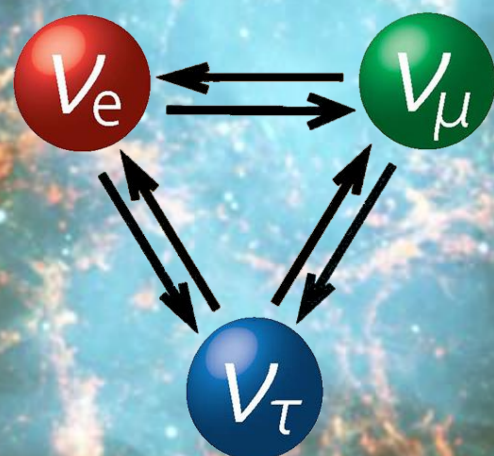
Self-Induced Flavor Conversion Without Neutrino Masses



Georg Raffelt, Max-Planck-Institut für Physik, München

PHYSICAL REVIEW D **72,** 045003 (2005)

Speed-up of neutrino transformations in a supernova environment

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(Received 8 April 2005; published 5 August 2005)

When the neutr ... current neutrino-neutrino interaction is treated completely, rather than as an
interaction among ... distributions, or as a set of flavor-diagonal effective potentials, the
result can be flavc ... nitude faster than that one would anticipate from the
measured neutrin ... ergy spectra of the three active species
of neutrinos eme

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week ending
26 FEBRUARY 2016

PRL **116,** 081101 (2016) PHYSICAL REVIEW LETTERS

Neutrino Cloud Instabilities Just above the Neutrino Sphere of a Supernova

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(Received 7 September 2015; revised manuscript received 2 January 2016; published 25 February 2016)

Most treatments of neutrino flavor evolution, above a surface of the last scattering, take identical angular
ributions on this surface for the different initial (unmixed) flavors, and for particles and antiparticles.
these distributions must be present, as a result of the species-dependent scattering cross
These lead to a new set of nonlinear equations, unstable even at the initial surface
reak all-over spherical symmetry. There could be important
for the neutrino pulse in the outer regions.

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

Self-induced neutrino flavor conversion without flavor mixing

S. Chakraborty,[a] R. S. Hansen,[b] I. Izaguirre[a] and G.G. Raffelt[a]

JCAP 03 (2016) 042

Self-Induced Flavor Conversion

Flavor conversion (vacuum or MSW) for a neutrino of given momentum p

- Requires lepton flavor violation by masses and mixing

$$\nu_e(p) \to \nu_\mu(p)$$

$$\boxed{\frac{\Delta m^2_{\rm atm}}{2E} = 10^{-10}\,\mathrm{eV} = 0.5\ \mathrm{km}^{-1}}$$

Pair-wise flavor exchange by ν–ν refraction (forward scattering)

- No net flavor change of pair
- Requires dense neutrino medium (collective effect of interacting neutrinos)
- Can occur without masses/mixing (and then does not depend on $\Delta m^2/2E$)
- Familiar as neutrino pair process $\mathcal{O}(G_{\rm F}^2)$ Here as coherent refractive effect $\mathcal{O}(G_{\rm F})$

$$\nu_e(p) + \overline{\nu}_e(k) \to \nu_\mu(p) + \overline{\nu}_\mu(k)$$

$$\nu_e(p) + \nu_\mu(k) \to \nu_\mu(p) + \nu_e(k)$$

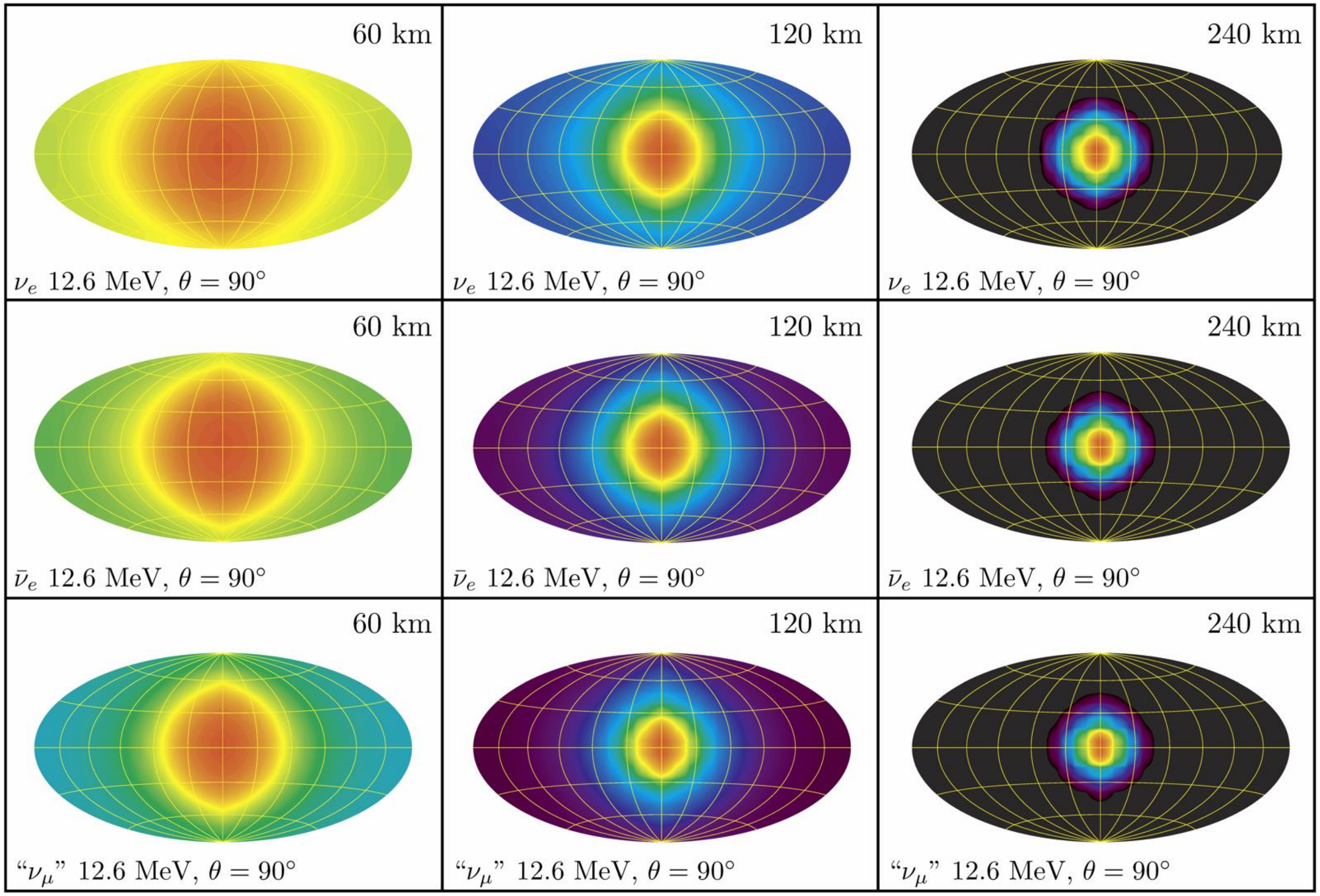
$$\boxed{\sqrt{2}G_{\rm F}n_\nu = 10^{-5}\,\mathrm{eV} = 0.5\ \mathrm{cm}^{-1}}$$

$$E = 12.5\ \mathrm{MeV}$$
$$R = 80\ \mathrm{km}$$
$$L_\nu = 40 \times 10^{51}\,\mathrm{erg/s}$$

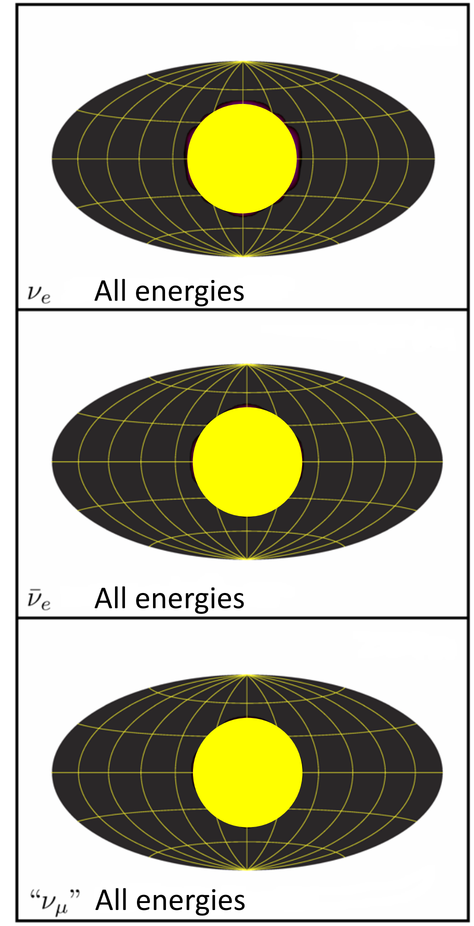
Neutrino Zenith-Angle Distribution

Examples for numerical angle distributions

(Ott, Burrows, Dessart & Livne, arXiv:0804.0239)



Bulb model
No limb darkening



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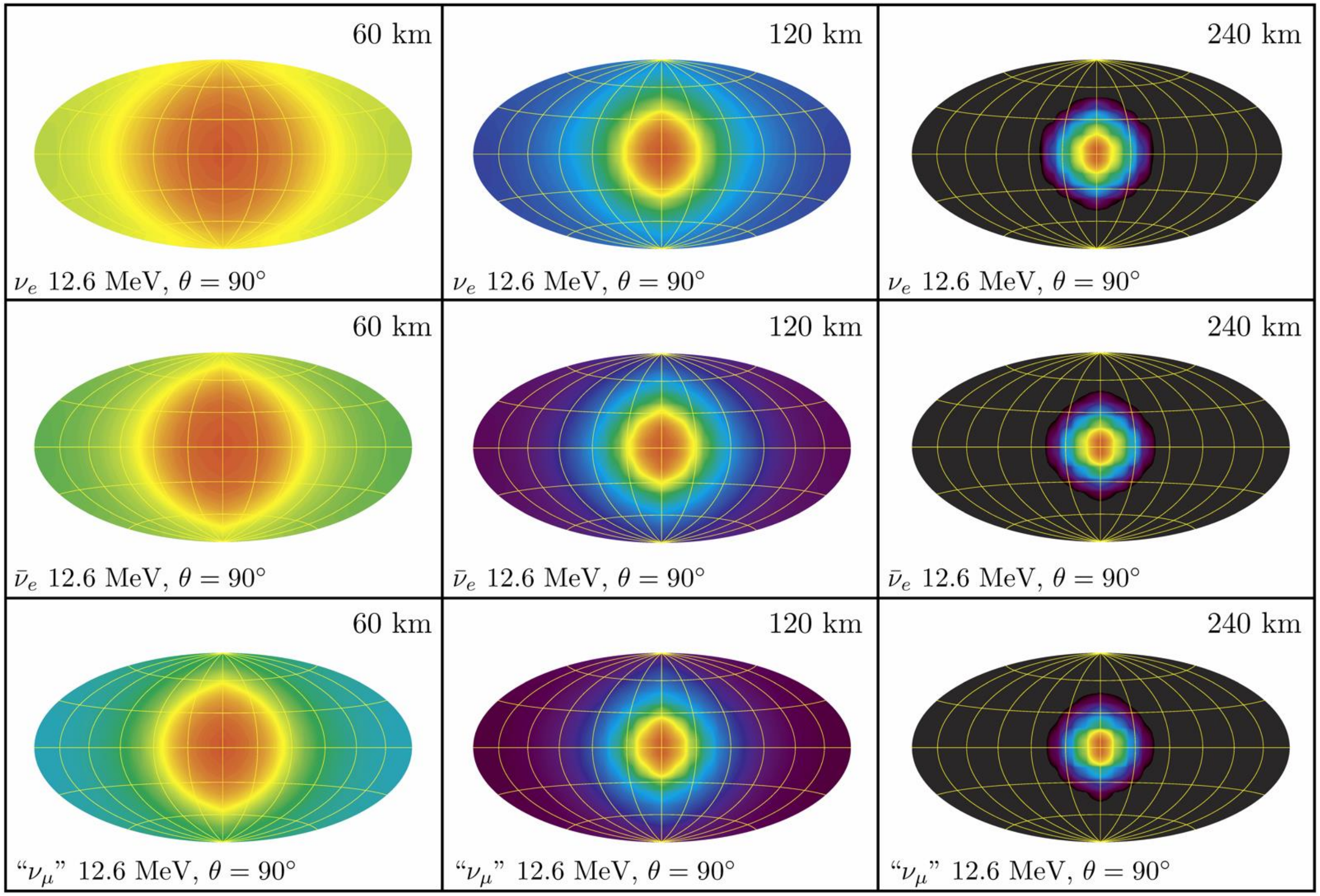

Bulb model
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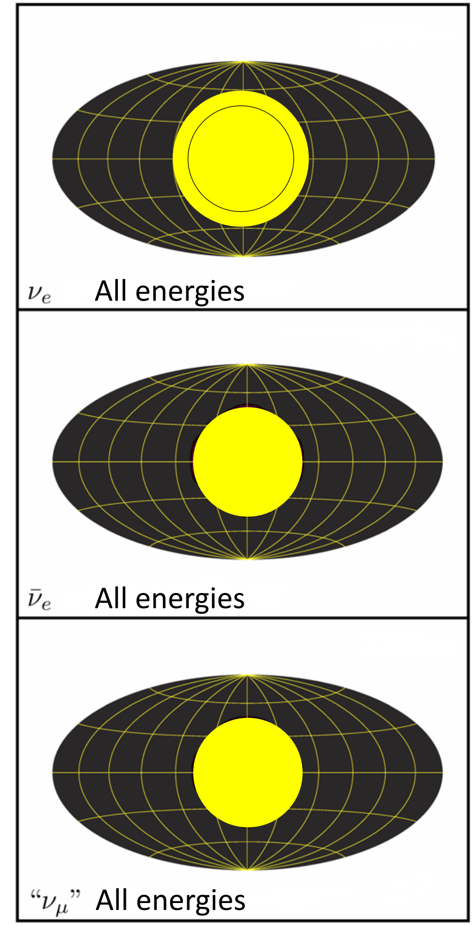
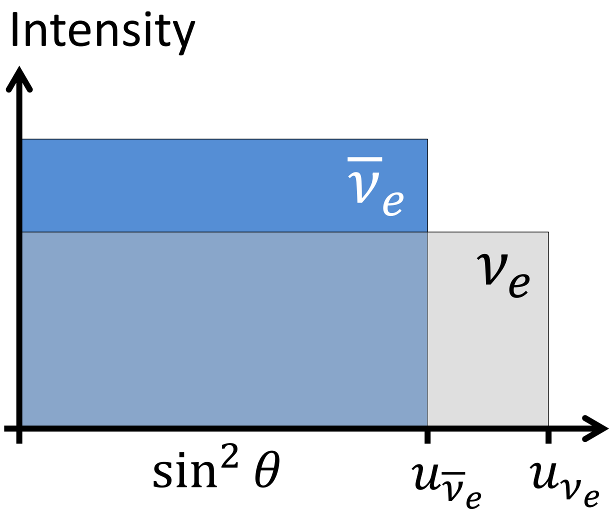


"Two-bulb" model:
Different size for ν_e and $\bar{\nu}_e$

Two-Bulb Supernova Model

Case proposed by R. Sawyer
arXiv:1509.03323

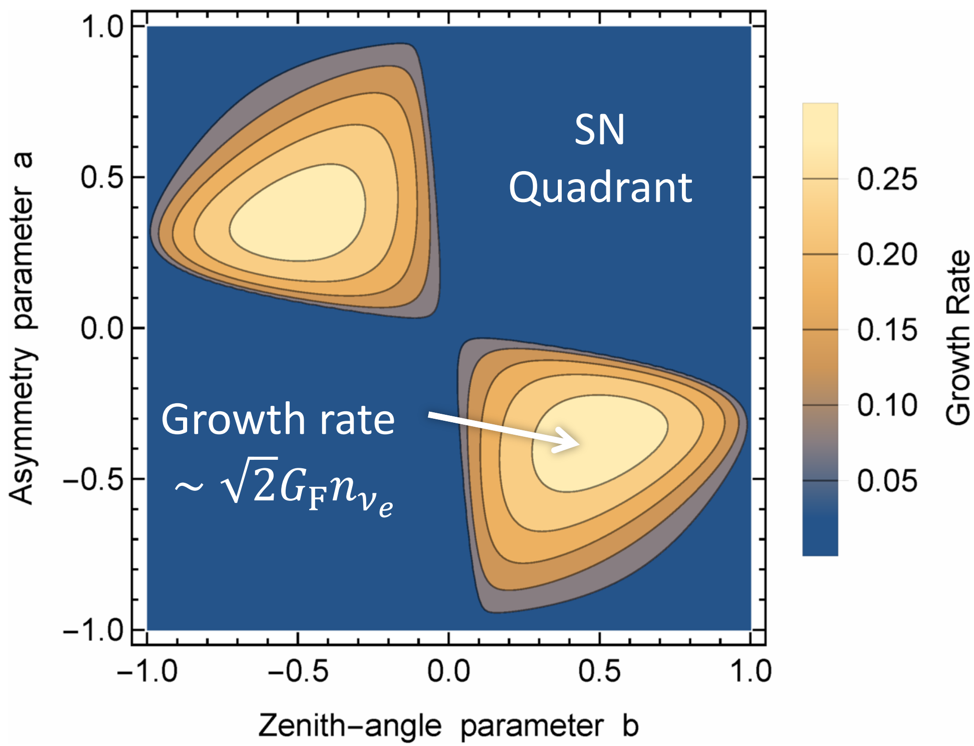


Lepton-number asymmetry $$a = \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_{\nu_e} + n_{\bar{\nu}_e}}$$

Bulb-size asymmetry $$b = \frac{u_{\nu_e} - u_{\bar{\nu}_e}}{u_{\nu_e} + u_{\bar{\nu}_e}}$$

Fast-instability growth rate

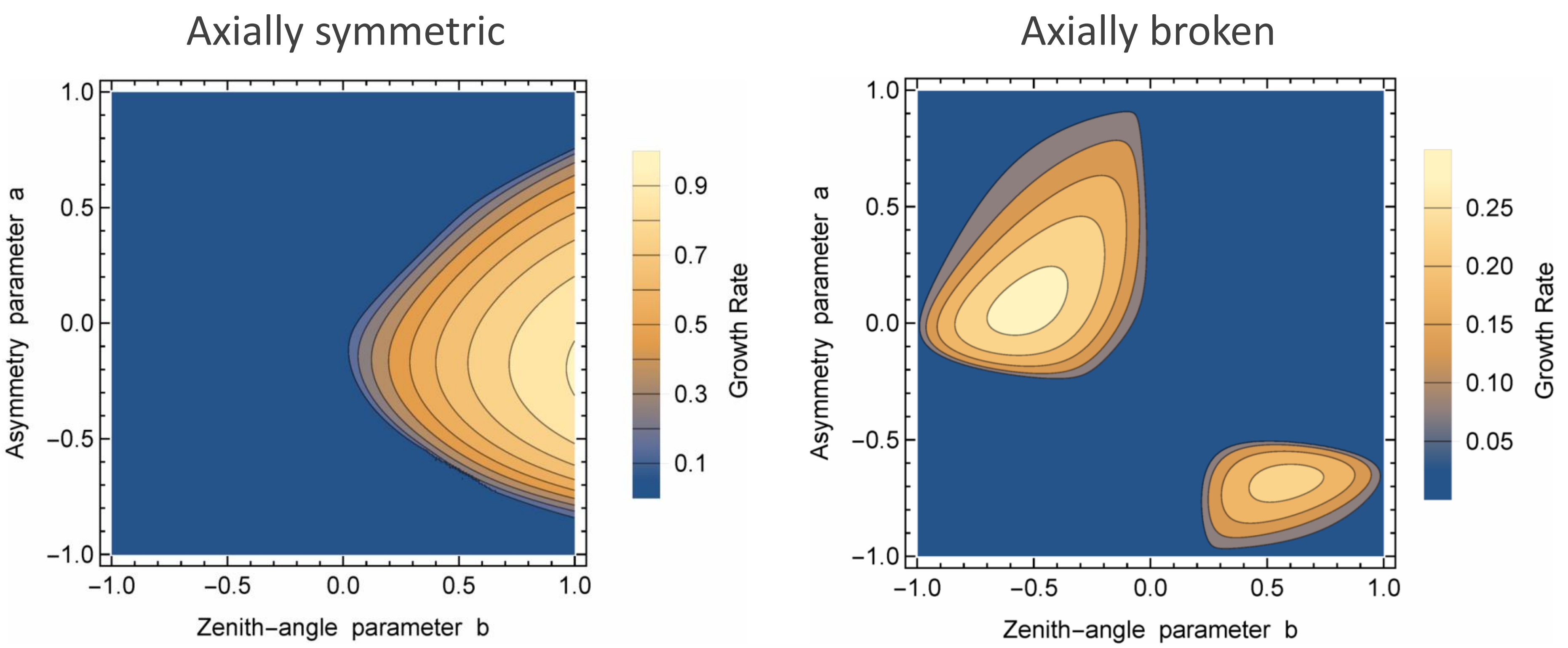
- Stationary solution
- No matter effect
- Breaks axial symmetry



Chakraborty, Hansen, Izaguirre, Raffelt, arXiv:1602.00698

Two-Bulb Model with Matter

- Stationary solutions (instability as a function of radius)
- Example with matter effect: $\sqrt{2}G_\mathrm{F}(n_e - n_{\bar{e}}) = \sqrt{2}G_\mathrm{F}(n_{\nu_e} - n_{\bar{\nu}_e})$



Chakraborty, Hansen, Izaguirre, Raffelt, arXiv:1602.00698

PHYSICAL REVIEW D **86**, 085010 (2012)

Flavor stability analysis of dense supernova neutrinos with flavor-dependent angular distributions

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(Received 1 August 2012; published 2 October 2012)

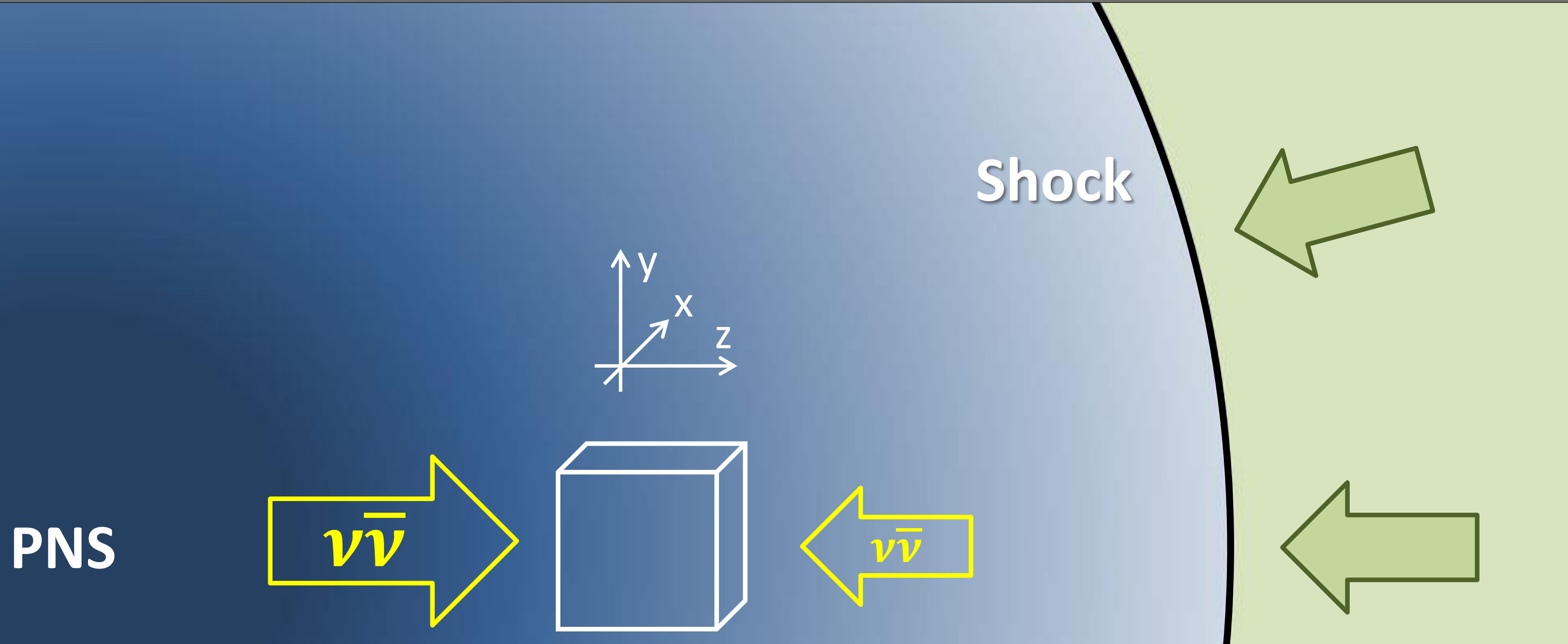
Numerical simulations of the supernova (SN) neutrino self-induced flavor conversions, associated with the neutrino-neutrino interactions in the deepest stellar regions, have been typically carried out assuming the "bulb model." In this approximation, neutrinos are taken to be emitted half-isotropically by a common neutrinosphere. In the recent [A. Mirizzi and P. D. Serpico, Phys. Rev. Lett. 108, 231102 (2012)] we have removed this assumption by introducing flavor-dependent angular distributions for SN neutrinos, as suggested by core-collapse simulations. We have found that in this case a novel multiangle instability in the self-induced flavor transitions can arise. In this work we perform an extensive study of this effect, carrying out a linearized flavor stability analysis for different SN neutrino energy fluxes and angular distributions, in both normal and inverted neutrino mass hierarchy. We confirm that spectra of different ν species which cross in angular space (where $F_{\nu_e} = F_{\nu_x}$ and $F_{\bar{\nu}_e} = F_{\bar{\nu}_x}$) present a significant enhancement of the flavor instability, and a shift of the onset of the flavor conversions at smaller radii with respect to the case of an isotropic neutrino emission. We also illustrate how a qualitative (and sometimes quantitative) understanding of the dynamics of these systems follows from a stability analysis.

DOI: 10.1103/PhysRevD.86.085010 PACS numbers: 14.60.Pq, 97.60.Bw

No fast flavor conversion observed

- Equal angle distributions for ν_e and $\bar{\nu}_e$, different for ν_x
- Different ν_e and $\bar{\nu}_e$ distributions required for fast flavor conversion if ν_x and $\bar{\nu}_x$ distributions are equal and thus cancel

Local Stability Analysis



Small test volume $\mathcal{O}(\mathrm{cm}^3)$

- Homogeneous conditions
- Need $f_\nu(E,\theta,\varphi)$ for all species
- Large mean free path
- What is flavor evolution (t,x,y,z) ?

Equation of Motion for Two-Flavor System

Liouville equation for 2×2 density matrices (phase-space densities)

$$\left(\partial_t + \vec{v}\cdot\vec{\nabla}\right)\varrho(t,\vec{r},\vec{p}) = -\mathrm{i}\left[\mathcal{H}(t,\vec{x},\vec{p}),\varrho(t,\vec{x},\vec{p})\right] + \mathcal{C}[\varrho(t,\vec{x},\vec{p})]$$

Free streaming:
collision term = 0

Flavor-dependent phase-space densities (occupation number matrices)

$$\varrho = \begin{pmatrix} f_{\nu_e} & f_{\langle\nu_e|\nu_x\rangle} \\ f_{\langle\nu_x|\nu_e\rangle} & f_{\nu_x} \end{pmatrix}$$

Antineutrinos: "Flavor isospin convention"
$E = -|\vec{p}|$ and $f_{\overline{\nu}}$ *negative* occupation number

Flavor evolution governed by "Hamiltonian matrix"

$$\mathcal{H} = \underbrace{\frac{\Delta m^2}{4E}\begin{pmatrix}\cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta\end{pmatrix}}_{\text{Vacuum oscillations}} + \underbrace{\sqrt{2}G_\mathrm{F}\begin{pmatrix} n_e & 0 \\ 0 & 0\end{pmatrix}}_{\text{MSW effect}} + \underbrace{\sqrt{2}G_\mathrm{F}\int\frac{d^3\vec{p}}{(2\pi)^3}(\varrho+\overline{\varrho})}_{\text{Nu-nu interaction term}}$$

Vacuum oscillations
$\Delta m^2/2E < 0$ for $\overline{\nu}$

MSW effect

Nu-nu interaction term
Nus feed back on each other

- **Flavor evolution is caused by off-diagonal $\mathcal{H}$ elements (vacuum or nu-nu term)**
- **For $\Delta m^2 = 0$, nu-nu term can still cause run-away modes!**

Linearized Stability Analysis

Flavor-dependent phase-space densities (occupation number matrices)

$$\varrho = \begin{pmatrix} f_{\nu_e} & f_{\langle\nu_e|\nu_x\rangle} \\ f_{\langle\nu_x|\nu_e\rangle} & f_{\nu_x} \end{pmatrix} = \frac{f_{\nu_e}+f_{\nu_x}}{2} + \frac{f_{\nu_e}-f_{\nu_x}}{2}\begin{pmatrix} s & S \\ S^* & -s \end{pmatrix} \qquad s^2+|S|^2=1$$

- Complex scalar function $S(t,\vec{x},\vec{p})$ contains all information if $|S| \ll 1$ and $s=1$
- $S(t,\vec{x},\vec{p})=0$ solves EOM identically if $\Delta m^2 = 0$
- Stable against small perturbations?

Linearized equations of motion for $\Delta m^2 = 0$

$$\mathrm{i}\big(\partial_t + \vec{v}\cdot\vec{\nabla}\big)S_{\vec{v}}(t,\vec{r}) = \big(\Lambda_0 + \Phi_0 - \vec{v}\cdot\vec{\Phi}\big)S_{\vec{v}}(t,\vec{r}) - \int\frac{d^3\vec{v}'}{4\pi}(1-\vec{v}\cdot\vec{v}')G_{\vec{v}'}S_{\vec{v}'}(t,\vec{r})$$

$\vec{v}$ is nu direction of motion (angle variable)

- Matter potential $\quad \Lambda_0 = \sqrt{2}G_F(n_{e^-}-n_{e^+})$
- Nu potential and current $\quad \Phi_0 = \sqrt{2}G_F\big(n_{\nu_e}-n_{\bar{\nu}_e}\big) \qquad \vec{\Phi} = \int\frac{d^3\vec{v}}{4\pi}\,\vec{v}\,G(\vec{v})$
- Lepton-number angle distribution for ν_e $\quad G_{\vec{v}} = \sqrt{2}G_F\int_0^\infty dE\,\frac{E^2}{2\pi^2}\left[f_{\nu_e}(E,\vec{v}) - f_{\bar{\nu}_e}(E,\vec{v})\right]$

Plane Wave Solutions (Fourier Space)

Look for plane-wave solutions of the form $S_{\vec{v}}(t,\vec{r}) \;= \tilde{S}_{\vec{v}}\big(\Omega,\vec{K}\big)\, e^{-\mathrm{i}\left(\Omega t-\vec{K}\cdot\vec{r}\right)}$

Eliminate differentials $\mathrm{i}\big(\partial_t+\vec{v}\cdot\vec{\nabla}\big)\, S_{\vec{v}} \to \big(-\Omega+\vec{v}\cdot\vec{K}\big)\,\tilde{S}_{\vec{v}}$

Eigenvalue equation for plane wave solutions:

$$\left[\Omega+\Lambda_0+\Phi_0-\vec{v}\cdot\big(\vec{\Phi}+\vec{K}\big)\right]\tilde{S}_{\vec{v}}=\sqrt{2}G_{\mathrm{F}}\int\frac{d^3\vec{v}'}{4\pi}(1-\vec{v}\cdot\vec{v}')\,g_{\vec{v}'}\,\tilde{S}_{\vec{v}'}$$

Potential and current from $(\nu_e - \overline{\nu}_e)$

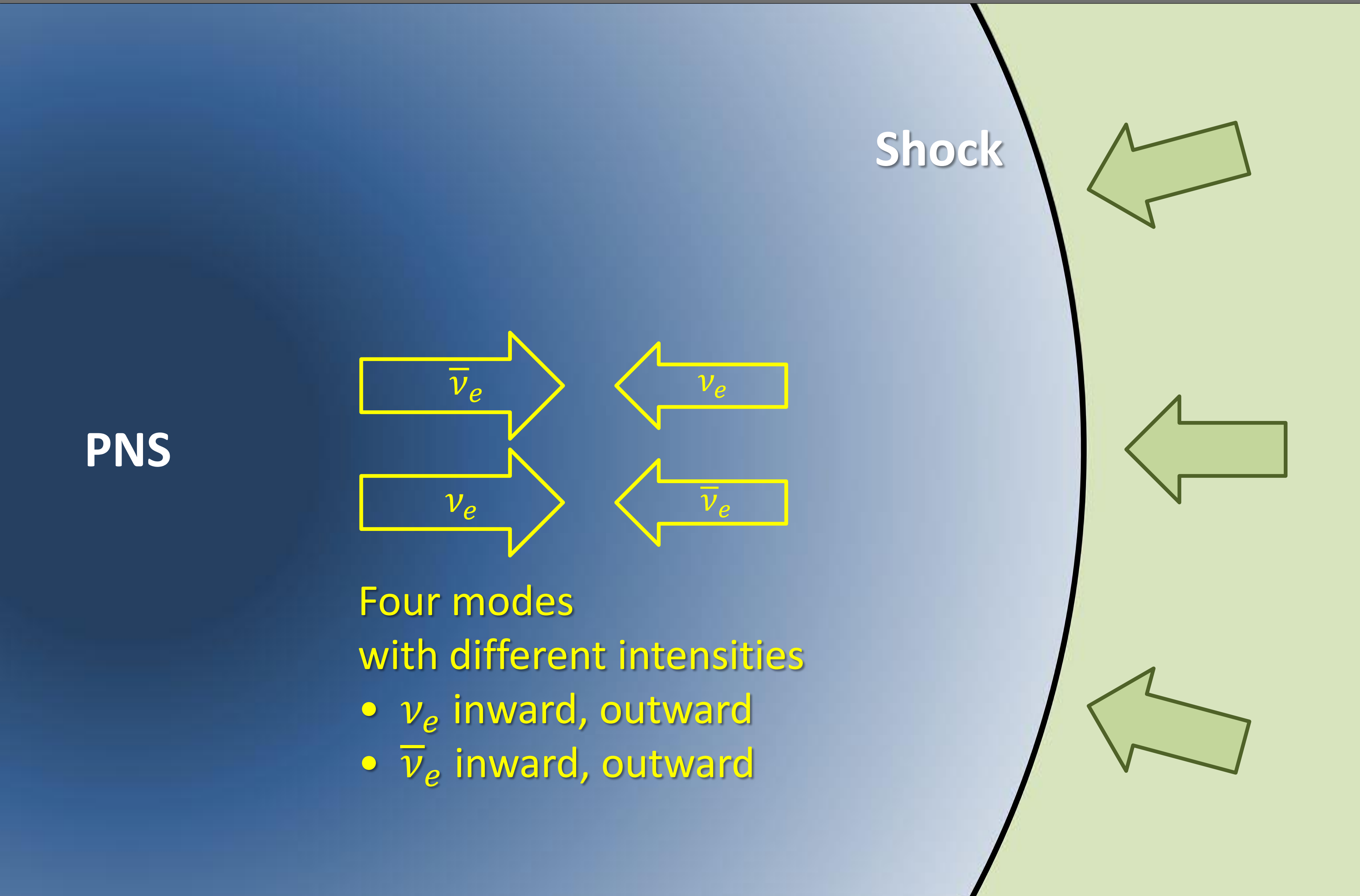
Angle distribution for $(\nu_e - \overline{\nu}_e)$
(“Intensities” of nu radiation field)

MSW potential from $(e^- - e^+)$

Task: Derive “dispersion relation” $\Omega=\Omega(\vec{K})$ or $\vec{K}=\vec{K}(\Omega)$

- Stationary solutions: $\Omega = 0$. Homogeneous solutions: $\vec{K}=0$.
- **In a SN, do not a priori assume $\boldsymbol{\Omega = 0}$** because $\Omega\neq 0$ can cancel $\Lambda_0+\Phi_0$
(Abbar & Duan, arXiv:1509.01538; Dasgupta & Mirizzi arXiv:1509.03171)

Colliding Beam Model

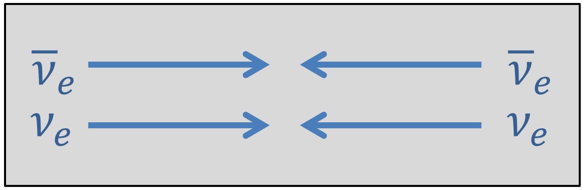


Four modes
with different intensities

- ν_e inward, outward
- $\bar{\nu}_e$ inward, outward

Colliding Beam Dispersion Relation

Parameters to describe the system:



- Matter potential: $\Lambda_e = \sqrt{2}G_\mathrm{F}(n_e - n_{\bar{e}})$
- Nu potential: $\Lambda_\nu = \sqrt{2}G_\mathrm{F}\left(n_{\nu_e} - n_{\bar{\nu}_e}\right)$
- Nu potential current: $\Phi_\nu = \sqrt{2}G_\mathrm{F}\left(n_{\nu_e} - n_{\bar{\nu}_e}\right)\langle\cos\theta\rangle$
- "Lepton velocity": $v_\nu = \frac{\phi_\nu}{\Lambda_\nu} = \frac{\left(n_{\nu_e} - n_{\bar{\nu}_e}\right)-\mathrm{Flux}}{\left(n_{\nu_e} - n_{\bar{\nu}_e}\right)-\mathrm{Density}}$
- No lateral variation: Spatial variation only along the beam

Dispersion relation for off-diagonal density matrix element

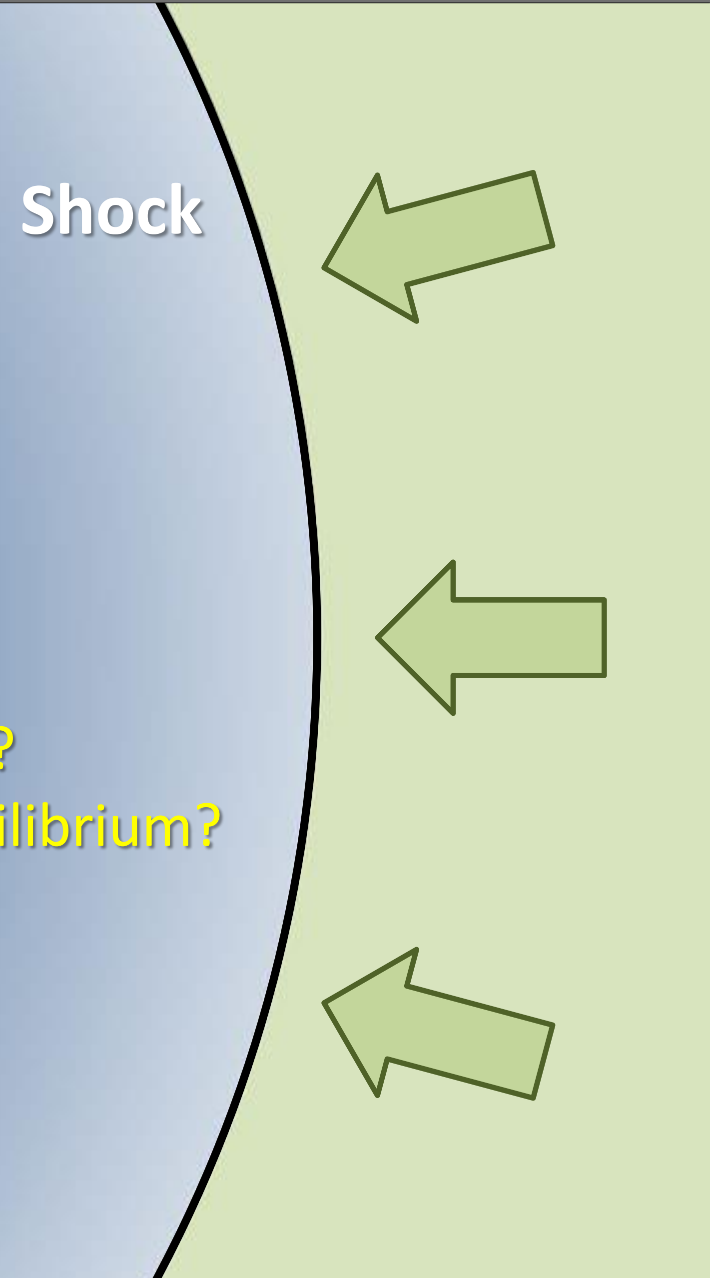
$$(\Omega - \Lambda_e - \Lambda_\nu)^2 - (K - \Lambda_\nu v_\nu)^2 = \Lambda_\nu^2(1 - v_\nu^2)$$

Assume real Ω, solve for wave number

$$K = \Lambda_\nu v_\nu \pm \sqrt{(\Omega - \Lambda_e - \Lambda_\nu)^2 - \Lambda_\nu^2(1 - v_\nu^2)}$$

- For $\Omega \sim \Lambda_e + \Lambda_\nu$ always exponentially growing solutions (spatially growing)
- For stationary assumption ($\Omega = 0$), net e and ν_e densities should essentially cancel.

Fast Flavor Conversion Generic ?



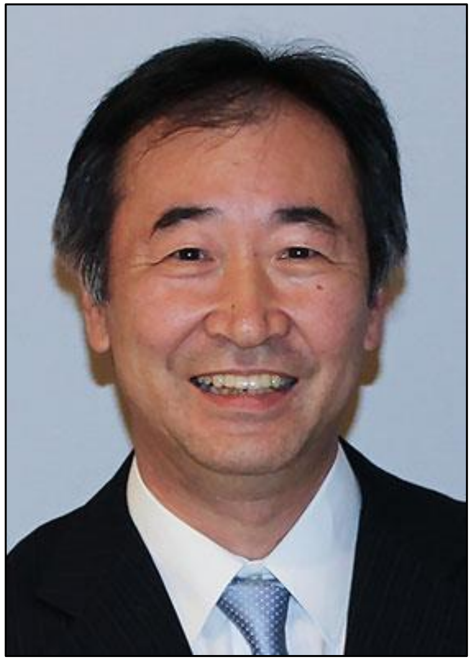
PNS

For any conditions
linearized stability analysis
finds unstable modes with

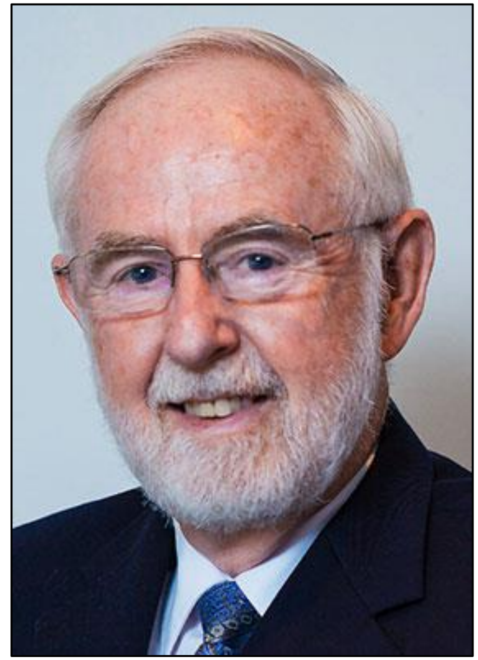
$$\Omega \sim \Lambda_e + \Lambda_\nu$$
$$\mathrm{Im}(K) \sim \Lambda_\nu$$

- Fast flavor conversion generic?
- Leads effectively to flavor equilibrium?
- Interpretation faulty?

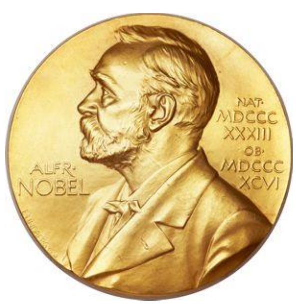
Flavor Conversion and Neutrino Masses



T. Kajita



A. McDonald



2015

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

More theory progress is needed to understand flavor conversion of supernova neutrinos!