Turbulence and neutrinos during the accretion phase

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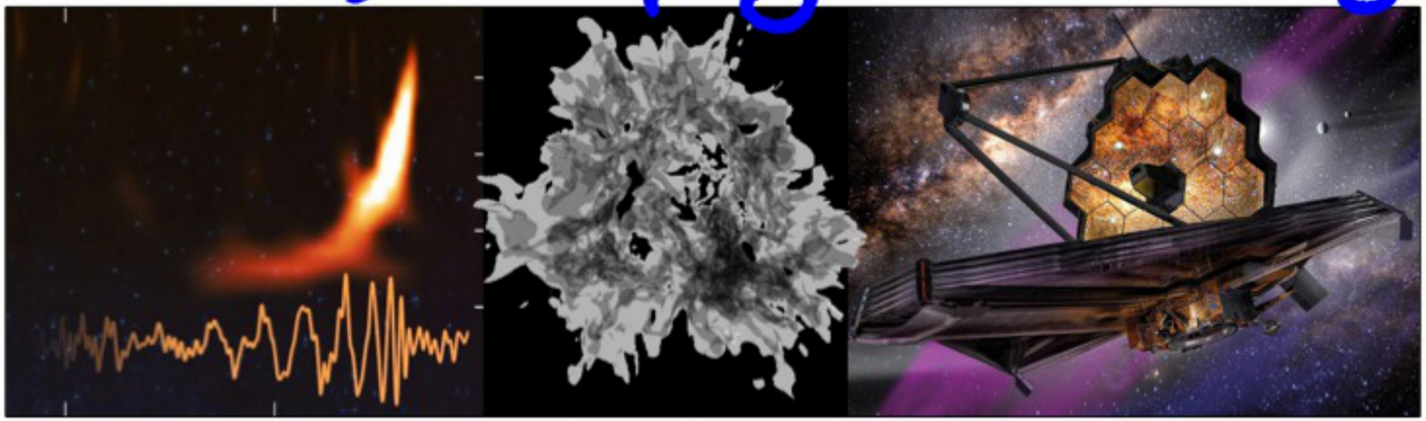
FOE17 Fifty-One Erg



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Welcome to:

FOE Fifty-One Erg

an international workshop on the physics and observations of supernovae, supernova remnants, and other cosmic explosive phenomena.

For information: email

The neutrino signal from supernovae

The time structure of the signal can be divided into four epochs:

- the precollapse emission (days,weeks),

see talk by Cecilia Lunardini

- the collapse epoch (50 milliseconds),
  - the emission during the precollapse and collapse phase is robust - the physics is 'simple' and reproduced by many groups.
- the accretion epoch (~ 1/2 - 1 second),
  - The most variation between simulations (and probably in Nature too) is during the accretion phase.

see talks by Bronson Messer, Kei Kotake and Thomas Janka

- the cooling epoch (10-100 seconds)
  - The emission during the cooling phase has not been extensively studied due to computational demands of multi-d simulations.

see talks by Luke Roberts & Shirley Li

Flavor transformation in supernovae

- The neutrino spectra emitted at the neutrinosphere are modified as the neutrinos propagate to Earth.
- The flavor structure changes due to five processes:
  - collective effects, (Huaiyu Duan, Georg Raffelt, Baha Balentekin)
  - a dynamic MSW effect,
  - turbulence, (Yue Yang)
  - decoherence,
  - Earth matter.

- Earth matter, decoherence and dynamic MSW effect are well understood.
- Not all effects are present at all epochs of neutrino emission.

Neutrino propagation

- The ν state at r is related to the initial state through a matrix S.
- The probability that an initial state j is detected as state i at r is

$$P(\nu_j \rightarrow \nu_i) \equiv P_{ij} = |S_{ij}|^2$$

- S obeys a differential equation

$$i \frac{dS}{d\lambda} = H\, S$$

- H is the Hamiltonian, λ is an affine parameter.

- The neutrino Hamiltonian is made up of several terms:
  - the vacuum H_V term,
  - the self-interaction H_{SI},
  - the matter potential H_M,
- H_{SI} has no consequence during the accretion phase.

Chakraborty et al., PRL **107** 151101 (2011)

- In the presence of matter the neutrinos gain a potential energy.
- For mixing between active flavors we only need consider the Charged Current potential.

$$H_M = \pm \begin{pmatrix} V_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V_{CC} = \sqrt{2}\, G_F Y_e n_N$$

Turbulence in supernovae

- There has been a lot of interest in turbulence in supernovae and how it changes the dynamics of the explosion.
- Questions under investigation include:
  - what is the source?
  - how isotropic is the turbulence?
  - what is the power spectrum?
  - what is the difference between 2D and 3D turbulence?
  - what are the consequences of the 'bottleneck' in 3D?

Murphy & Meakin, ApJ, **742**, 74 (2011)

Dolence, Burrows, Murphy & Nordhaus, ApJ, **765**, 110 (2013)

Muller, & Janka, MNRAS, **448**, 2141 (2015)

Couch & Ott, ApJ, **799**, 5 (2015)

…

Turbulence and neutrinos

- It is known the flavor evolution of a neutrino is affected by turbulence during the cooling phase of a SN.

Sawyer, PRD, **42** 3908 (1990)

Loreti *et al.*, PRD, **52**, 6664 (1995)

Fogli *et al.*, JCAP, **0606**, 012 (2006)

Friedland & Gruzinov, arXiv:astro-ph/0607244

Choubey, Harries & Ross, PRD, **76** 073013(2007)

Kneller & Volpe, PRD **82** 123004 (2010)

Lund & Kneller, PRD, **88**, 023008 (2013)

…

Modeling turbulence

- We do not yet posses suitable high resolution, long duration multi-d simulations which can be used (without modification) to study the effect of turbulence.
- We take a supernova profile from a 1D hydrodynamical supernova simulation and add turbulence to it i.e.

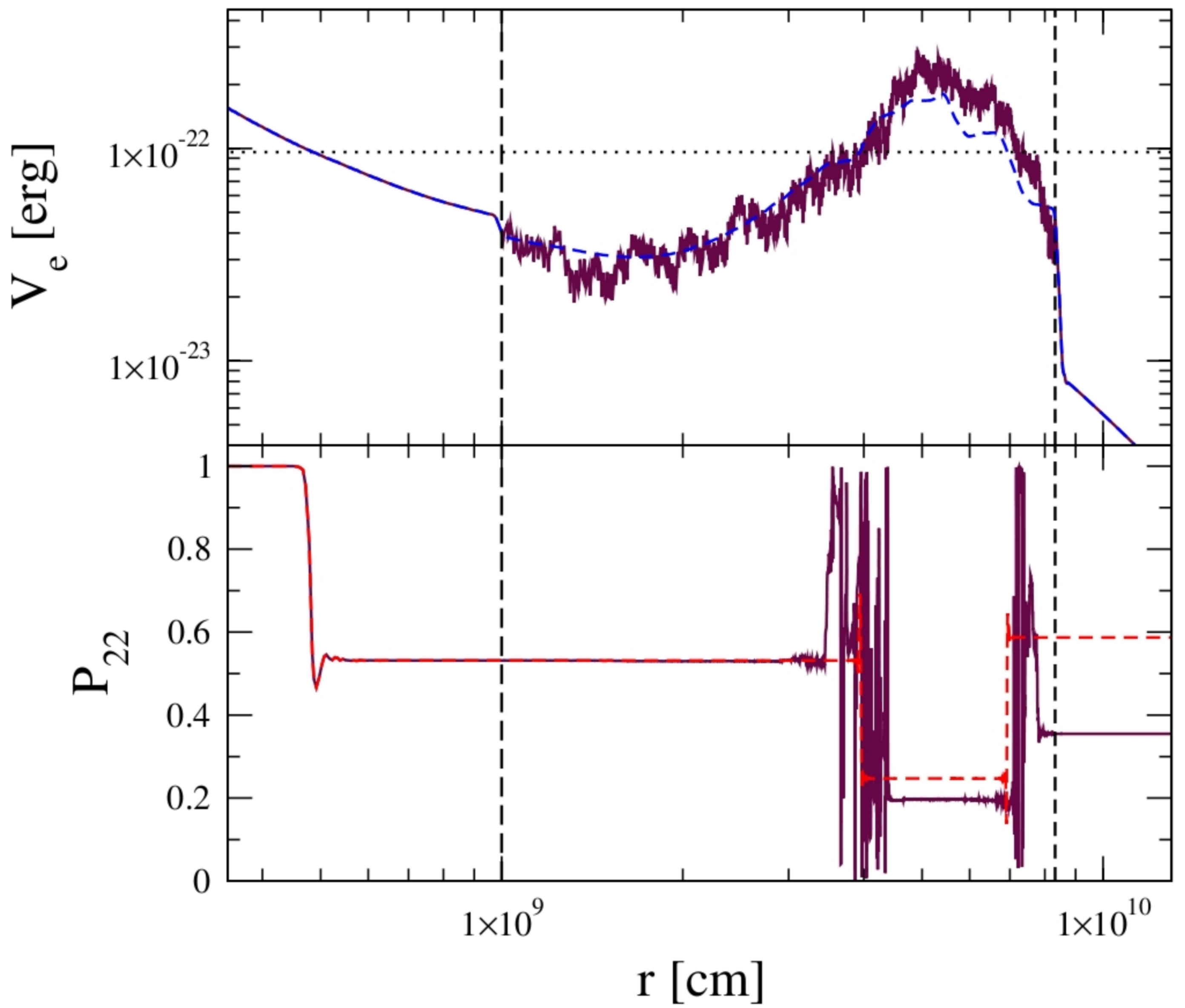
$$V_{CC}(r)=(1+F(r))\langle V_{CC}(r)\rangle$$

- where *F(r)* is a Gaussian random field with rms amplitude C_* and power spectrum *E(q)*
  - e.g. an IPL with index $\alpha=5/3$ and cutoff q_{cut}.
- Realizations of *F* are constructed with a Fourier series.

$$F(r)\propto C_* \sum_{n=1}^{N_q} \{ A_n \cos(q_n r)+B_n \sin(q_n r)\}$$

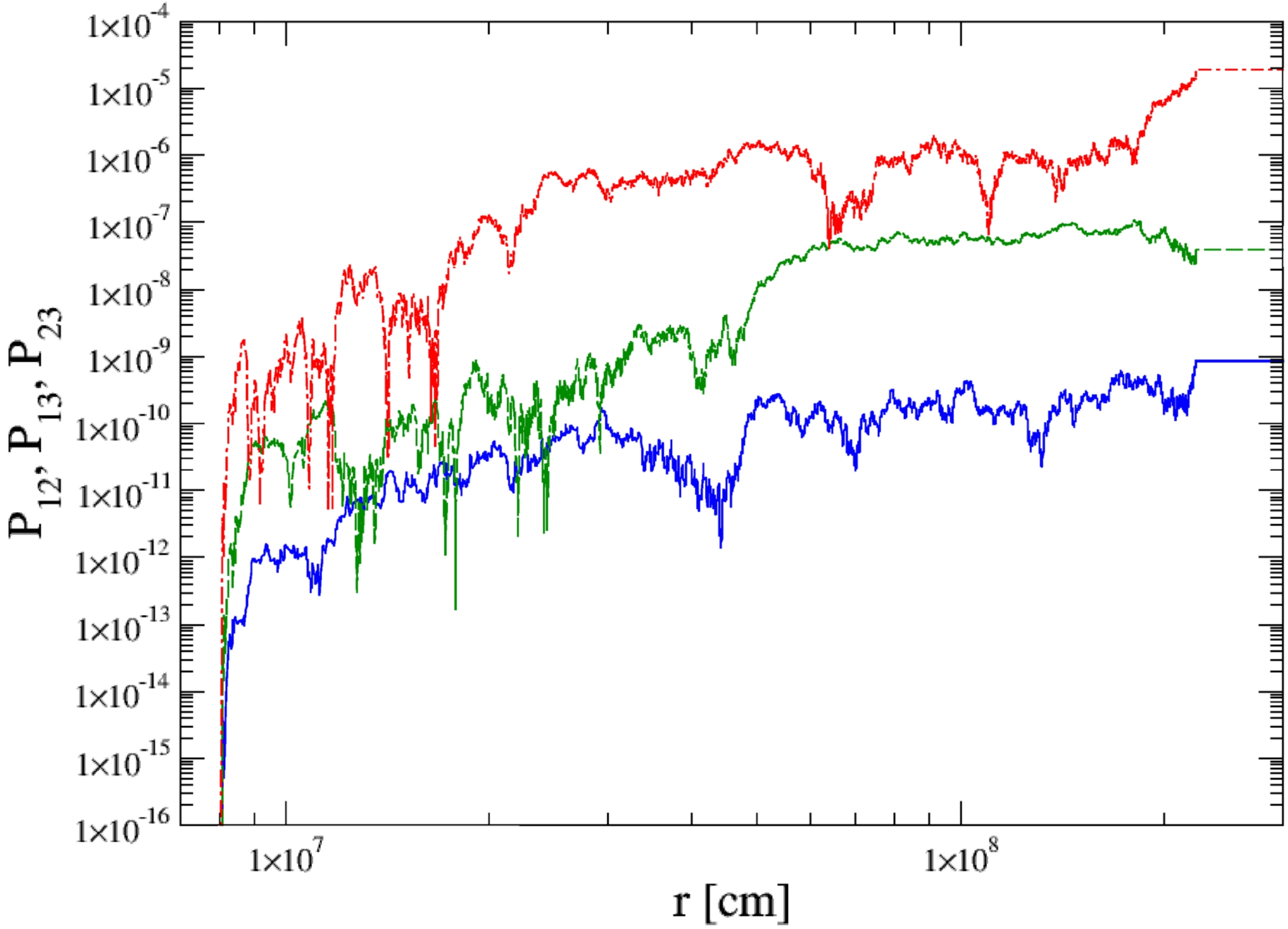
- The sets {A}, {B} and {q} are random variates.

- For example: using a density profile from the cooling phase.



Kneller & Volpe, PRD **82** 123004 (2010)

- Does the turbulence during the accretion phase affect the neutrino evolution (and feedback upon the hydrodynamics)?



- Apparently not.

The six scales of turbulence

- Amazingly turbulence is amenable to theory.

- The effect of turbulence upon neutrinos depends upon six different scales:
  - the cutoff scale – the longest wavelength Fourier mode
  - the dissipation scale – the shortest wavelength Fourier mode
  - the potential scale height – the distance over which the potential changes
  - the splitting scale – the wavelengths corresponding to the splitting between pairs of eigenvalues of the neutrino Hamiltonian,
  - the transition scale – the wavelength of the transitions between pairs of neutrino eigenstates
  - the suppression scale – the wavelength of the mode which sends the transition scale to infinity essentially suppressing transitions

The cutoff scale

- The cutoff scale is the longest wavelength in the turbulence.
  - In 3D it is the scale at which the turbulence is driven.
  - It is approximately the size of the turbulent region i.e. ~ the shock radius.

- If the power spectrum is only defined for wavenumbers greater than q_{cut} the cutoff is

$$\lambda_{cut} = \frac{2\pi}{q_{cut}}$$

The dissipation scale

- The dissipation scale is the shortest wavelength in the turbulence.
  - In the absence of magnetic fields, the dissipation scale is estimated to be in the range of μm to nm.

- If the power spectrum is only defined for wavenumbers less than q_{diss} the dissipation scale is

$$\lambda_{diss} = \frac{2\pi}{q_{diss}}$$

The potential scale height

- The potential changes over a distance h_{cc} defined to be

$$h_{CC} = \frac{V_{CC}}{|dV_{CC}/dr|}$$

- The potential scale height measures how fast the potential changes.

Stimulated transitions

Patton, Kneller & McLaughlin, PRD **91** 025001 (2015)

Patton, Kneller & McLaughlin, PRD **89** 073022 (2014)

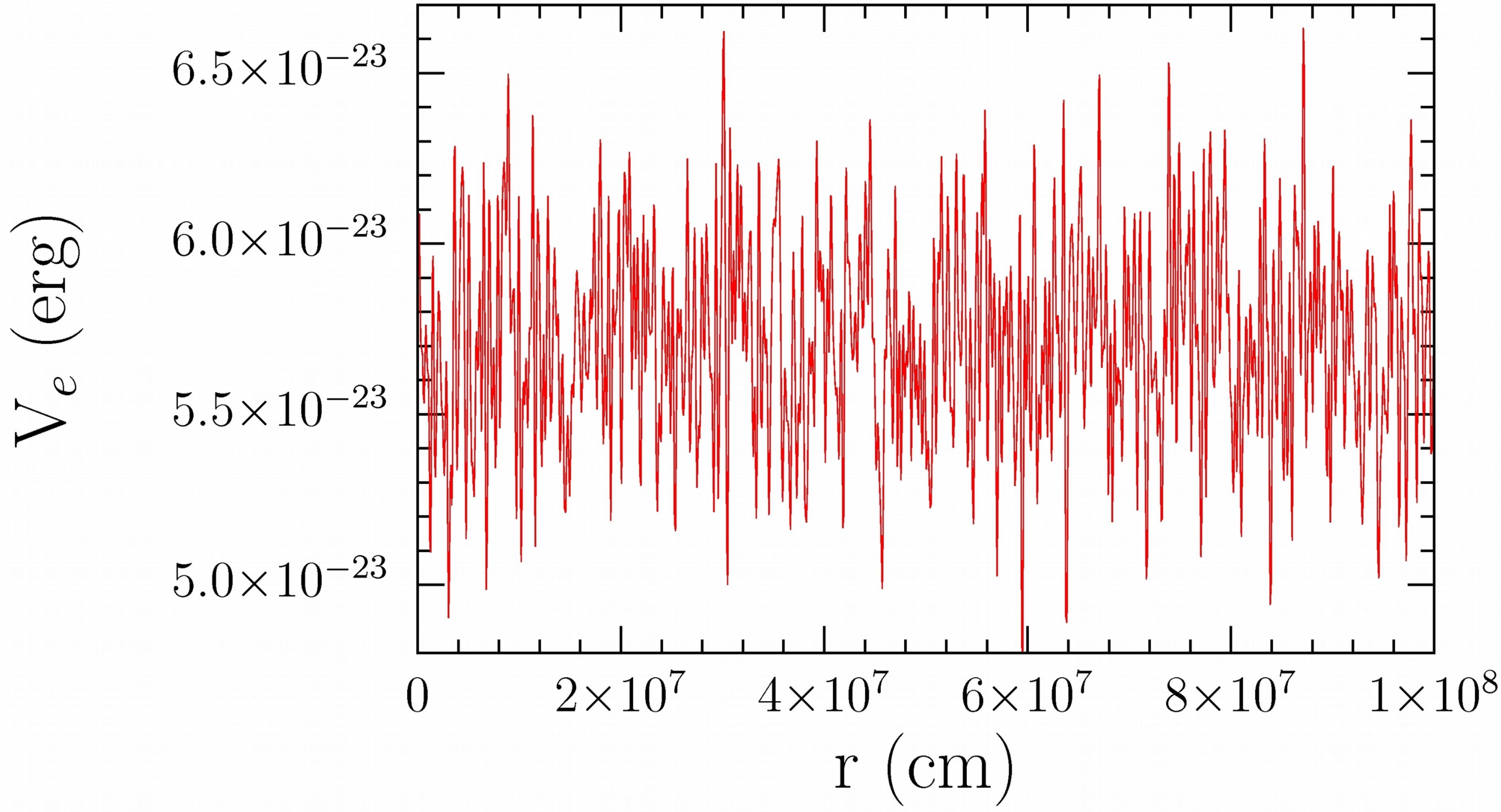
Kneller, McLaughlin & Patton, JPG **40** 055002 (2013)

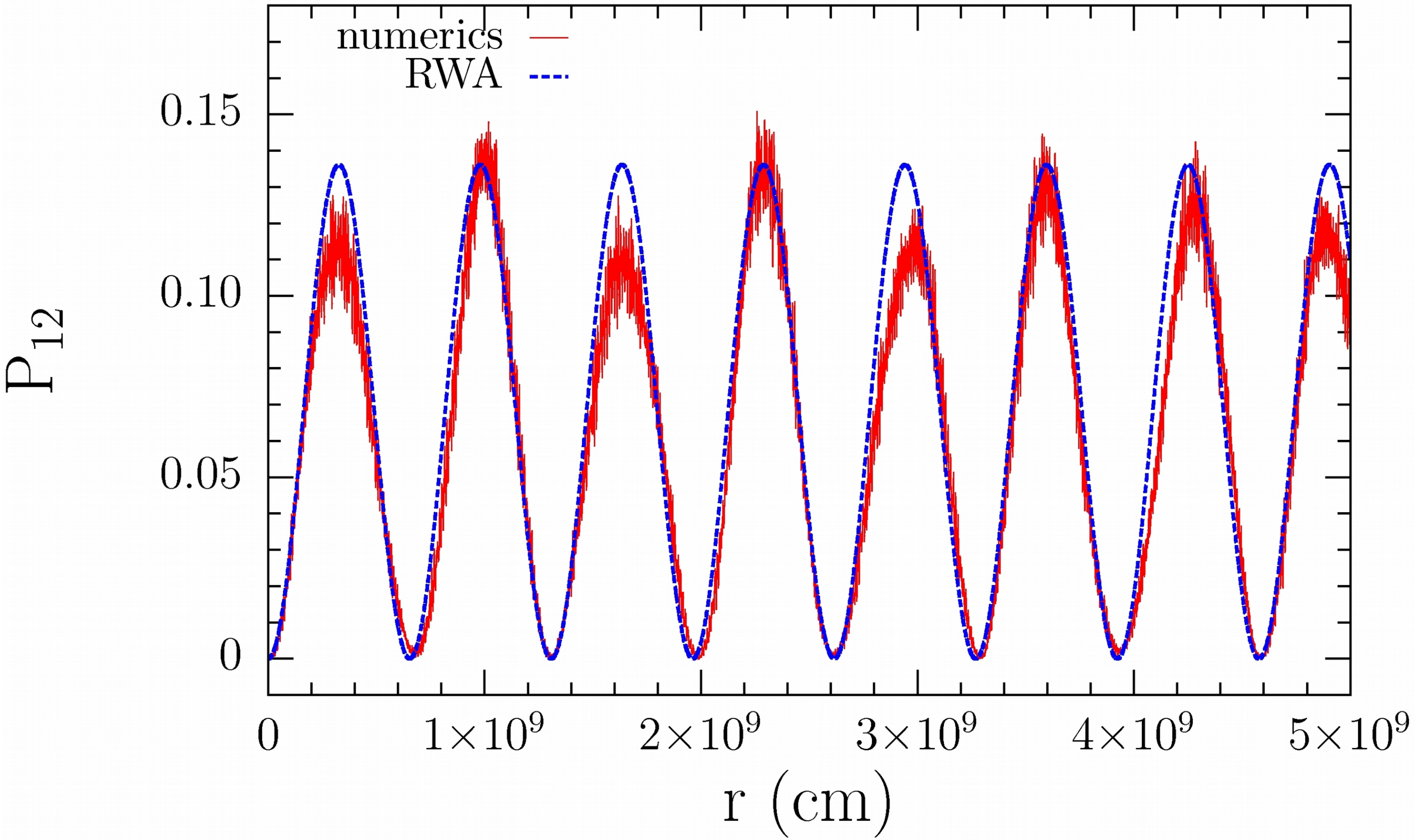
Yang, Kneller & Perkins, arXiv:1510.01998

- The evolution of a neutrino traveling through a fluctuating matter potential is well described by the Stimulated Transition model.
  - The basic idea is to treat the neutrino like a polarized molecule and the turbulence as an external perturbing potential.
- H is composed of two terms:
  - the vacuum contribution,
  - the underlying smooth matter potential,
  - the perturbing (turbulent) potential δH.
- Use time-dependent perturbation theory to calculate the effect of turbulence.

A realization of turbulence created using 50 Fourier modes.

Patton, Kneller & McLaughlin, PRD **89** 073022 (2014)





numerics
RWA
0.15
0.10
0.05
0
P_{12}
0
1×10^9
2×10^9
3×10^9
4×10^9
5×10^9
r (cm)

The splitting scale

- The unperturbed Hamiltonian H_0 has eigenvalues k_1, k_2, ...

- The differences between the eigenvalues of the unperturbed Hamiltonian define the splitting scale.
- For eigenstates i and j with eigenvalues k_i and k_j, the splitting scale is simply

$$\lambda_{split} = \frac{2\pi}{|k_i - k_j|}$$

- Given the eigenvalues we can also define a 'mixing matrix' defined to be

$$H_0 = U\,K\,U^\dagger$$

- The square magnitude of the elements of U are the flavorness of the eigenstates.
  - e.g. $|U_{e1}|^2$ is the electron neutrinoness of eigenstate 1.

The transition scale

- Transitions between two eigenstates occur over some finite distance: the transition scale.
- The formula for this scale is complicated and requires a lengthy explanation.

Empty space where all the math goes ….

- For two flavors the transition probability is

$$P_{12} = \frac{\kappa^2}{Q^2} \sin^2(Q r)$$

- The transitions occur over a distance given by

$$\lambda_Q = \frac{2\pi}{Q}$$

- We can make a couple of (reasonable) approximations:
  - there is a Fourier mode which exactly matches the eigenvalue splitting
  - the amplitude of every mode is small

- On resonance Q is proportional to a product of Bessel functions:

$$Q \propto J_{n_1} J_{n_2} J_{n_3} \ldots$$

- The RWA integers { n_i } are { 0,0,…,1,0,0,... } i.e. zero for every mode except the one which is on resonance.
- For small amplitudes $J_0(z) \sim 1$ and $J_1(z) \sim z/2$

- Using these approximations we find, on resonance, the transition wavelength between states i an j satisfy

$$C_a \lambda_Q > \lambda_{trans} = \frac{4\pi}{|U_{ei} U_{ej}| V_{CC}}$$

- where C_a is the amplitude of the resonant mode.

- λ_{trans} is a lower limit on the actual transition wavelength.

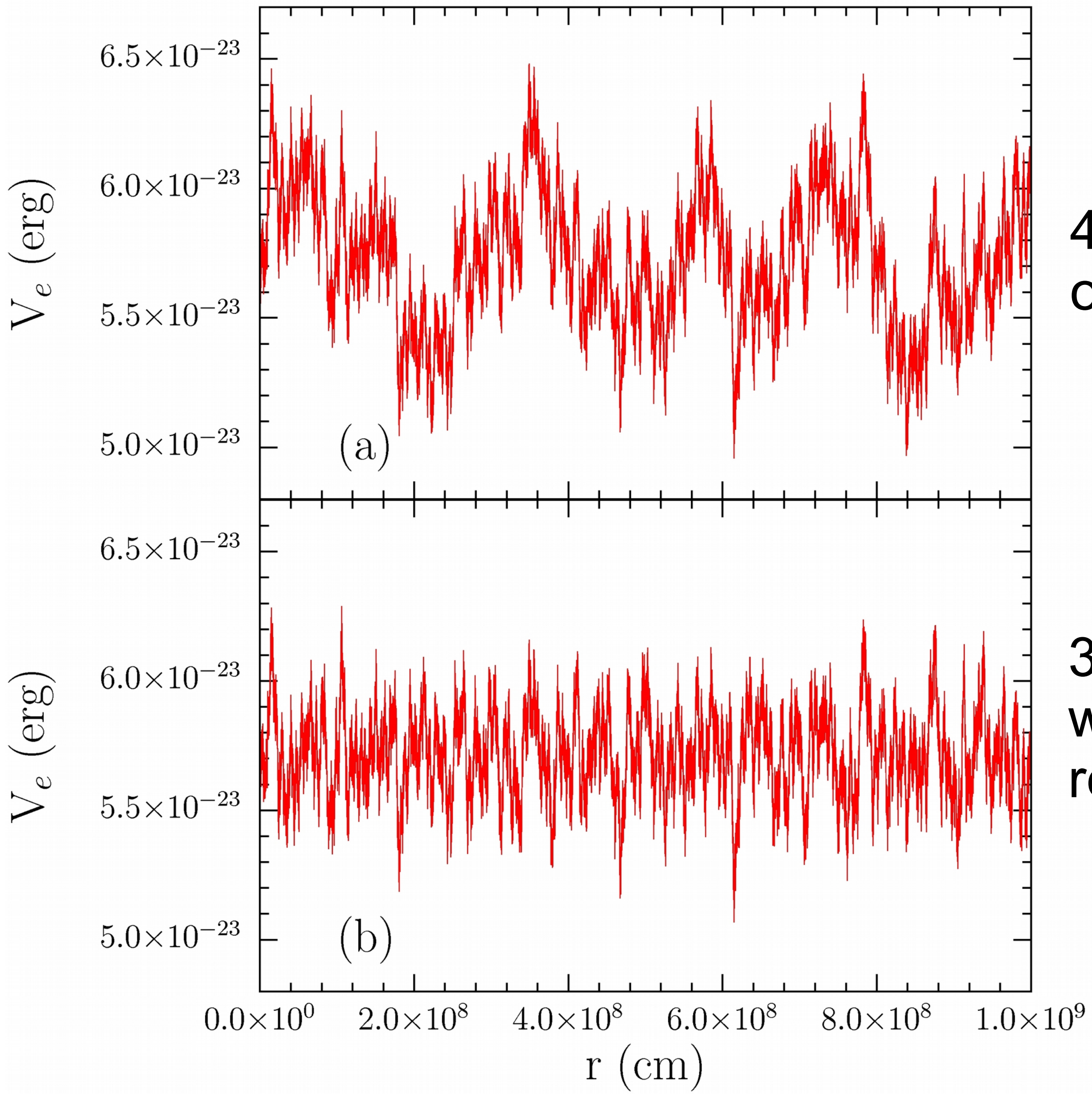
The suppression scale

- The transition wavelength depends upon <u>all</u> the Fourier modes, not just the one on resonance.
- The small amplitude approximation is not always valid.
  - There are some modes for which $J_0 \nrightarrow 1$.

- For a mode a with amplitude C_a and wavenumber q_a the argument z of the Bessel function for states i and j is

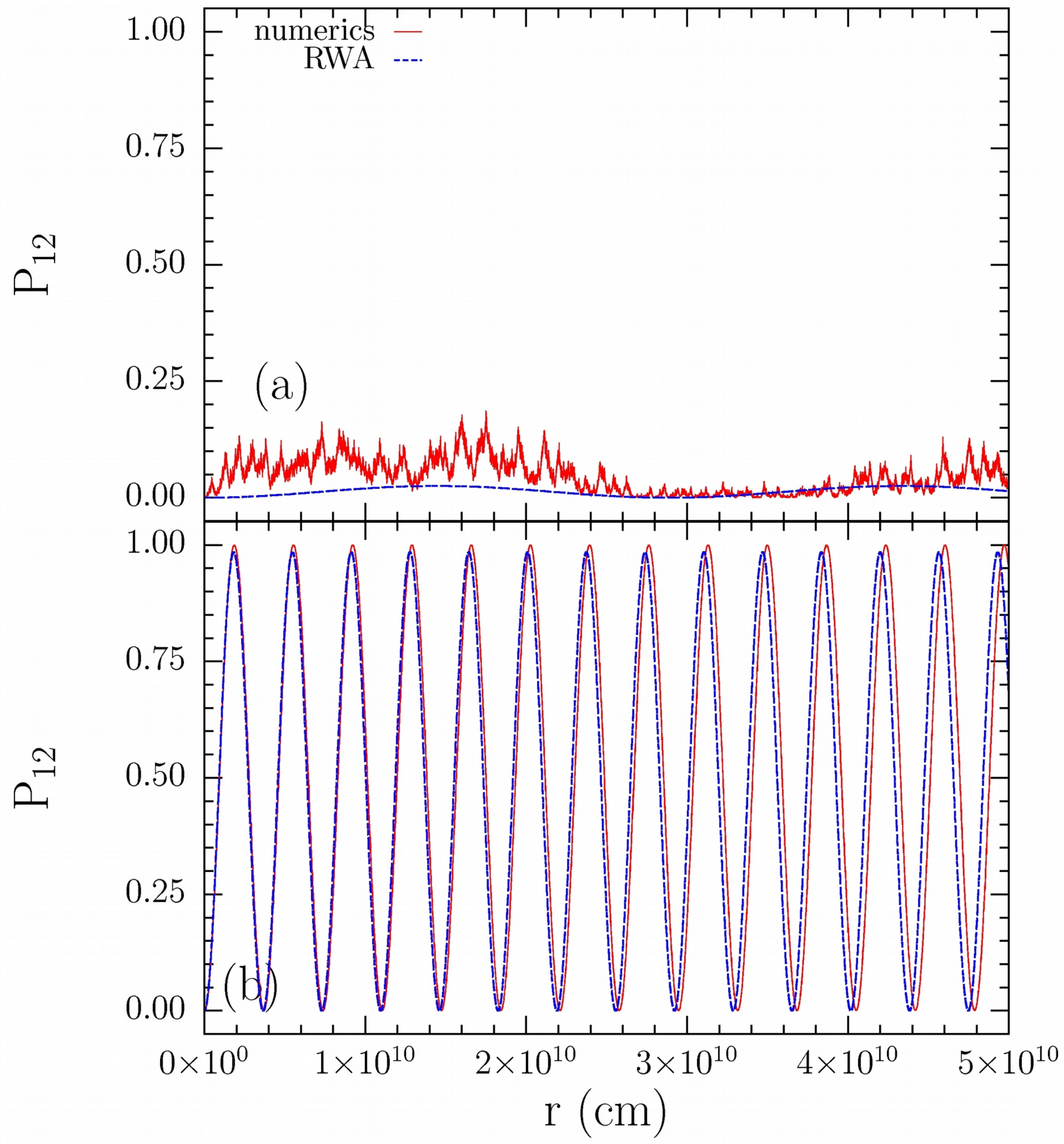
$$z = \frac{C_a V_{CC}}{q_a} \left(|U_{ei}|^2 - |U_{ej}|^2 \right)$$

- If z = 2.405.. then $J_0(z) = 0$ which causes $Q \rightarrow 0$ causing the transition wavelength to go to infinity.



40 modes:
only one is resonant

35 modes: (5 longest
wavelengths removed)
resonant mode still present



The five long wavelength modes suppress the transition.

- This amplitude suppression effect defines a scale called λ_{ampl}

$$\lambda_{ampl} = \frac{4.9096\,\pi}{|V_{CC}(|U_{ei}|^2 - |U_{ej}|^2)|}$$

- If there is any Fourier mode a such that the product of its amplitude and wavelength satisfies

$$C_a \lambda_a > \lambda_{ampl}$$

- then the suppression effect occurs.
- The longest wavelength Fourier mode λ_{cut} usually has the largest amplitude which is usually of order C_*.
- Transitions are suppressed if

$$C_* \lambda_{cut} > \lambda_{ampl}$$

Scale Hierarchy

- In order for turbulence to have an effect the 6 scales have to satisfy three conditions
  - the splitting scale must lie between the dissipation scale and the cutoff scale

$$\lambda_{diss} < \lambda_{split} < \lambda_{cut}$$

  - if C_a is the amplitude of the mode matching the splitting scale then transition scale must be smaller than the potential scale height

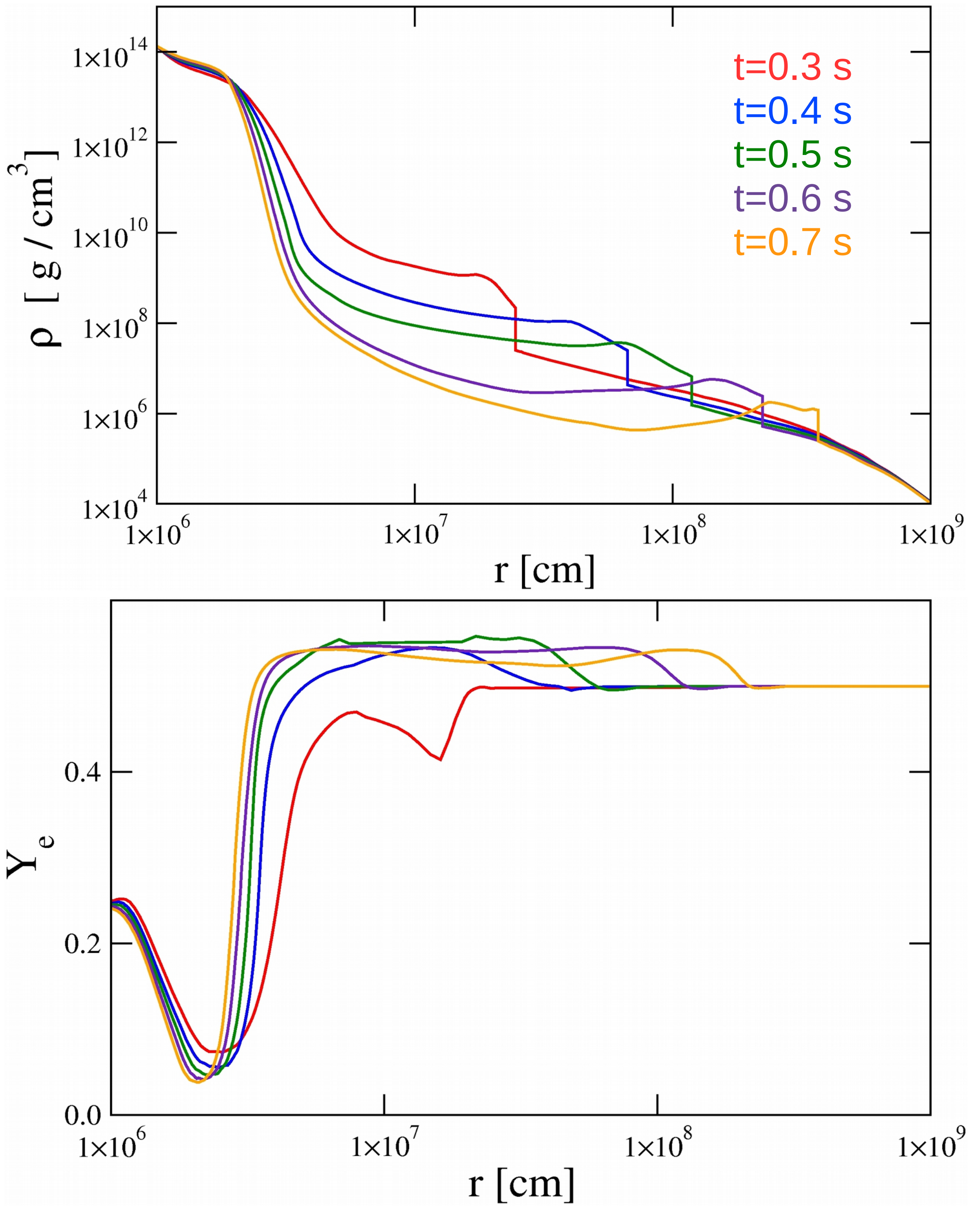
$$\lambda_{trans} < C_a h_{CC}$$

  - the cutoff scale must be shorter than the suppression scale.

$$C_a \lambda_a < \lambda_{ampl} \; \forall \, a$$

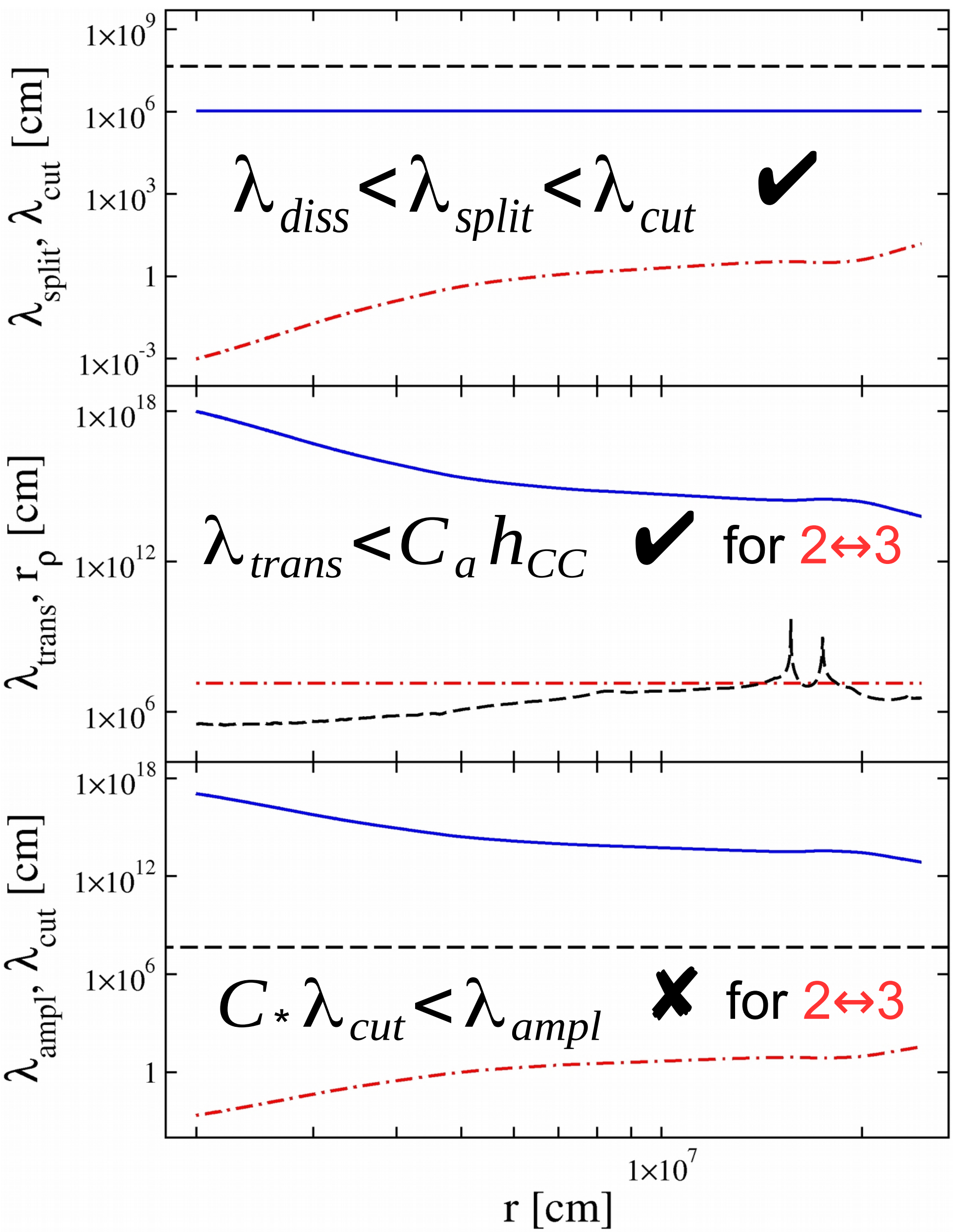
$$C_* \lambda_{cut} < \lambda_{ampl}$$

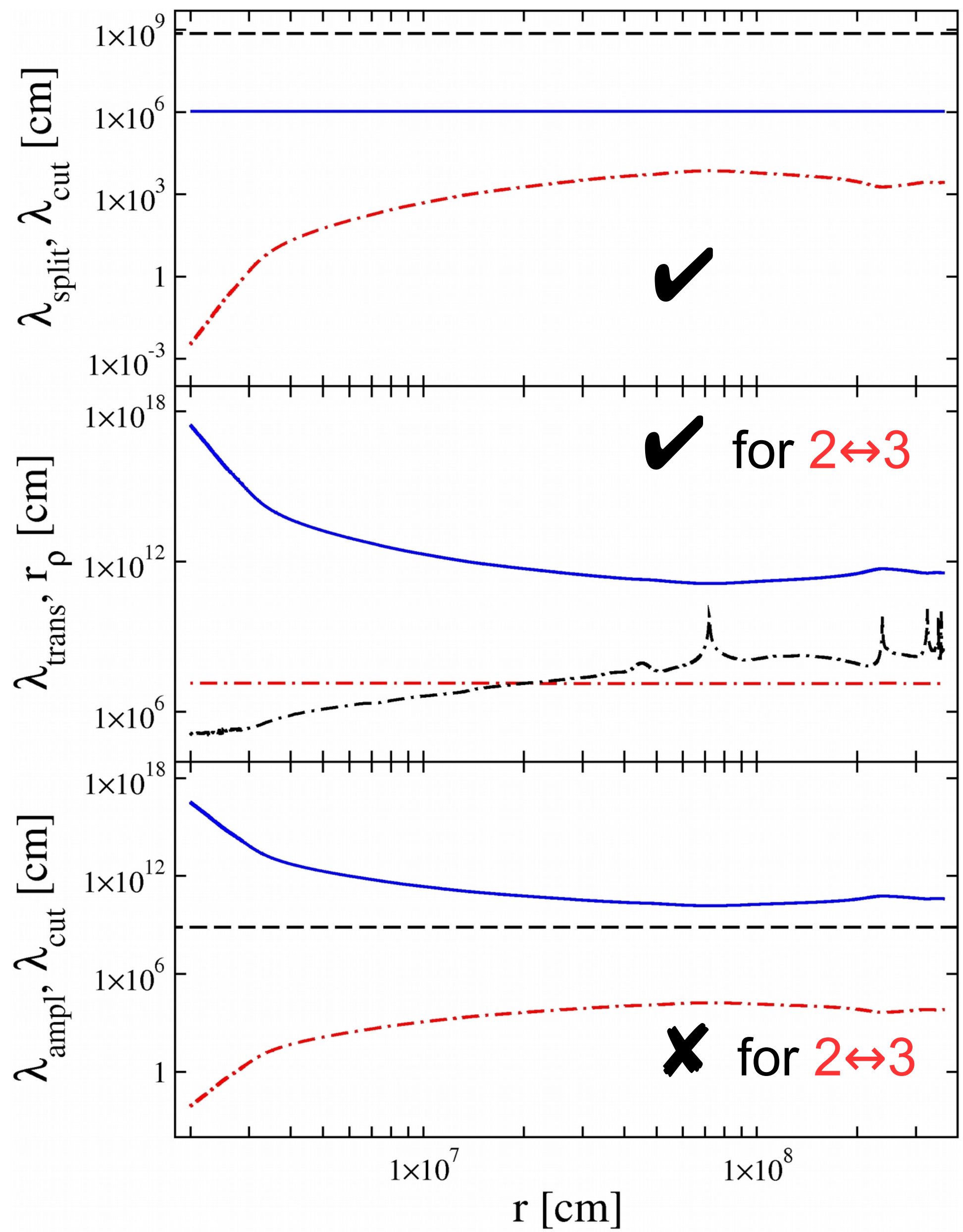
Snapshots of the 10.8 model by Fischer et al.



We use a normal hierarchy:

1↔2 blue, 2↔3 red, left panel is t = 0.3 s, right is t =0.7 s.





- The reason turbulence has no effect is because the suppression scale is so small.
  - There is no dependence upon the shape of the turbulence power spectrum in this conclusion.

$$\lambda_{ampl} = \frac{4.9096\,\pi}{\left|V_{CC}\left(|U_{ei}|^2 - |U_{ej}|^2\right)\right|}$$

- Turbulence effects emerge at low density and/or closer to an MSW resonance density.

Summary

- Turbulence effects do not occur during the accretion phase*.

*assuming self-interaction can be ignored.

We transform to the eigenbasis of H_0 using a matrix U and solve for the evolution matrix S_0 in that basis.

If H_0 is a constant then $S_0 = \exp(-i K r)$ where K is the diagonal matrix of eigenvalues of H_0,

We consider an arbitrary perturbing Hamiltonian of the form

$$\delta H^{(f)} = \sum_{a=1}^{N_q} C_a e^{-i q_a r} + C_a^\dagger e^{i q_a r}$$

In the eigenbasis of H_0, we write $S = S_0 A$ and find A evolves according to

$$i \frac{dA}{dr} = \sum_a \left\{ e^{i K r} U^\dagger \left[C_a e^{i q_a r} + C_a^\dagger e^{-i q_a r} \right] U e^{-i K r} \right\} A = H^{(A)} A$$

We pull out the diagonal elements of C_a and write them as

$$diag\left(U^{\dagger} C_a U\right) = \frac{1}{2i} e^{i\Phi_a} F_a$$

Where $\Phi_a = diag(\varphi_{a,1}, \varphi_{a,2}, \ldots)$ and $F_a = diag(F_{a,1}, F_{a,2}, \ldots)$.

We now write A as A = W B where the diagonal matrix W given by

$$W = \exp\left(-i \sum_a \Xi_a\right)$$

$$\Xi_a = \frac{F_a}{q_a}\left[\cos \Phi_a - \cos\left(\Phi_a + q_a r\right)\right]$$

Ξ_a is also a diagonal matrix: $\Xi_a = diag(\xi_{a,1}, \xi_{a,2}, \ldots)$.

The purpose of W is to remove the diagonal elements of $H^{(A)}$.

We also define the matrix G_a by

$$offdiag\left(U^{\dagger} C_a U\right) = G_a$$

The matrix B evolves according to

$$i \frac{dB}{dr} = e^{iKr + i\sum_b \Xi_b} \left(\sum_a \left\{ G_a e^{iq_a r} + G_a^{\dagger} e^{-iq_a r} \right\} \right) e^{-iKr - i\sum_b \Xi_b} B = H^{(B)} B$$

The element ij of $H^{(B)}$ is

$$H_{ij}^{(B)} = \sum_a \left\{ G_{a,ij} e^{i\left(\left[q_a + (k_i - k_j)\right] r + \sum_b \left[\xi_{b,i} - \xi_{b,j}\right]\right)} + c.c \right\}$$

The term $\exp(i[\xi_{b,i} - \xi_{b,j}])$ needs attention.

In full this term is

$$\xi_{b,i} - \xi_{b,j} = \frac{\left[F_{b,i}\cos\phi_{b,i} - F_{b,j}\cos\phi_{b,j}\right]}{q_b}\left(1 - \cos\left(q_b r\right)\right)$$

$$+ \frac{\left[F_{b,i}\sin\phi_{b,i} - F_{b,j}\sin\phi_{b,j}\right]}{q_b}\sin\left(q_b r\right)$$

which can be simplified by introducing $x_{b,ij}$ and $y_{b,ij}$, and then rewriting it using $(z_{b,ij})^2 = (x_{b,ij})^2 + (y_{b,ij})^2$ and $\tan \Psi_{b,ij} = y_{b,ij} / x_{b,ij}$

$$\xi_{b,i} - \xi_{b,j} = x_{b,ij} - z_{b,ij}\cos\left(q_b r + \psi_{b,ij}\right)$$

The term $\exp(i[\xi_{b,i} - \xi_{b,j}])$ can be expanded using Jacobi-Anger

$$e^{i\left[\xi_{b,i} - \xi_{b,j}\right]} = e^{i x_{b,ij}} \sum_{m_b = -\infty}^{+\infty} (-i)^{m_b} J_{m_b}\left(z_{b,ij}\right) e^{i m_b\left[q_b r + \psi_{b,ij}\right]}$$

And the element ij of H^(B) is

$$H_{ij}^{(B)} = -i\, e^{i[k_i - k_j]r} \sum_a \left(\sum_{m_a} \kappa_{a m_a, ij}\, e^{i m_a q_a r} \left\{ \prod_{b \neq a} \sum_{m_b} \lambda_{b m_b, ij}\, e^{i m_b q_b r} \right\} \right)$$

$$\kappa_{a m_a, ij} = (-i)^{m_a} e^{i[x_{a,ij} + m_a \psi_{a,ij}]} \left[G_{a,ij}\, e^{-i \psi_{a,ij}} J_{m_a - 1} - G_{a,ij}^{*}\, e^{i \psi_{a,ij}} J_{m_a + 1} \right]$$

$$\lambda_{b m_b, ij} = (-i)^{m_b} e^{i[x_{b,ij} + m_b \psi_{b,ij}]} J_{m_b}$$

The Hamiltonian for B looks simple ☺ but we cannot obtain a solution for B without making the Rotating Wave Approximaiton.

We assume that for each Fourier mode there is only one* important contribution to the series – n_a.

$$H_{ij}^{(B)} = -i\, e^{i[k_i - k_j]r} \sum_a \kappa_{a n_a, ij}\, e^{i n_a q_a r} \prod_{b \neq a} \lambda_{b, n_b, ij}\, e^{i n_b q_b r}$$

This Hamiltonian has an exact solution for any number of Fourier modes and any number of neutrino flavors.

For the case of two flavors:

$$B=\begin{pmatrix} e^{ipr}\left[\cos Qr - i\frac{p}{Q}\sin Qr\right] & -i\,e^{ipr}\frac{\kappa}{Q}\sin Qr \\ -i\,e^{-ipr}\frac{\kappa^*}{Q}\sin Qr & e^{-ipr}\left[\cos Qr + i\frac{p}{Q}\sin Qr\right] \end{pmatrix}$$

where

$$\kappa=\sum_a \kappa_{a,n_a}\prod_{b\neq a}\lambda_{b,n_b}$$

$$2\mathrm{p}=k_1-k_2+\sum_a n_a q_a$$

$$Q^2=p^2+\kappa^2$$

The transition probability in the eigenbasis of H_0 is

$$P_{12}=\frac{\kappa^2}{Q^2}\sin^2(Qr)$$