Stimulated neutrino transformation: Three flavor effects.

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Inside a supernova

- Within 1000km, neutrino-neutrino interaction dominates

  H Duan & GM Fuller & YZ Qian, PRD 74 (2006); H Duan & A Friedland, PRL (2011); S Hannestad & G Raffelt et al. PRD 74 (2006).

- Beyond that, matter effects take over

  Dighe & Smirnov arXiv (2001); Schirato & Fuller arXiv (2002); R Tomas et al. JCAP (2004)

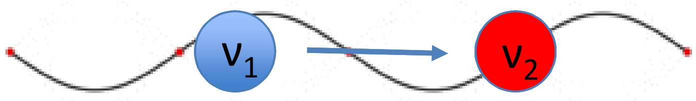
- Stimulated neutrino transformations caused by turbulence

  Balentekin & Fetter PRD 54 (1996); Friedland & Gruzinov arXiv (2006); Kneller & Volpe PRD 82 (2010); Patton et al. PRD 89 (2014).

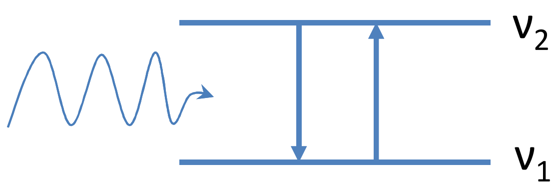
- All effects should be taken in to account.

Stimulated neutrino transformation

- Stimulated neutrino transformation is induced by an oscillating potential (parametric resonance).

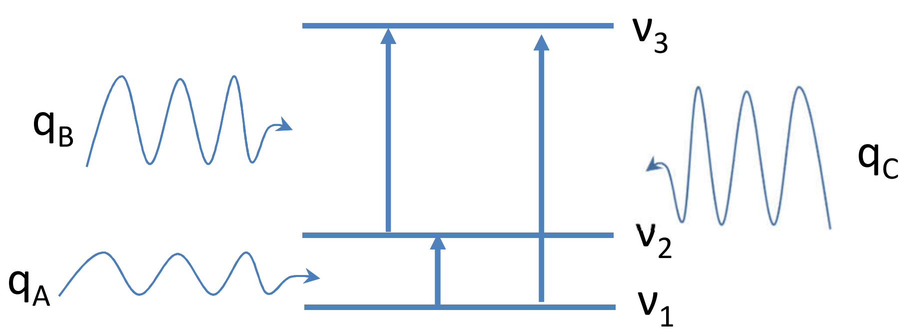


- Similar to the quantum transformation of a two-level system between its two eigenstates.



When we need 3-flavor mixing model

- With 3 mass eigenstates, neutrino will be responsive to multiple frequencies.



$$\Delta m_{21}^2 \sim q_A,\ \ \Delta m_{32}^2 \sim q_B,\ \ \Delta m_{31}^2 \sim q_C$$

Time-dependent perturbation theory

- The evolution equation in flavor basis:

$$i\frac{dS^{(f)}}{dr} = H^{(f)}S^{(f)} = \left[H_0^{(f)} + \delta H^{(f)}\left(r\right)\right]S^{(f)},$$

- The eigenbasis of $H_0^{(f)}$:

$$i\frac{dS^{(u)}}{dr} = H^{(u)}S^{(u)}, \;\; H^{(u)} = K^{(u)} + U^{\dagger}\delta H^{(f)}U,$$

$$S^{(f)} = US^{(u)}U^{\dagger}, \;\; K^{(u)} = \mathrm{diag}\left(k_1, k_2 \ldots\right)$$

Time-dependent perturbation theory

- Isolating the interaction part

$$i\frac{dS_0}{dr} = K^{(u)}S_0, \quad S^{(u)} = S_0 A,$$

- The equation for A-matrix is

$$i\frac{dA}{dr} = \left(S_0 U \; \delta H^{(f)} U S_0\right) A, \qquad A = e^{-i\Xi}B,$$

- Eliminating diagonal elements

$$i\frac{dB}{dr} = e^{i\Xi}\left(S_0^\dagger U^\dagger \delta H^{(f)} U S_0 - \frac{d\Xi}{dr}\right) e^{-i\Xi} B \equiv H^{(B)} B.$$

Fourier-decomposable potential

- A Fourier-decomposable perturbation can be generally expressed as

$$\delta H^{(f)}\left(r\right) = \sum_{a}\left(C_{a} e^{i q_{a} r} + C_{a} e^{-i q_{a} r}\right),$$

- The perturbation can be both periodic or aperiodic(turbulent), depending on the ratios between wavenumbers of different modes.

Jacobi-Anger expansion

- For the N-flavor case, expanded $H^{(B)}$ looks like $\left(\delta k_{ij} = k_i - k_j\right)$

$$H^{(B)} = \sum_{\{n_a\}} \begin{pmatrix} 0 & -i\kappa_{12,\{n_a\}} e^{i\left(\delta k_{12} + \sum_a n_a q_a\right) r} & -i\kappa_{13,\{n_a\}} e^{i\left(\delta k_{13} + \sum_a n_a q_a\right) r} & \cdots \\ i\kappa^*_{12,\{n_a\}} e^{-i\left(\delta k_{12} + \sum_a n_a q_a\right) r} & 0 & -i\kappa_{23,\{n_a\}} e^{i\left(\delta k_{23} + \sum_a n_a q_a\right) r} & \cdots \\ i\kappa^*_{13,\{n_a\}} e^{-i\left(\delta k_{13} + \sum_a n_a q_a\right) r} & i\kappa^*_{23,\{n_a\}} e^{-i\left(\delta k_{23} + \sum_a n_a q_a\right) r} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

- The kappa for channel i→j is determined by

$$\kappa_{ij,\{n_a\}} \Leftarrow \left\{\left\{n_a\right\}, \left\{C_a\right\}, \left\{q_a\right\}\right\}.$$

Rotating Wave Approximation

- Take the 2-flavor case as an example,

$$i\frac{dB}{dr} = \sum_{\{\mathrm{n}_a\}} \begin{pmatrix} 0 & -i\kappa_{\{\mathrm{n}_a\}} e^{i\left(\delta k_{12} + \sum_a n_a q_a\right)r} \\ i\kappa^*_{\{\mathrm{n}_a\}} e^{-i\left(\delta k_{12} + \sum_a n_a q_a\right)r} & 0 \end{pmatrix} B \equiv H^{(B)} B,$$

- All possible combinations of $\{n_a\}$ keep all the information of the solution.

Rotating Wave Approximation

- Keeping only the combination fulfilling the resonance condition: $\delta k_{12} + \sum_a n_a q_a \approx 0$

$$i\frac{dB}{dr} = \begin{pmatrix} 0 & -i\kappa_{\{\mathrm{n}_a\}} e^{i\left(\delta k_{12} + \sum_a n_a q_a\right) r} \\ i\kappa^*_{\{\mathrm{n}_a\}} e^{-i\left(\delta k_{12} + \sum_a n_a q_a\right) r} & 0 \end{pmatrix} B,$$

- Define some quantities

$$\kappa_{\{n_a\}} = \kappa,\ \ 2p = \delta k_{12} + \sum_a n_a q_a,\ \ Q^2 = p^2 + \left|\kappa\right|^2.$$

2-flavor solution

- The analytical solution for B-matrix will be

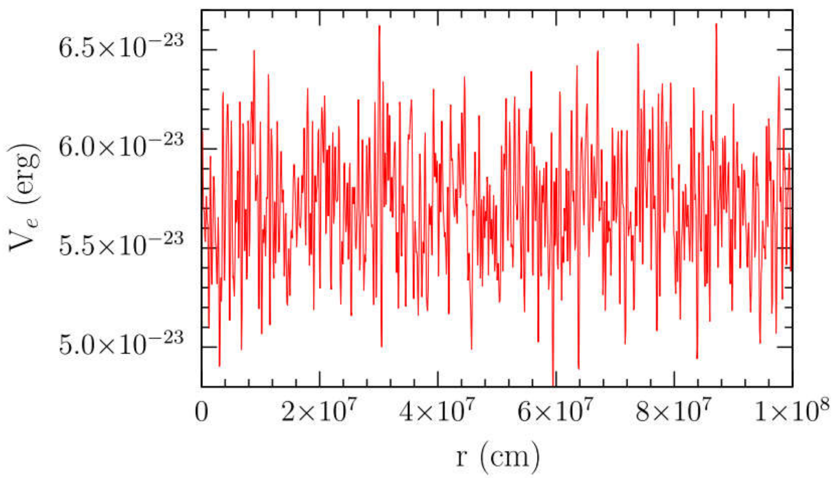
$$B=\begin{pmatrix} e^{ipr}\left[\cos\left(Qr\right)-i\dfrac{p}{Q}\sin\left(Qr\right)\right] & -e^{ipr}\dfrac{\kappa}{Q}\sin\left(Qr\right) \\ e^{-ipr}\dfrac{\kappa^{*}}{Q}\sin\left(Qr\right) & e^{-ipr}\left[\cos\left(Qr\right)+i\dfrac{p}{Q}\sin\left(Qr\right)\right] \end{pmatrix}$$

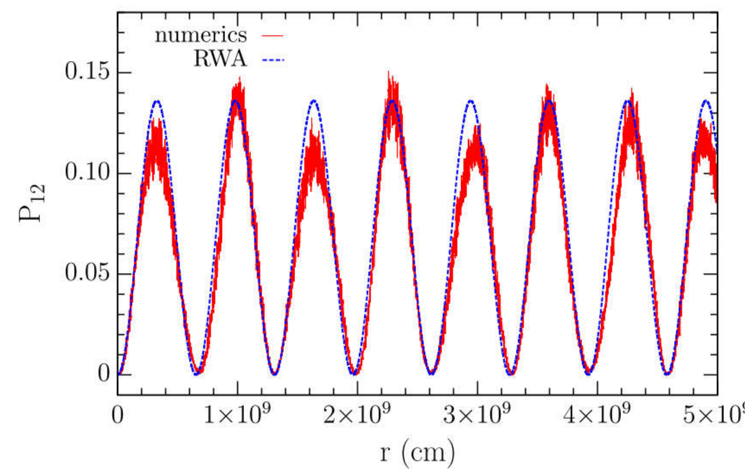
- The transition probability will be

$$P_{12}\left(r\right)=\left|B_{12}\right|^{2}=\frac{\left|\kappa\right|^{2}}{Q^{2}}\sin^{2}\left(Qr\right).$$

Numeric vs RWA

- The match between numeric and RWA results





KM Patton et al. PRD (2014)

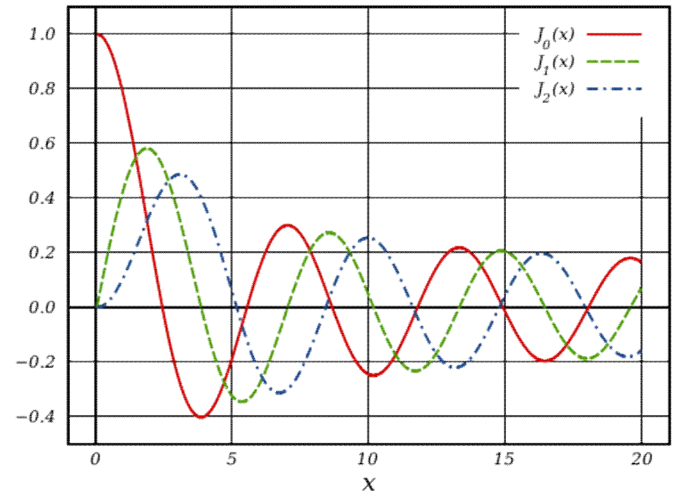
Suppressed transition

- A matter potential with 2 Fourier modes:

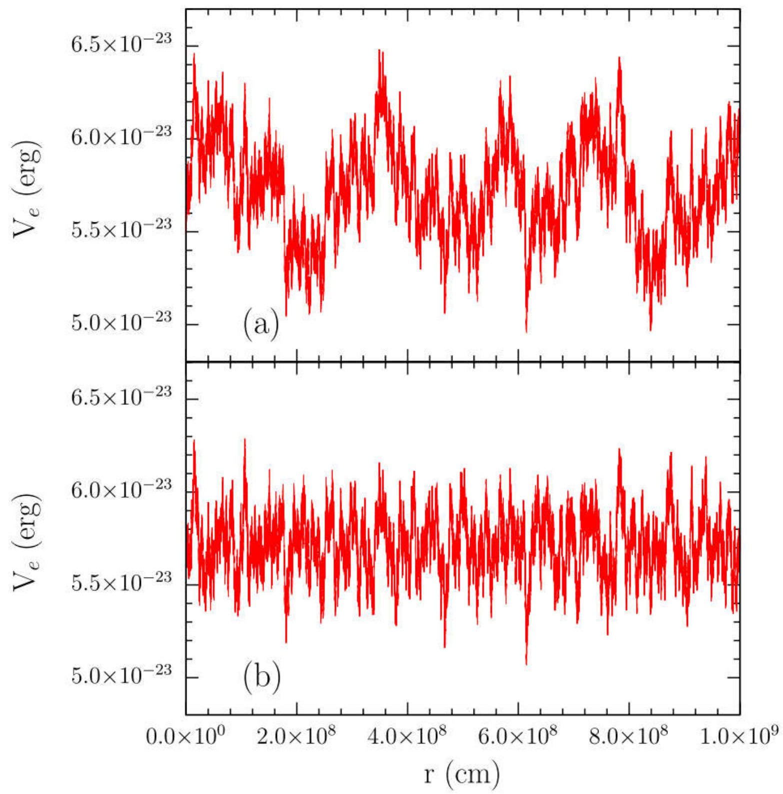
$$\delta V(r) = V_0 \left[A_1 \cos\left(q_1 r + \phi_1\right) + A_2 \cos\left(q_2 r + \phi_2\right) \right],$$

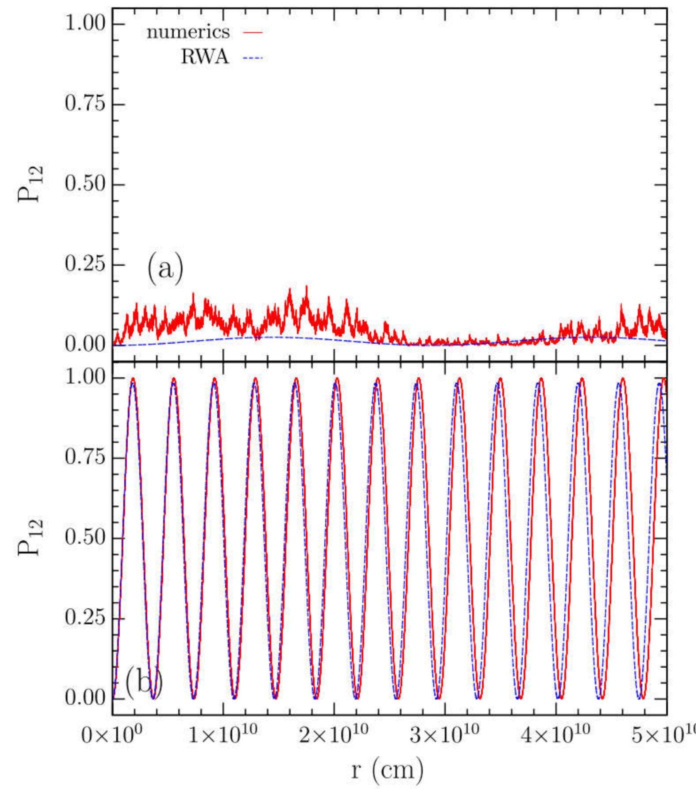
$$\delta k_{12} + n_1 q_1 + n_2 q_2 \sim 0, \quad q_1 \sim \left|\delta k_{12}\right|, \quad q_2 \ll \left|\delta k_{12}\right|,$$
$$\Rightarrow \left\{n_1, n_2\right\} = \left\{1, 0\right\}.$$

$$\kappa = (const.)\left(n_1 q_1 + n_2 q_2\right) J_1\left(z_1\right) J_0\left(z_2\right),$$
$$z_a = (const.)\left|\frac{V_a}{q_a}\right|. \left(z_1 \sim 0, \quad z_2 \sim 2.4\right)$$



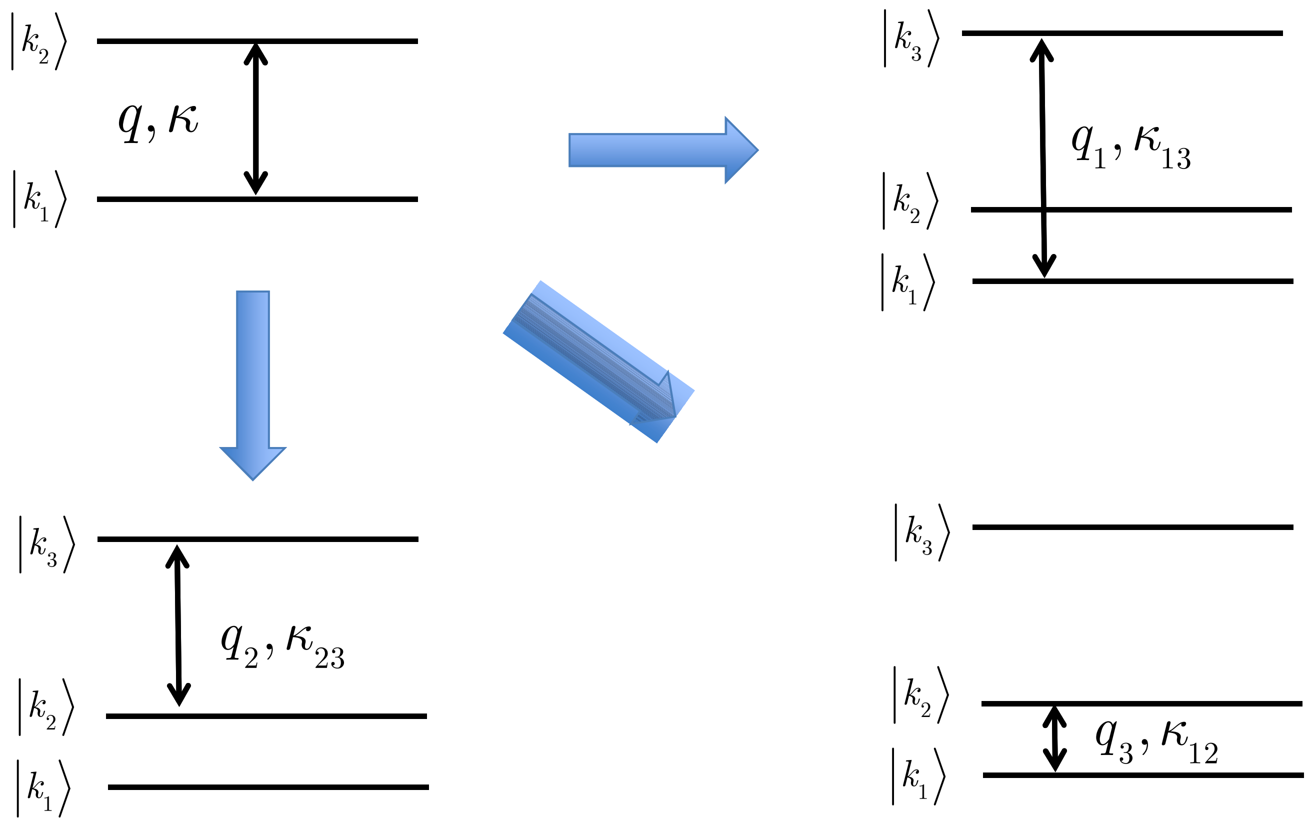
Suppressed transition





KM Patton et al. PRD (2014)

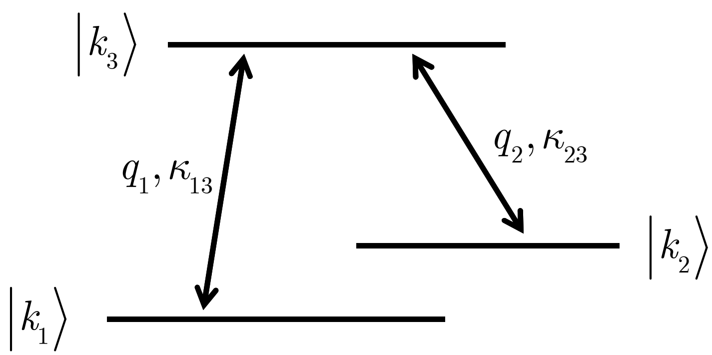
From 2 eigenstates to 3 eigenstates



Competing modes

- 2 Fourier modes

$$\delta V(r) = V_0\left[A_1\cos(q_1 r+\phi_1)+A_2\cos(q_2 r+\phi_2)\right],\ \ q_1\sim|\delta k_{13}|,\, q_2\sim|\delta k_{23}|.$$

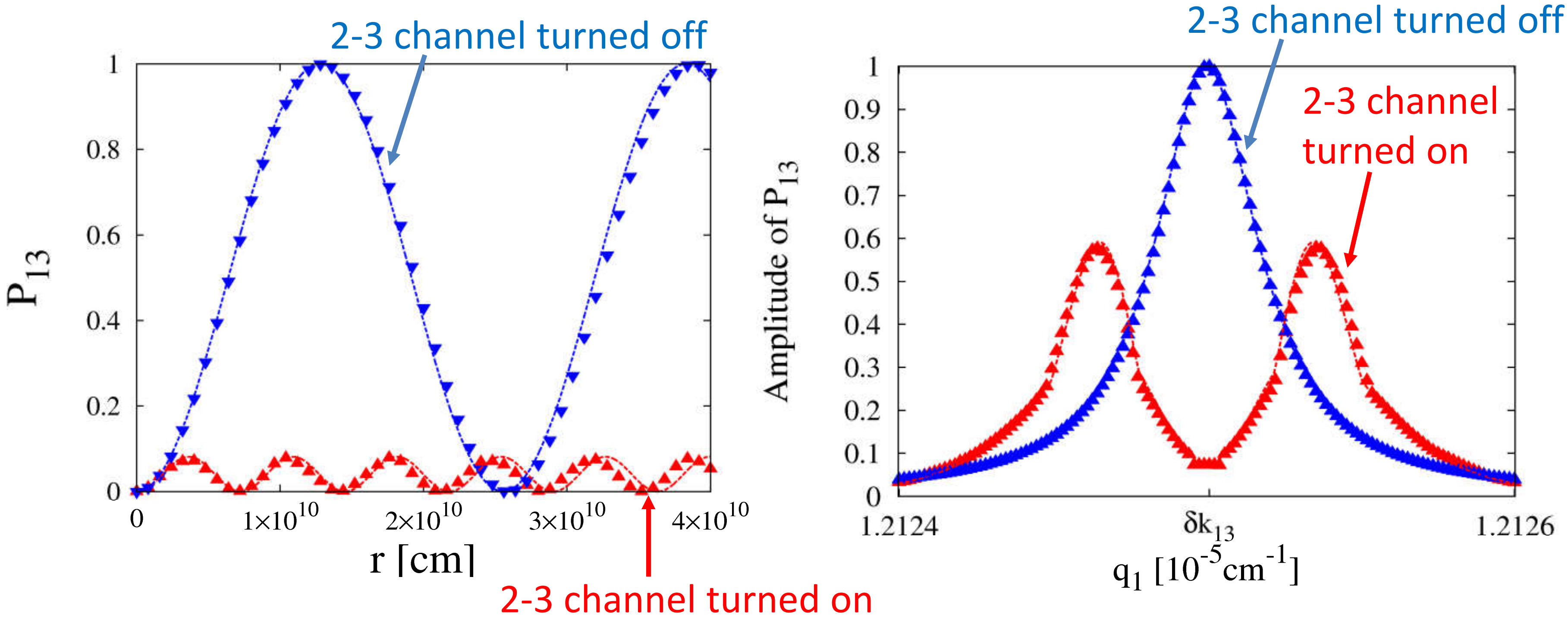


$$B=\begin{pmatrix} \frac{|\kappa_{23}|^2}{Q^2}+\frac{|\kappa_{13}|^2}{Q^2}\cos(Qr) & \frac{\kappa_{13}\kappa_{23}^*}{Q^2}\left[\cos(Qr)-1\right] & -\frac{i\kappa_{13}}{Q}\sin(Qr) \\ \frac{\kappa_{23}\kappa_{13}^*}{Q^2}\left[\cos(Qr)-1\right] & \frac{|\kappa_{13}|^2}{Q^2}+\frac{|\kappa_{23}|^2}{Q^2}\cos(Qr) & -\frac{i\kappa_{23}}{Q}\sin(Qr) \\ -\frac{i\kappa_{13}^*}{Q}\sin(Qr) & -\frac{i\kappa_{23}^*}{Q}\sin(Qr) & \cos(Qr) \end{pmatrix},$$

$$P_{13}(r)=|B_{13}|^2=\frac{|\kappa_{13}|^2}{Q^2}\sin^2(Qr)=\frac{|\kappa_{13}|^2}{|\kappa_{13}|^2+|\kappa_{23}|^2}\sin^2(Qr).$$

Competing modes

- By turning on the 2→3 channel, 1→3 channel is suppressed. (Dots/lines are numeric/analytical results.)



Channel manipulation

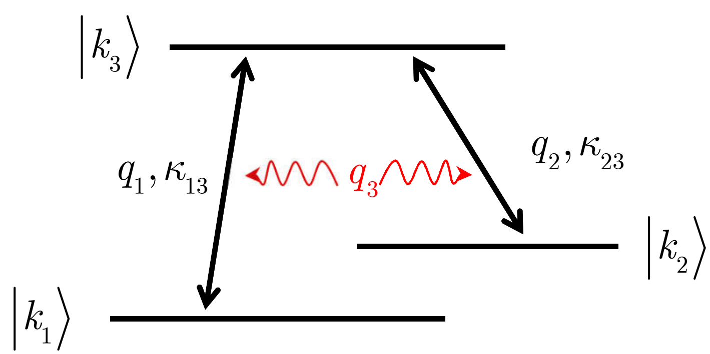
- 3 Fourier modes:

$$\delta V(r) = V_0\left[A_1\cos\left(q_1 r+\phi_1\right)+A_2\cos\left(q_2 r+\phi_2\right)+A_3\cos\left(q_3 r+\phi_3\right)\right],\quad q_3 \ll \left|\delta k_{12}\right|$$

$$\kappa_{13} = (\cdots)J_0\left(z_{13,3}\right),\quad \kappa_{23} = (\cdots)J_0\left(z_{23,3}\right)\sim 0,$$

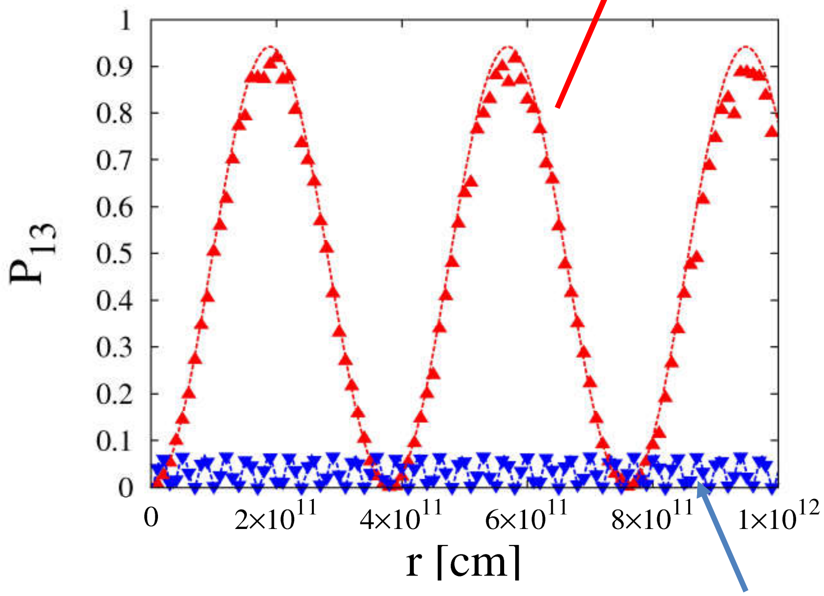
$$z_{ij,a} = (const.)\left(\frac{V_a}{q_a}\right)\left(\left|U_{ei}\right|^2-\left|U_{ej}\right|^2\right).$$

$$\left(z_{13,3}\neq 2.4,\quad z_{23,3}\sim 2.4\right)$$



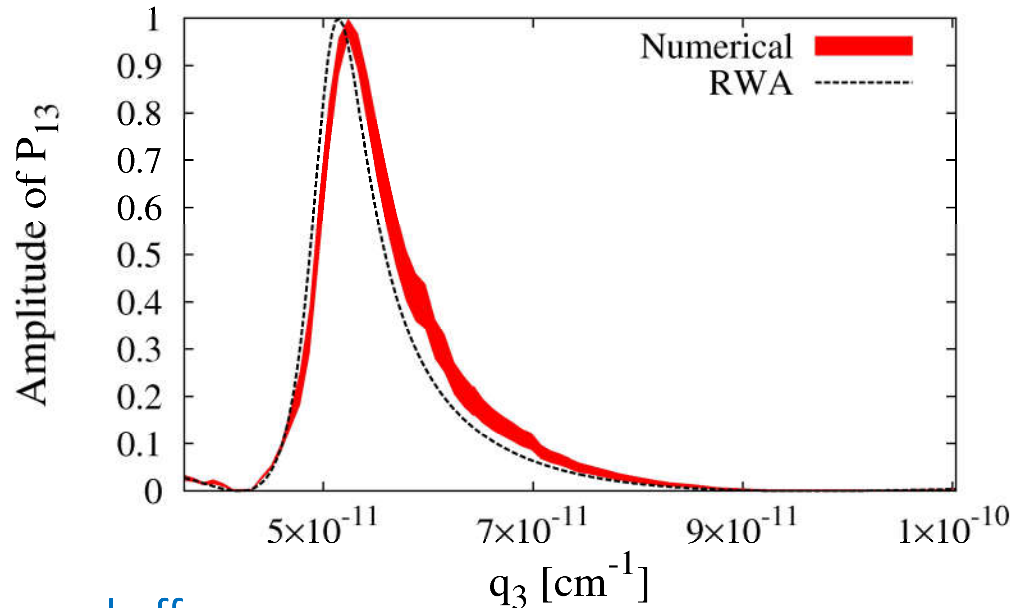
Channel manipulation

Mode 3 turned on



Mode 3 turned off

V0=20,A1=0.01,A2=0.05,A3=0.1



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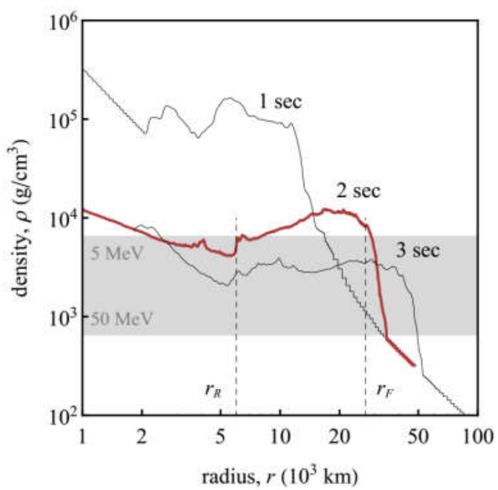
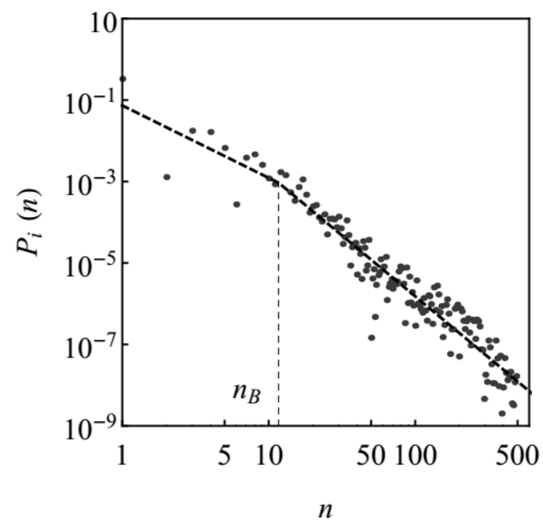
Turbulence

- High resolution spectrum





E. Borriello et al. JCAP (2014)

- Power law spectrum (longer wavelength comes with larger amplitude)

Conclusion

- Stimulated neutrino transformation caused by turbulence may play an important role.
- Rotating Wave Approximation (RWA) is good tool to investigate stimulated transformations analytically.
- Some novel effects could take place in a 3-flavor-mixing model.
- Numeric study using real turbulence spectrum is needed.

Thank you very much!