INT Program INT-16-2b, The Phases of Dense Matter
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Constraining the symmetry energy and effective mass splitting from HICs

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INT, Seattle, Aug. 9, 2016

Outline

1, Symmetry energy constraints and its uncertainty

2, Sensitivity study of parameter *x* and observables *O* in transport model

3, Constraints on L, ms* and fi from the HIC observables

- CI-DR(n/p) at 120AMeV, R_{diff} at 35, 50AMeV

4, Summary and outlook

Isospin asymmetric Equation of State

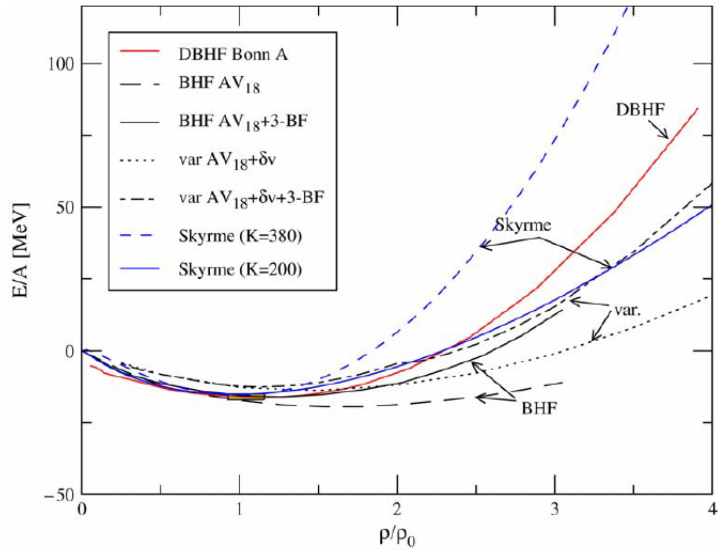
$$E(\rho,\delta)=E(\rho,\delta=0)+S(\rho)\delta^2+O(\delta^4)$$

It is a fundamental properties of nuclear matter, and is very important for understanding

- *properties of nuclear structure*
- *properties of neutron star*
- *properties of heavy ion reaction mechanism*

Theoretical predictions on the properties of nuclear matter

C. Fuchs / Progress in Particle and Nuclear Physics 56 (2006) 1–103



- Effective field theory approaches (Based on Chiral perturbation theory, ……)

- Ab initio approaches (Based on the high precision free space nucleon-nucleon interaction) DBHF, SCGF, QMC

• Phenomenological density functional (Based on Gogny or **Skyrme** force or RMF)

E. Chabanat et al. / Nuclear Physics A 627 (1997) 710–746

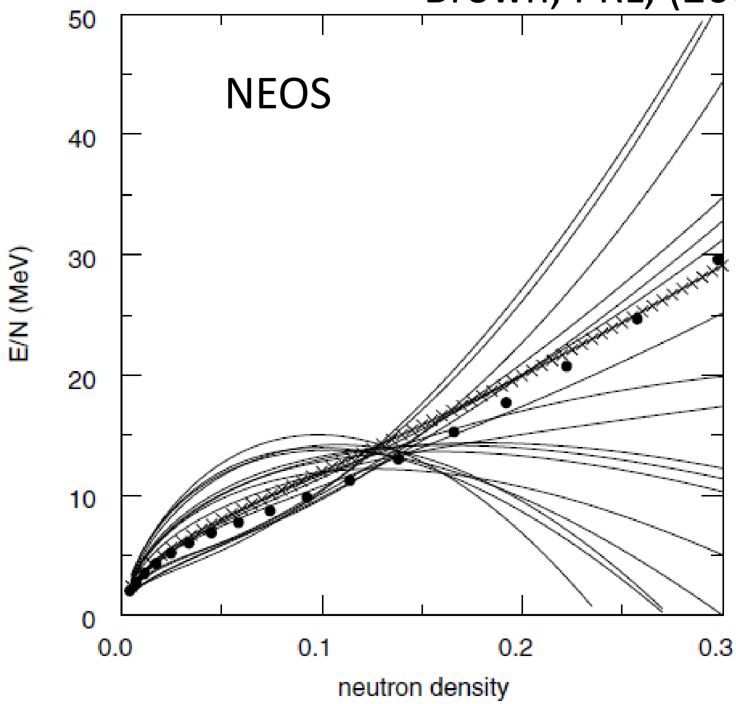
Effective Skyrme

$$
\begin{aligned}
V(\boldsymbol{r}_1,\boldsymbol{r}_2) = \; & t_0\,(1+x_0P_\sigma)\,\delta(\boldsymbol{r}) && \text{central term} \\
& +\frac{1}{2}t_1\,(1+x_1P_\sigma)\left[\boldsymbol{P}'^2\delta(\boldsymbol{r})+\delta(\boldsymbol{r})\boldsymbol{P}^2\right] && \\
& +t_2\,(1+x_2P_\sigma)\,\boldsymbol{P}'\cdot\delta(\boldsymbol{r})\boldsymbol{P} && \text{non-local terms} \\
& +\frac{1}{6}t_3\,(1+x_3P_\sigma)\left[\rho(\boldsymbol{R})\right]^\sigma\delta(\boldsymbol{r}) && \text{density-dependent term} \\
& +iW_0\boldsymbol{\sigma}\cdot\left[\boldsymbol{P}'\times\delta(\boldsymbol{r})\boldsymbol{P}\right] && \text{spin–orbit term} .
\end{aligned}
$$

Skyrme force (Skyrme, 1956):

- simple and widely used
- effectively taken into account the complicated correlations
- parameter are determined by fitting, *rho_0, K_0, m_s*, kappa, mass, ……*
- **Many applications in nuclear structure studies**
- ***parameterization is not unique and there exists >240 sets (drawback)***

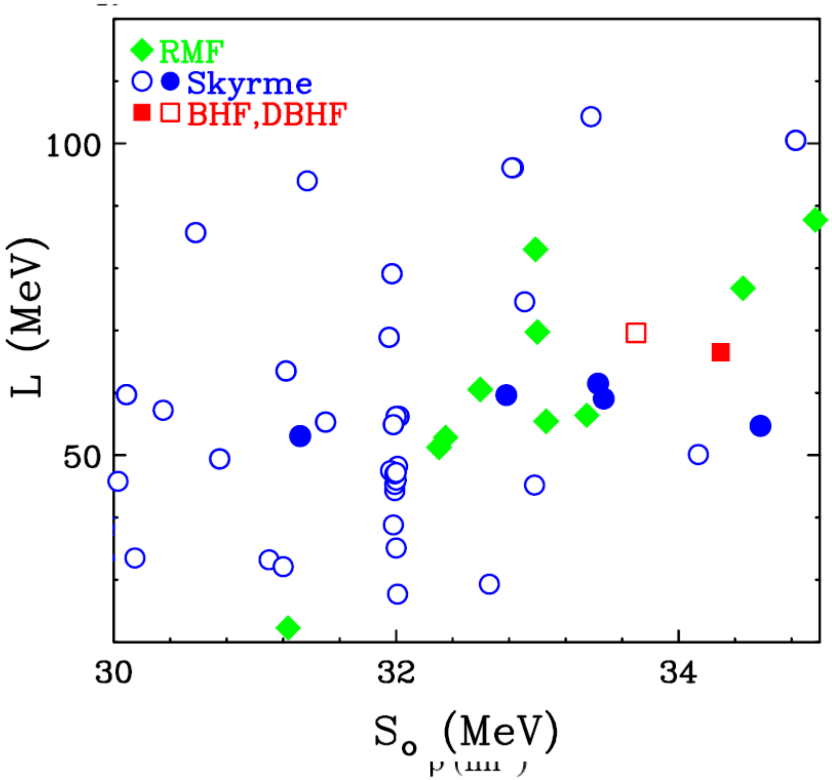
Brown, PRL, (2000)



Density dependent of symmetry energy

$$S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho - \rho_o}{\rho_o}\right) + \frac{K_{\text{sym}}}{18}\left(\frac{\rho - \rho_o}{\rho_o}\right)^2 + \ldots,$$

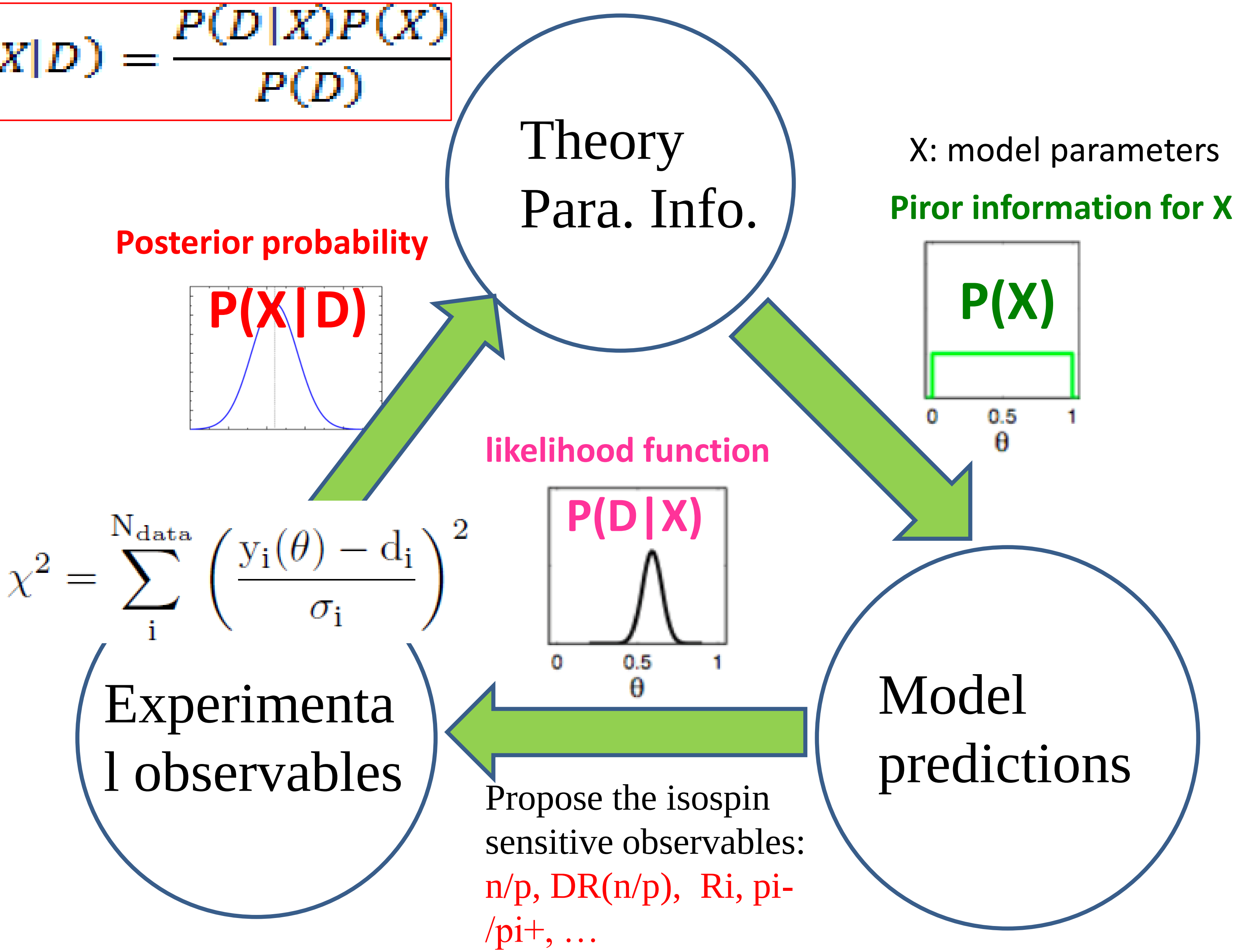
S_0: symmetry energy coefficient
L: slope of density dependent of symmetry energy
K_{sym}: curvature of density dependent of symmetry energy



S(ρ) is the density dependence of symmetry energy, it is a key ingredient of the isospin asymmetric *EOS. However, S(ρ)* uncertainty

Strategy for constraining the symmetry energy

$$P(X|D) = \frac{P(D|X)P(X)}{P(D)}$$

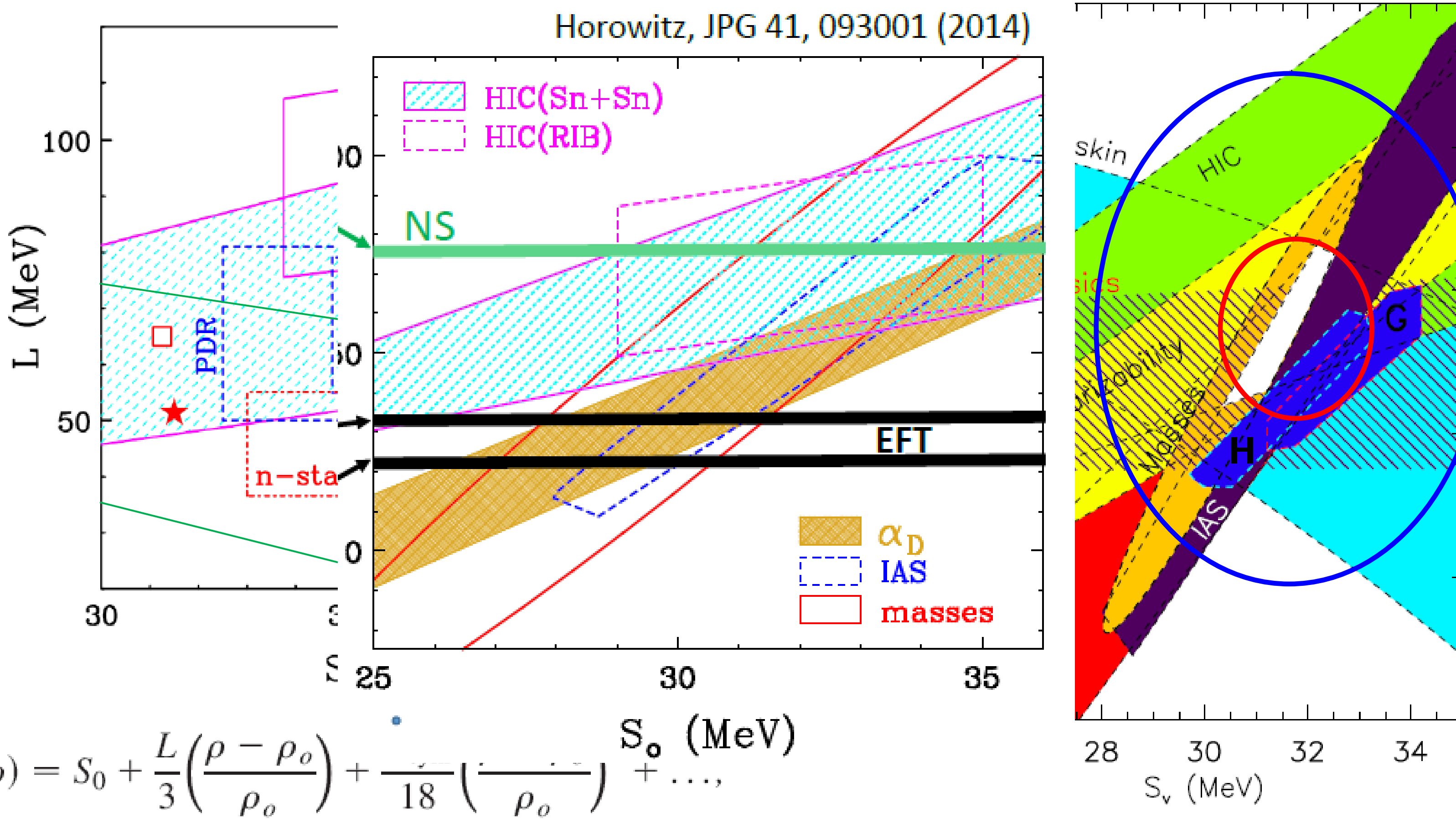


$$\chi^2 = \sum_{i}^{N_{data}} \left(\frac{y_i(\theta) - d_i}{\sigma_i} \right)^2$$

Progress on the constraints of symmetry energy

M.B.Tsang, Yingxun Zhang, et.al., PRL2009

Lattimer, EPJA 50 (2014) 40



$$S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho-\rho_o}{\rho_o}\right) + \frac{\cdots}{18}\left(\frac{\cdots}{\rho_o}\right) + \ldots,$$

Consensus on symmetry energy have been obtained at subsaturation density.

Uncertainties on the constraints still need to be understand and improved! (L+- ΔL)

$$\chi^2 = \sum_{i}^{N_{data}} \left(\frac{y_i(\theta) - d_i}{\sigma_i} \right)^2 \quad +1$$

(Minimize chi2, to find best θ, uncertainty of the constrained parameter θ)

Origin of the uncertainty:

- **Experimental uncertainty (need high precision experiment data)**
- **Model uncertainty (transport code comparison, talk by H. Wolter) or missed physics (??)**
- **sensitivity and parameters correlation in symmetry energy constraints (this talk)**

Writing group
J. Xu
L.W. Chen
B. Tsang
H. Wolter
Y.X. Zhang

Transport Code Comparison Project

Boltzmann-like (9)

	Code Name	Who did?
1	BLOB	P. Napolitani
2	SMF	M. Colonna
3	GiBUU(Sky)	J. Weil/U. Mosel
4	GiBUU(RMF)	J. Weil
5	RVUU	Taesoo Song
6	IBUU(04)	Jun Xu
7	IBL	Wen-Jie Xie
8	RBUU	Kyungil Kim
9	pBUU	P. Danielewicz

MD-like (9)

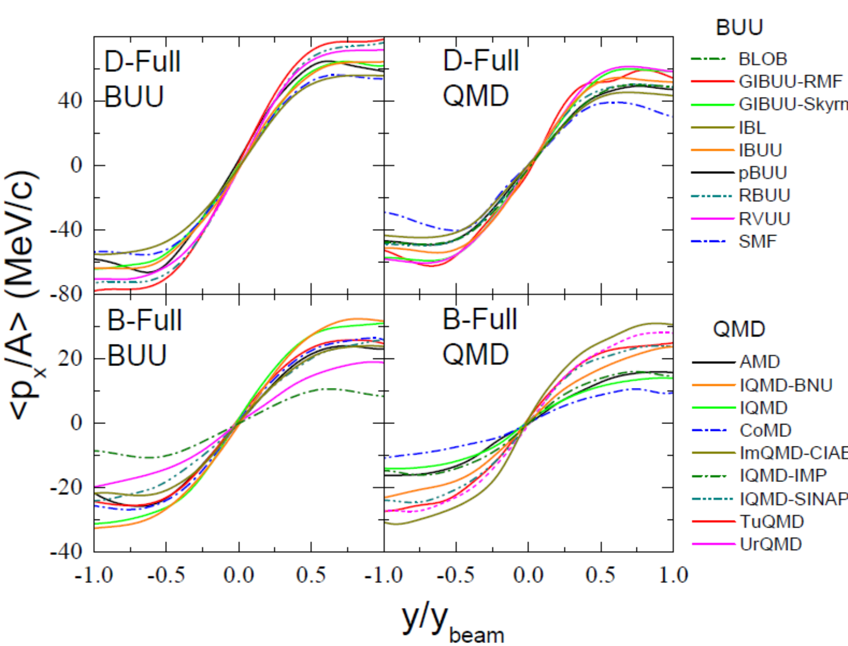
	Code Name	Who did?
1	AMD	Akira Ono
2	CoMD	Maximo Papa
3	ImQMD-CIAE	Ying-Xun Zhang
4	IQMD	Ch. Hartnack
5	IQMD-BNU	Jun Su
6	IQMD-SINAP-	Guo-Qiang Zhang
7	LQMD-IMP	Zhao-Qing Feng
8	UrQMD (L=1)	Yong-Jia Wang
9	TuQMD	Dan Cozma

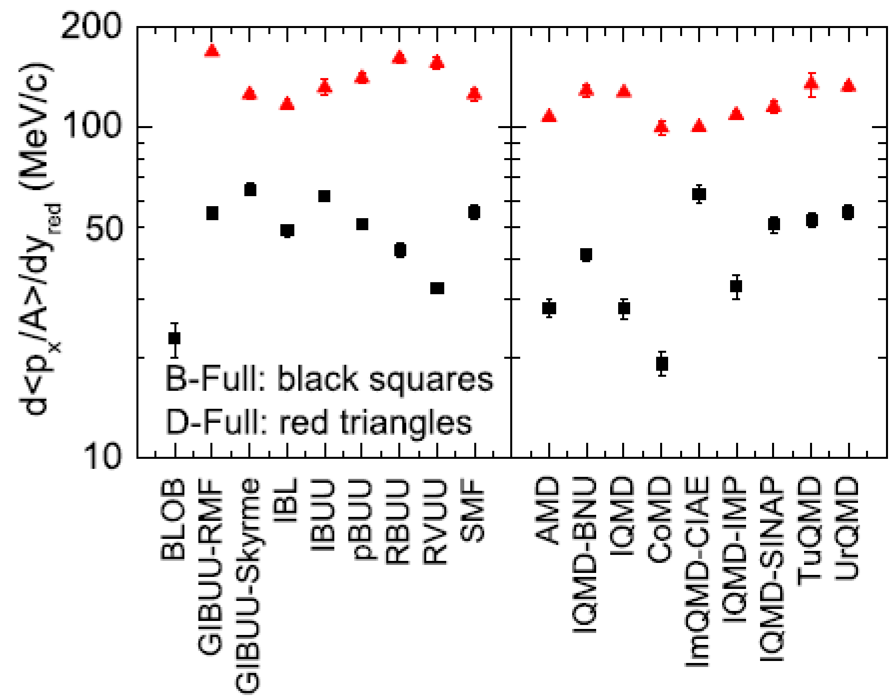
Efforts on this direction, since 2004, 2009, 2014, 2015,2016, ….

PHYSICAL REVIEW C 93, 044609 (2016)

Understanding transport simulations of heavy-ion collisions at 100*A* and 400*A* MeV: Comparison of heavy-ion transport codes under controlled conditions

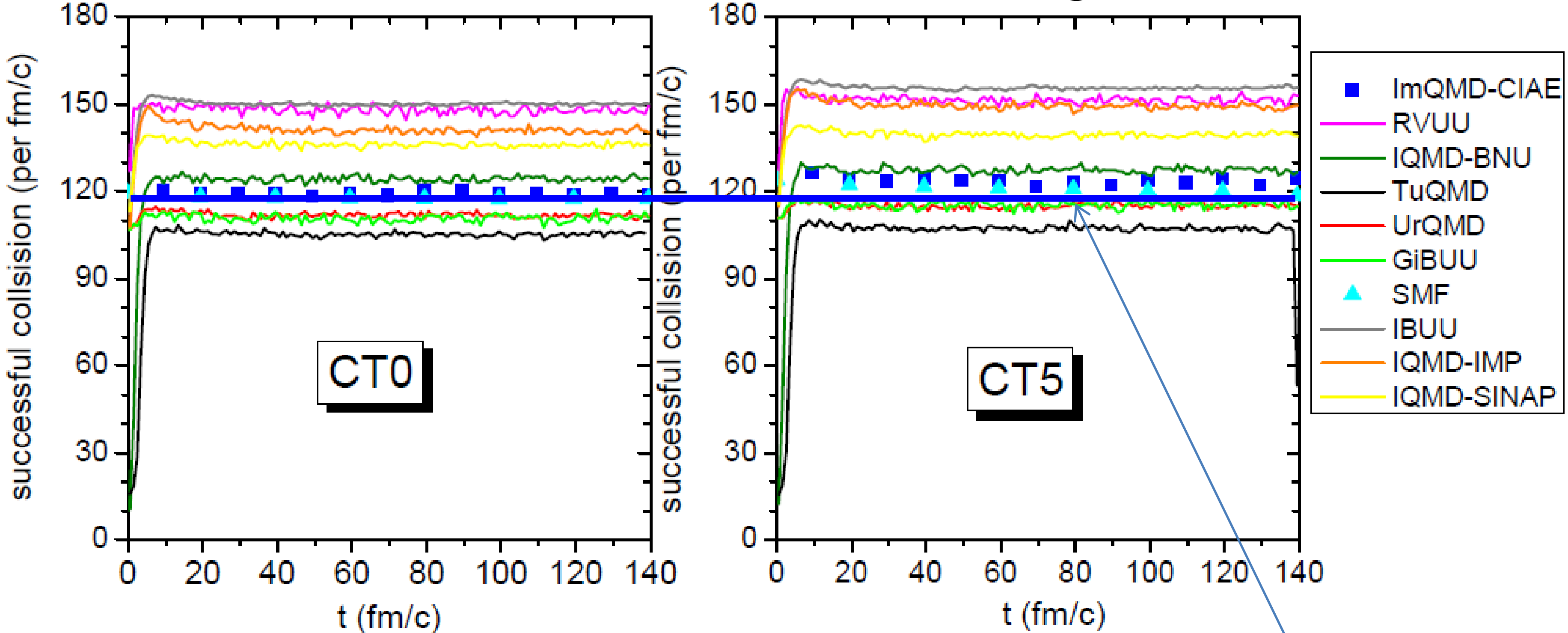
Jun Xu,[1,*] Lie-Wen Chen,[2,†] ManYee Betty Tsang,[3,‡] Hermann Wolter,[4,§] Ying-Xun Zhang,[5,∥] Joerg Aichelin,[6] Maria Colonna,[7] Dan Cozma,[8] Pawel Danielewicz,[3] Zhao-Qing Feng,[9] Arnaud Le Fèvre,[10] Theodoros Gaitanos,[11] Christoph Hartnack,[6] Kyungil Kim,[12] Youngman Kim,[12] Che-Ming Ko,[13] Bao-An Li,[14] Qing-Feng Li,[15] Zhu-Xia Li,[5] Paolo Napolitani,[16] Akira Ono,[17] Massimo Papa,[18] Taesoo Song,[19] Jun Su,[20] Jun-Long Tian,[21] Ning Wang,[22] Yong-Jia Wang,[15] Janus Weil,[19] Wen-Jie Xie,[23] Feng-Shou Zhang,[24] and Guo-Qiang Zhang[1]





where the differences come from? Box simualtions

Cascade mode without Pauli blocking



Analytical value: ~118

Reason for the differences in Cascade without Pauli-blocking (A.Ono, Y.Zhang, J.Xu,,…) :

- **Spurious collision made those difference and time step dependence**

Particle *i* and *j* collide more than once without colliding with others,

S(ρ) and correlations between L, ms* and fi in Skyrme-HF

Density dependent of symmetry energy from SHF

$$S(\rho) = \frac{1}{3}\frac{\hbar^2}{2m}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3} - \frac{1}{8}t_0(2x_0+1)\rho - \frac{1}{48}t_3(2x_3+1)\rho^{\sigma+1} - \frac{1}{24}(\frac{3\pi^2}{2})^{2/3}(3\Theta_v - 2\Theta_s)\rho^{5/3} \quad \text{(C8)}$$

$$S(\rho) = \frac{1}{3}\epsilon_F\rho^{2/3} + A_{sym}\rho + B_{sym}\rho^{\sigma+1} + C_{sym}(m_s^*, m_v^*)\rho^{5/3} \quad \text{(C11)}$$

Density dependent of symmetry depends not only on density, effective mass splitting but also isoscalar effective mass

$$f_I = \frac{1}{2\delta}(\frac{m}{m_n^*} - \frac{m}{m_p^*}) = \frac{m}{k}\frac{\partial((U_n - U_p)/2\delta)}{\partial k} = \frac{\partial U_{sym}}{\partial E_k}.$$

$$\begin{aligned} f_I &= \frac{m}{8\hbar^2}[t_2(2x_2+1) - t_1(2x_1+1)]\frac{\rho}{2} \\ &= m/8\hbar^2(\Theta_s - 2\Theta_v)\rho \\ &= (m/m_s^* - m/m_v^*) \end{aligned}$$

$$\begin{aligned} \frac{\Delta m*}{m} &= \frac{m_n^* - m_p^*}{m} = -\frac{\delta}{m}\frac{2m_s^*m_v^*(m_v^* - m_s^*)}{m_v^{*2} - \delta^2(m_v^* - m_s^*)^2} \\ &= -2\frac{m_s^*}{m}[\frac{m_v^* - m_s^*}{m_v^*}\delta + (\frac{m_v^* - m_s^*}{m_v^*})^2\delta^2 + \cdots \end{aligned}$$

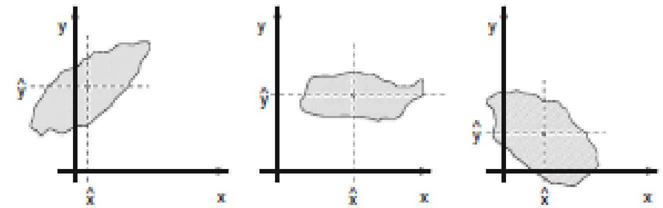
• Correlations between nuclear matter parameters from Skyrme sets

$$C_{AB} = \frac{cov(A,B)}{\sigma(A)\sigma(B)} \quad (1)$$

$$cov(A,B) = \frac{1}{N-1}\sum_i (A_i - \langle A \rangle)(B_i - \langle B \rangle) \quad (2)$$

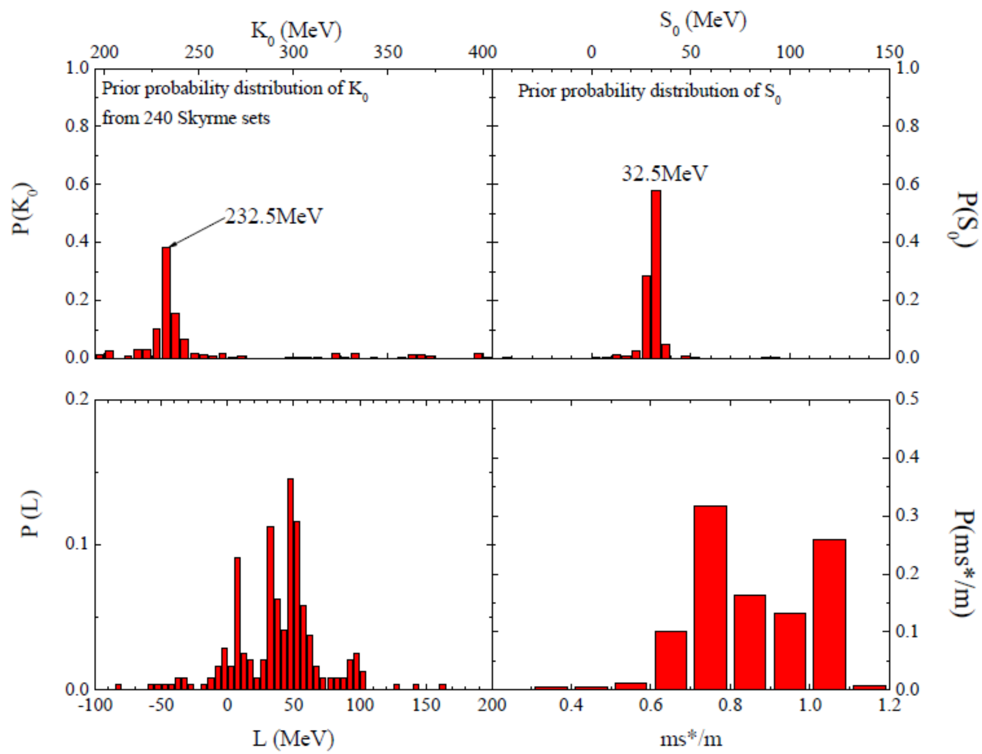
$$\sigma(X) = \sqrt{\frac{1}{N-1}\sum_i (X_i - \langle X \rangle)^2}, X = A, B \quad (3)$$

$$\langle X \rangle = \frac{1}{N}\sum_i X_i, i = 1, N. \quad (4)$$



C_AB=1 0 -1

$$\langle \delta x_i \delta x_j \rangle$$



Parameter probability distribution from 120 Skyrme sets

C_AB between pairs of variables from 120 Skyrme sets

C_{AB}	K_0	S_0	L	Ms*	Mv*
K_0	1	0.003	**-0.161**	**-0.131**	**-0.295**
S_0	0.003	1	**0.764**	**-0.397**	**-0.228**
L	**-0.161**	**0.764**	1	**-0.460**	**-0.212**
Ms*	**-0.131**	**-0.397**	**-0.460**	1	**0.715**
Mv*	**-0.295**	**-0.228**	**-0.212**	**0.715**	1

Ms*, strongly influence L

Mv*, strongly influence ms*

So, we need sensitivity study in the model since the parameters are correlated and complicate the constraints

How a given observable $y=O$ changes due to the change of a parameter x_i ?

$$C_{AB} = \frac{cov(A,B)}{\sigma(A)\sigma(B)}$$

$$\mathrm{Cov}(y_i, x_j)$$

Model: transport code, Improved Quantum Molecular Dynamics model

7 Nuclear matter parameters:

$$\{\rho_0,\ E_0,\ K_0,\ S_0,\ L,\ m_s^*,\ m_v^*\}$$

fi

Observables: isospin sensitive observables

5 Isospin sensitive observables:

$\{ R_2(n/p), DR(n/p), R_{21}(n/n), R_{21}(p/p), R_{diff} \}$

High kinetic energy part of R(n/p) or DR(n/p), Ek>40MeV

How do NM parameters enter into the transport models ?

Schrodinger Equation for N-body system with spinless particles,

$$i\hbar\frac{\partial\psi(x_1,...,x_n)}{\partial t} = (-\sum_{i=1}^{N}\frac{\hbar^2}{2m}\nabla_i^2 + V(x_1,...,x_n))\psi(x_1,...,x_n)$$

$$(\frac{\partial}{\partial t} + \sum_i v_i\cdot\nabla_i - \sum_i \nabla_i V(R_1,R_2,...,R_N)\cdot\nabla_{p_i})f_N(R_1,p_1,...,R_N,p_N,t)$$

$$= \frac{2}{\hbar}\sum_\lambda\ i\ (\frac{\hbar}{2i})^{\lambda_1+\ldots+\lambda_n}\frac{1}{\lambda_1!...\lambda_n!}\frac{\partial^{\lambda_1+...+\lambda_n}}{\partial_{R_1}^{\lambda_1}...\partial_{R_n}^{\lambda_n}}V(R_1,R_2,...,R_N)\frac{\partial^{\lambda_1+...+\lambda_n}}{\partial_{p_1}^{\lambda_1}...\partial_{p_n}^{\lambda_n}}f_N(R_1,p_1,...R_N,p_N,t)$$

$$\frac{\partial < p_i >}{\partial t} = - < \nabla_i V(R_1,...R_N) > \approx -\frac{\partial U(\bar{R}_1...\bar{R}_N)}{\partial\bar{R}_i}$$

$$\frac{d < R_i >}{dt} = \frac{< p_i >}{m}.$$

Collision part,

Cascade

In the new version of ImQMD code, nucleons are represented by Gaussian wavepackets

• ImQMD with standard Skyrme interaction

the potential energy U that includes the full Skyrme potential energy without the spin-orbit term:

$$U = U_\rho + U_{md} + U_{coul} \quad (2)$$

and U_{coul} is the Coulomb energy. The nuclear contributions are represented in a local form with

$$U_{\rho,md} = \int u_{\rho,md} d^3 r$$

and

$$u_\rho = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\eta+1}\frac{\rho^{\eta+1}}{\rho_0^\eta} + \frac{g_{sur}}{2\rho_0}(\nabla\rho)^2 + \frac{g_{sur,iso}}{\rho_0}[\nabla(\rho_n - \rho_p)]^2 + A_{sym}\rho^2\delta^2 + B_{sym}\rho^{\eta+1}\delta^2$$

Y.X. Zhang, M.B.Tsang, Z.X. Li, HLiu, PLB732,186 (2014)

$$u_{\rm md} = \frac{1}{2\rho_0}\sum_{N_1,N_2={\rm n,p}}\frac{1}{16\pi^6}\int {\rm d}^3p_1{\rm d}^3p_2 f_{N_1}(\mathbf{p}_1)f_{N_2}(\mathbf{p}_2) \times 1.57\,[\ln(1.+5.\times 10^{-4}(\Delta p)^2)]^2, \quad (4)$$

$$\delta(r_1 - r_2)(p_1 - p_2)^2$$

$$\begin{aligned} u_{md} &= u_{md}(\rho\tau) + u_{md}(\rho_n\tau_n) + u_{md}(\rho_p\tau_p) \quad (5)\\ &= C_0\int d\vec{p}\, d\vec{p}'\, f(\vec{r},\vec{p})f(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2 \\ &+ D_0[\int d\vec{p}\, d\vec{p}'\, f_n(\vec{r},\vec{p})f_n(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2 + \int d\vec{p}\, d\vec{p}'\, f_p(\vec{r},\vec{p})f_p(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2] \end{aligned}$$

Skyrme type MDI

Nuclear matter parameters in transport model

$$g_{\rho\tau} = \frac{3}{5}\left(\frac{m}{m_s^*} - 1\right)\frac{\hbar^2}{m}\epsilon_F^0$$

$$\gamma = \frac{K_0 + \frac{6}{5}\epsilon_F^0 - 10 g_{\rho\tau}}{\frac{9}{5}\epsilon_F^0 - 6 g_{\rho\tau} - 9E_0}$$

$$\beta = \frac{\left(\frac{1}{5}\epsilon_F^0 - \frac{2}{3}g_{\rho\tau} - E_0\right)(\gamma + 1)}{\gamma - 1}$$

$$\alpha = E_0 - \epsilon_F^0 - \frac{8}{3}g_{\rho\tau} - \beta$$

$$C_{sym} = -\frac{1}{24}\left(\frac{3\pi^2}{2}\right)^{2/3}(3\Theta_v - 2\Theta_s)\rho^{5/3}$$

$$B_{sym} = \frac{3S_0 - L - \frac{1}{3}\epsilon_F^0 + 2C_{sym}(m_s^*, m_v^*)}{-3\sigma}$$

$$A_{sym} = S_0 - \frac{1}{3}\epsilon_F^0 - B_{sym} - C_{sym}(m_s^*, m_v^*)$$

$$\Theta_s = \left(\frac{m}{m_s^*} - 1\right)\frac{8\hbar^2}{m}\frac{1}{\rho_0}, \quad \Theta_v = \left(\frac{m}{m_v^*} - 1\right)\frac{4\hbar^2}{m}\frac{1}{\rho_0}.$$

$$C_0 = \frac{1}{16\hbar^2}[t_1(2 + x_1) + t_2(2 + x_2)] = \frac{1}{16\hbar^2}\Theta_v$$

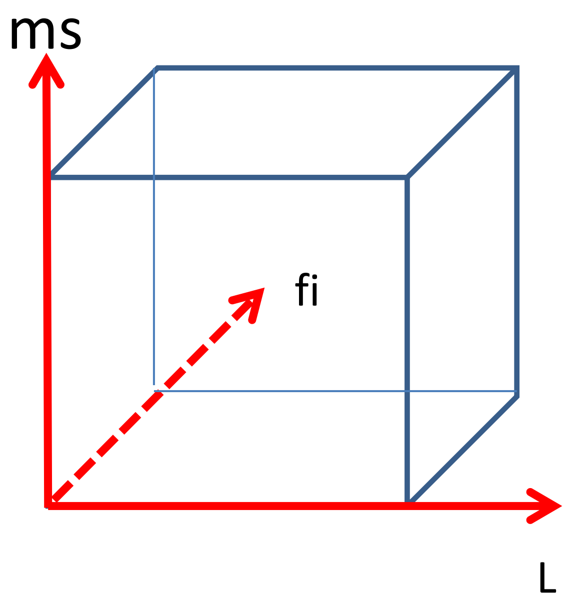
$$D_0 = \frac{1}{16\hbar^2}[t_2(2x_2 + 1) - t_1(2x_1 + 1)] = \frac{1}{16\hbar^2}(\Theta_s - 2\Theta_v)$$

7 (5) model parameters:

$$\{\rho_0,\ E_0,\ K_0,\ S_0,\ L,\ m_s^*,\ m_v^*\}$$

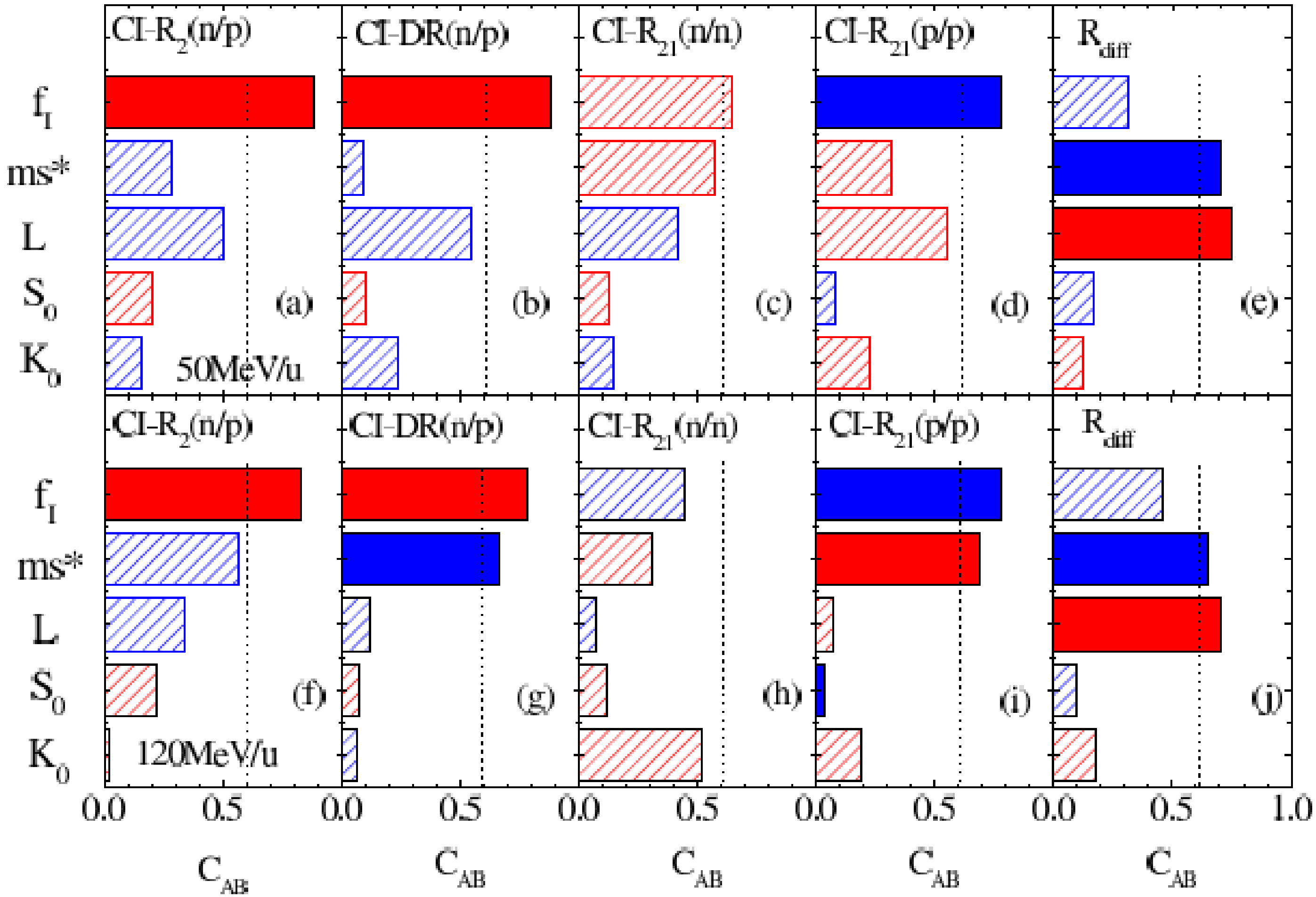
Table 1: List of twelve parameters used in the ImQMD calculations. $\rho_0 = 0.16 fm^{-3}$, $E_0 = -16 MeV$, and $g_{sur} = 24.5 MeV fm^2$, $g_{sur,iso} = -4.99 MeV fm^2$

Para.	K_0 (MeV)	S_0(MeV)	L (MeV)	m_s^*/m	f_I
1	**230**	**32**	**46**	**0.7**	**-0.238**
2	**280**	32	46	0.7	-0.238
3	**330**	32	46	0.7	-0.238
4	230	**30**	46	0.7	-0.238
5	230	**34**	46	0.7	-0.238
6	230	32	**60**	0.7	-0.238
7	230	32	**80**	0.7	-0.238
8	230	32	**100**	0.7	-0.238
9	230	32	46	**0.85**	-0.238
10	230	32	46	**1.00**	-0.238
11	230	32	46	0.7	**0.0**
12(SLy4)	230	32	46	0.7	**0.178**



Correlation coefficient of x={} and O={}

Y.X.Zhang, M.B.Tsang, Z.X.Li, PLB749,262(2015)



Blue:
negative correlation

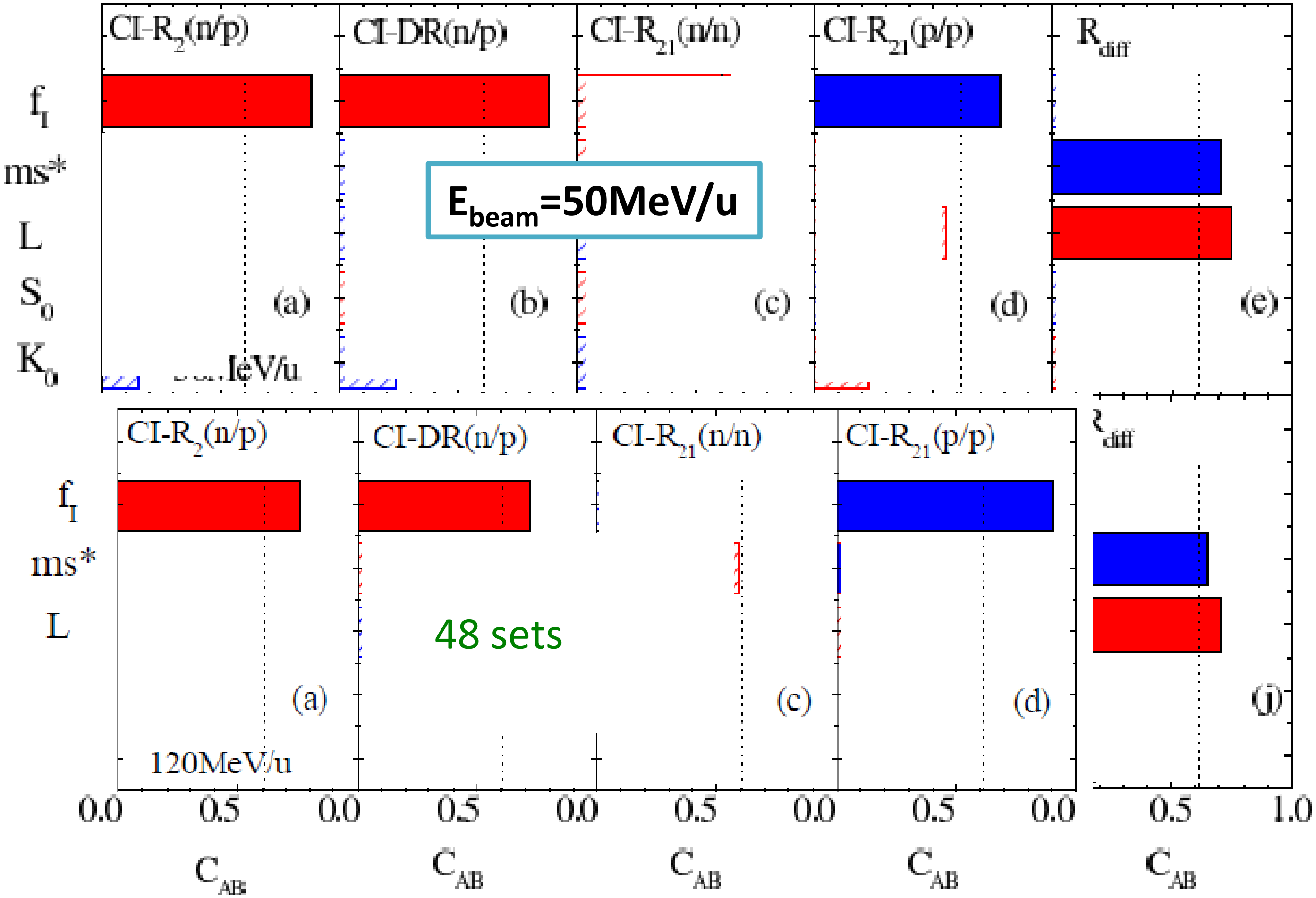
Red:
Positive correlation

The ratios are constructed with Ek>40MeV

- **Ms* also play important roles for isospin diffusion, and neutron to proton yield ratio observables at 120MeV/u.**

Correlation coefficient of A={} and B={}

Y.X.Zhang,M.B.Tsang, Z.X.Li, PLB749,262 (2015)



Blue:
negative correlation

Red:
Positive correlation

The ratios are constructed with Ek>40MeV

- **Ms* also play important roles for isospin diffusion, and neutron to proton yield ratio observables at 120MeV/u, one can reasonable determine {L,ms*, fi} by combination analysis.**

Constraints on L, ms* and fi

$$P(X|D) = \frac{P(D|X)P(X)}{P(D)}$$

Prior distribution of L, ms* and fi

Range of L

$$P(L) \propto \exp\left(-\frac{(L-L_0)^2}{2\sigma_L^2}\right), L_0 = 60MeV, \sigma_L^2 = 30^2$$

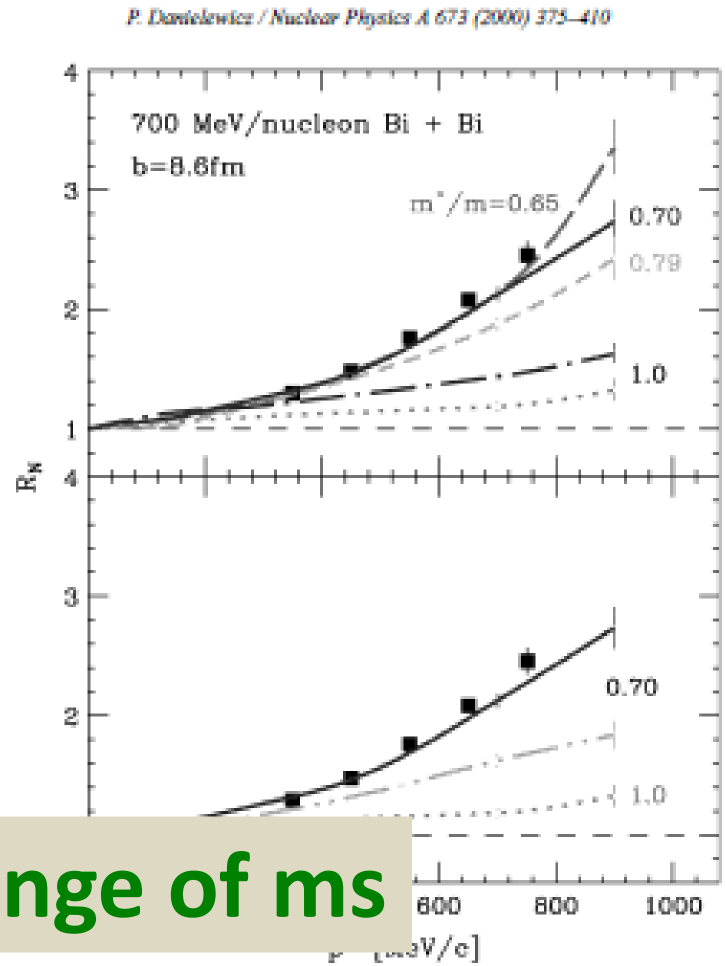
- Information of Effective mass and effective mass splitting

ms*/m~0.65-0.9,

F.Chappert, PLB668(2008)402,

Table 2
Infinite and semi-infinite nuclear matter properties of the D1S and D1N interactions compared to empirical values

	D1S	D1N	Emp. values
ρ_0 (fm^{-3})	0.16	0.16	0.17 ± 0.02
E_0/A (MeV)	-15.9	-16.0	-16 ± 1
K_∞ (MeV)	210	230	220 ± 10
m^*/m	0.70	0.75	0.70 ± 0.05
E_{sym} (MeV)	32.0	29.3	30 ± 2
E_{surf} (MeV)	20.0	19.3	21 ± 2



P. Klupfel, et.al., PRC79, 0343310(2009)

GQR Pb208, ms*/m=0.9

X.H.Li, et.al., PLB743(2015)408

Table 4
Nucleon isoscalar effective mass m_0^*/m and the neutron-ting m_{n-p}^* from the three cases studied in this work.

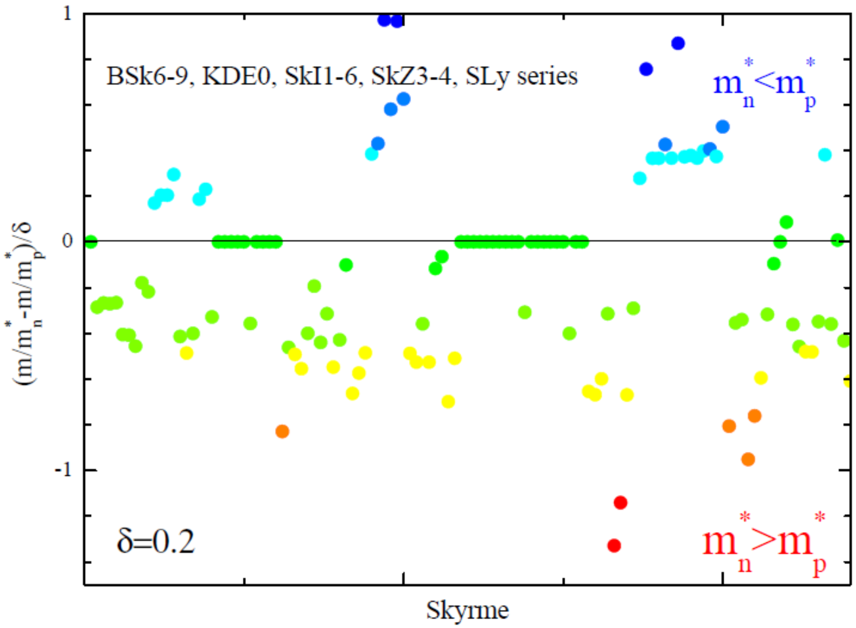
Case	m_0^*/m
I	0.65 ± 0.05
II	0.67 ± 0.06
III	0.65 ± 0.06

Range of ms

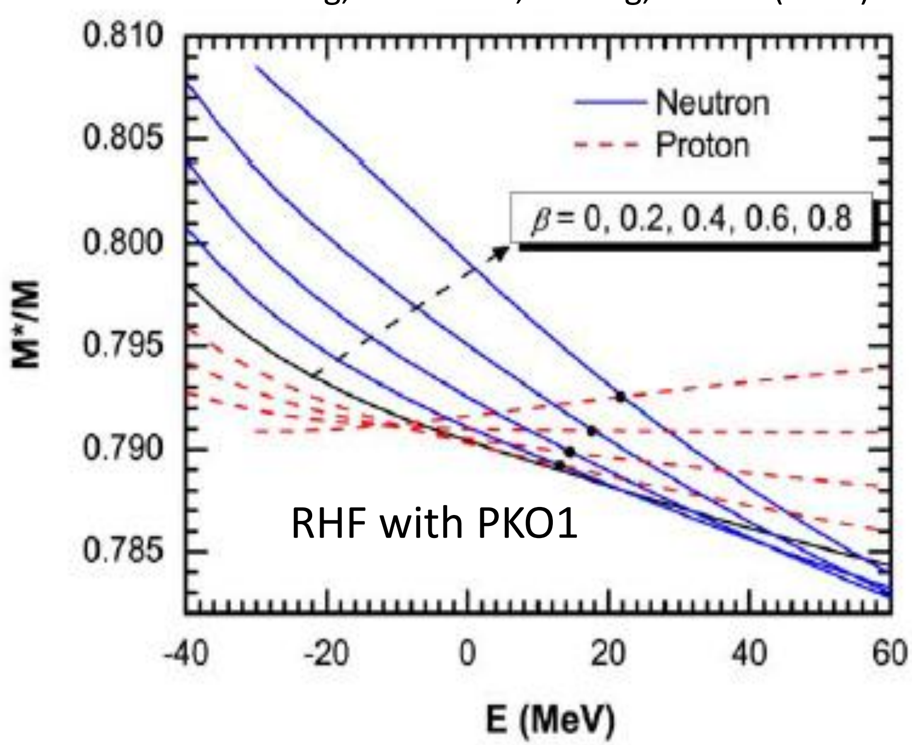
$$P(ms) \propto \exp\left(-\frac{(ms - ms_0)^2}{2\sigma_{ms}^2}\right), ms = 0.7, \sigma_{ms}^2 = 0.15^2$$

Effective mass splitting

$$\frac{m}{m_n^*} - \frac{m}{m_p^*} = \frac{\partial\left(U_n - U_p\right)}{\partial E_k}$$



WHLong, N Van Giai, J.Meng, PLB640(2008)150



B.A.Li, C. Xu, et.al., PRC2006,2010,
X.H.Li, PLB743(2015)408)

At normal density,

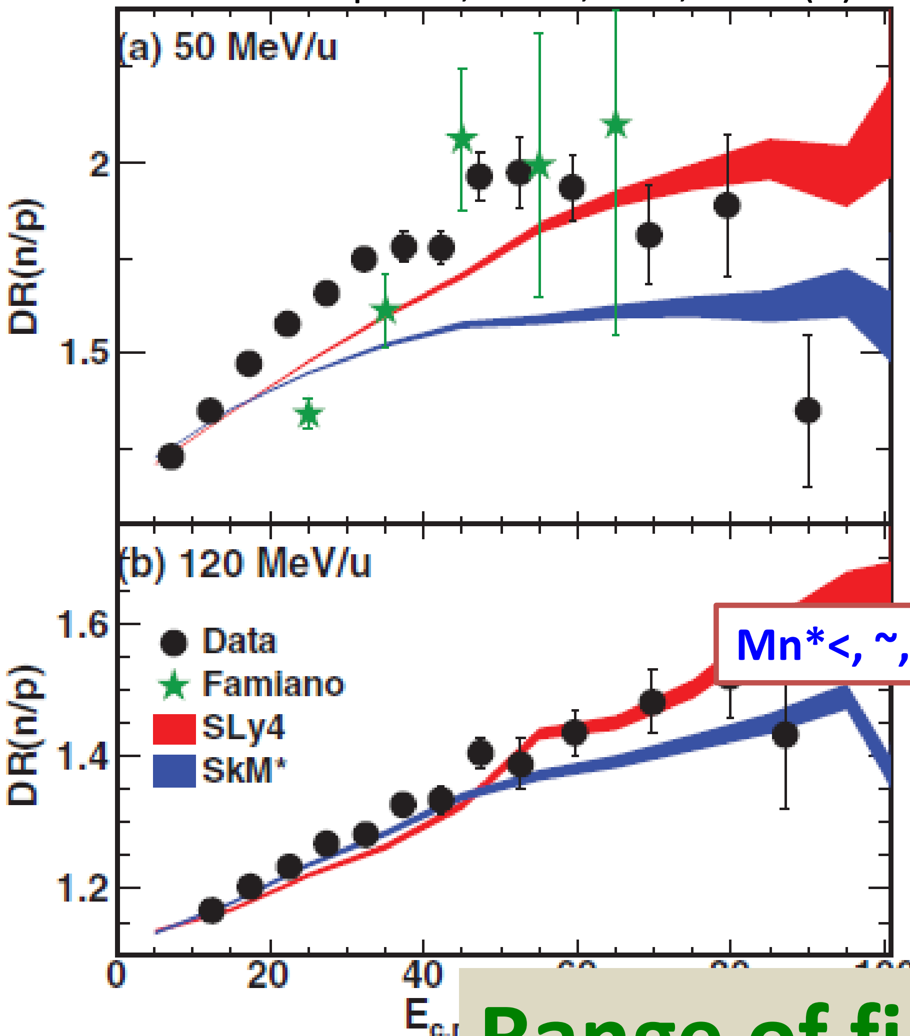
$(m_n^*-m_p^*)/m=0.32\delta$,
$(m_n^*-m_p^*)/m=(0.41+-0.15)\delta$

Effective mass splitting depends not only on the in-medium density but also on momentum or energy ??

Information on fi from HICs

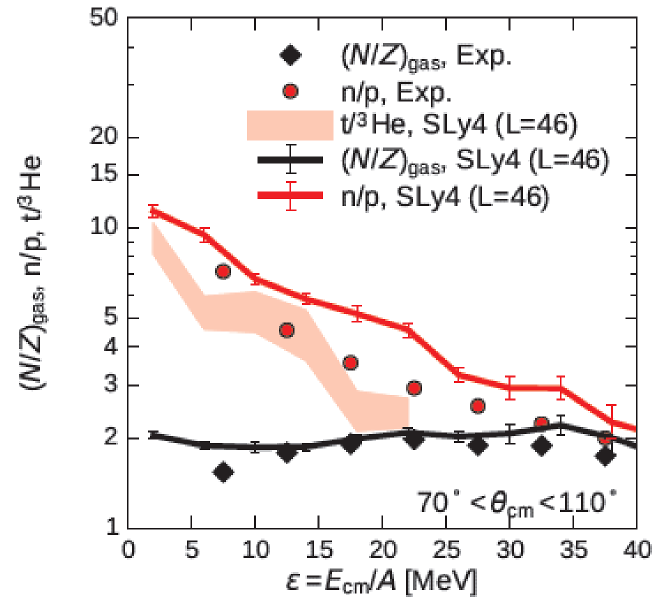
Ono, Talk at NN2015

D.D.S.Coupland, et al., PRC, 2016(R)

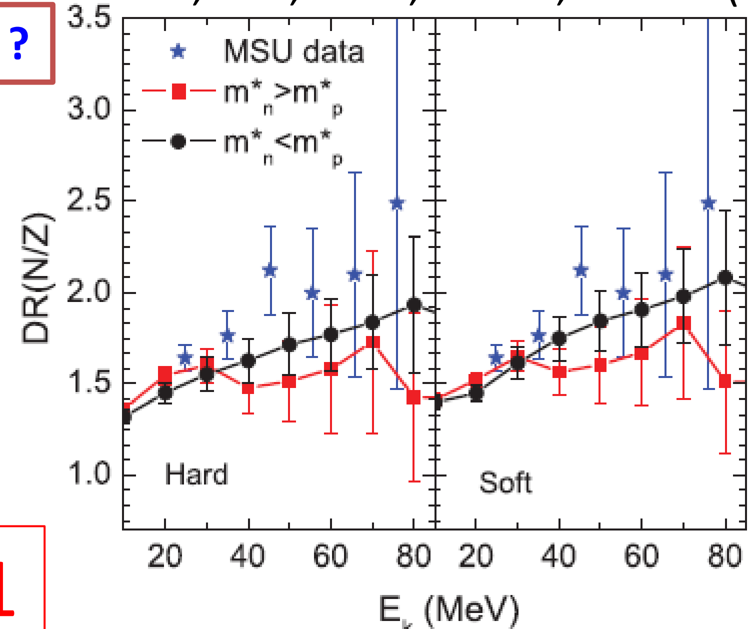


Mn*<, ~, mp* ?

Range of fi



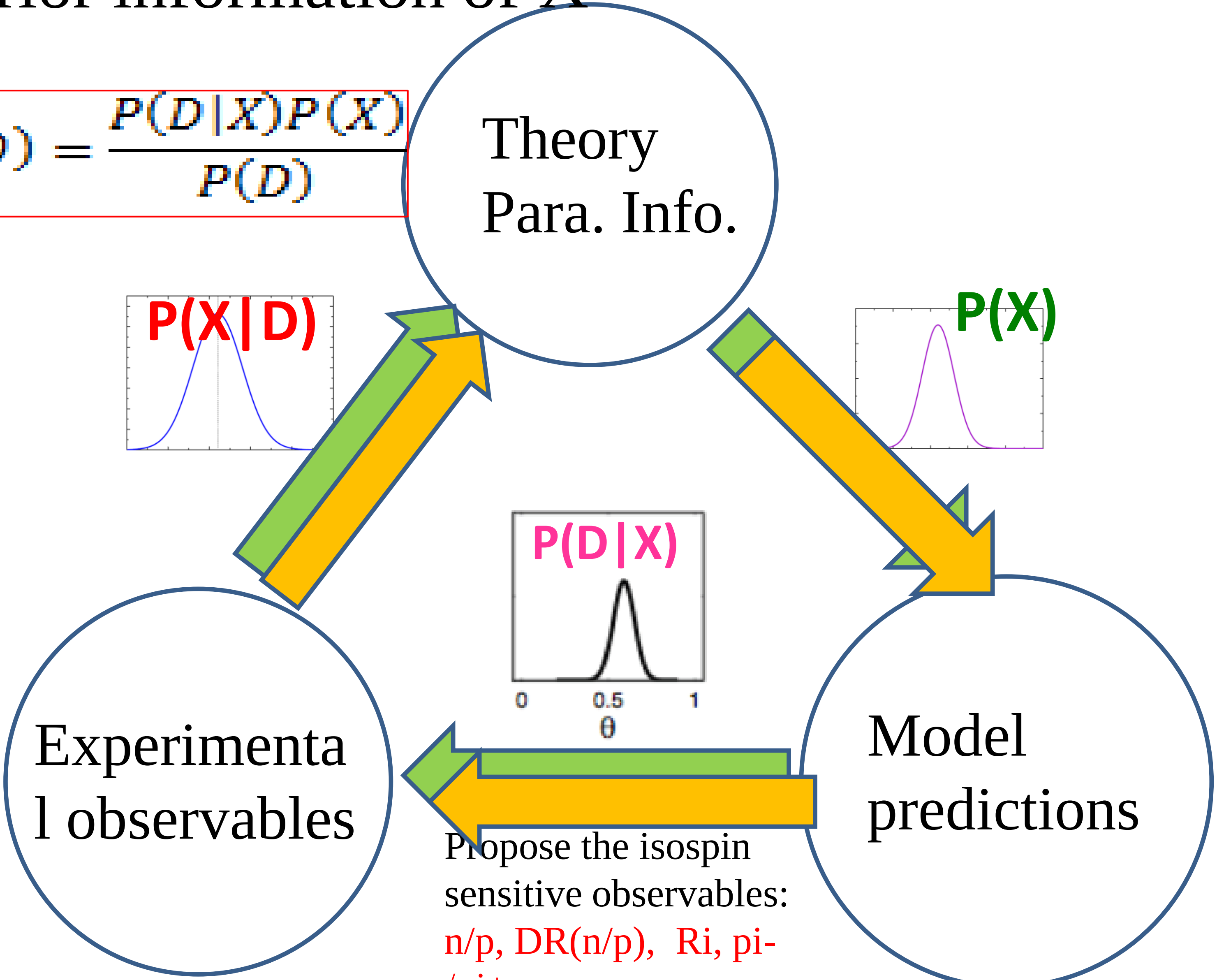
W.J.Xie, J.Su, et al., PRC88, 061601(2013)



Assume: P(fi)=1

constraining the symmetry energy consider prior information of X

$$P(X|D) = \frac{P(D|X)P(X)}{P(D)}$$



Prior information of L, ms* and fi

$$P(L) \propto \exp\left(-\frac{(L-L_0)^2}{2\sigma_L^2}\right), L_0 = 60MeV, \sigma_L^2 = 30^2$$

$$P(ms) \propto \exp\left(-\frac{(ms-ms_0)^2}{2\sigma_{ms}^2}\right), ms = 0.7, \sigma_{ms}^2 = 0.15^2$$

$$P(f_i) = 1$$

Rho, eta, alpha, correlation coefficients

$$P(L, ms*) \propto \exp\left(-\frac{1}{1-\rho^2}\left(\frac{(L-L_0)^2}{2\sigma_L^2} - \frac{2\rho(L-L_0)}{2\sigma_L}\frac{(ms-ms_0)}{2\sigma_{ms}} - \frac{(ms-ms_0)^2}{2\sigma_{ms}^2}\right)\right)$$

$$P(L, f_i) \propto \exp\left(-\frac{1}{1-\eta^2}\left(\frac{(L-L_0)^2}{2\sigma_L^2} - \frac{2\eta(L-L_0)}{2\sigma_L}\frac{(f_i-f_{i0})}{2\sigma_{fi}} - \frac{(f_i-f_{i0})^2}{2\sigma_{fi}^2}\right)\right)$$

$$P(ms, f_i) \propto \exp\left(-\frac{1}{1-\alpha^2}\left(\frac{(ms-ms_0)^2}{2\sigma_{ms}^2} - \frac{2\alpha(ms-ms_0)}{2\sigma_{ms}}\frac{(f_i-f_{i0})}{2\sigma_{fi}} - \frac{(f_i-f_{i0})^2}{2\sigma_{fi}^2}\right)\right)$$

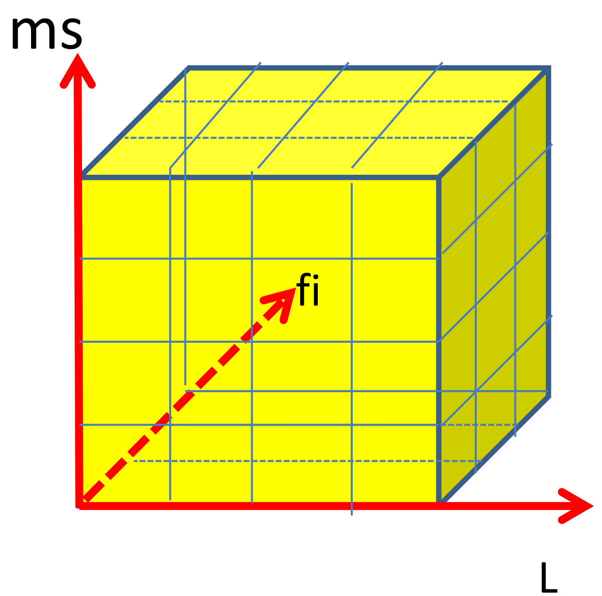
Likelihood function

$$L(x) \sim \exp\left(-\sum_a \frac{\left(y_{M,a}(x) - y_a^{exp}\right)^2}{2\sigma_a^2}\right)$$

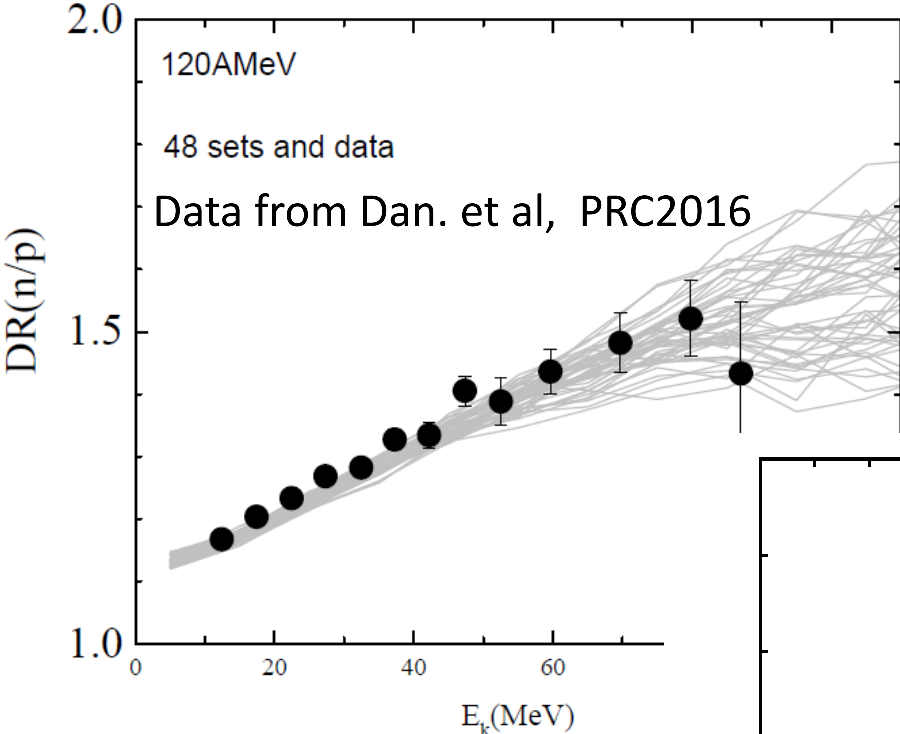
Use K_0=230 and S_0=32MeV

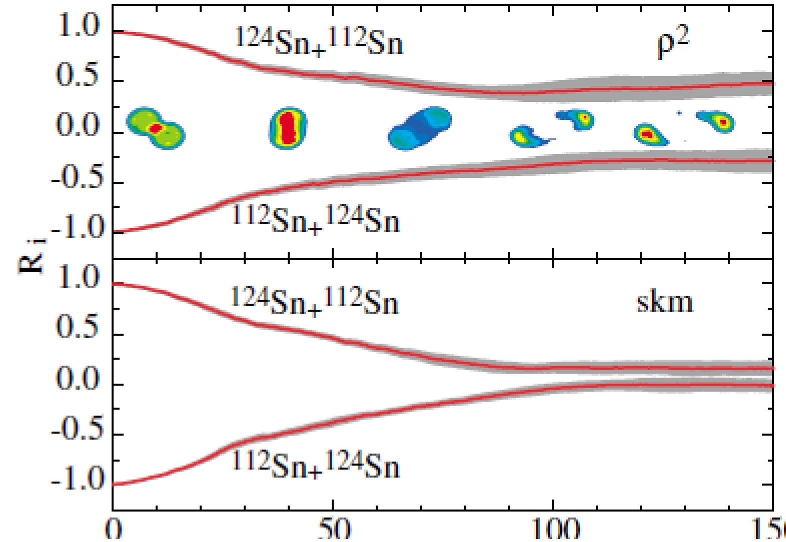
48 set points

- L=46-100MeV,
- m_s*/m=0.6-1.0
- f_i=-0.238-0.178



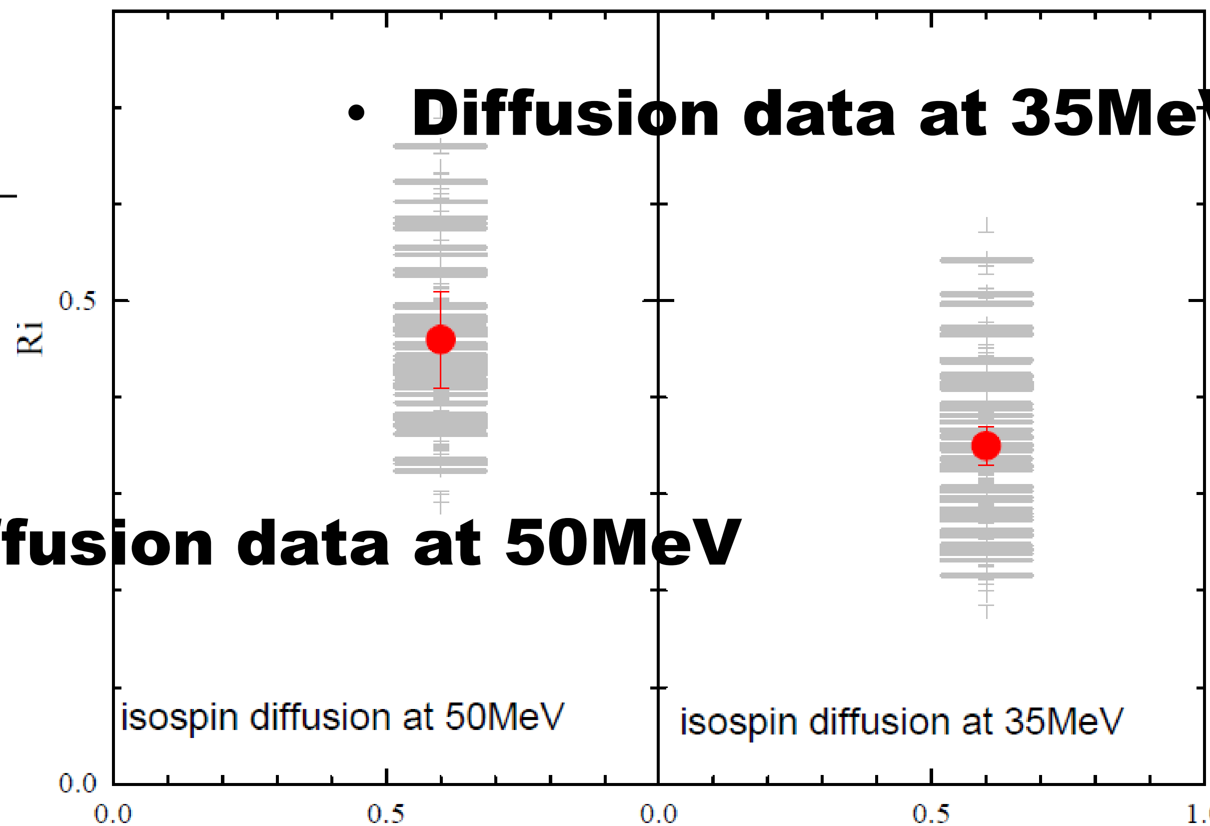
- **DR data at 120MeV**





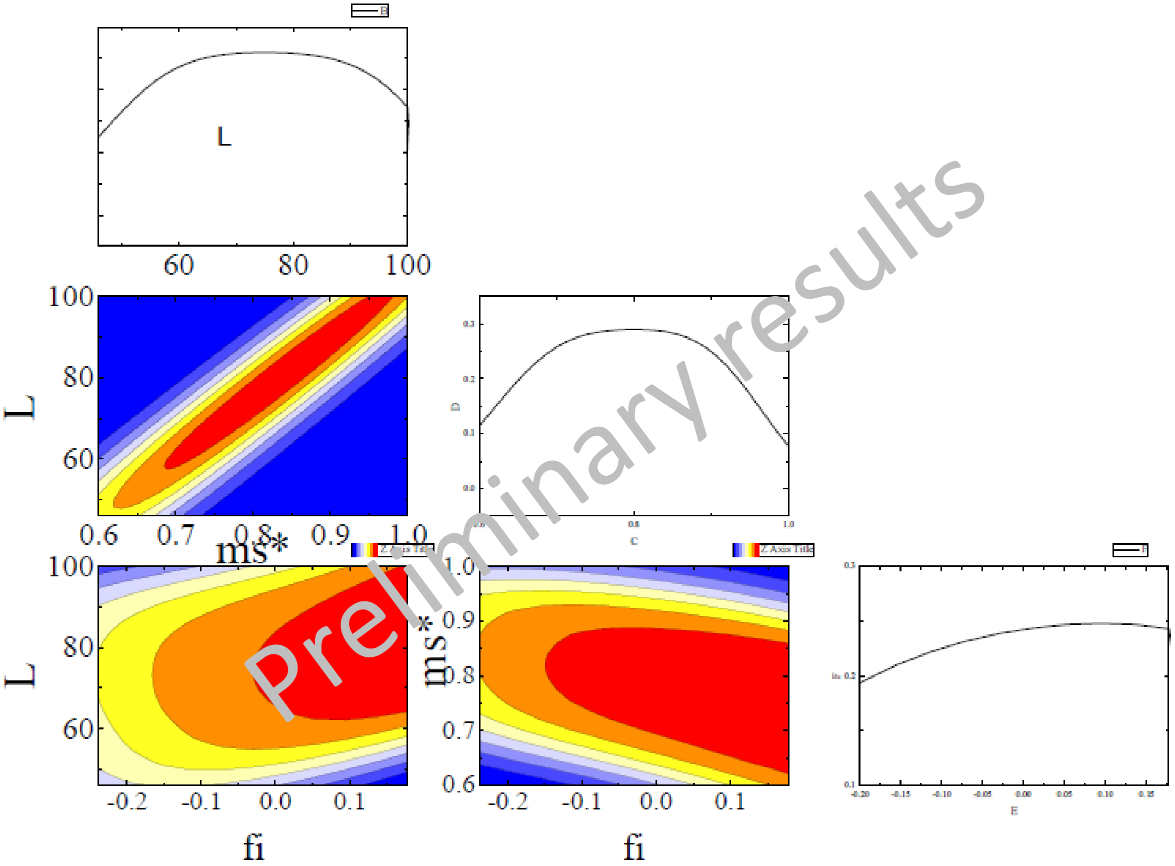
- **Diffusion data at 35MeV**

- **Diffusion data at 50MeV**



One- and two-dimensional likelihood projection

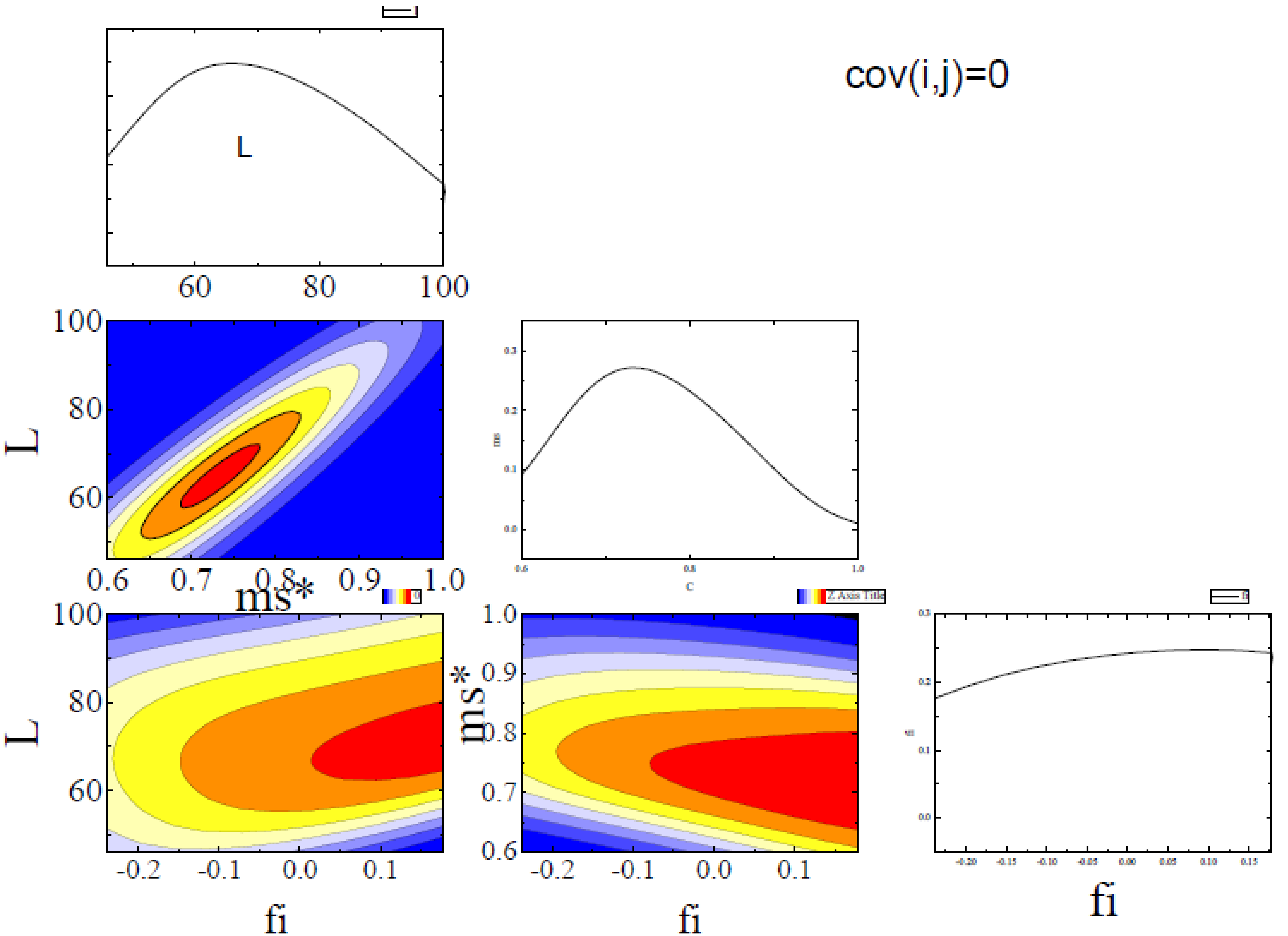
$$L(x) \sim \exp\left(-\sum_a \frac{\left(y_{M,a}(x) - y_a^{exp}\right)^2}{\sigma_a^2}\right)$$



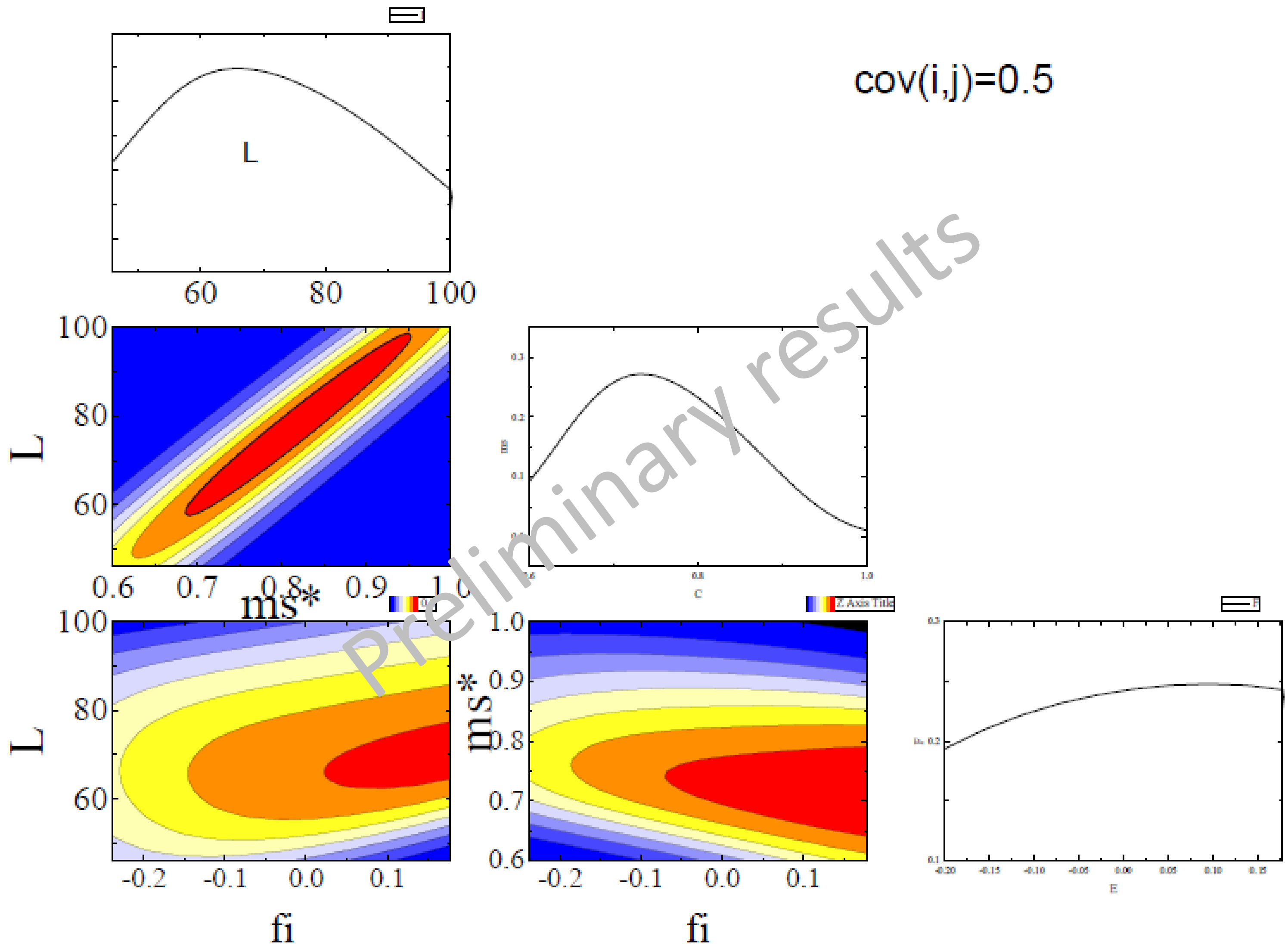
- Use large ms* in the model, lead to larger L constraints
- Use large ms*, lead to smaller fi
- Use smaller L, lead to smaller f_i

•In one-dimension figure, {L~70, ms*~0.8, fi~0.01} give largest probability of constraints

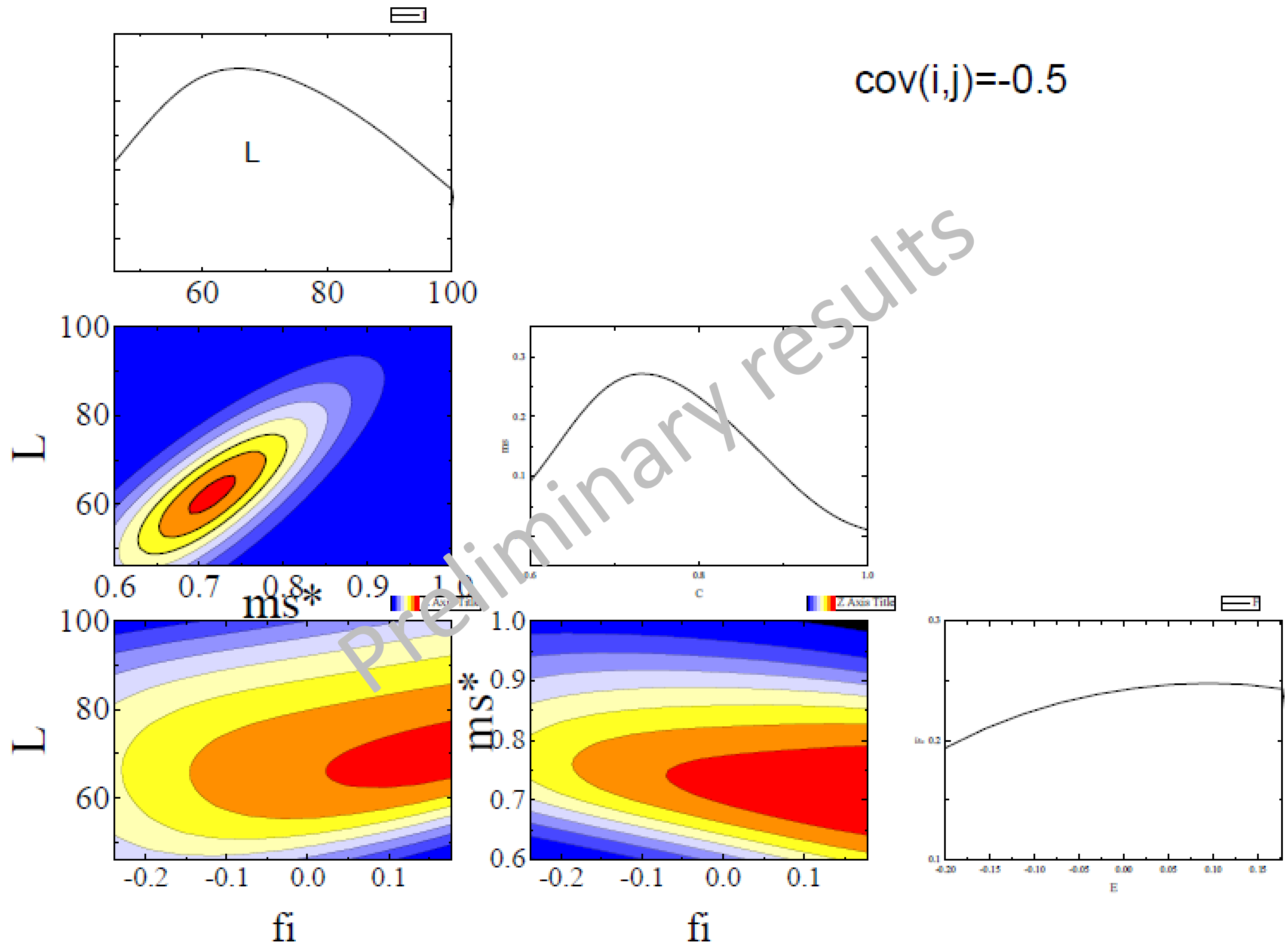
If ρ, η, α =0, i.e. the L, ms* and fi are independent.



If ρ= η= α =0.5, i.e. the L, ms* and fi are correlated.



If ρ= η= α =-0.5, i.e. the L, ms* and fi are correlated.



4, Summary and outlook

1, The results have identified the importance of ms* on isospin sensitive observables as well as L and f_I.

2, The larger ms* used in transport model lead to larger L values in the constraints on symmetry energy, or smaller fi.

3, L, ms and fi can be extracted and work is in progress

4, Model or method uncertainties are necessary in future.

Thanks for your attention!