Chiral Transport of Neutrinos in Supernovae

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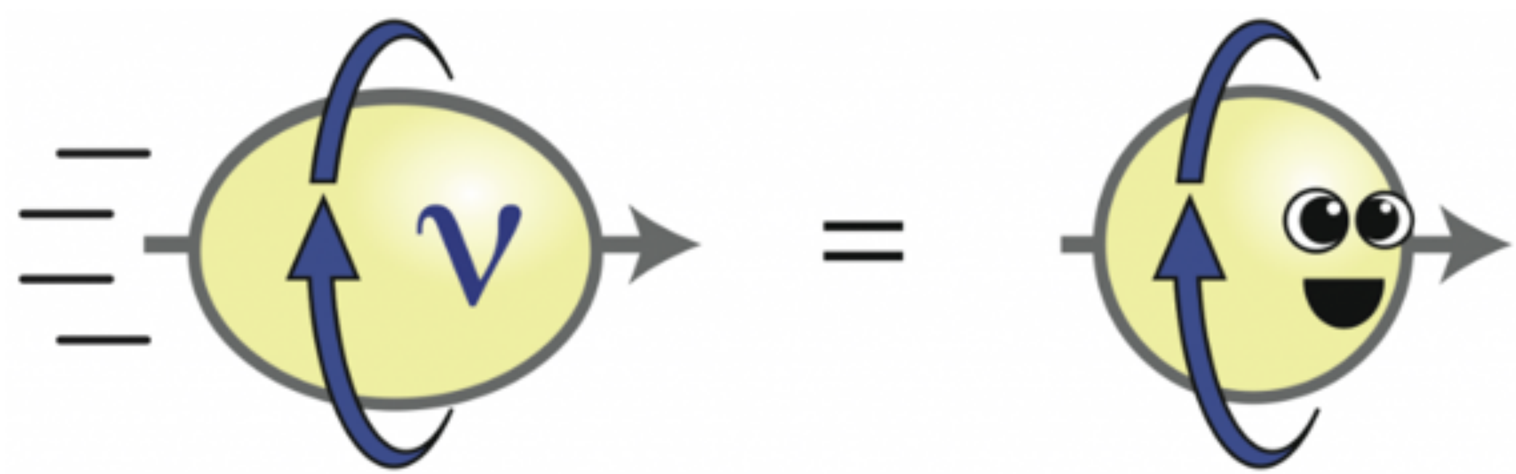
"Flavor Observations with Supernova Neutrinos"
August 18, 2016 @ INT

Key property of neutrinos



http://www.quantumdiaries.org/2012/03/04/particle-paparazzi-the-private-lives-of-the-standard-model-particles-summary/

Neutrinos are left-handed.

Why is "God" left-handed?

The laws of physics are left-right symmetric except for the weak interaction that acts only on left-handed particles.

W. Pauli

"God is just a weak left-hander."

From micro to macro

Microscopic parity violation is reflected in macroscopic behavior:

Micro Chirality of neutrinos in Standard Model

↓

Chiral Kinetic Theory (Boltzmann eq.)

Son-NY, PRL (2012); Stephanov-Yin, PRL (2012)

↓

Macro Evolution of core-collapse supernovae (giant P violation)

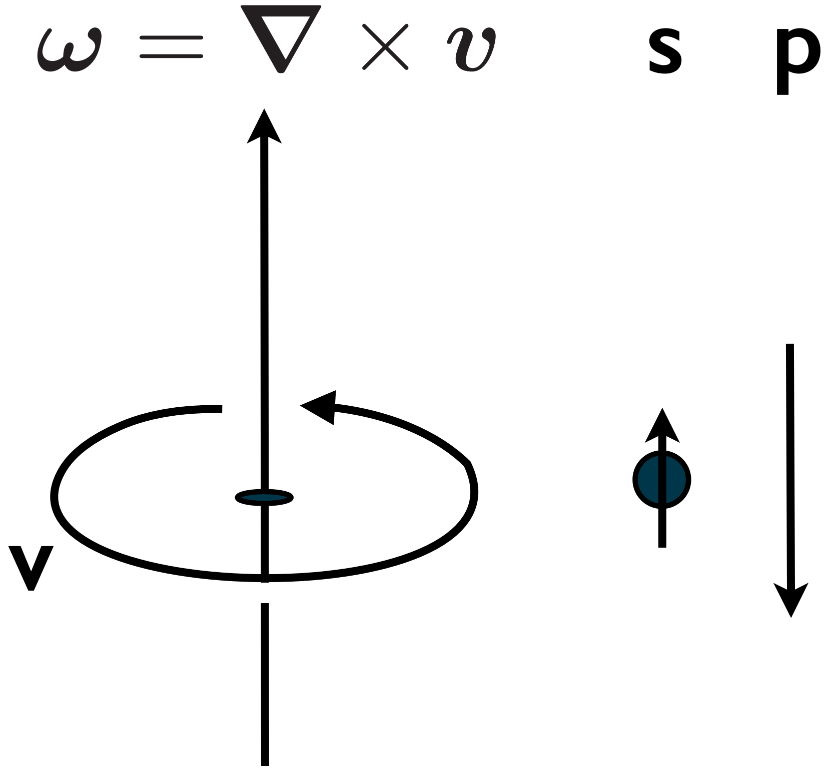
NY, PRD (2016)

Supernova = Giant parity breaker

$$p + e^{\mathbf{L}} \rightarrow n + \nu_e^{\mathbf{L}}$$

Chiral Vortical Effect (CVE)

$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$



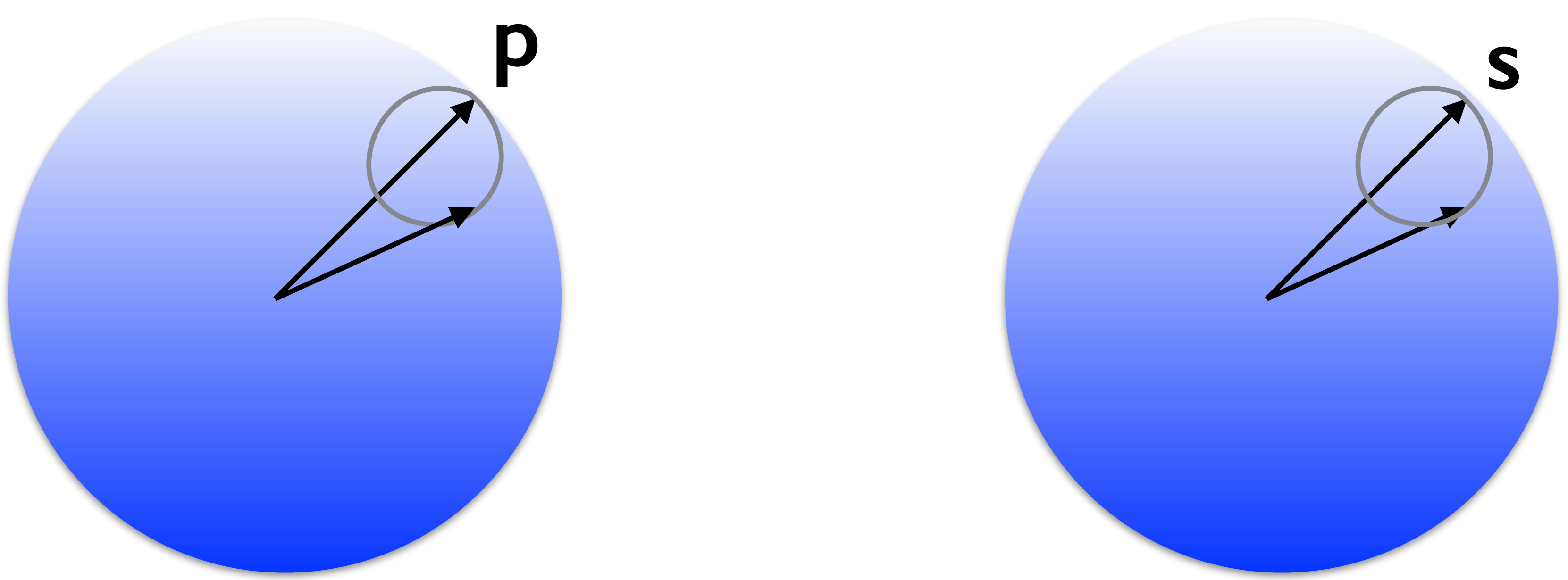
$$\boldsymbol{j} = -\left(\frac{\mu^2}{8\pi^2} + \frac{T^2}{24}\right)\boldsymbol{\omega}$$

Vilenkin (1979); Erdmenger et al. (2009); Banerjee et al. (2011); Son-Surowka (2009); K. Landsteiner et al. (2011)

Chiral Transport Theory

Chirality and topology

Right-handed fermions



momentum space

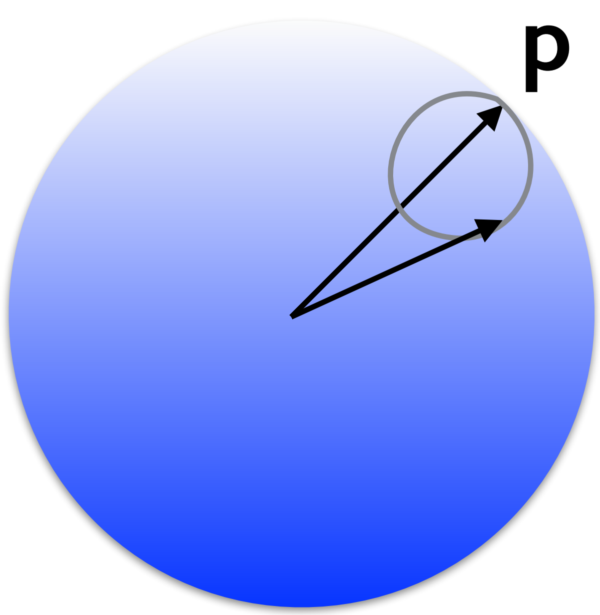
spin space

Mapping: S^2 (p-space) → S^2 (spin space)

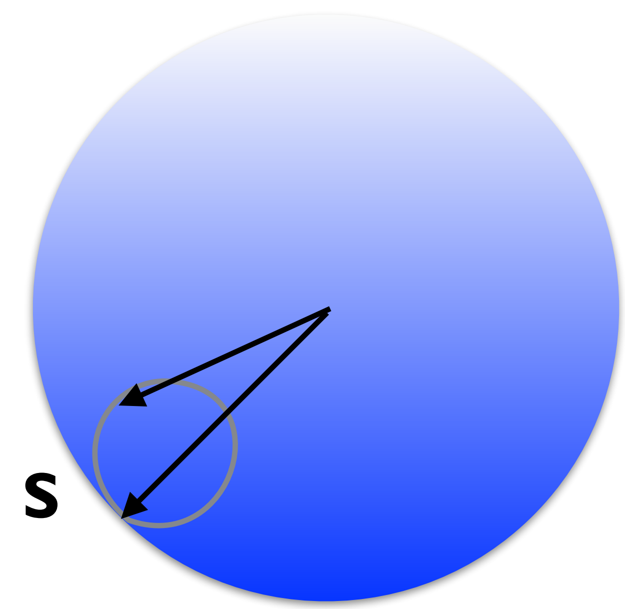
winding number +1

Chirality and topology

Left-handed fermions



momentum space

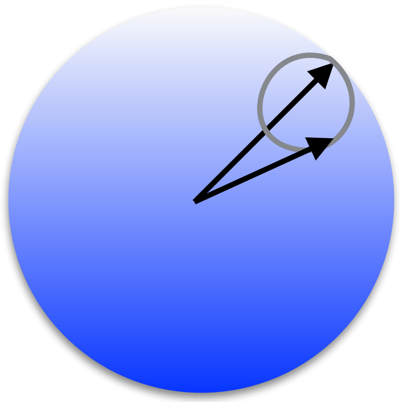


spin space

Mapping: S^2 (p-space) → S^2 (spin space)

winding number -1

Topology and Berry curvature



- $\pi_2(S^2)=\pm 1$ → Monopole at $\boldsymbol{p}$=**0**
- "Magnetic field" of monopole = Berry curvature $\boldsymbol{\Omega_p} = \pm\frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|^2}$
- $\boldsymbol{\Omega}_p$ modifies the equation of motion

ν has the Berry curvature; it modifies non-equilibrium dynamics

Equations of motion
(for charged *chiral* particles)

- Action: $S = \int [(\boldsymbol{p} + \boldsymbol{A}) \cdot d\boldsymbol{x} - (\epsilon_{\boldsymbol{p}} + \phi) dt - \boldsymbol{a}_{\boldsymbol{p}} \cdot d\boldsymbol{p}]$
- Berry curvature: $\boldsymbol{\Omega}_{\boldsymbol{p}} \equiv \boldsymbol{\nabla}_{\boldsymbol{p}} \times \boldsymbol{a}_{\boldsymbol{p}} = \pm \frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|^2}$
- Dispersion relation: $\epsilon_{\boldsymbol{p}} = |\boldsymbol{p}|(1 - \boldsymbol{\Omega}_{\boldsymbol{p}} \cdot \boldsymbol{B})$ Son-NY, PRD (2013)
- Equations of motion:

$$\dot{\boldsymbol{x}} = \frac{\partial \epsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{\Omega}_{\boldsymbol{p}}$$

$$\dot{\boldsymbol{p}} = -\frac{\partial \epsilon_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}$$

Chiral kinetic theory

(for charged *chiral* particles)

Substitute modified EOM into

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \underline{\dot{\boldsymbol{x}}} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \underline{\dot{\boldsymbol{p}}} \cdot \frac{\partial f}{\partial \boldsymbol{p}} = C[f]$$

$$(1 + \boldsymbol{B} \cdot \boldsymbol{\Omega_p}) \frac{\partial f}{\partial t} + \left[\tilde{\boldsymbol{v}} + \tilde{\boldsymbol{E}} \times \boldsymbol{\Omega_p} + (\tilde{\boldsymbol{v}} \cdot \boldsymbol{\Omega_p}) \boldsymbol{B} \right] \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \left[\tilde{\boldsymbol{E}} + \tilde{\boldsymbol{v}} \times \boldsymbol{B} + (\tilde{\boldsymbol{E}} \cdot \boldsymbol{B}) \boldsymbol{\Omega_p} \right] \cdot \frac{\partial f}{\partial \boldsymbol{p}} = C[f]$$

$$\boldsymbol{\Omega_p} = \pm \frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|^2}, \quad \tilde{\boldsymbol{v}} \equiv \frac{\partial \epsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}}, \quad \tilde{\boldsymbol{E}} \equiv \boldsymbol{E} - \frac{\partial \epsilon_{\boldsymbol{p}}}{\partial \boldsymbol{x}}$$

Son-NY, PRL (2012); Stephanov-Yin, PRL (2012)

Current and stress tensor

(for neutrinos)

- Current: $\boldsymbol{j} = \int_{\boldsymbol{p}} (\hat{\boldsymbol{p}} f - |\boldsymbol{p}| \boldsymbol{\Omega}_{\boldsymbol{p}} \times \boldsymbol{\nabla}_{\boldsymbol{x}} f)$

- Energy-momentum tensor:

$$T^{ij} = \int_{\boldsymbol{p}} |\boldsymbol{p}| \left(\hat{p}^i \hat{p}^j f - \frac{1}{2} p^i \epsilon^{jk\ell} \Omega^k_{\boldsymbol{p}} \partial_\ell f - \frac{1}{2} p^j \epsilon^{ik\ell} \Omega^k_{\boldsymbol{p}} \partial_\ell f \right)$$

Corrections due to chirality

Son-NY, PRD (2013); Chen-Son-Stephanov, PRL (2015)

- Radiation *chiral* hydrodynamics: $\nabla_\alpha (T^{\alpha\beta}_{\rm hyd} + T^{\alpha\beta}_{\nu}) = 0$

NY, in preparation

Applications to core-collapse supernovae

ν medium in SN = chiral matter
$\to$ chiral kinetic theory

NY, PRD (2016) [arXiv:1511.00933]

Energy cascade of turbulence

Direct cascade

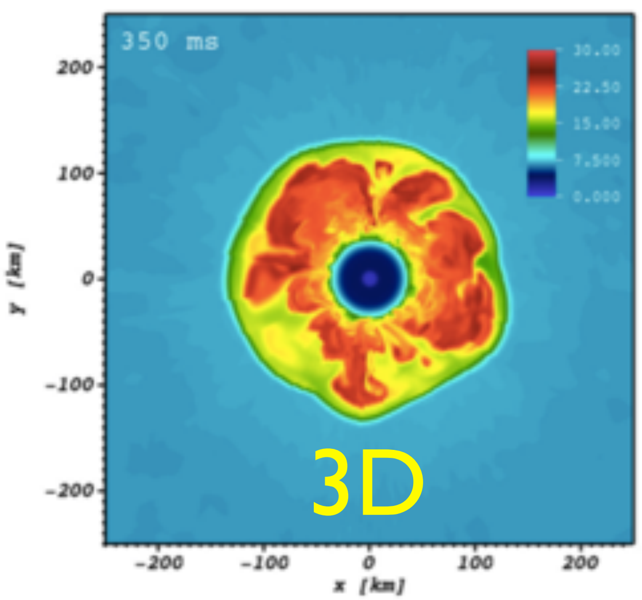
(3D *usual* matter)

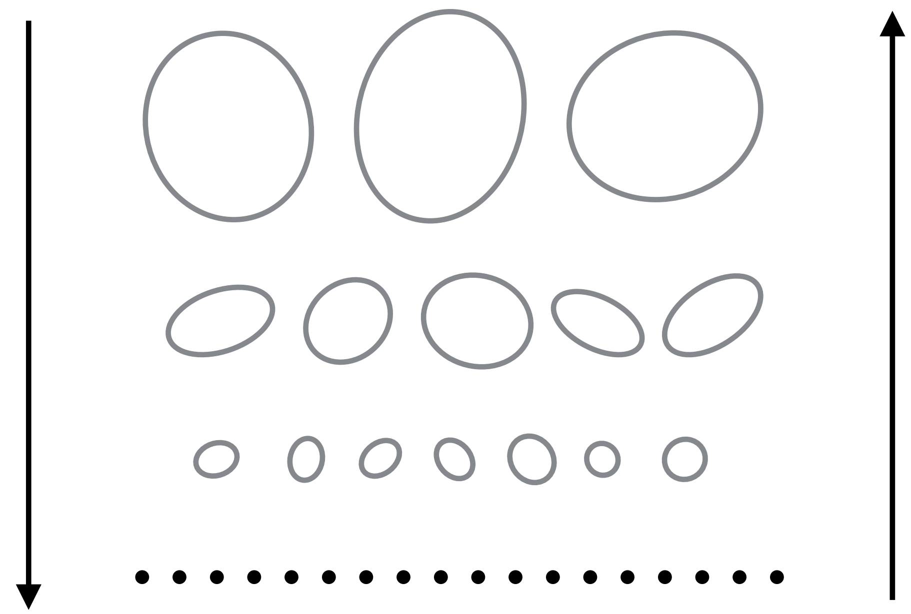
explosion difficult

Inverse cascade

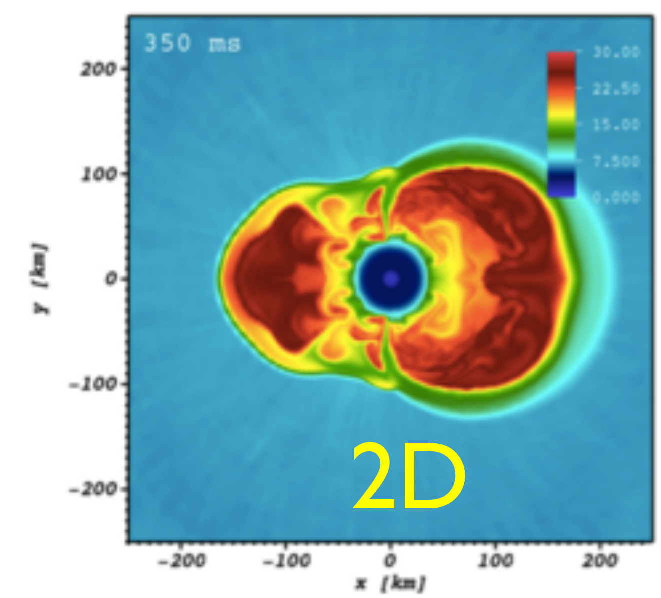
(2D usual matter)

explosion easier





F. Hanke (2014)



3D pure *chiral* matter in the hydro regime: inverse cascade

Chiral turbulence

NY, PRD (2016) [arXiv:1603.08864]

- Chiral hydrodynamic equations:

$$(\epsilon + P)(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = -\boldsymbol{\nabla}P + \nu\boldsymbol{\nabla}^2\boldsymbol{v}$$

$$\partial_t\left(n + \underline{\kappa\boldsymbol{v}\cdot\boldsymbol{\omega}}\right) + \boldsymbol{\nabla}\cdot\boldsymbol{j} = 0, \qquad \boldsymbol{j} = n\boldsymbol{v} + \underline{\kappa\boldsymbol{\omega}}$$

P violation

- Unique scaling symmetry in the inertial range:

$$\boldsymbol{x} \to l\boldsymbol{x}, \quad t \to l^{1-h}t, \quad \boldsymbol{v} \to l^h\boldsymbol{v}, \quad \mu \to l^p\mu$$

$$\boxed{h = 0, \qquad p = -1}$$

- Correlation length: $\xi_v(t) = \xi_v(t_s)\left(\dfrac{t}{t_s}\right)$ inverse energy cascade

Inverse Energy Cascade in Three-Dimensional Isotropic Turbulence

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We study the statistical properties of homogeneous and isotropic three-dimensional (3D) turbulent flows. By introducing a novel way to make numerical investigations of Navier-Stokes equations, we show that all 3D flows in nature possess a subset of nonlinear evolution leading to a reverse energy transfer: from small to large scales. Up to now, such an inverse cascade was only observed in flows under strong rotation and in quasi-two-dimensional geometries under strong confinement. We show here that energy flux is always reversed when mirror symmetry is broken, leading to a distribution of helicity in the system with a well-defined sign at all wave numbers. Our findings broaden the range of flows where the inverse energy cascade may be detected and rationalize the role played by helicity in the energy transfer process, showing that both 2D and 3D properties naturally coexist in all flows in nature. The unconventional numerical methodology here proposed, based on a Galerkin decimation of helical Fourier modes, paves the road for future studies on the influence of helicity on small-scale intermittency and the nature of the nonlinear interaction in magnetohydrodynamics.

Summary

- Relevance of ν chiral transport in supernova explosions.
- Chirality of ν gives the tendency to inverse cascade in 3D.
- Should be checked with neutrino radiation *chiral* hydro.
- Possible observables of the chiral effects?