Neutrino Interactions in Dense Matter

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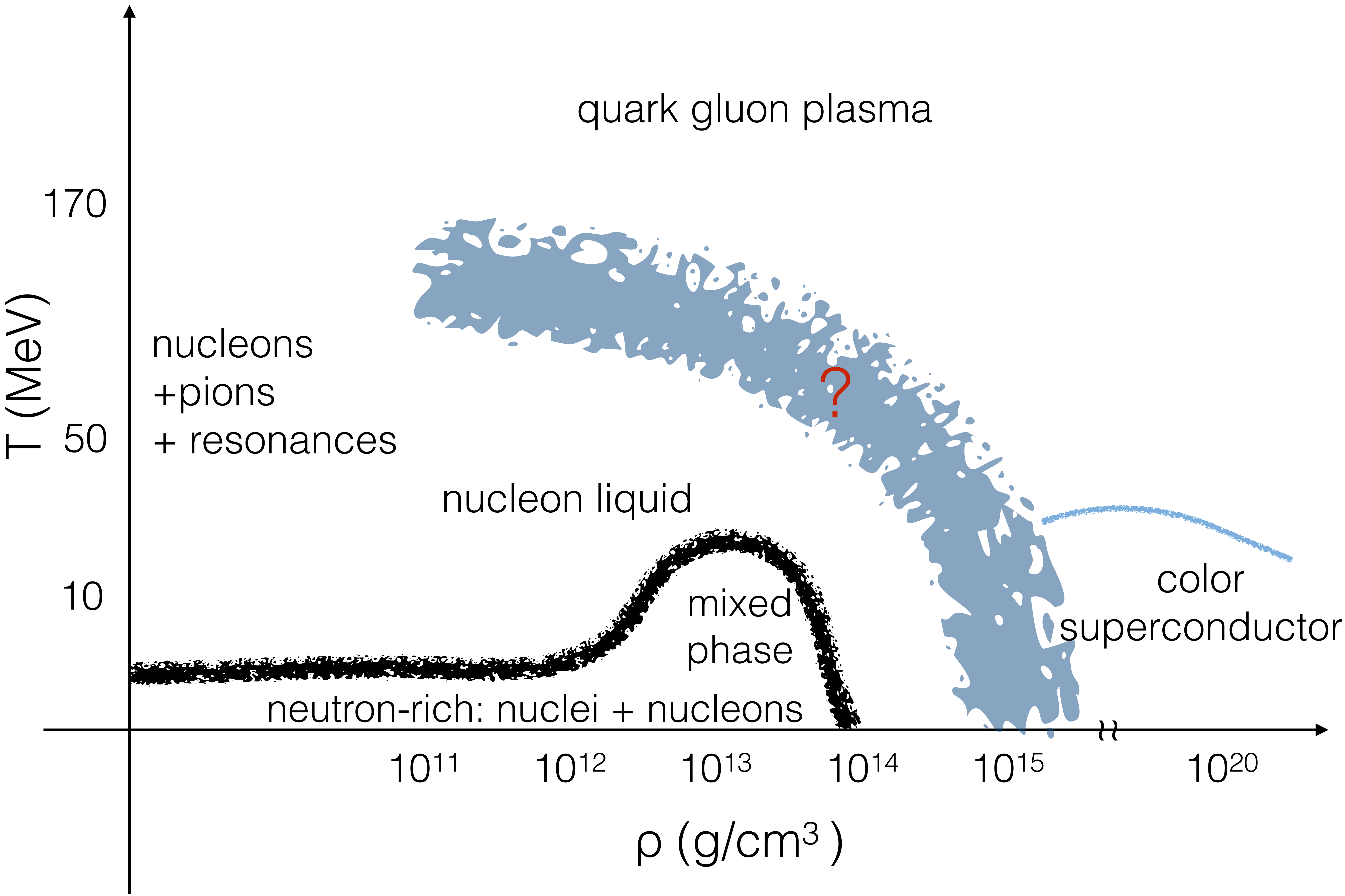
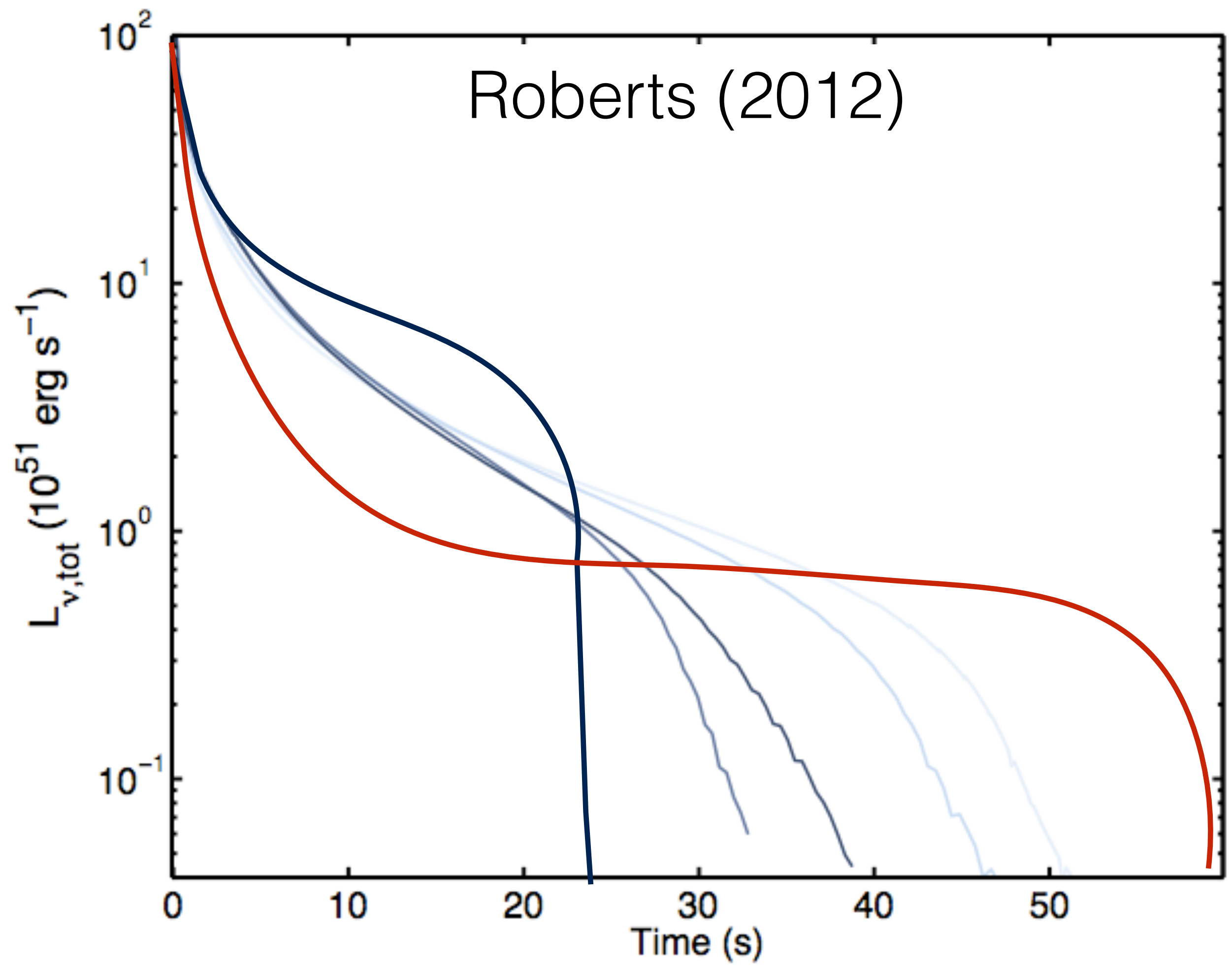
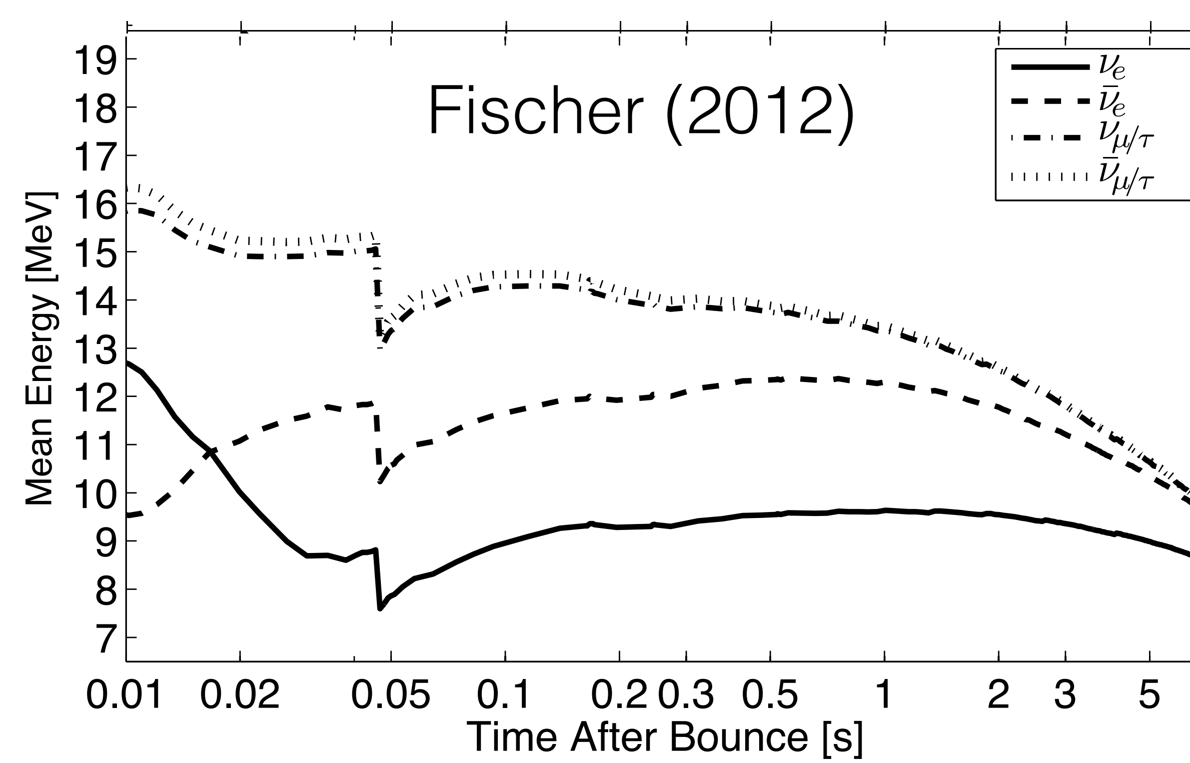
Phase Diagram of Hot and Dense Matter

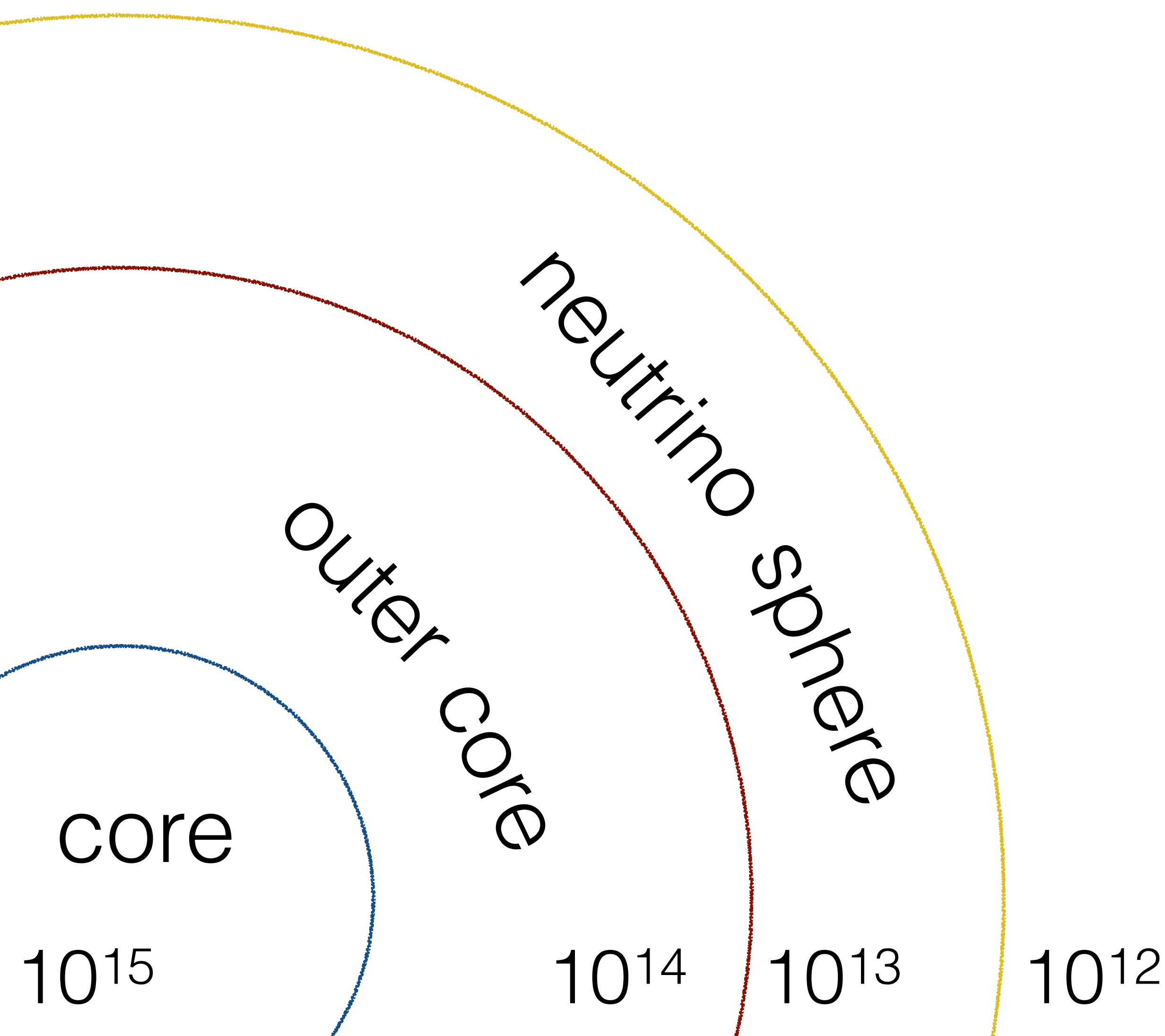


Neutrino Interactions and Observables

- Neutrino diffusion and convection in the outer and inner core influence the temporal structure.
- Neutrino interactions in the neutrino-sphere determine the spectra.





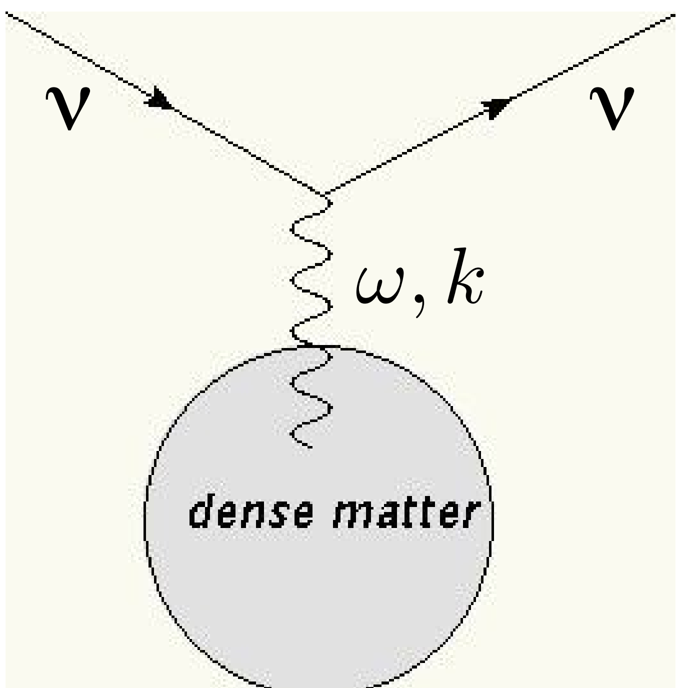


Low Energy Neutrino Scattering in Non-Relativistic Matter

Neutral-current coupling in the non-relativistic limit

$$\mathcal{L}_W = -\frac{G_F}{2\sqrt{2}}\ l^\mu\ J_\mu$$

$$J_\mu = \bar{N}\ \Gamma_\mu\ N \simeq N^\dagger\ (C_V\ \delta^0_\mu - C_A \delta^i_\mu\ \sigma_i)\ N$$



Neutrinos scatter from density and spin fluctuations.

Cannot resolve individual nucleons when

$ka \leq 1$ and/or $\omega\tau \leq 1$

a: nucleon correlation length

τ: nucleon collision frequency

Sawyer (1975, 1989)
Iwamoto & Pethick (1982)
Horowitz & Wehrberger (1991)
Raffelt & Seckel (1995)
Reddy et al. (1999)
Burrows & Sawyer (1999)

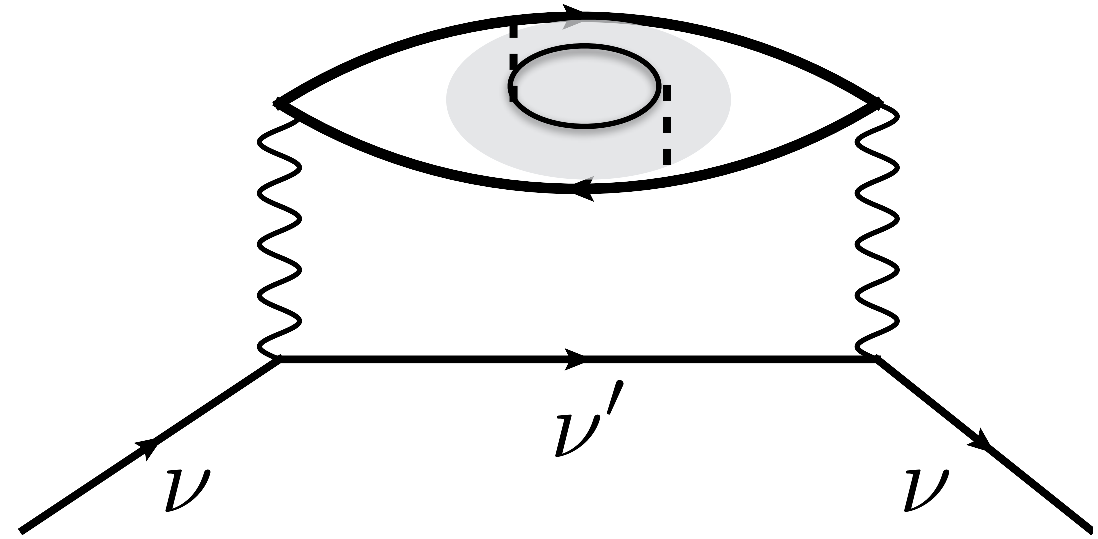
Scattering rate:

$$\frac{d\Gamma(E_\nu)}{d\cos\theta\ d\omega} = \frac{G_F^2}{4\pi^2}\ (E_\nu - \omega)^2 (1 - f_\nu(E_\nu - \omega)) \times \mathcal{R}(\omega, k)$$

$$\mathcal{R}(\omega, k) = C_V^2\ (1 + \cos\theta)\ S_\rho(\omega, k) + C_A^2\ (3 - \cos\theta)\ S_\sigma(\omega, k)$$

Response and Correlation Functions

$$\Pi_{\mu\nu}(\omega,\vec{k}) = \int \frac{d^4p}{(2\pi)^4}\ \mathrm{Tr}[J_\mu(p_0,\vec{p})\ J_\nu(p_0+\omega,\vec{p}+\vec{k})]$$



Density response: $$S_\rho(\omega,\vec{k}) = \frac{1}{1-\exp(-\beta\omega)}\ \mathrm{Im}\ \Pi_0(\omega,\vec{k})$$

Spin response: $$S_\sigma(\omega,\vec{k}) = \frac{\delta_{ij}}{1-\exp(-\beta\omega)}\ \mathrm{Im}\ \Pi_{ij}(\omega,\vec{k})$$

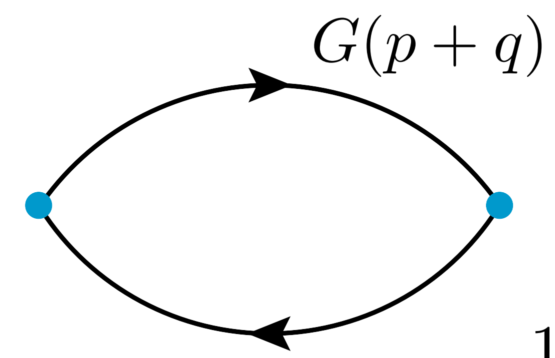
Information about many-nucleon dynamics is contained in these correlation functions.

$$\Pi_0(\omega,|\vec{k}|) = -i\int d^4x\ e^{-i(\vec{k}\cdot\vec{x}-\omega t)}Tr(\rho_G\ [\rho(x,t),\rho(0,0)])$$

$$\Pi_{ij}(\omega,|\vec{k}|) = -i\int d^4x\ e^{-i(\vec{k}\cdot\vec{x}-\omega t)}Tr(\rho_G\ [\sigma_i(x,t),\sigma_j(0,0)])$$

Diagrammatic Calculations: Mean Field + RPA

Correlations functions on the non-interacting gas:

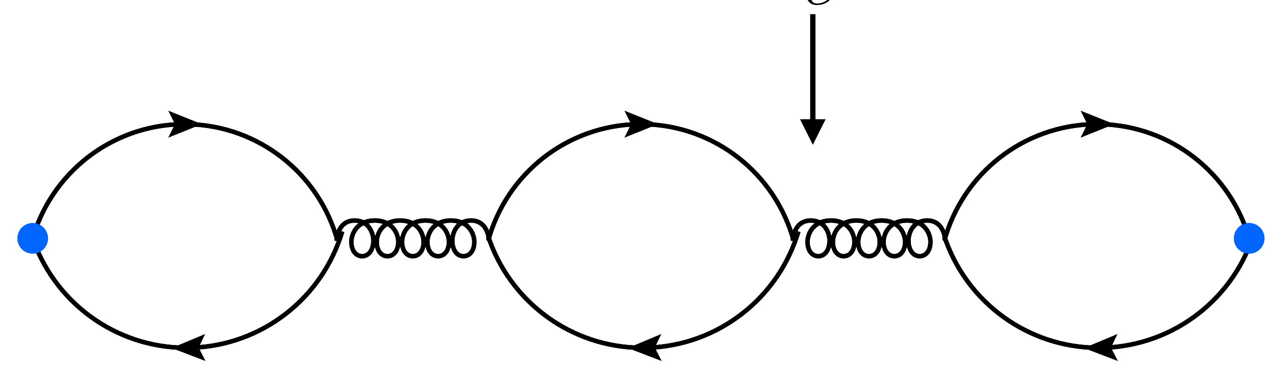


$$\Pi^0(q_0, q) = i \int \frac{d^4p}{(2\pi)^2}\; G(p)\; G(p+q)$$

$$G(p) = \frac{1}{p_0 - \mu - (p^2/2M)}$$

Response functions in RPA which includes particle-hole screening to all orders.

$$\Pi^{\rm RPA} = \Pi^0 + \Pi^{\rm RPA}\; V_c\; \Pi^0$$



Recovers the long-wavelength properties of the mean field ground state.

$$S_{\rm RPA}(q_0, q) = \frac{1}{1 - \exp\left(-\beta\omega\right)}\; {\rm Im}[\Pi^{\rm RPA}]$$

$$\Pi^{\rm RPA} = \left[\frac{\Pi^0(q_0, q)}{1 - V_c(q)\; \Pi^0(q_0, q)}\right]$$

Self-consistent approximation to a mean-field ground state.

Low Energy Neutrino Scattering in Hydrodynamic Limit

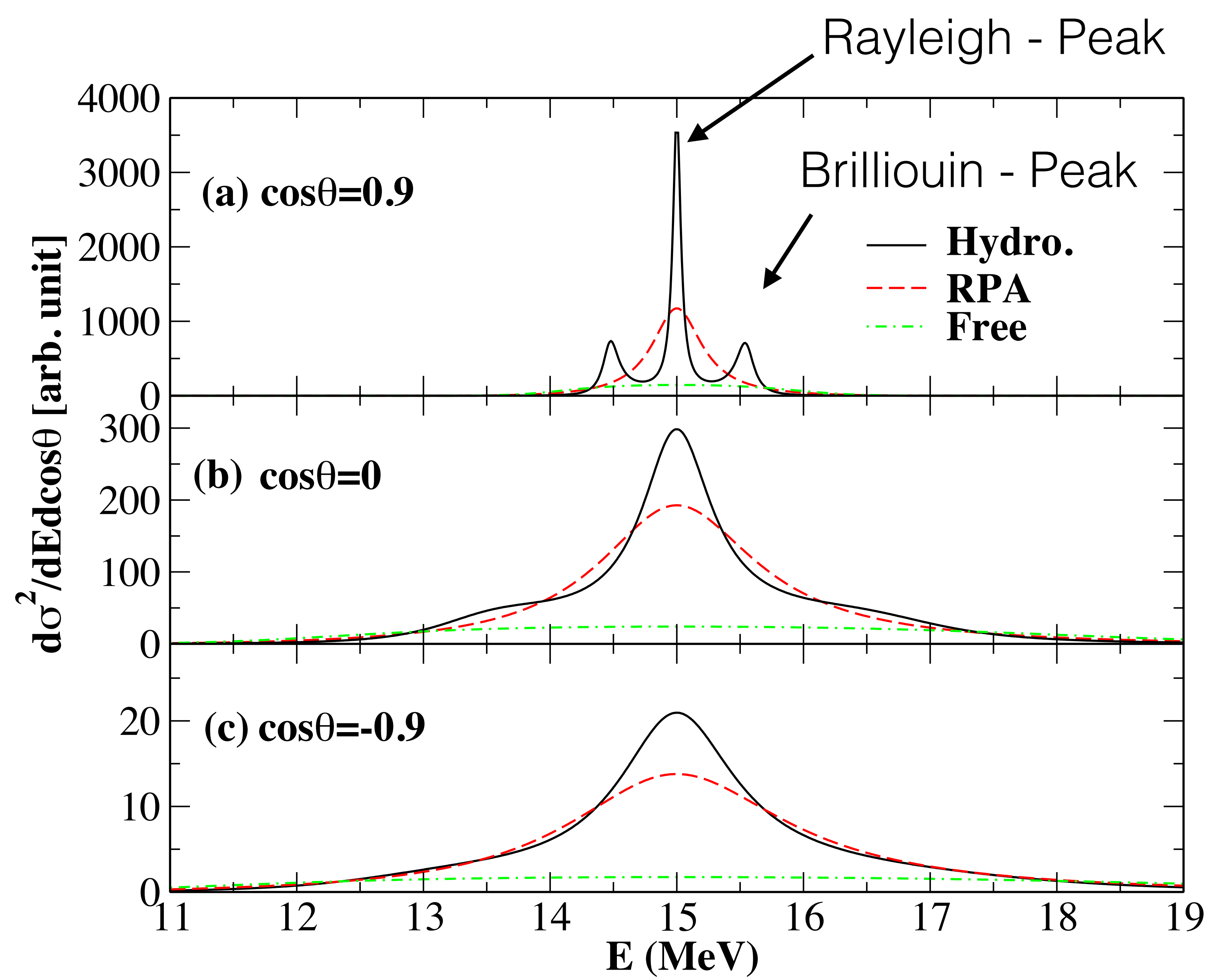
When the energy transfer is small compared to the collision frequency, the response is determined by hydrodynamic fluctuations $\omega\ \tau < 1$

$$\mathrm{Im}\chi(\mathbf{q}, q_0) = \frac{2F_2}{3m^2c^2}\left[\frac{q_0(\gamma-1)\Gamma_\kappa}{q_0^2+\Gamma_\kappa^2} + \frac{2q_0\Gamma\Omega^2}{(q_0^2-\Omega^2)^2+(2q_0\Gamma)^2} - \frac{q_0\Gamma_\kappa(\gamma-1)(q_0^2-\Omega^2)}{(q_0^2-\Omega^2)^2+(2q_0\Gamma)^2}\right]$$

Hydro Response is determined by the equation of state (speed of sound), thermal conductivity and shear viscosity of matter.

Hydro naturally incorporates multi-particle hole excitations.

RPA with a simple nucleon-nucleon interaction provides a fair description at moderate momentum transfer.



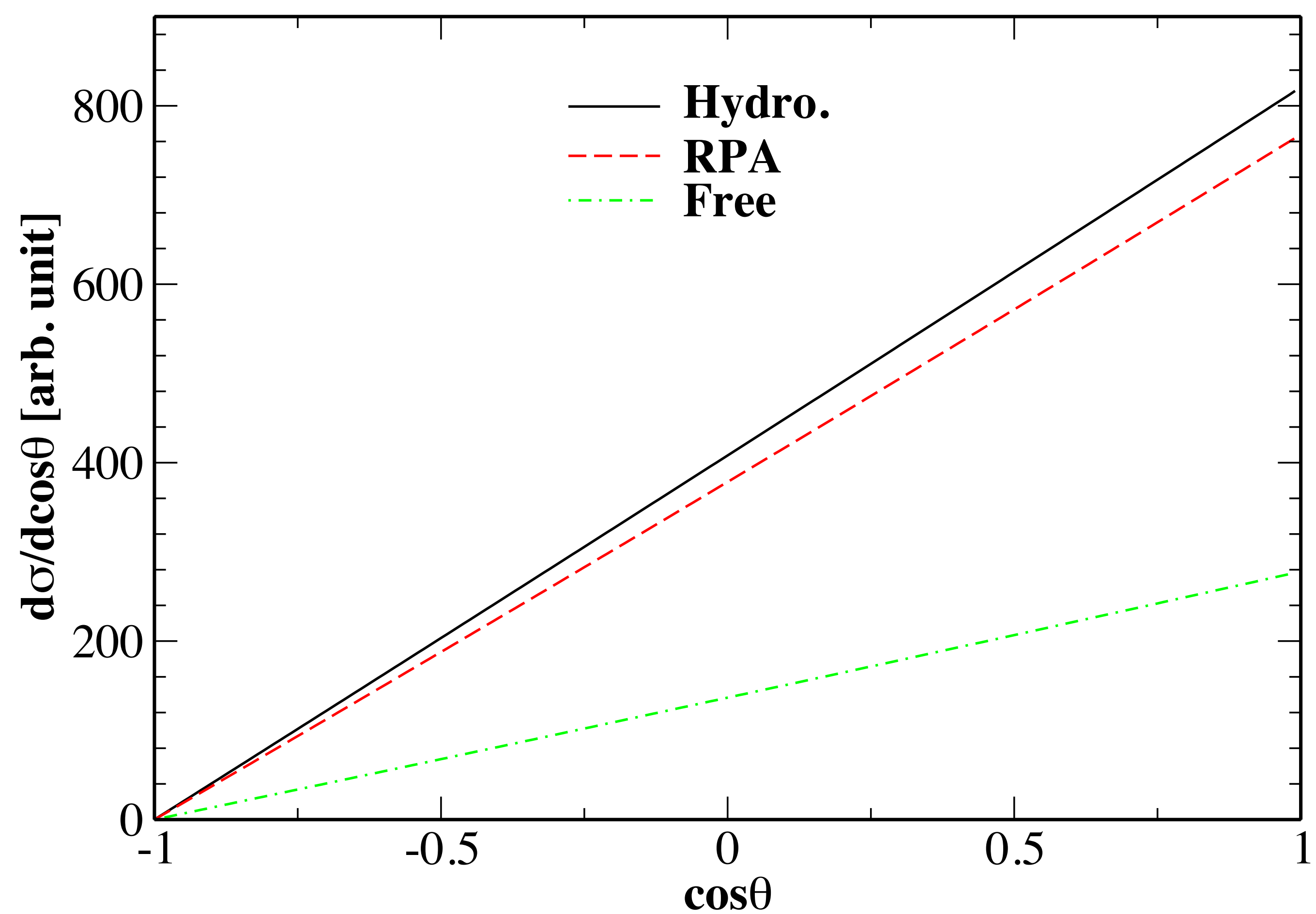
Shen & Reddy (2014)

Enhanced Density Response in Low Density Neutron Matter

Density fluctuations are enhanced by attractive nuclear interactions in low density matter.

Hydro and RPA capture the essential physics.

Redistribution of strength due to collisional damping changes opacity at 10-20%.



$$\lambda^{-1}(E_\nu) = \int d\theta\ (1-\cos\theta)\ \frac{d\sigma}{d\cos\theta}$$

Neutrino mean free path

Free Gas	3.6 km
Hydro	1.1 km
RPA	1.2 km

$$\rho_n = 10^{13}\ \mathrm{g/cm^3} \quad T = 5\ \mathrm{MeV} \quad E_\nu = 15\mathrm{MeV}$$

(inclusion of protons leads to formation of large nuclei (pasta), talk by Zidu Lin)

Neutrino rates and the nuclear spin response function

$$\mathcal{R}(\omega,k) = C_V^2\ (1+\cos\theta)\ S_\rho(\omega,k) + C_A^2\ (3-\cos\theta)\ S_\sigma(\omega,k)$$

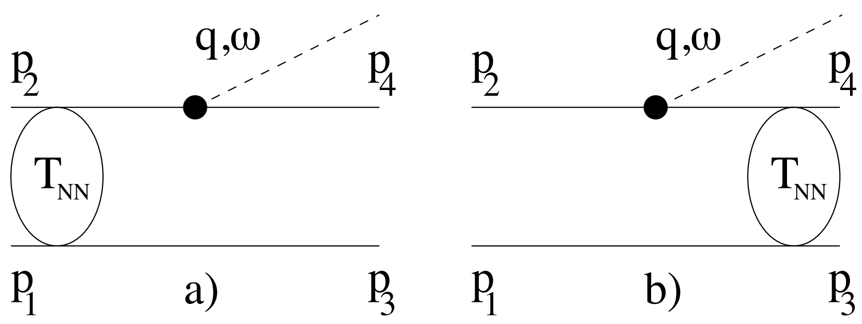
Quasi-particle response in Fermi Liquid Theory. It incorporates the Landau-Pomeranchuk-Migdal suppression.

$$S_\sigma(\omega) = \frac{N(0)}{n\pi}\ \frac{\omega\tau_\sigma}{(1+G_0)^2 + (\omega\tau_\sigma)^2}$$

Lykasov, Pethick, Schwenk (2006)
(see also Raffelt & Seckel (1995))

Spin-relaxation time: $\tau_\sigma = 2\pi \sum_{2,3,4} |\mathcal{M}|^2 \times \text{Phase Space}$

Spin-flip transitions: $|\mathcal{M}|^2 = \frac{1}{12} \sum_j \mathrm{Tr}\left[\mathrm{T}_{nn}\sigma_1^j\ \left[(\sigma_1+\sigma_2)^j, \mathrm{T}_{nn}\right]\right]$



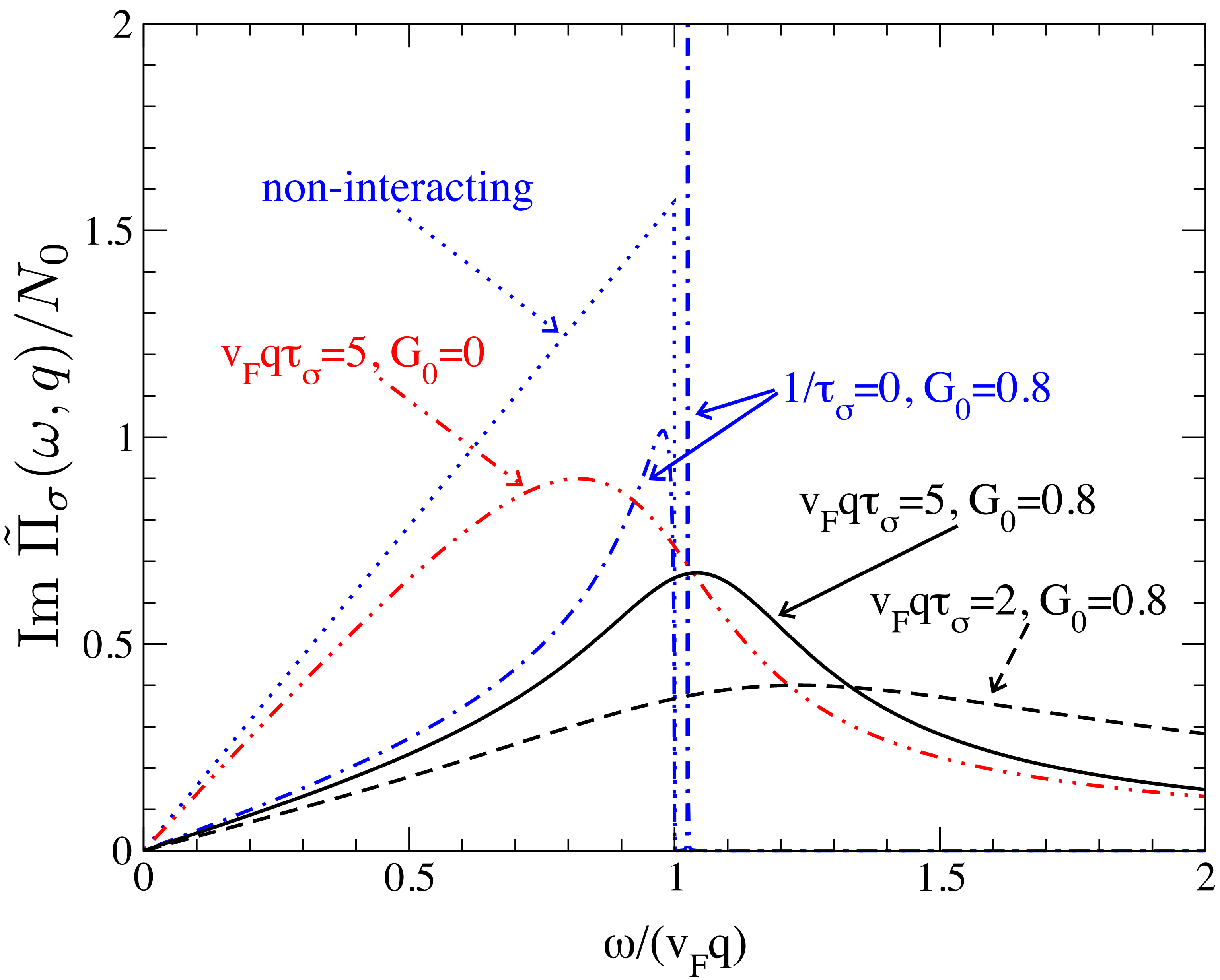
Spin is not conserved in nuclear interactions. Non-central interactions play an important role.

Hanhart, Phillips & Reddy (2001), Bacca, Hally, Pethick, Schwenk (2009), Shen, Gandolfi , Carlson, Reddy (2012)

Spin Response: Screening and Damping Effects

- Captures key aspects of the response (screening, damping and collectivity).
- Combines single-pair and multi-pair excitations and RPA correlations.
- Response is broadened and pushed to higher energy.

$$S_\sigma(\omega) = \frac{N(0)}{n\pi} \frac{\omega\tau_\sigma}{(1+G_0)^2 + (\omega\tau_\sigma)^2}$$



Lykasov, Olsson, Pethick (2005)
Lykasov, Pethick, Schwenk (2006)

Non-perturbative Effects in the Spin-Response of Neutron Matter

Going beyond Fermi-Liquid Theory or diagrammatic approaches with sum-rules calculated with QMC.

$$S_\sigma^n = \int_0^\infty S_\sigma(\omega, \boldsymbol{q}=0)\ \omega^n\ d\omega$$

$$S_\sigma^0 = 1 + \lim_{q\to 0} \frac{4}{3N} \sum_{i\neq j}^{N} \langle 0| e^{-i\mathbf{q}\cdot(\mathbf{r_i}-\mathbf{r_j})} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j |0\rangle$$

$$S_\sigma^{-1} = \frac{\chi_\sigma}{2n}$$

$$S_\sigma^{+1} = -\frac{4}{3N} \lim_{q\to 0} \langle 0| [H_N, \boldsymbol{s}(\mathbf{q})] \cdot \boldsymbol{s}(-\mathbf{q}) |0\rangle$$

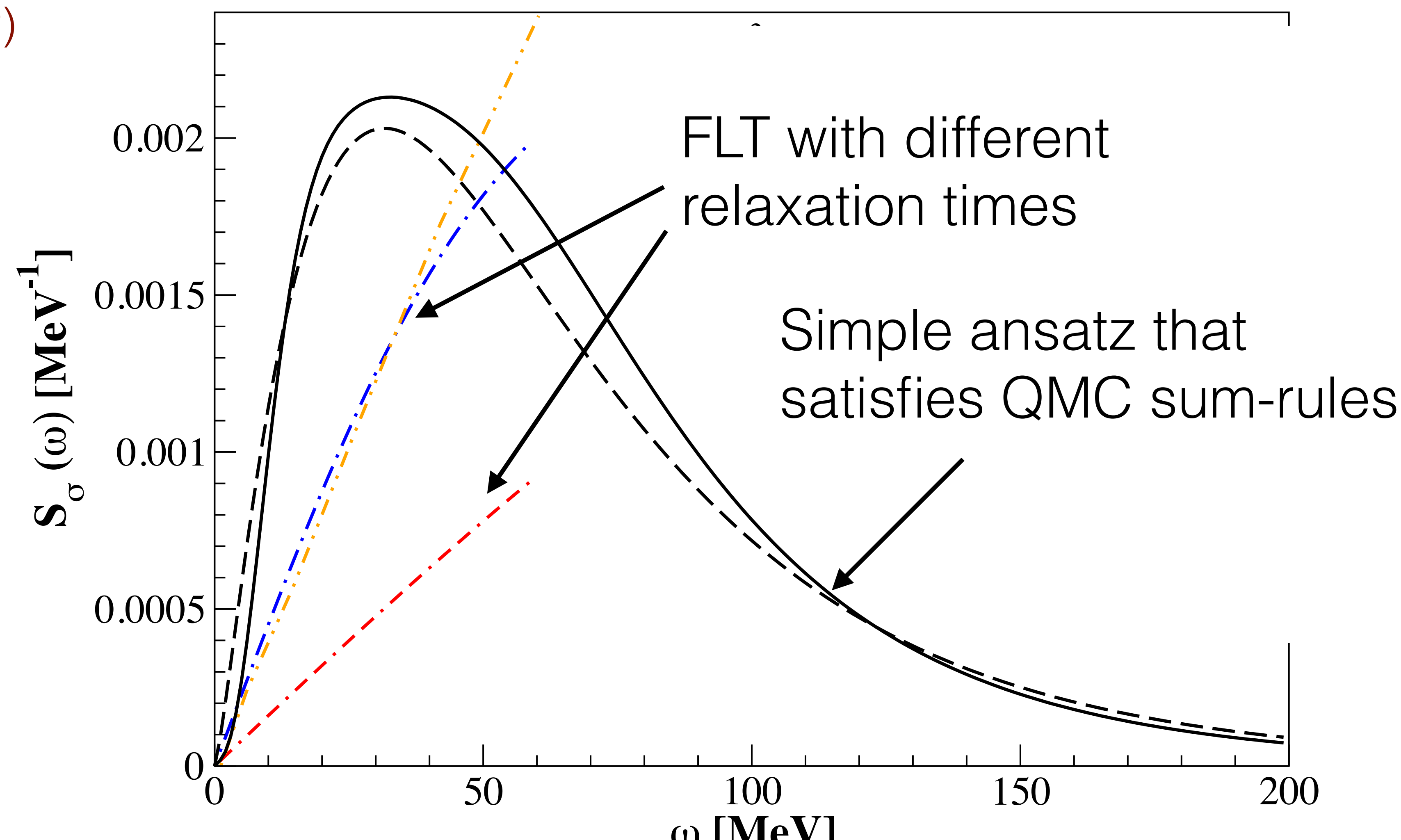
Density (fm^{-3})	S_σ^{-1} (MeV^{-1})	S_σ^0	S_σ^{+1} (MeV)	$\bar\omega_0$ (MeV)	$\bar\omega_1$ (MeV)
$n = 0.12$	0.0057(9)	0.20(1)	8(1)	35(9)	40(8)
$n = 0.16$	0.0044(7)	0.20(1)	11(1)	46(11)	55(8)
$n = 0.20$	0.0038(6)	0.18(1)	14(1)	47(12)	78(10)

Shen, Gandolfi , Carlson, Reddy (2012)

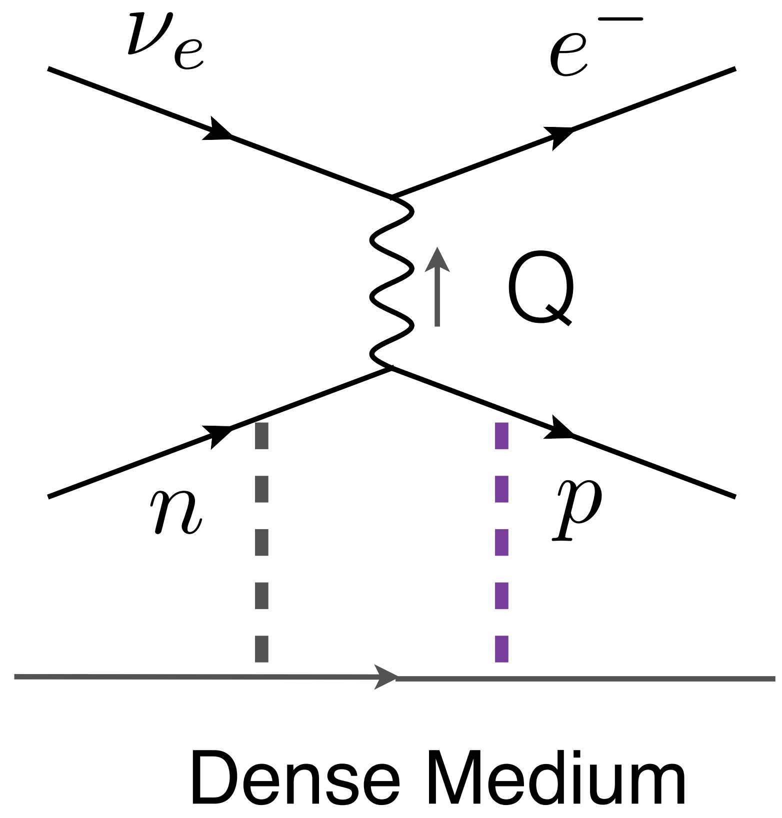
In the vicinity of nuclear density QMC sum-rules indicate significant strength at

$$\omega \simeq 30 - 50\ \text{MeV}$$

what is this energy scale ?

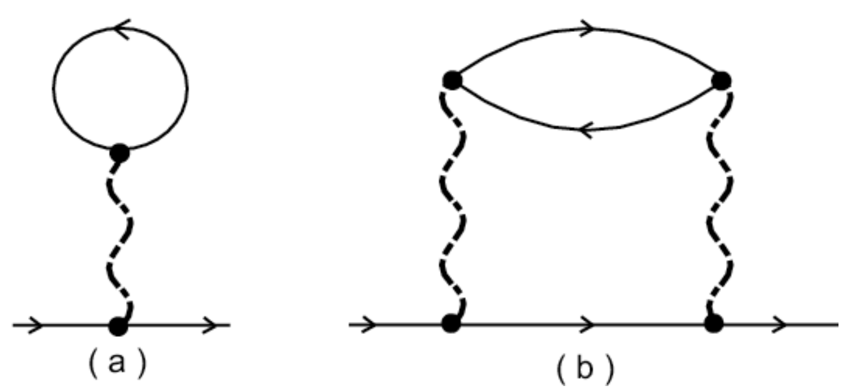


Charged Current Reactions in Neutron-Rich Matter

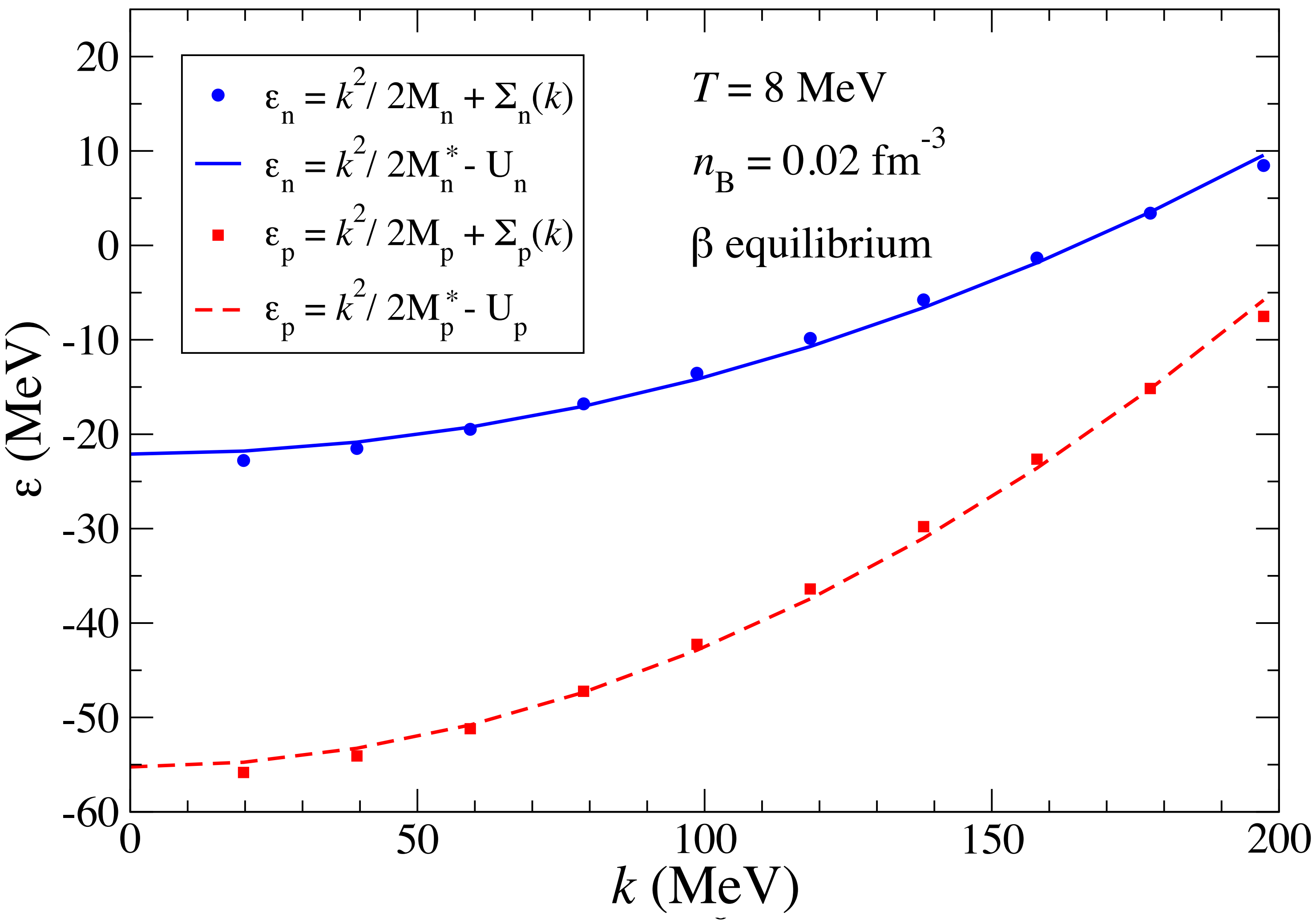


Potential energy difference between neutrons and protons is large - symmetry energy.

Large Q needed to overcome electron final state blocking



$$Q = E_n(p) - E_p(p+q)$$



Reddy, Prakash & Lattimer (1998), Martinez-Pinedo et al. (2012), Roberts & Reddy (2012), Rrapaj, Bartl, Holt, Reddy, Schwenk (2015)

Mean Field & Collisional Broadening

Ansatz for the spin-isospin charge-exchange response function in hot matter.

$$S_{\sigma\tau^-}(q_0,q) = \frac{1}{1-\exp\left(-\beta(q_0+\mu_n-\mu_p\right)}\mathrm{Im}\left[\frac{\tilde{\Pi}(q_0,q)}{1-V_{\sigma\tau}\tilde{\Pi}(q_0,q)}\right]$$

$V_{\sigma\tau} \simeq 200-220\ \mathrm{MeV/fm}$ G. Bertsch, D. Cha, and H. Toki (1984)

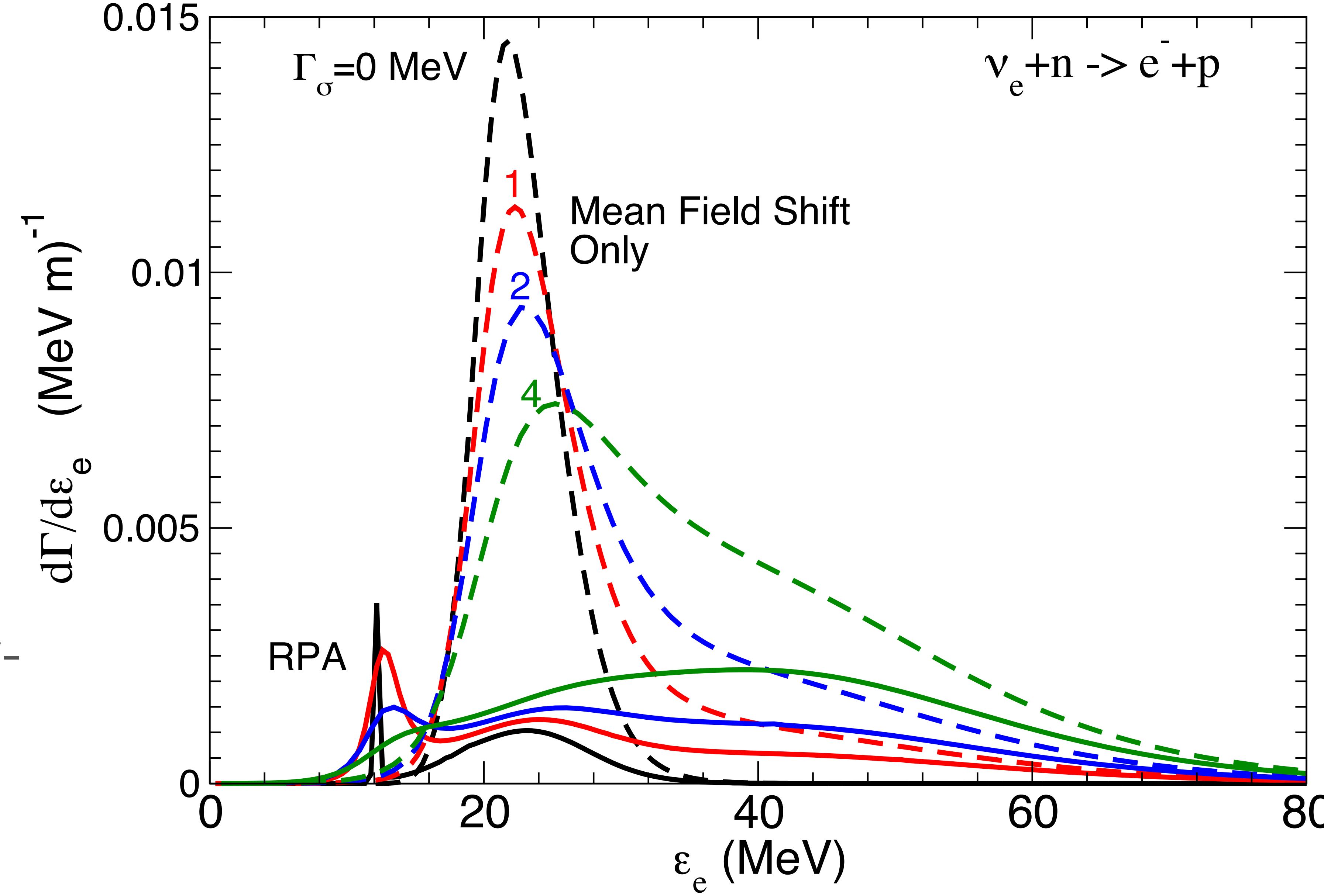
Collisional broadening introduced in the relaxation time approximation:

$$\mathrm{Im}\tilde{\Pi}(q_0,q) = \frac{1}{\pi}\int\frac{d^3p}{(2\pi)^3}\frac{f_p(\epsilon_{p+q})-f_n(\epsilon_p)}{\epsilon_{p+q}-\epsilon_p+\hat{\mu}}\,\mathcal{I}(\Gamma)$$

$$\mathcal{I}(\Gamma) = \frac{\Gamma}{(q_0+\Delta U-(\epsilon_{p+q}-\epsilon_p))^2+\Gamma^2}$$

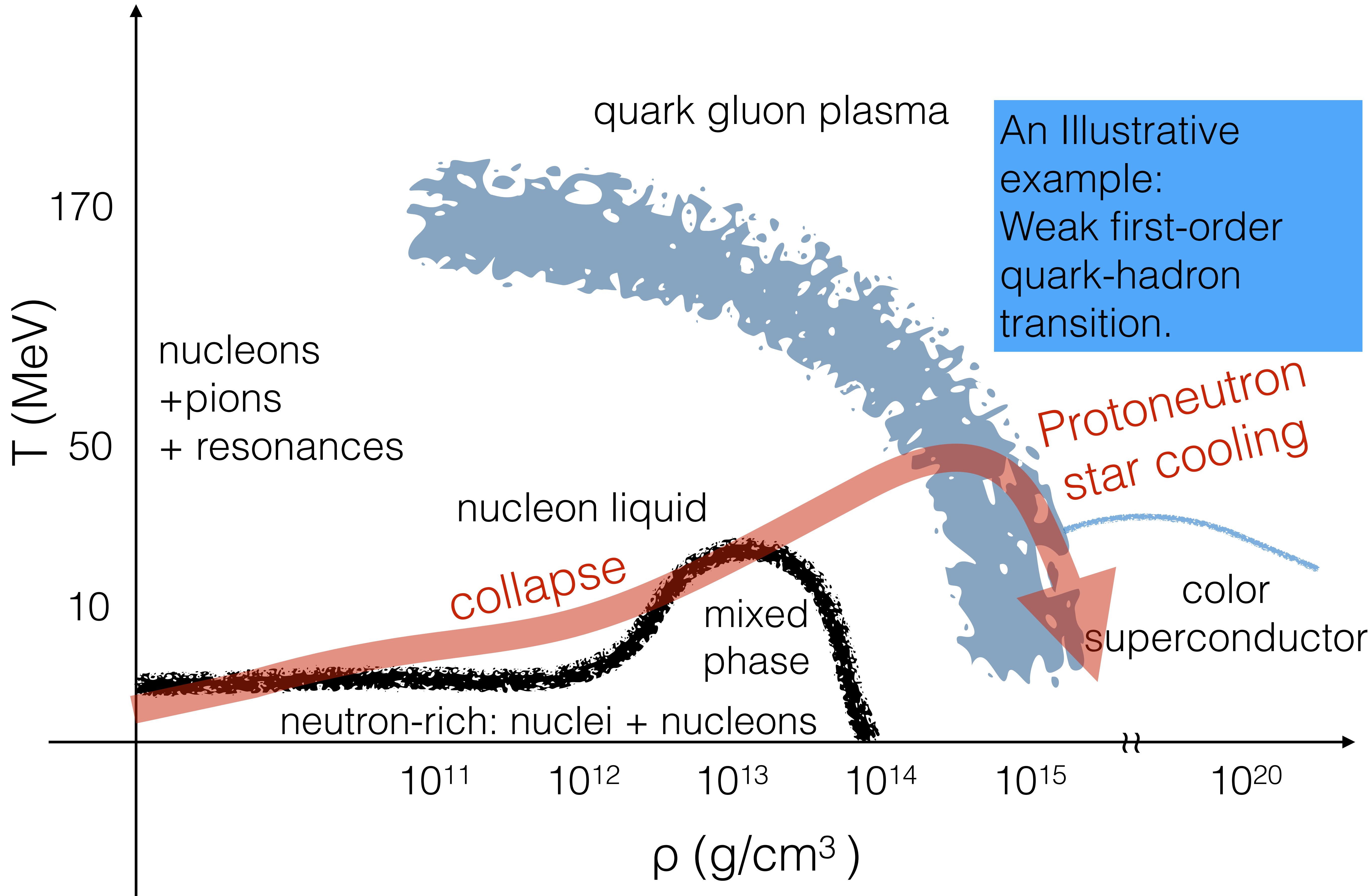
$$\Gamma = \tau_\sigma^{-1}$$

Energy shifts, the Gamow-Teller resonance, and collisional broadening all play a role.



Shen, Roberts, Reddy (2013)

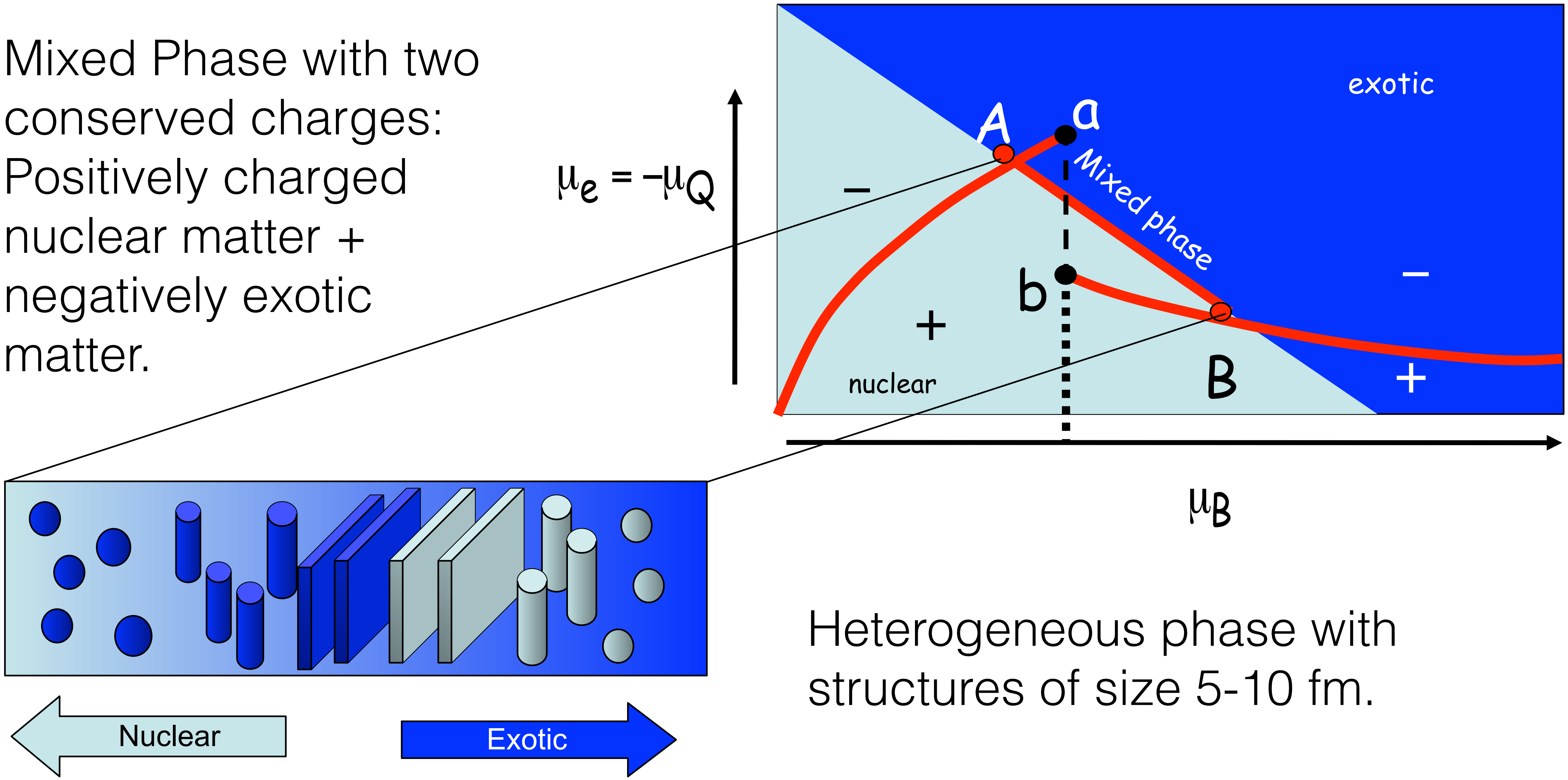
Phase transitons at supra-nuclear density



Weak First-Order Transitions

In dense matter first order transitions with low surface tension generically lead to phase co-existence. Glendenning (1996), Norsen & Reddy (2001)

Mixed Phase with two conserved charges: Positively charged nuclear matter + negatively exotic matter.

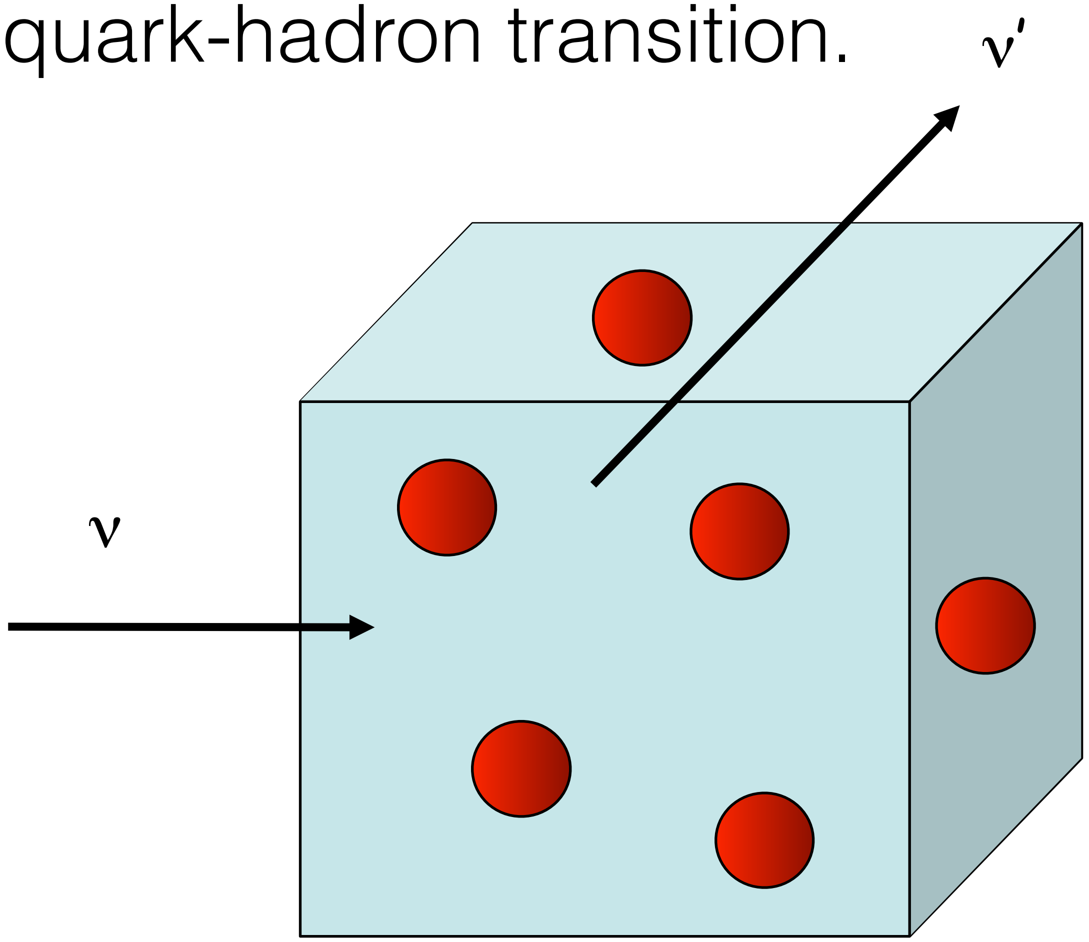


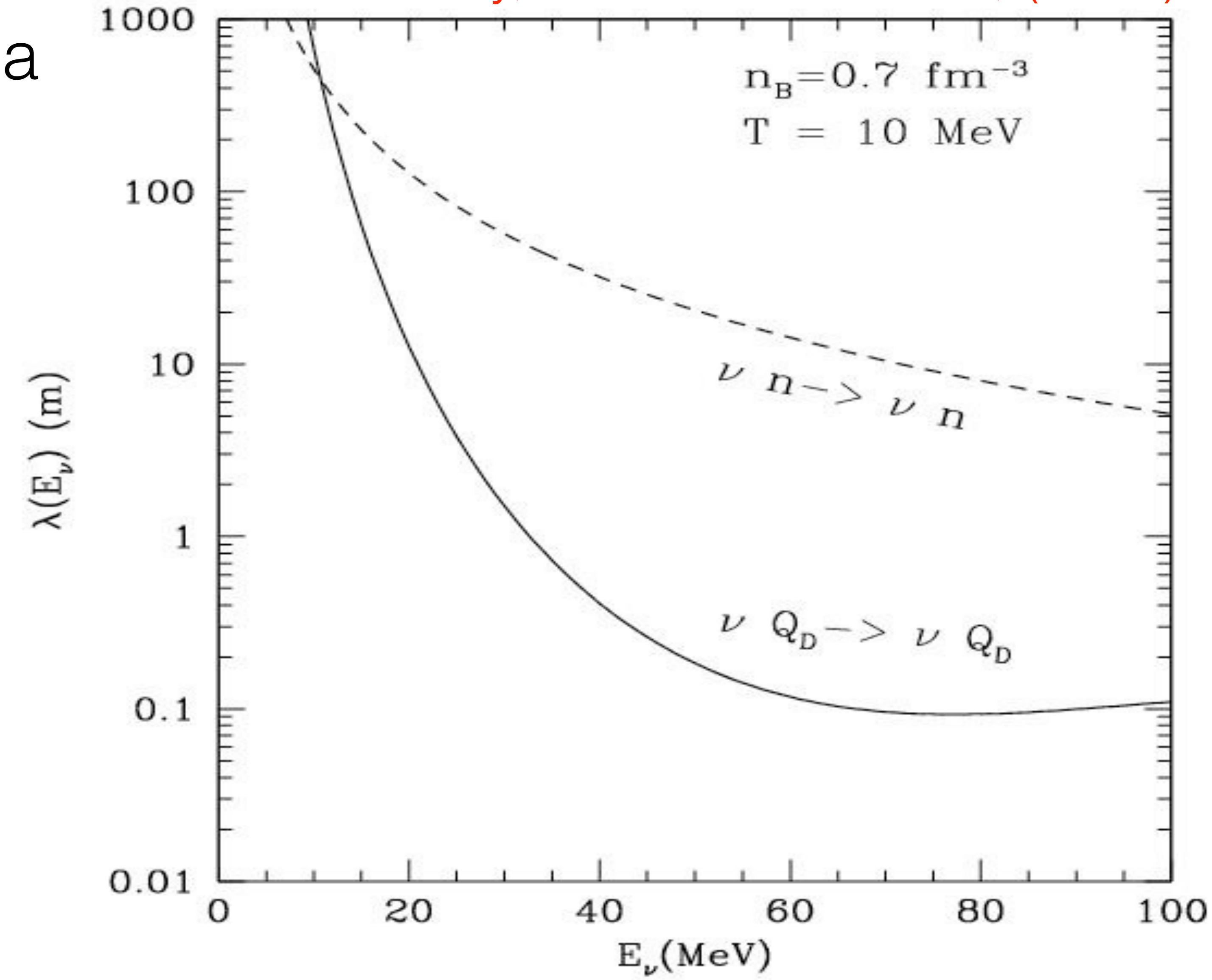
Heterogeneous phase with structures of size 5-10 fm.

Neutrino Mean Free Path in a Mixed Phase

Reddy, Bertsch & Prakash, (2000)

Scattering from quark droplets in a quark-hadron transition.





Coherent scattering: Neutrino wavelength comparable to size of droplets.

$$\frac{d\sigma}{d\cos(\theta)} = N_D \frac{G_F^2}{16\pi} S_q \, Q_W^2 E_\nu^2 (1+\cos(\theta))$$

number density of droplets — static structure factor — weak charge of the droplet

Summary and Outlook

- Expected changes to the neutrino opacities in the neutrino sphere and mantle are large. Likely to lead to observable effects.
- Effects due to screening, damping and energy shifts of nucleons are important.
- Spin response of nucleons is suppressed by nuclear interactions.
- Error estimates are needed. QMC and other ab-initio methods can provide sum-rules to helping this regard.
- Opacities at supra-nuclear density largely unknown. Phase transitions can lead to large modifications.
- Neutron star tomography may be possible with next galactic supernova.

Correlations in Neutrino Interactions in Nuclear Matter & Nuclei