Relativistic Hartree Response for nu-N Interactions in Supernovae

INT Workshop INT-16-61W

“Flavor Observations with Supernova Neutrinos”

INT – August 17th 2016

Andreas Lohs (Univ. Basel)

Universität Basel

TECHNISCHE UNIVERSITÄT DARMSTADT

Neutrino-Nucleon-Interactions: Two Regimes

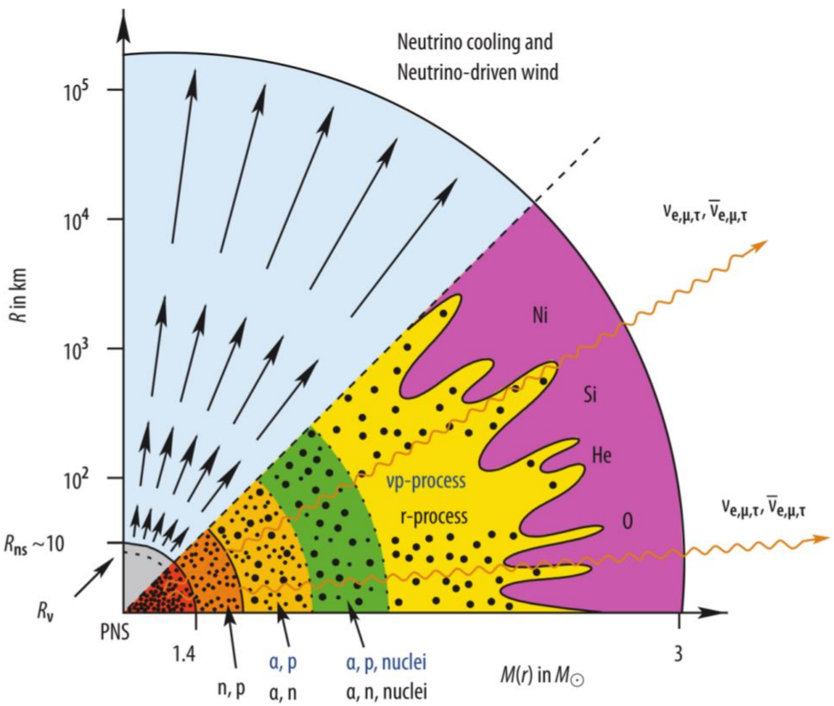
Interior of neutron star:

Neutrino spectra formation

$$p + e^- \leftrightarrow \nu_e + n$$

$$n + e^+ \leftrightarrow \bar{\nu}_e + p$$

$$\nu/\bar{\nu} + N \rightarrow \nu/\bar{\nu} + N$$



Surface of neutron star:

Neutrino absorption heats matter

Ejection of Neutrino Driven Wind

$$\nu_e + n \rightarrow p + e^-$$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

Spectrum determines composition

Overview

Motivation

- Why should we use precise neutrino nucleon interactions?

Relativistic Hartree Response

- Definition
- Alternative ansatz for computation

Comparison to nonrelativistic expressions

- Elastic approximation
- Recoil and weak magnetism corrections
- Inelastic opacities in nonrelativistic limit

Neutrino-Nucleon Microphysics: Explosions in 3D

NEUTRINO-DRIVEN EXPLOSION OF A 20 SOLAR-MASS STAR IN THREE DIMENSIONS ENABLED BY STRANGE-QUARK CONTRIBUTIONS TO NEUTRINO-NUCLEON SCATTERING

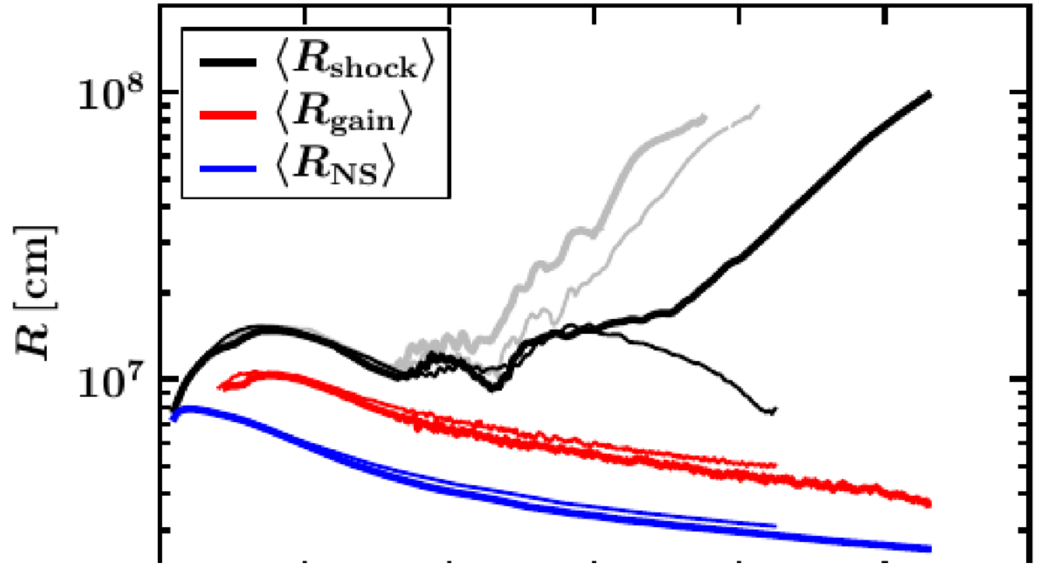
TOBIAS MELSON[1,2], HANS-THOMAS JANKA[1], ROBERT BOLLIG[1,2], FLORIAN HANKE[1,2], ANDREAS MAREK[3], AND BERNHARD MÜLLER[4]

[ApJ 808 (2015) no.2]

Strange contribution to neutral weak axial coupling to parametrize general uncertainties

$$c_A = \frac{1}{2}\left(\pm g_a - g_A^s\right)$$

$$\rightarrow g_A^S = -0.2$$

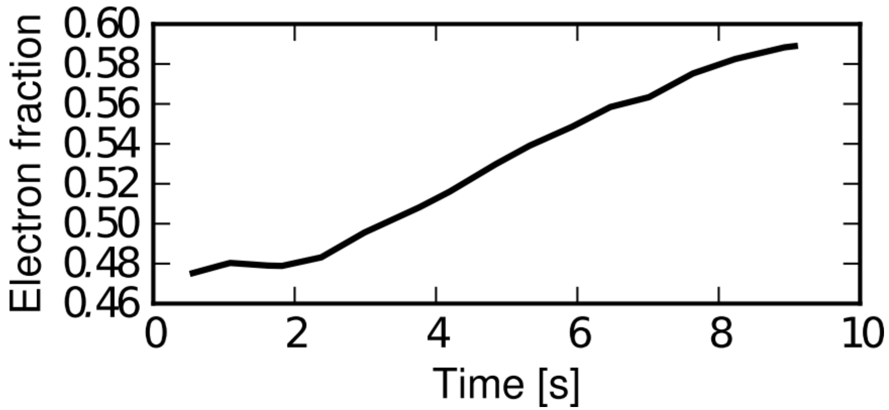


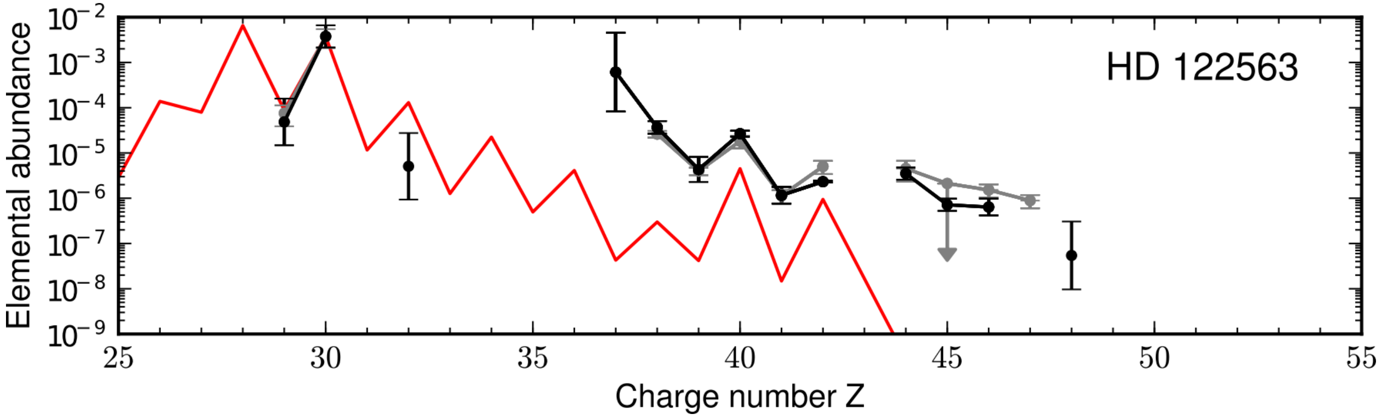
„…the outcome of multi-dimensional core-collapse simulations that marginally explode or fail can sensitively depend on effects on the 10% level in the neutral-current neutrino-nucleon interactions.“

Neutrino-Nucleon Microphysics: Weak r-Process

Nucleon potential differences at high densities strongly affect charged-current rates

- NDW initially neutron rich?
- Sensitive to changes in rates





[Martinez-Pinedo,Fischer,AL,Huther, PRL 109 (2012) 251104]

[Martinez-Pinedo,Fischer,Huther, JPhG 41 (2014) 044088]

[Roberts,Reddy,Shen, PRC 86 (´12) 065083]

[Horowitz,Shen,O'Connor,Ott, PRC 86 (´12) 065806]

Mean Free Path for Neutrino Absorption (CC)

Structure function from RPA / Linear response theory

- Fully consistent with RMF-EOS, correlations (can be) included
- Requires 3-D numerical integrals to obtain $\lambda(E_\nu)$
- „Gold standard" for nu-N interactions in CCSNe

Opacities with correct kinematics/degeneracy

- **Relativistic (Hartree) response with full matrix element**
- **2-D/3-D numerical integrals to obtain** $\lambda(E_\nu)$

Elastic Approximation

- Lowest order expression for nonrelativistic nucleons
- Simplified degeneracy to obtain analytic formula for $\lambda(E_\nu)$

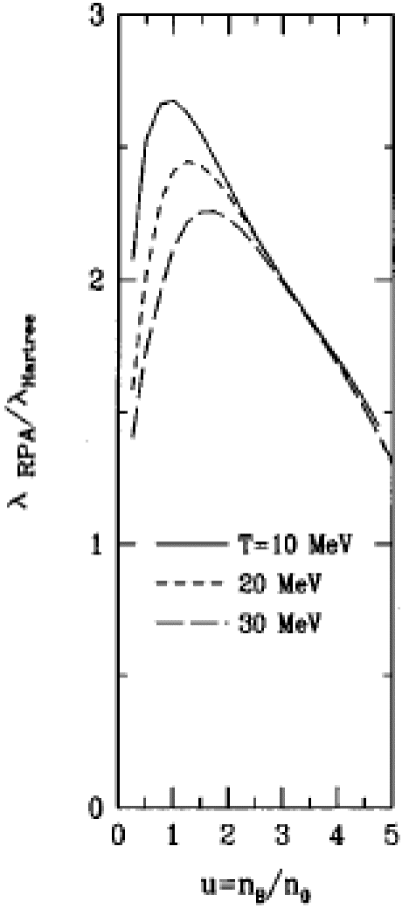
(Relativistic) RPA and correlations

Formalism for relativistic RPA has already been developed, but not yet widespread in CCSNe-simulation

[Horowitz,Wehrberger, PhysLettB266 (´91) 236]

[Reddy,Prakash,Lattimer,Pons, PRC59 (´99) 2888]

[Horowitz,Pérez-Garcia, PRC68 (´03) 025803]



Effect below $n_0/4$ rarely published

Suppression up to several 10% at 10^{13} g/ccm

ρ (g cm^{-3})	Y_ν	T (MeV)	Y_e	S_A	S_V
1.40×10^{13}	0.067	15	0.258	0.790	0.840

[Burrows,Sawyer, PRC59 (´99) 510]

Beyond p-h-correlations: multi-particle scattering

[Roberts,Reddy,Shen, PRC86 (´12) 065083]

Relativistic Hartree response in supernova matter

Hartree approximation for nucleon response:

- nucleon-nucleon interaction described by RMF-potentials and effective masses
- nucleons are quasi-free particles with modified energy

$$E_{n,p} = \sqrt{\mathbf{p}^2 + m^{*2}_{n,p}} + U_{n,p}$$

- relativistic kinematics, „full“ matrix element, weak magnetism
- „Exact“ for densities below 10^{13} g/ccm
- Usually computed from medium polarization

$$\frac{1}{\lambda(E_l)} \sim \int \frac{d^3p'_l}{E'_l} (1 - f_{l'}) L_{\mu\nu} \Im(\Pi^{\mu\nu}_R) \left[1 - \exp\left(-\frac{q_0 + \mu_N - \mu'_N}{T}\right)\right]$$

Alternative ansatz for relativistic Hartree response

Describe nucleons by effective quasi-particle spinors

$$\psi_i^* = \frac{1}{\sqrt{(E_i - U_i)^{*2} + m_i^{*2}}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_i^* + m_i^*}\chi_s \end{pmatrix}$$

Compute interaction rate from Fermi's Golden Rule

$$\lambda(E_\nu)^{-1} \sim \int d^3p_e \left[1 - f_e(E_e)\right] \int d^3p_n \int d^3p_p \frac{\left\langle |M|^2 \right\rangle}{16 E_\nu E_n E_e E_p} f_n(E_n) \left[1 - f_p(E_p)\right] \delta^4$$

- Equivalent to medium polarization [Leinson,Perez,PLB 518 (´01)]

Analytically integrate all angles to obtain 2D-numerical rate

$$\frac{1}{\lambda} = \frac{G^2}{4\pi^3} \frac{1}{E_1^2} \int_{E_{2-}}^{\infty} dE_2 \int_{m_3}^{E_{3+}} dE_3 f_2 \left[1 - f_3\right] \left[1 - f_4\right] I_{tot}.$$

[Steiner,Prakash, Lattimer, PLB 509 (´01)]

- Finite lepton mass considered (absorption of muon neutrinos)
- Cannot be extended to RPA formalism
- Not suited for angular dependent neutrino-nucleon scattering

Corrections to Opacities in Elastic Approximation

Elastic approximation

- yields analytic opacities in nonrelativistic limit
- highly simplified matrix element and kinematics

$$E_n - E_p \simeq m_n - m_p + U_n - U_p$$

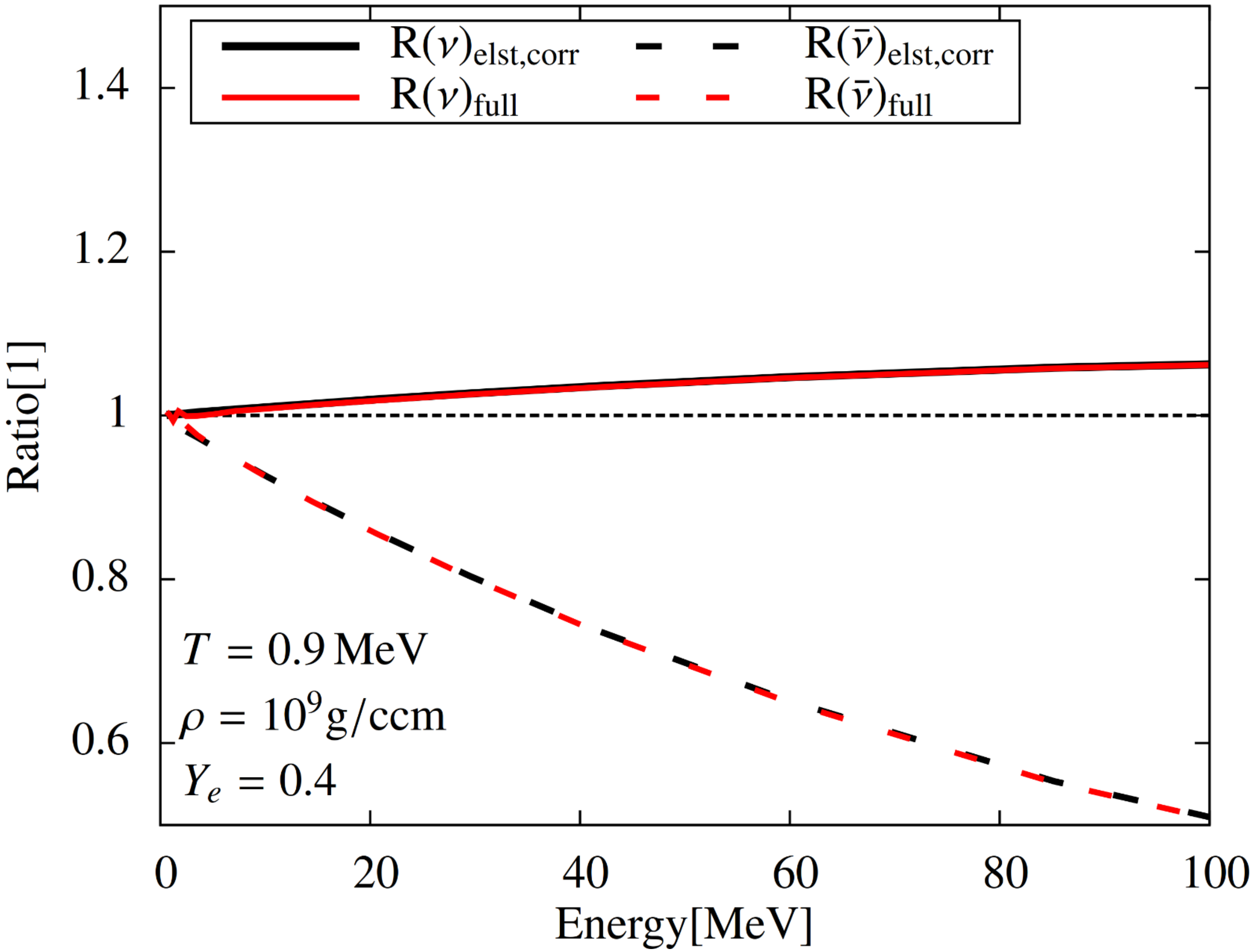
[Horowitz, PRD 65 (2002) 043001] pointed out:

- Kinematics/Recoil can be treated relativistically

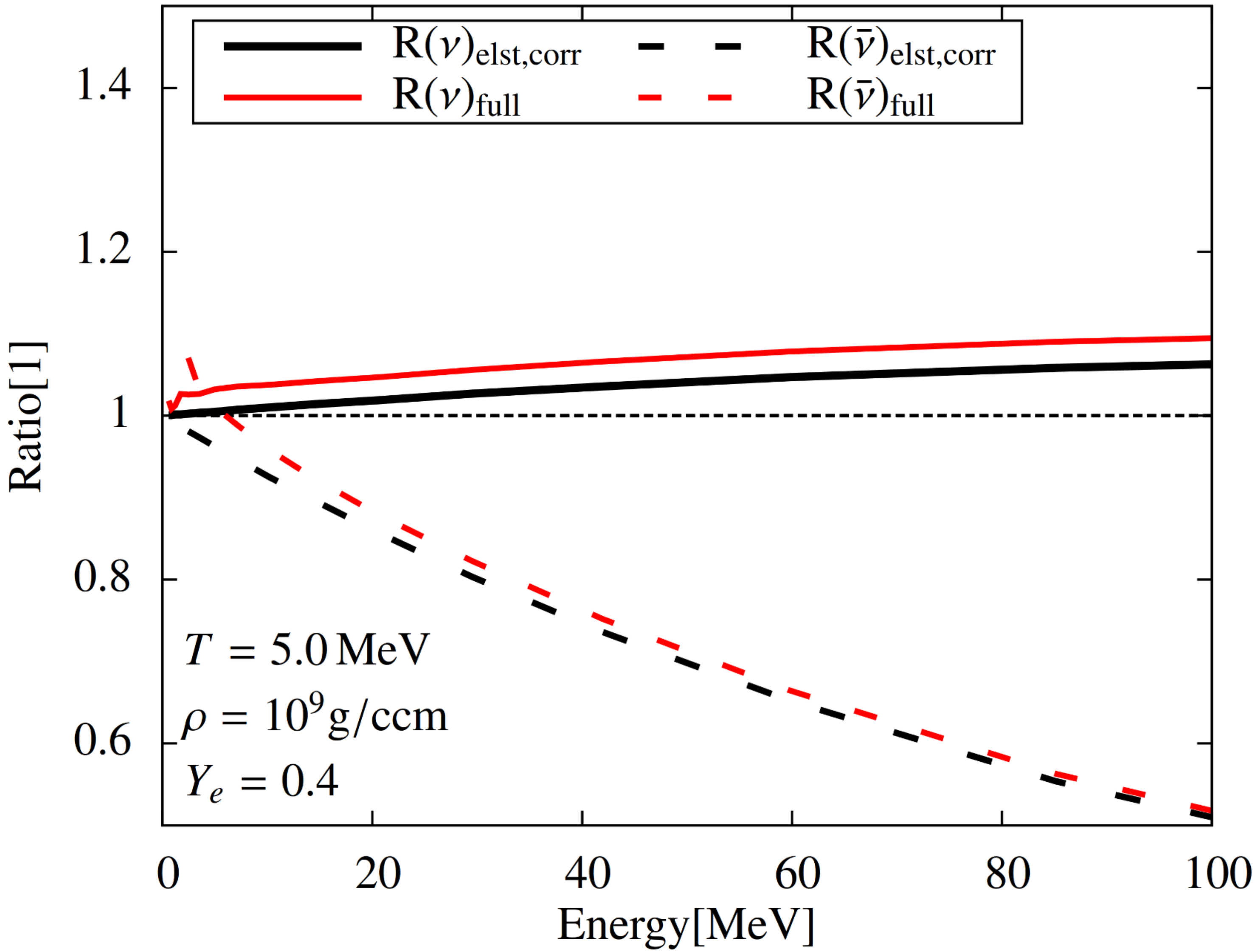
$$E_n = m_n \Rightarrow E_e = \frac{E_\nu}{1 + \frac{E_\nu}{m_n}(1-x)}$$

- Gives rise to analytic correction factor for cross-section

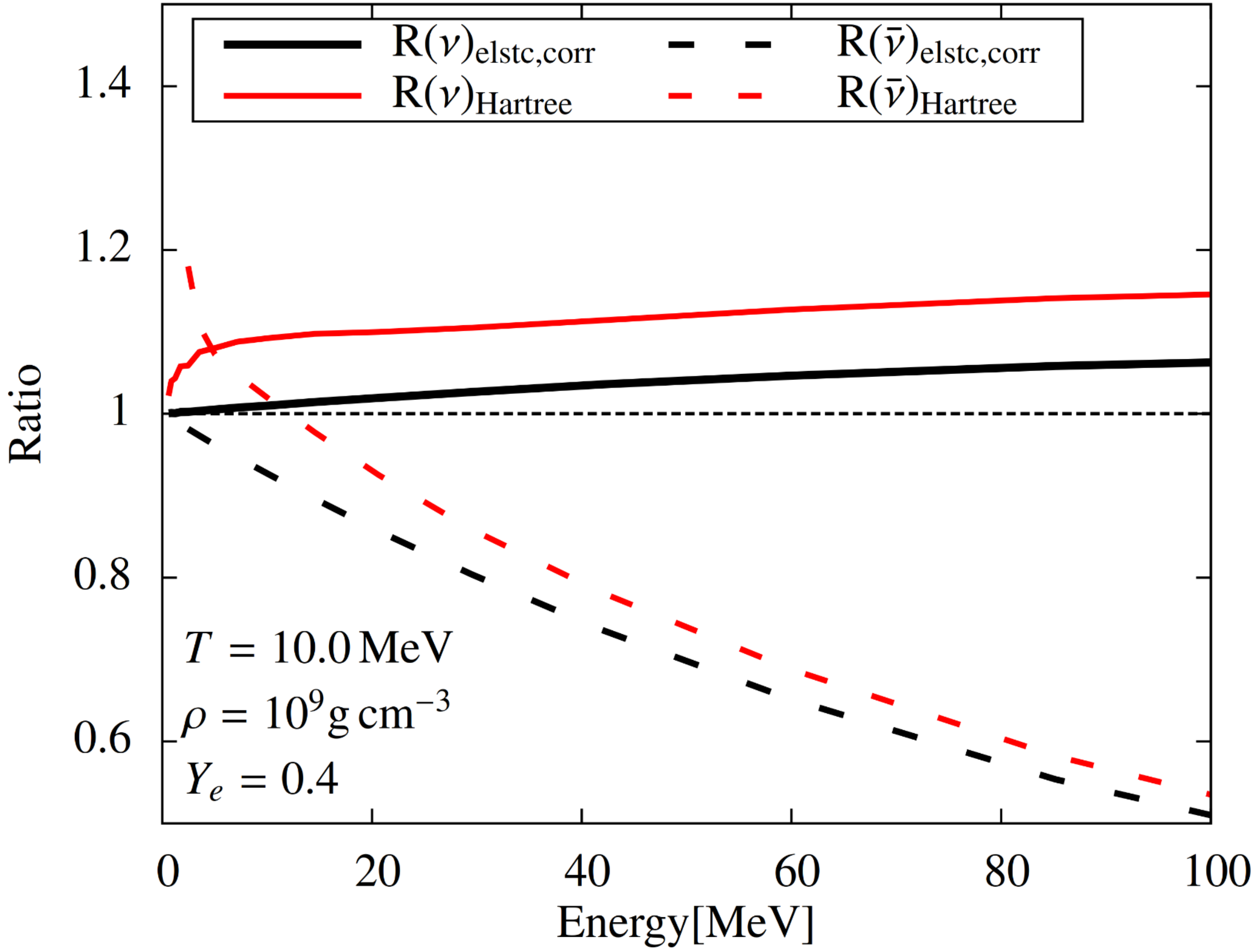
Elastic Approximation at low *T* and *ρ*



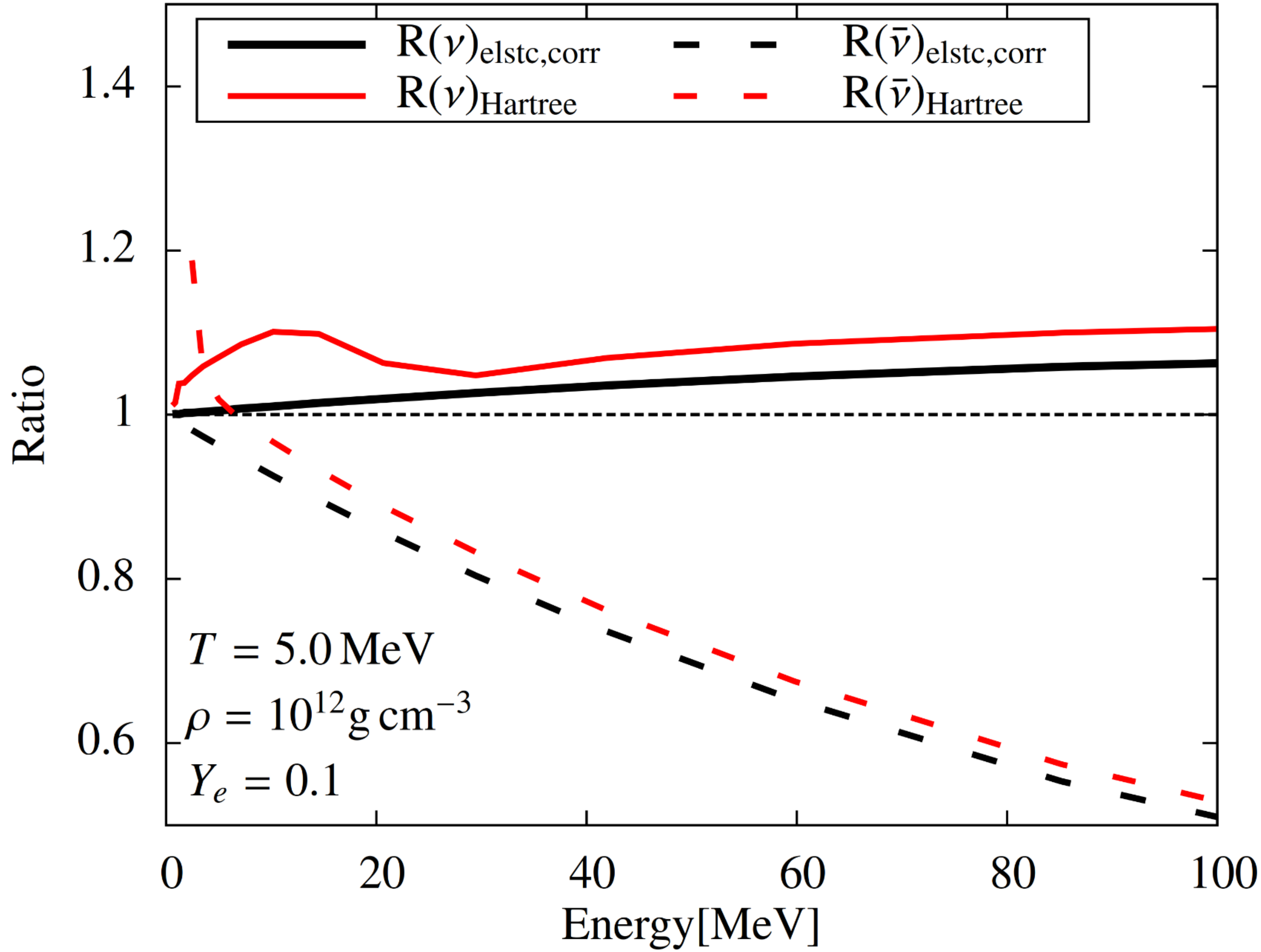
Elastic Approximation at high *T* and low *ρ*



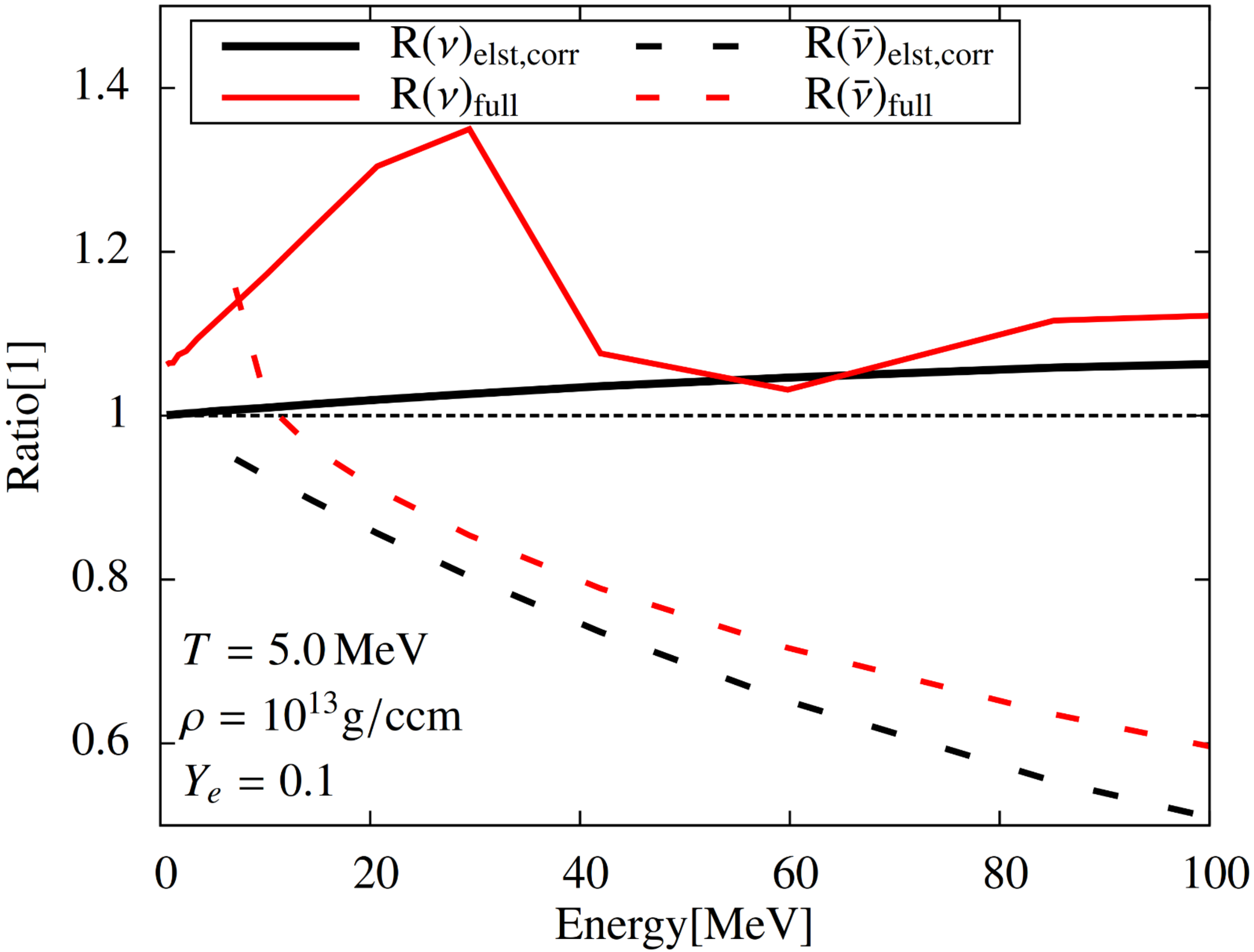
Elastic Approximation at high T and low ρ



Elastic Approximation at high T and high ρ



Elastic Approximation at high T and high ρ



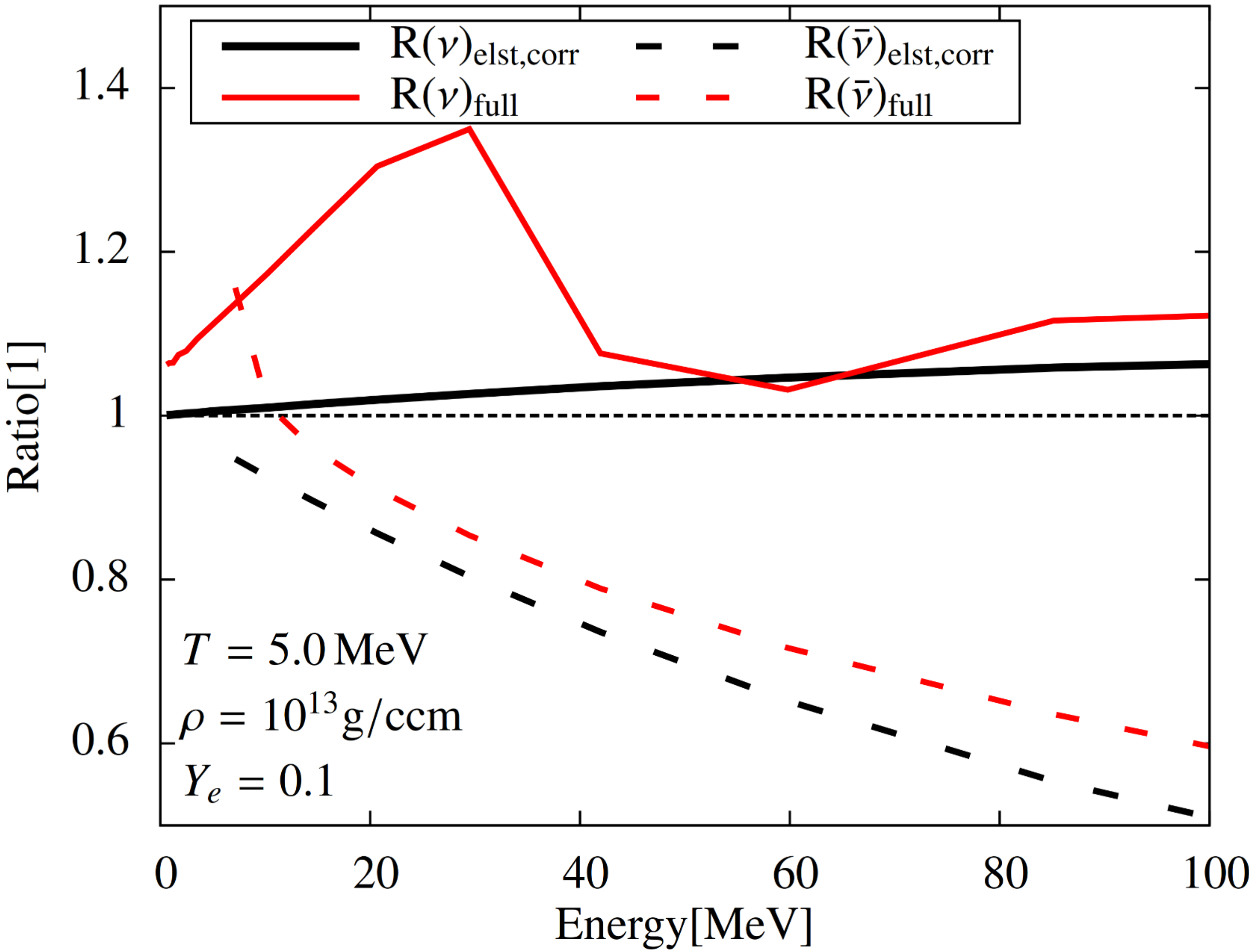
Limits of Elastic Approximation

- Elastic opacities with weak magnetism corrections are indeed very good for low temperatures and densities (NDW?)

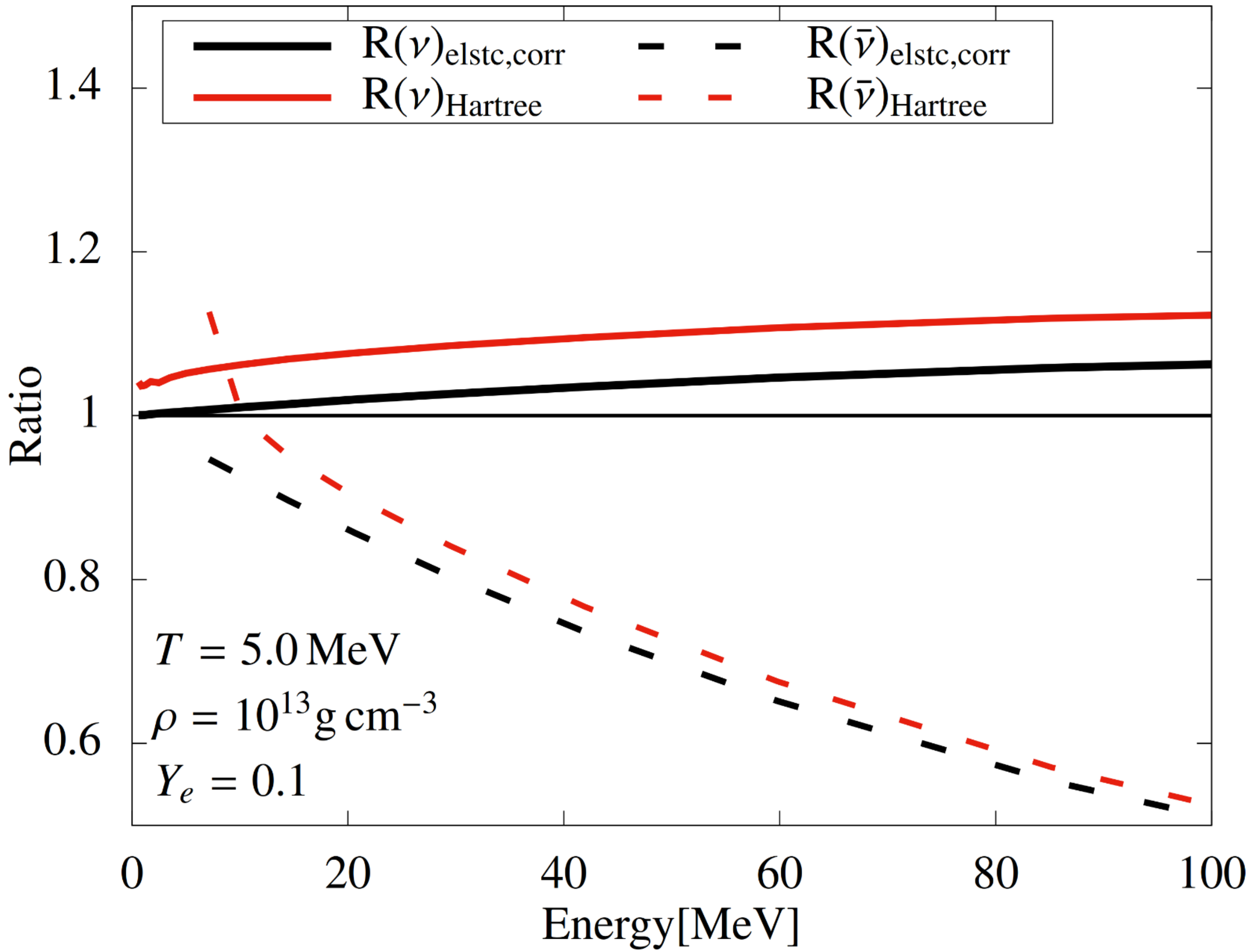
- For temperatures of several MeV, approximation underestimates opacities (~10%)
- At higher densities, additional significant deviations for neutrino energies of several 10 MeV

- Approximation „fails“ at the level of weak magnetism corrections for higher densities/temperatures
  - What is the reason for the failure? (target at rest; inelasticity; relativity)
  - Is there a „cure“?

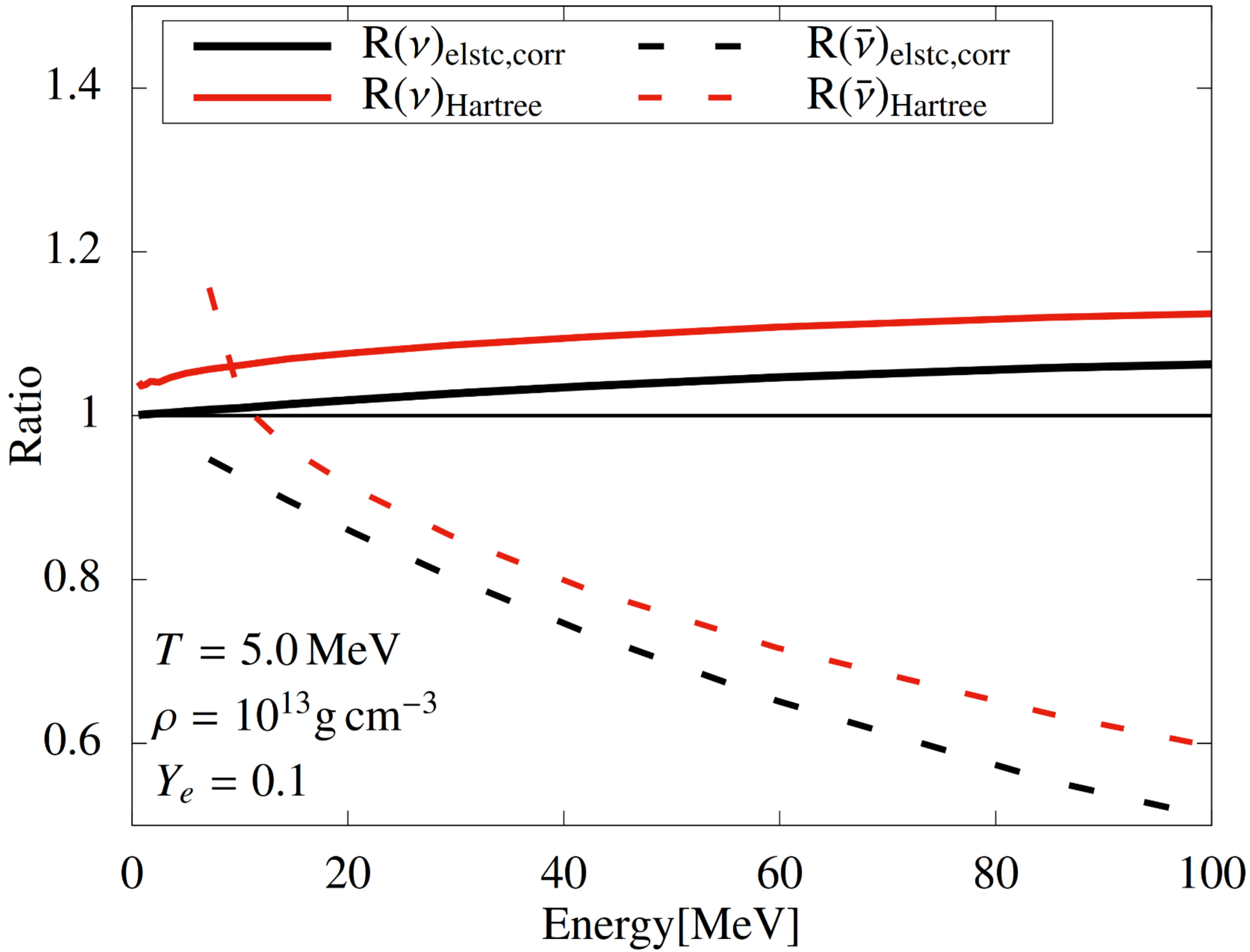
Elastic Approximation at high T and high ρ



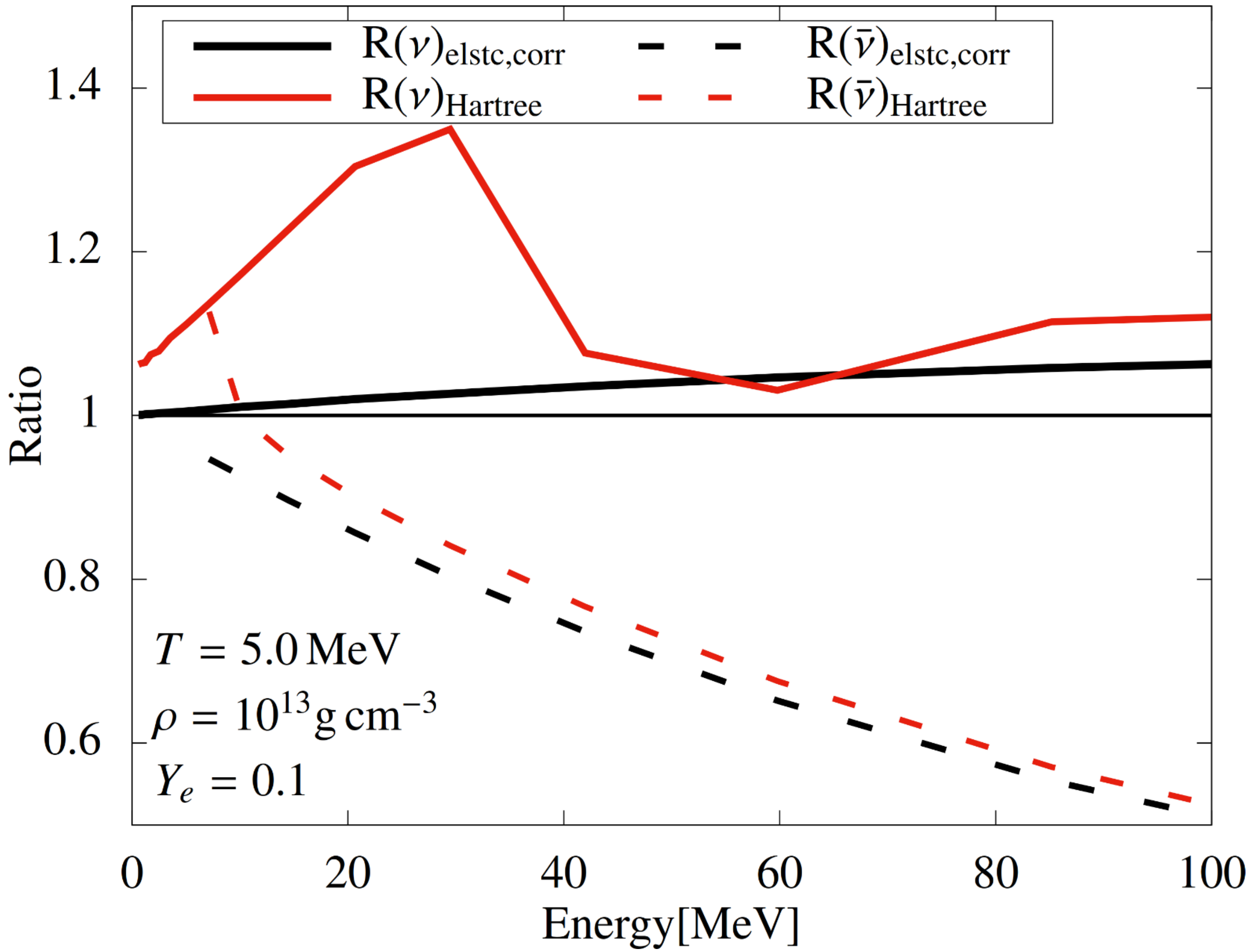
No final state blocking at high T and ρ



No final state lepton blocking at high T and ρ



No final state nucleon blocking at high T and ρ



Elastic vs. Inelastic Opacities

- Elastic approximation feels too large blocking since it does not allow recoil.
- Probably no „*cure*“ since decoupling of final state blocking of leptons and nucleons is the core of the elastic approximation.

Alternative approximation:

- Inelastic opacities with non-relativistic kinematics and simplified matrix element

[Reddy,Prakash,Lattimer, PRD 58 (´98) 013009]

- Apply appropriate correction factor

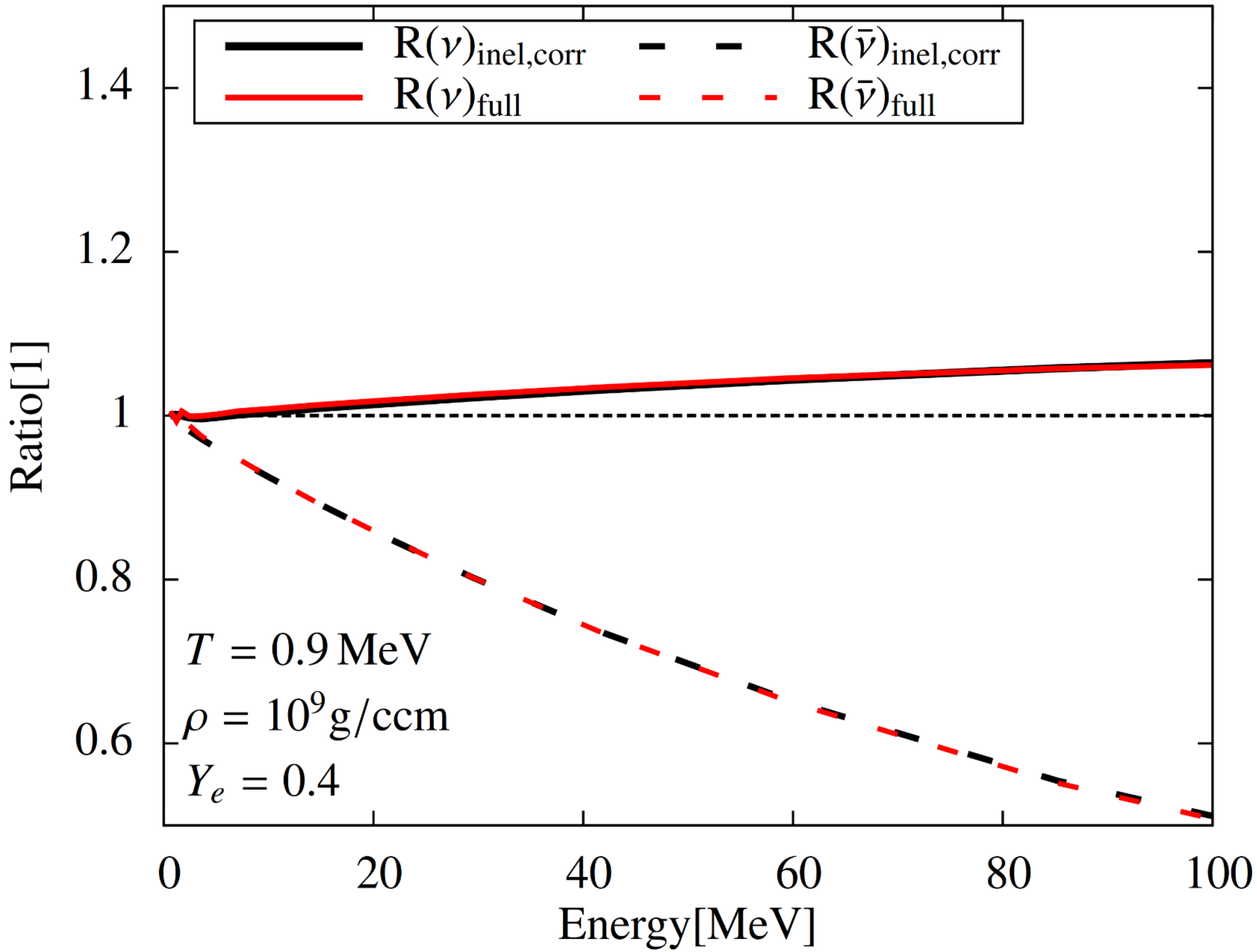
[Horowitz, PRD 65 (2002) 043001]

- Can include p-h-correlations in nonrelativistic RPA

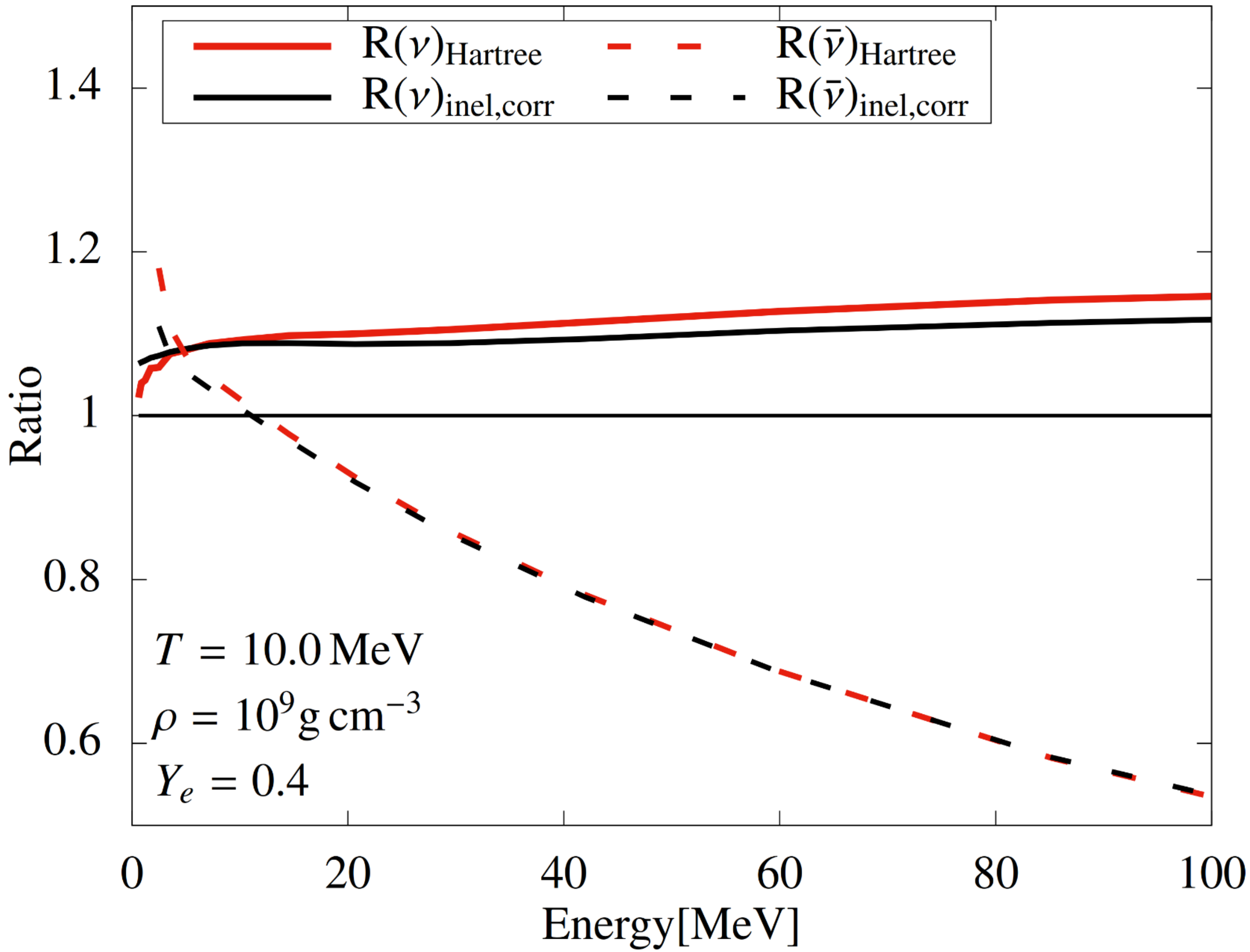
[Burrows,Sawyer, PRC 59 (´99) 510]

[Buras et al., A&A447 (´06) 1049]

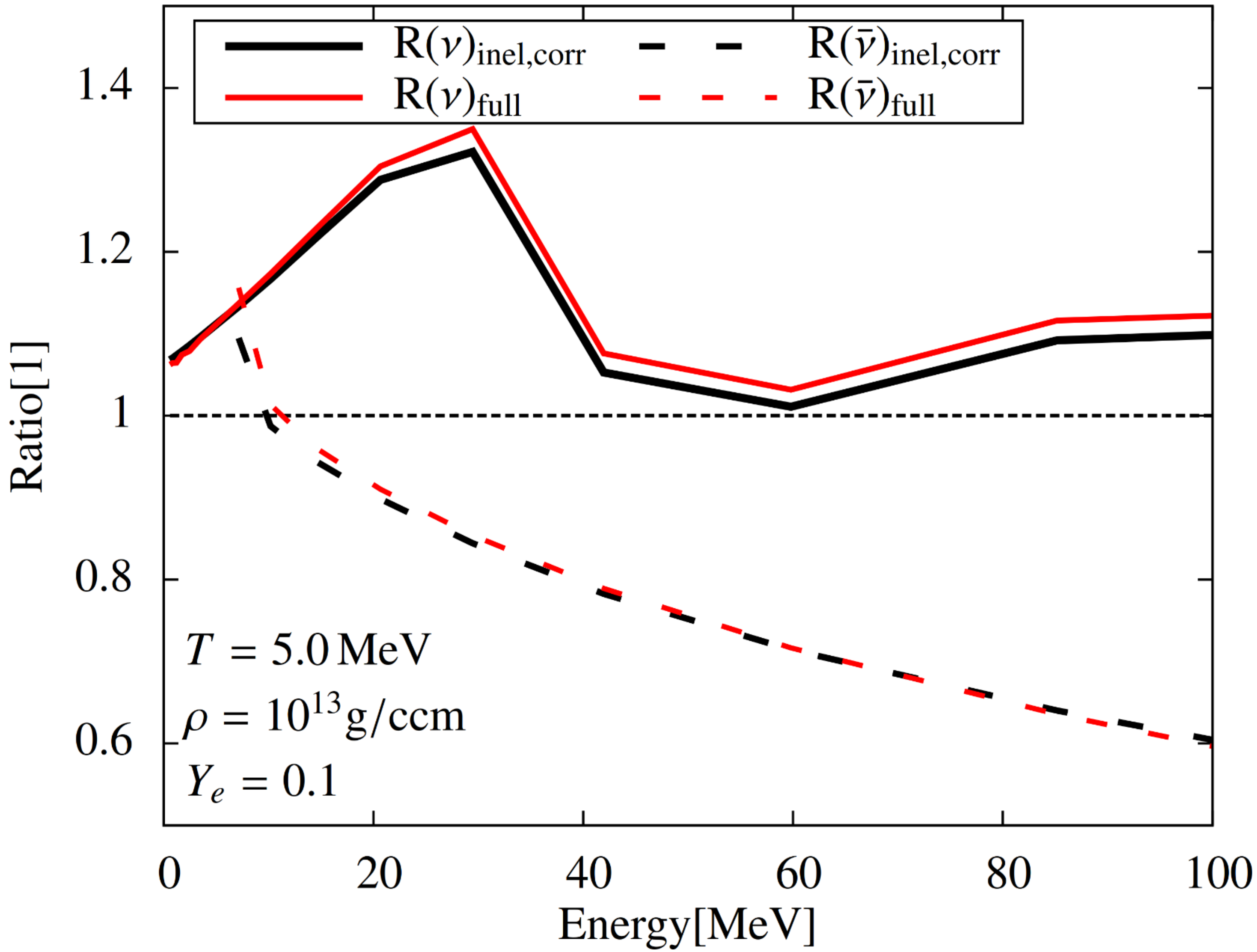
Inelastic Opacity at low *T* and *ρ*



Inelastic Opacity at high T and low ρ



Inelastic Opacity at high T and ρ



Summary & Conclusion

- For densities up to NDW-conditions and temperatures below several MeV, exact neutrino opacities can be reproduced by elastic approximation + correction factors.
- For higher temperatures or for neutrinosphere densities, the elastic approximation „fails“ at the level of the correction.
- For inelastic opacities, „good“ corrections can be found also at higher densities and temperatures.
- Calculation of simplified, inelastic opacities equally demanding as „exact“ relativistic Hartree response
  - For precision at 10% level, relativistic Hartree response favourable over elastic approximation
  - When interested in p-h-correlations, inelastic but approximated opacity + corrections can be suitable

Summary and Conclusion

Collaborators:

Gabriel Martinez-Pinedo (TU Darmstadt / GSI)

Tobias Fischer (Univ. Wroclaw, Poland)

Matthias Hempel (Univ. Basel, CH)

Stefan Typel (TU Darmstadt / GSI)

THANK YOU FOR YOUR ATTENTION!

q^2-dependence of effective weak hadronic coupling constants

q^2-Dependence of Weak Hadronic Couplings

Effective couplings of nucleons depend on momentum transfer

$$q^2 = 2E_\nu E_e \left(v_e \cos\theta - 1\right) + m_e^2$$

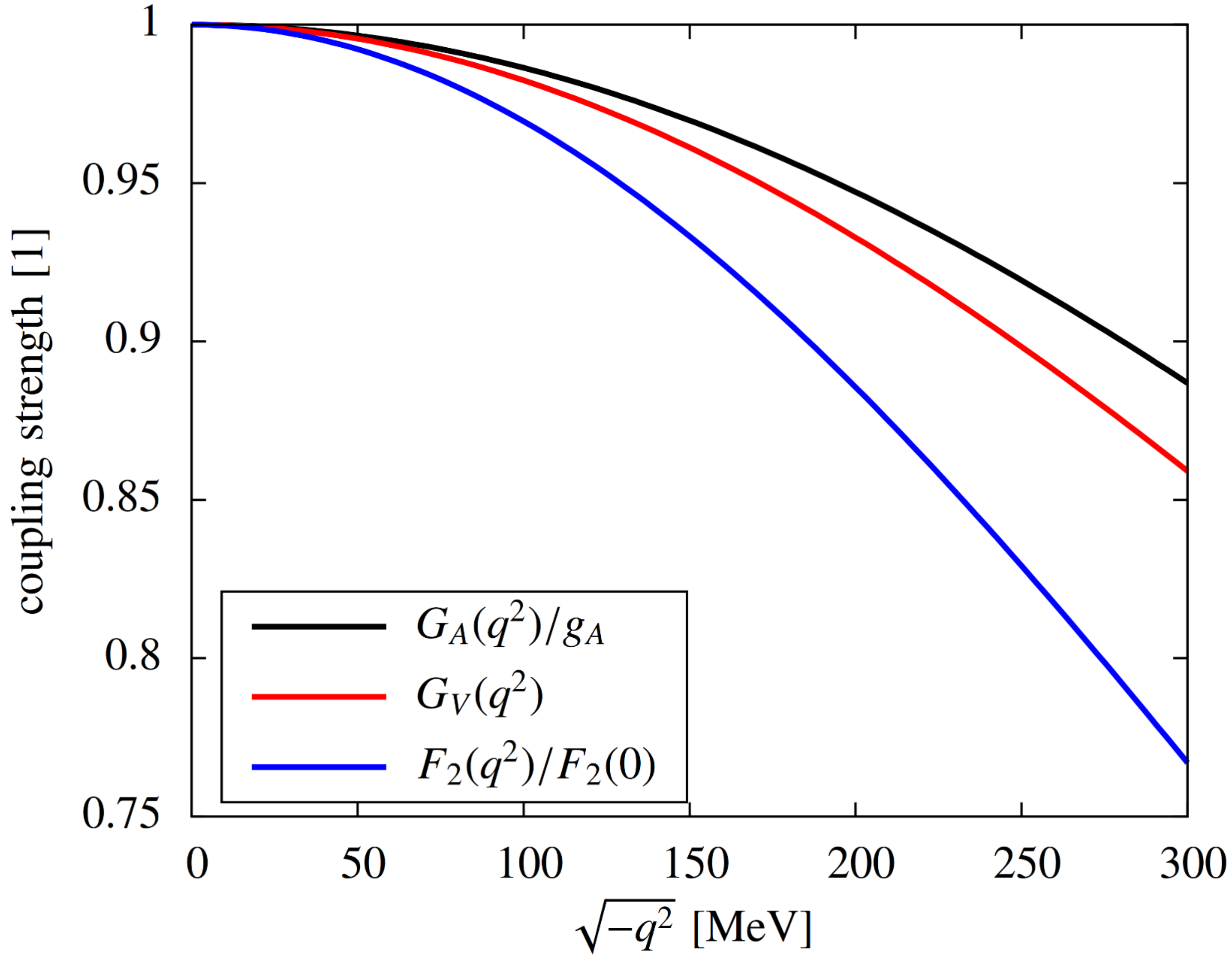
Neutrino transport in *CCSN* usually neglects q^2-dependence

$$G_A(q^2) = g_A \left(1 - q^2/m_A^2\right)^{-2}$$

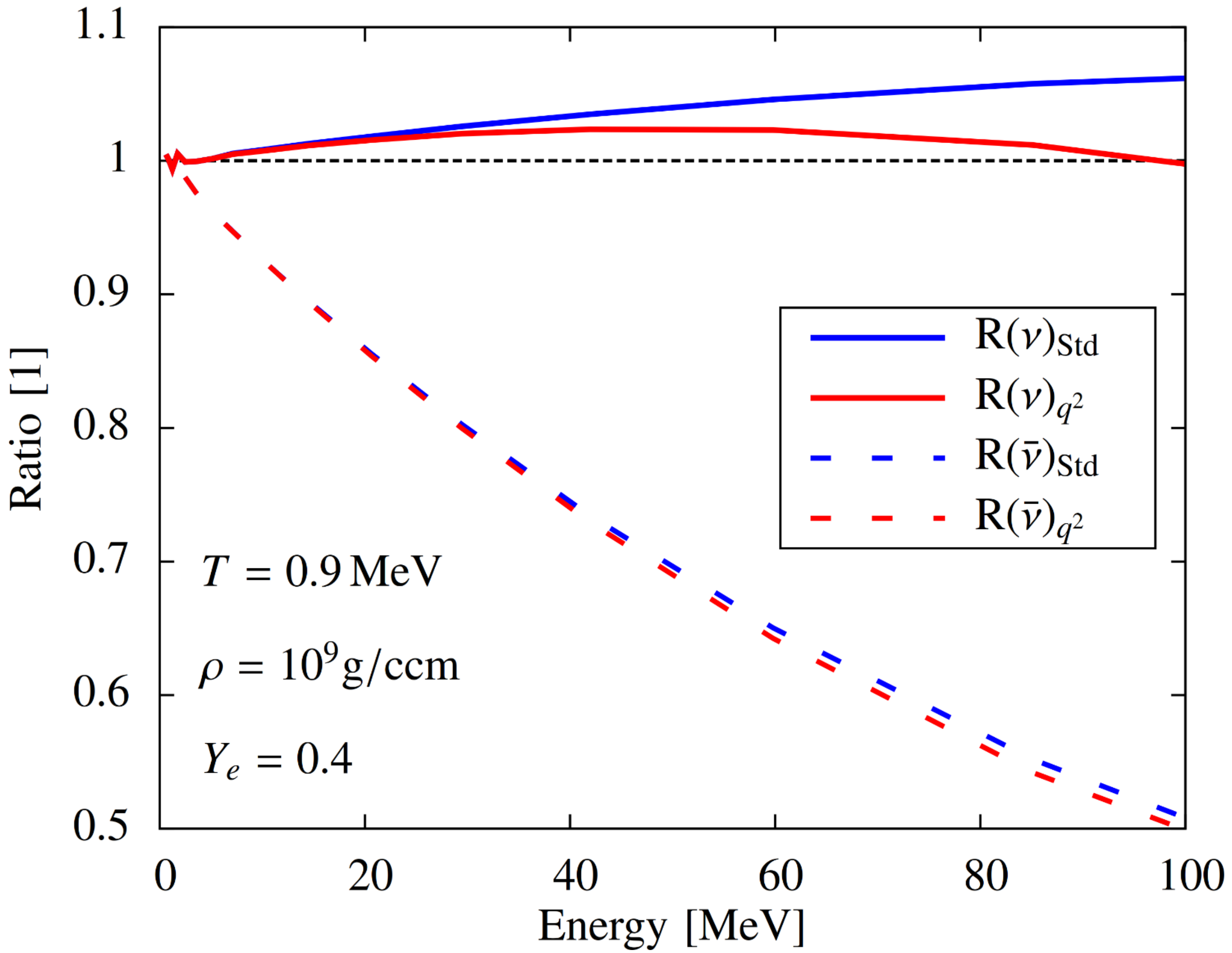
$$G_V(q^2) = \left[1 - \left(F_2(0) + 1\right)\frac{q^2}{4m_N^2}\right]\left(1 - \frac{q^2}{4m_N^2}\right)^{-1}\left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

$$F_2(q^2) = F_2(0)\left(1 - q^2/4m_N^2\right)^{-1}\left(1 - q^2/M_V^2\right)^{-2}$$

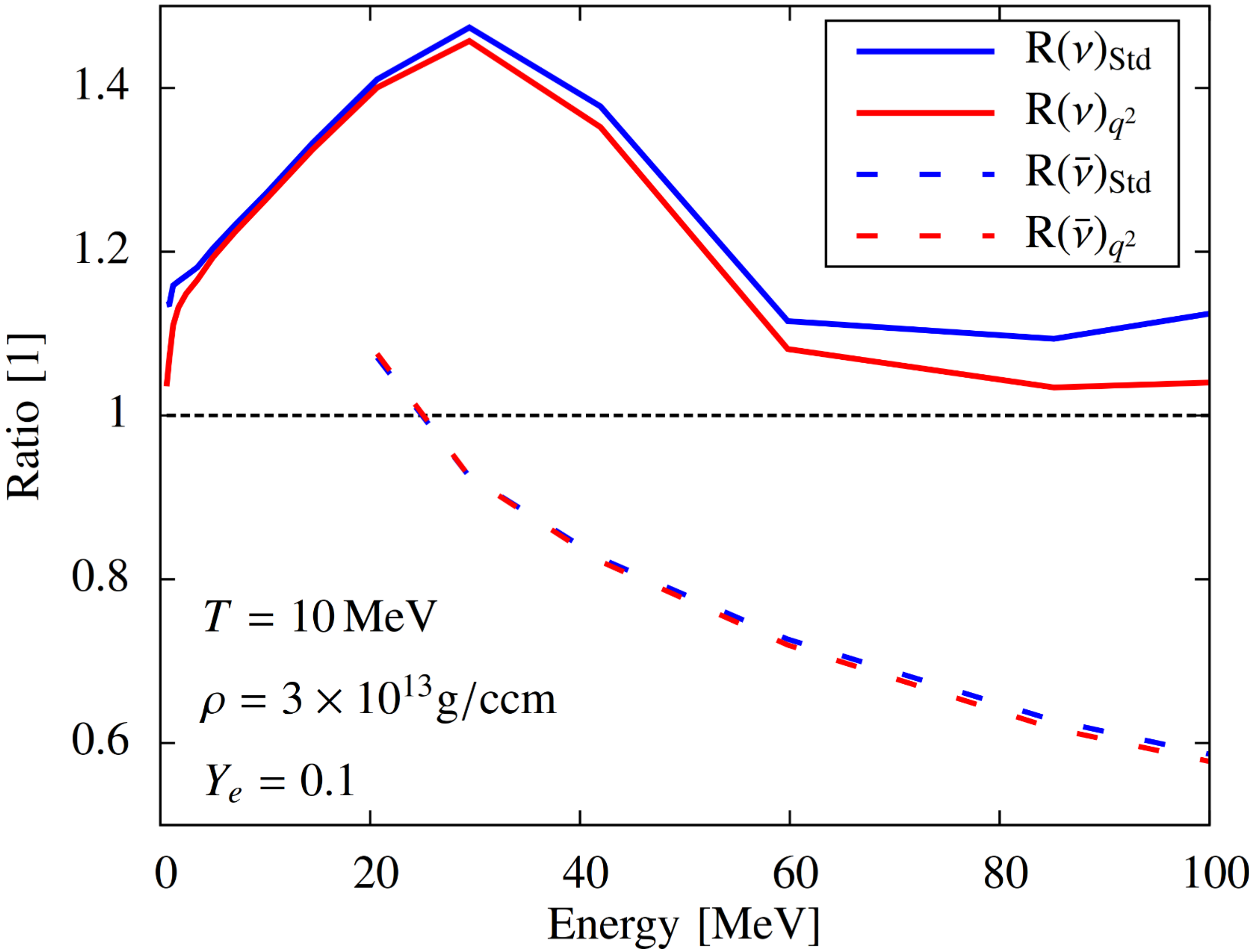
q^2-Dependence of Weak Hadronic Couplings



Opacities with q^2-dependent Couplings



Opacities with q^2-dependent Couplings



Medium modification of analytic correction for nucleon recoil and weak magnetism

Recoil and Weak Magnetism Corrections

[Horowitz, PRD 65 (2002) 043001] pointed out:

- “Elastic Approximation“ is more simplified than necessary
- Kinematics/Recoil can be treated relativistically

$$E_n = m_n \Rightarrow E_e = \frac{E_\nu}{1 + \frac{E_\nu}{m_n}(1-x)}$$

- To include in phase space factor and matrix element
- Gives rise to analytic correction factor for cross-section

$$R = \left\{ G_V^2 \left(1 + 4e + \frac{16}{3}e^2\right) + 3G_A^2 \left(1 + \frac{4}{3}e\right)^2 \pm 4G_A \left(G_V + F_2\right) e \left(1 + \frac{4}{3}e\right) + \frac{8}{3} G_V F_2 e^2 + \frac{1}{3} F_2^2 e^2 (5 + 2e) \right\} / \left[(1+2e)^3 \left(G_V^2 + 3G_A^2\right)\right]$$

Improvement: Consider Mass and Potential Differences

- Masses and strong interaction potentials of nucleons differ
- At large densities effective masses decrease

$$E_e = \frac{E_\nu + \frac{M_*^2 - m_p^{*2}}{2M_*}}{1 + \frac{E_\nu}{M_*}(1-x)} \qquad M_* = m_n^* + U_n - U_p$$

- Analytic correction factor can still be derived the same way
- In the matrix element, additional terms can be included
- For neutrino scattering, only difference is exchange of rest mass with effective mass

Improved Correction Factor

$$R = \Big\{ G_V^2 \left[1 + 4e_* + \frac{16}{3}e_*^2 + \frac{4}{3}e_*\xi + \left(1 + \frac{2}{3}e_*\right)(\xi - q_*) \right]$$

$$+ G_A^2 \left[3 + 8e_* + \frac{16}{3}e_*^2 - \frac{4}{3}e_*\xi - \left(1 + \frac{2}{3}e_*\right)(\xi + q_*) \right]$$

$$\pm G_A \left[G_V + F_2 \frac{M_*}{m_N} \left(1 - \frac{\xi}{2}\right) \right] \left[4e_* + \frac{16}{3}e_*^2 + q_* \left(2 + \frac{4}{3}e_*\right) \right]$$

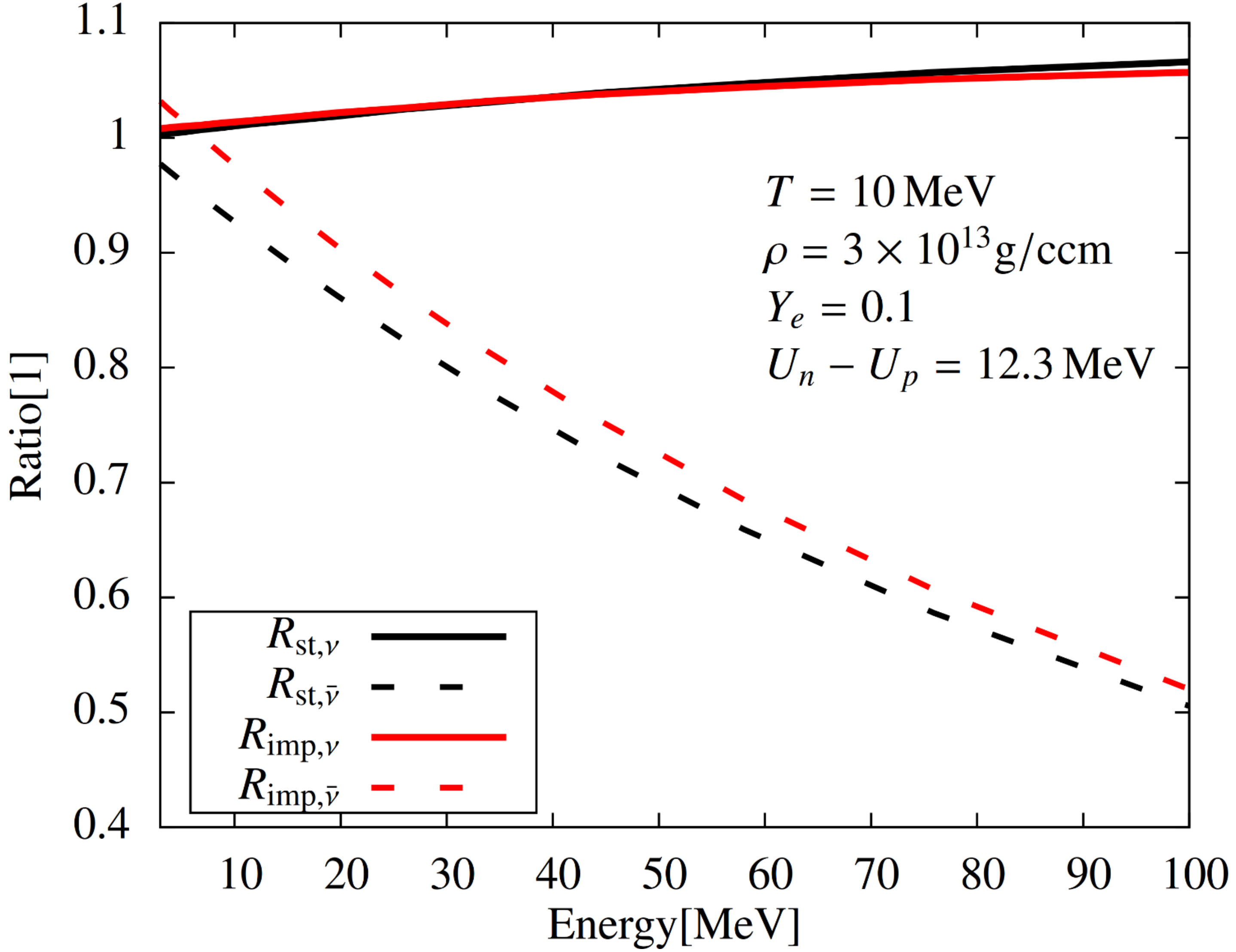
$$+ G_V F_2 \frac{M_*}{m_N} \left[\left(1 + \frac{q_*}{e_*} - \frac{\xi}{2}\right) \frac{8}{3}e_*^2 + \xi q_* \left(1 + 2e_* + \frac{4}{3}e_*^2\right) \right]$$

$$+ F_2^2 \frac{M_*^2}{m_N^2} \left[\frac{5}{3}e_*^2 + \frac{2}{3}e_*^3 + \left(\frac{1}{2} + e_*\right)\tilde{A} + \left(\frac{1}{2} + \frac{1}{3}e_*\right)\tilde{B} + \frac{2}{3}e_*\tilde{C} \right] \Big\}$$

$$/ \left[(1 + 2e)^3 \left(G_V^2 + 3G_A^2\right) \right]$$

$$\xi = \frac{\Delta m^* + \Delta U}{M_*}, \quad q = \frac{m_n^{*2} - m_p^{*2}}{2M_*^2}, \quad q_* = \frac{M_*^2 - m_p^{*2}}{2M_*^2}$$

Improved Correction Factor at High Densities



(Improved) Correction Factors at High Densities