Instabilities in collective neutrino transformations

Huaiyu Duan

University of New Mexico

U.S. DEPARTMENT OF ENERGY | Office of Science

Introduction

- ~10^{58} neutrinos in ~10 seconds => dense neutrino medium => collective neutrino flavor transformation, a quantum collective phenomenon mediated by the weak force on scales ~10-1000 km.
- Based on Standard Model physics.
- Can affect many aspects of SN physics (neutrino signals, nucleosynthesis, dynamics? …).

Neutrino Flavor Transport in a Dense Medium

$$(\partial_t + \hat{\mathbf{v}} \cdot \boldsymbol{\nabla})\rho = -\mathrm{i}[\mathsf{H},\, \rho] + \mathcal{C}$$

mass matrix

electron density

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2} G_{\mathrm{F}}\, \mathrm{diag}[n_e, 0, 0] + \mathsf{H}_{\nu\nu}$$

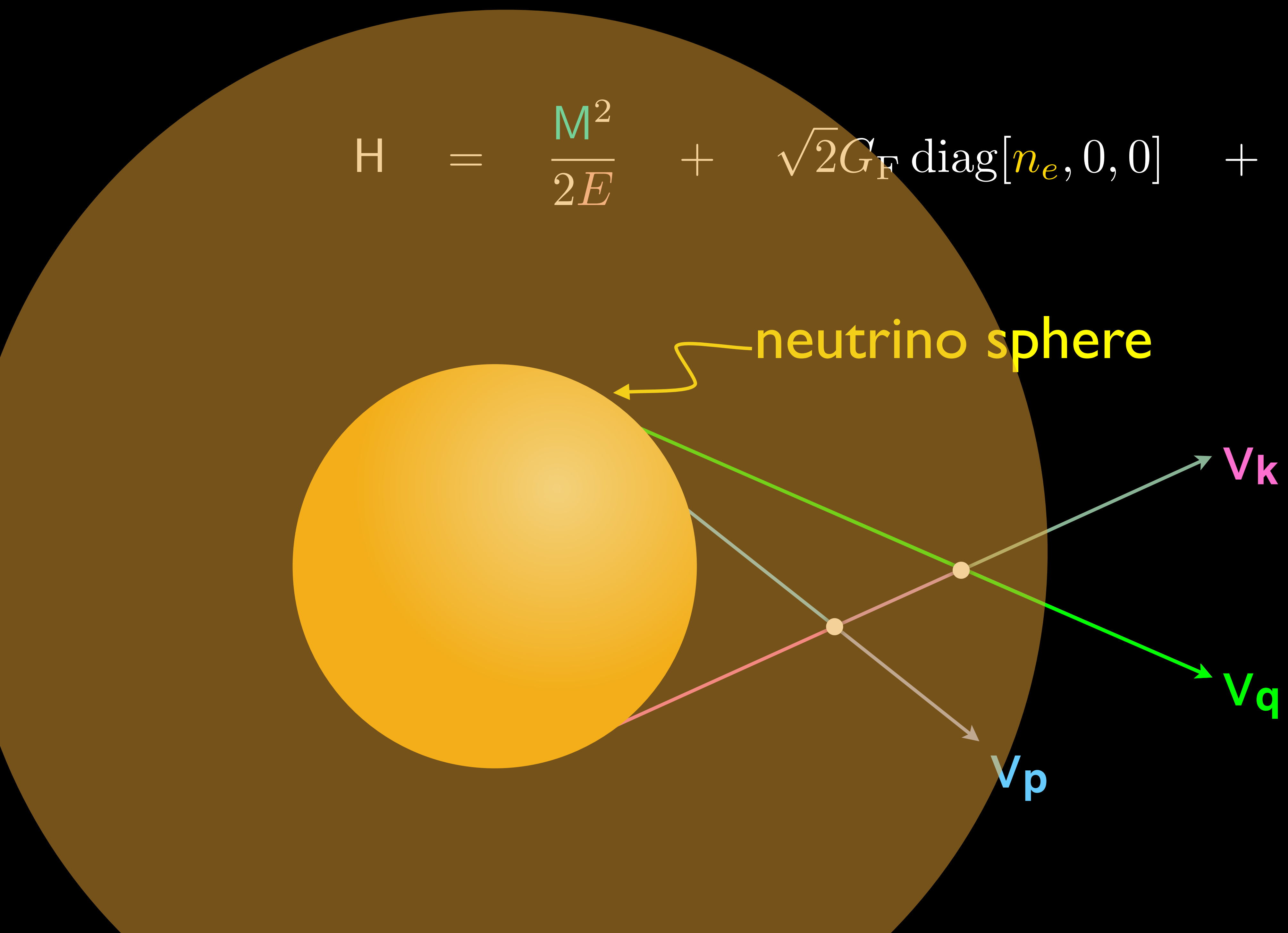
neutrino energy

v-v forward scattering
(self-coupling)

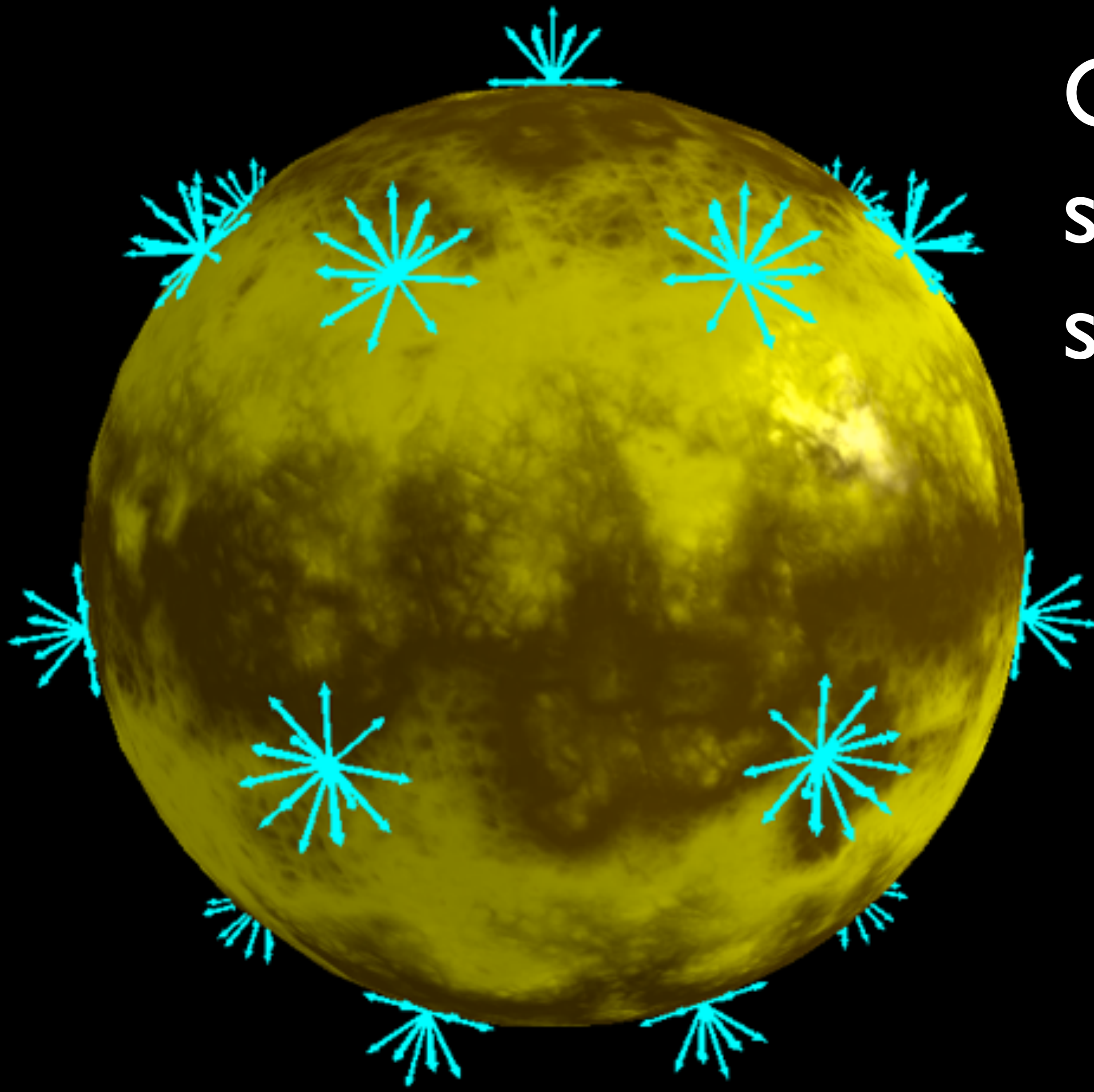
$$\mathsf{H}_{\nu\nu} = \sqrt{2} G_{\mathrm{F}} \int \mathrm{d}^3\mathbf{p}' (1 - \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}')(\rho_{\mathbf{p}'} - \bar{\rho}_{\mathbf{p}'})$$

Oscillations in SN

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_{\mathrm{F}}\,\mathrm{diag}[n_e, 0, 0] + \mathsf{H}_{\nu\nu}$$



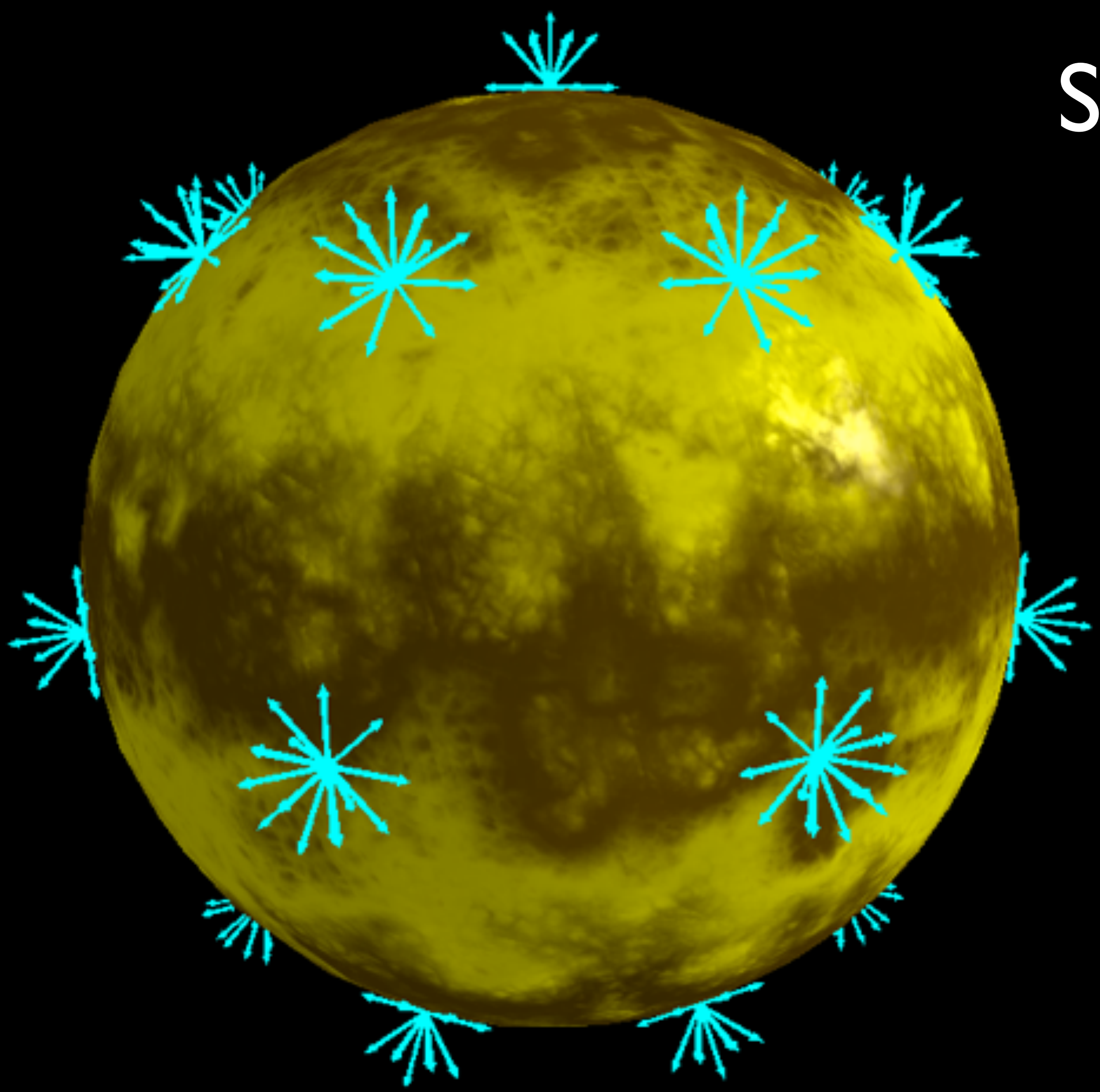
Numerical Models



Coherent forward scattering outside neutrino sphere

$$\rho(t; r, \Theta, \Phi; E, \vartheta, \varphi)$$

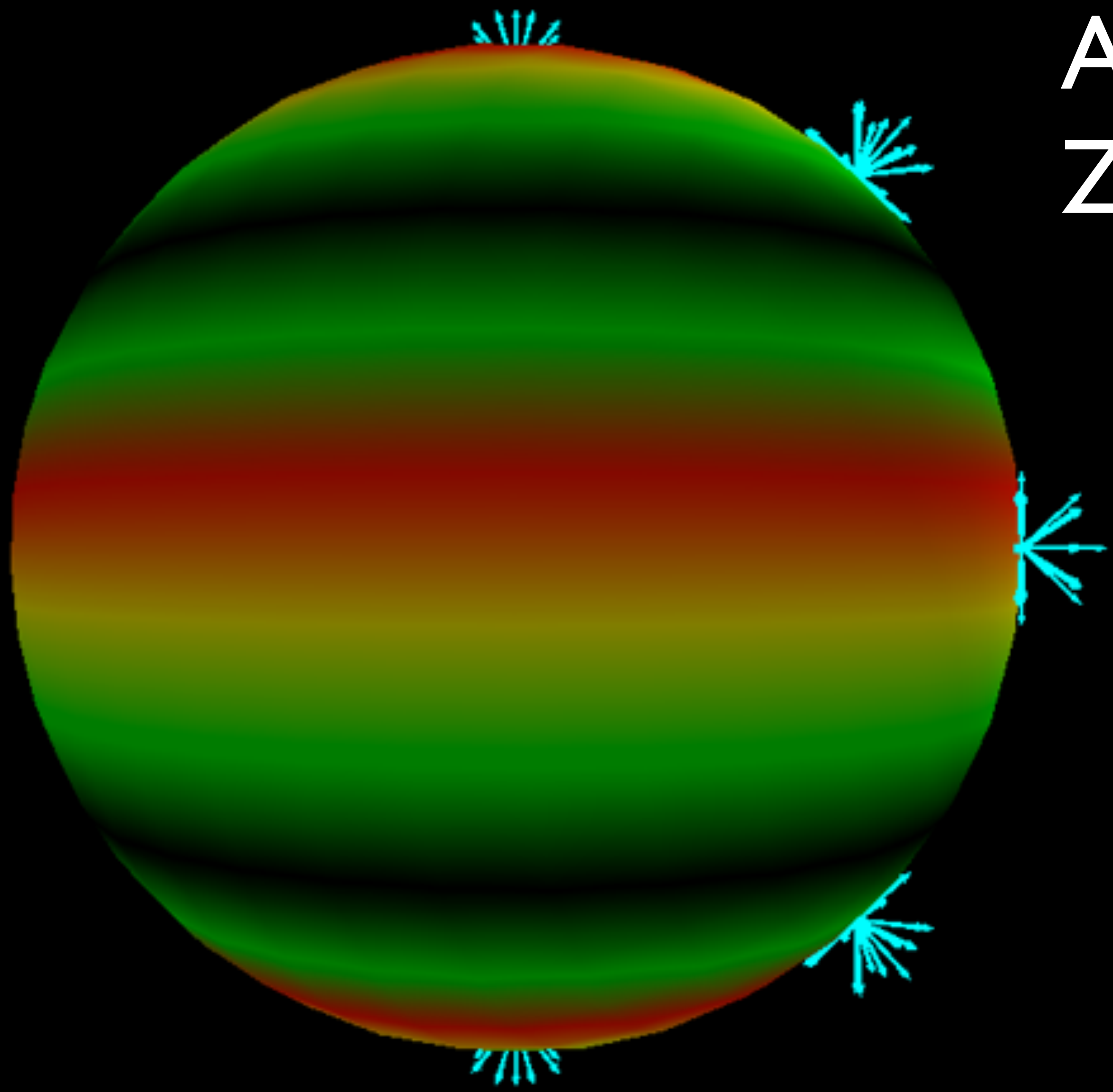
Numerical Models



Stationary emission

$$\rho(r, \Theta, \Phi; E, \vartheta, \varphi)$$

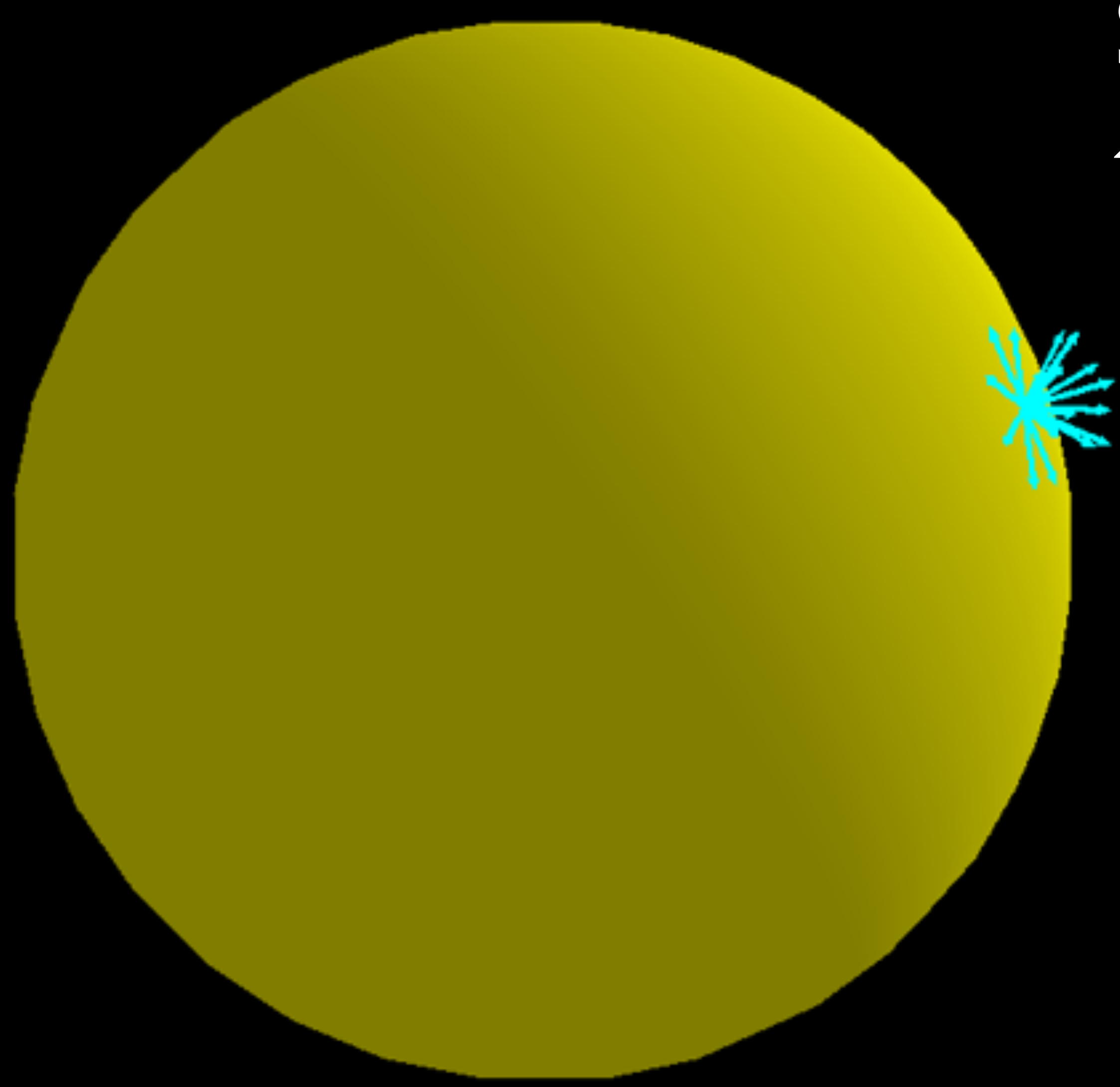
Numerical Models



Axial symmetry around the Z axis

$$\rho(r, \Theta; E, \vartheta, \varphi)$$

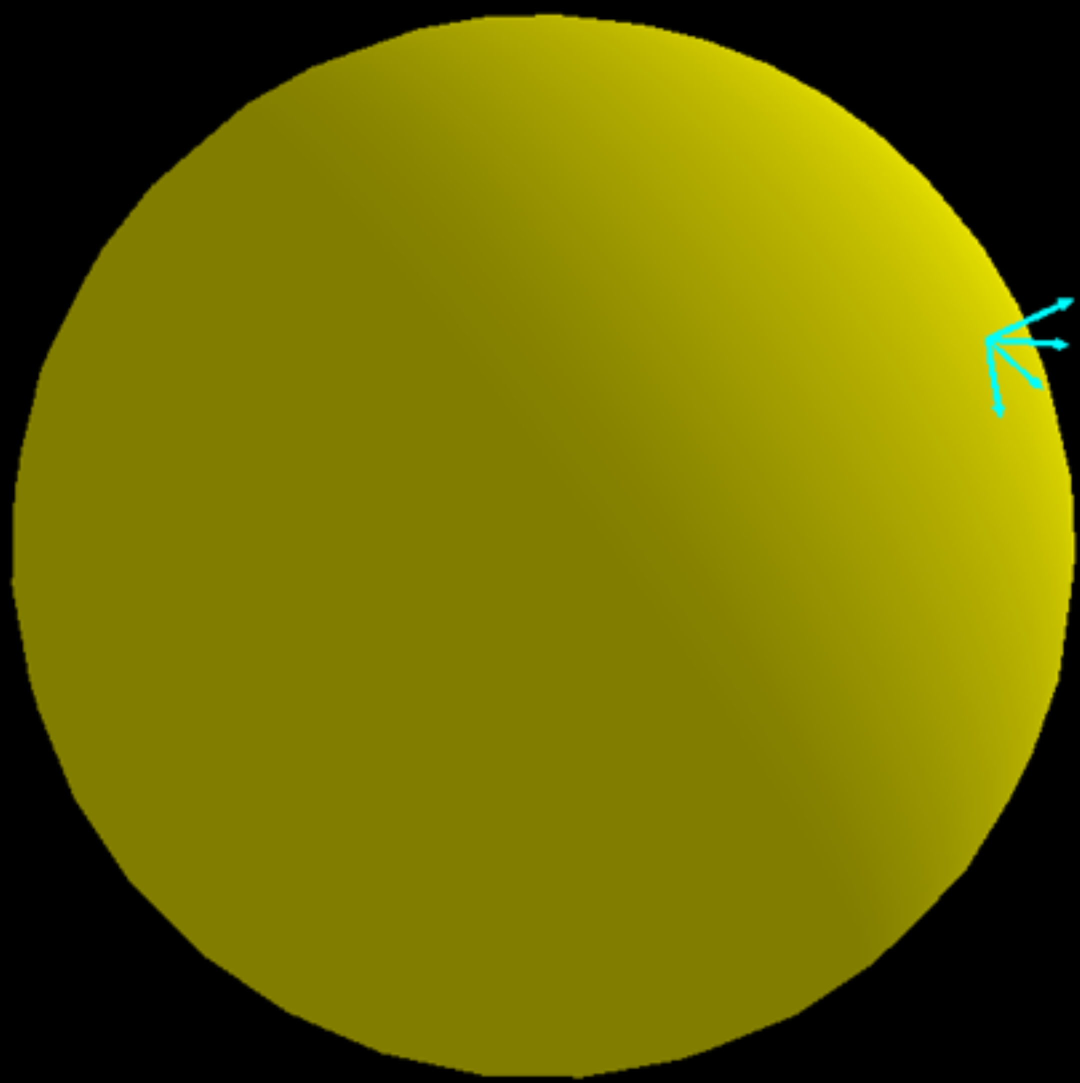
Numerical Models



Spherical symmetry about the center (inconsistent?)

$$\rho(r; E, \vartheta, \varphi)$$

Numerical Models

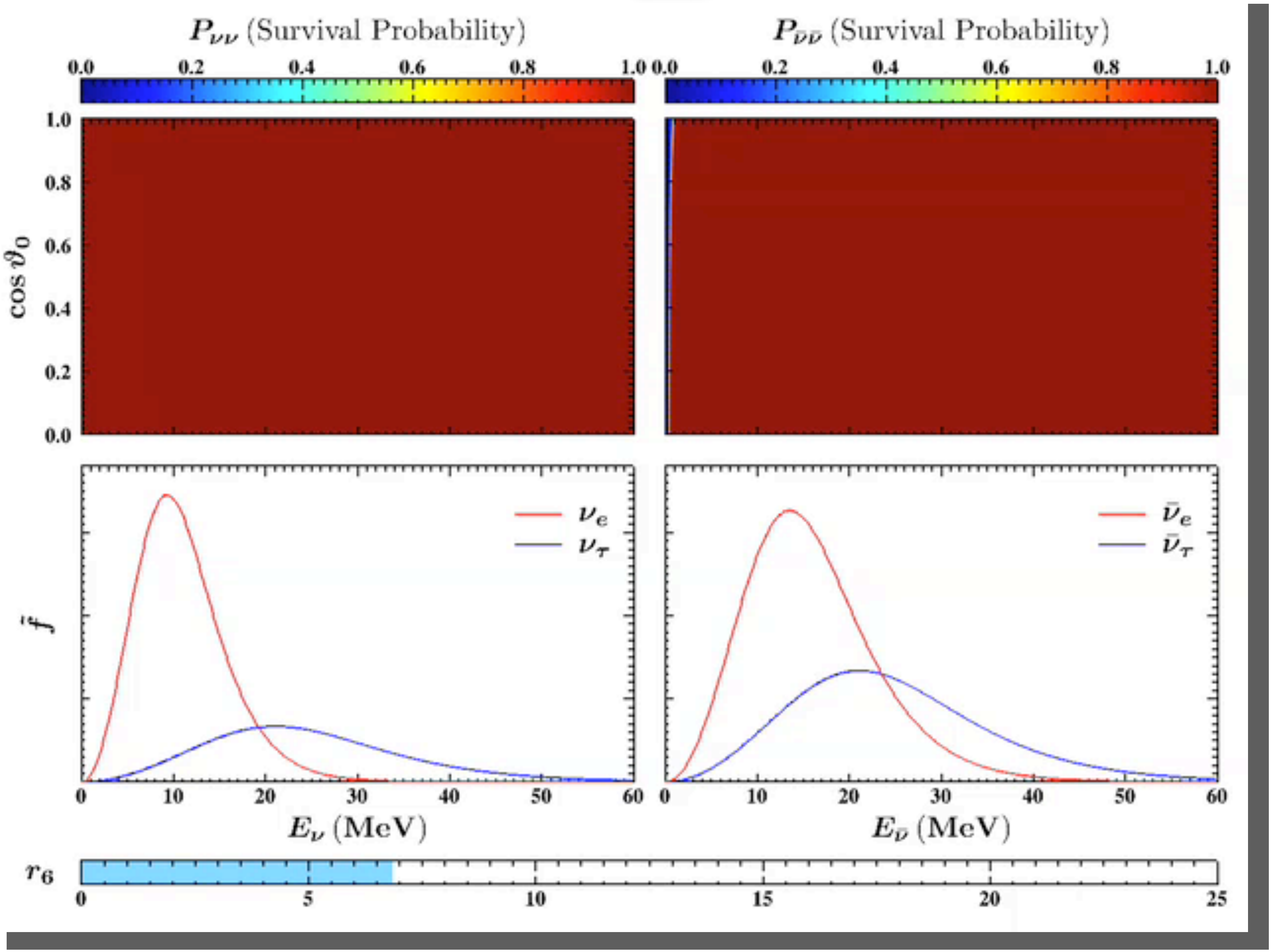


Azimuthal symmetry around any radial direction

$$\rho(r; E, \vartheta)$$

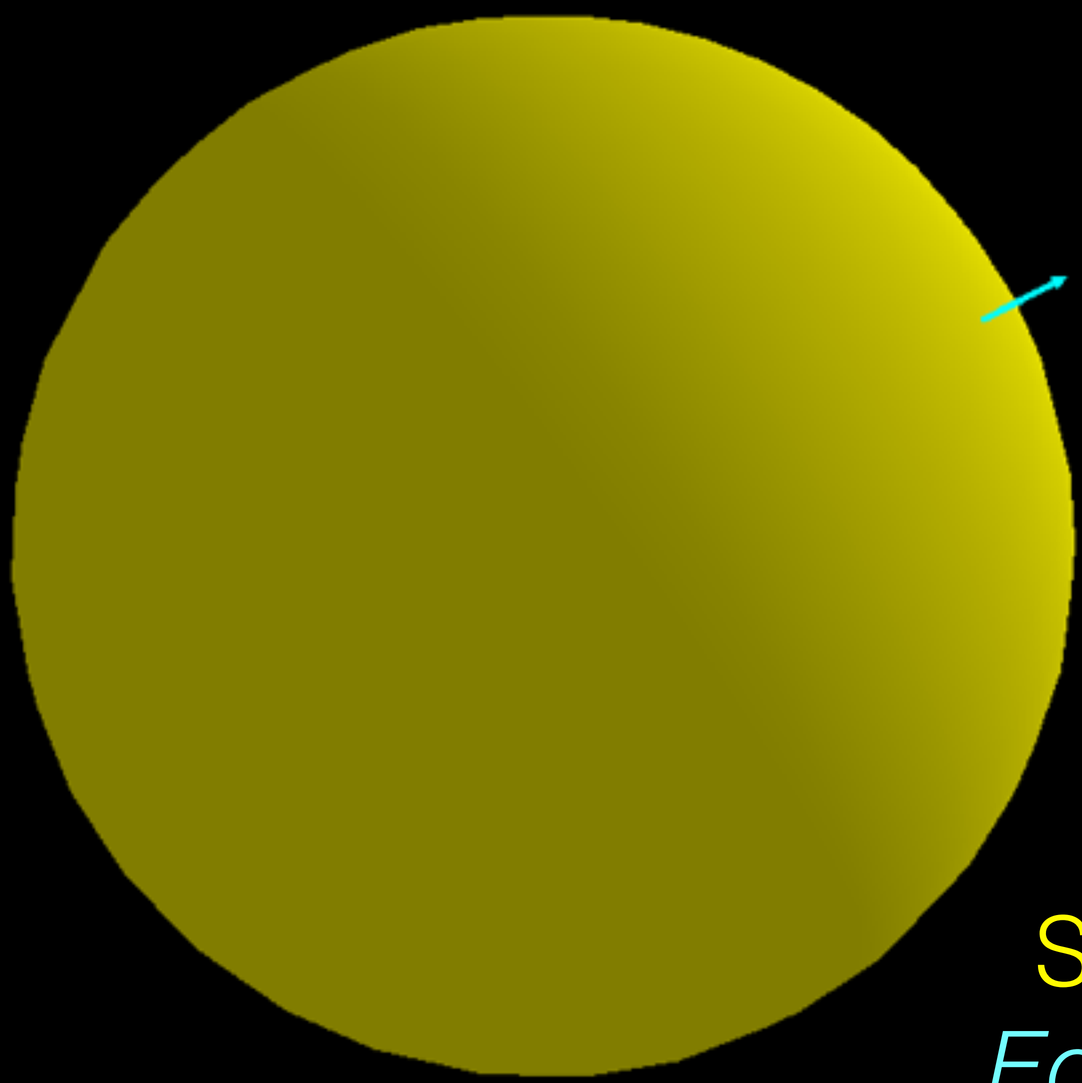
Bulb model

$$\delta m^2 = -3 \times 10^{-3}\,\mathrm{eV}^2 \simeq \delta m^2_{\mathrm{atm}},\ \theta_{\mathrm{v}} = 0.1,\ L_\nu = 10^{51}\,\mathrm{erg/s}$$



Duan, Fuller, Carlson & Qian (2006)

Numerical Models

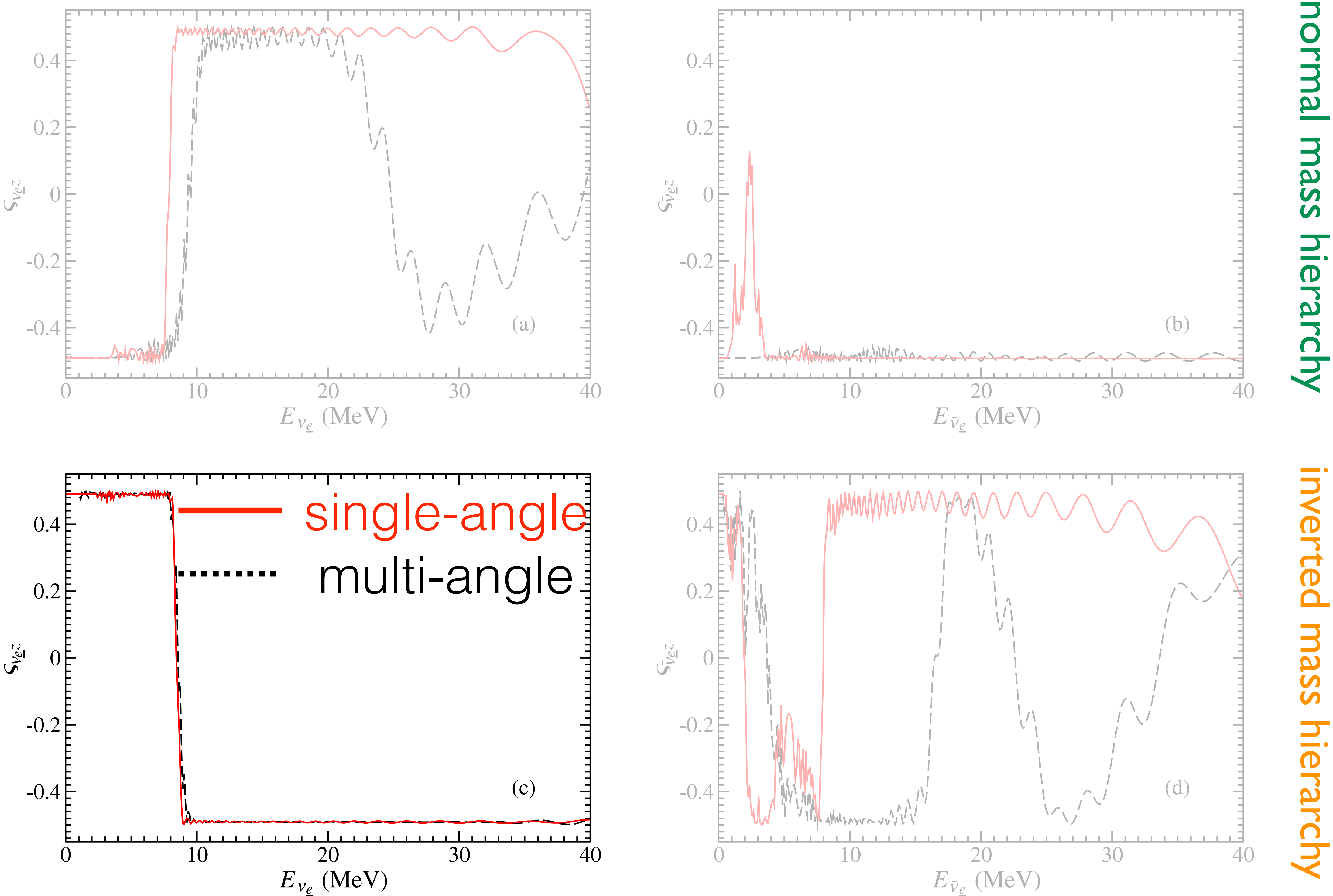


Trajectory independent
neutrino flavor evolution

$$\rho(r; E)$$

Single-angle model

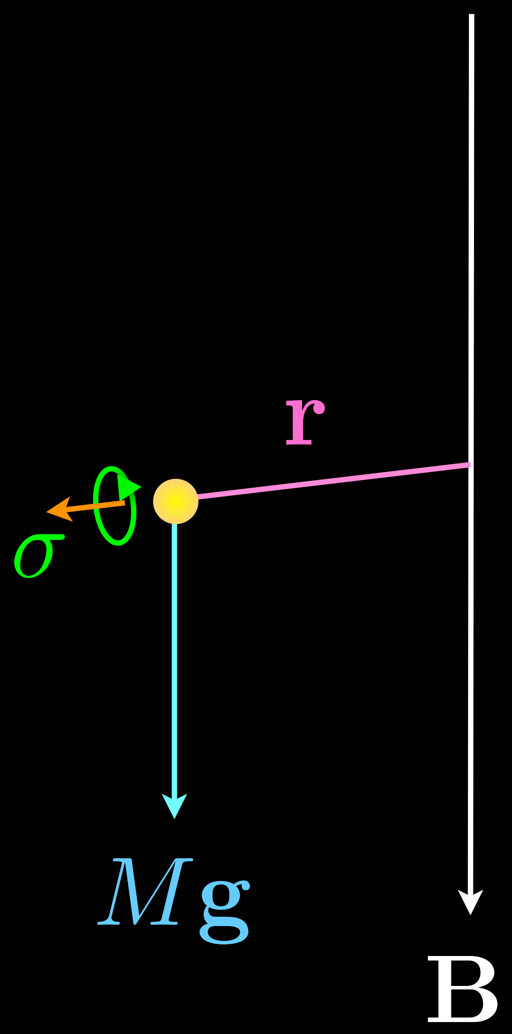
Equivalent to the expansion of a homogeneous, isotropic gas neutrino antineutrino

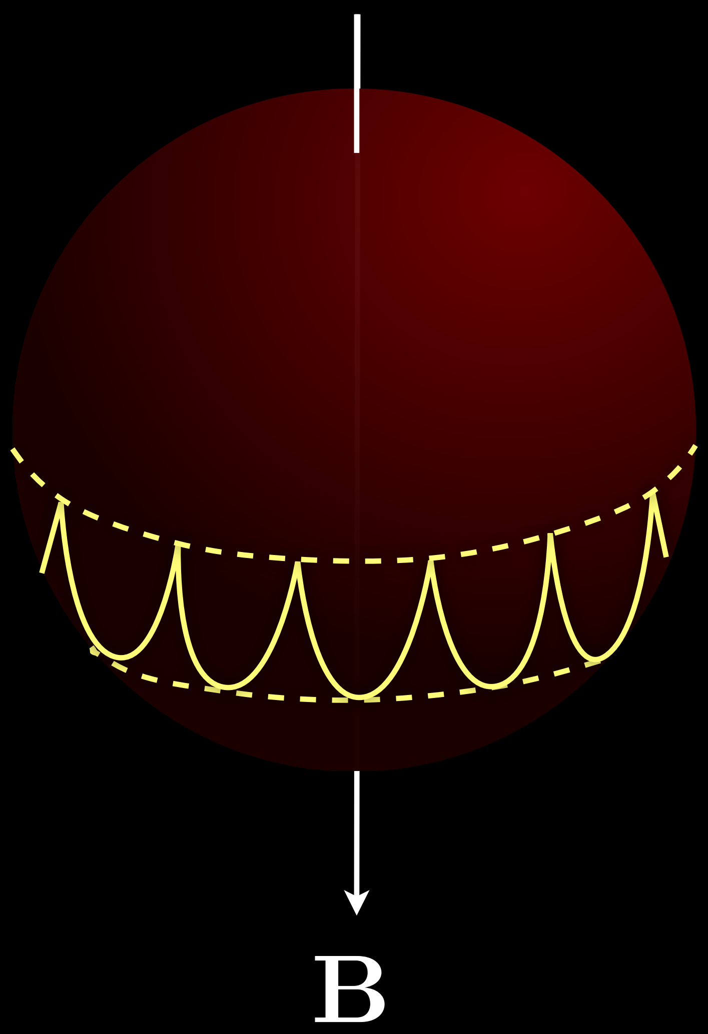


Duan+ (2006)

Flavor pendulum

Mono-energetic ν-$\bar{\nu}$ gas





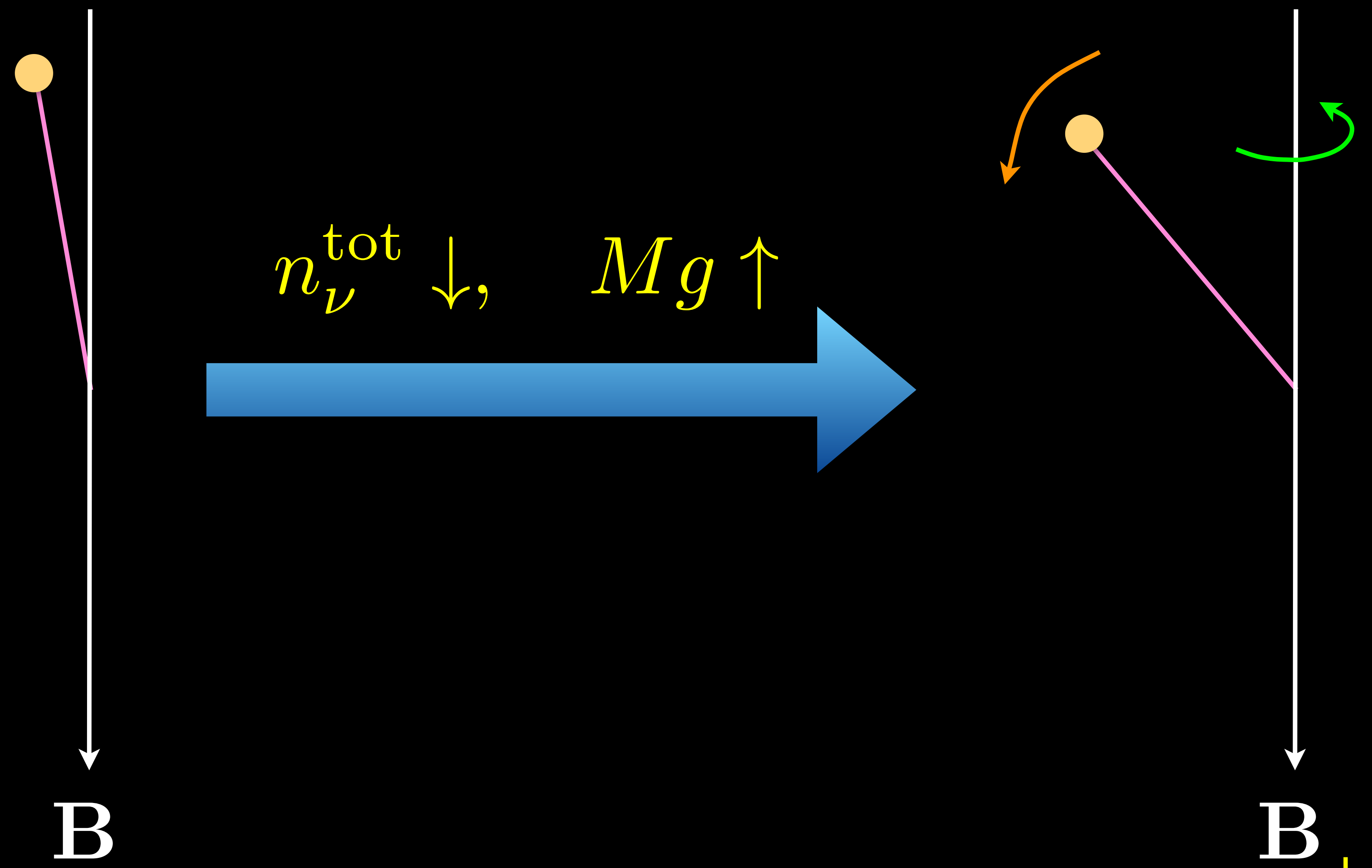
$$\sigma \sim \frac{n_\nu - n_{\bar{\nu}}}{n_\nu + n_{\bar{\nu}}}$$

$$M\mathbf{g} \sim \frac{\mathbf{B}}{n_\nu + n_{\bar{\nu}}}$$

Hannestad+ (2006)
Duan+ (2007)

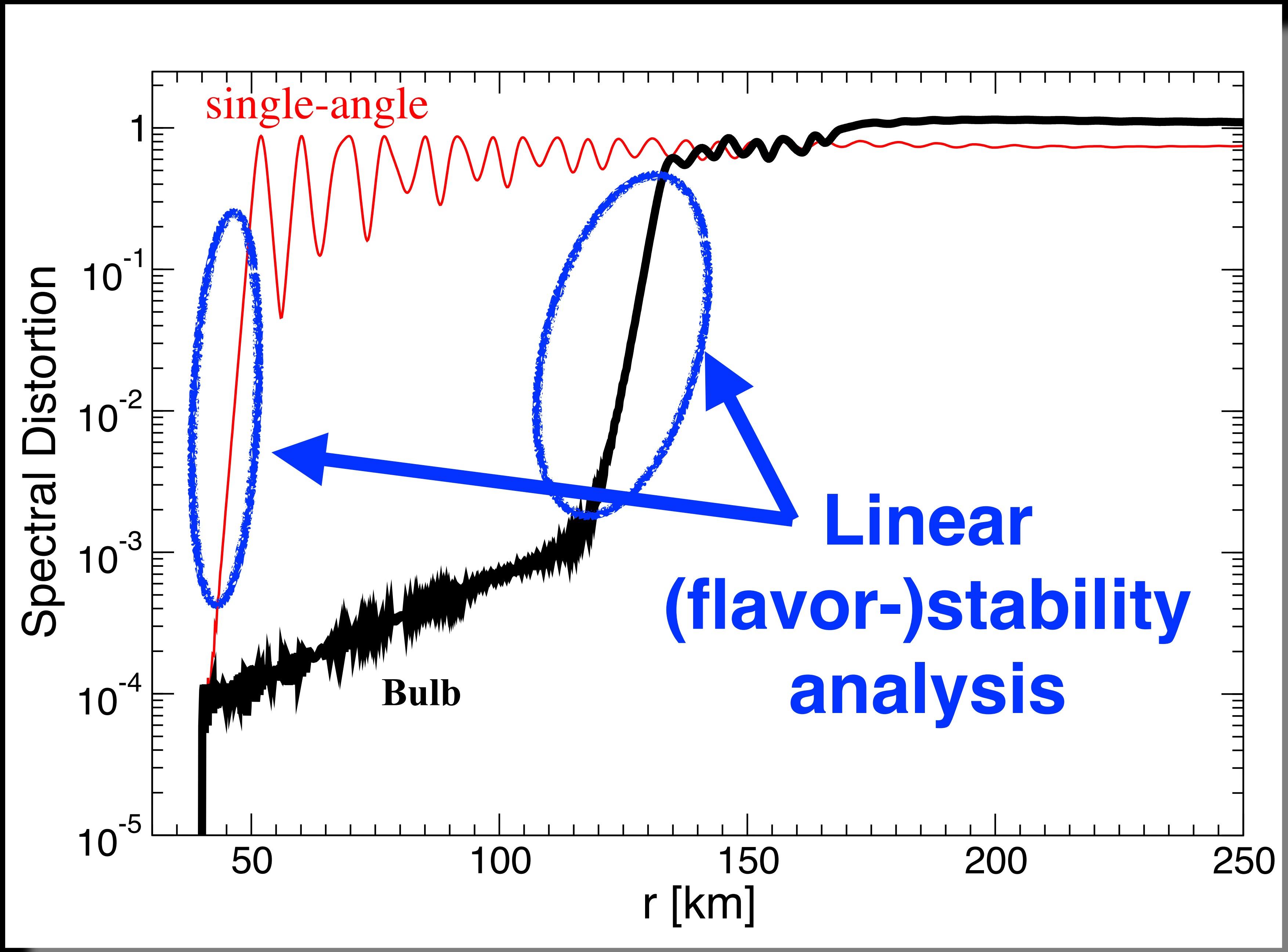
Flavor pendulum

Inverted Mass Hierarchy



Hannestad+ (2006)
Duan+ (2007)

Dimension matters



Duan & Friedland (2010)

Flavor Instabilities

Electron flavor neutrinos and antineutrinos initially

$$\rho \propto \begin{bmatrix} 1 & \epsilon \\ \epsilon^* & 0 \end{bmatrix} \qquad \bar{\rho} \propto \begin{bmatrix} 1 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$$

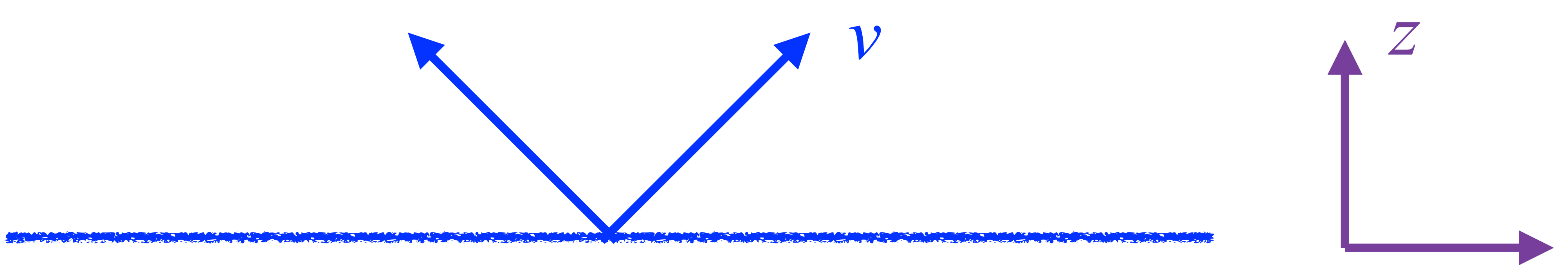
$$\mathrm{i}\partial_z \begin{bmatrix} \epsilon \\ \bar{\epsilon} \end{bmatrix} = v_z^{-1} \begin{bmatrix} -\eta\omega - \cancel{\alpha\mu} & \cancel{\alpha\mu} \\ -\cancel{\mu} & \eta\omega + \cancel{\mu} \end{bmatrix} \begin{bmatrix} \epsilon \\ \bar{\epsilon} \end{bmatrix}$$

η Hierarchy

ω Osc. Freq.

$\alpha = n_{\bar{\nu}} / n_{\nu}$

~~$\mu \propto n_{\nu}$~~



Banerjee+ (2011), Duan (2015)

Flavor Instabilities

Electron flavor neutrinos and antineutrinos initially

$$\rho \propto \begin{bmatrix} 1 & \epsilon \\ \epsilon^* & 0 \end{bmatrix} \qquad \bar{\rho} \propto \begin{bmatrix} 1 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$$

$$\mathrm{i}\partial_z \begin{bmatrix} \epsilon \\ \bar{\epsilon} \end{bmatrix} = v_z^{-1} \begin{bmatrix} -\eta\omega - \alpha\mu & \alpha\mu \\ -\mu & \eta\omega + \mu \end{bmatrix} \begin{bmatrix} \epsilon \\ \bar{\epsilon} \end{bmatrix}$$

η Hierarchy

ω Osc. Freq.

$\alpha = n_{\bar{\nu}}/n_{\nu}$

$\mu \propto n_{\nu}$

- Normal modes —> Collective oscillations $(\epsilon, \bar{\epsilon} \sim e^{-\mathrm{i}\Omega z})$
- κ=Im(Ω) > 0 —> Flavor instabilities

Banerjee+ (2011), Duan (2015)

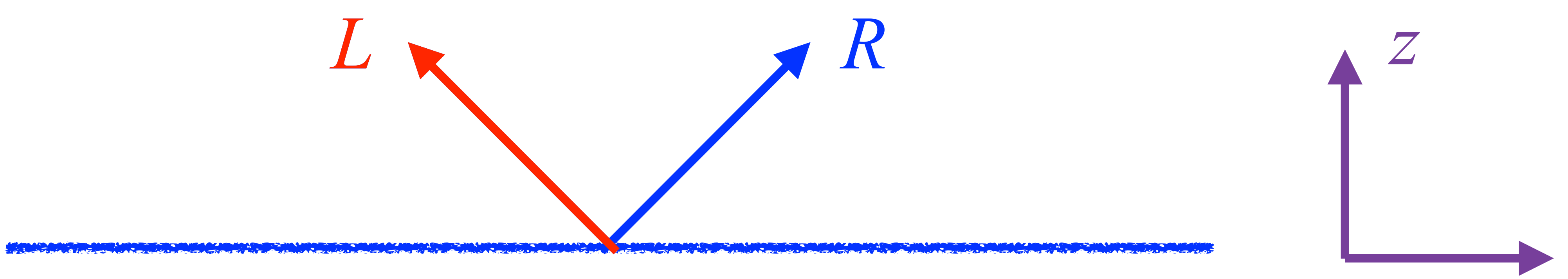
Directional Symmetry

time independent, x translation symmetry, ~~left-right symmetry~~

$$\epsilon_\pm = \frac{\epsilon_L \pm \epsilon_R}{2}$$

$$\mathrm{i}\partial_z \begin{bmatrix} \epsilon_+ \\ \bar{\epsilon}_+ \\ \epsilon_- \\ \bar{\epsilon}_- \end{bmatrix} = \begin{bmatrix} \Lambda_+ & \\ & \Lambda_- \end{bmatrix} \begin{bmatrix} \epsilon_+ \\ \bar{\epsilon}_+ \\ \epsilon_- \\ \bar{\epsilon}_- \end{bmatrix}$$

Break left-right symmetry



Raffelt+ (2013), Duan (2015)

Directional Symmetry

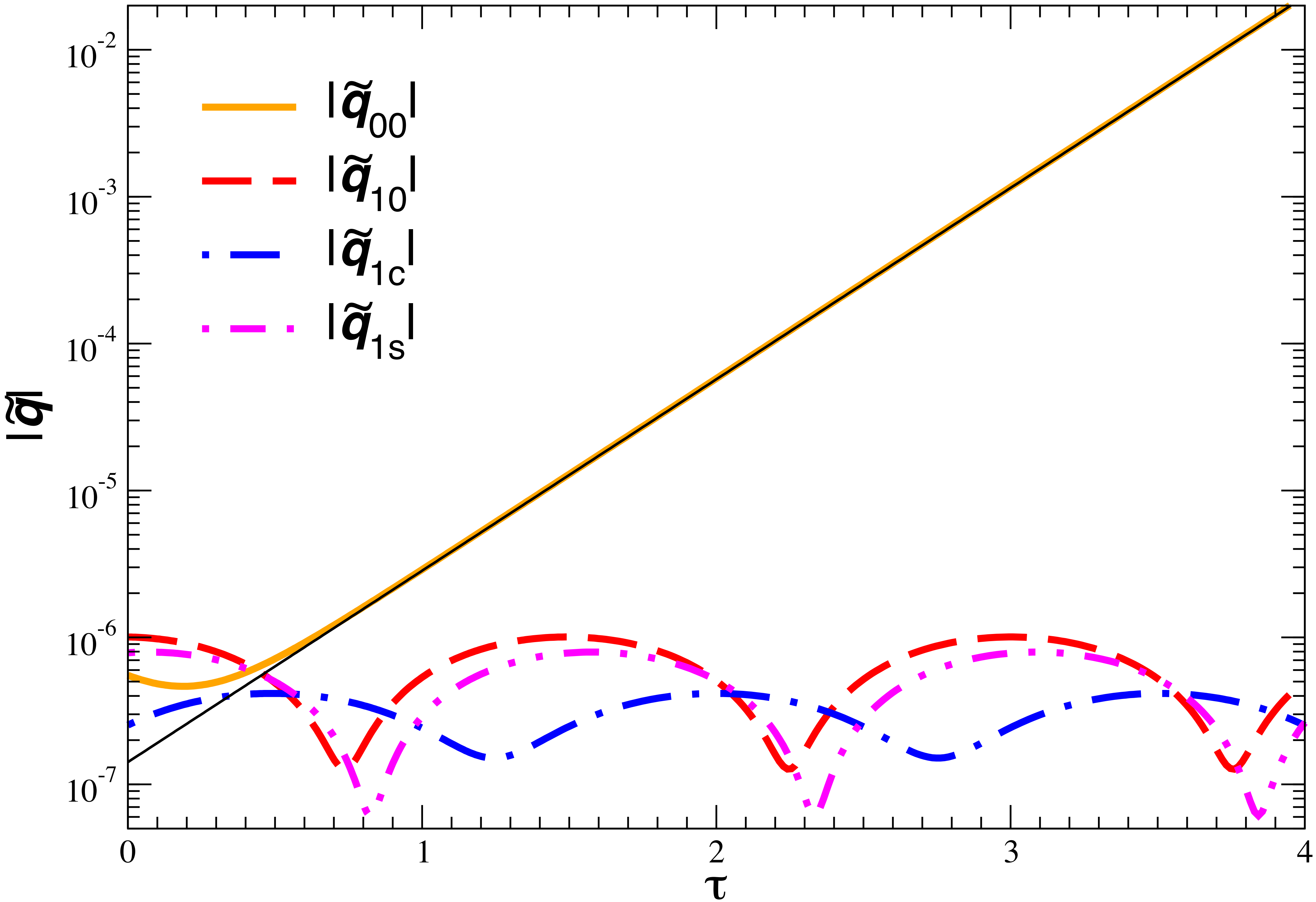
$$\mathsf{H}_{\nu\nu} = \sqrt{2}G_\mathrm{F}\int \mathrm{d}^3\mathbf{p}'(1-\hat{\mathbf{v}}\cdot\hat{\mathbf{v}}')(\rho_{\mathbf{p}'}-\bar{\rho}_{\mathbf{p}'})$$

$$(1-\hat{\mathbf{v}}\cdot\hat{\mathbf{v}}') = 4\pi\left[Y_{0,0}(\hat{\mathbf{v}})Y^*_{0,0}(\hat{\mathbf{v}}') - \frac{1}{3}\sum_{m=0,\pm1} Y_{1,m}(\hat{\mathbf{v}})Y^*_{1,m}(\hat{\mathbf{v}}')\right]$$

- Monopole (l=0) and dipole (l=1) modes are unstable in opposite neutrino mass hierarchies.
- Unstable dipole (l=1) modes break the directional symmetry.

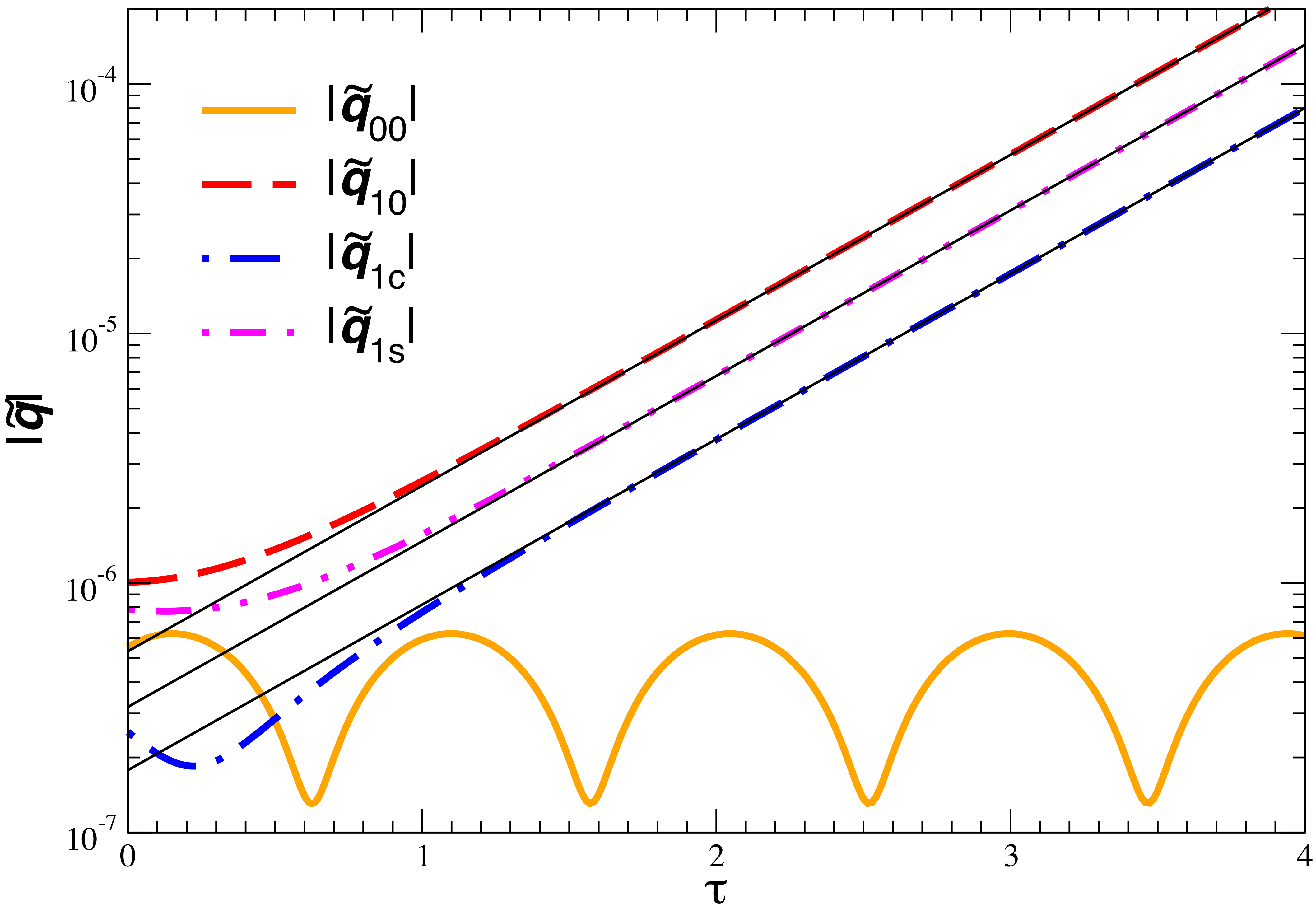
Duan (2013)

Inverted Hierarchy



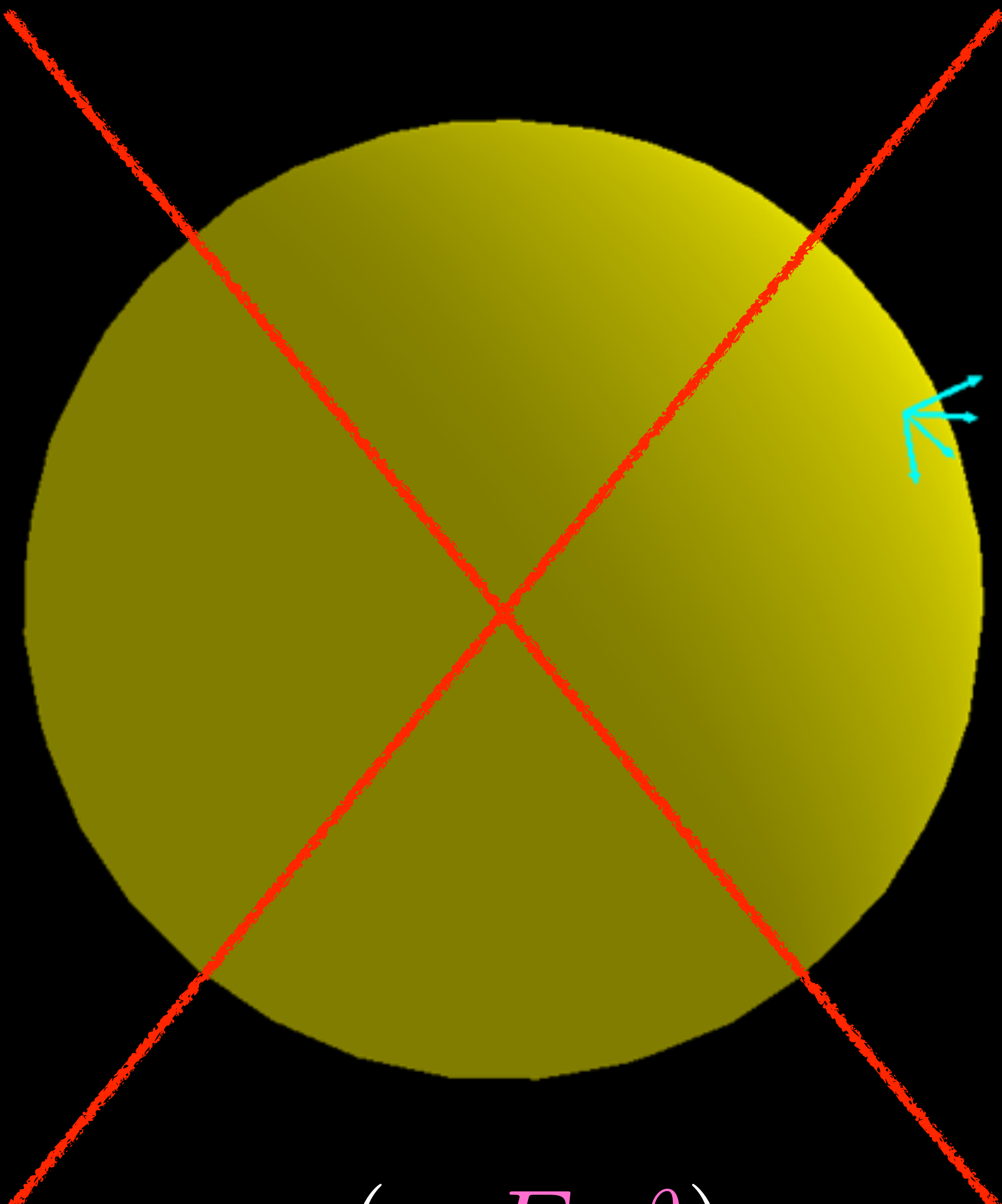
Duan (2013)

Normal Hierarchy

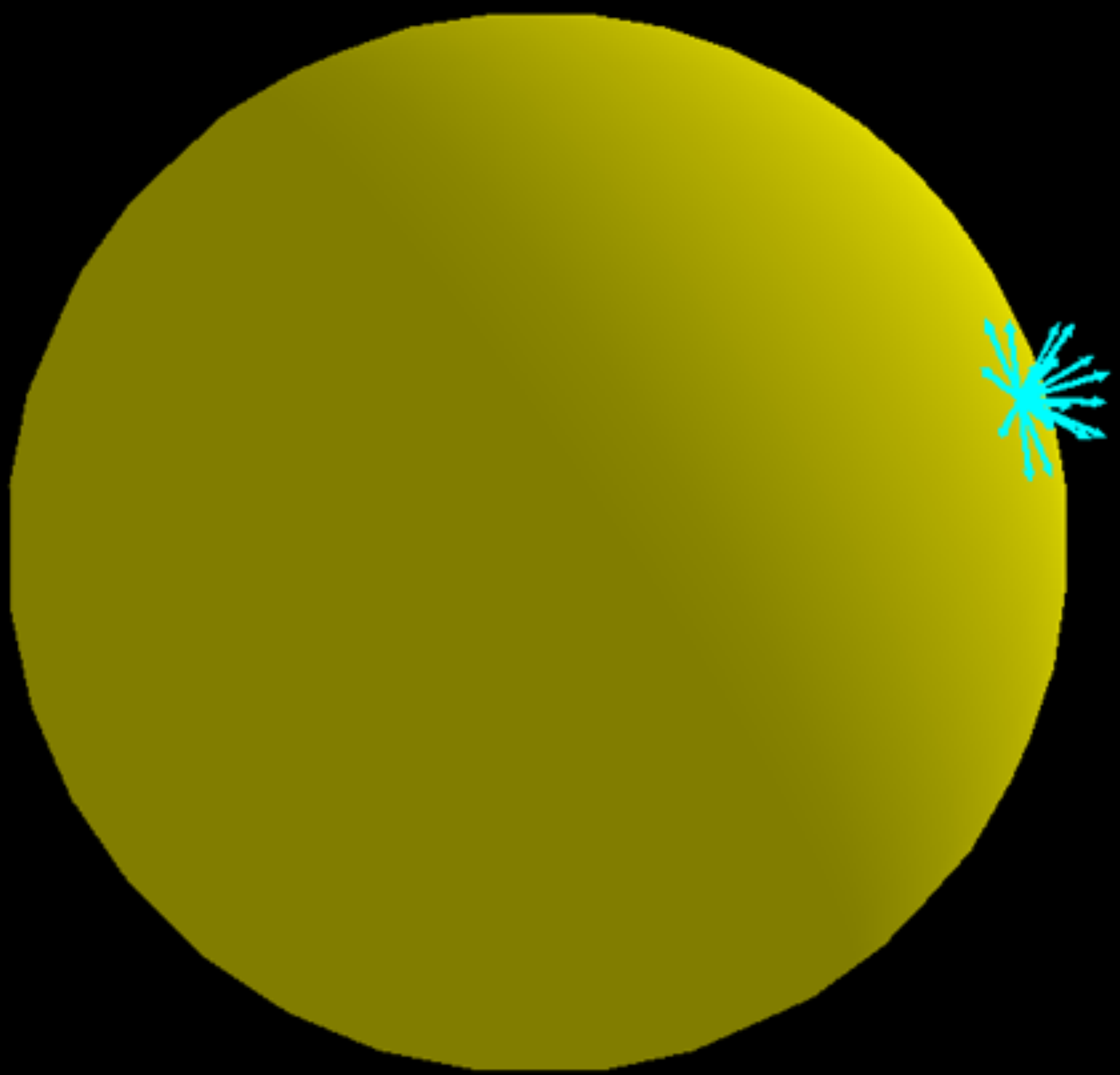


Duan (2013)

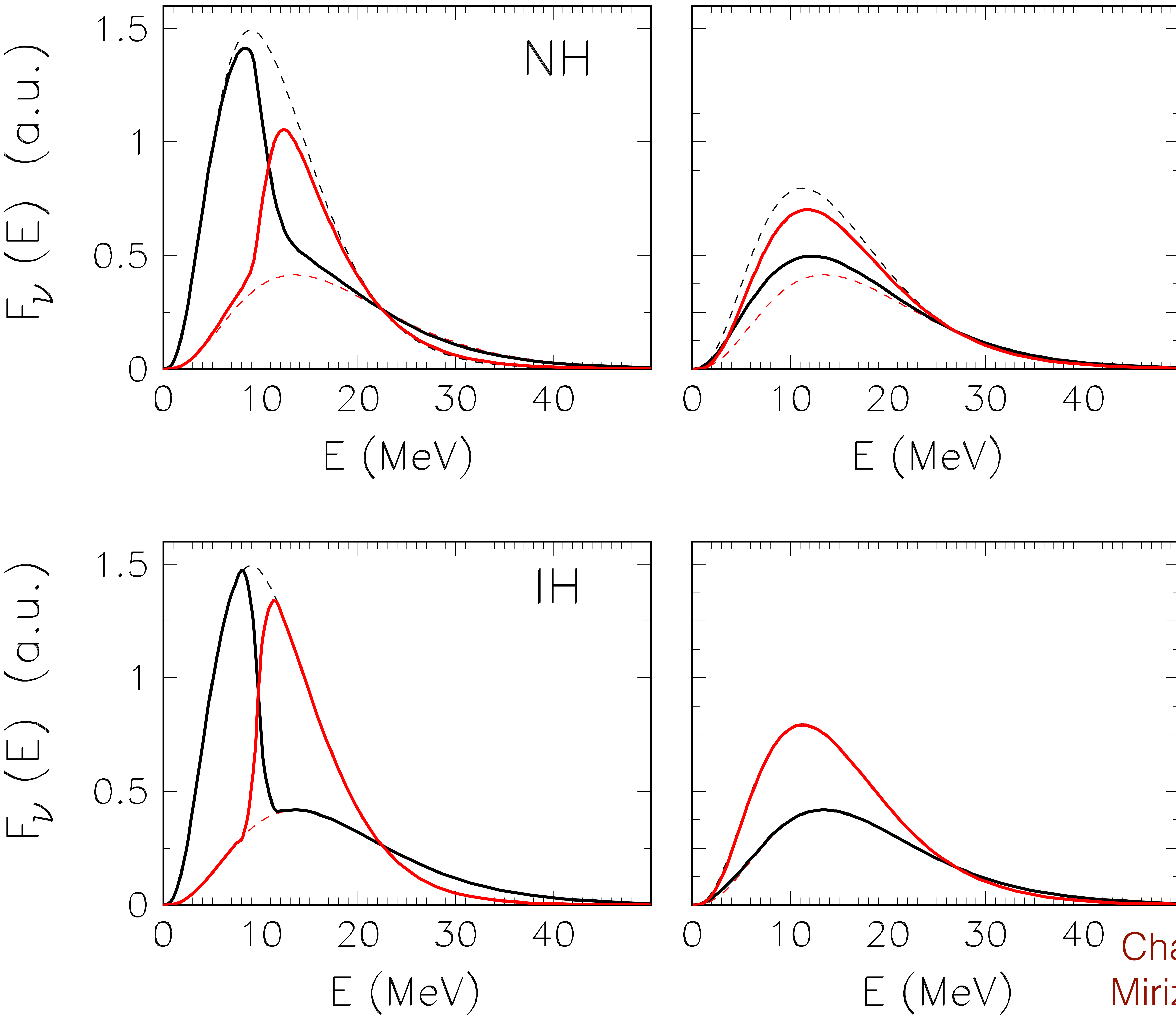
Directional Symmetry



$\rho(r; E, \vartheta)$



$\rho(r; E, \vartheta, \varphi)$



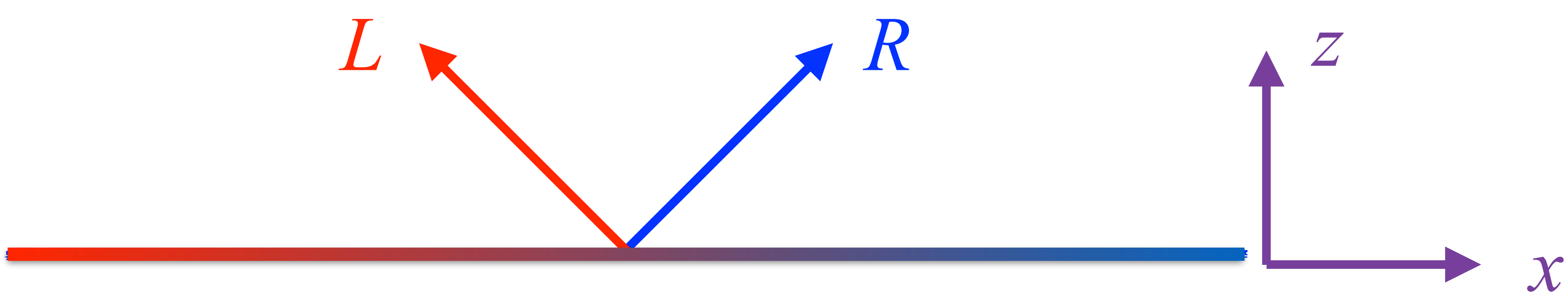
Chakraborty, Mirizzi (2013)

Spatial Symmetry

time independent, ~~x translation symmetry, left-right symmetry~~

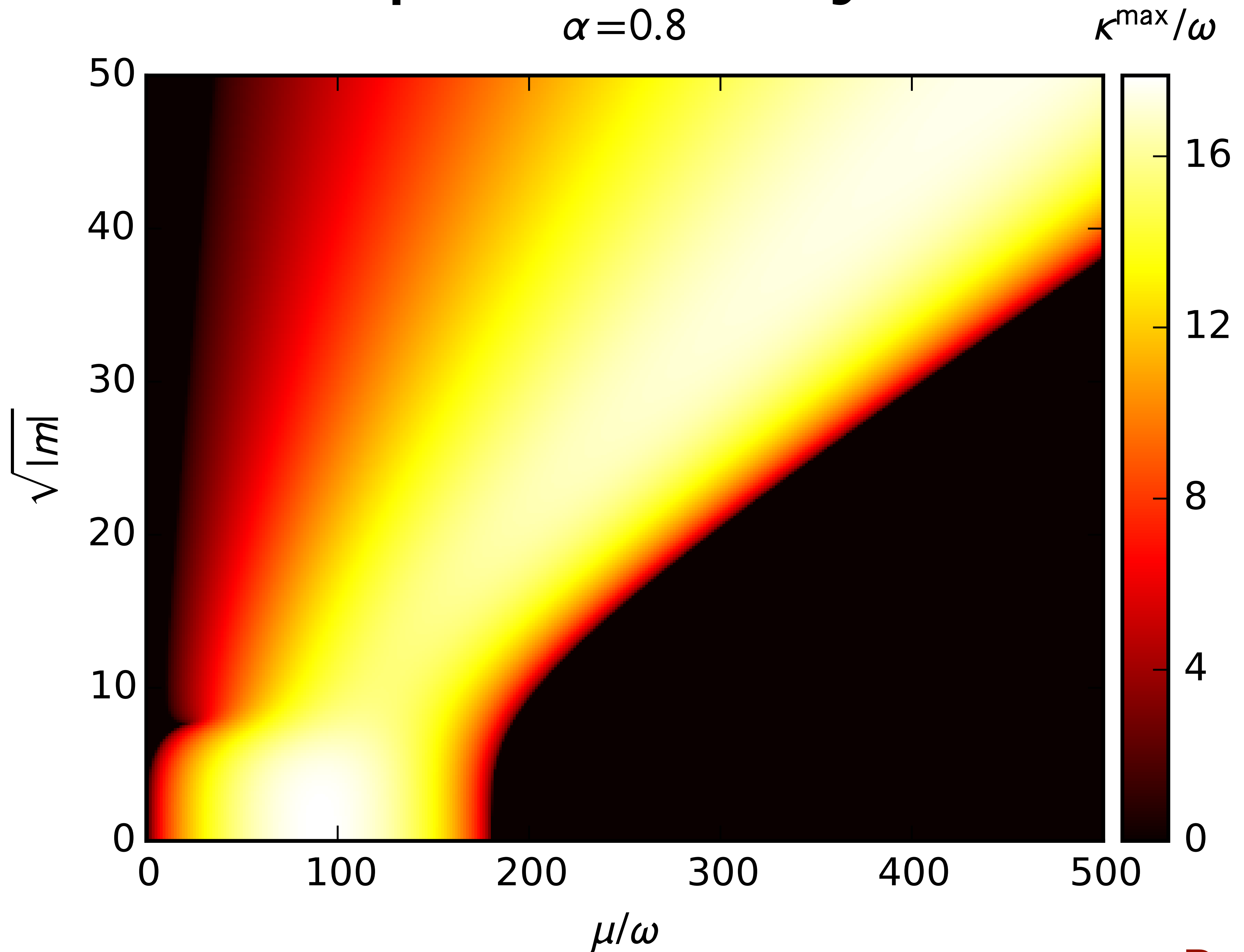
$$\epsilon_m^{\pm} = \frac{1}{L}\int_0^L \left[\frac{\epsilon_L(x) \pm \epsilon_R(x)}{2}\right] e^{-\mathrm{i}(2m\pi)(x/L)}\,\mathrm{d}x$$

$$\mathrm{i}\partial_z \begin{bmatrix} \epsilon_m^+ \\ \bar{\epsilon}_m^+ \\ \epsilon_m^- \\ \bar{\epsilon}_m^- \end{bmatrix} = \Lambda_m \cdot \begin{bmatrix} \epsilon_m^+ \\ \bar{\epsilon}_m^+ \\ \epsilon_m^- \\ \bar{\epsilon}_m^- \end{bmatrix}$$



Duan & Shalgar (2015)

Spatial Symmetry

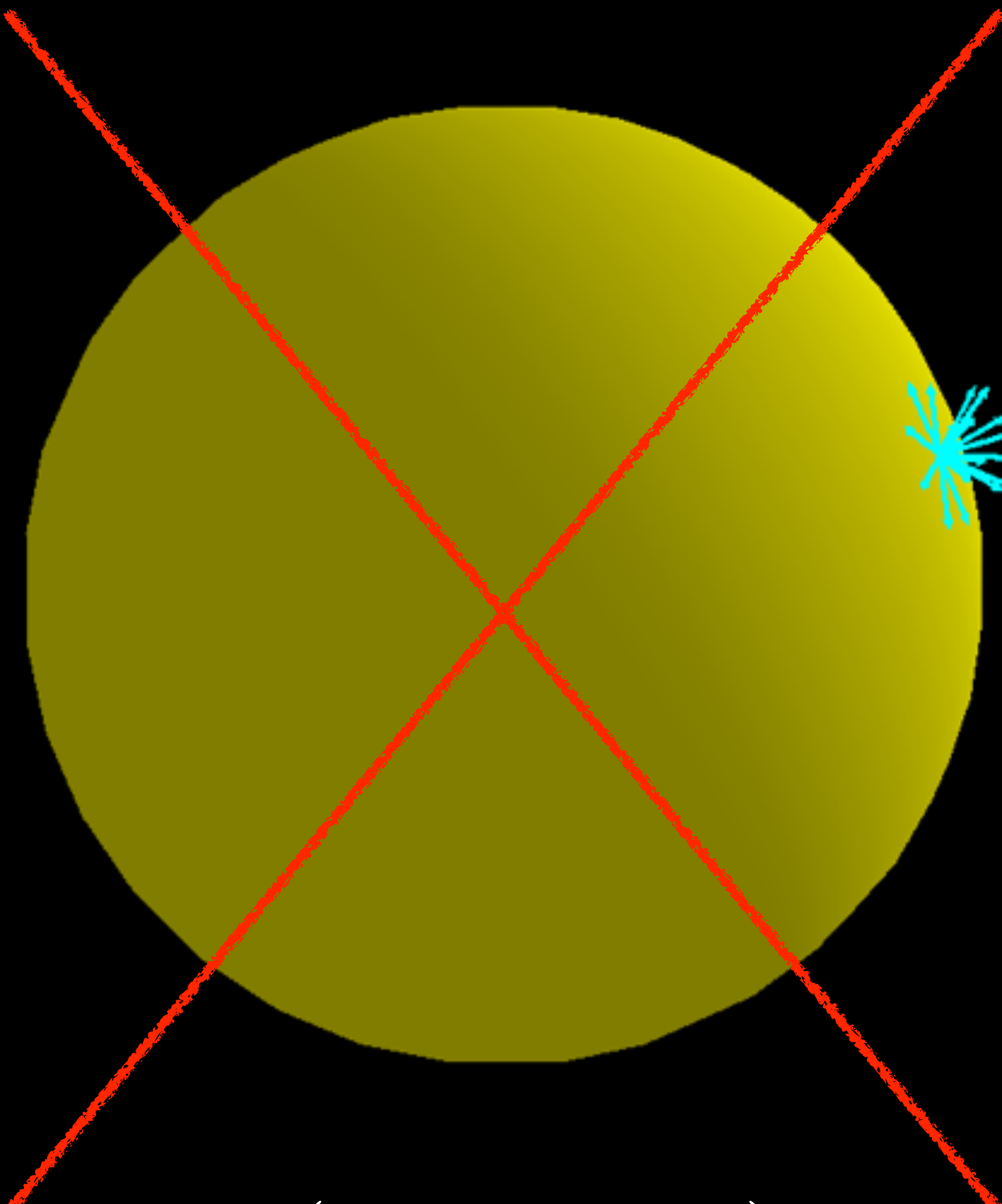


$\alpha = n_{\bar{\nu}}/n_{\nu}$

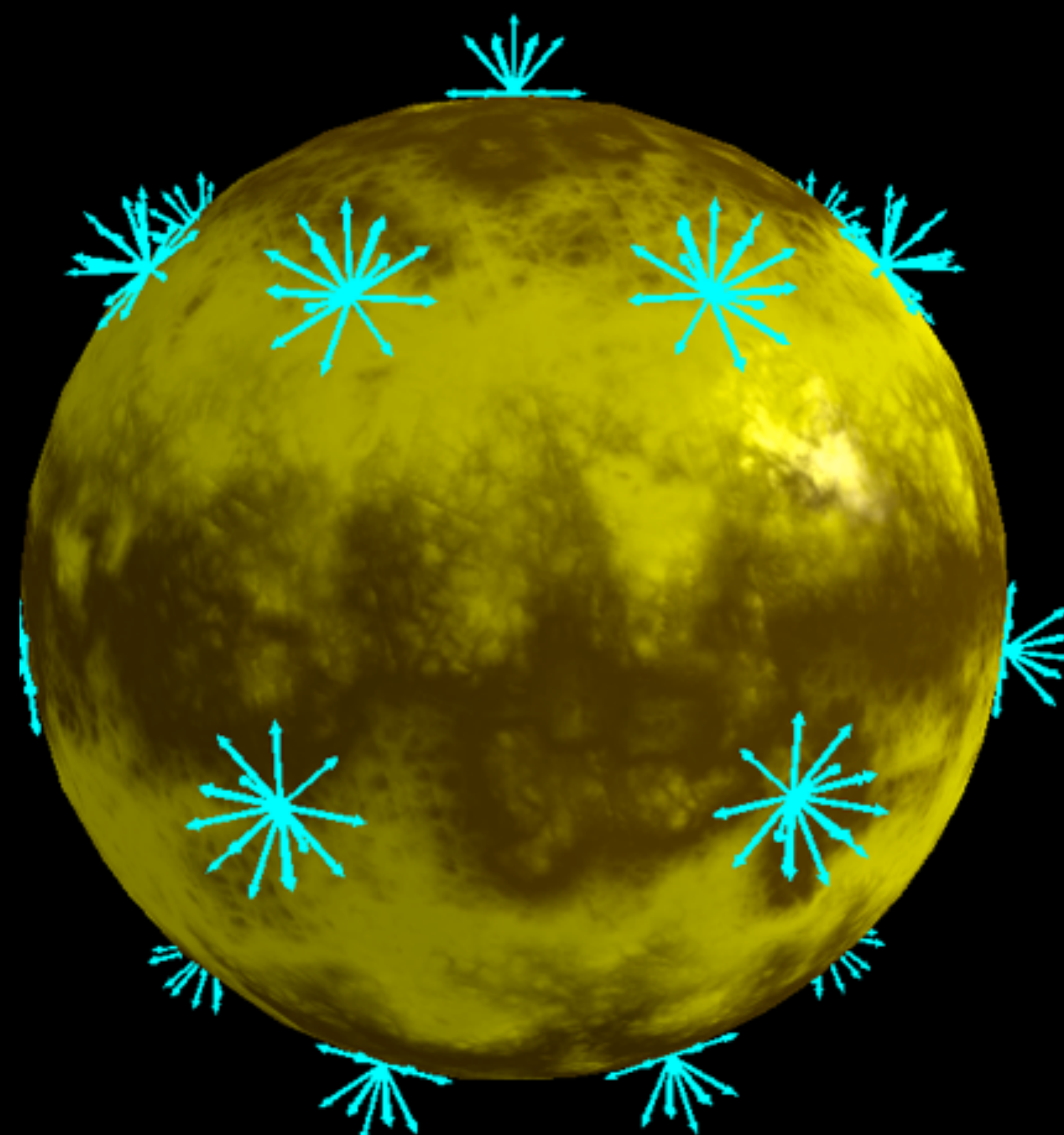
$\mu \propto G_{\mathrm{F}} n_{\nu}$

Duan & Shalgar (2015)

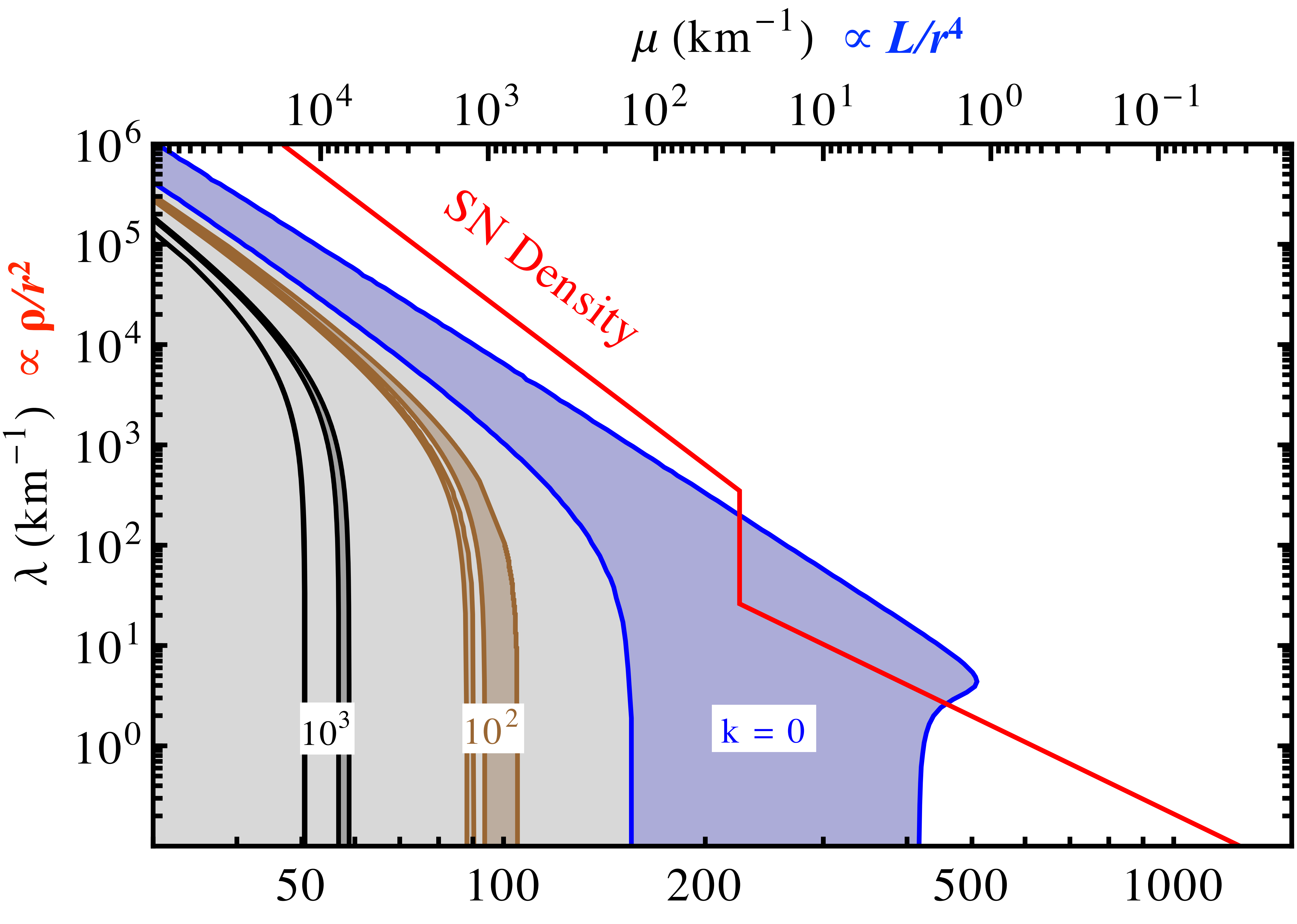
Spatial Symmetry



$\rho(r; E, \vartheta, \varphi)$

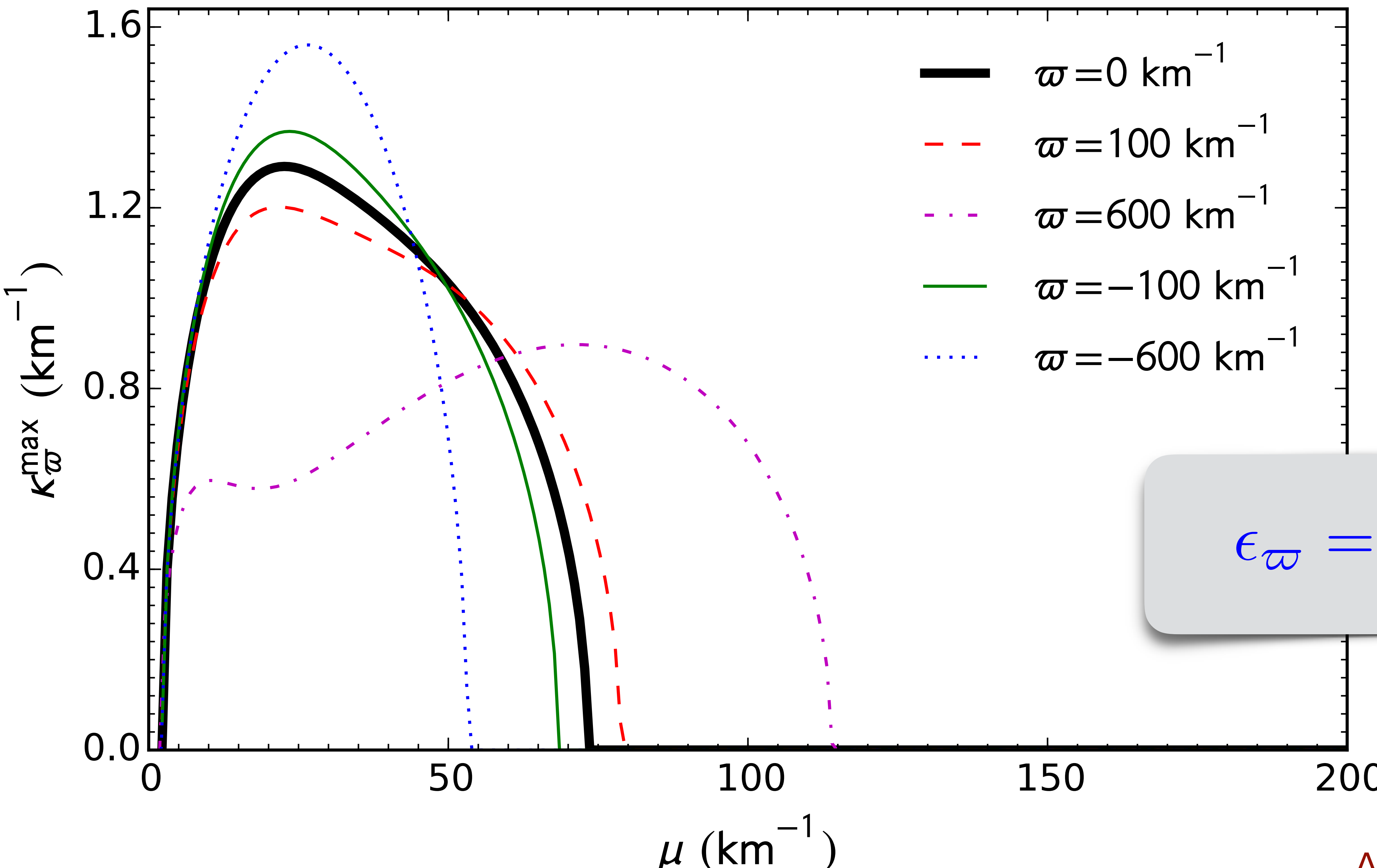


$\rho(r, \Theta, \Phi; E, \vartheta, \varphi)$



Chakraborty+ (2015)

Temporal Symmetry



$$\epsilon_{\varpi} = \int \epsilon(t)\, e^{\mathrm{i}\varpi t}\, \mathrm{d}t$$

Abbar & Duan (2015)
Dasgupta & Mirizzi (2015)

Summary

- Research of collective neutrino oscillations in SNe is at a major turning point.
  - Linear flavor-stability analysis suggests nontrivial results in multi-D models.
  - The existence of “neutrino halo” (Cherry?) and fast modes (Raffelt) can further enrich the problem.
  - Simulations with multi-spatial dimensions are necessary.