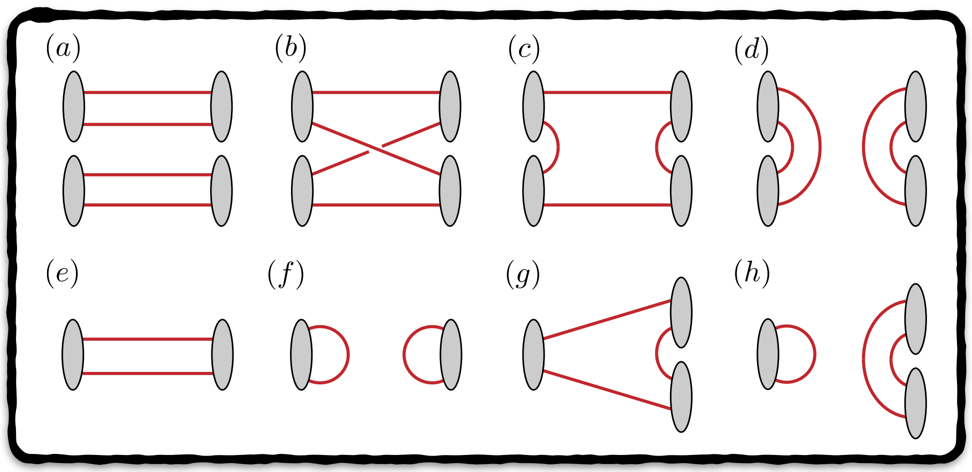
Extracting the spectrum

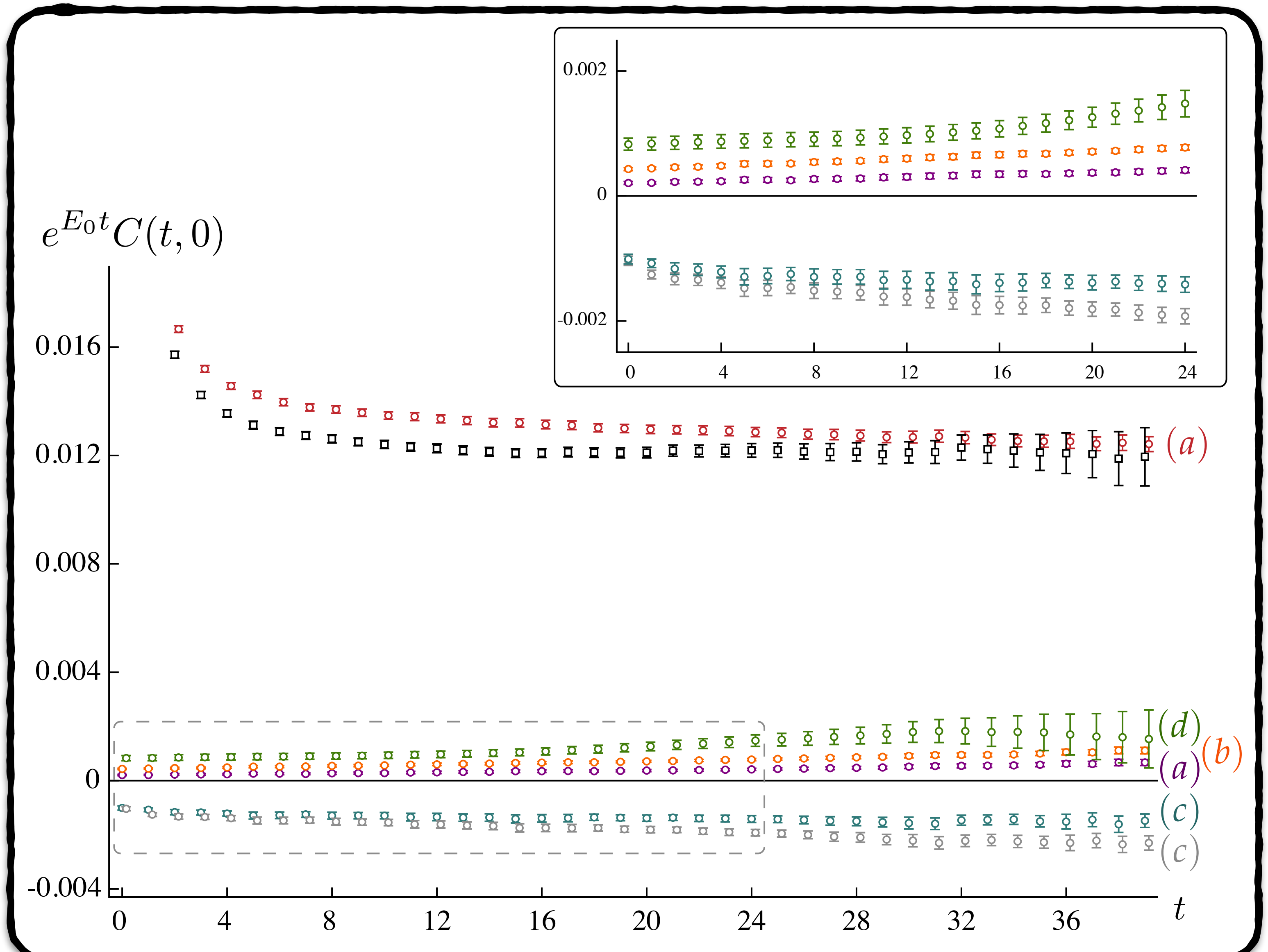
Two-point correlation functions:

$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

- Evaluate **all** Wick contraction

e.g. isoscalar: $\pi_{[000]}\pi_{[110]}$, $m_\pi = 236$ MeV





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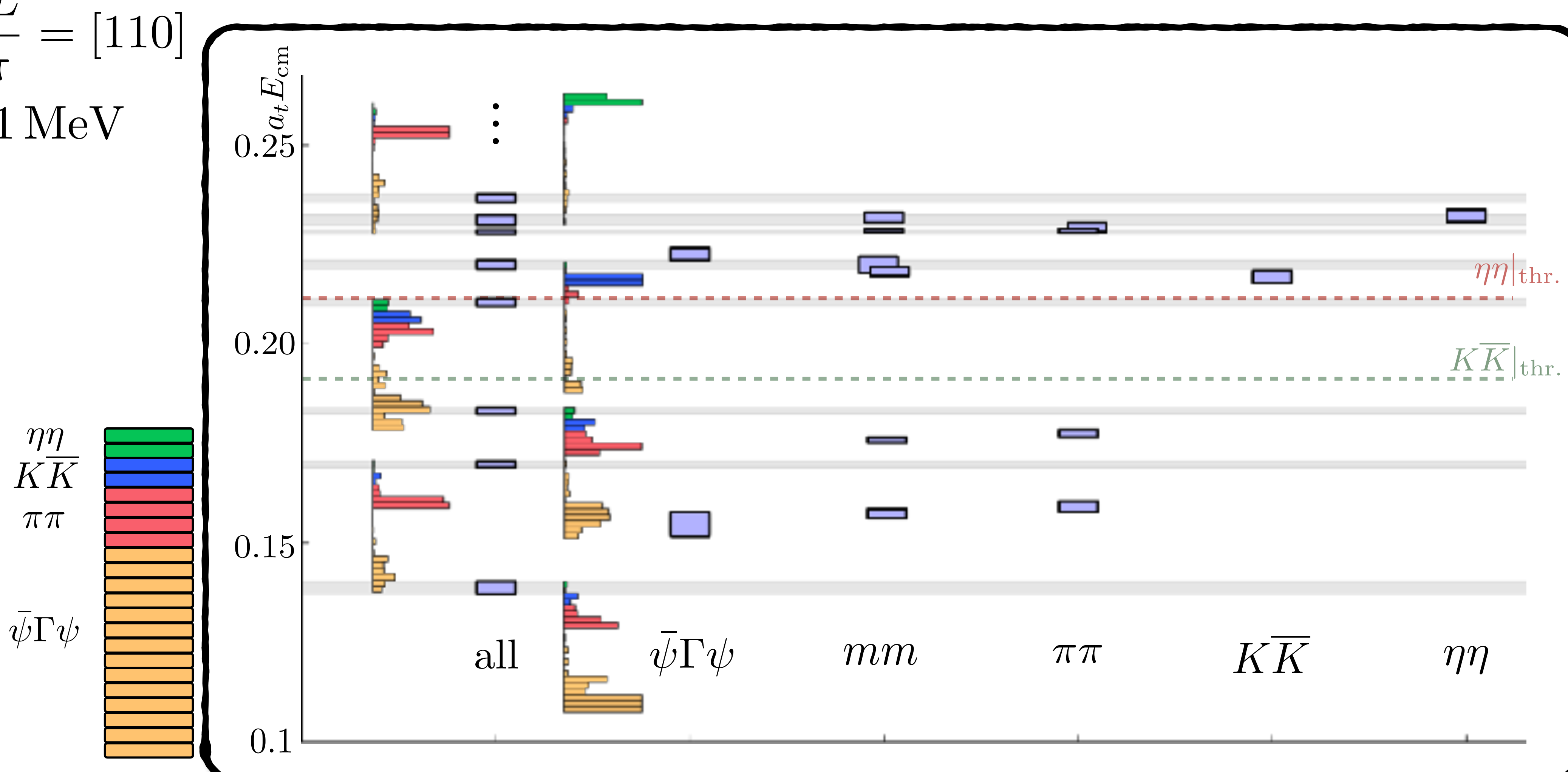
- Evaluate **all** Wick contraction
- Use a large basis of operators with the same quantum numbers
- ‘Diagonalize’ correlation function *variationally*

e.g. $\vec{d} = \frac{\vec{P}L}{2\pi} = [110]$

$m_\pi = 391\,\text{MeV}$

$L/a_s = 24$



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- Evaluate **all** Wick contraction - [distillation - Peardon, *et al.* (Hadron Spectrum, 2009)]
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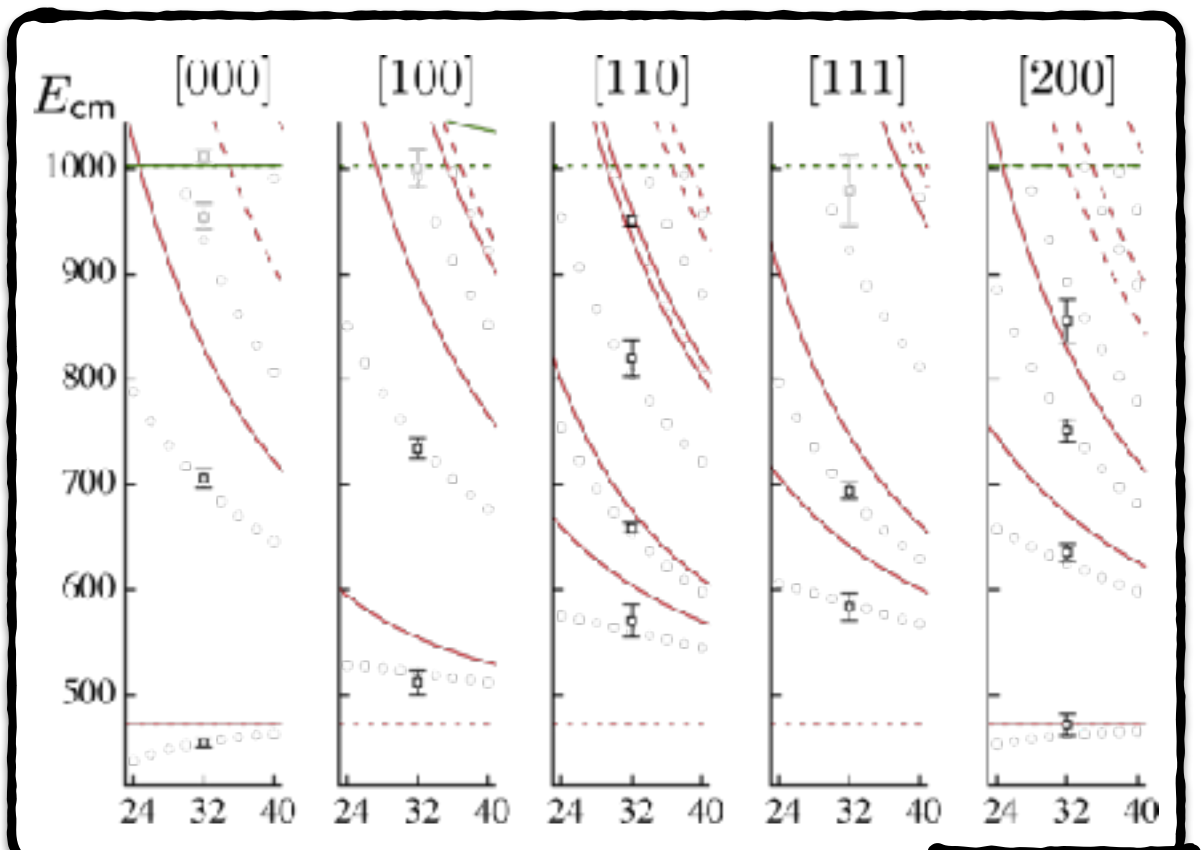
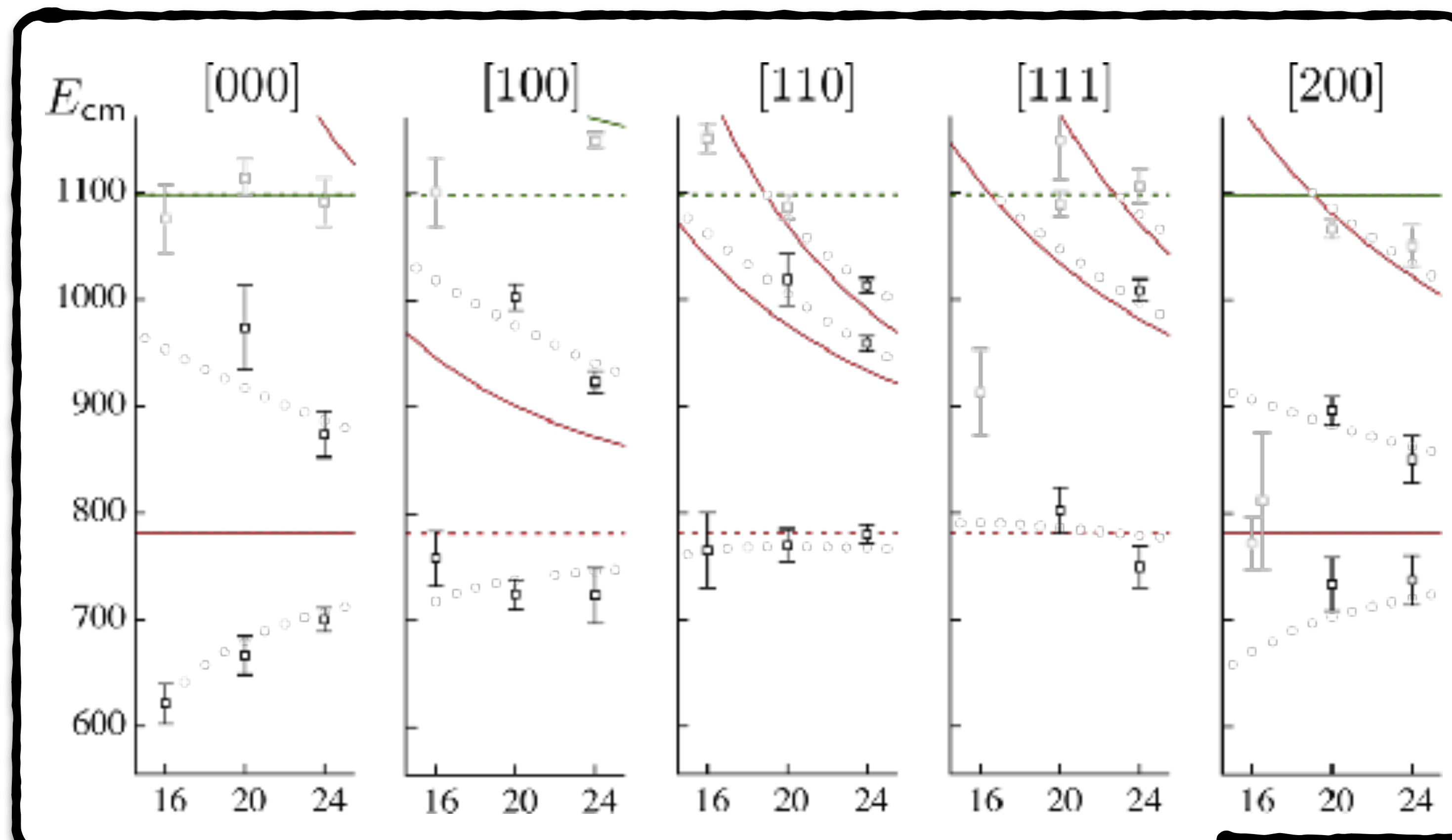
- 'Diagonalize' correlation function *variationally*

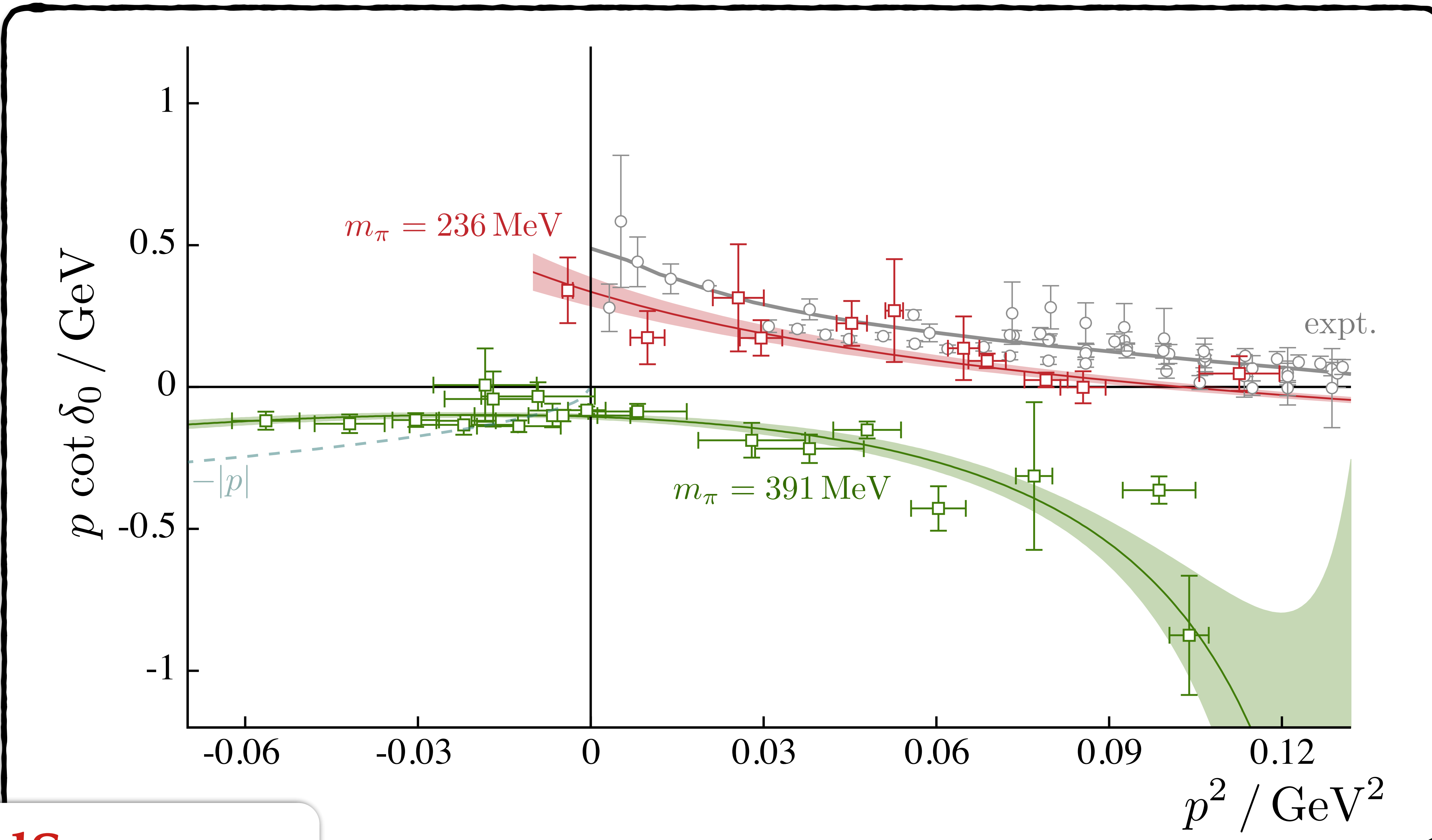


m_π=236 MeV



m_π=391 MeV

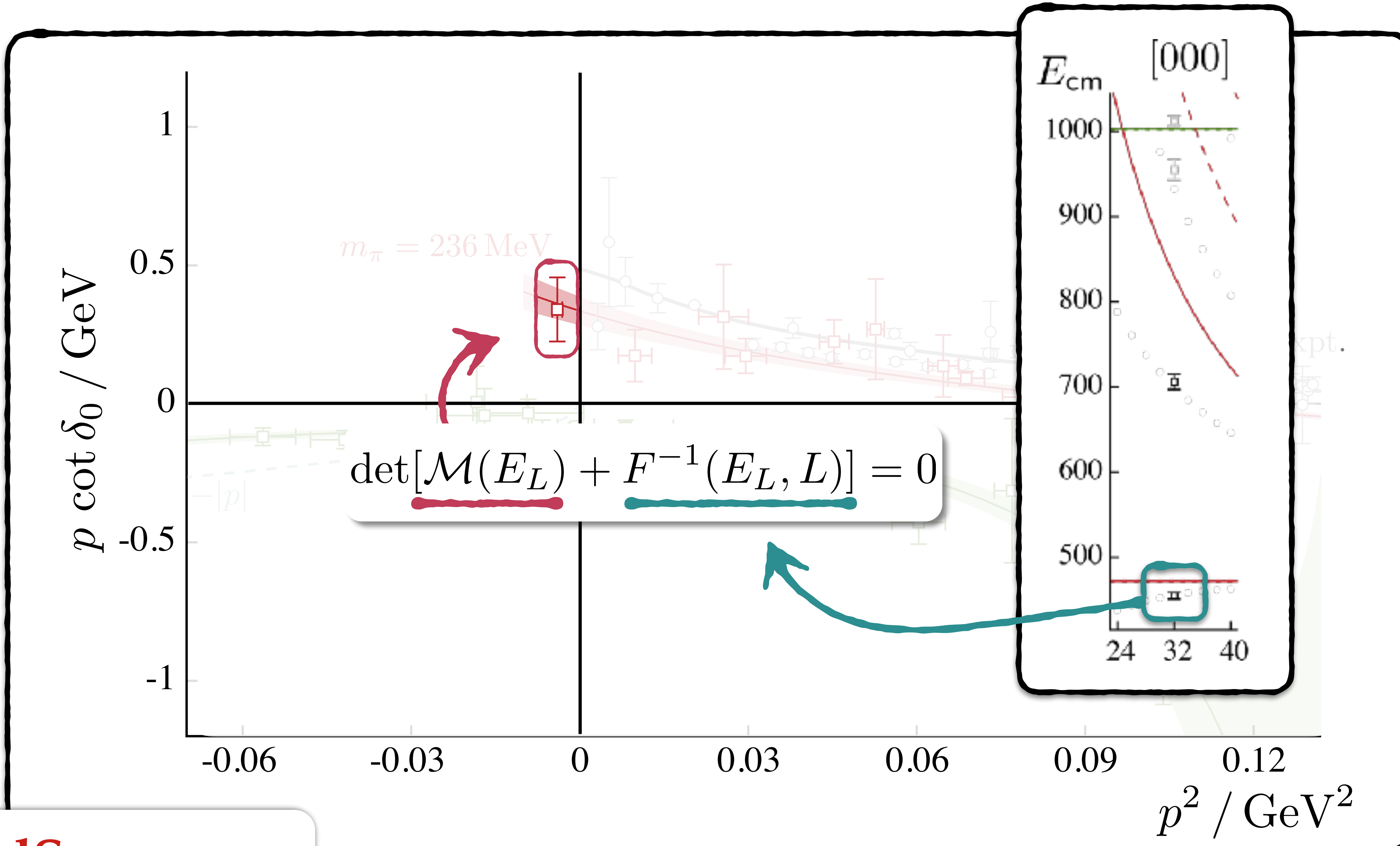
Isoscalar $\pi\pi$ scattering



HadSpec Collaboration

RB, Dudek, Edwards & Wilson (2016)

Isoscalar $\pi\pi$ scattering

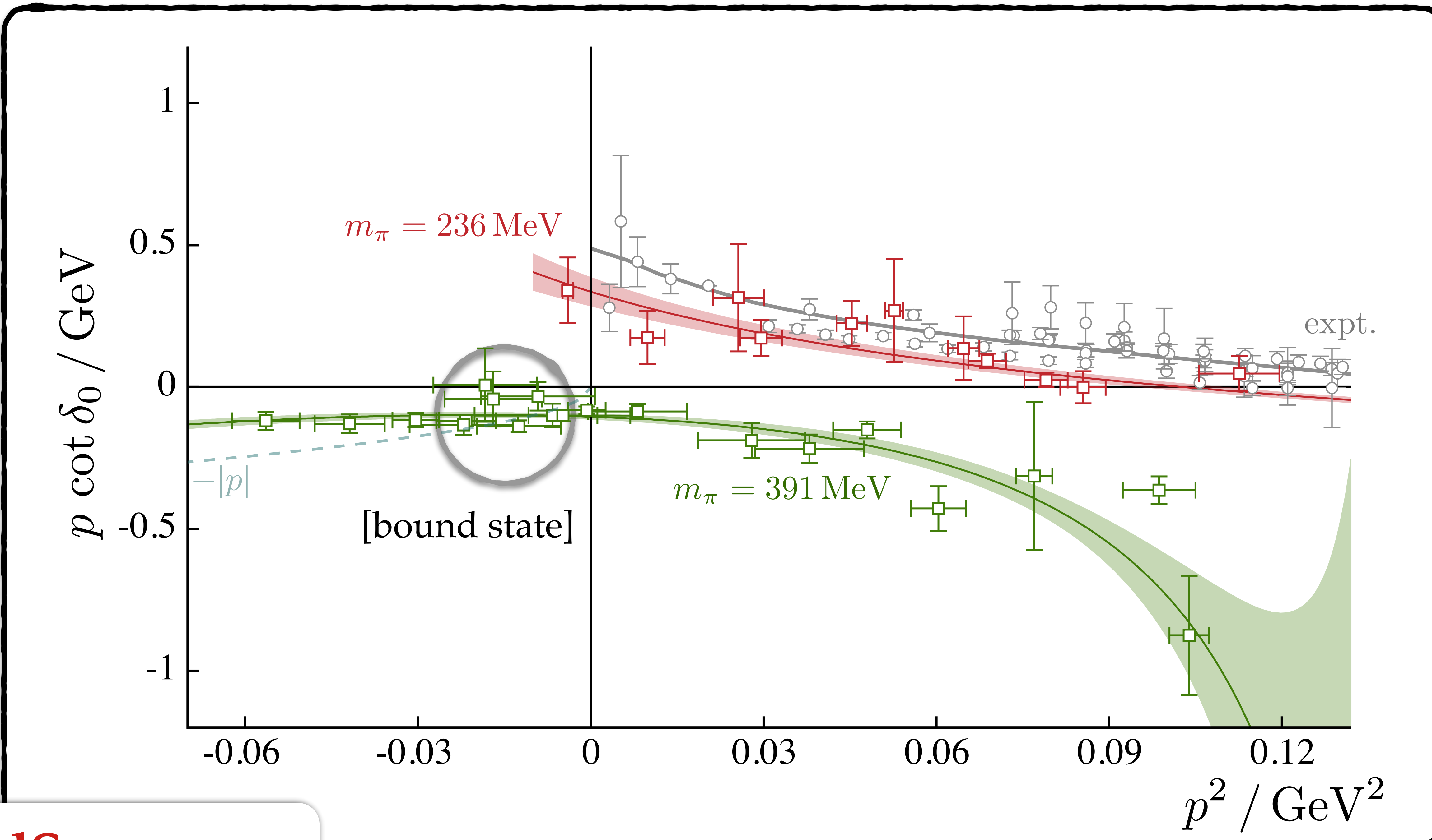


$$\det[\mathcal{M}(E_L) + F^{-1}(E_L, L)] = 0$$

HadSpec Collaboration

$$\mathcal{M}_0 = \frac{16\pi E_{\mathrm{cm}}}{p\cot\delta_0 - ip}$$

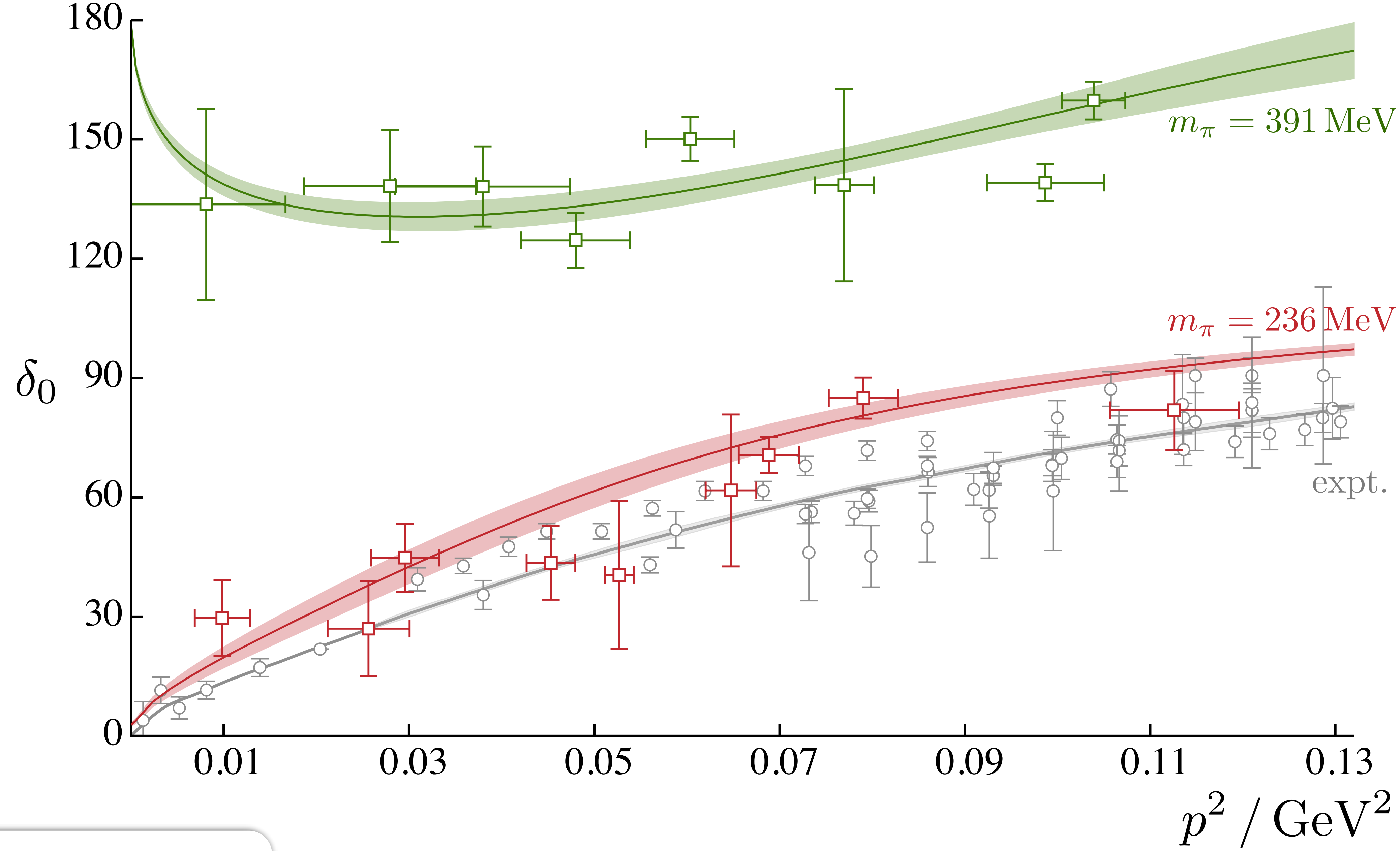
Isoscalar $\pi\pi$ scattering



HadSpec Collaboration

$$\mathcal{M} \sim \frac{1}{p \cot \delta_0 - ip} \to \frac{1}{p \cot \delta_0 + |p|}$$

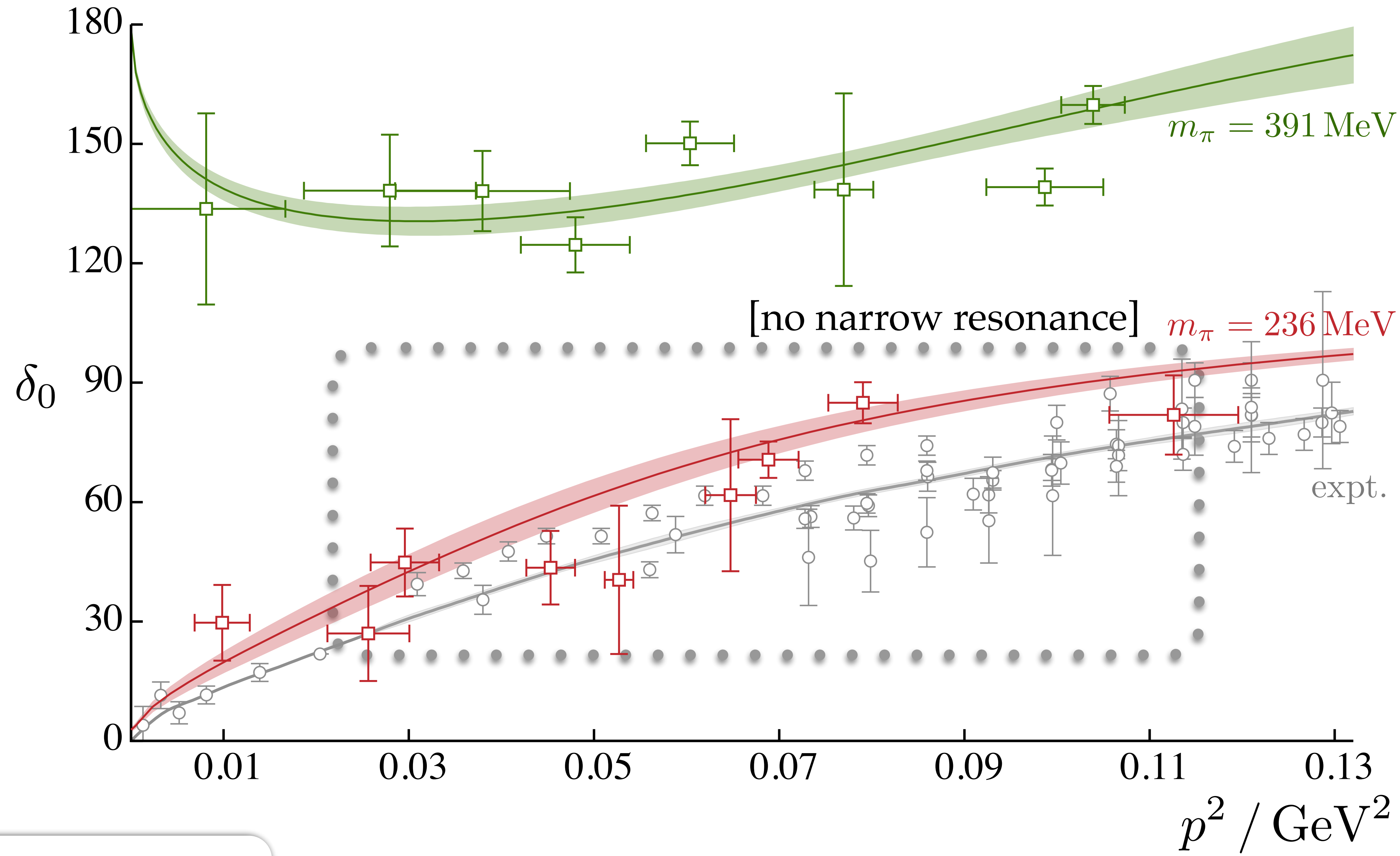
Isoscalar $\pi\pi$ scattering



HadSpec Collaboration

RB, Dudek, Edwards & Wilson (2016)

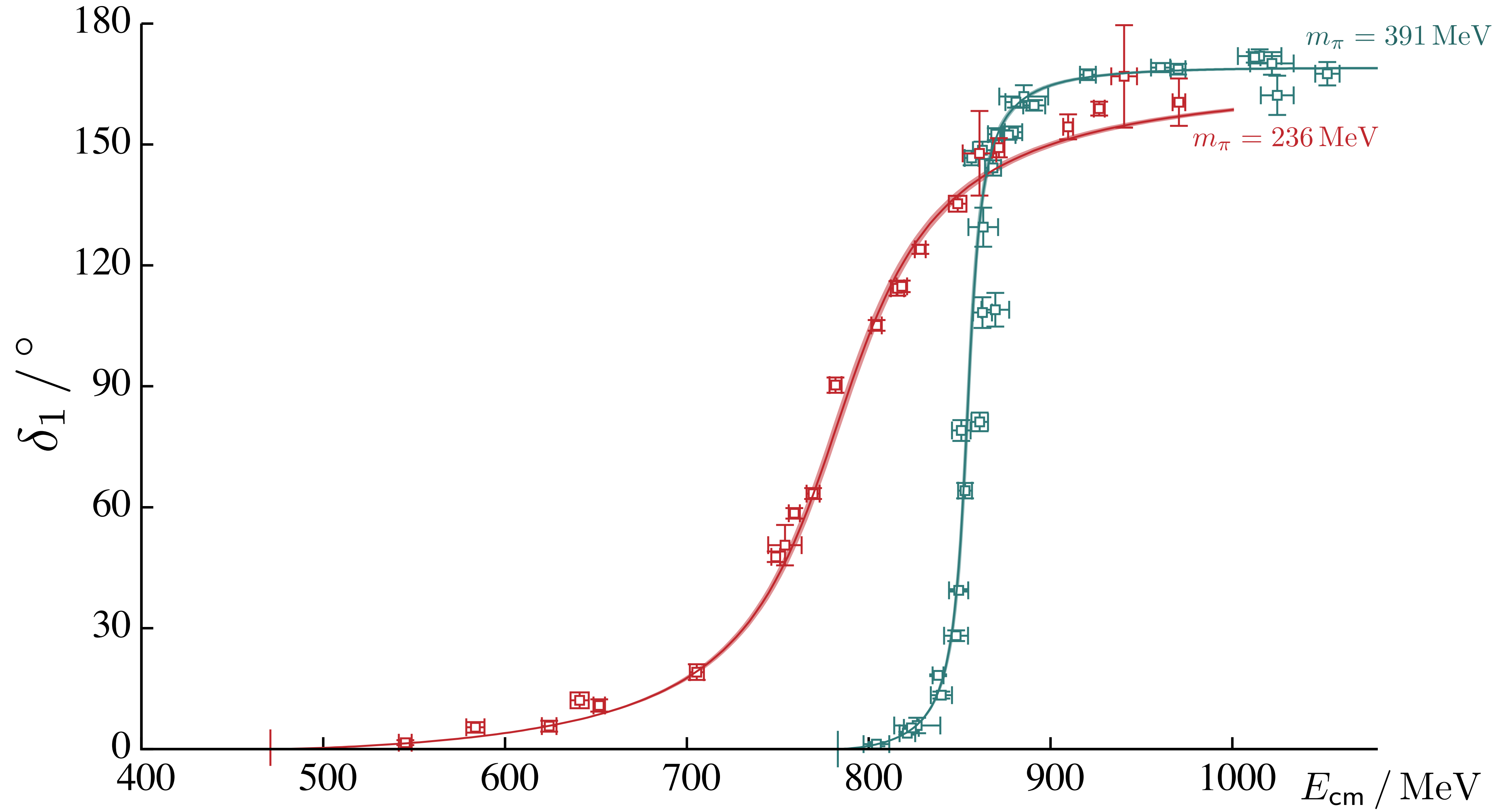
Isoscalar $\pi\pi$ scattering



HadSpec Collaboration

RB, Dudek, Edwards & Wilson (2016)

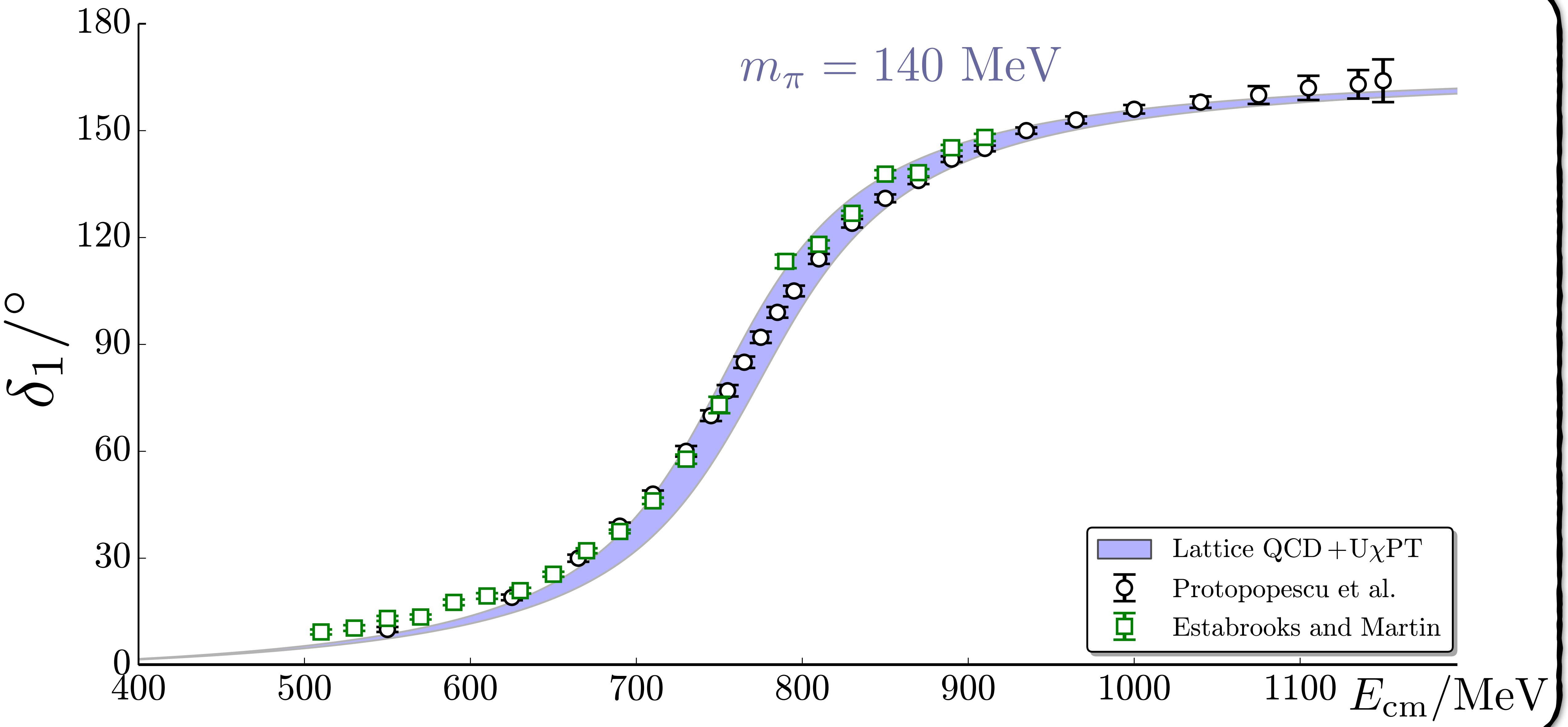
Isovector $\pi\pi$ scattering



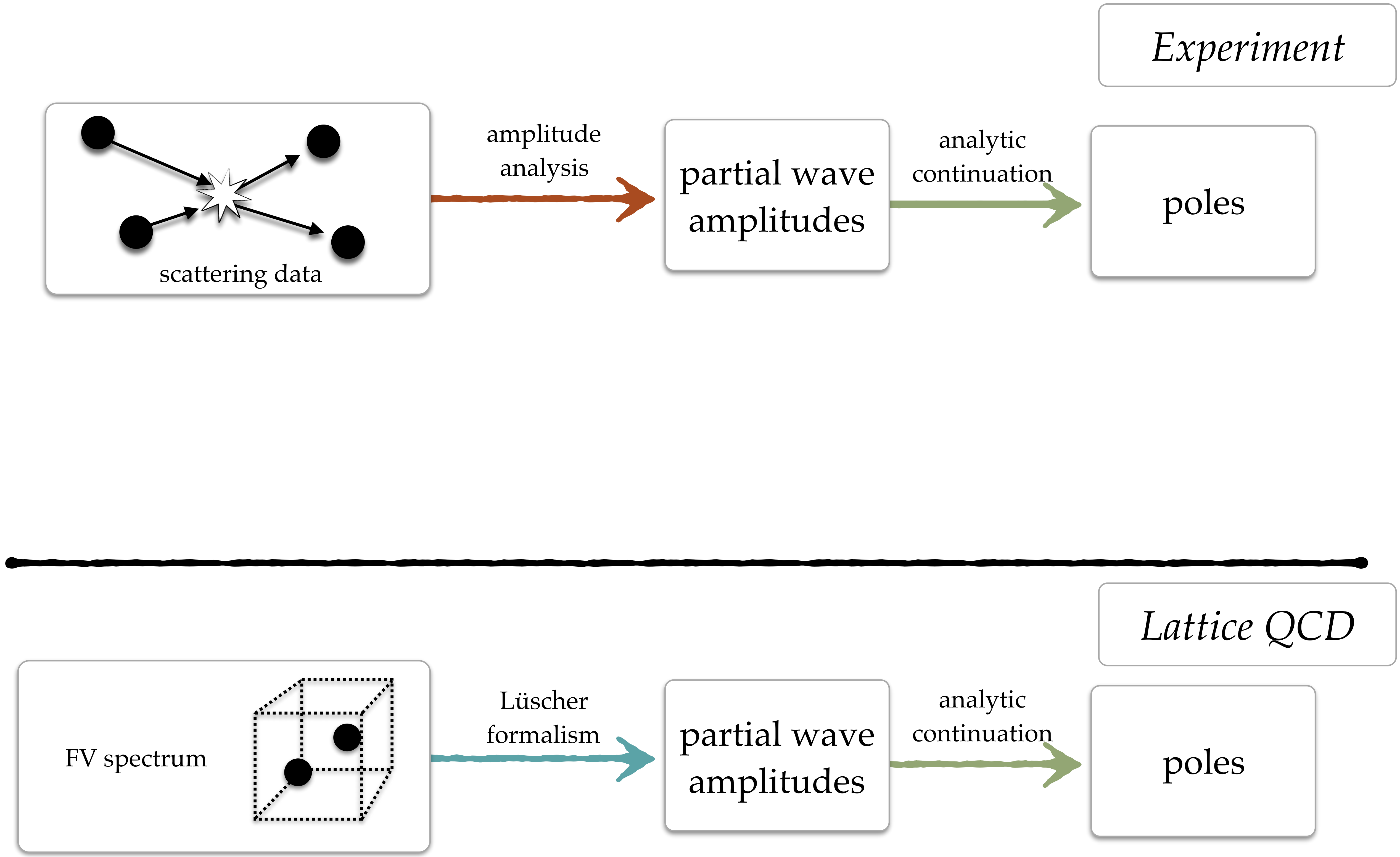
HadSpec Collaboration

Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

Comparison with experiment



Bolton, RB & Wilson Phys.Lett. B757 (2016) 50-56.


Experiment
scattering data
amplitude analysis
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Lattice QCD
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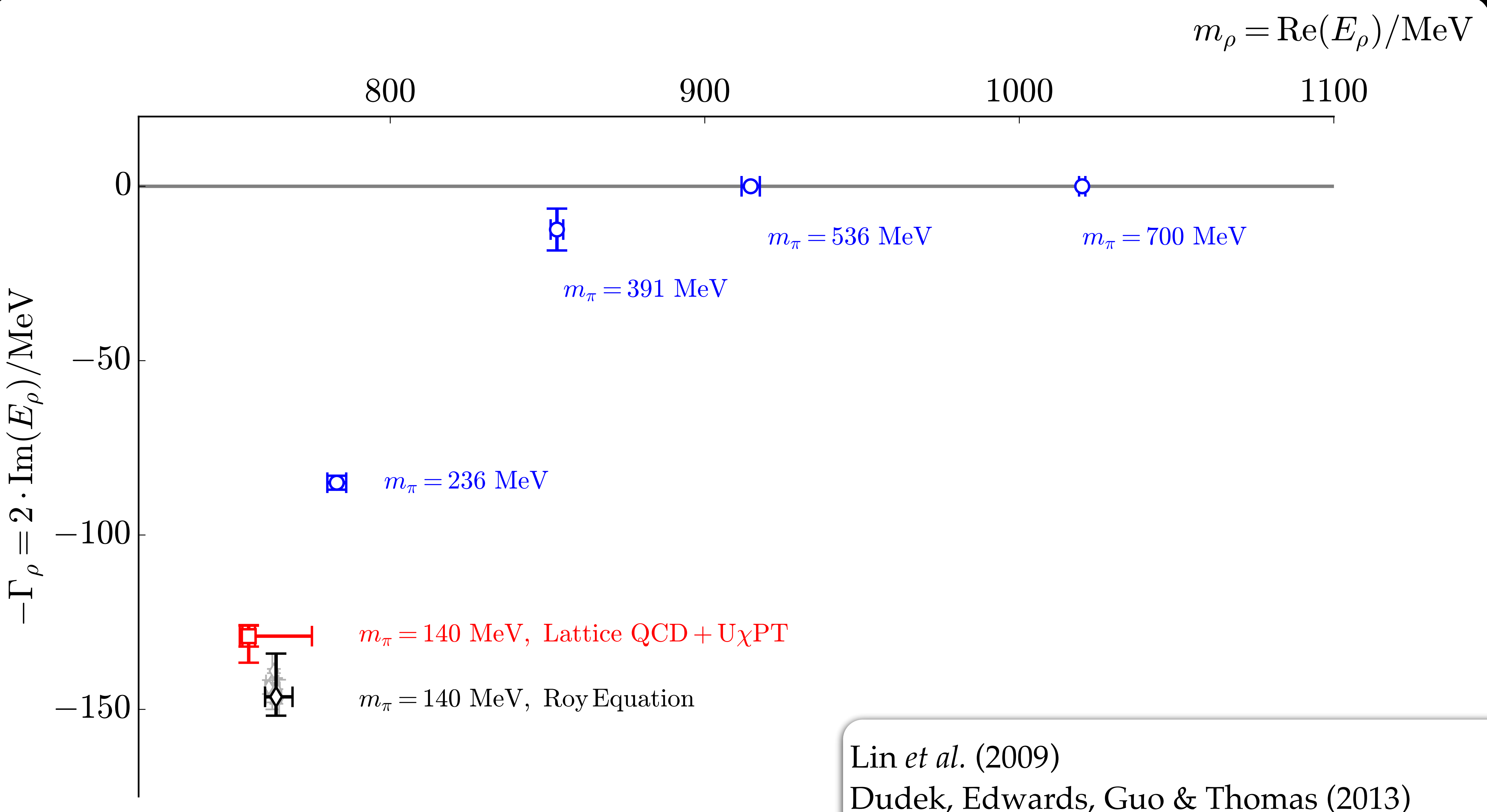
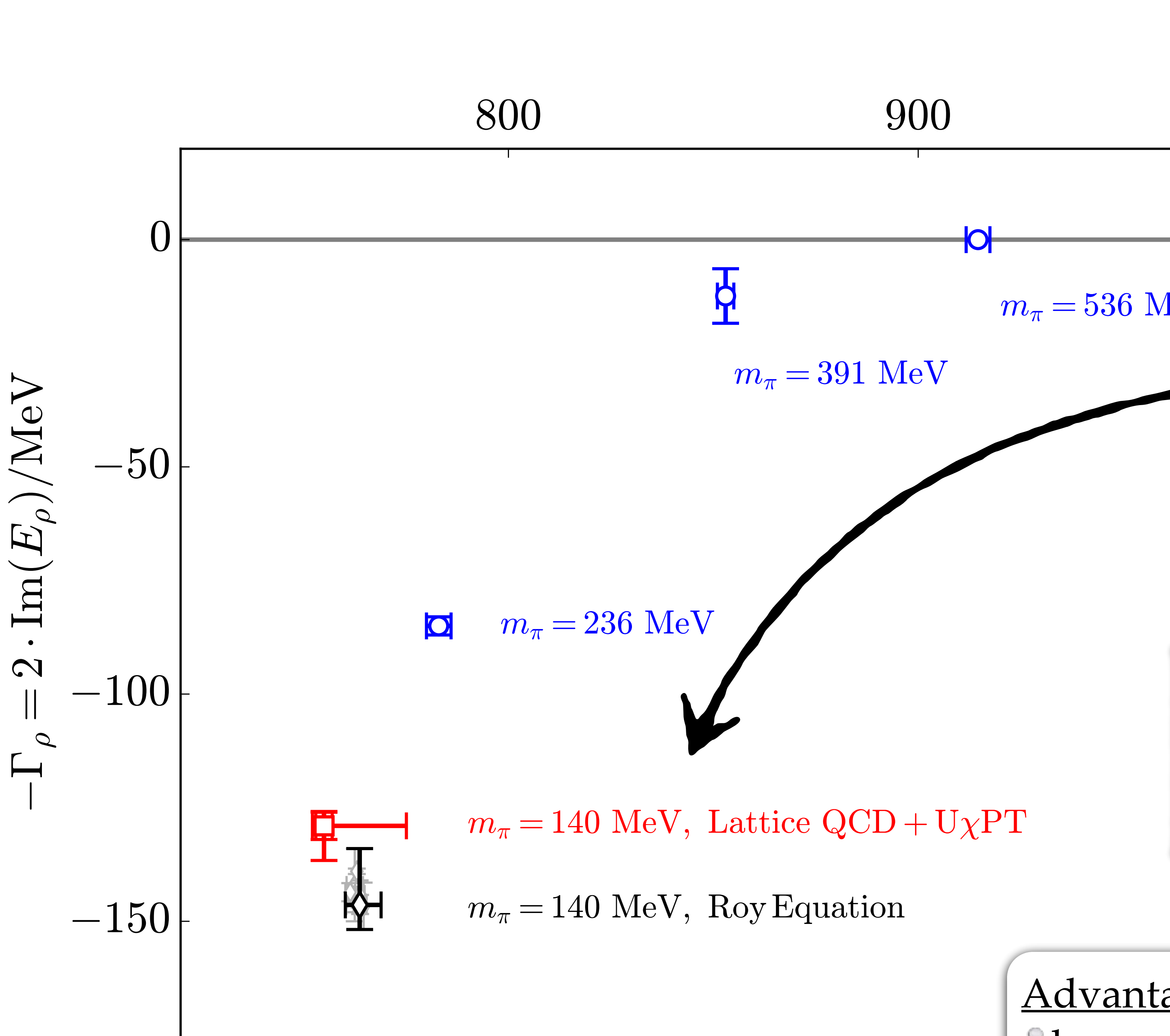
The ϱ vs m_π



Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

The ϱ vs m_π



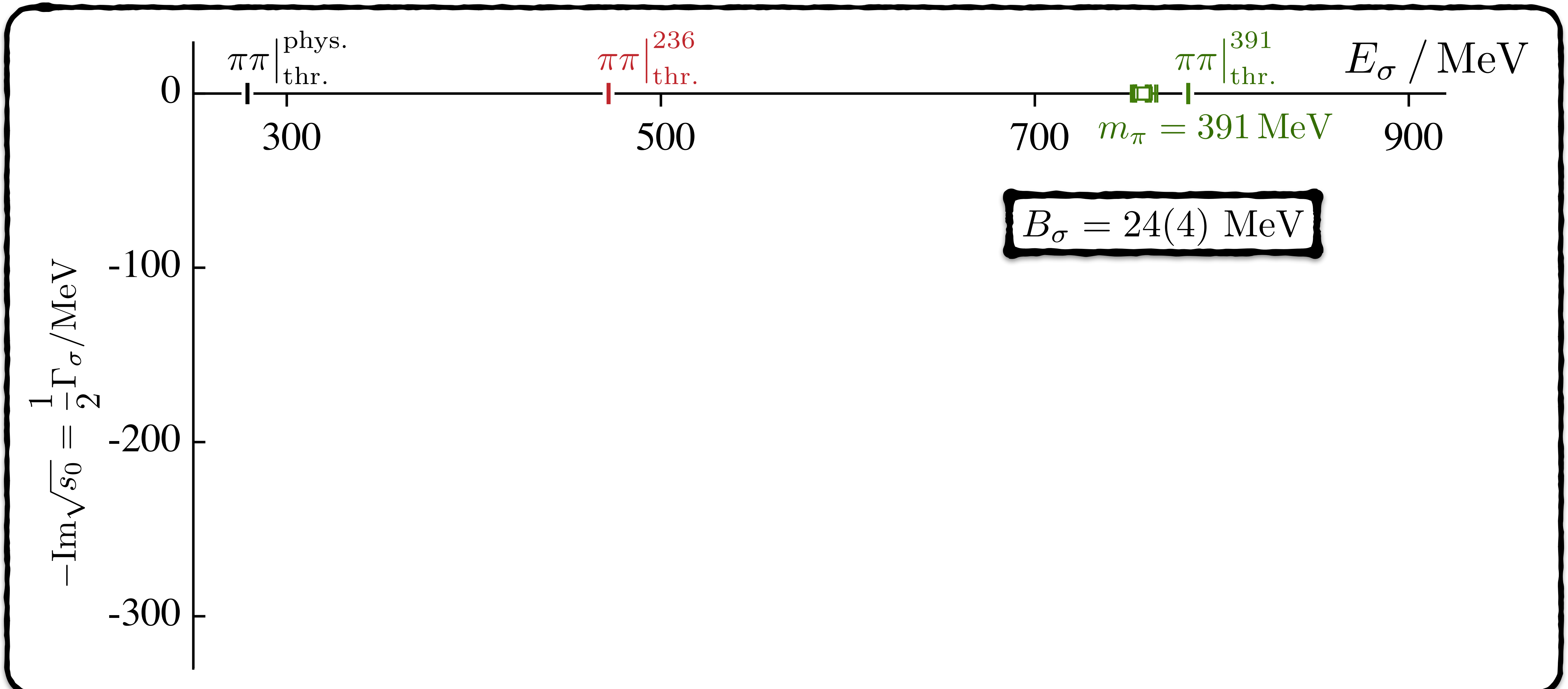
$$\mathcal{M}(s \sim s_0) \sim \frac{g^2}{s_0 - s}$$

$$g = g(m_\pi),\ s_0 = s_0(m_\pi)$$

<u>Advantage over experiment:</u>

- heavy quarks make broad resonances bound
- **unambiguously track poles in complex plane**

The σ / $f_0(500)$ vs m_π



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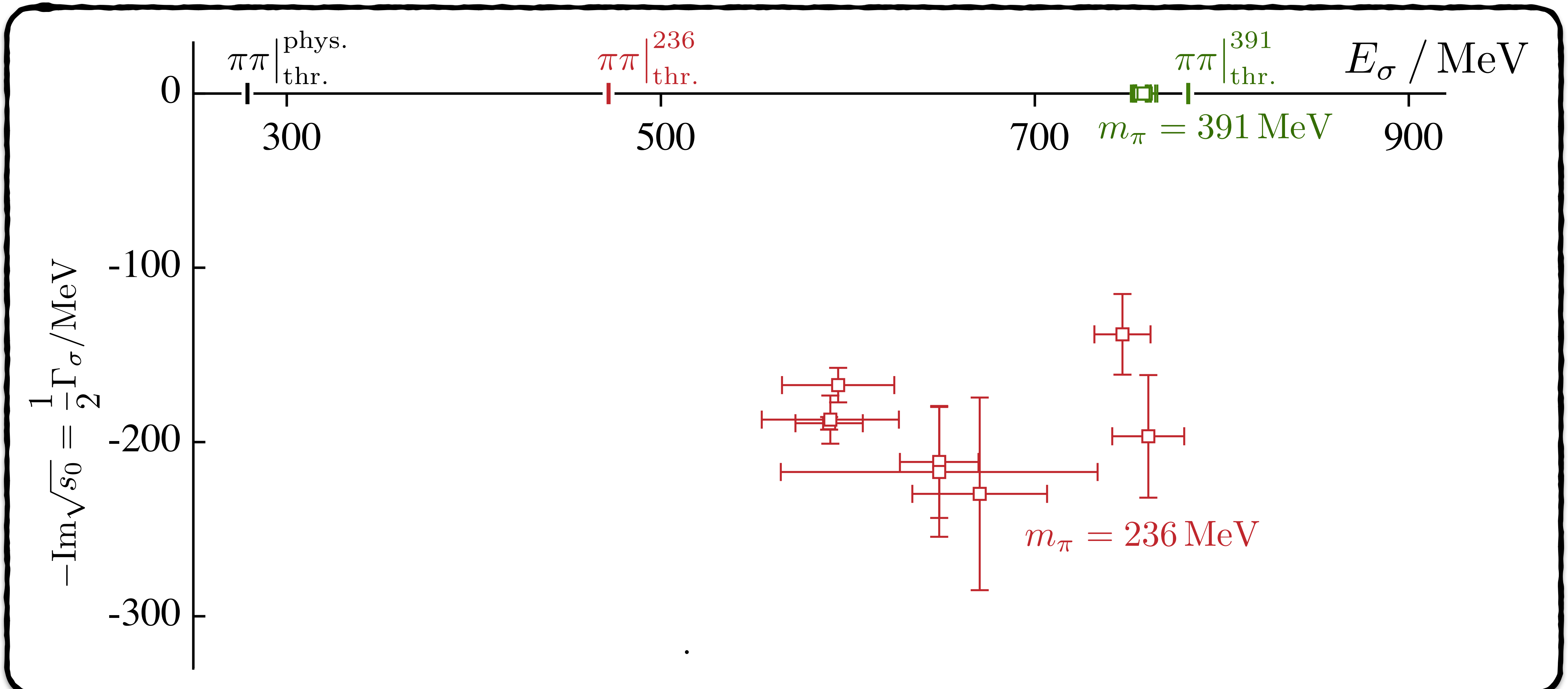
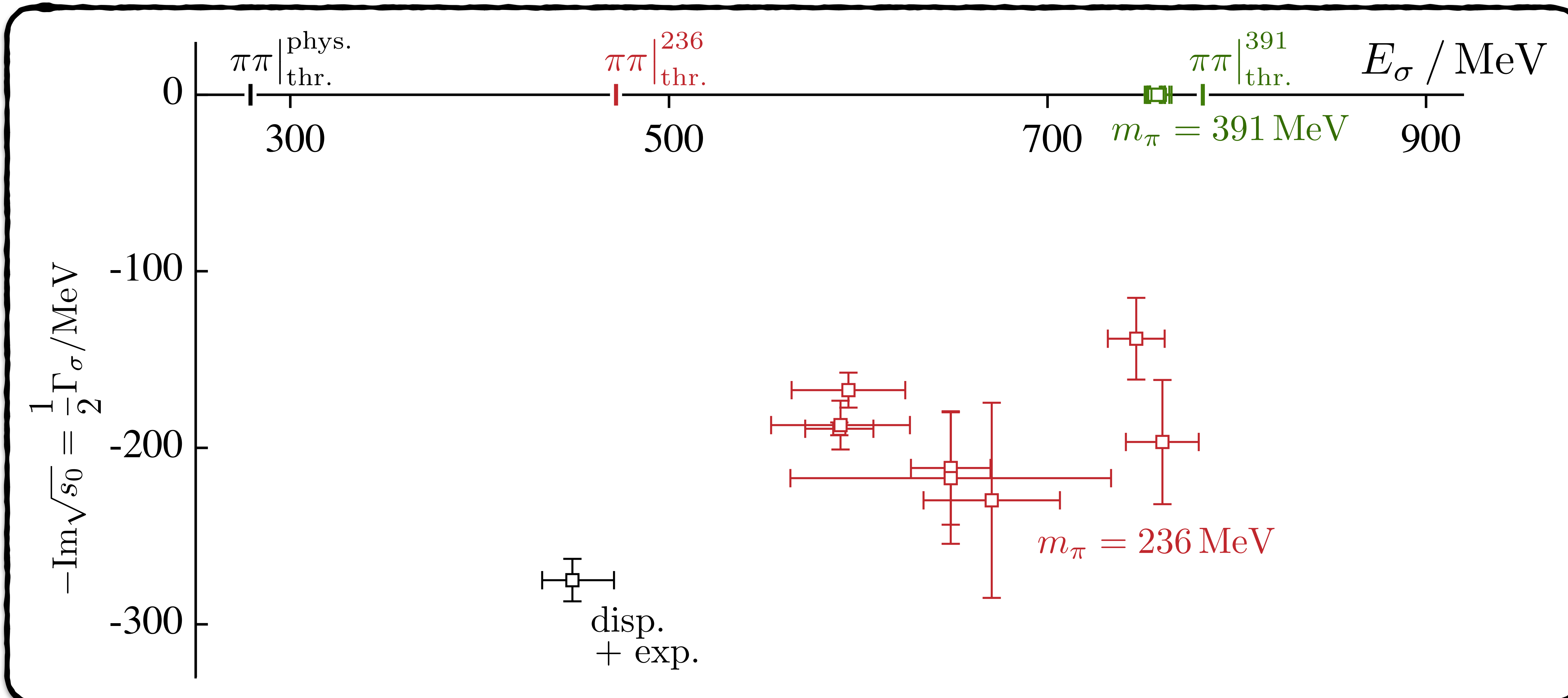
The σ / $f_0(500)$ vs m_π



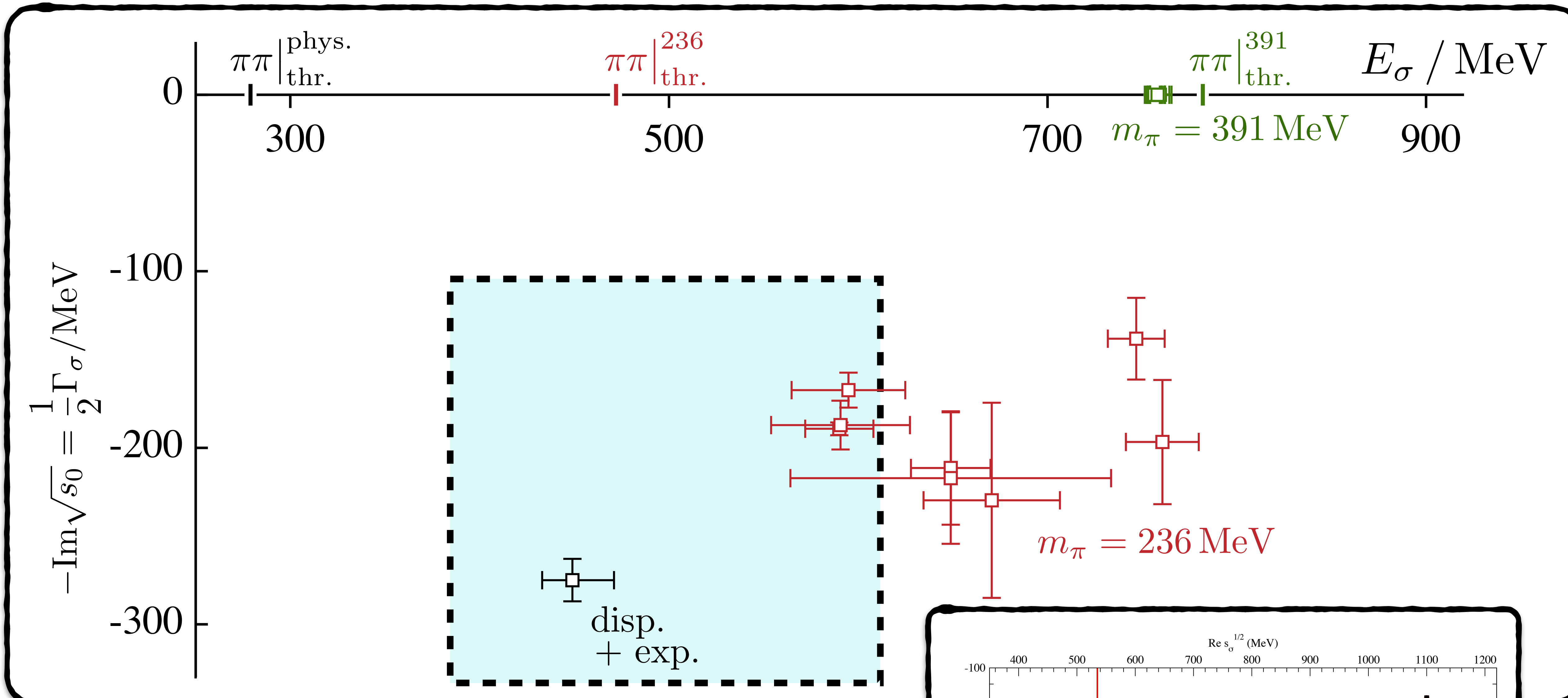
RB, Dudek, Edwards & Wilson (2016)

The σ / $f_0(500)$ vs m_π

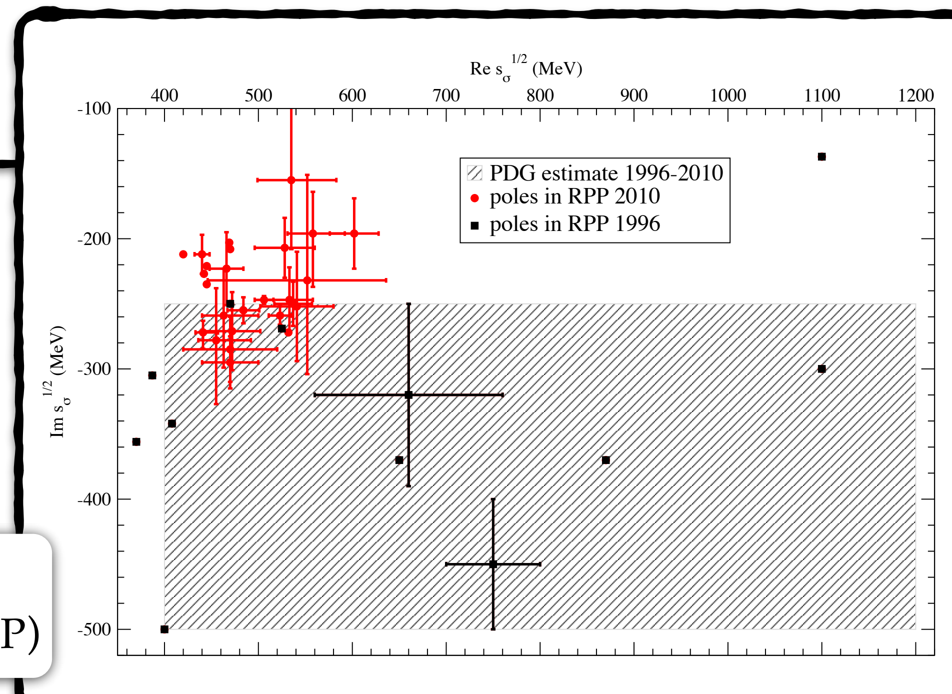


RB, Dudek, Edwards & Wilson (2016)

The σ / $f_0(500)$ vs m_π

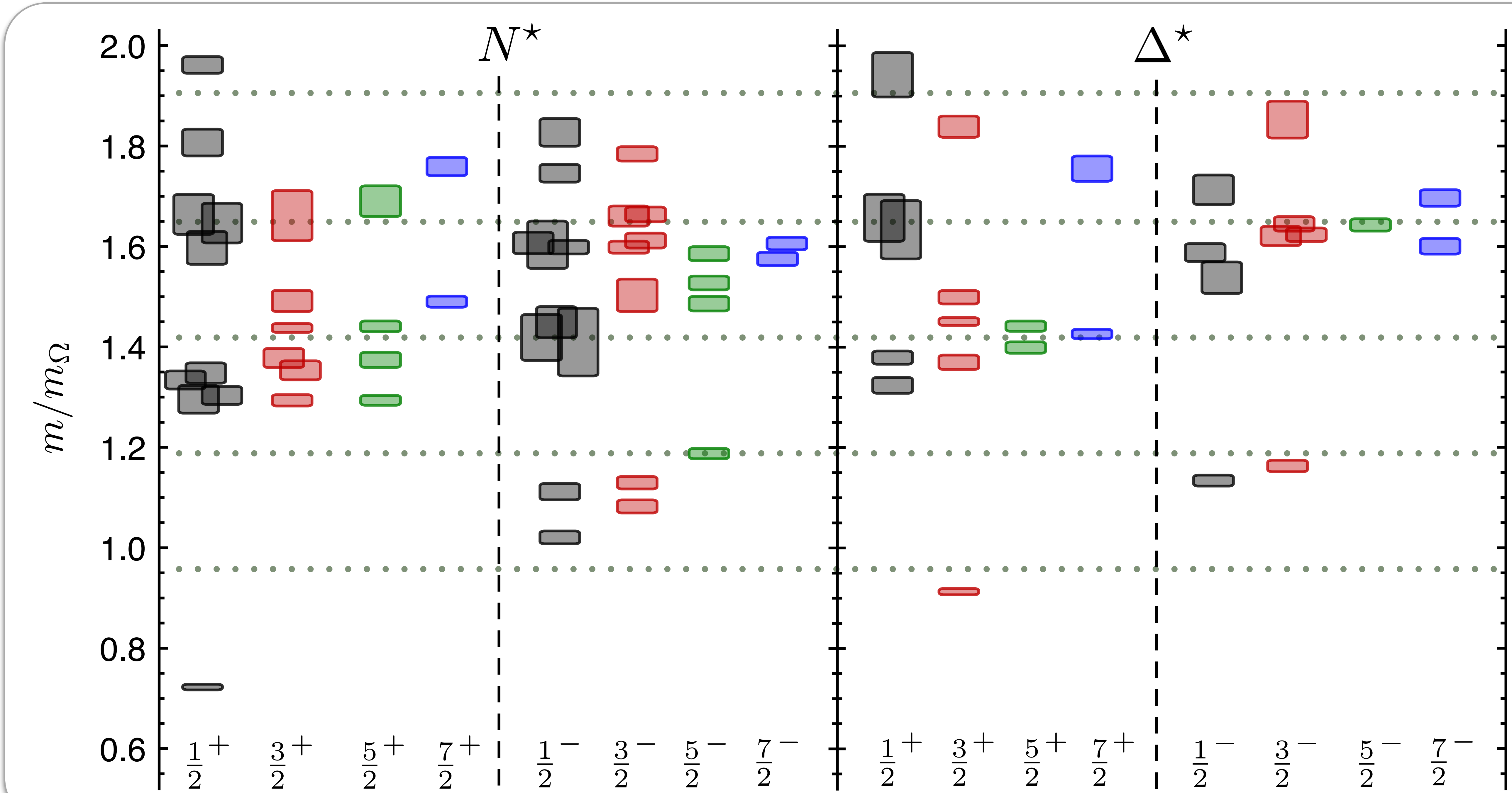


RB, Dudek, Edwards & Wilson (2016)



J. R. Peláez (2015)
Review of Particle Physics (RPP)

Going higher in energy



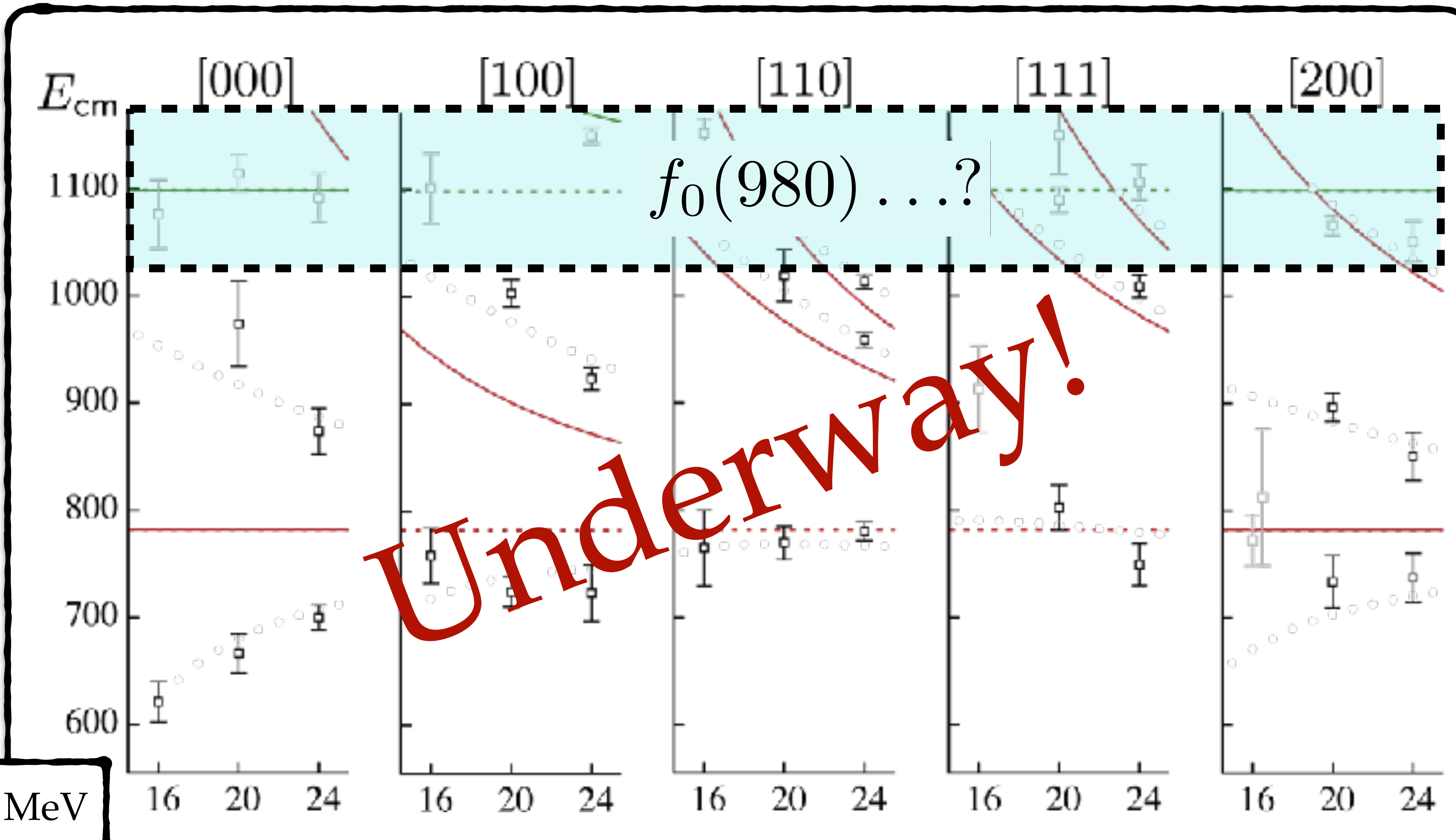
m_π=391 MeV

Edwards, Dudek, Richards, Wallace [Hadspec Collab.] (2011)

Going higher in energy

- *Coupled channels:* $$\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F_{K\overline{K}}^{-1} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$

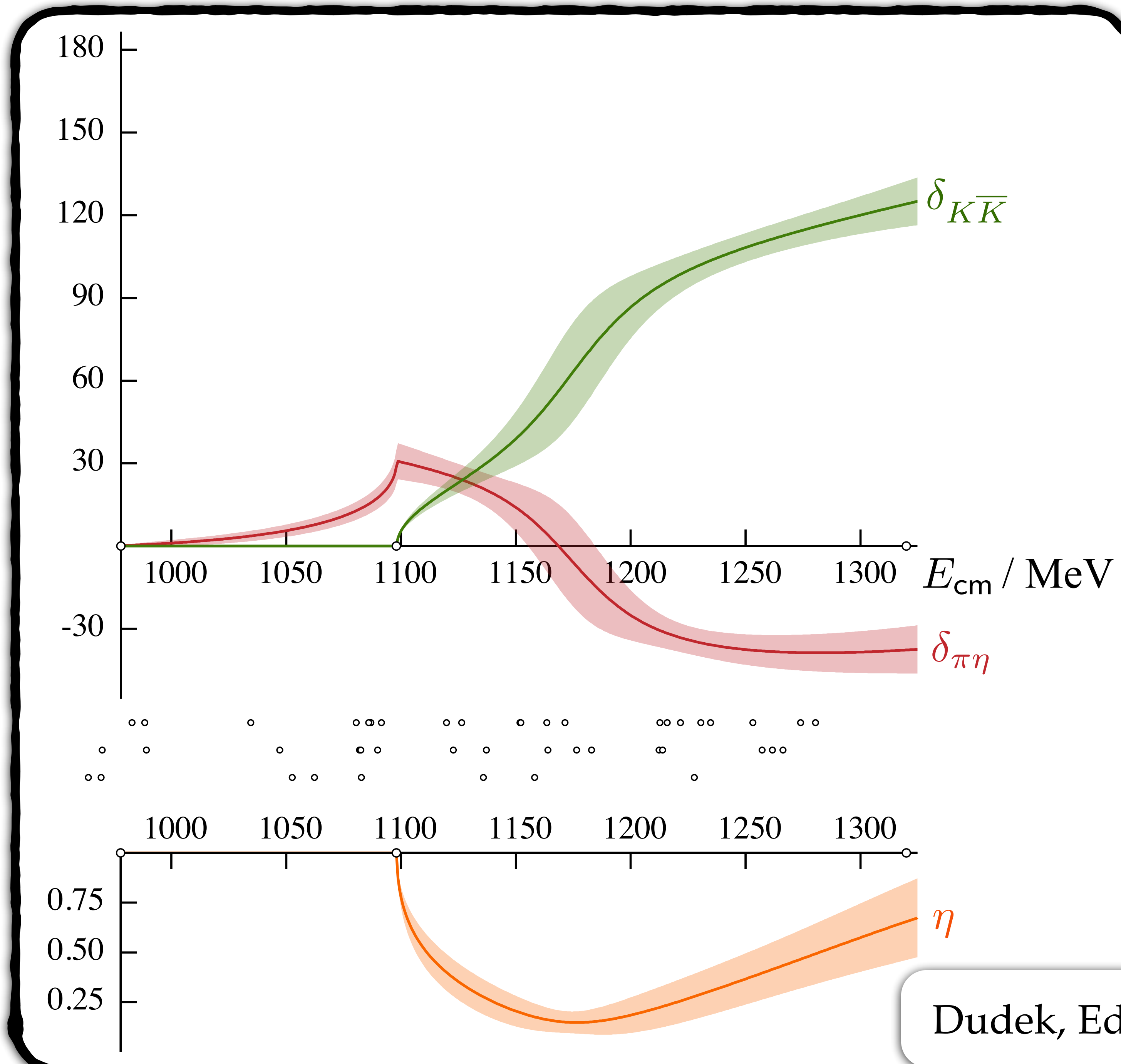
Hansen & Sharpe / RB & Davoudi (2012)
RB (2014) / RB & Hansen (2015)



m_π=391 MeV

Going higher in energy

- *Coupled channels:* e.g., $\pi\eta$, $K\overline{K}$



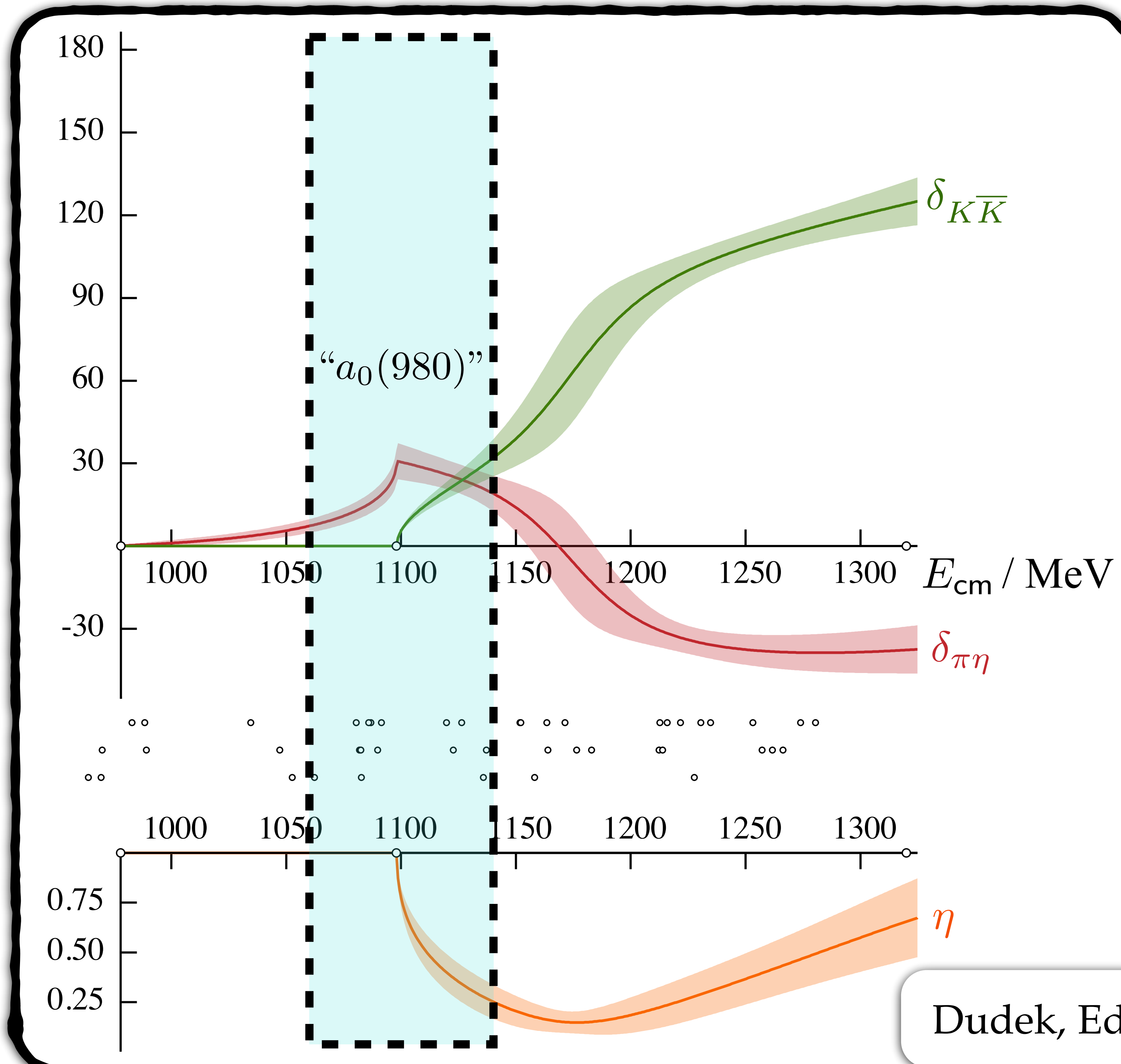
m$_\pi$=391 MeV

RB

Dudek, Edwards & Wilson (2016)

Going higher in energy

- *Coupled channels:* e.g., $\pi\eta$, $K\overline{K}$



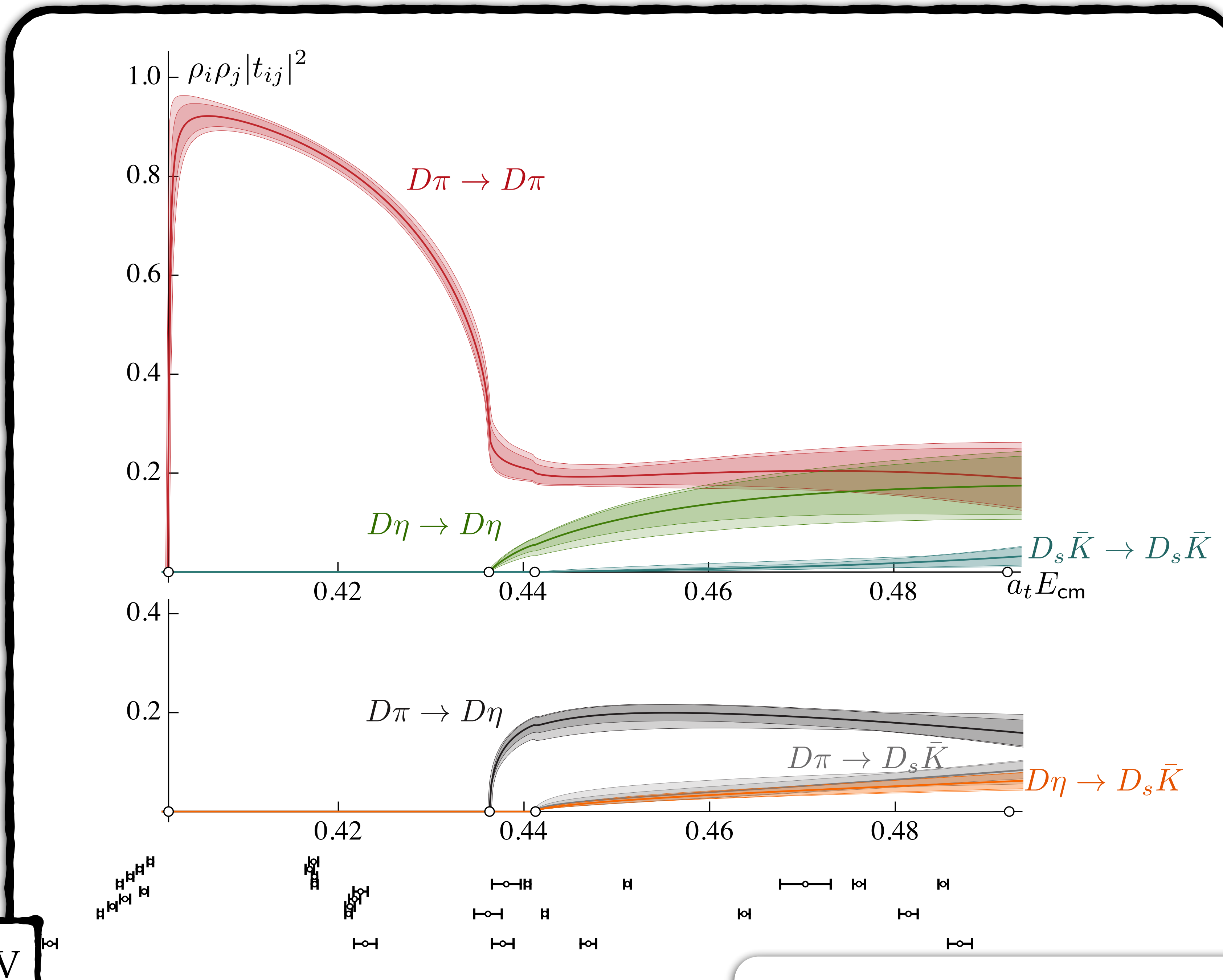
m$_\pi$=391 MeV

RB

Dudek, Edwards & Wilson (2016)

Going higher in energy

- *Coupled channels:* e.g., $D\pi$, $D_s\overline{K}$



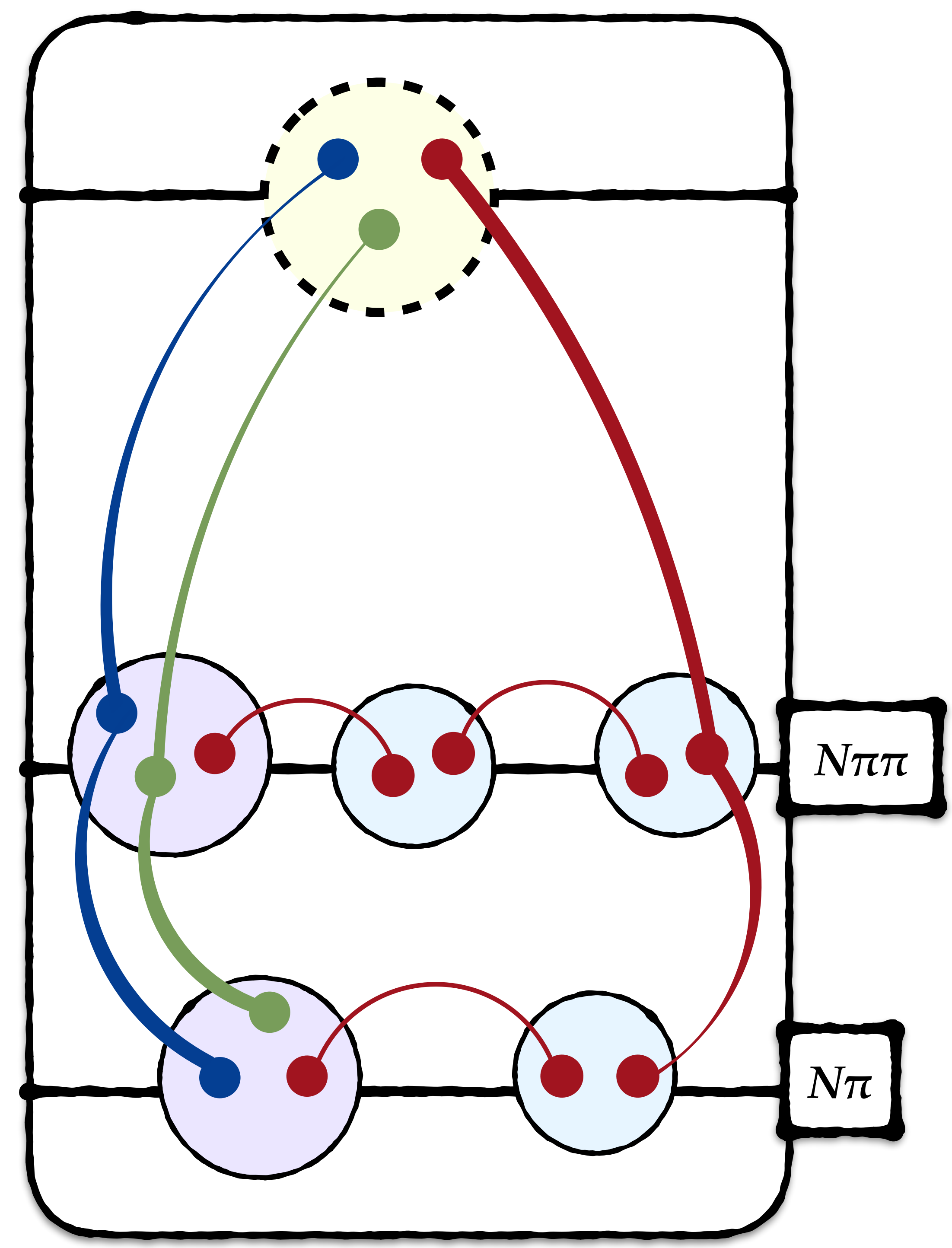
m$_\pi$=391 MeV

Moir, Peardon, Ryan, Thomas, Wilson (2016)

~~RB~~

Going higher in energy

- *Coupled channels*
- *Beyond two particles:*



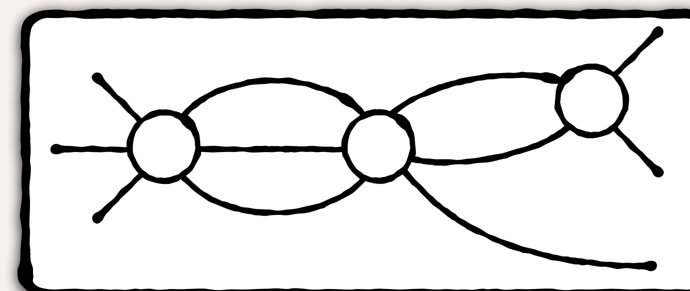
Going higher in energy

- *Coupled channels*

- *Beyond two particles:*

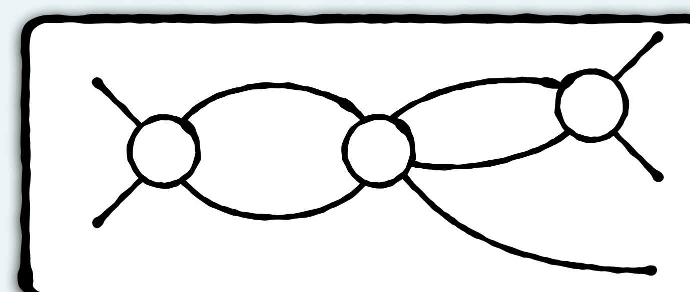
$$\det\left[1 + F_3 \mathcal{K}_{\text{df},3}\right] = 0$$

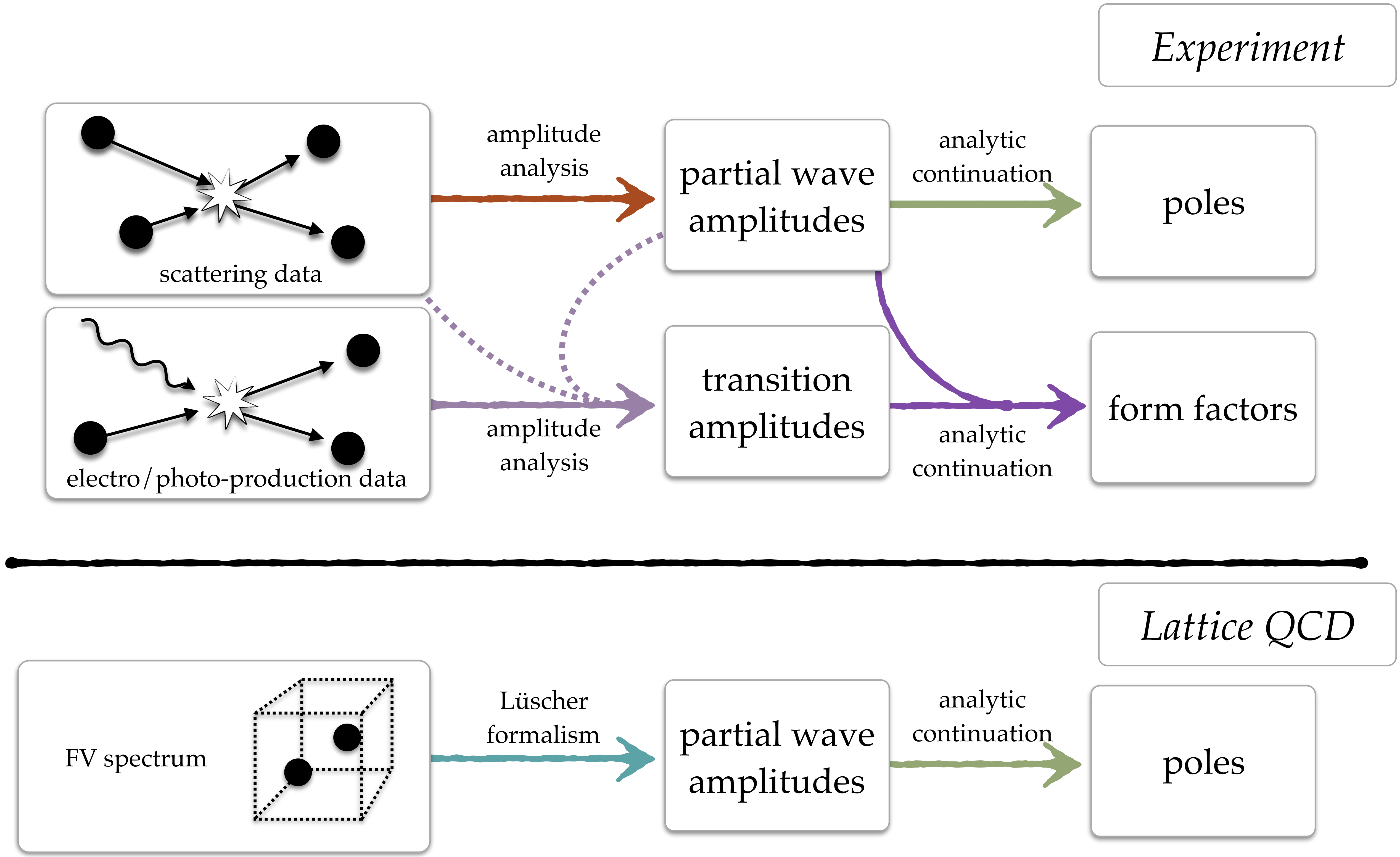
Hansen & Sharpe (2014)



$$\det\left[1 + \begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix} \begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\text{df},3} \end{pmatrix}\right] = 0$$

RB, Hansen & Sharpe (2016)




Experiment
scattering data
amplitude
analysis
partial wave
amplitudes
analytic
continuation
poles
electro/photo-production data
amplitude
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transition
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analytic
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Lattice QCD
FV spectrum
Lüscher
formalism
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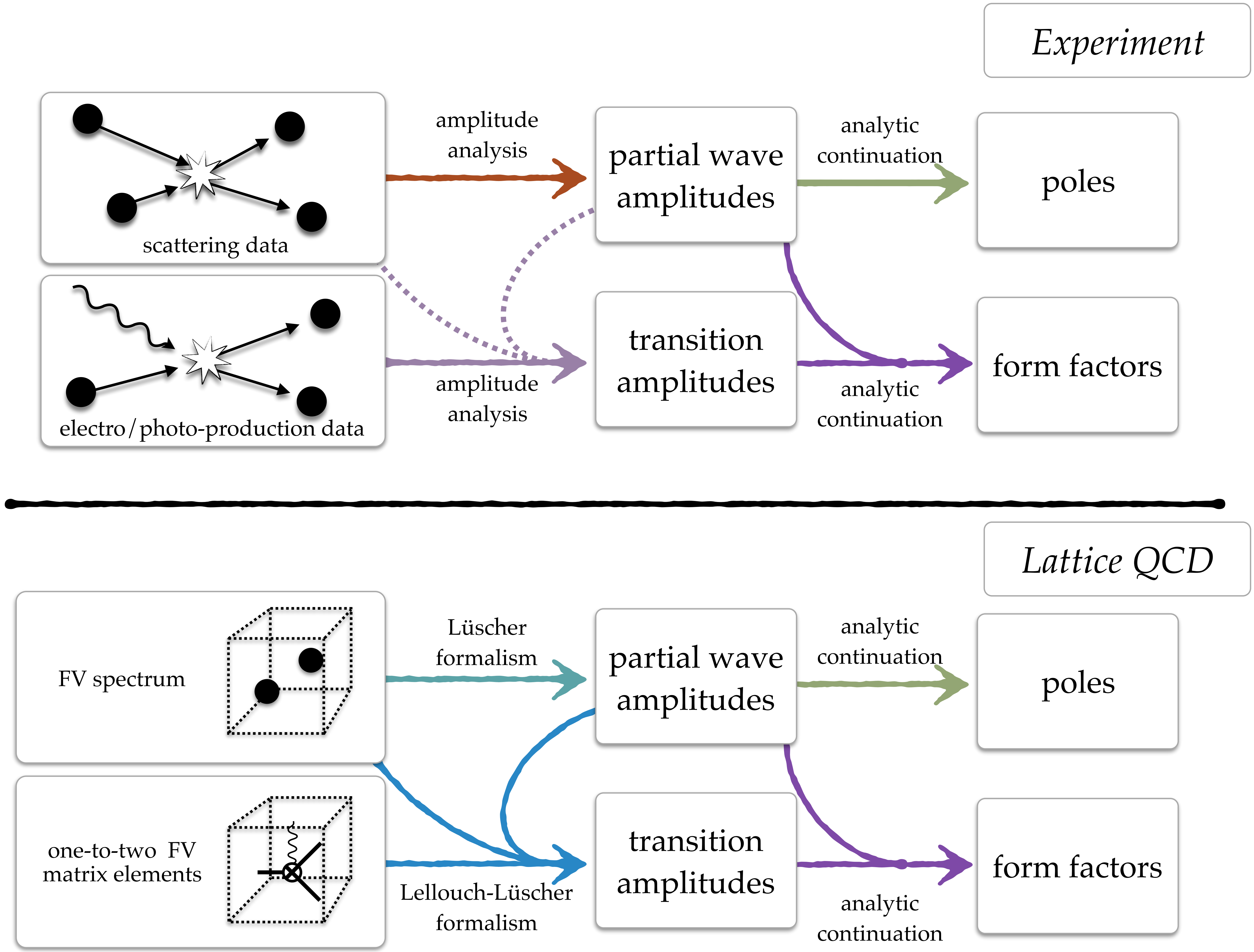
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Lüscher formalism
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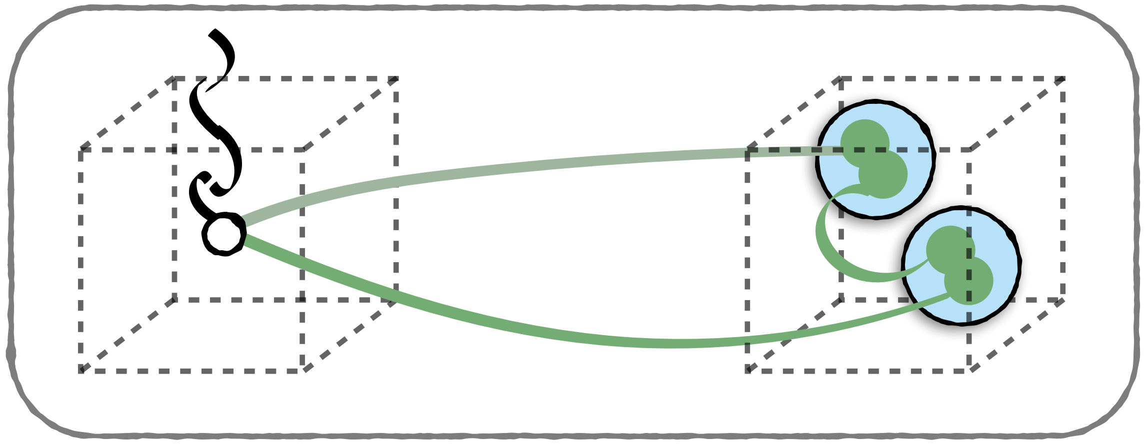
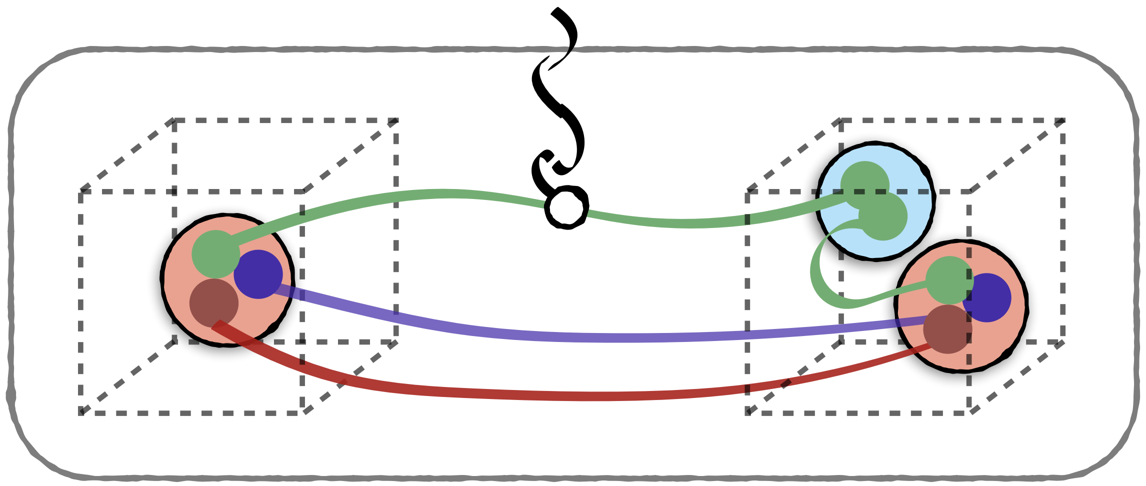
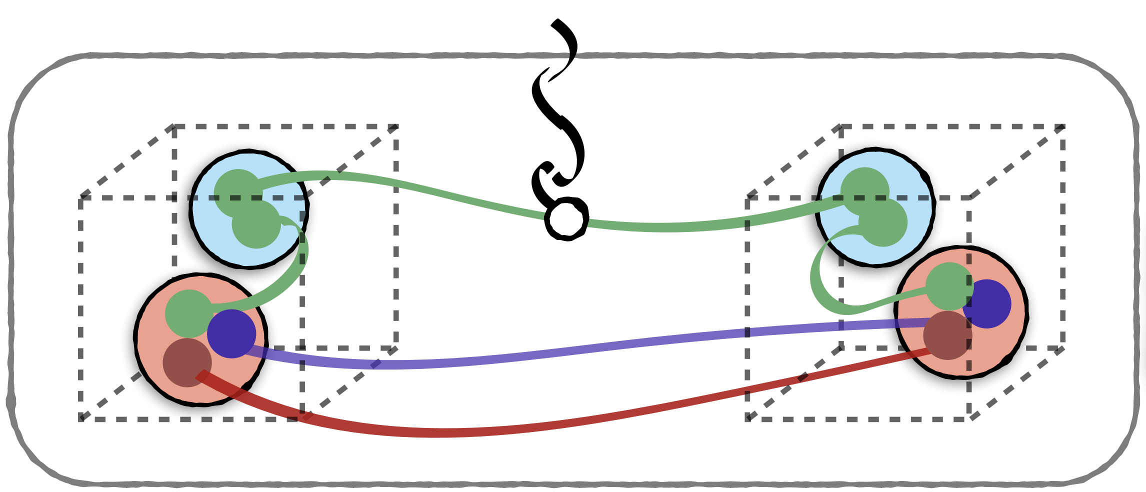
Beyond spectroscopy



$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{0}\rangle_L\right| = \sqrt{L^3}\sqrt{\mathcal{V}\ \mathcal{R}\ \mathcal{V}}$$



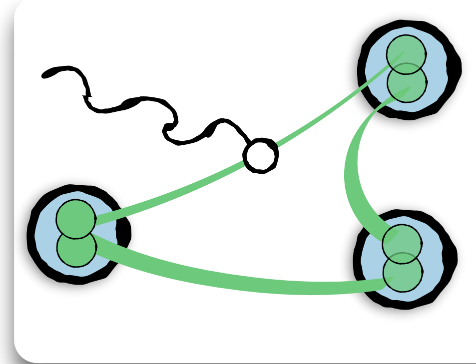
$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right| = \sqrt{\mathcal{H}\ \mathcal{R}\ \mathcal{H}}$$



$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{2}\rangle_L\right| = \frac{1}{\sqrt{L^3}}\sqrt{\mathrm{Tr}\left[\mathcal{R}\ \mathcal{W}_{L,\mathrm{df}}\ \mathcal{R}\ \mathcal{W}_{L,\mathrm{df}}\right]}$$

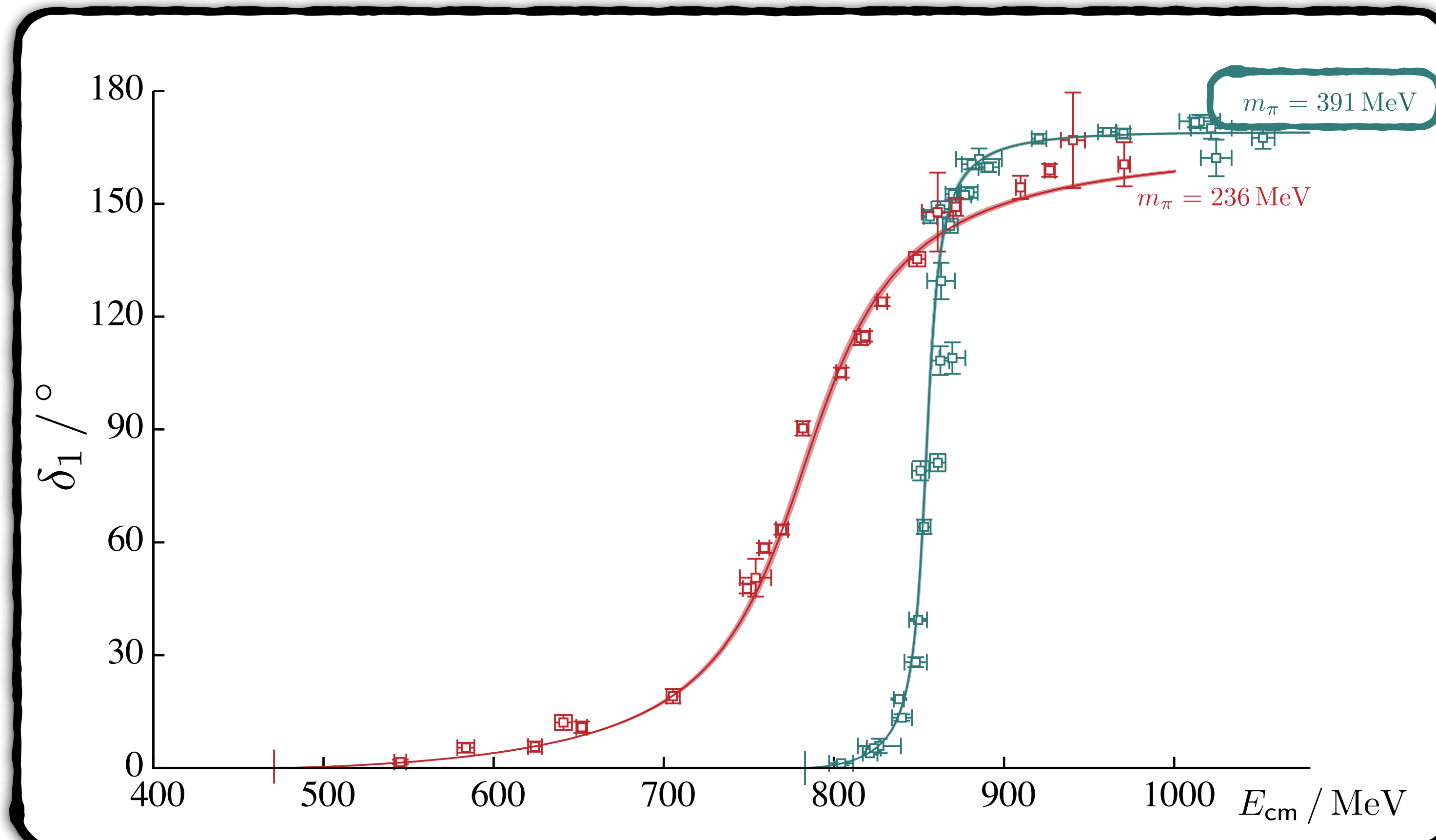
RB, Hansen (2016)
RB, Hansen (2015)
RB, Hansen, Walker-Loud (2014)

$\pi\gamma^*$-to-$\pi\pi$



Exploratory $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ρ calculation:

- $m_\pi = 391$ MeV



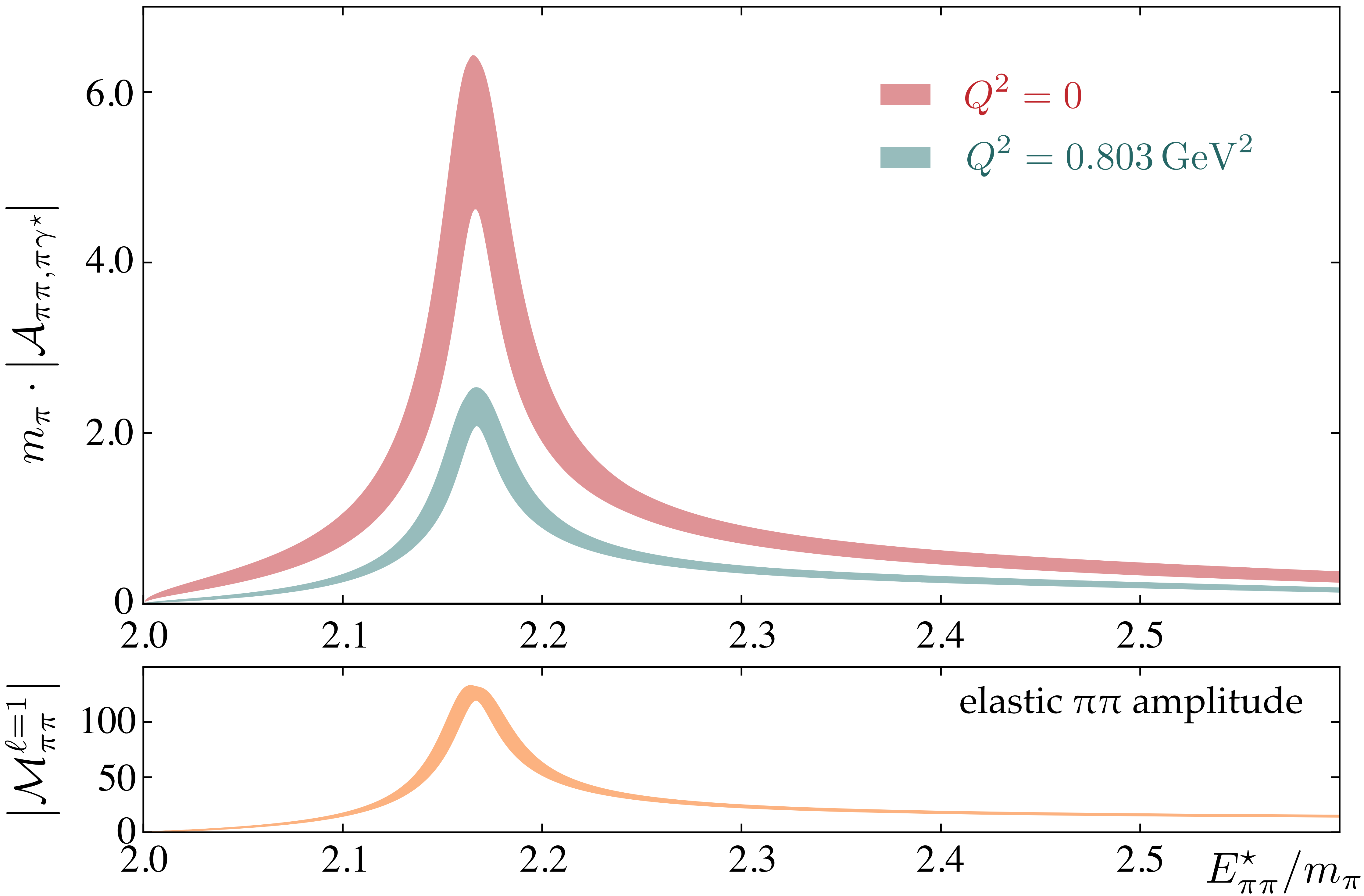
- Matrix element determined in **42** kinematic point: $(E_{\pi\pi}, Q^2)$

- Lorentz decomposition:

$$\mathcal{H}^{\mu}_{\pi\pi,\pi\gamma^\star} = \epsilon^{\mu\nu\alpha\beta}\, P_{\pi,\nu}\, P_{\pi\pi,\alpha}\, \epsilon_\beta(\lambda_{\pi\pi}, \mathbf{P}_{\pi\pi})\, \frac{2}{m_\pi} \mathcal{A}_{\pi\pi,\pi\gamma^\star}$$

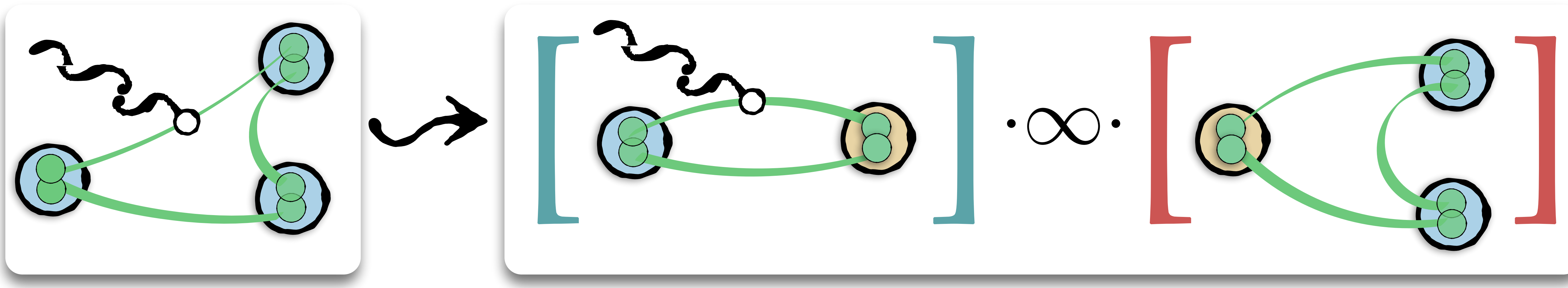
$\pi\pi/\rho$ polarization · *$\pi\pi/\rho$ helicity* · *Lorentz scalar*

$\pi\gamma$*-to-$\pi\pi$ amplitude



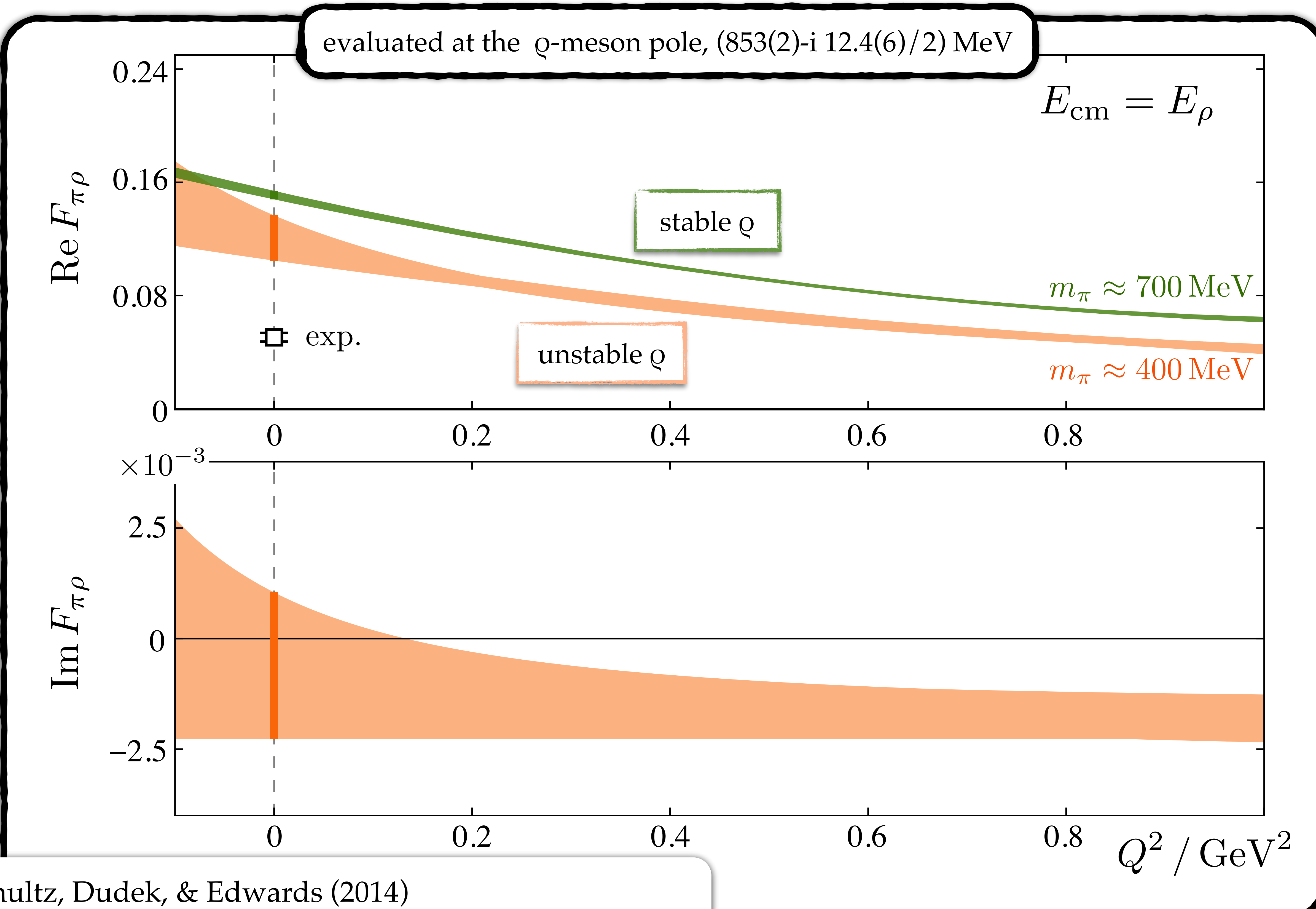
Form factor at ϱ pole

- The residue encodes the $\pi\gamma^*$-to-ϱ form factor

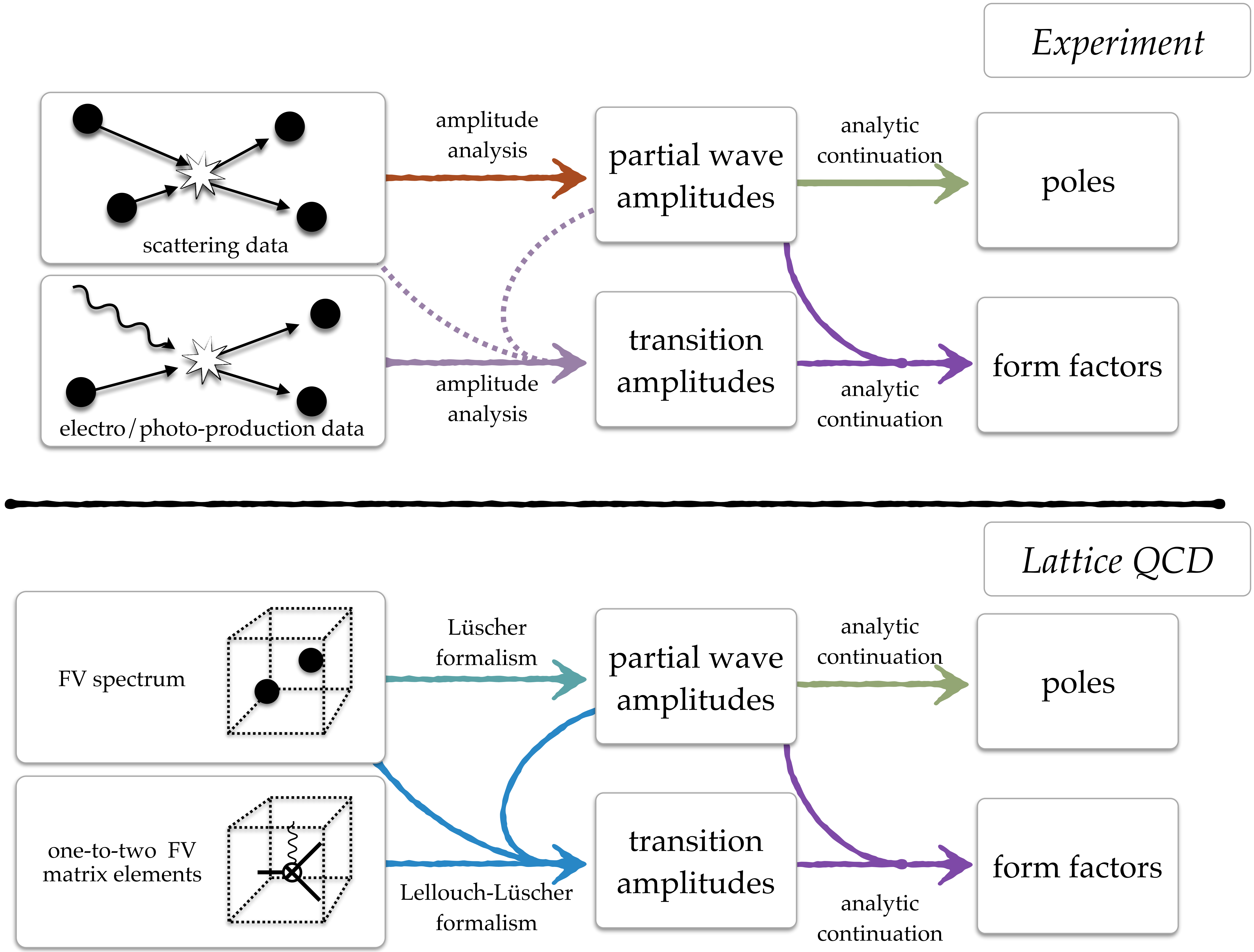


$$\mathcal{A}_{\pi\pi,\pi\gamma^\star}(E_{\pi\pi},Q^2) = F(E_{\pi\pi},Q^2) \times \left[\frac{1}{\cot\delta_1(E_{\pi\pi}) - i}\right] \times \sqrt{\frac{16\pi}{q_{\pi\pi}\,\Gamma(E_{\pi\pi})}}$$

Form factor at ϱ pole



Shultz, Dudek, & Edwards (2014)
RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)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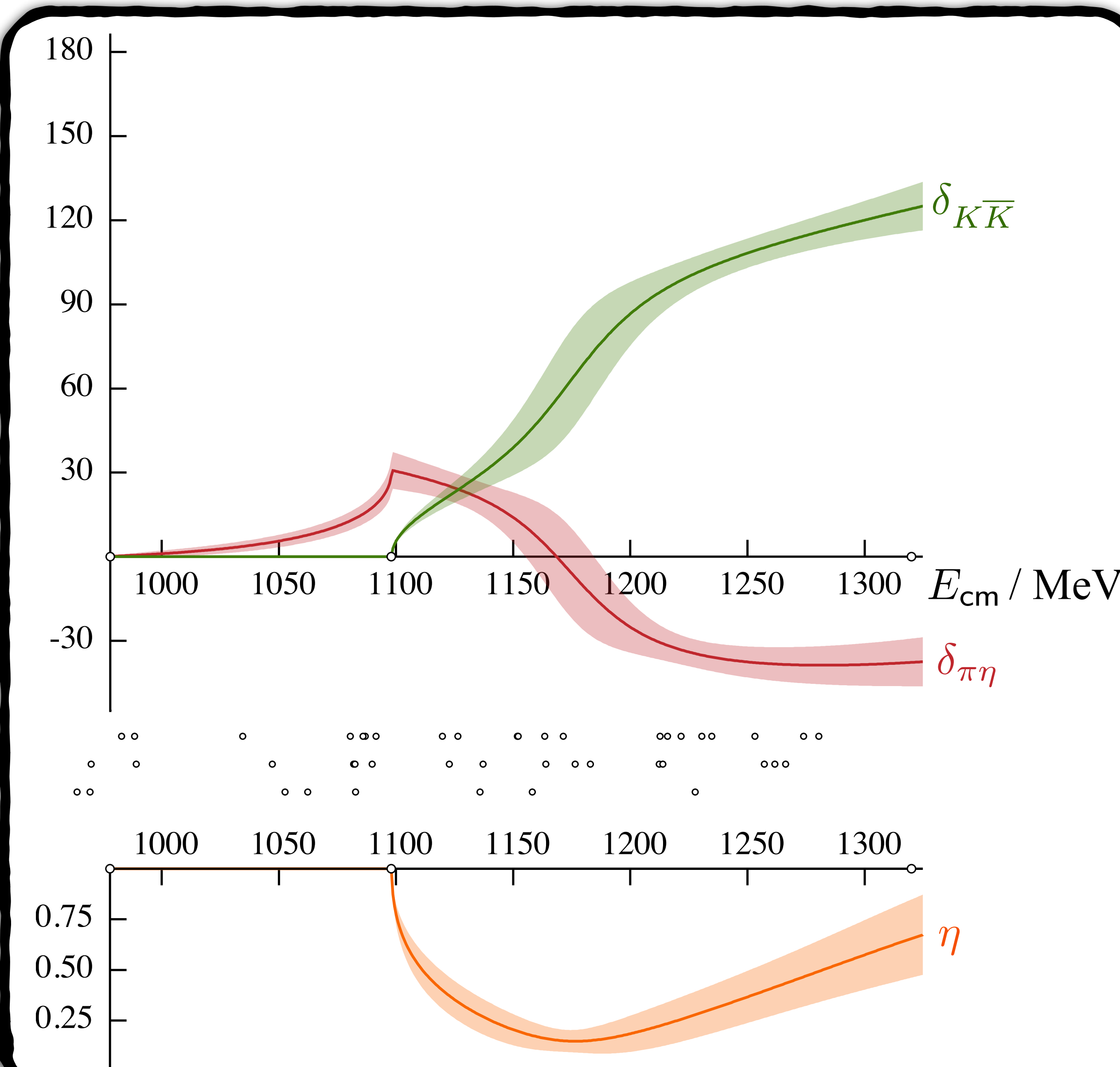
Summary / outlook

- *Coupled channels*

formalism understood:

Hansen & Sharpe / RB & Davoudi (2012)
RB (2014) / RB & Hansen (2015)

few implementations to date by HadSpec



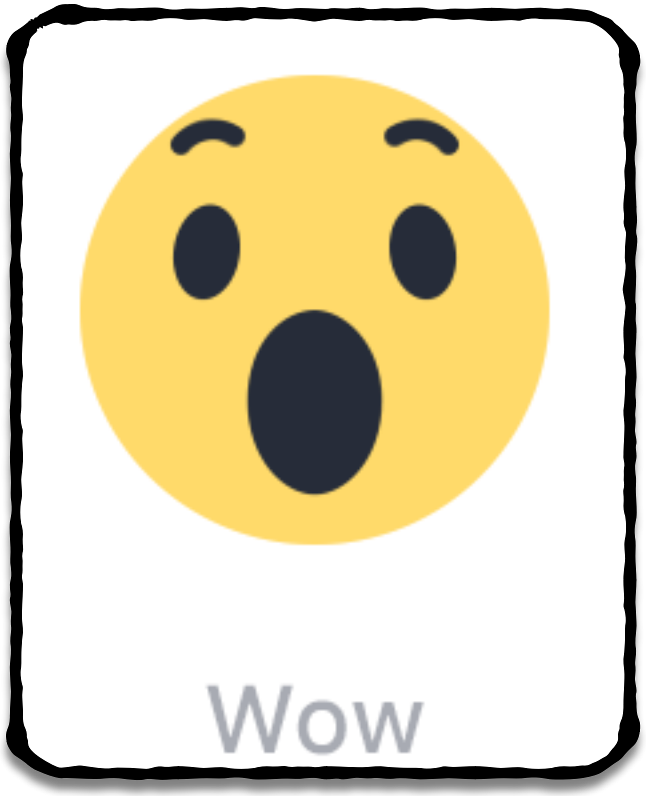
~~RB~~

Dudek, Edwards & Wilson (2016)

Summary / outlook

- *Coupled channels*
- *Baryons*

no implementation to date!



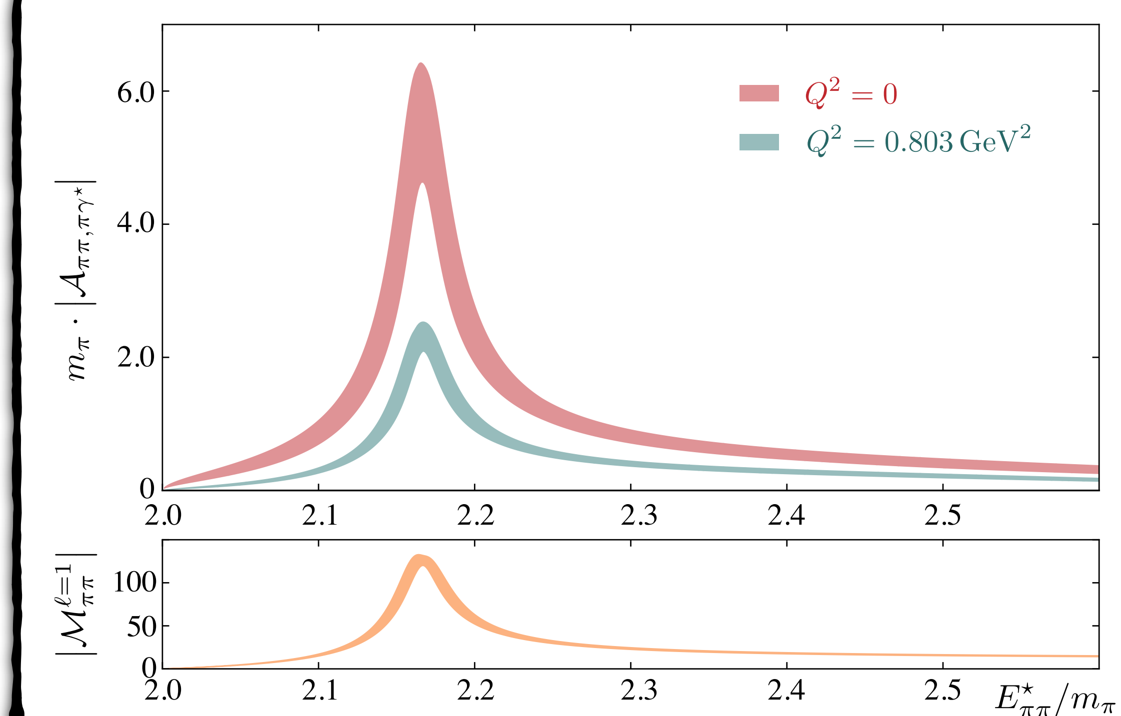
formalism understood:

RB (2014) / RB & Hansen (2015)

Summary / outlook

- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*

first implementation: $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ϱ



formalism understood:

RB, Hansen (2016)
RB, Hansen (2015)
RB, Hansen, Walker-Loud (2015)

RB, Dudek, Edwards, Thomas, Shultz, Wilson (2015, 2016)
RB, Dudek, Edwards, Thomas, Shultz, Wilson (2015, 2016)

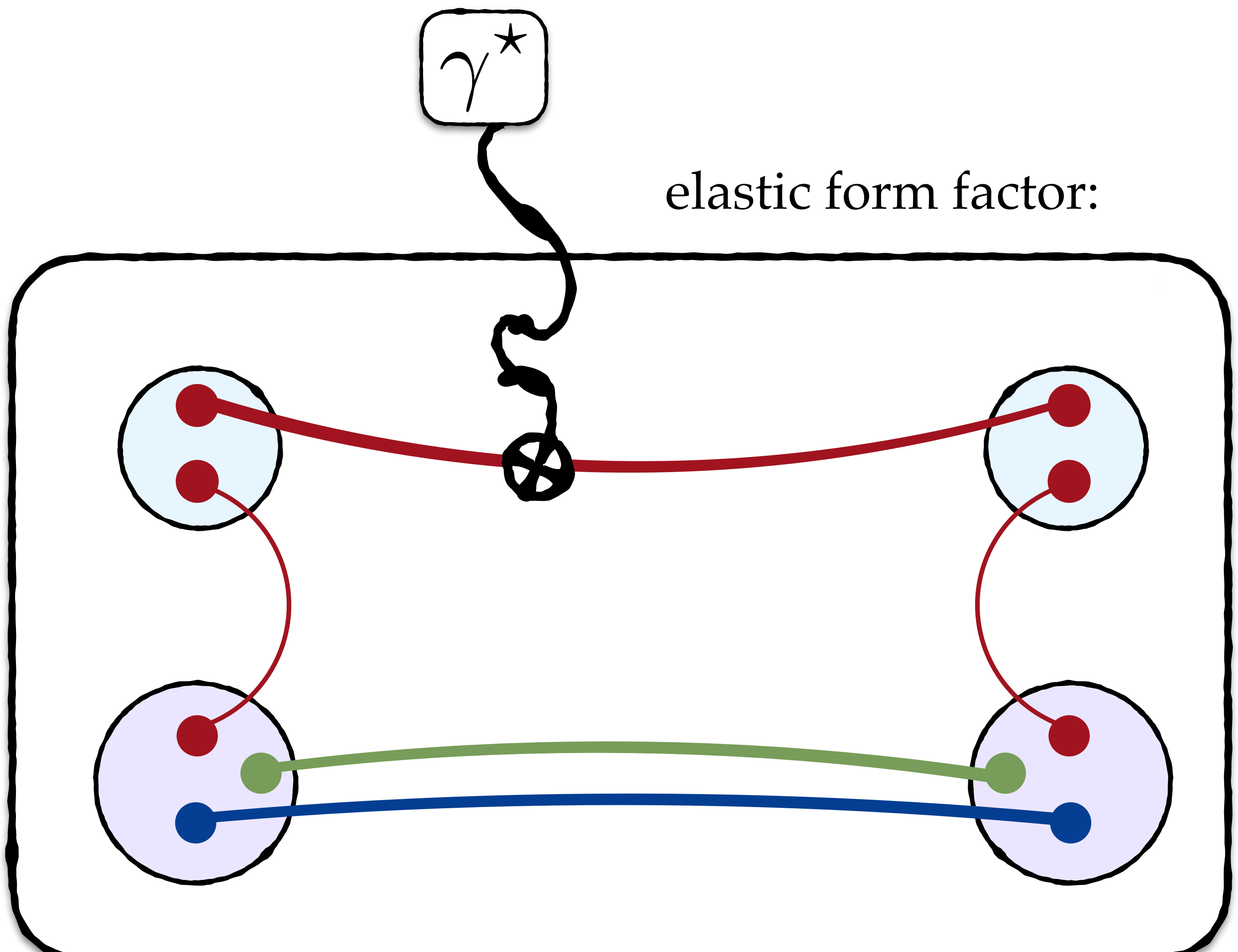
Summary / outlook

- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*

formalism understood:

RB, Hansen (2016)
RB, Hansen (2015)
RB, Hansen, Walker-Loud (2015)

elastic form factor:



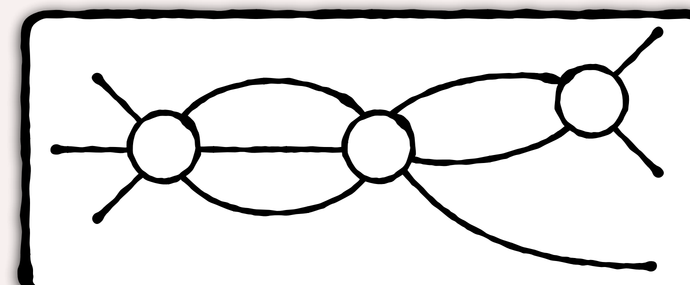
Summary / outlook

- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems*

formalism under construction:

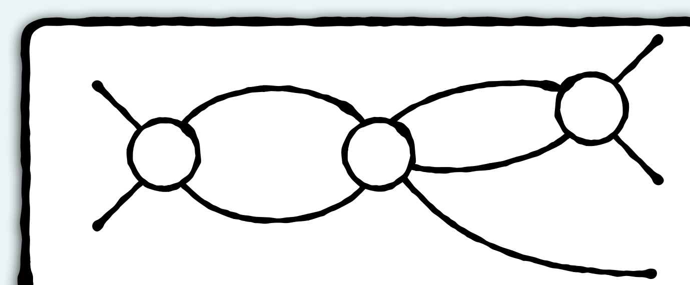
$$\det\left[1 + F_3 \mathcal{K}_{\mathrm{df},3}\right] = 0$$

Hansen & Sharpe (2014)



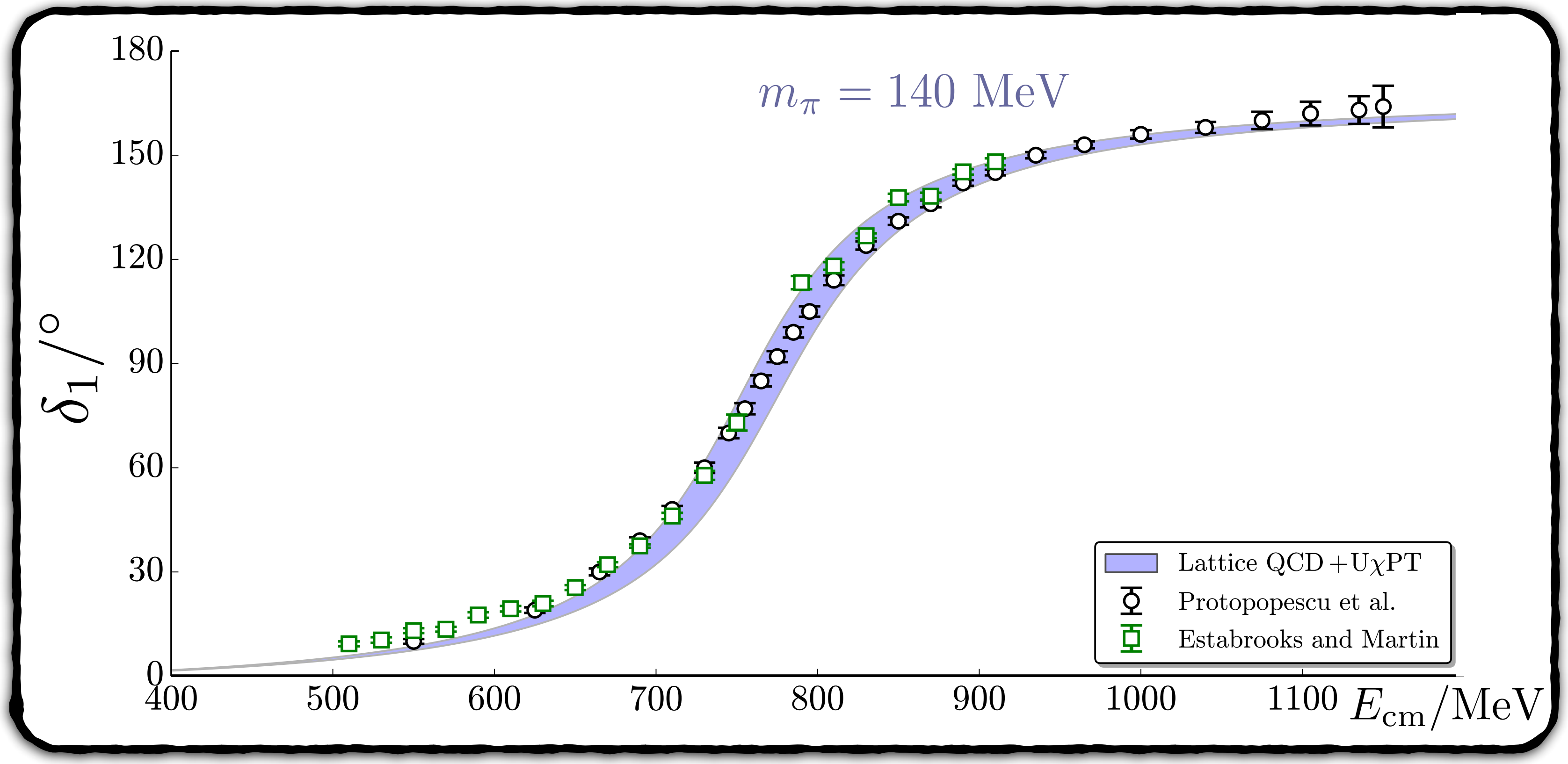
$$\det\left[1 + \begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix} \begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\mathrm{df},3} \end{pmatrix}\right] = 0$$

RB, Hansen & Sharpe (2016)



Summary / outlook

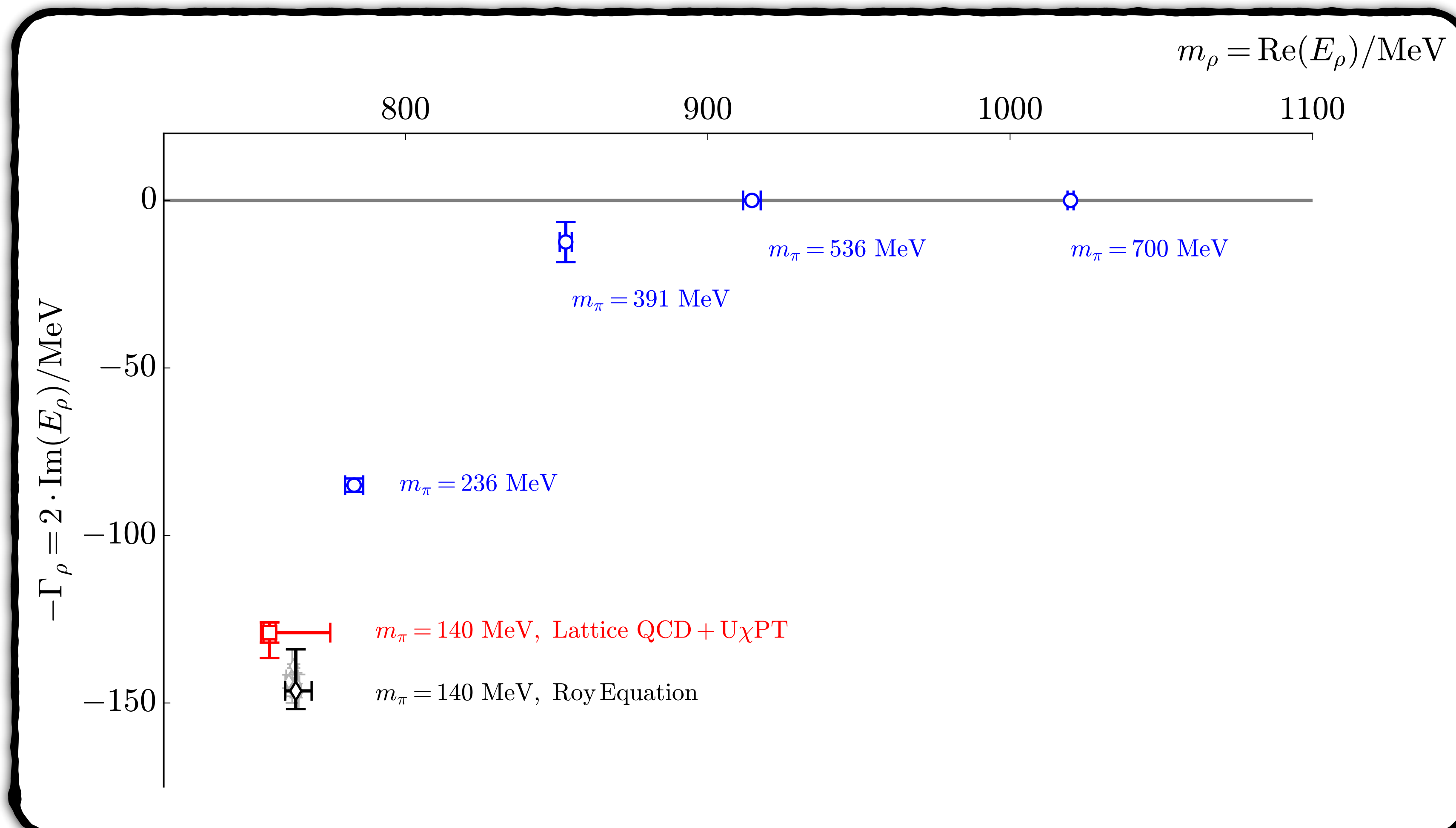
- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems*
- *Physical point, chiral extrapolation?*



Bolton, RB & Wilson Phys.Lett. B757 (2016) 50-56.

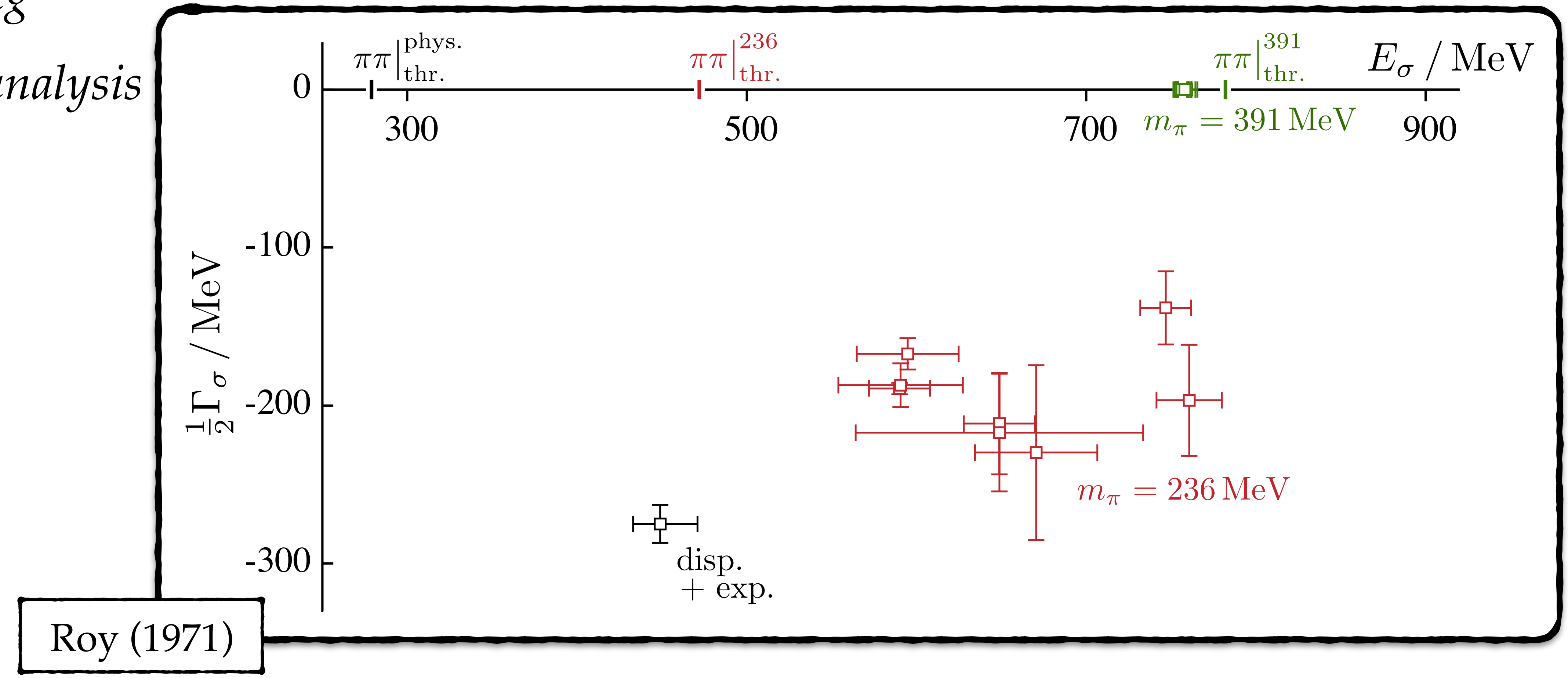
Summary / outlook

- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems*
- *Physical point, chiral extrapolation?*
- *pole tracking*



Summary / outlook

- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems*
- *Physical point, chiral extrapolation?*
- *pole tracking*
- *dispersive analysis*

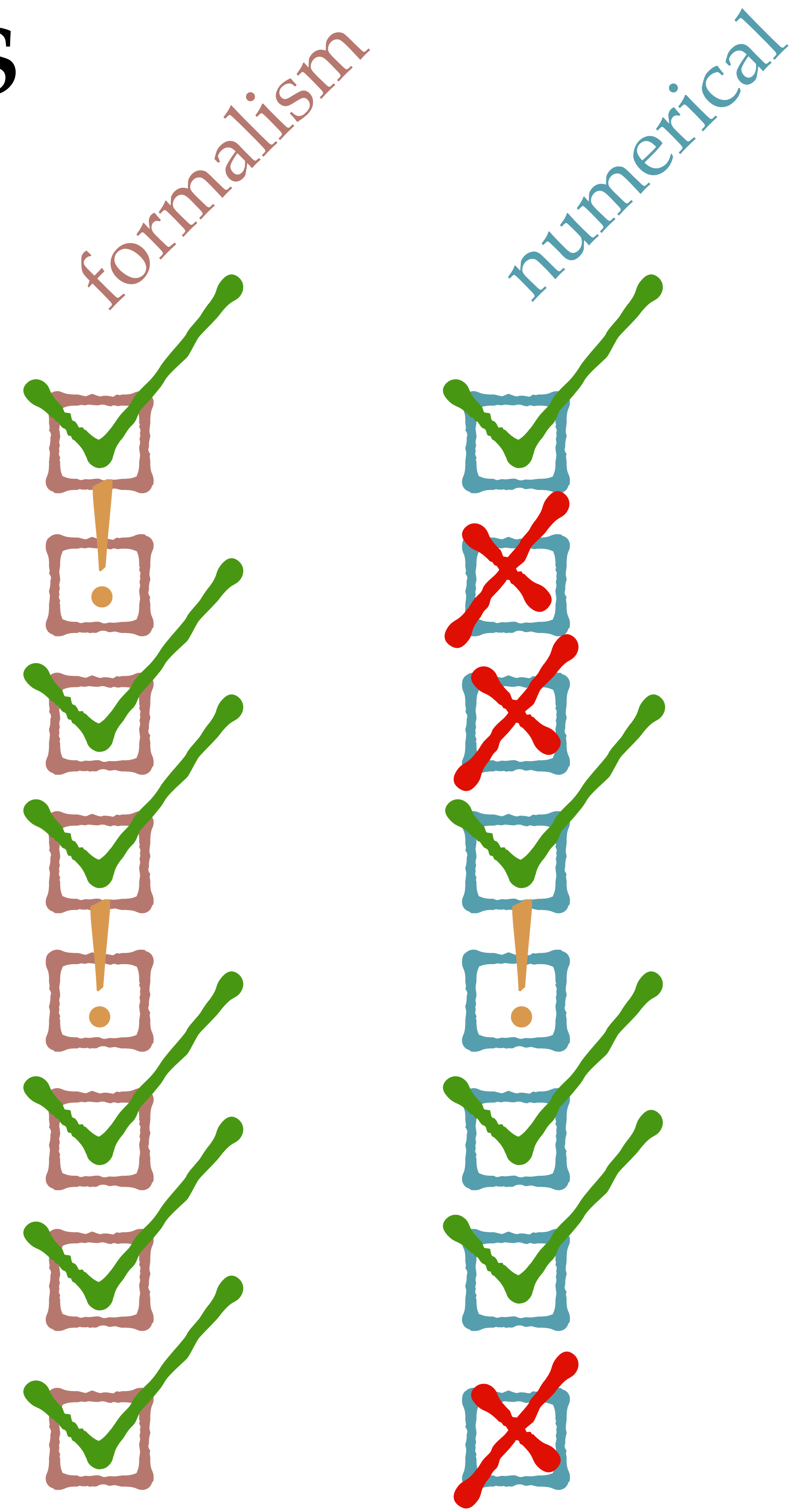


Broad goals

	formalism	numerical
Strongly coupled **2**-body	☐	☐
Strongly coupled **2, 3**-body	☐	☐
Spin-dependent amps.	☐	☐
Narrow resonances	☐	☐
Broad resonances	☐	☐
Photo-, electro-production	☐	☐
Transition form factors	☐	☐
Elastic form factors	☐	☐

Broad goals

- Strongly coupled **2**-body
- Strongly coupled **2, 3**-body
- Spin-dependent amps.
- Narrow resonances
- Broad resonances
- Photo-, electro-production
- Transition form factors
- Elastic form factors



Broad goals

- Strongly coupled **2**-body
- Strongly coupled **2, 3**-body
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- Narrow resonances
- Broad resonances
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- Elastic form factors

Broad goals

- Strongly coupled **2**-body
- Strongly coupled **2, 3**-body
- Spin-dependent amps.
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- Broad resonances
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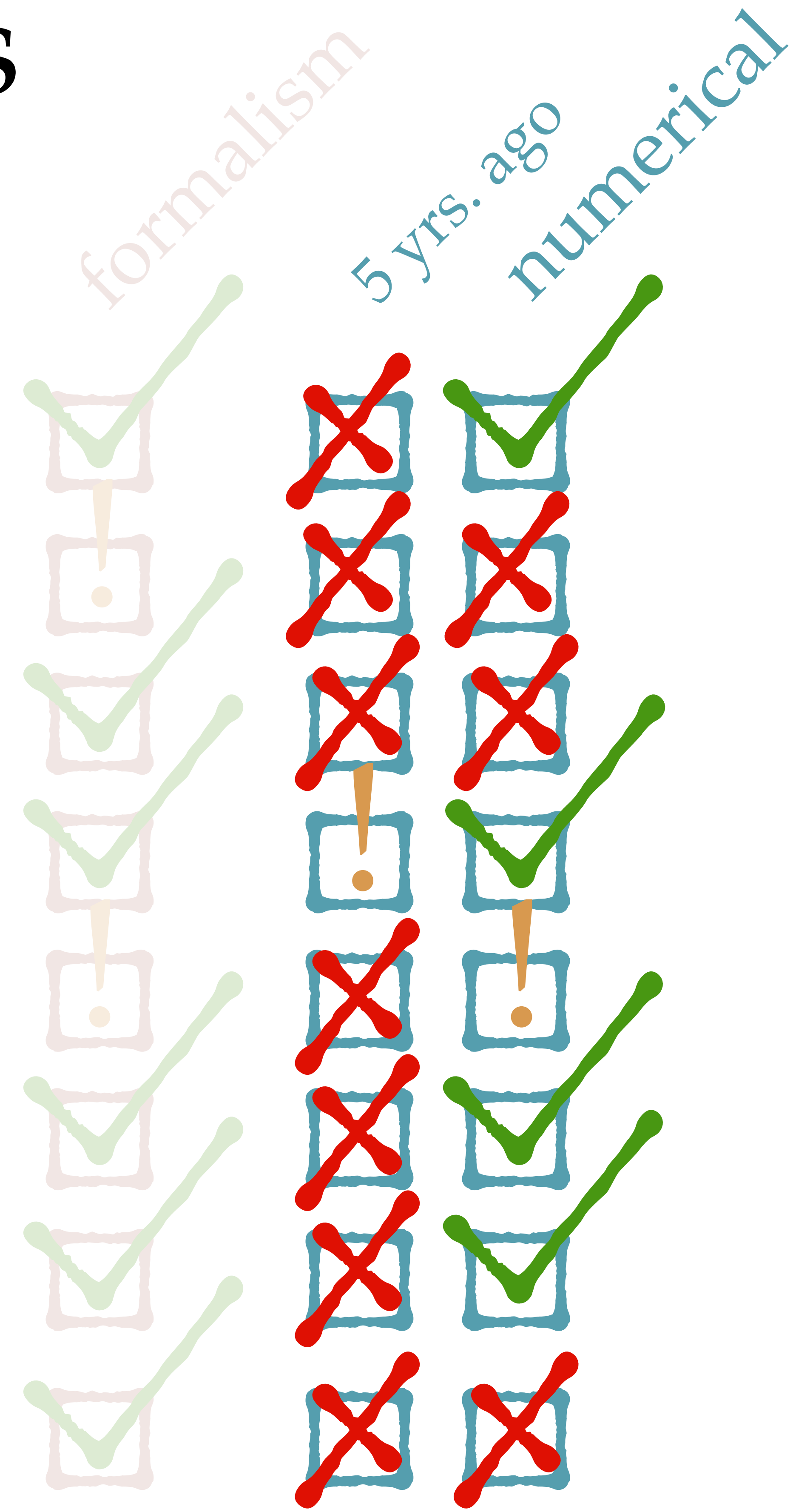
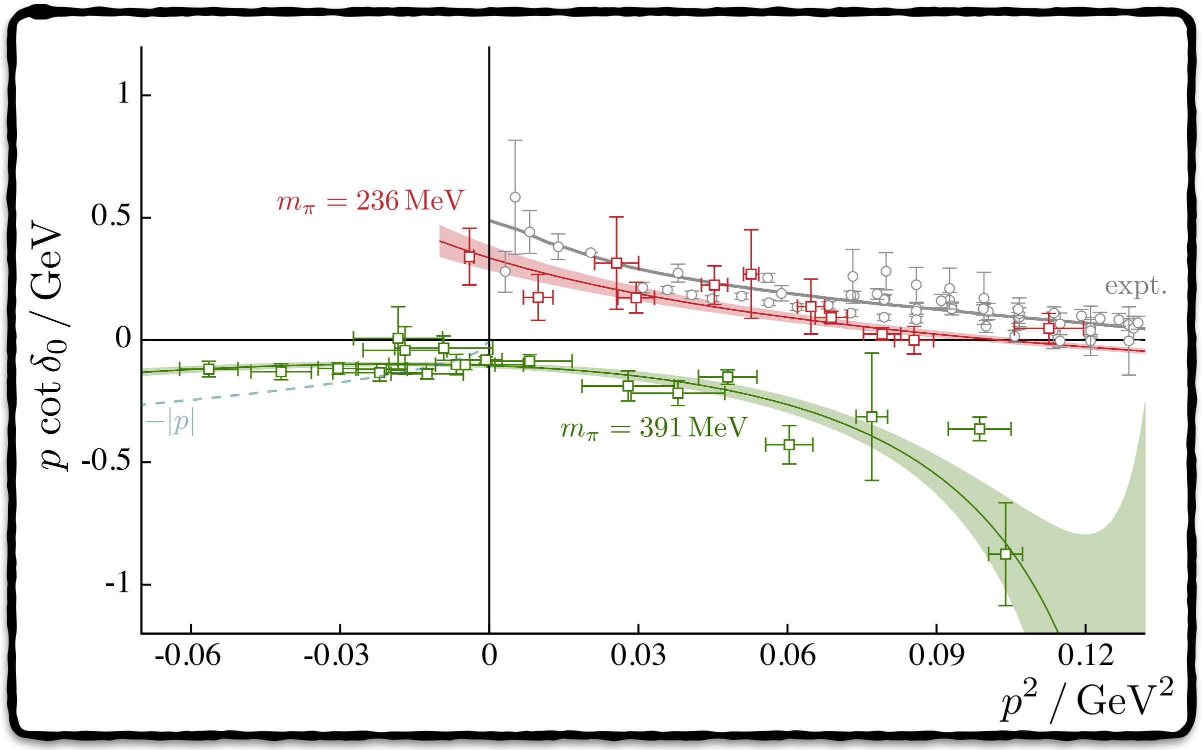
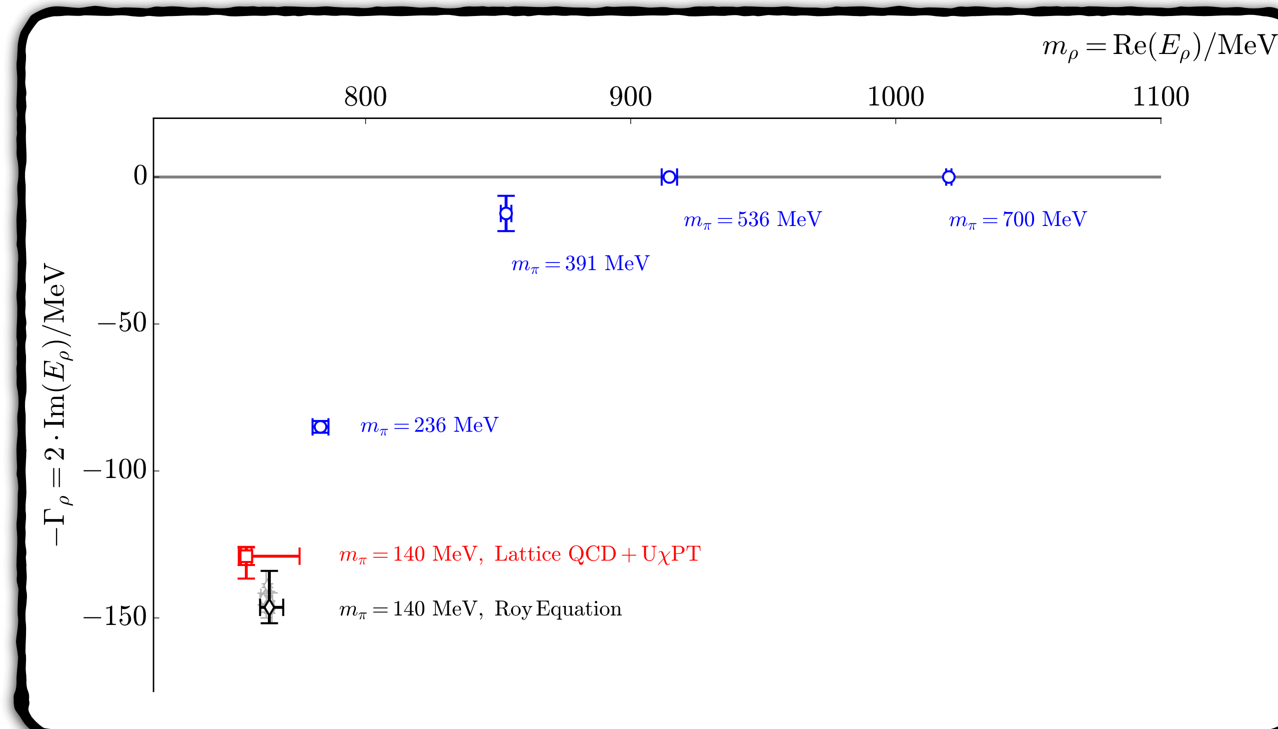
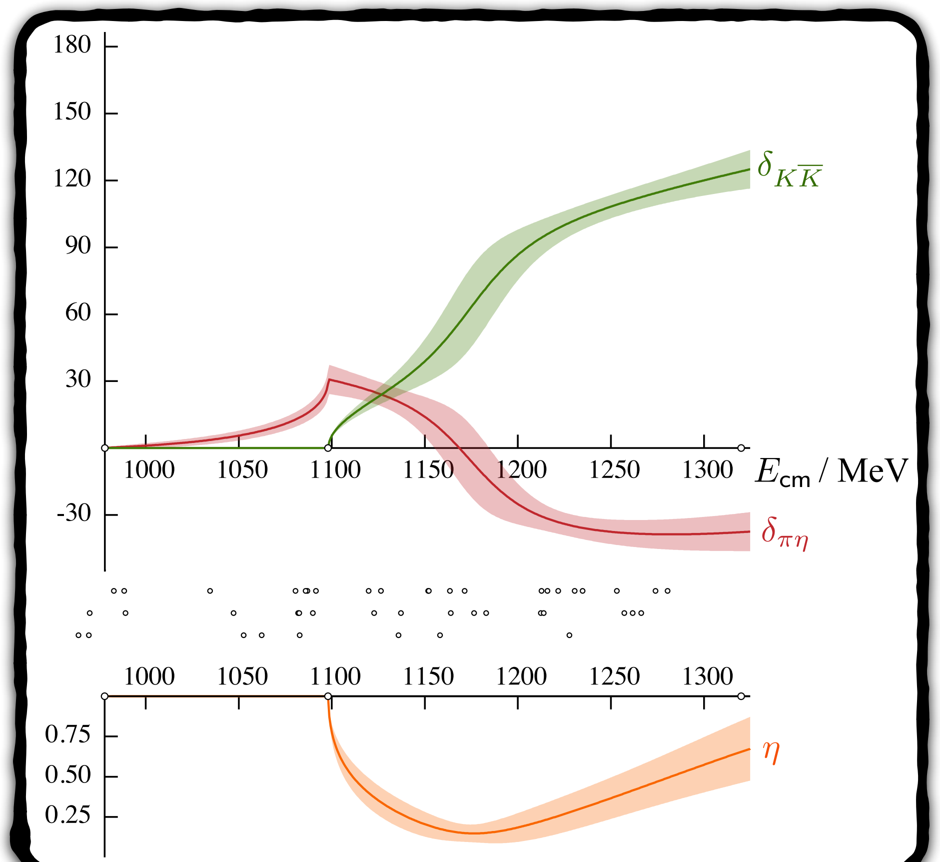
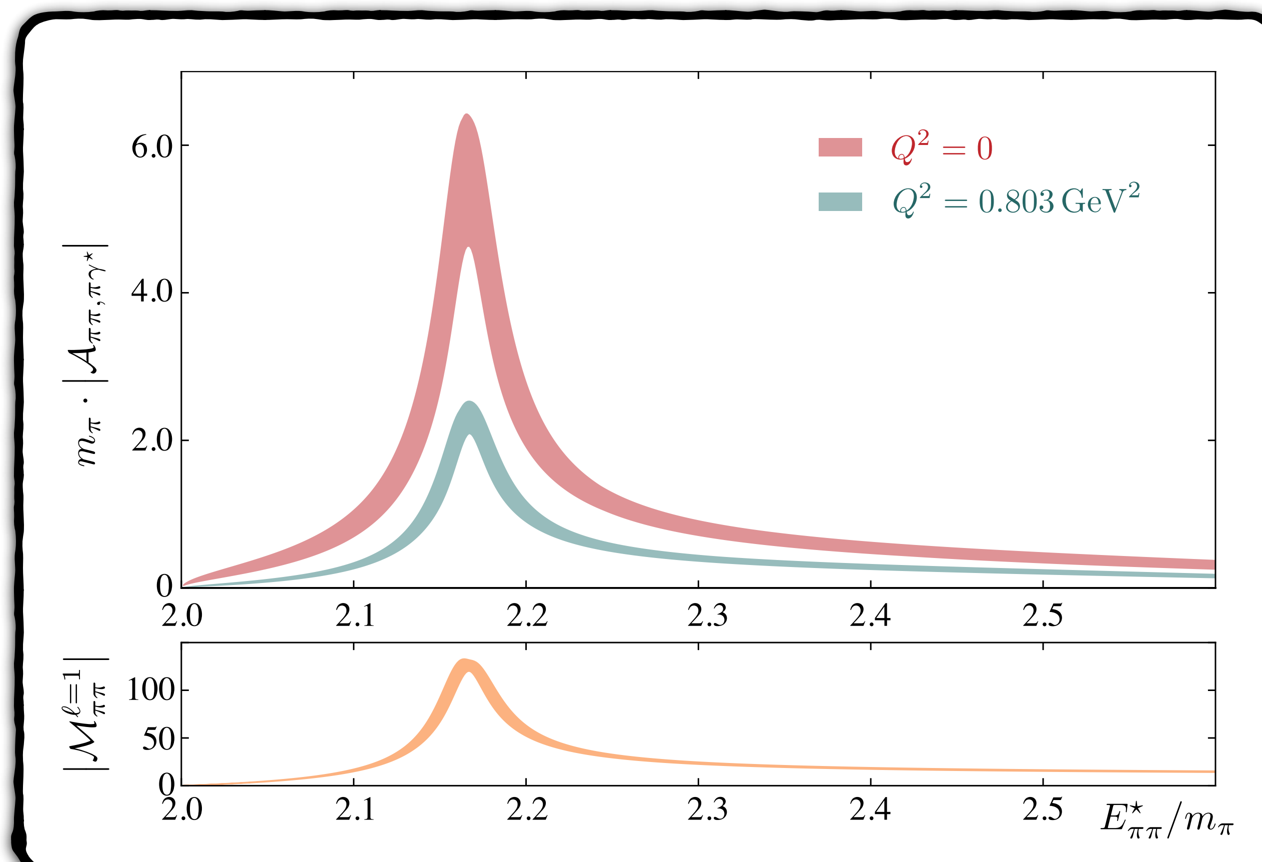


The big picture!









Collaborators & references

formalism

Hansen

Walker-Loud

Sharpe

numerical

Wilson

Shultz

Thomas

Dudek

Edwards

HadSpec Collaboration

Bolton

RB, Hansen, Sharpe - arXiv:1609.09805 [hep-lat] (2016)
RB, Hansen - Phys.Rev. D94 (2016) no.1, 013008 .
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RB, Dudek, Edwards, Wilson - arXiv:1607.05900 [hep-ph].
Moir, Peardon, Ryan, Thomas, Wilson - arXiv:1607.07093 [hep-lat].
RB, Dudek, Edwards, Thomas, Shultz, Wilson - Phys.Rev. D93 (2016) 114508.
RB, Dudek, Edwards, Thomas, Shultz, Wilson - Phys.Rev.Lett. 115 (2015) 242001
Wilson, RB, Dudek, Edwards, Thomas - Phys.Rev. D92 (2015) no.9, 094502