

Hot and dense matter beyond relativistic mean field theory

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X.Z. and Madappa Prakash (Ohio U.), PRC.93.055805 (2016) [arXiv: 1602.07665]

INT Program: INT-16-2b “The Phases of Dense Matter”, summer, 2016

Outline

- Motivations
- Mean field theory (MFT) and two-loop (TL) calculations in the framework of quantum hadro-dynamics (QHD): zero temperature
- Finite temperature calculations
- Summary

Motivations

- Dense matter equation of state (EOS) is needed in astrophysics simulations
- QHD's MFT approximation is widely used to model the EOS, but TL terms have not been well studied
- Goal is to compare MFT and MFT+TL on EOS and nucleon optical potential (relevant for medium energy heavy ion collisions)

Lagrangian

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$$\mathcal{L}_N = \bar{N} \left[i\gamma^\mu (\partial_\mu + ig_\rho \rho_\mu + ig_v V_\mu) + \frac{g_A}{f_\pi} \gamma^\mu \gamma_5 \partial_\mu \pi - M + g_s \phi \right] N$$

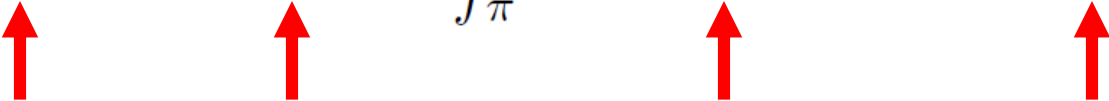
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(p,n) fields

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I=1 vector, I=0 vector, I=1 pseudoscalar, I=0 scalar

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Pseudovector coupling due to Chiral symmetry

$$\mathcal{L}_N = \overline{N} \left[i\gamma^\mu (\partial_\mu + ig_\rho \rho_\mu + ig_v V_\mu) + \frac{g_A}{f_\pi} \gamma^\mu \gamma_5 \partial_\mu \pi - M + g_s \phi \right] N$$

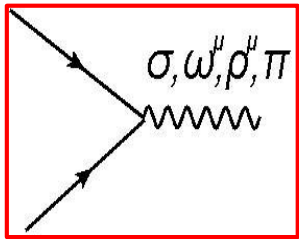
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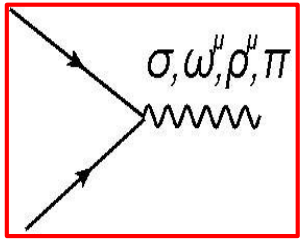
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$$\mathcal{L}_{\text{meson}} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \left(\frac{1}{2} + \frac{\kappa_3 g_s \phi}{3! M} + \frac{\kappa_4 g_s^2 \phi^2}{4! M^2} \right) m_s^2 \phi^2 + \frac{1}{2} \partial^\mu \pi^i \partial_\mu \pi_i - \frac{1}{2} m_\pi^2 \pi^i \pi_i$$

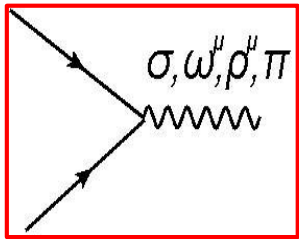
$$- \frac{1}{4} V^{\mu\nu} V_{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu - \frac{1}{4} \rho_{\mu\nu}^i \rho_i^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu^i \rho_i^\mu,$$

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Masses are given as known, 5 couplings need to be fixed

g_s^2

g_v^2

g_ρ^2

κ_3

κ_4

MFT approximation

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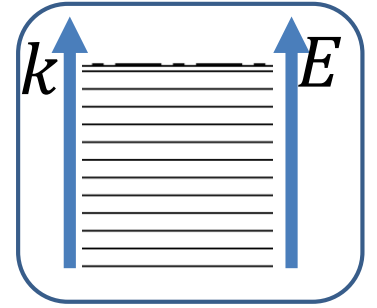
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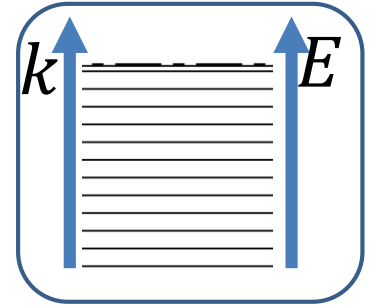
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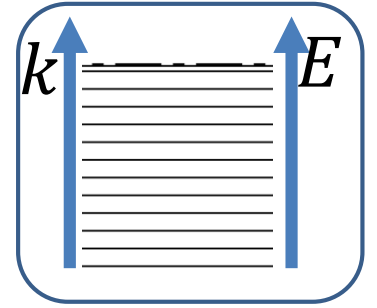
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$$g_v \bar{V} = \frac{g_v^2}{m_v^2} \rho_B, \quad g_\rho \bar{b} = \frac{g_\rho^2}{m_\rho^2} \frac{\rho_p - \rho_n}{2}, \quad \text{and}$$

$$\rho_{s,p} + \rho_{s,n} \equiv \gamma_s \sum_i \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{M^*}{E^*(k)} n_i(k) = - \frac{\partial \mathcal{V}(\bar{\phi})}{\partial M^*}$$

Fix Couplings

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Five empirical saturation properties, including saturation density, binding energy, symmetry energy, incompressibility, and Landau mass, are used to **solve** the couplings.

ρ_0 (fm ⁻³)	B_0 (MeV)	$S_{2,0}$ (MeV)	$K_{v,0}$ (MeV)	m_0^*/M
0.16	16.0	35.0	250	0.73

$$K_v \equiv 9 \left. \frac{\partial P}{\partial \rho_B} \right|_x \quad S_2 \equiv \frac{1}{8\rho_B} \left. \frac{\partial^2 \mathcal{E}}{\partial x^2} \right|_{x=0.5, \rho_B}$$

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$M = 939$ MeV, $m_s = 550$ MeV, $m_v = 783$ MeV, $m_\rho = 770$ MeV, $m_\pi = 138$ MeV, $f_\pi = 93$ MeV, and $g_A = 1.26$.

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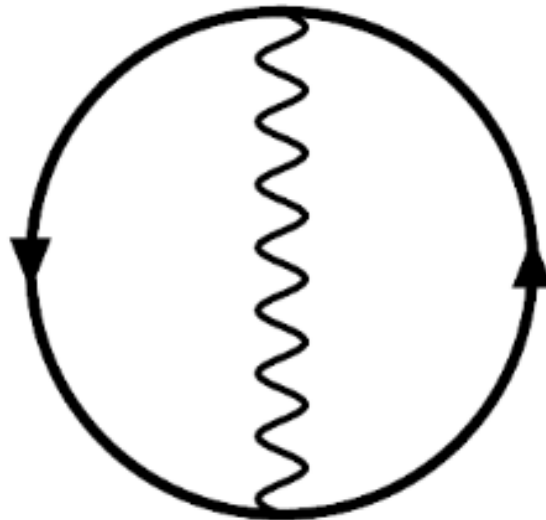
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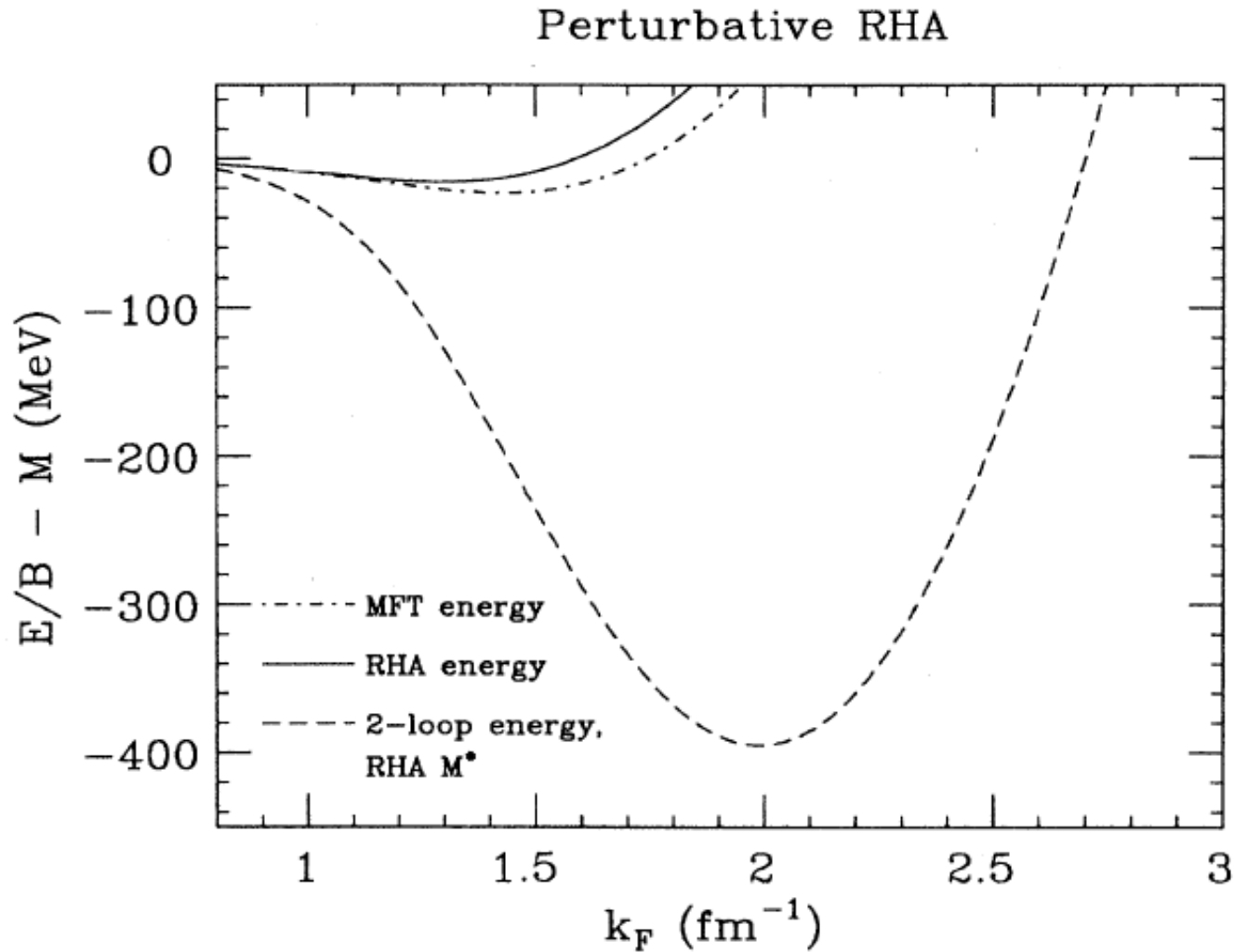
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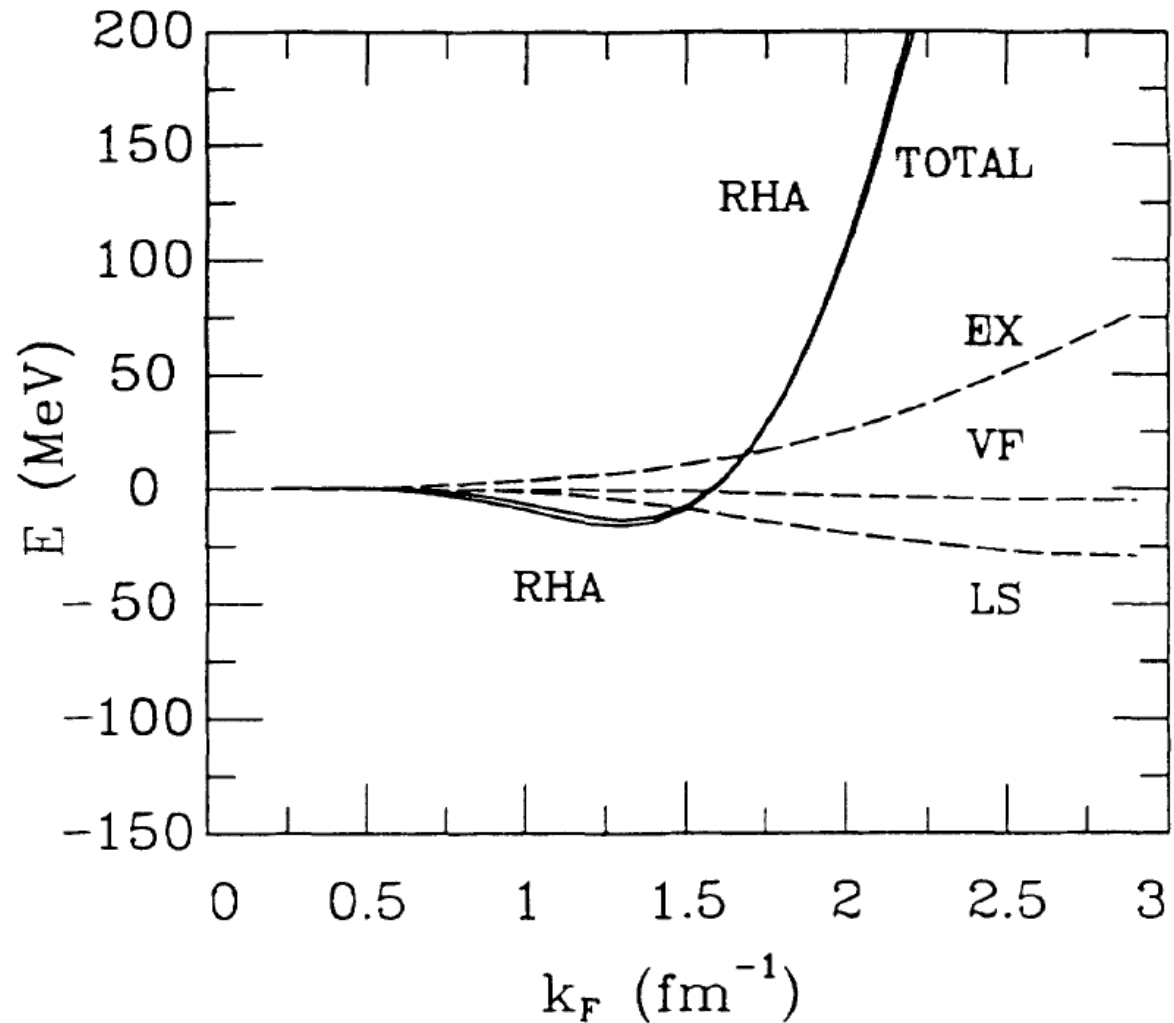
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- M. Prakash, P. J. Ellis, and J. I. Kapusta, Phys. Rev. C **45**, 2518 (1992)

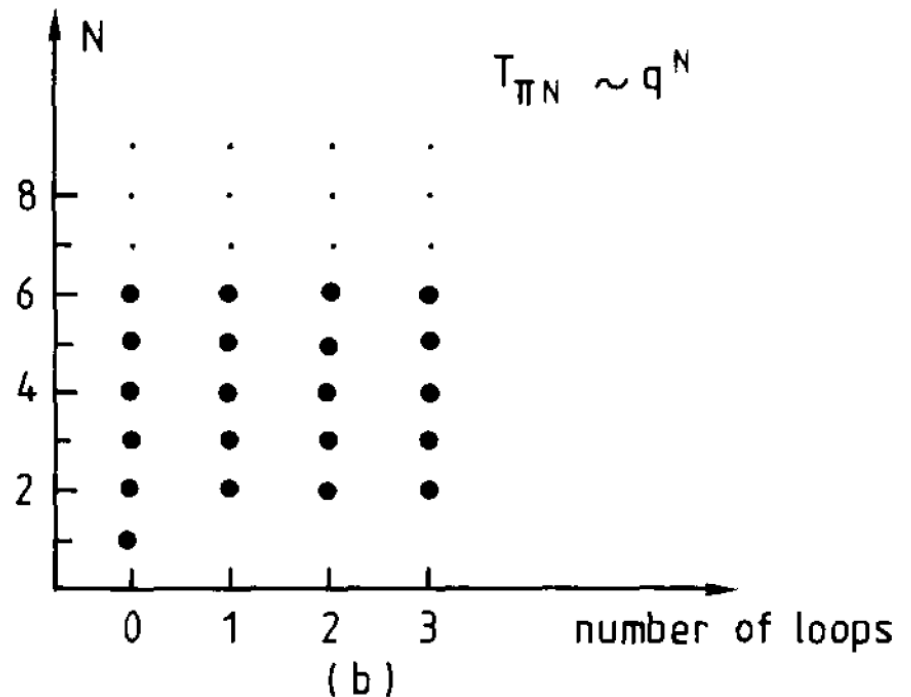
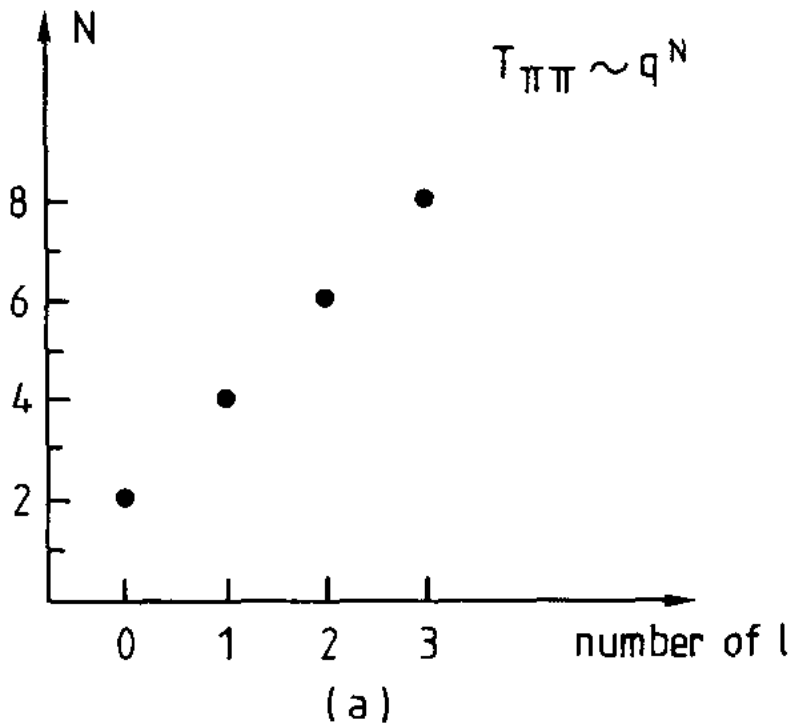
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- M. Prakash, P. J. Ellis, and J. I. Kapusta, Phys. Rev. C **45**, 2518 (1992)



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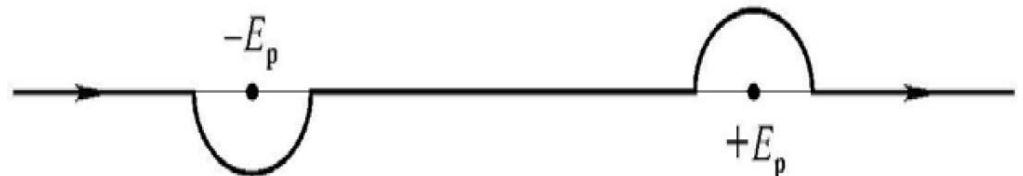
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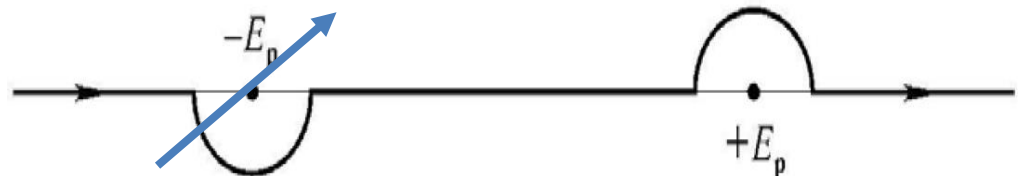
- R. J. Furnstahl, R. J. Perry, and B. D. Serot, Phys. Rev. C **40**, 321 (1989)
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- **Infrared regularizations**: P. J. Ellis and H.-B. Tang, Phys. Rev. C **57**, 3356 (1998); T. Becher and H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999)

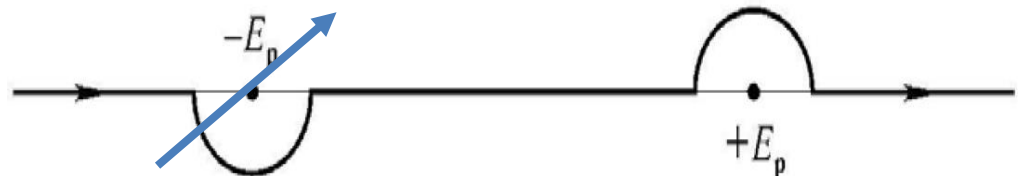
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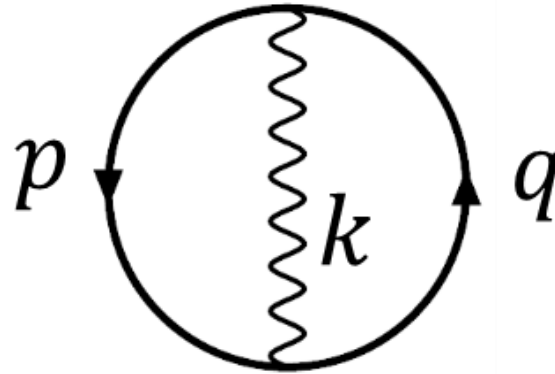
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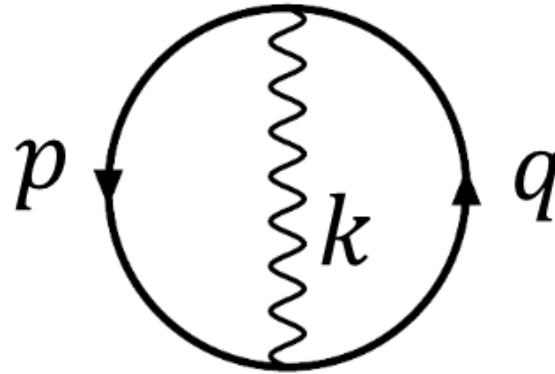
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- **Many body**: Y. Hu, J. McIntire, and B. Serot, Nucl. Phys. **A794**, 187 (2007).



TL contributions

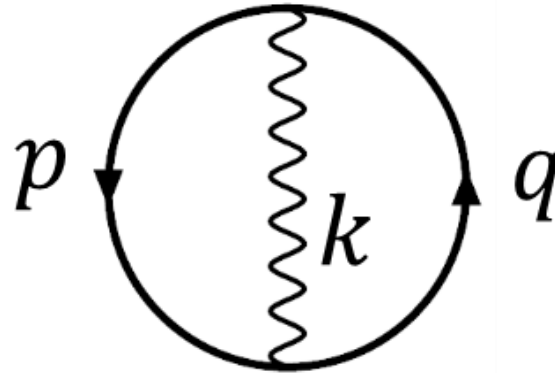


TL contributions



$$\frac{\gamma_s}{4} g^2 \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \text{Tr} [G(q)\Gamma(p - q)G(p)\Gamma(q - p)] D(q - p)$$

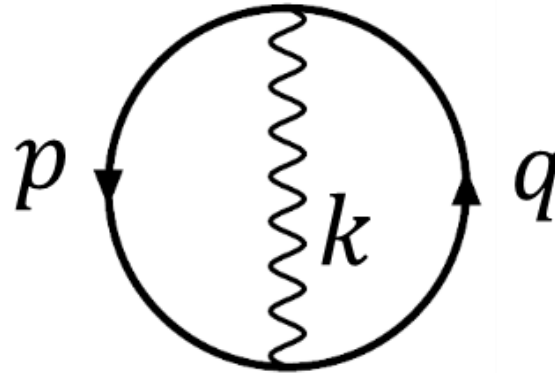
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Y. Hu, J. McIntire, and
B. Serot, Nucl. Phys.A
794, 187 (2007)

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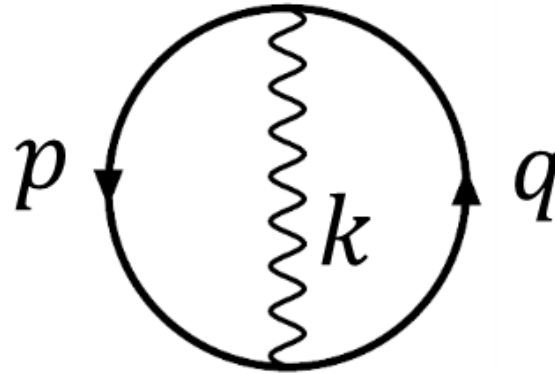


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Y. Hu, J. McIntire, and
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Absorbed into
contact terms
in lagrangian

TL contributions

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$$d\tau_{\mathbf{p}} \equiv d^3\mathbf{p} / [(2\pi)^3 2E^*(p)], \quad f_s(p^*, q^*) \equiv 4(p^* \cdot q^* + M^{*2}), \quad D(k; m) \equiv \frac{1}{k \cdot k - m^2}$$

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 Impact EOS, and $\epsilon_{(1),i}(p) = \epsilon_{(0),i}(p) + \delta\epsilon_{(1),i}(p)$

Note: TLs also modifies bg's dependence on density

Couplings

	g_s^2	g_v^2	g_ρ^2	κ_3	κ_4	L (MeV)
MFT	96.36	118.45	70.13	2.08	-6.77	103.62
TL(235)	71.26	49.58	60.72	5.94	-2.48	83.66
TL(250)	74.03	56.58	57.97	4.84	-4.47	85.09
TL(270)	74.65	61.45	58.06	3.70	1.97	84.51

TL calculations with different incompressibilities, 235, 250, 270 MeV

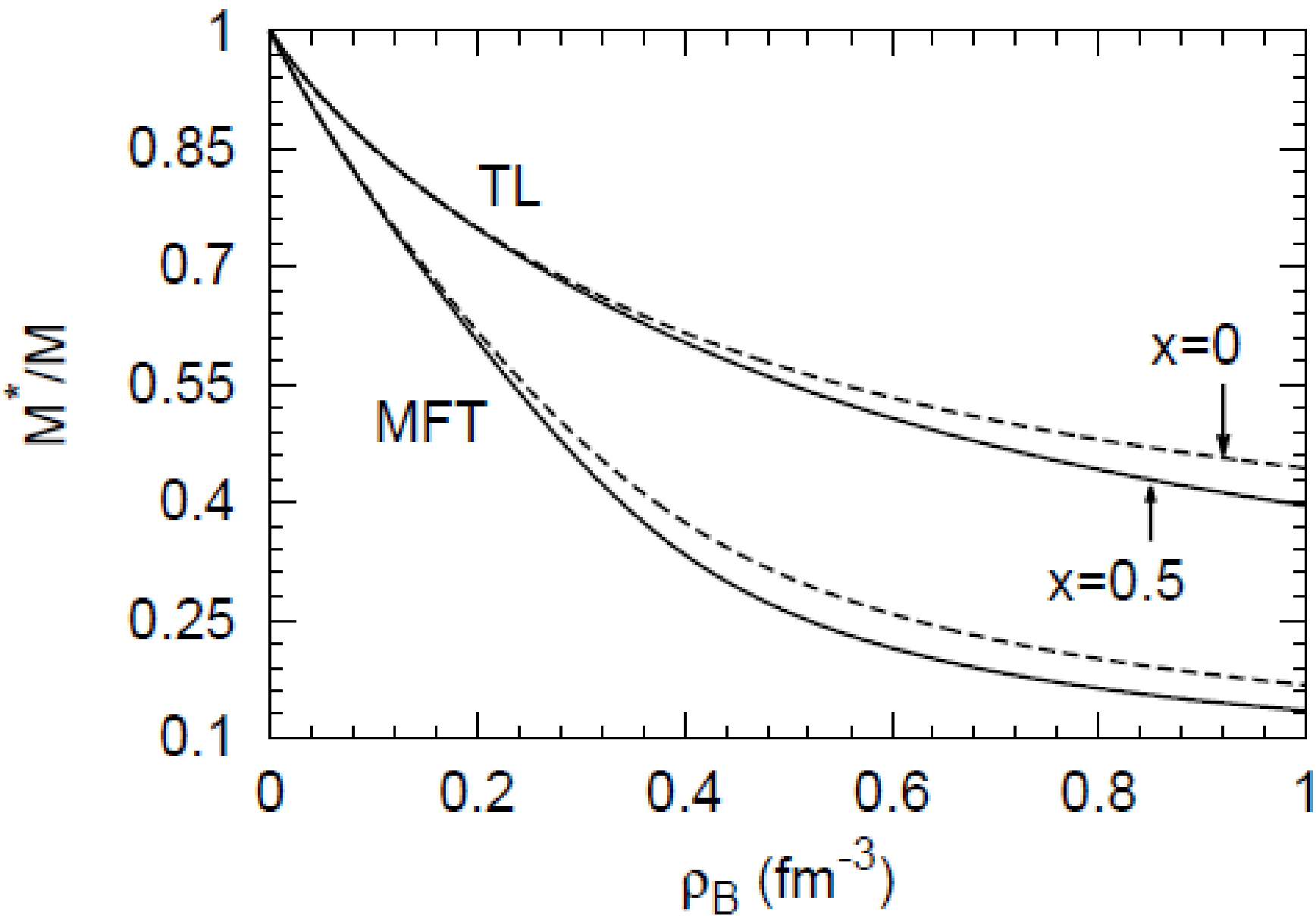
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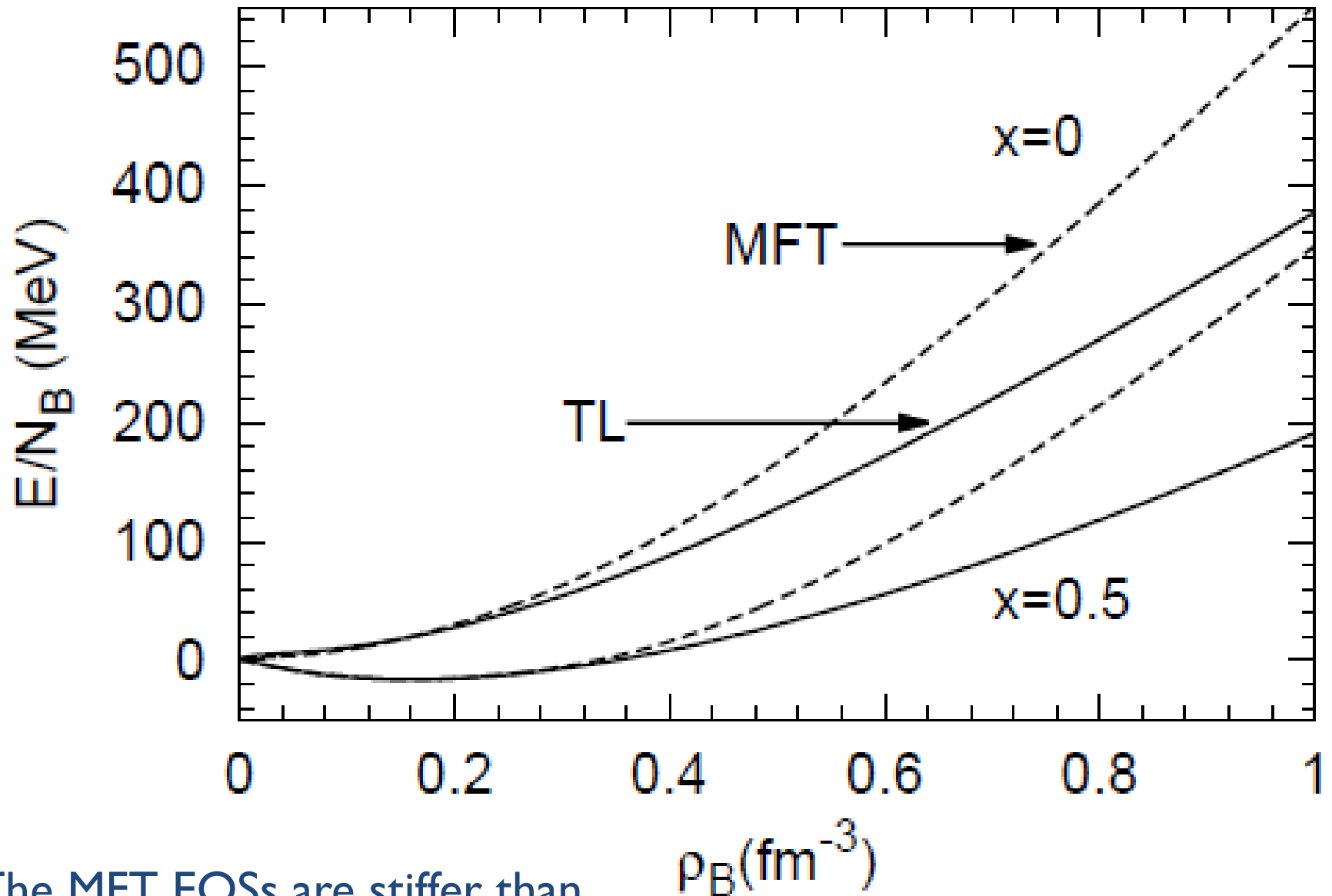
TL calculations with different incompressibilities, 235, 250, 270 MeV

- In the MFT, the scalar coupling is smaller than the vector coupling, while in the TL calculation the former is larger
- The TL calculation's L is smaller than the MFT's

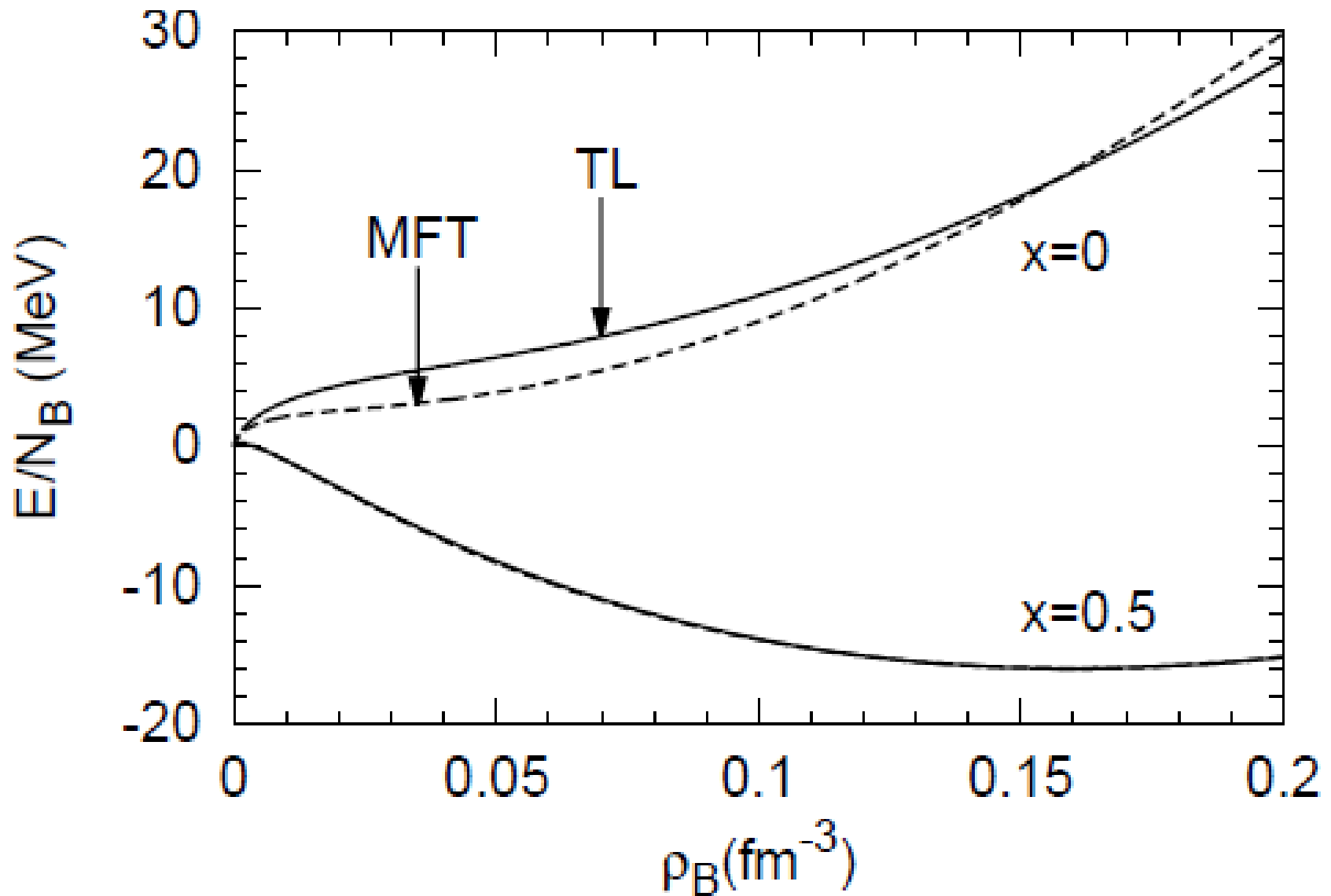
Zero T results



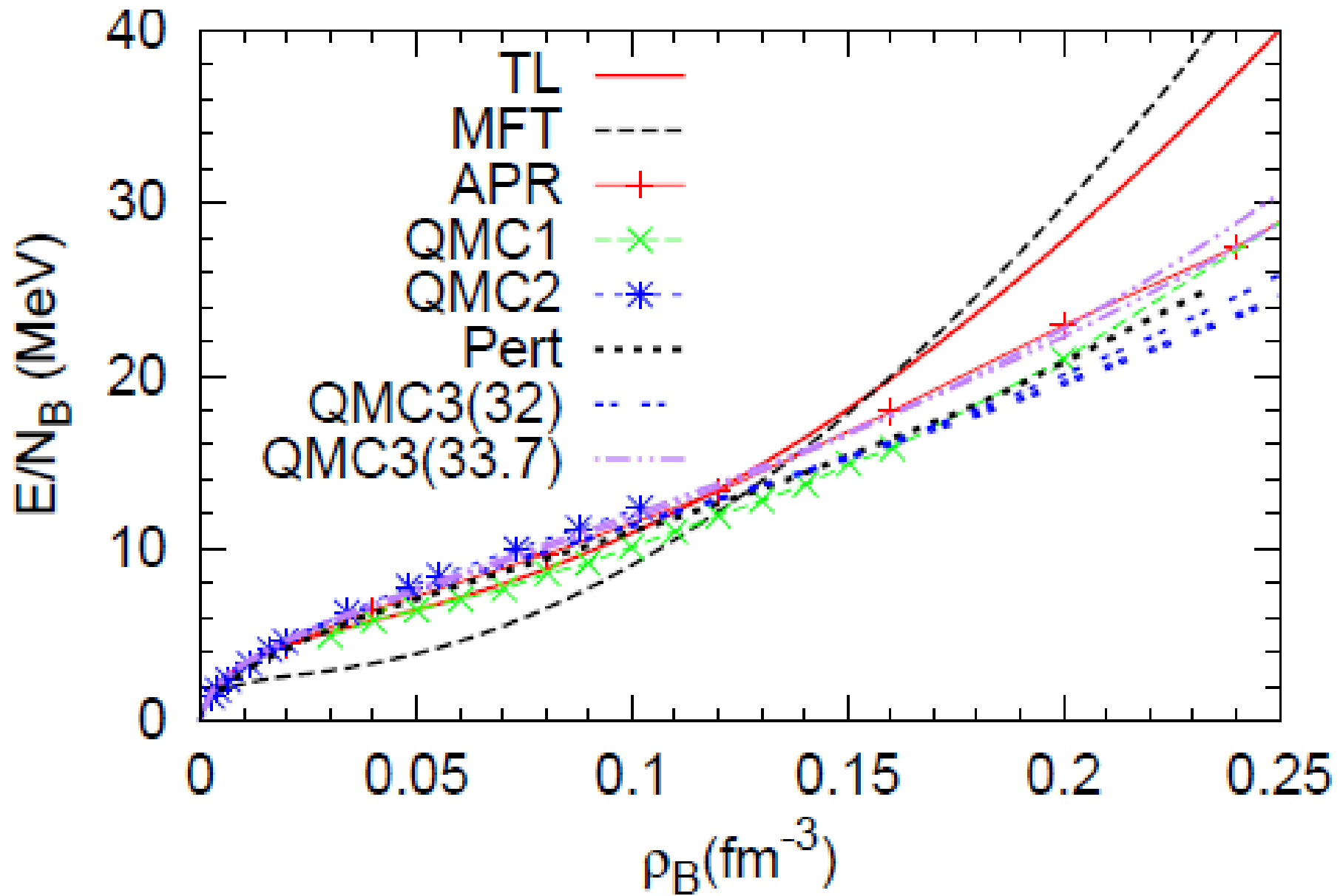
The TL calculation is more nonrelativistic than the MFT, for both pure neutron matter (PNM) and symmetrical nuclear matter (SNM)



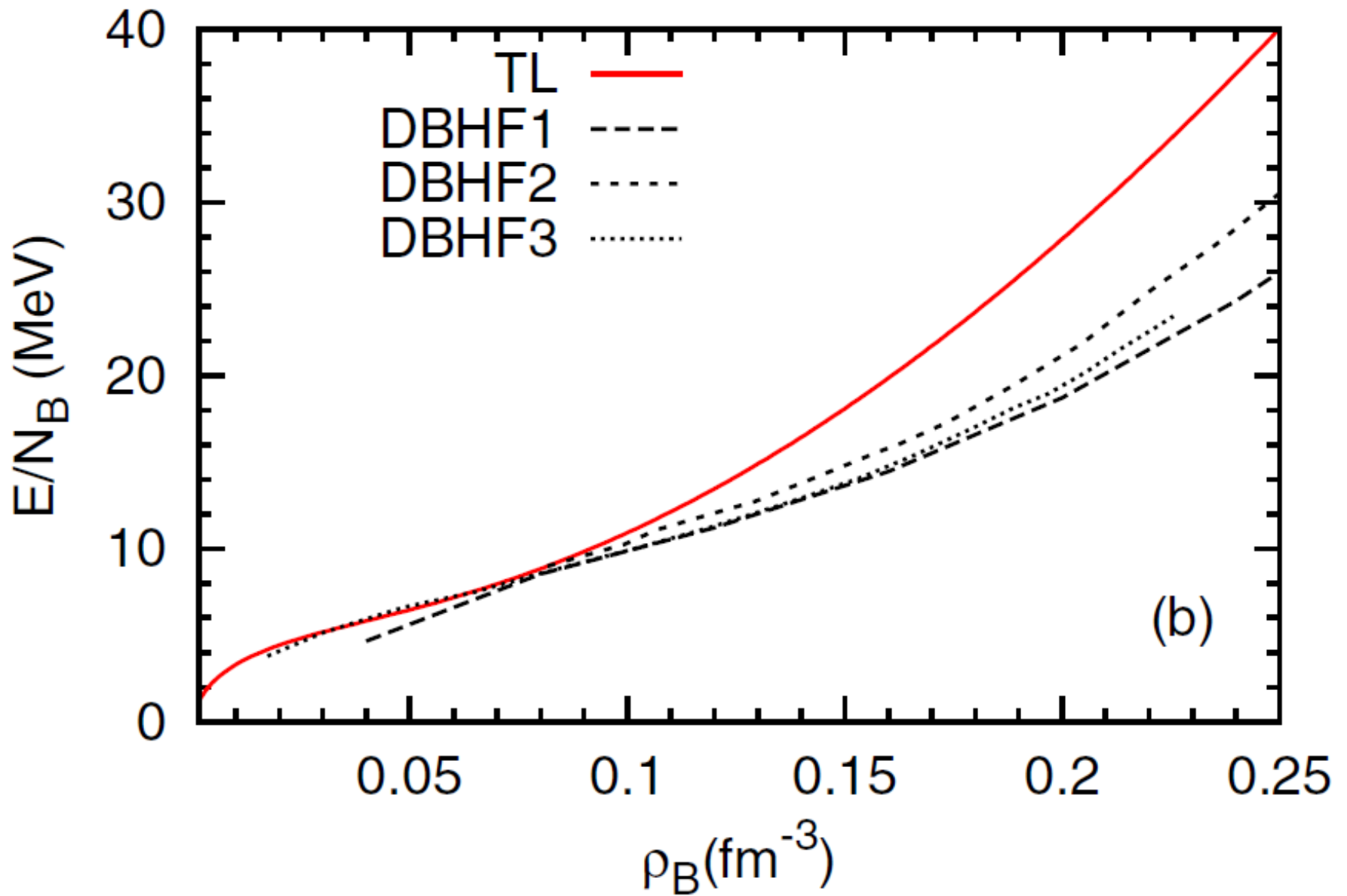
The MFT EOSs are stiffer than the TLs beyond saturation density



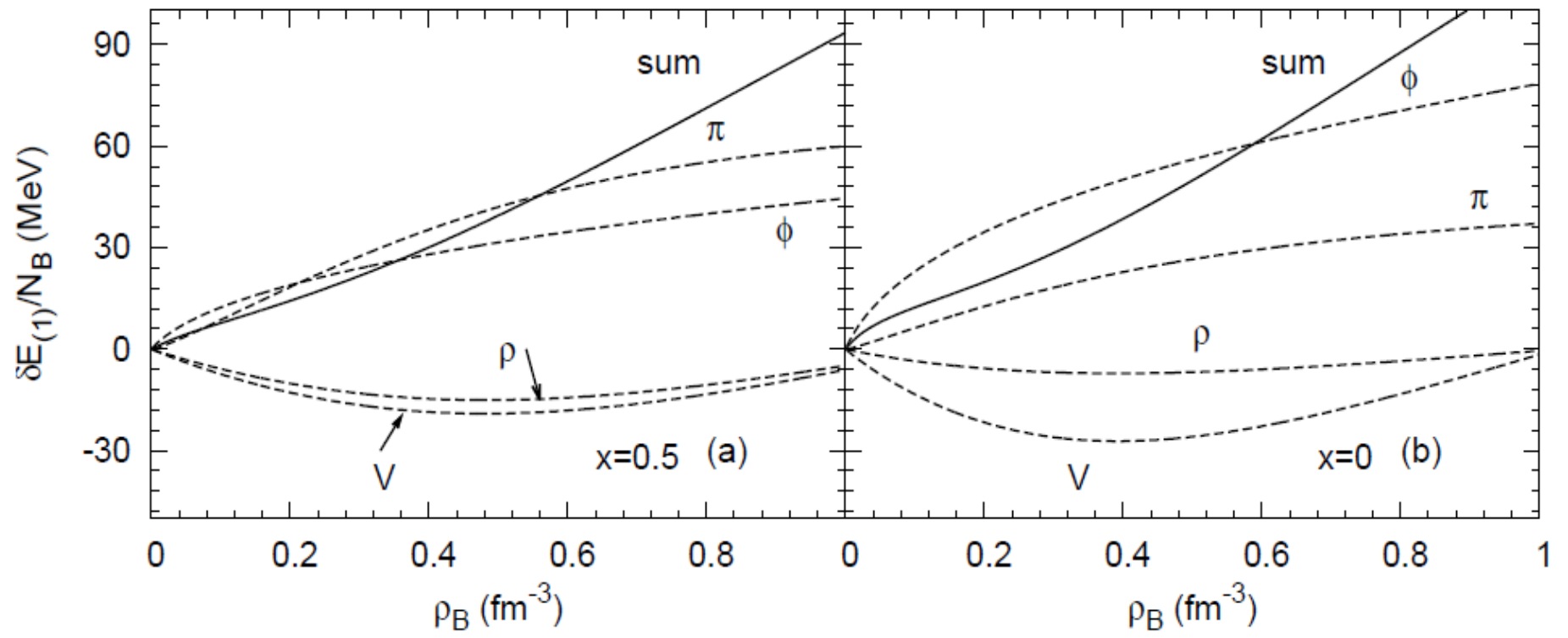
For SNM, the TL and the MFT are close around and below saturation density, but for PNM, the two differ

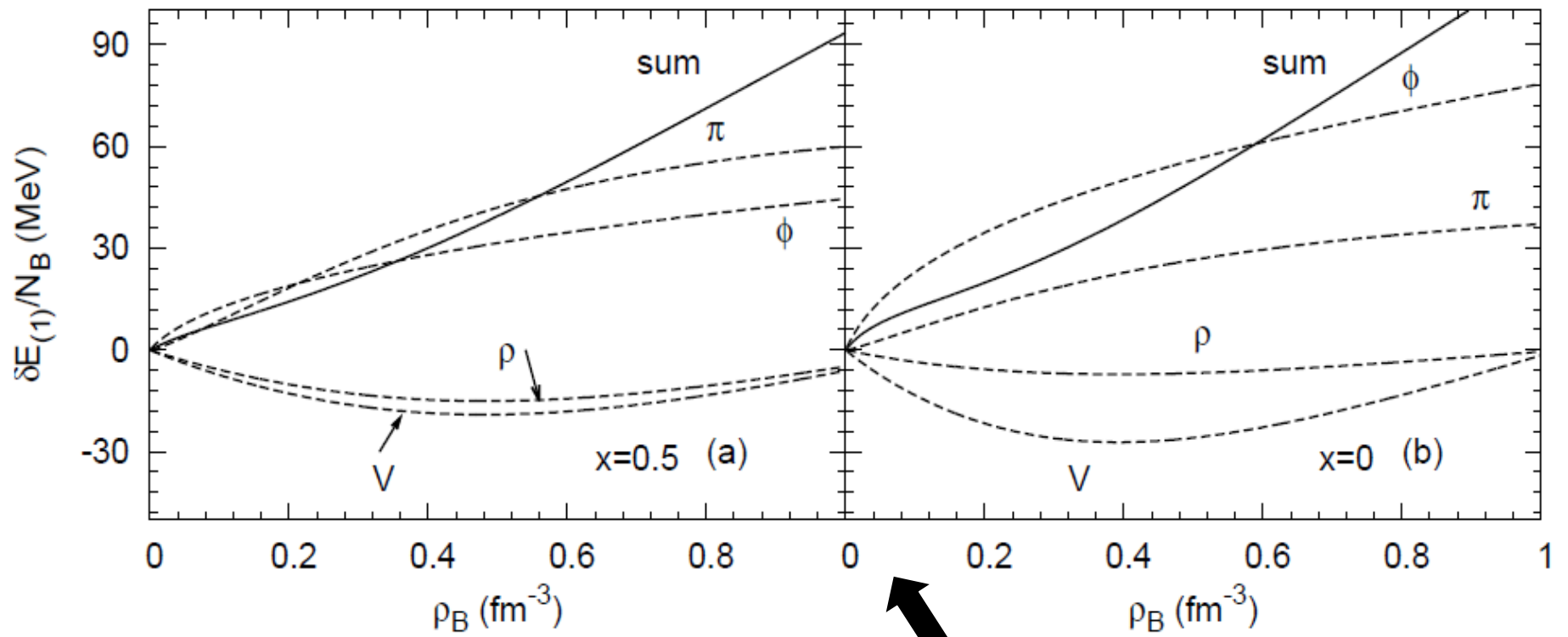


A. Akmal, *et al.*, PRC **58**, 1804 (1998); P. Armani, *et al.*, J. Phys. Conf. Ser. **336**, 012014 (2011);
 G. Wlazlowski, *et al.*, PRL **113**, 182503 (2014); L. Coraggio, *et al.*, PRC **87**, 014322 (2013);
 S. Gandolfi, *et al.*, PRC **85**, 032801 (2012)

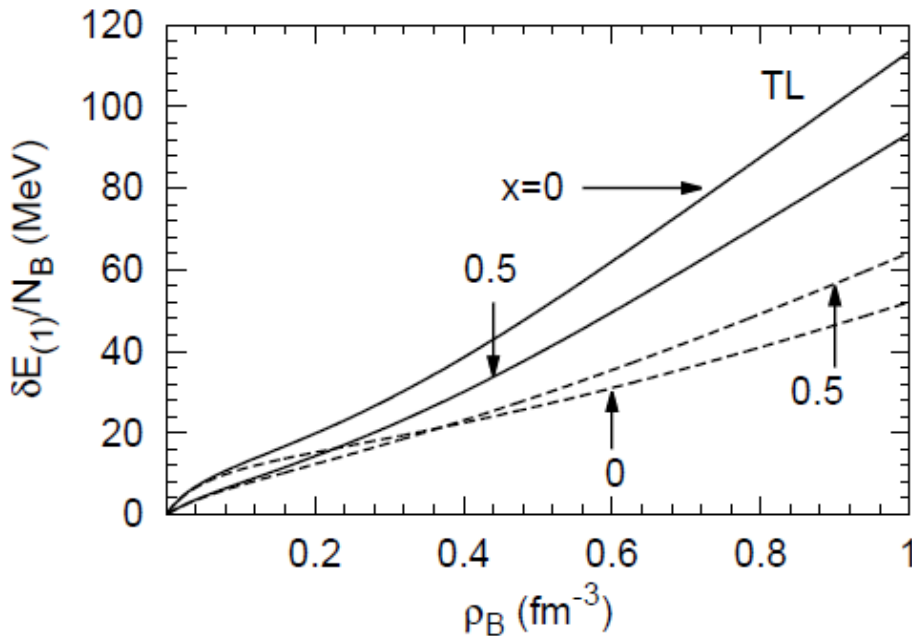
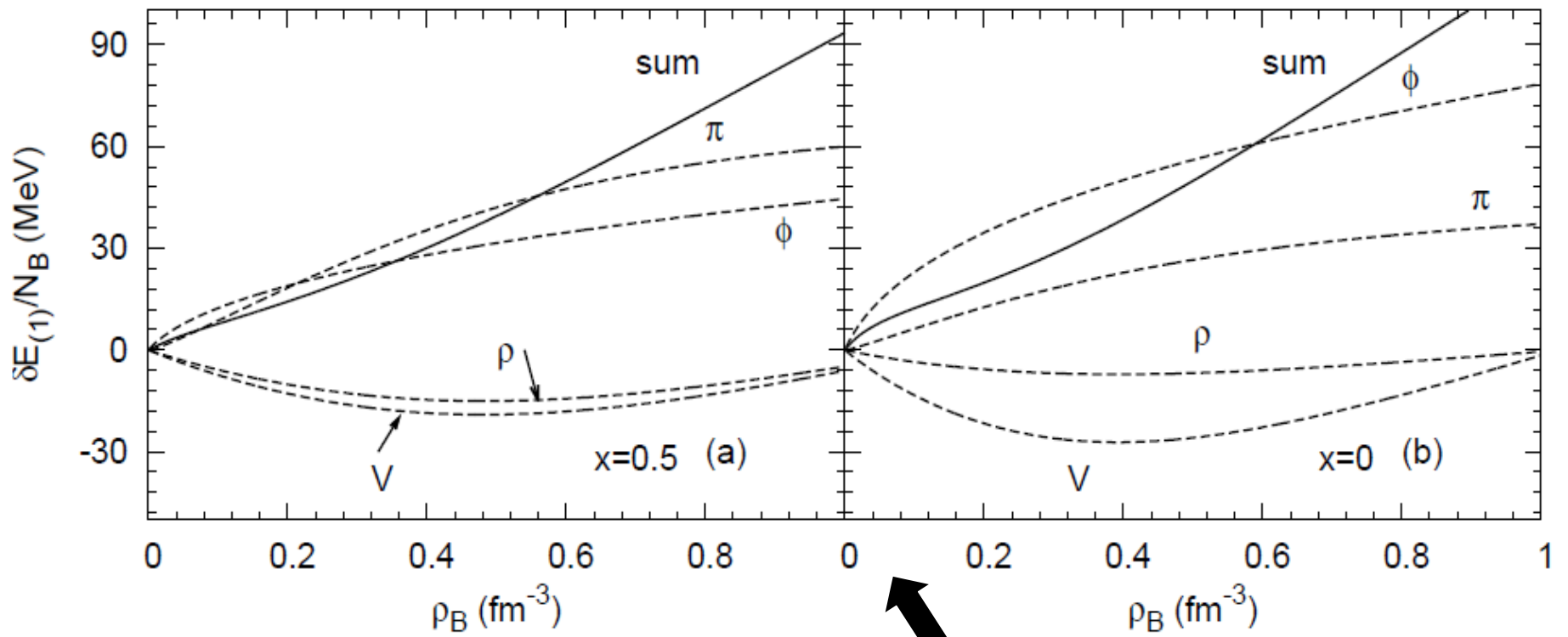


H. Muether, M. Prakash, and T. L. Ainsworth, PLB **199**, 469 (1987); E. N. E. van Dalen, C. Fuchs, and A. Faessler, PRC **72**, 065803 (2005); F. Sammarruca, B. Chen, L. Coraggio, N. Itaco, and R. Machleidt, PRC **86**, 054317 (2012).



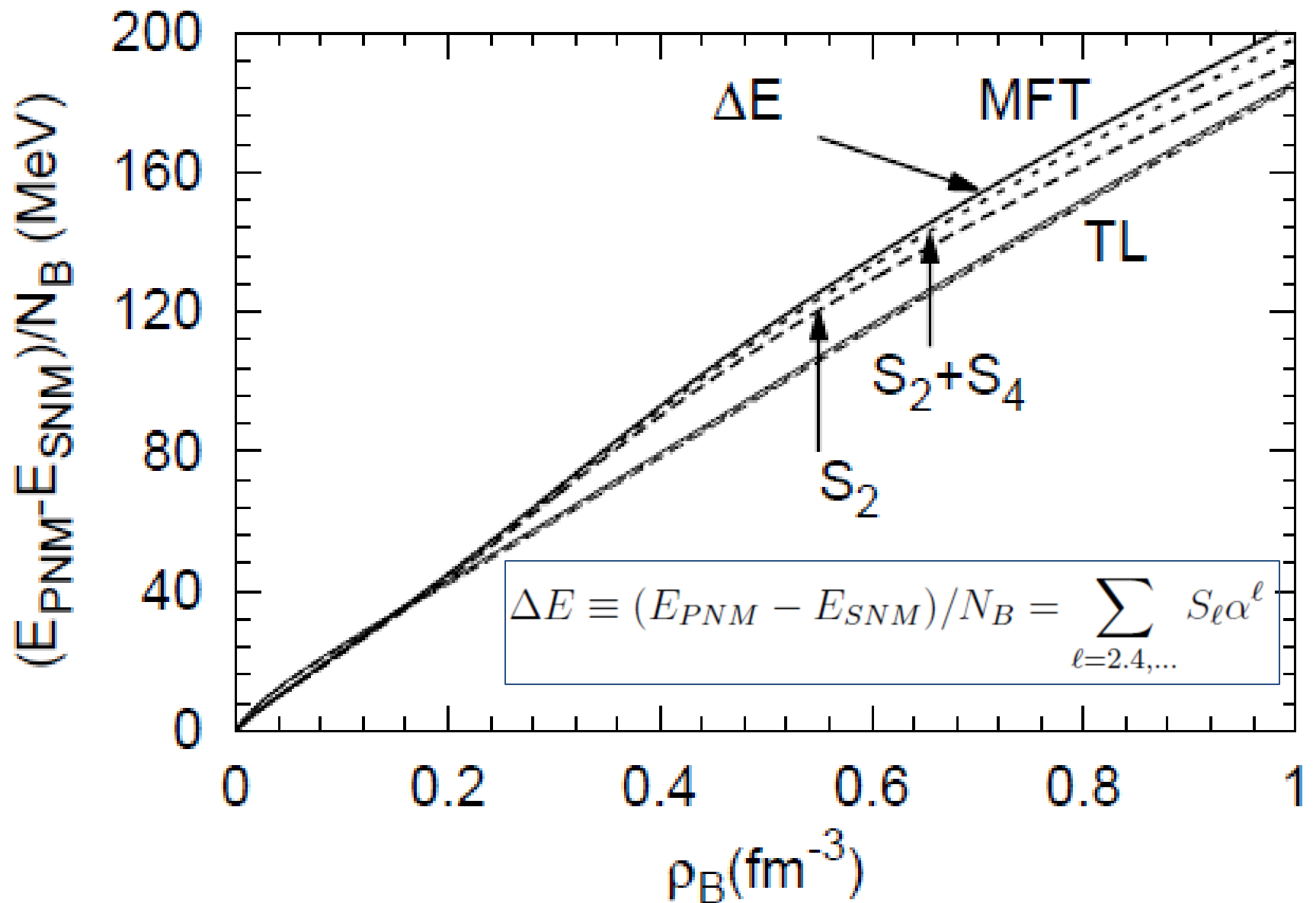


Strong cancellations among mesons

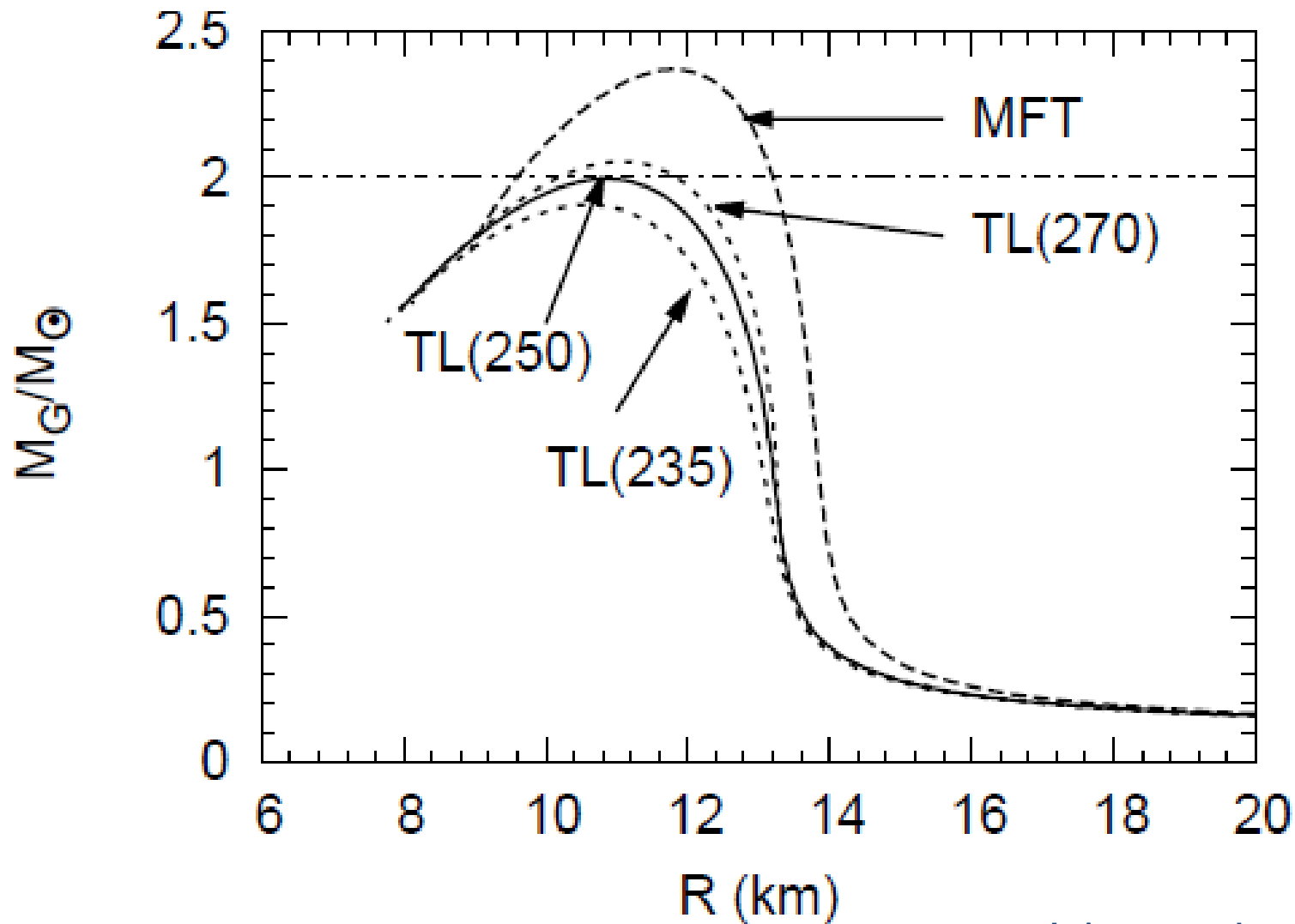


Strong cancellations among mesons

Nonrelativistic reduction of the full TL contributions

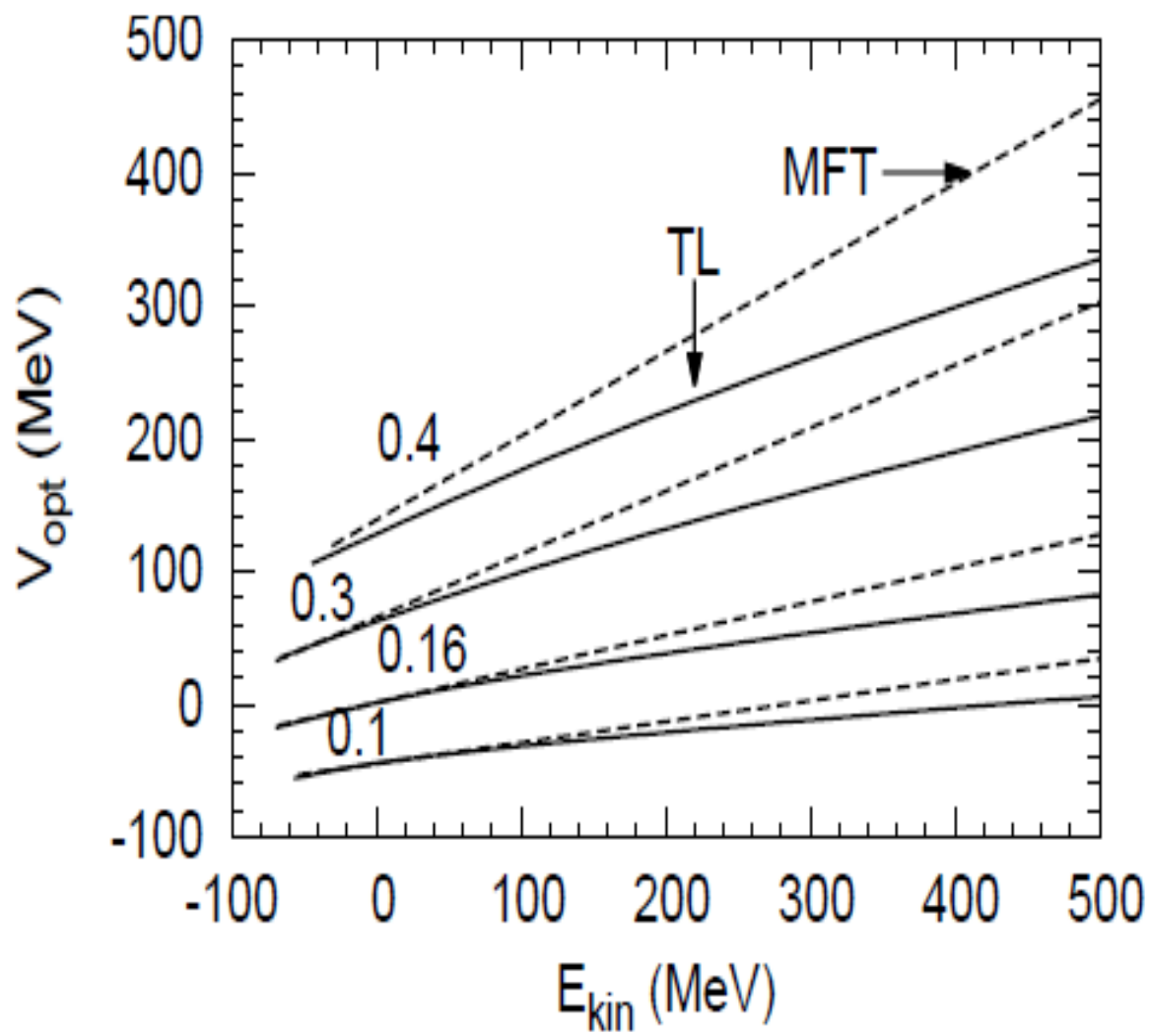


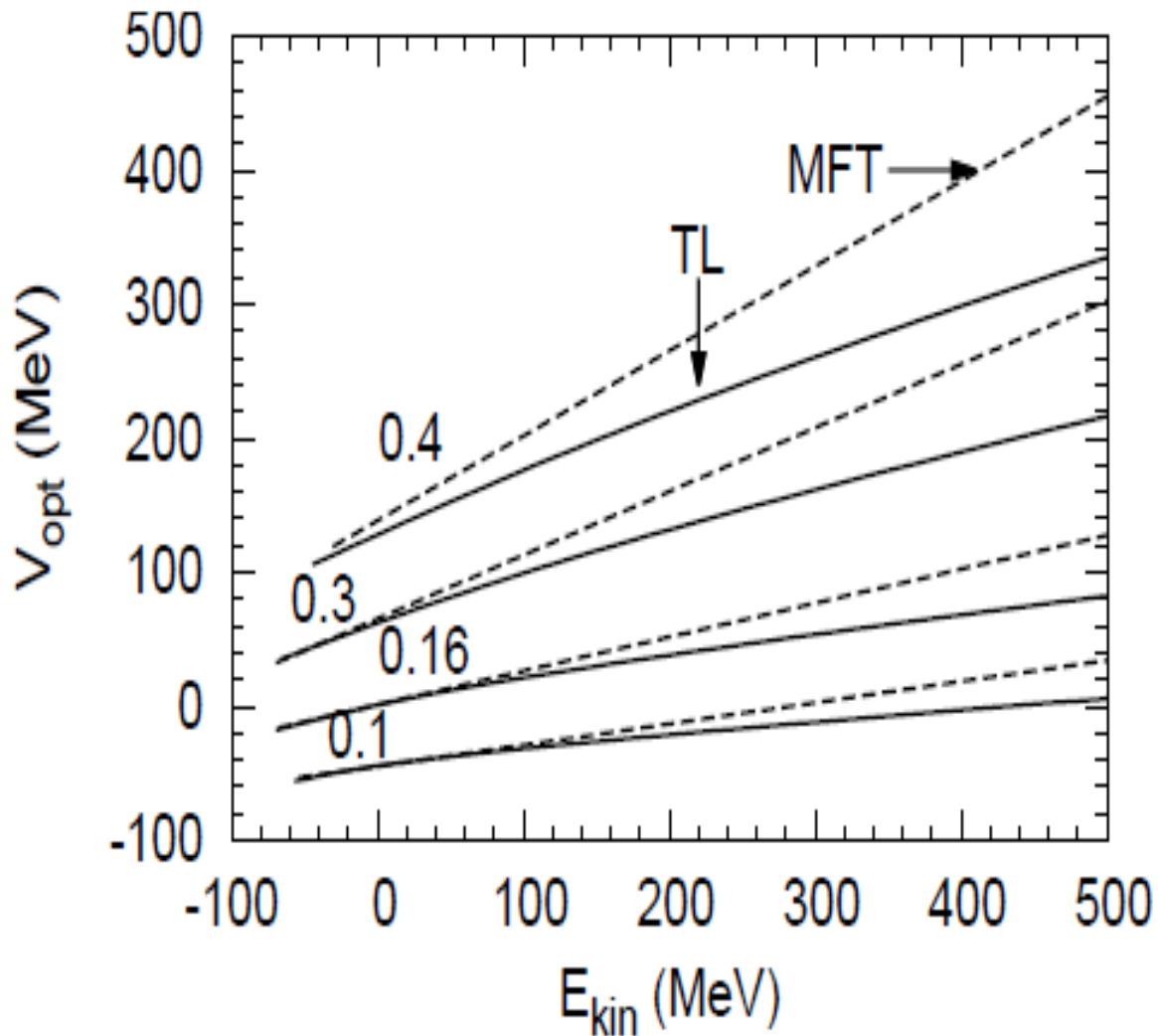
- Symmetry energy (S₂) saturates the PNM and SNM energy difference
- The TL S₂ is lower below saturation density and higher above the density than the MFT



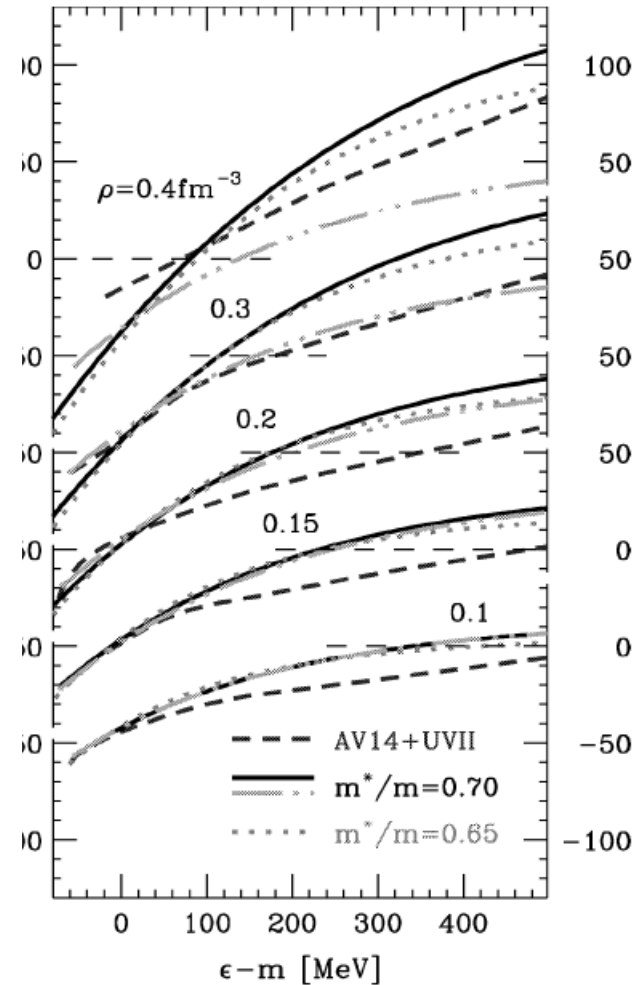
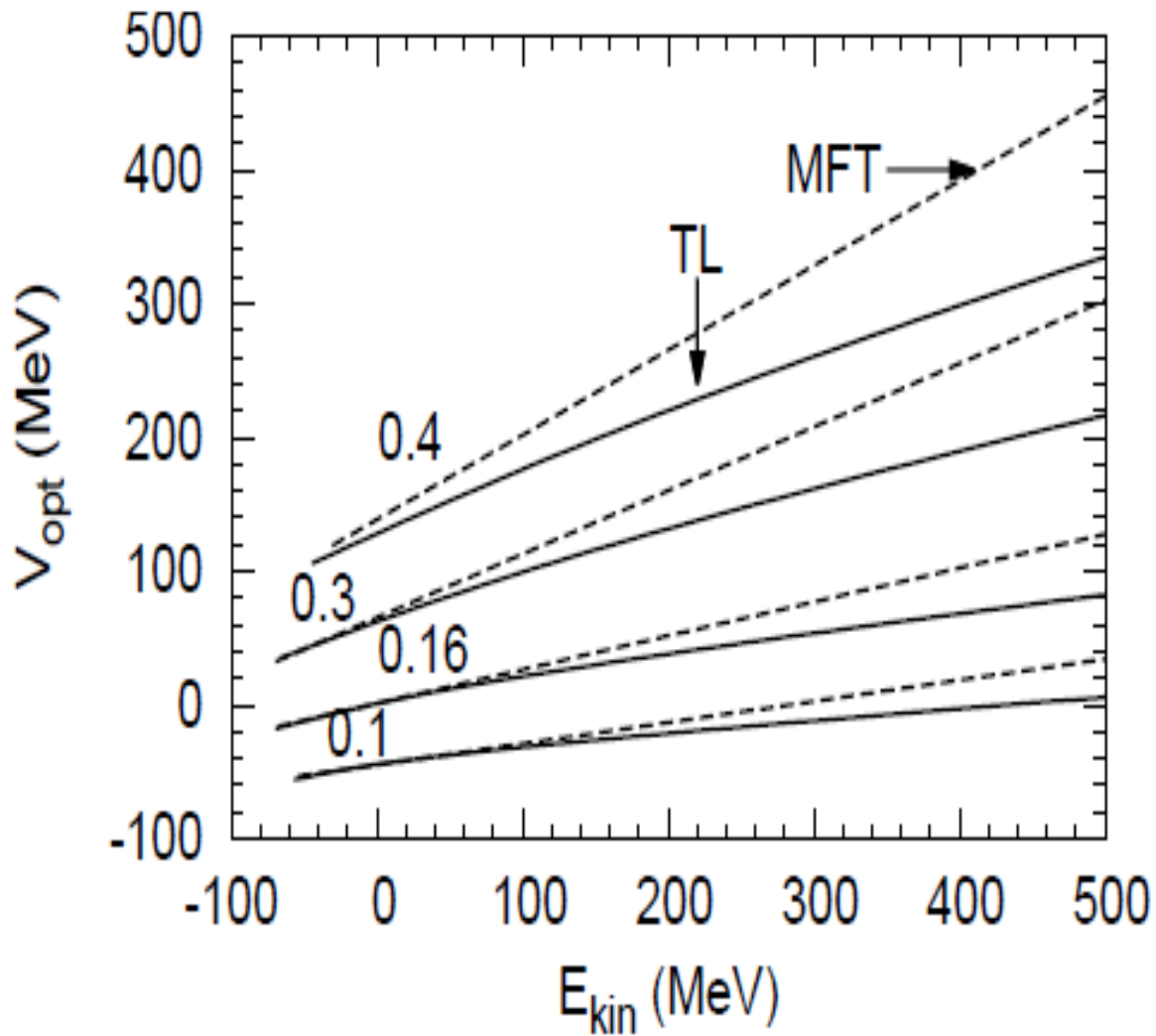
- Neutron star (NS) maximum mass observed:
 $2.01 \pm 0.04 M_\odot$, $R_{max} = 11.0 \pm 1.0$ km and
 $R_{1.4} = 11.5 \pm 0.7$ km
- Based on beta-equilibrium EOS

J. Antoniadis, et.al.,
 Science **340**, 448 (2013)
 J. M. Lattimer and M.
 Prakash, Phys. Rept **621**,
 127 (2016)





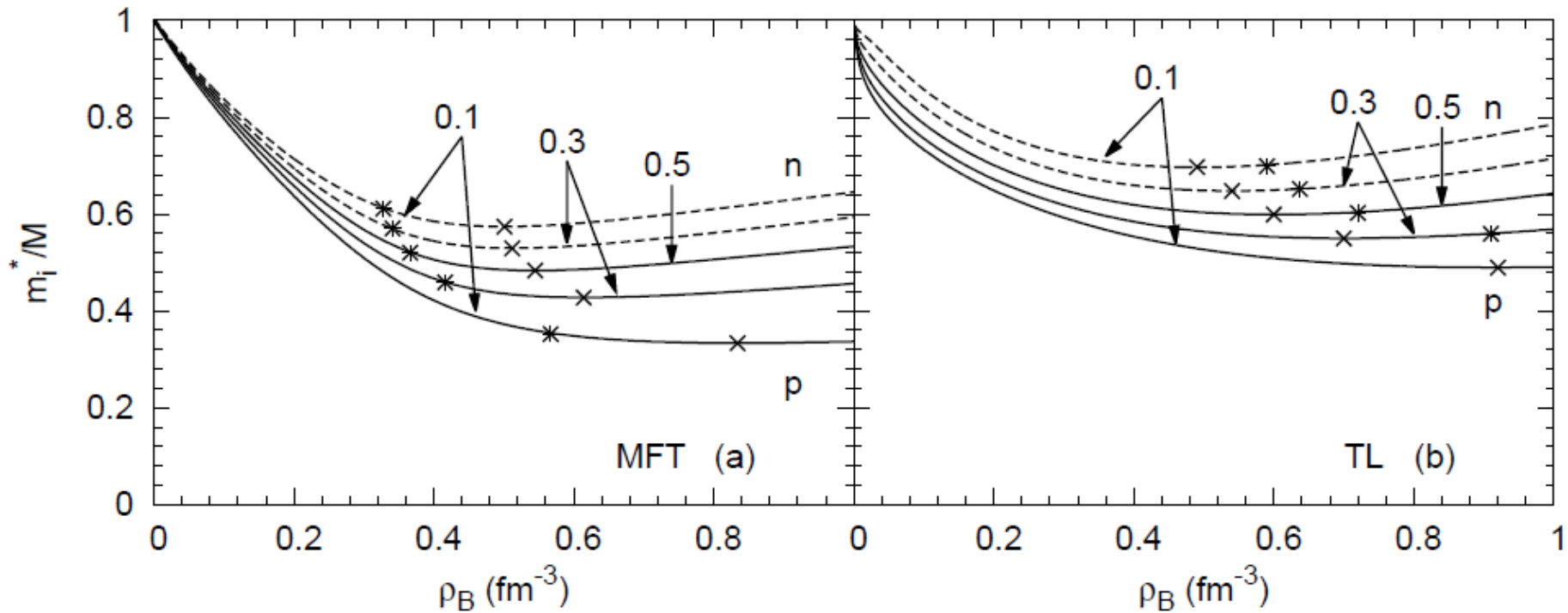
- The MFT grows linearly with E_{kin} , much faster than the TL
- The TL agrees better with the microscopic calculation and extraction from medium-energy heavy-ion collisions



P. Danielewicz, NPA
673, 375 (2000)

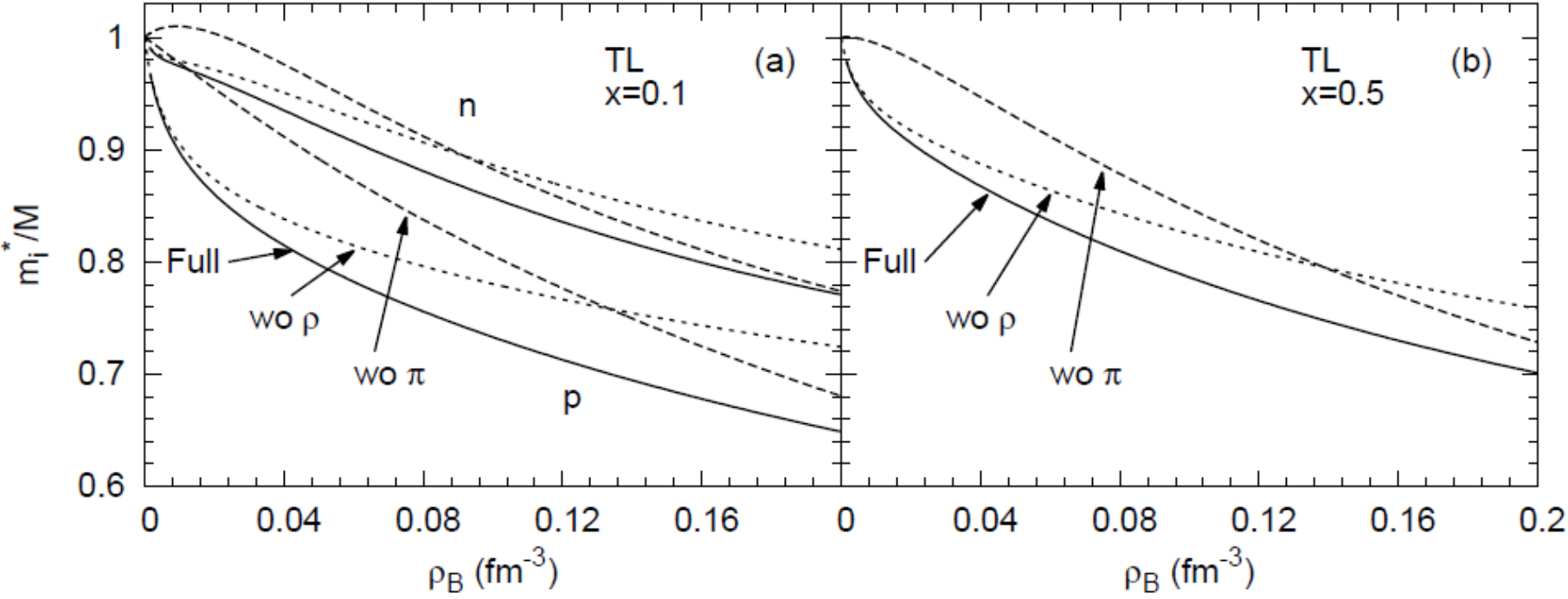
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$$m_i^*(\rho_B, x) \equiv k_{F,i} \left(\frac{\partial \epsilon_i(k)}{\partial k} \right)^{-1} \Big|_{k=k_{F,i}}$$



- The TL Landau masses are larger than the MFTs
- At x -density, Fermi momentum $=M^*$.
- At x -density, Landau mass first derivative $=0$

Pion plays an important role in the low density behavior of the Landau mass



Finite T results

Formalisms: perturbative

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$$Z \equiv \text{Tr} \exp [-\beta (H - \mu_p N_p - \mu_n N_n)] \equiv \exp [-\beta \Omega (T, V, \mu_{p,n}; \bar{\phi}, \bar{V}, \bar{b})]$$

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At MFT level, this approach is the same as the perturbative one.
At two-loop level, they are different.

Landau's Fermi liquid theory

In the degenerate limit,

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$$\frac{E_{th}}{N_B} = \frac{T^2}{\rho_B} \sum_i a_i \rho_i \quad [Q_{th} \equiv Q(\rho_B, x, T) - Q(\rho_B, x, T = 0)]$$

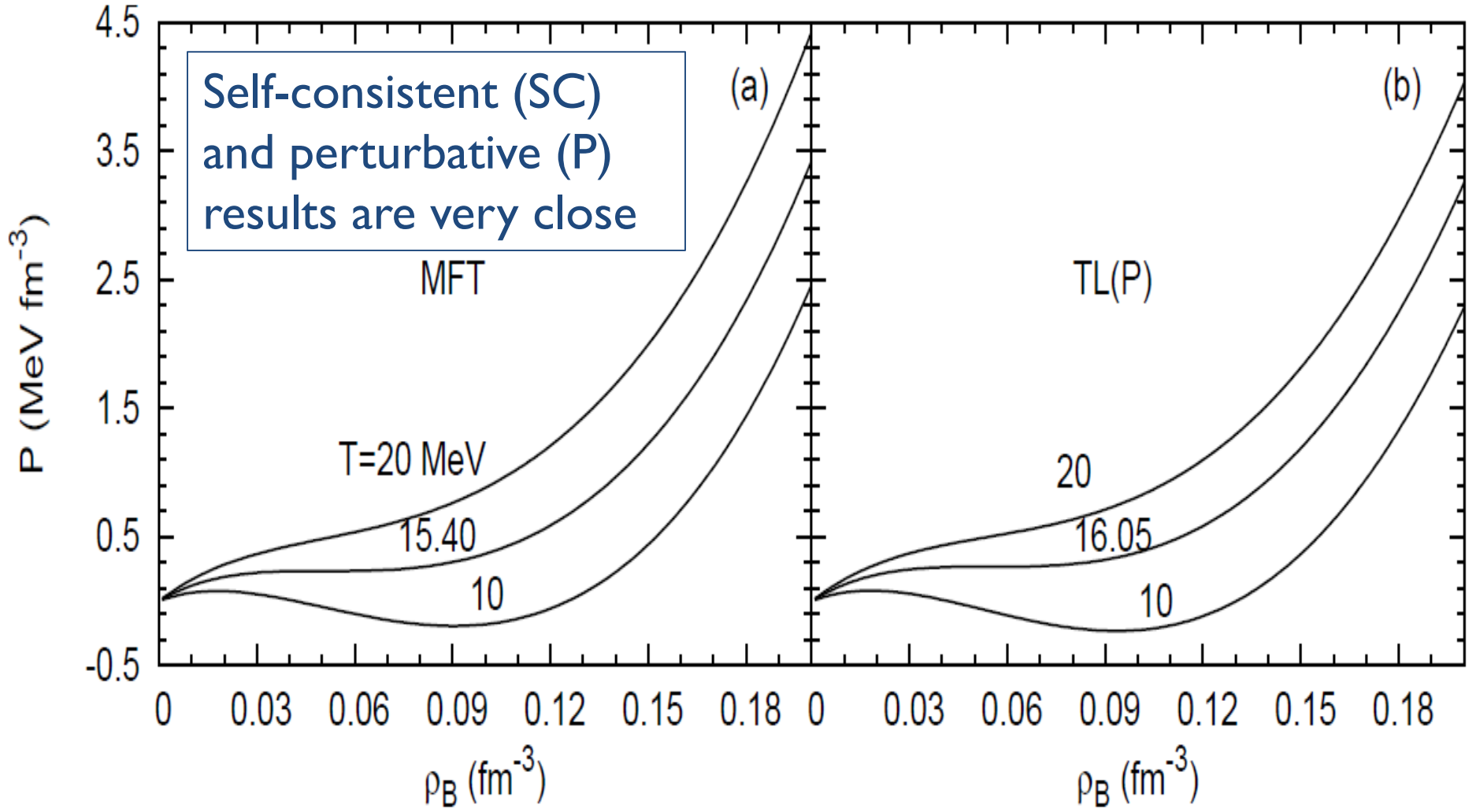
Landau's Fermi liquid theory

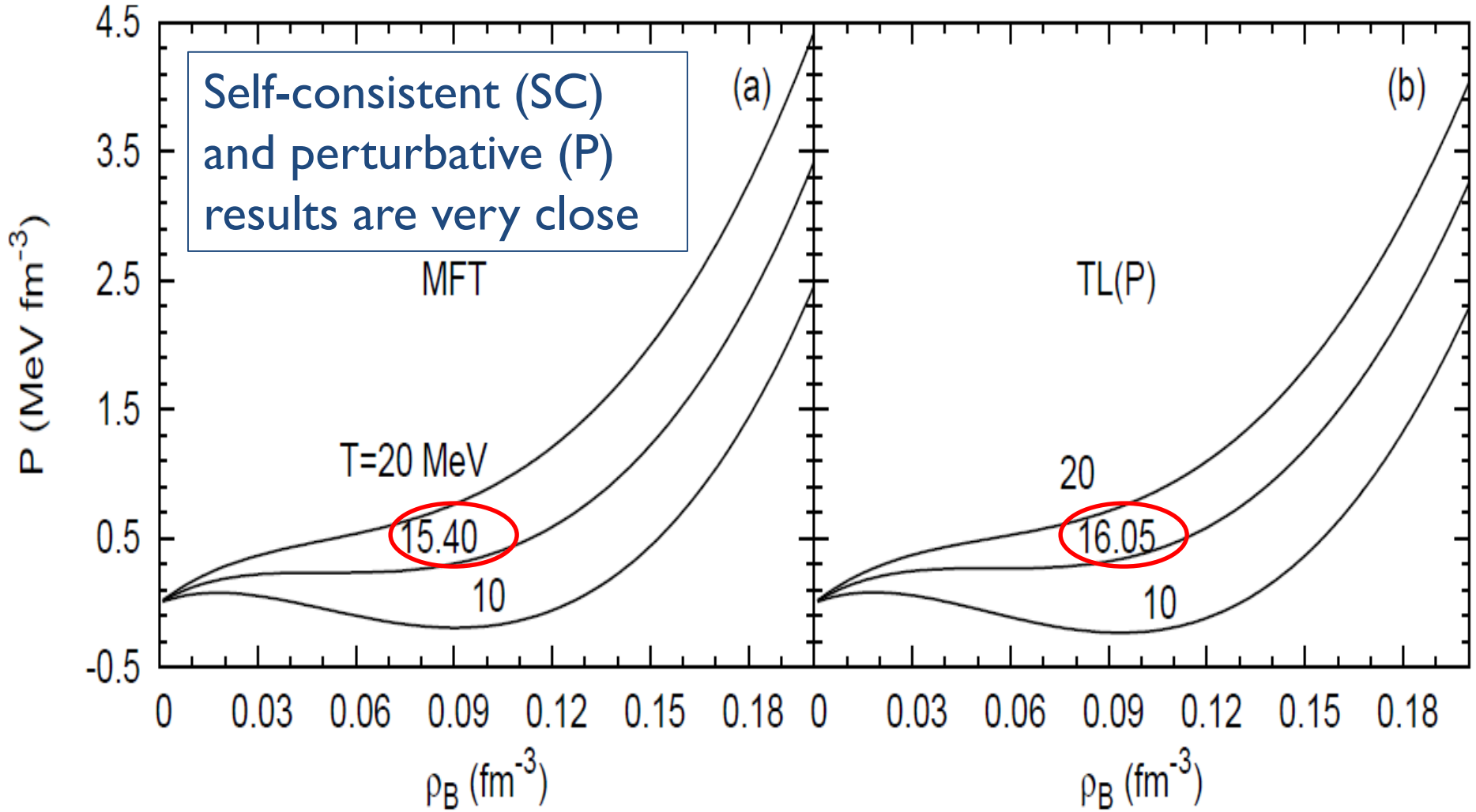
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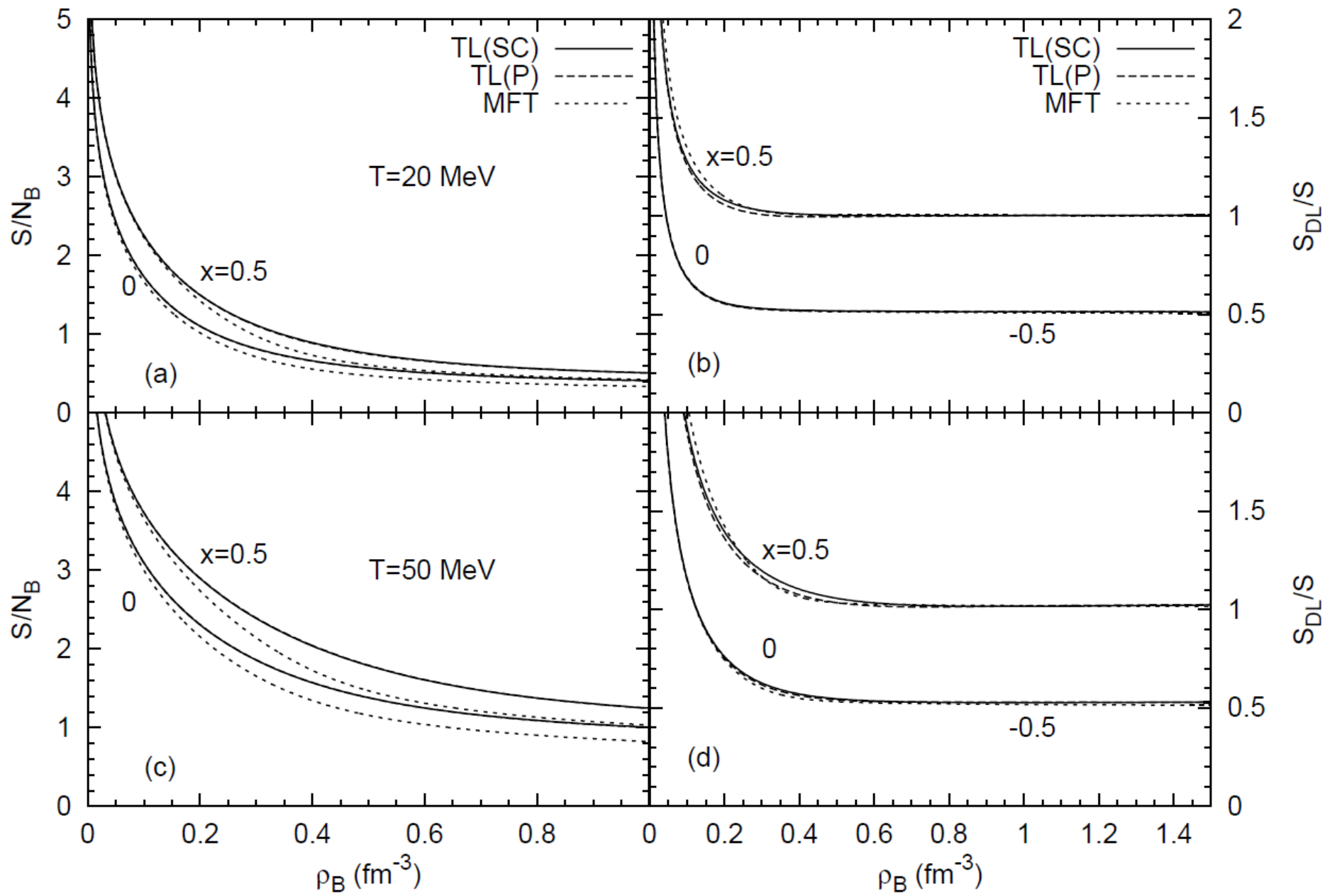
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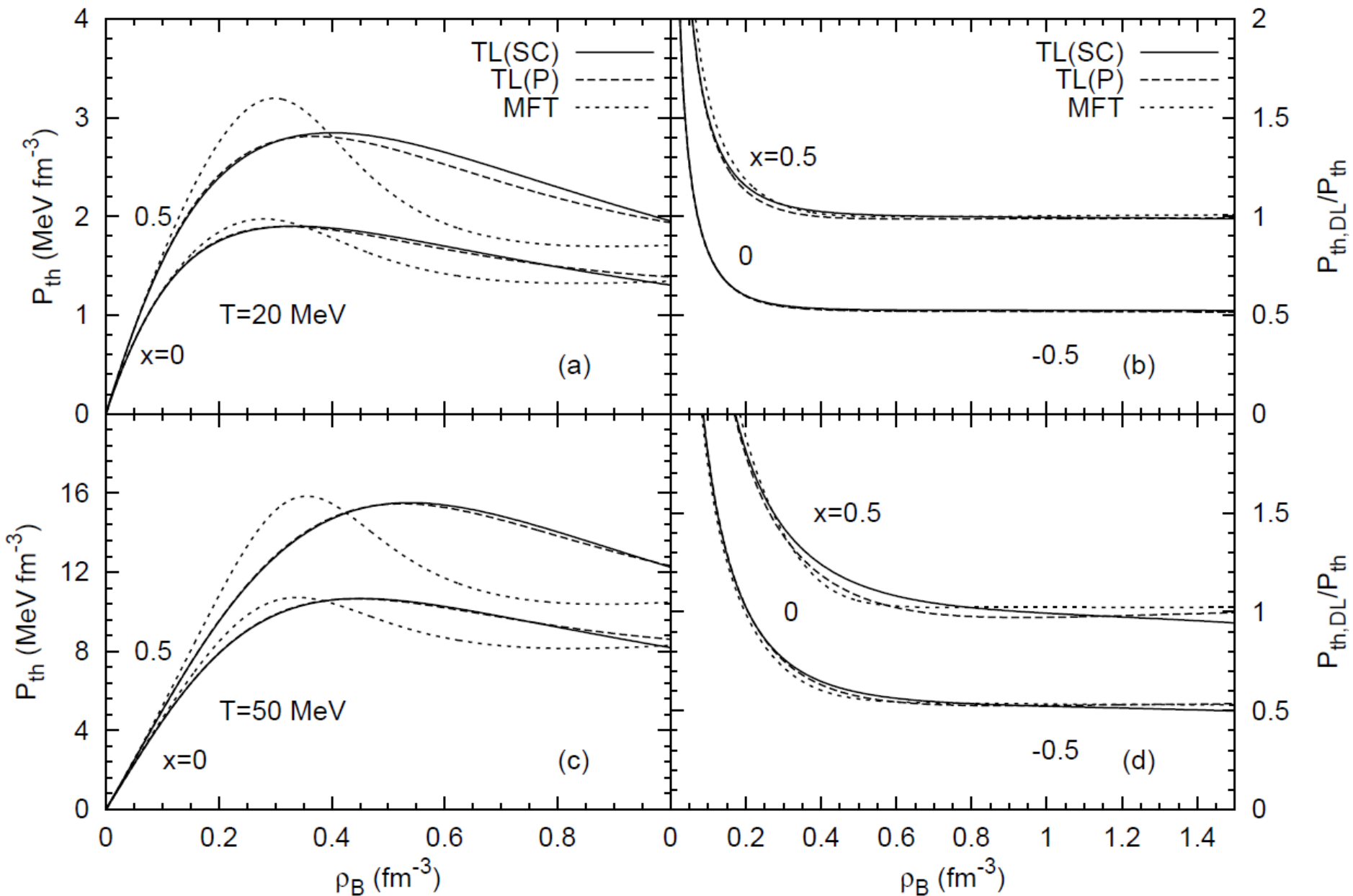
A good way to test our calculation





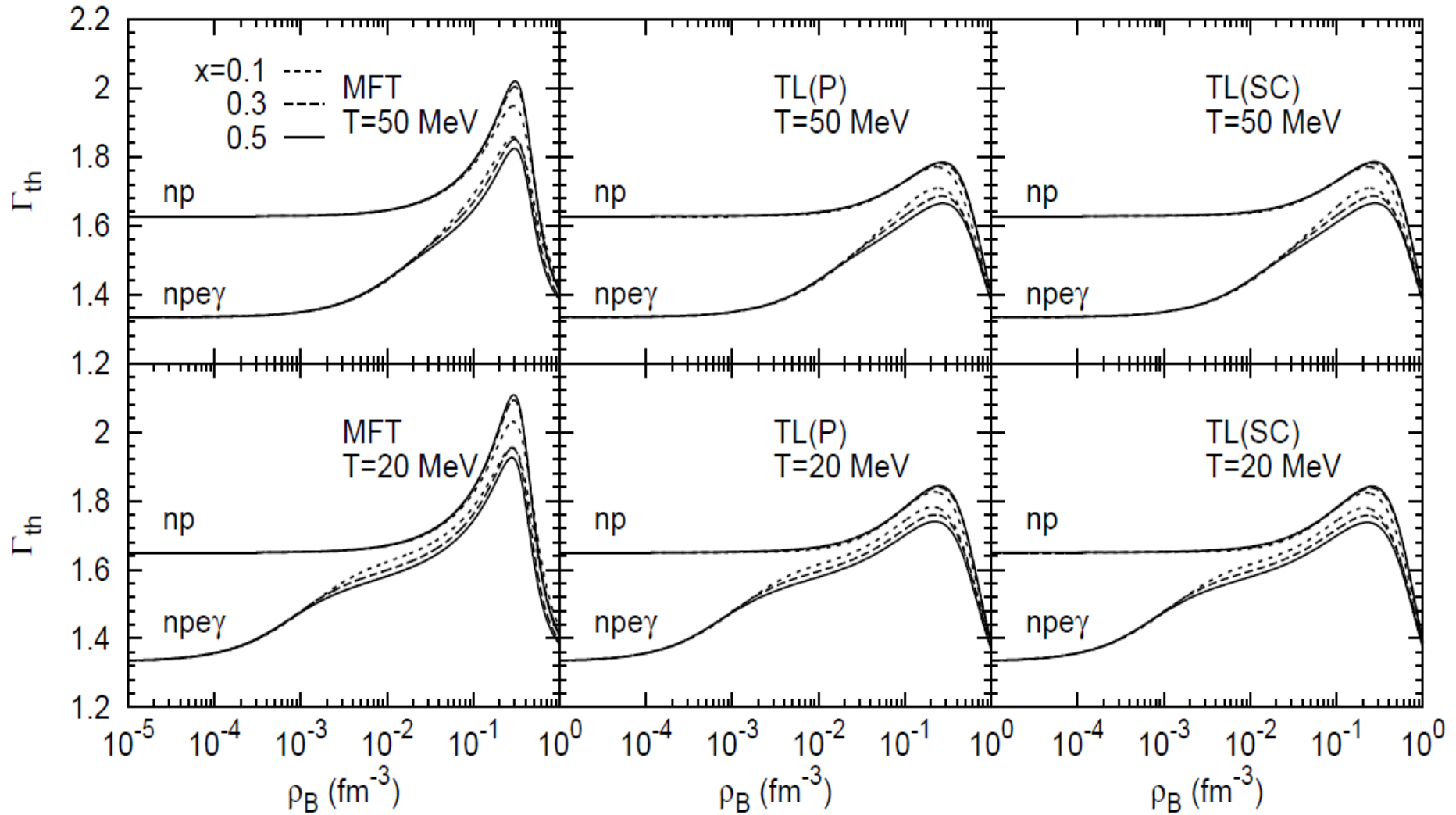
Critical temperatures





SC and P differ by less than 3%. MFT has a characteristic bump at 0.3 fm^{-3} .

$$\Gamma_{\text{th}} \equiv 1 + \frac{P_{\text{th}}}{\mathcal{E}_{\text{th}}}$$



The SC and P results are different from the MFT: no bump!
 Low density limits: non-relativistic and relativistic gas

Summary

- Including TL contribution to the MFT terms improves PNM EOS at sub-nuclear density
- TL softens EOS, but supports 2-solar mass neutron star
- TL improves nucleon optical potential
- TL's impacts on zero and finite-T bulk properties are significant
- Improved relativistic calculations can extrapolate the constrained EOS to high density region

Back up

$$m_i^* = E_{F,i}^*$$

Fix couplings

$$M_0^* = \sqrt{m_0^{*2} - k_{F0}^2} = 0.674M$$

$$\mu_0 - E_{F0}^* = \frac{g_v^2}{m_v^2} \rho_0 \quad \text{with} \quad E_{F0}^* = \sqrt{k_{F0}^2 + M_0^{*2}}$$

$$S_{2,0} - \frac{k_{F0}^2}{6E_{F0}^*} = \frac{\rho_0}{8} \frac{g_\rho^2}{m_\rho^2}$$

$$M - B_0 - \frac{\mathcal{E}_{kin0}}{\rho_0} = \frac{\mathcal{V}(\bar{\phi}_0)}{\rho_0} + \frac{1}{2} \frac{g_v^2}{m_v^2} \rho_0$$

$$\frac{1}{g_s \bar{\phi}_0} \frac{\rho_{s,0}}{m_s^2} = \frac{1}{g_s^2} + \frac{\kappa_3}{g_s^2} \frac{1}{2} \frac{g_s \bar{\phi}_0}{M} + \frac{\kappa_4}{g_s^2} \frac{1}{6} \left(\frac{g_s \bar{\phi}_0}{M} \right)^2$$

$$K_{v,0} - 3 \frac{k_{F0}^2}{E_{F0}^*} = 9\rho_0 \left[\frac{g_v^2}{m_v^2} + \frac{M_0^*}{E_{F0}^*} \frac{\partial M^*}{\partial \rho_B} \Big|_{x=0.5} \right]$$

Fix couplings

$$\mu_0 - E_{F0}^* = \frac{g_v^2}{m_v^2} \rho_0 + \left. \frac{\partial \delta \mathcal{E}_{(1)}}{\partial \rho_B} \right|_{\bar{\phi}_0, x=0.5}$$

$$S_{2,0} - \frac{k_{F0}^2}{6E_{F0}^*} = \frac{\rho_0}{8} \frac{g_\rho^2}{m_\rho^2} + \delta S_{2,(1)}$$

$$M - B_0 - \frac{\mathcal{E}_{kin0}}{\rho_0} = \frac{\mathcal{V}(\bar{\phi}_0) + \delta \mathcal{E}_{(1)}}{\rho_0} + \frac{1}{2} \frac{g_v^2}{m_v^2} \rho_0$$

$$K_{v,0} - 3 \frac{k_{F0}^2}{E_{F0}^*} = 9\rho_0 \left[\frac{g_v^2}{m_v^2} + \frac{M_0^*}{E_{F0}^*} \left. \frac{\partial M^*}{\partial \rho_B} \right|_{x=0.5} \right] + \delta K_{v,(1)}$$

$$\frac{E_{F0}^*}{m_0^*} = 1 + \frac{E_{F0}^*}{k_{F0}} \left. \frac{\partial \delta \epsilon_{(1)}(k)}{\partial k} \right|_{k=k_{F0}},$$

