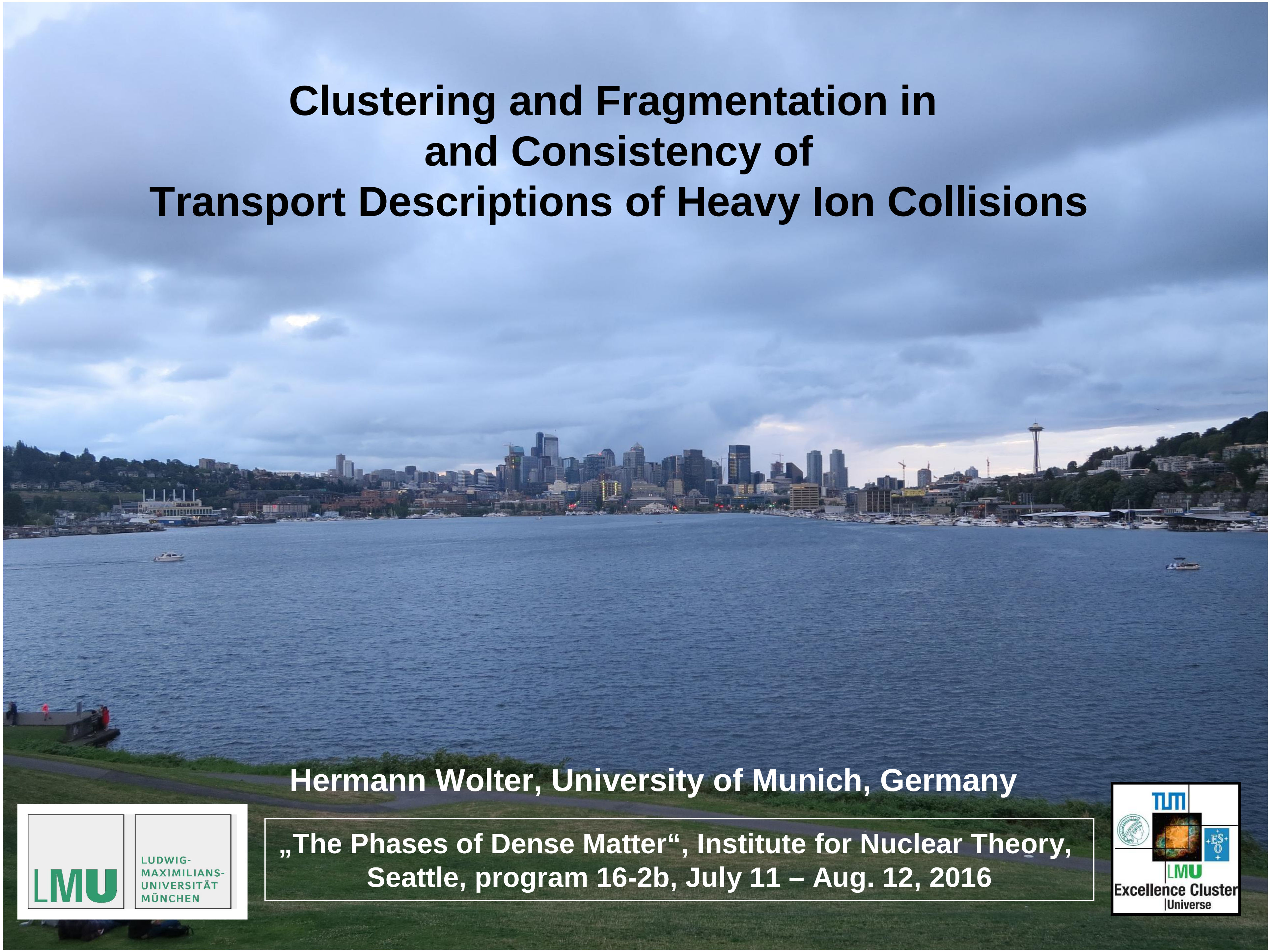

Clustering and Fragmentation in and Consistency of Transport Descriptions of Heavy Ion Collisions

Hermann Wolter, University of Munich, Germany

„The Phases of Dense Matter“, Institute for Nuclear Theory,
Seattle, program 16-2b, July 11 – Aug. 12, 2016

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Clustering and Fragmentation in
and Consistency of Transport Descriptions of Heavy Ion Collisions

A complicated title!
What is the idea of the talk?
Heavy Ion Collisions (HIC) have been mentioned often in the workshop to give information on the Equation-of-State (EoS), particularly of Dense Matter.

I will not try to collect all these results and evaluate them.

Rather I would like to discuss some of the issues that are involved in extracting information from the interpretation of HIC experiments

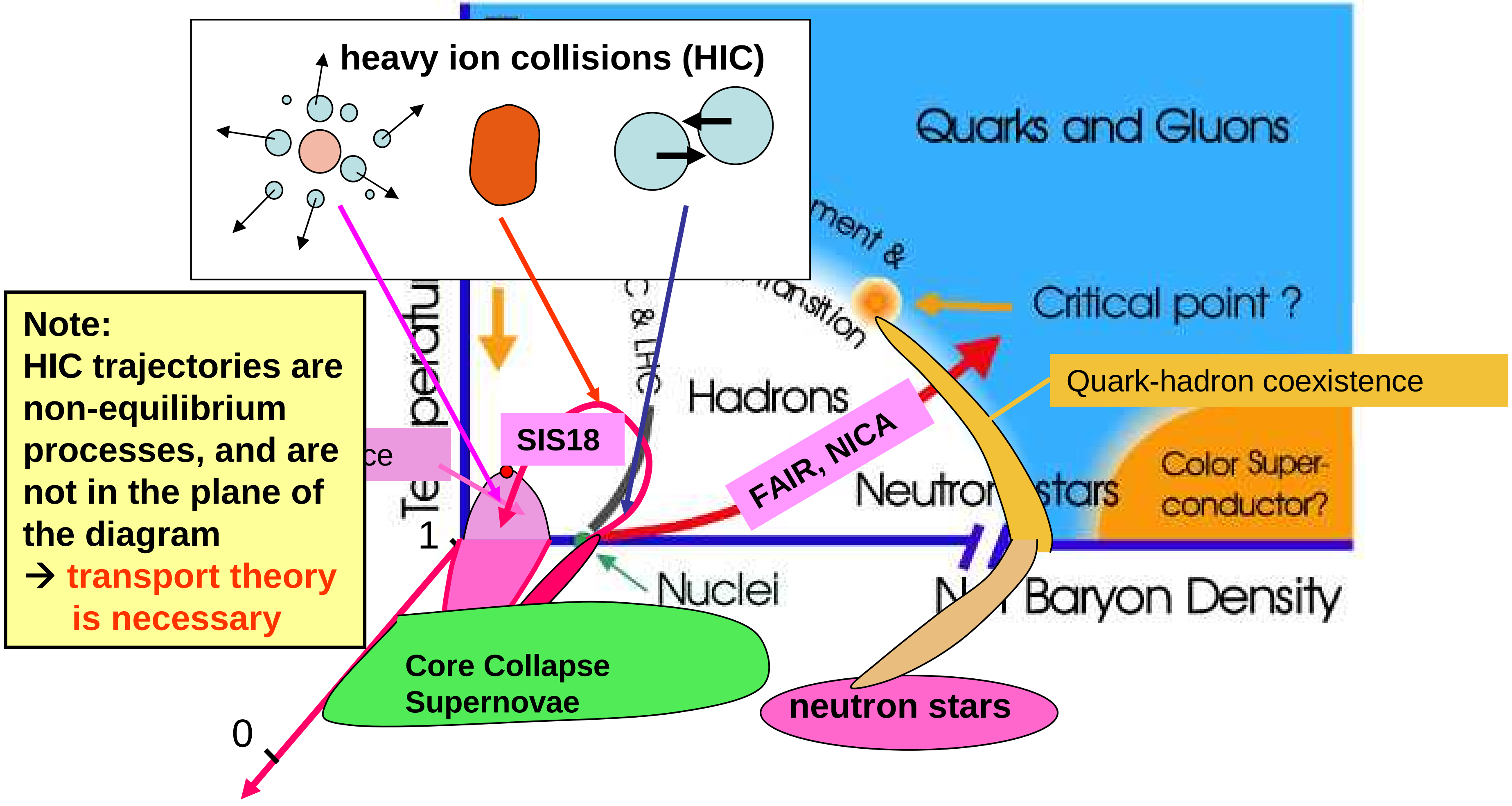
1. Attempts to quantify and to improve the consistency of transport calculations: Code comparison project (work in progress)

2. Many observables rely on measuring clusters and fragments from HIC. But transport descriptions have difficulties to go beyond the single particle information. Thus, I discuss methods to treat clusters (light) and fragments (intermediate) in transport calculations
(personal assessment of this problem and of ways of improvement).

3. Some remarks on the status of the determination of the high density symmetry energy in HIC (some newer results and their problems, continued in the next talk)

General aim in Heavy Ion Reactions:

The Phase Diagram of Strongly Interacting Matter



Note:
HIC trajectories are non-equilibrium processes, and are not in the plane of the diagram
→ transport theory is necessary

Isospin degree of freedom

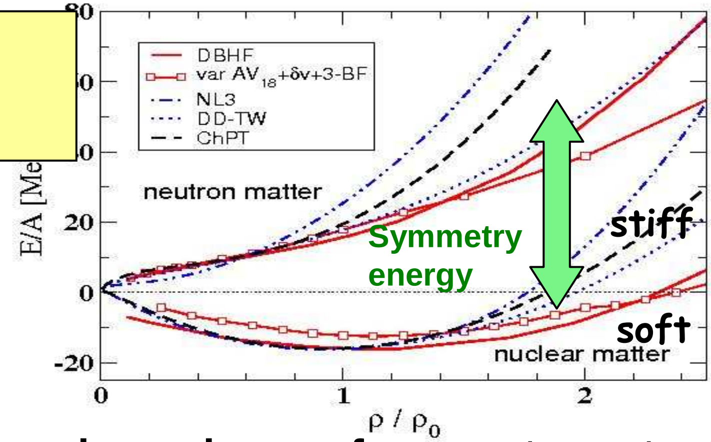
Equation-of-State and Symmetry Energy

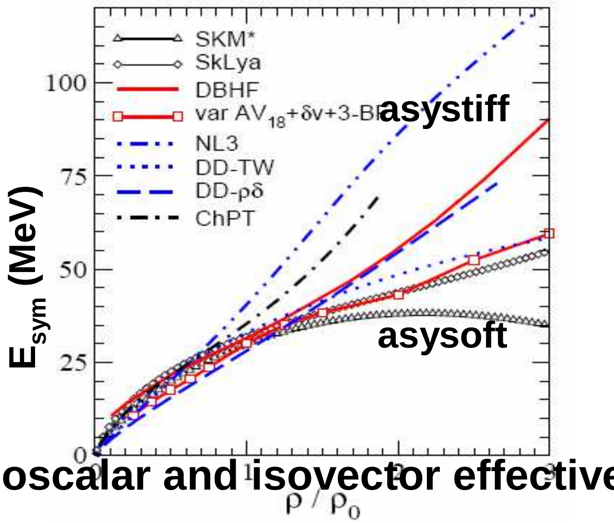
density-asymmetry dep. of nucl.matt.

$$E(\rho_B,\delta)/A = E_{nm}(\rho_B) + \boxed{E_{sym}(\rho_B)}\,\delta^2 + O(\delta^4) + \ldots \qquad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Many-Body calculations:

Rel, Brueckner
Variational
Rel. Mean field
Chiral perturb.

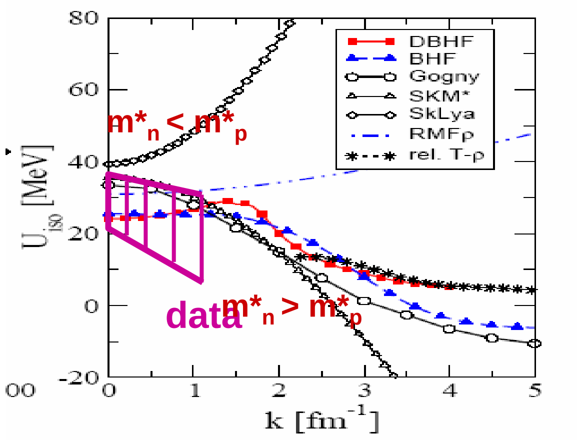




C. Fuchs, H.H. Wolter, EPJA 30(2006)5

Momentum dependence of symmetry potential (isoscalar and isovector effective mass)

$$U(\rho,k;\delta) = \frac{\partial\varepsilon(\rho,\delta)}{\partial f(\rho,k)} = \underbrace{U_0(\rho,k) + U_{sym}(\rho,k)(\tau\delta)}_{U_\tau(\rho,k)} + \ldots \qquad \frac{m^*_\tau}{m} = \left(1 + \frac{m}{\hbar^2 k}\frac{\partial U_\tau}{\partial k}\right)^{-1}$$



effective mass splitting also uncertain from many-body theories and not very constrained from data on optical potentials.

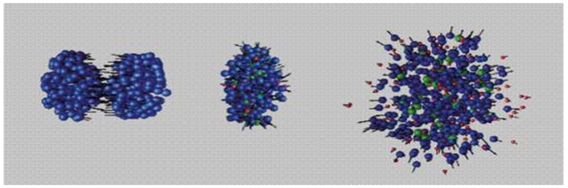
Why is symmetry energy so uncertain??
→ Short range isovector tensor correlations; 3-body forces

Thus one way to obtain information on the EoS in heavy in collisions – but HIC are complex processes

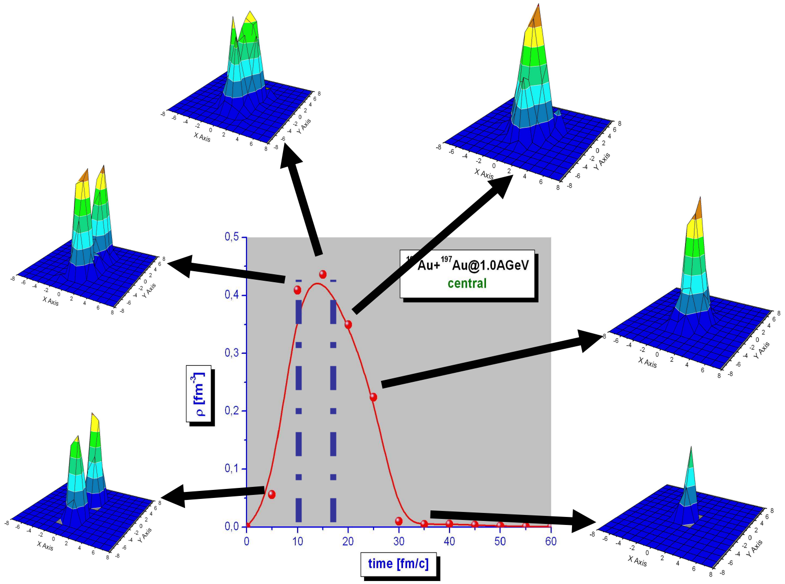
Fermi energies: (multi)-fragmentation in central collisions

from M. Colonna

Intermediate energies: several 100 MeV/A to several GeV/A
Vaporization, production of new particles, like pions, strangeness (kaons, hyperons), etc,



Au+Au, 1.8AGeV, b=2fm



non-equilibrium ! → Transport theory

Heavy ion collisions (in the hadronic regime)

finite, highly dynamic

↑

transport theory

↓

Investigation of the EoS

- as a function of density, asymmetry, temperature
- medium properties of hadrons and clusters
- phase transitions

} infinite, static system
astrophysical connection

Two main transport approaches

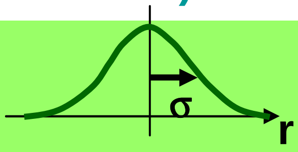
Boltzmann-Vlasov-like (BUU/BL/BLOB)

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\vec{\nabla}^{(r)}-\vec{\nabla}U(r)\,\vec{\nabla}^{(p)}\right)f(\vec{r},\vec{p};t)$$
$$=I_{coll}[\sigma^{in-med}]+\delta I_{fluct}$$

Dynamics of the 1-body phase space distribution function f with 2-body dissipation + fluctuations

Molecular-Dynamics-like (QMD/AMD)

$$|\Phi\rangle = A\prod_{i=1}^{A}\varphi(r;r_i,p_i)|0\rangle$$



$$\dot{r}_i=\{r_i,H\};\quad \dot{p}_i=\{p_i,H\};\quad H=\sum_i t_i+\sum_{i,j}V(r_i-r_j)$$

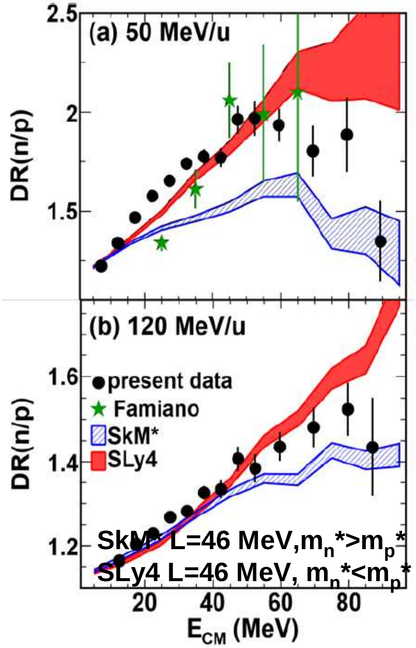
TD-Hartree(-Fock) with classical many body correlations $\{r_i(t), p_i(t)\}$, plus stochastic NN collisions

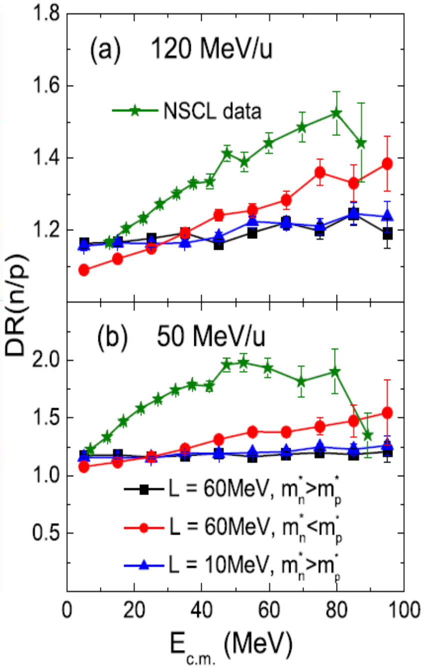
Solved by simulations (motion of (test) particle coordinates):
- physical input: mean field U (2-body interaction V), in-medium collision cross section
- technical choices: discretization, averaging (coarse graining), Pauli blocking, etc

Do these choices influence the result and hence the physical conclusions???

Code Comparison:
A need for more consistency in HI simulations: examples

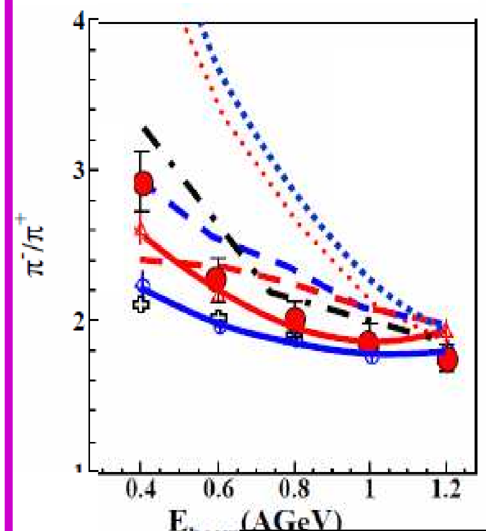
double ratio of n/p pre-equilibrium emiss.

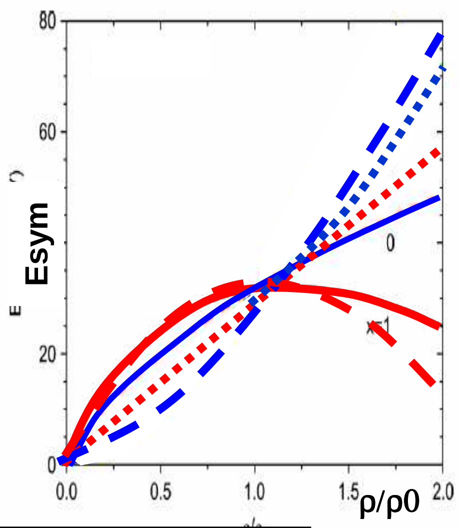




D.D.S.Coupland, et al.,arXiv1406.4546
H.J.Kong, et al., PRC91,047601 (2015)

ratio of pion yields, Au+Au,0.4-1.2 GeV/A





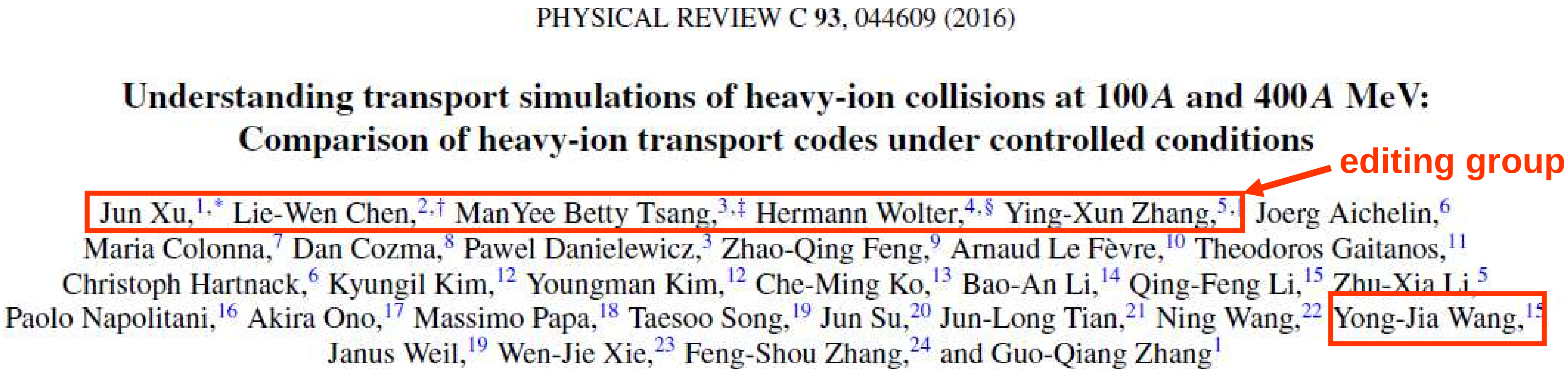
various models
blue: stiffer symm energy
red: softer symm energy
→ no consensus on ordering

Reasons for differences often not clear, since calculations slightly different in the physical parameters.
→ therefore comparison of calculations with same physical input, i.e. under controlled conditions

Code Comparison Project

Idea: Comparison of transport simulations
Determine a kind of - measure for the reliability
- i.e. a systemtic theoretical error

History:
Workshop in Trento 2004 (1 AGeV regime, mainly particle production π,K
E. Kolomeitsev, et al., J. Phys. G 31 (2005) S741)
Workshop in Trento 2009 (100, 400 AMeV)
Workshops in Shanghai and Lanzhou 2014, Shanghai 2015 → paper just published



PHYSICAL REVIEW C 93, 044609 (2016)

Understanding transport simulations of heavy-ion collisions at 100*A* and 400*A* MeV: Comparison of heavy-ion transport codes under controlled conditions

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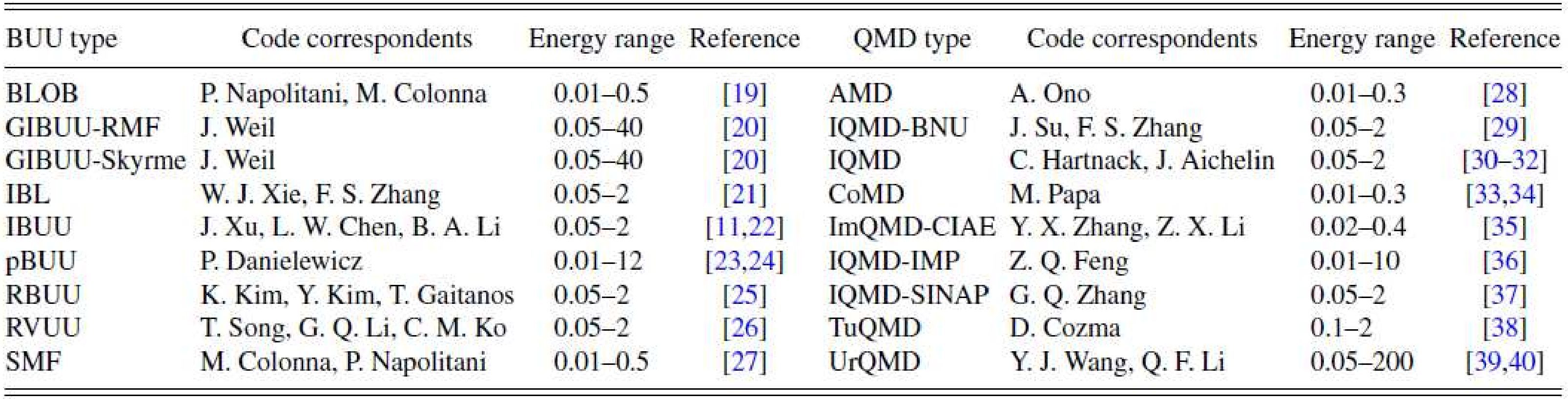
editing group

Codes participating in the code comparison



BUU type	Code correspondents	Energy range	Reference	QMD type	Code correspondents	Energy range	Reference
BLOB	P. Napolitani, M. Colonna	0.01–0.5	[19]	AMD	A. Ono	0.01–0.3	[28]
GIBUU-RMF	J. Weil	0.05–40	[20]	IQMD-BNU	J. Su, F. S. Zhang	0.05–2	[29]
GIBUU-Skyrme	J. Weil	0.05–40	[20]	IQMD	C. Hartnack, J. Aichelin	0.05–2	[30–32]
IBL	W. J. Xie, F. S. Zhang	0.05–2	[21]	CoMD	M. Papa	0.01–0.3	[33,34]
IBUU	J. Xu, L. W. Chen, B. A. Li	0.05–2	[11,22]	ImQMD-CIAE	Y. X. Zhang, Z. X. Li	0.02–0.4	[35]
pBUU	P. Danielewicz	0.01–12	[23,24]	IQMD-IMP	Z. Q. Feng	0.01–10	[36]
RBUU	K. Kim, Y. Kim, T. Gaitanos	0.05–2	[25]	IQMD-SINAP	G. Q. Zhang	0.05–2	[37]
RVUU	T. Song, G. Q. Li, C. M. Ko	0.05–2	[26]	TuQMD	D. Cozma	0.1–2	[38]
SMF	M. Colonna, P. Napolitani	0.01–0.5	[27]	UrQMD	Y. J. Wang, Q. F. Li	0.05–200	[39,40]

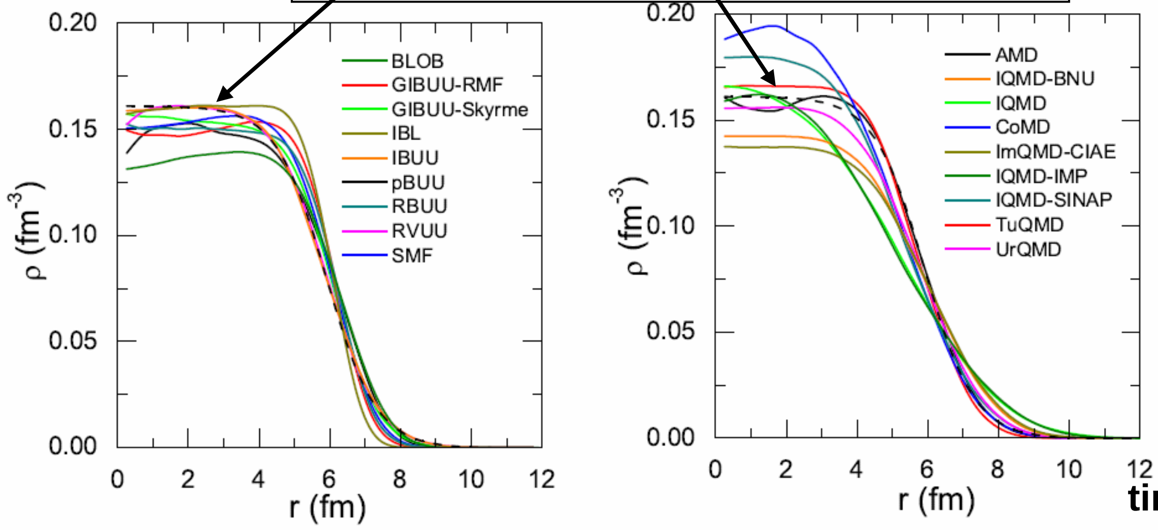
→ BUU- and QMD-type
→ non-rel. and relativistic codes
→ antisymmetrized QMD code: AMD, CoMD
→ BUU codes with explicit fluctuations: SMF, BLOB
→ many new Chinese codes: (I)QMD-XXX: much new activity in China, often originally closely related

Set-up of code comparison („homework“)

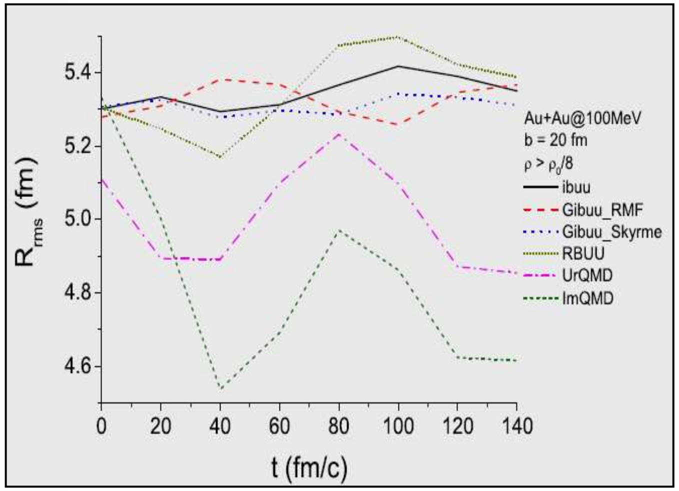
- typical reaction in low and intermediate energy: Au+Au, 100 and 400 AMeV
- impact parameter 20 fm (no collision, stability of initialization) and 7 fm (midcentral)
- simple physics case (not necessarily realistic)
  standard Skyrme mean field, momentum independent, equlivalent RMF
  constant cross section, no inelastic collisions
- „close“ initialization of colliding nuclei
  prescribed density profile, momentum in local Fermi sphere
- collision and blocking procedures as in standard use of code
- different „modes“: Vlasov (only mean field), Cascade (only collisions),„full“
- monitor: (test) particle motion, nmber and energy and time of collisions,
  Pauli-blocking, observables (rapidity, flow)

Initialization and Stability

dashed curve ≡ prescribed density profile



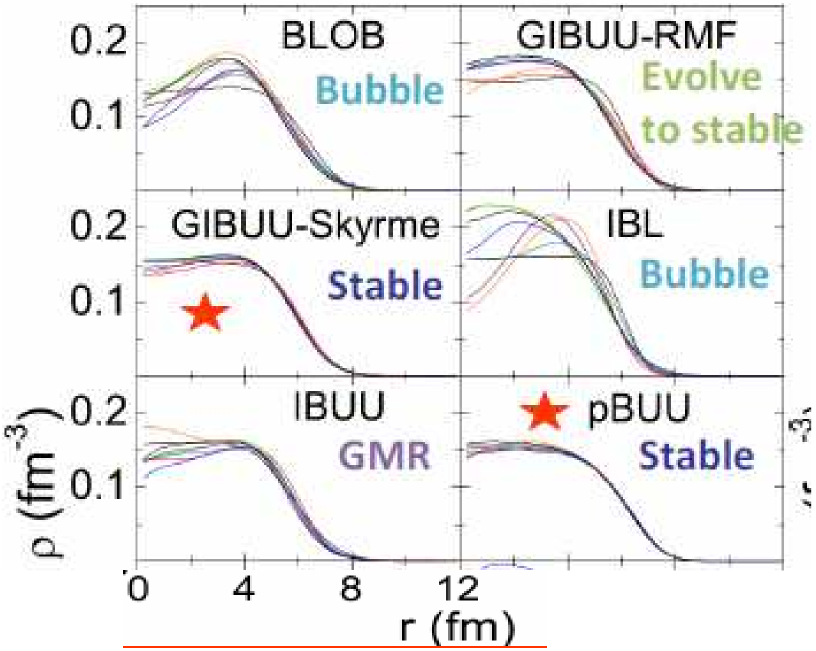
mean square radius as a fct of time



- „identical“ initialization difficult, since it depends also on repesentation of (test) particles
- prescribed density profile is not neccessarily ground state and may be non-stationary
- diff. initializations affect evolution also in case of a collision

- suggestion: stability of initialization more important than identical profile. Use consistent method to generate initial nucleus, e.g. Thomas-Fermi

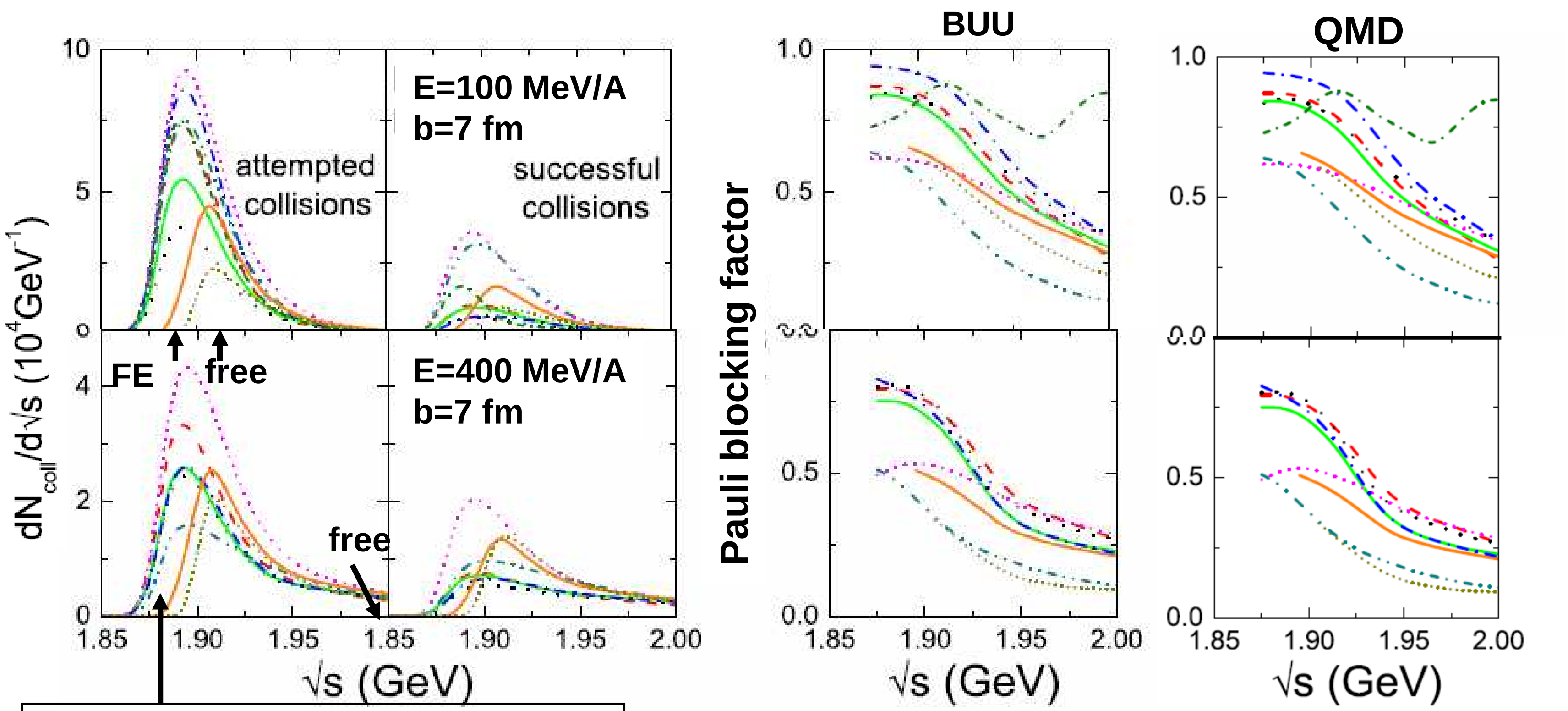
time evolution of isolated nucleus(examp)



★ Dynamical initialization (Thomas-Fermi)

NN Collision rates per energy bin

attempted unblocked blocking factor



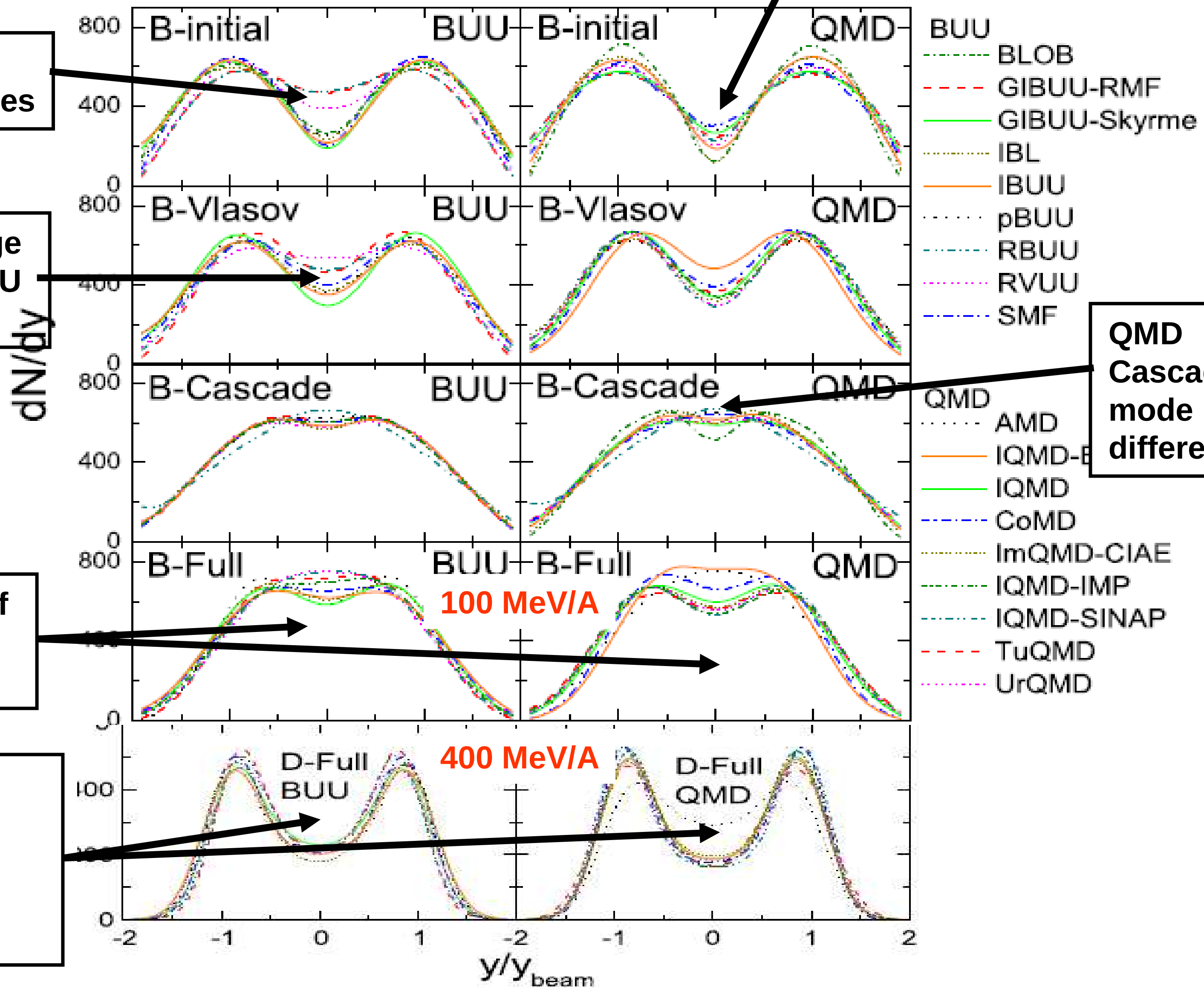
threshold for low energy collisions

Considerable difference both for :
- attempted collisions, mostly low energy(!)
  (depends on strategy for finding collision pairs)
- blocking factor (depends on occupation of final state)
- better consistency for higher energy

- not much difference for BUU and QMD

Observables: rapidity distribution

difference because of
larger difference in initial distrib.



rel. BUU codes,
correct eff. masses

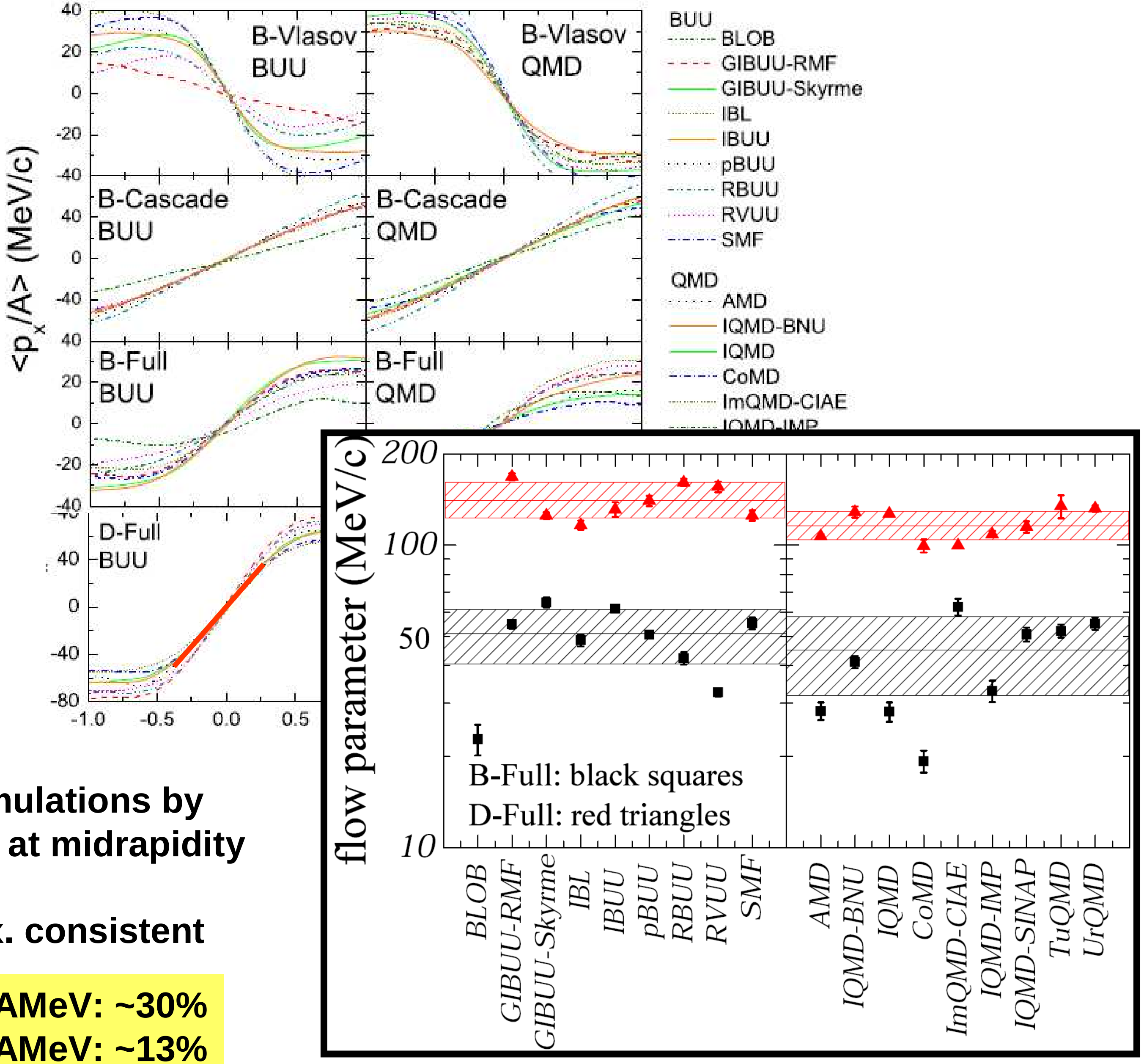
surprisingly large
difference in BUU
Vlasov

QMD
Cascade
mode rather
different

combination of
Cascade and
Vlasov

better
consistency at
higher energy,
mean field less
important

Observables: directed flow



Vlasov and Cascade opposite slope:
~ balance energy at 100 MeV, sensitive region,
→large discrepancies

at higher energy more consistent

quantify spread of simulations by value of „flow“=slope at midrapidity

BUU and QMD approx. consistent

uncertainity

100 AMeV: ~30%
400 AMeV: ~13%

Intermediate Conclusion:

Obtained estimate of systematic error of transport simulations

100 AMev: ~30%
400 AMeV: ~13%

reasons: initialization, collisions, blocking, mean field propagation
difficult to separate quantitatively,
difficult to say what is correct, no democratic principle!

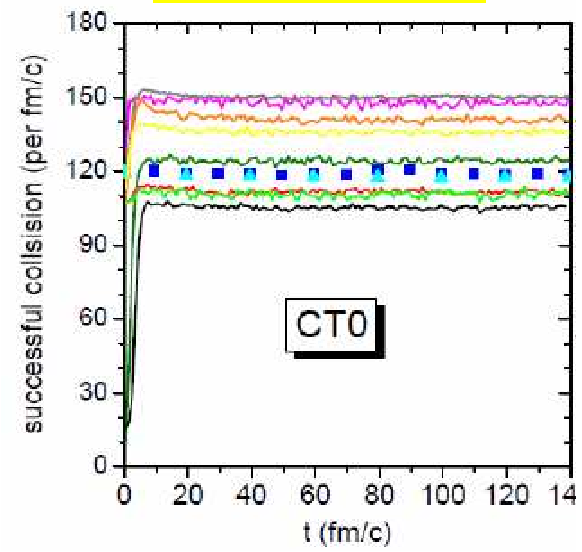
Should be improved!!

try to assess separately: not collision but simulation of static system of nuclear matter
→ calculation in a periodic box
in progress, first results, Workshop planned MSU Nov 2016
coordinators: Maria Colonna, Akira Ono, Jun Xu, Yongjia Wang, Yingxun Zhang

Set-up:
- box (20fm) with periodic boundary conditions (not reflecting!)
- can be implemented by a new metric: $dr_{k,new} = \text{modulo}(dr_k + L_k/2, L_k) - L_k/2$, k=x,y,z
- density ρ_0, T= 0, 5 MeV (stable situation)
- physical input (force, cross section) as before
- modes:
  - Cascade (only collisions) with/without blocking
  - Vlasov (only mean field): propagate wave of given λ: check dispersion relation
- Extensions: fragmentation, pion production

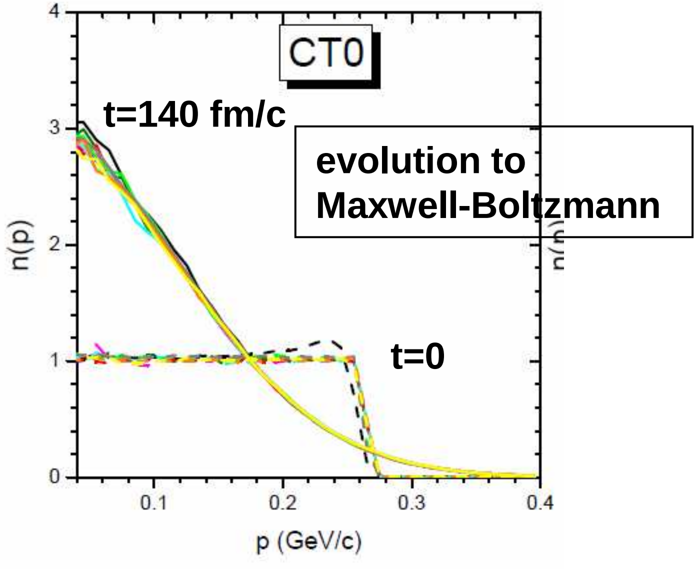
Cascade, no blocking

Collision rates



analytical result 119 $(\mathrm{fm}/c)^{-1}$

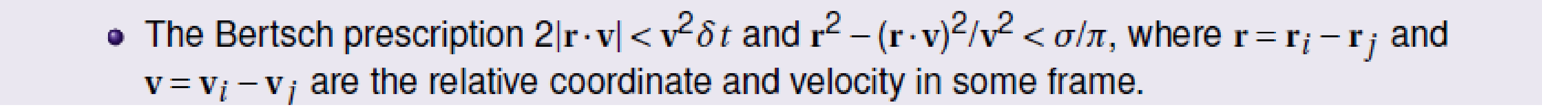
momentum distribution



evolution to Maxwell-Boltzmann

Discrepancy has been understood:

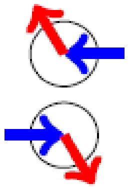
simulation of collision integral: two particles attempt a collision in a time step Δt, if



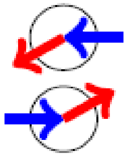
- The Bertsch prescription $2|\mathbf{r}\cdot\mathbf{v}| < \mathbf{v}^2\delta t$ and $\mathbf{r}^2 - (\mathbf{r}\cdot\mathbf{v})^2/\mathbf{v}^2 < \sigma/\pi$, where $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$ and $\mathbf{v} = \mathbf{v}_i - \mathbf{v}_j$ are the relative coordinate and velocity in some frame.

Can the particle collide again in the same Δt? yes, but not immediately with the same particle. but this can happen, since the collision is not local

not here



but here the Bertsch criterion is fullfilled again:

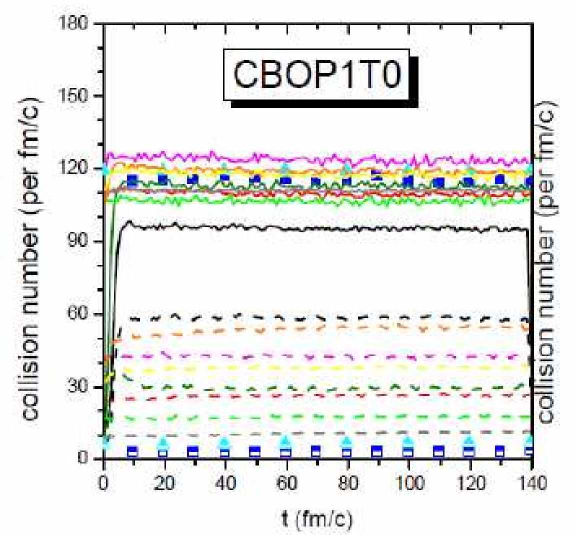


Effect in dN_{coll}/dt for CT0: 124 (avoid spurious) → 161 (allow spurious, $\Delta t = 0.5$ fm/c)

Cascade, with blocking

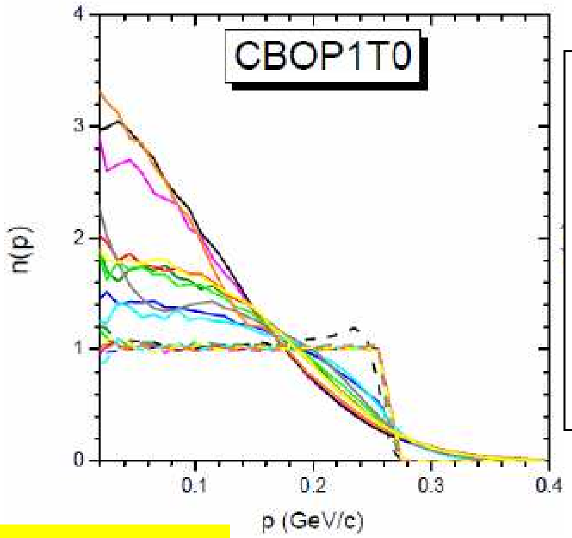
Collision rates

standard blocking



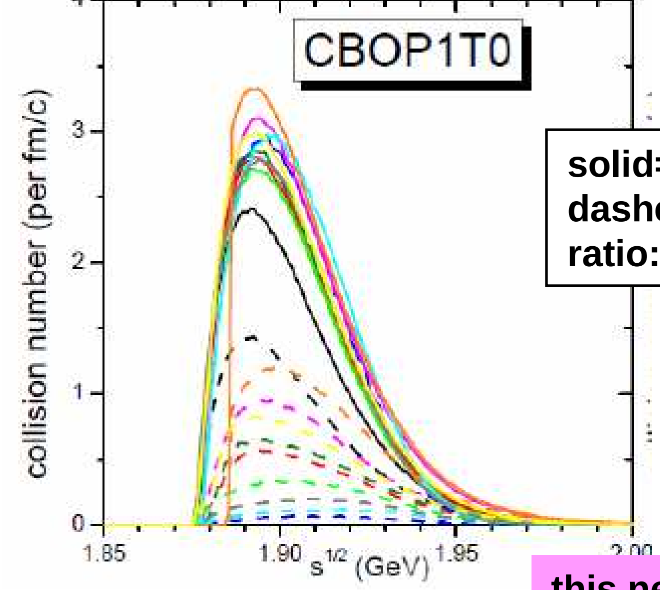
solid:attempted
dashed: unblocked
should be 0!
depends on calculation of phase space occupancy

momentum distribution

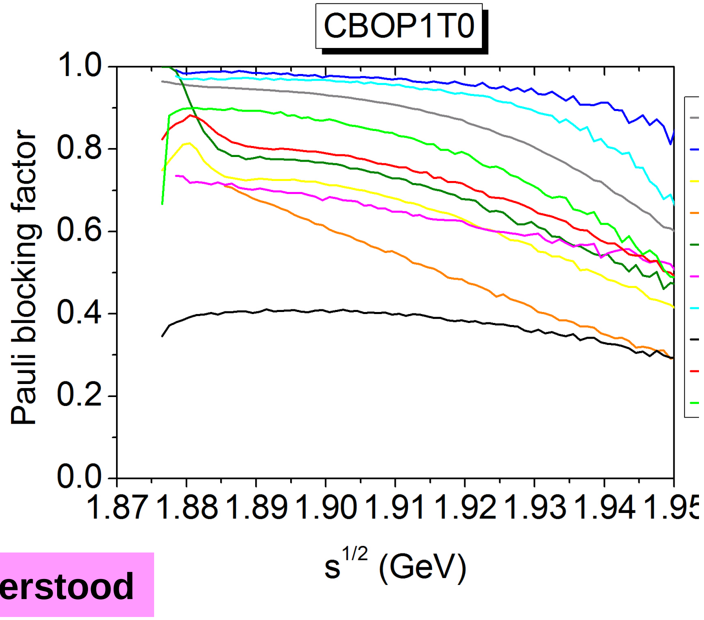


Fermionic character of system rather qickly lost, but depending on amount of spurios collisions

Energy distribution of collisions and blocking



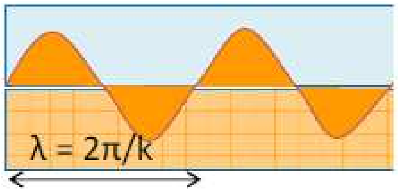
solid=attempted coll
dashed=unblocked coll
ratio: blocking factor



this needs to be better understood

Box Homework 2: propagation (Vlasov)
Initialize standing wave, and check dispersion relation

2.2 Box simulations: test of m.f. dynamics



$$\rho(z,t=t_0)= \rho_0 + a_\rho \sin(k_i z)$$

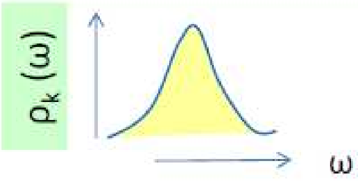
$$k_i = n_i\, 2\pi/L \qquad a_\rho = 0.2\, \rho_0$$

- Study the time evolution of ρ(z)
- Extract the Fourier transform in space and in time

$$\rho_k(t) = \int dz \sin(kz)\, \rho(z,t) \qquad \rho_k(\omega) = \int dt \cos(\omega t)\, \rho_k(t)$$

➢ significant contribution only for $k = k_i$ (to be checked)

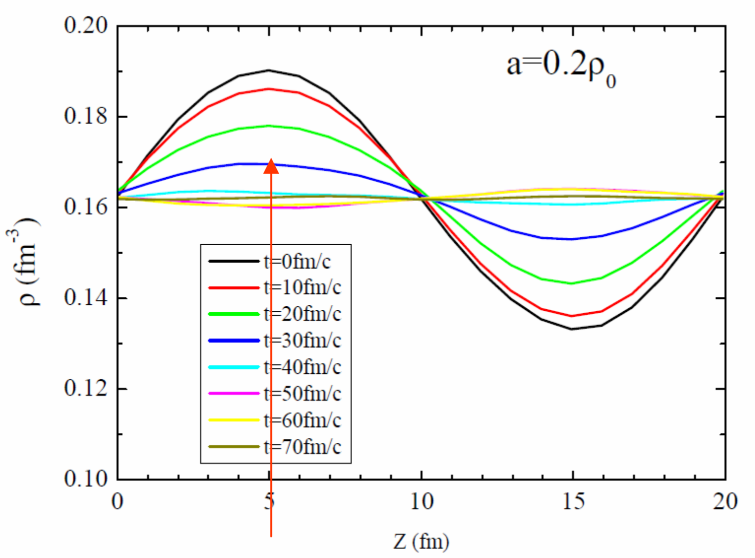
damped oscillations are expected



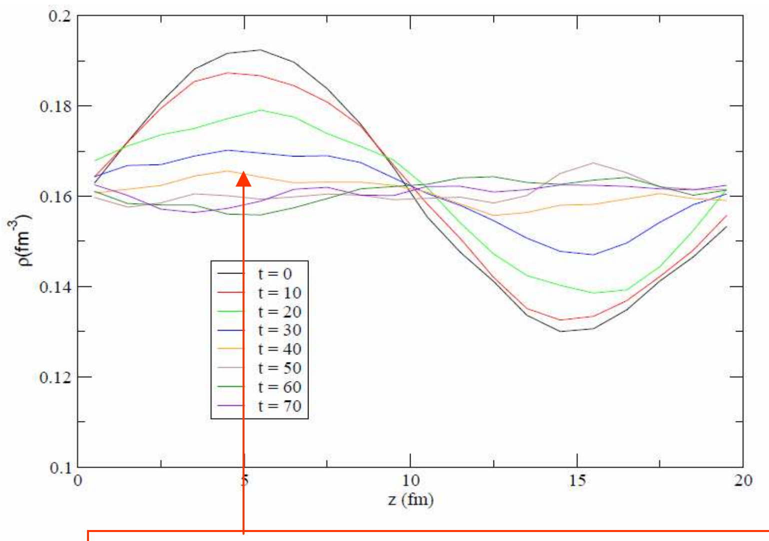
- check with the dispersion relation $\omega(k_i)$ for the maximum,
dispersion realtion is given analytically for the given EoS
- width of the distribution ís given by fluctuations in the different codes
interesting for the description of clustering and fragmentatio
- first results, not completed but can already see some effects

Time evolution of a standing wave

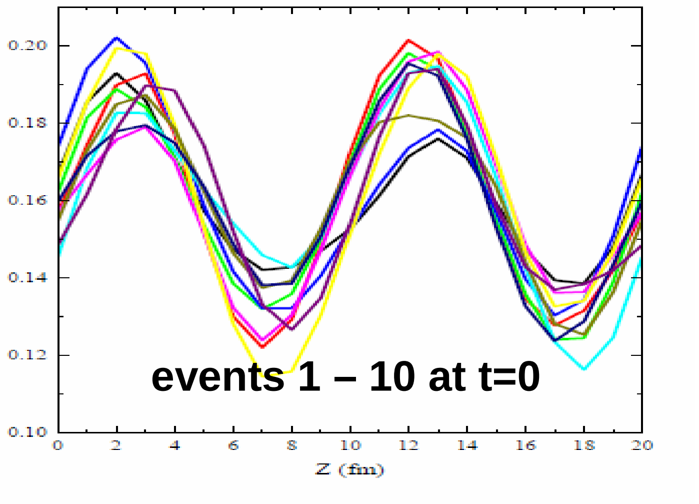
QMD (here ImQMD-CIAE)



BUU (here SMF)

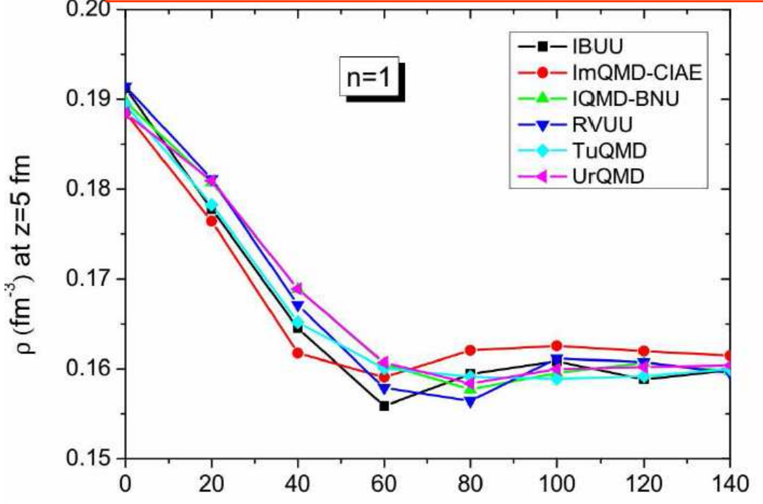


large fluctuation in QMD in different „events“ at t=0 fm/c



events 1 – 10 at t=0

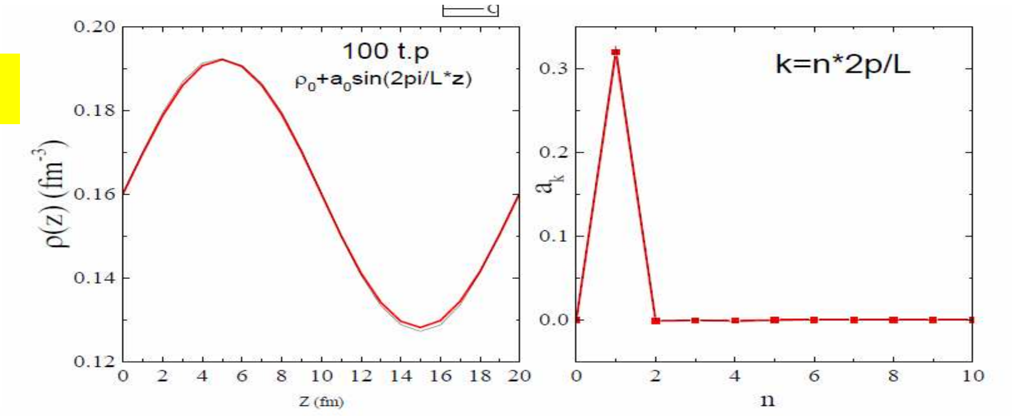
time dependence of maximum for different codes→ reasonably similar



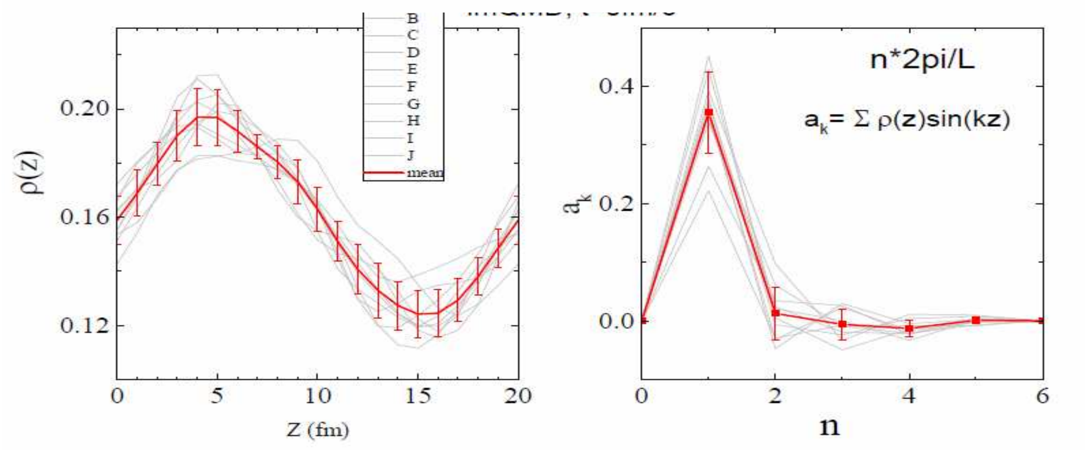
Fourier series to momentum space

BUU: Initial wave very stable
small fluctuations (too small!)

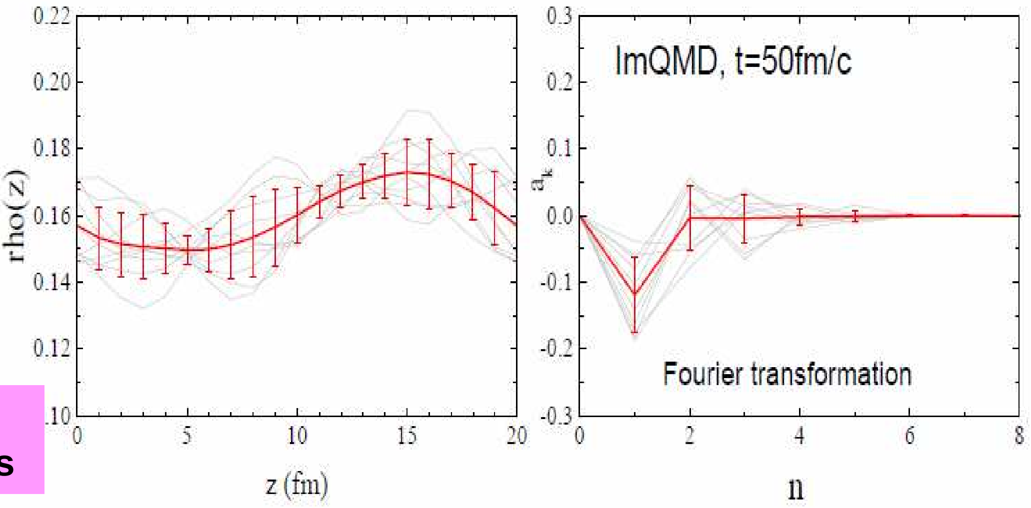
t=0



QMD: fluctuation is seen in
width of fourier distribution



QMD: fluctuation increase with time
t=50 fm/c



further evaluation are in progress
interesting because of role of fluctuations

Further Steps in Code Comparison

- Box calculations obviously very useful to understand different behaviors of codes
- in many cases there are analytical limits, which allow to point to places of improvement
- one can check the different ingredients in a simulation of a HIC separately :

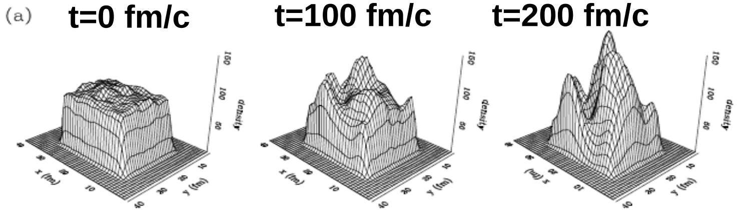
→ collision rates, blocking (Cascade): homework #1:
results, but now deviations from exact values need to be better understood

→ propagation (Vlasov), homework #2:
first results, but need to be evaluated. Check dispersion relation,
differences in fluctuation seen
further possible modes, isospin

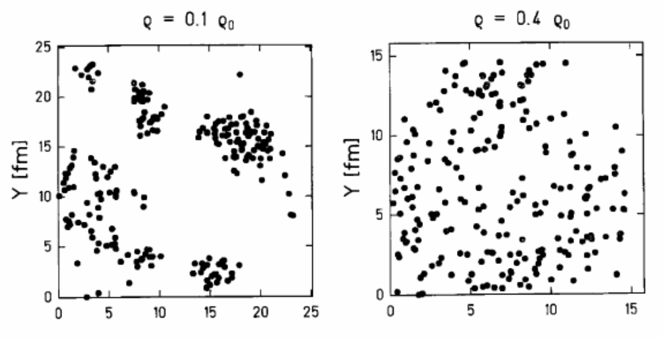
→ check fluctuation and fragmentation:
initialize system in spinodal region. Then system should evolve into fragments and gas.
compare fragment distributions and time constants

example: $\rho=\rho_0/3$, T=5 MeV, $\delta=0$

t=0 fm/c t=100 fm/c t=200 fm/c



→ Formation of „clusters (fragments)“, from small (physical) fluctuations in the density. (V.Baran, et al., Phys.Rep.410,335(05))



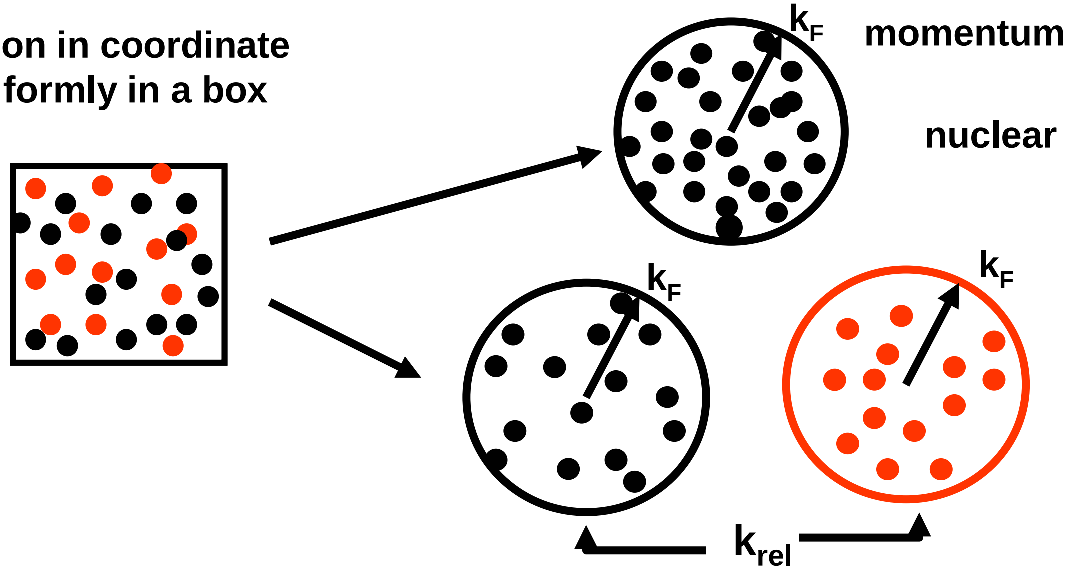
→ same thing for asymmetric system and check isospin fractionation or destillation

Further Possibilities for Box Calculations:

A possible argument: box calculations for T=0 contain only momenta up to the Fermi momentum and are therefore not characteristic of HIC. Two possibilities

→ High temperatures, but not very characteristic for the initial phase of a HIC

→ box calculations of colliding nuclear matter



initialization in coordinate space uniformly in a box

momentum initialization

nuclear matter in a box

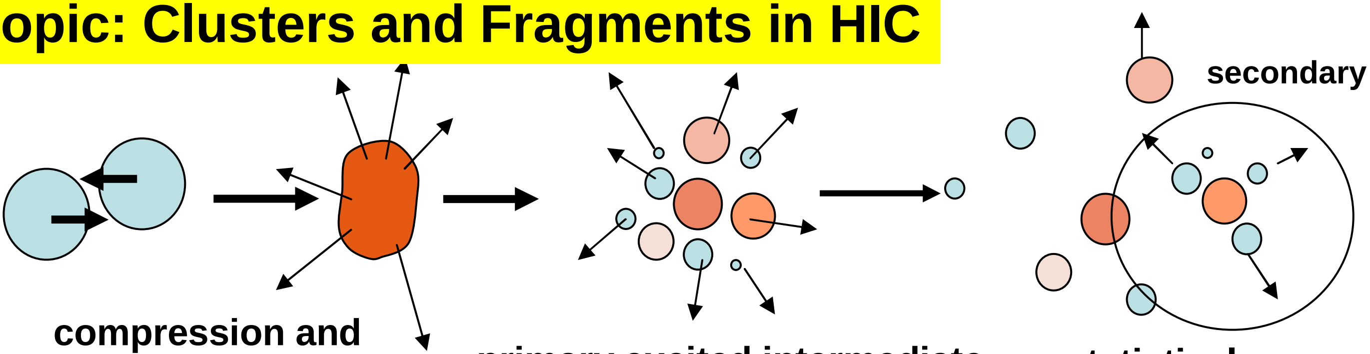
counter-streaming nuclear matter (colliding nuclear matter) with different incident energies k_{rel}.

many questions to investigate, keeping the advantage of separating the different effects:

- collisions and blocking
- propagation with momentum dependent fields and effective masses
- equilibration of momentum:
  longitudinal momentum distribution, stopping
  transverse momentum distributions, „flow“
- all the above with asymmetric systems and symmetry potentials

finally pion production (if the collisions and blocking are under control)
involves new physics: π,Δ dynamics independent of the evolution of the system
check different ingredients (π,Δ potentials, threshold effects, detailed balance, etc. step by step

Next topic: Clusters and Fragments in HIC



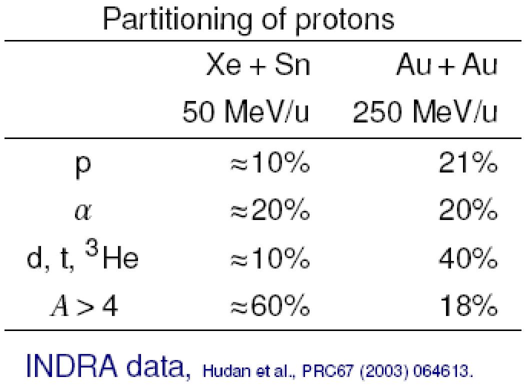
secondary decay

compression and pre-equil. emission of light clusters (LC) n,p,d,t,3He,α (A≤4)

primary excited intermediate mass fragments (IMF) (5≤A≤≈30)

statistical secondary decay of fragments

large fractions of particles in clusters, e.g.

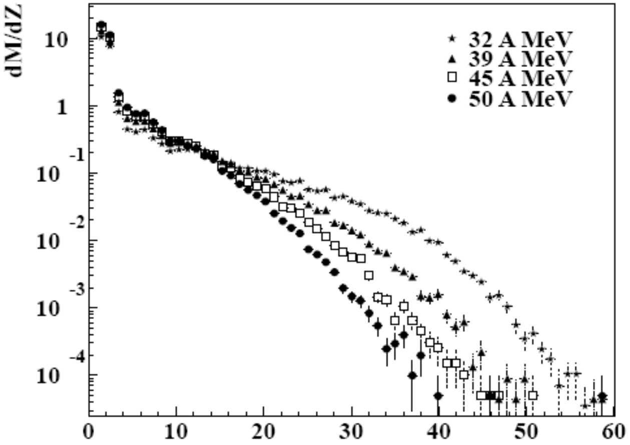


Partitioning of protons

	Xe + Sn 50 MeV/u	Au + Au 250 MeV/u
p	≈10%	21%
α	≈20%	20%
d, t, ^{3}He	≈10%	40%
$A > 4$	≈60%	18%

INDRA data, Hudan et al., PRC67 (2003) 064613.

^{136}Xe+^{124}Sn, E = 32,..,150 AMeV



S. Hudan, et al., PRC67, 064613

dynamically correlations and fluctuations are seeds to formation of clusters and fragments but often also treated with statistical models with considerable success

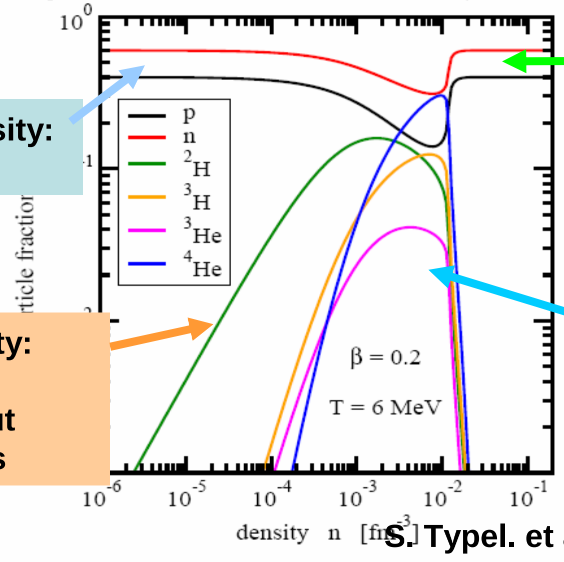
But: **BUU** **no correlation (mean field)**
no fluctuation (exc. numerical)

QMD: only classical correlations
fluctations depend on parameter, wave packet width

Way out??

Remark about statistical application: Clustering of very dilute nuclear matter

composition as fct of density; x=0.2, T=6 MeV



very low density: p,n

Increasing density: clusters arise: deuteron first, but then α dominates

Mott density: clusters melt, homogeneous p,n matter;

here heavier nuclei (embedded into a gas) become important here

S. Typel. et al., PRC 81(2010)

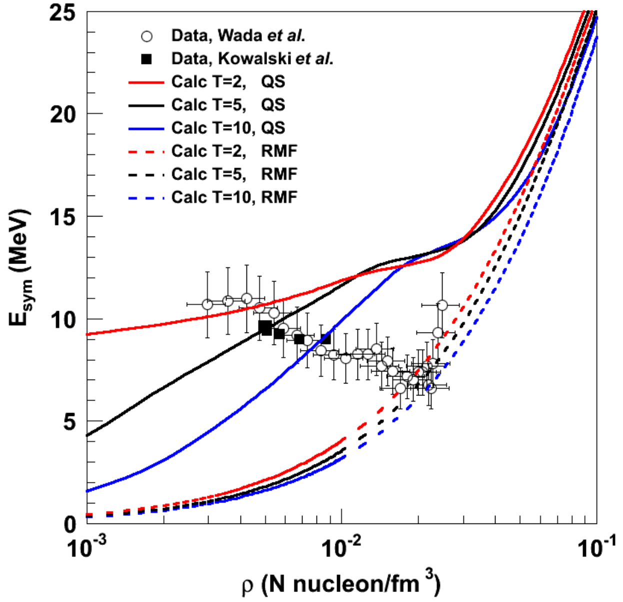
Can be investigated in heavy ion collisions

Semi-central heavy ion collisions, (^{64}Zn+^{92}Mo,^{197}Au at 35MeV/A) and time-resolved measurement of light fragments from decay of fireball:

S. Kowalski +, PRC75 014601 (2007)

J. Natowitz, G. Röpke+ PRL 104, 202501 (2010)

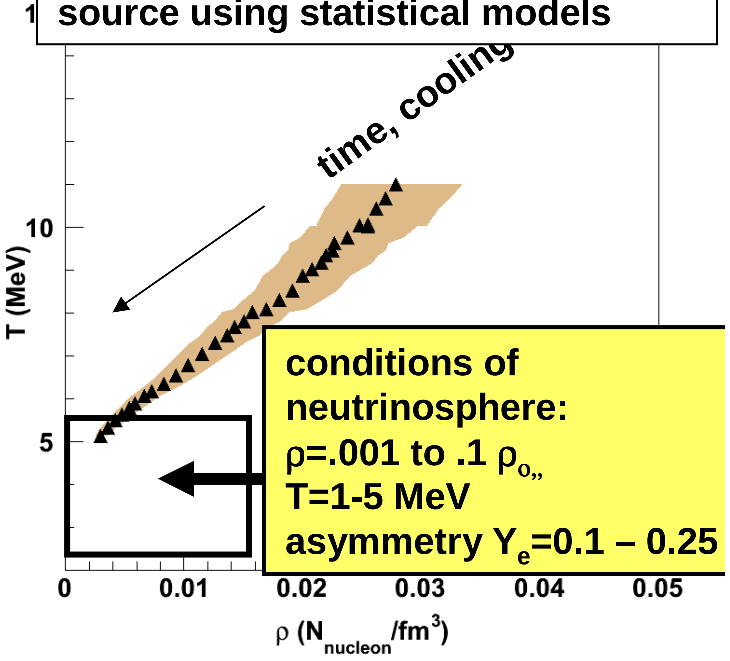
extract symmetry energy and compare with quantumstat. calculation of clustered matter



Should be verified in dynamical description,

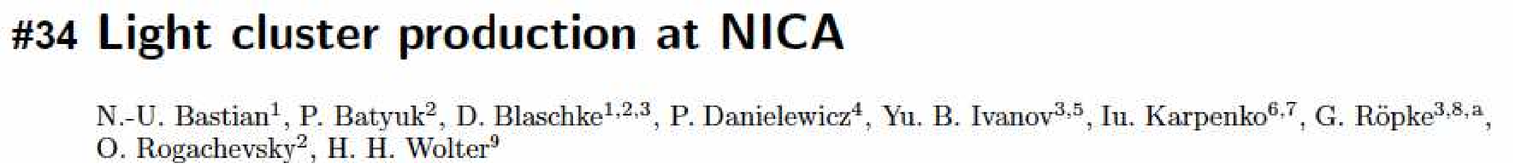
Relevance for Supernovae physics? HIC ⟺ Femto-Nova?

determine „trajectory“ of evolution density and temp. of expanding source using statistical models



conditions of neutrinosphere:
ρ=.001 to .1 ρ_o„
T=1-5 MeV
asymmetry Y_e=0.1 – 0.25

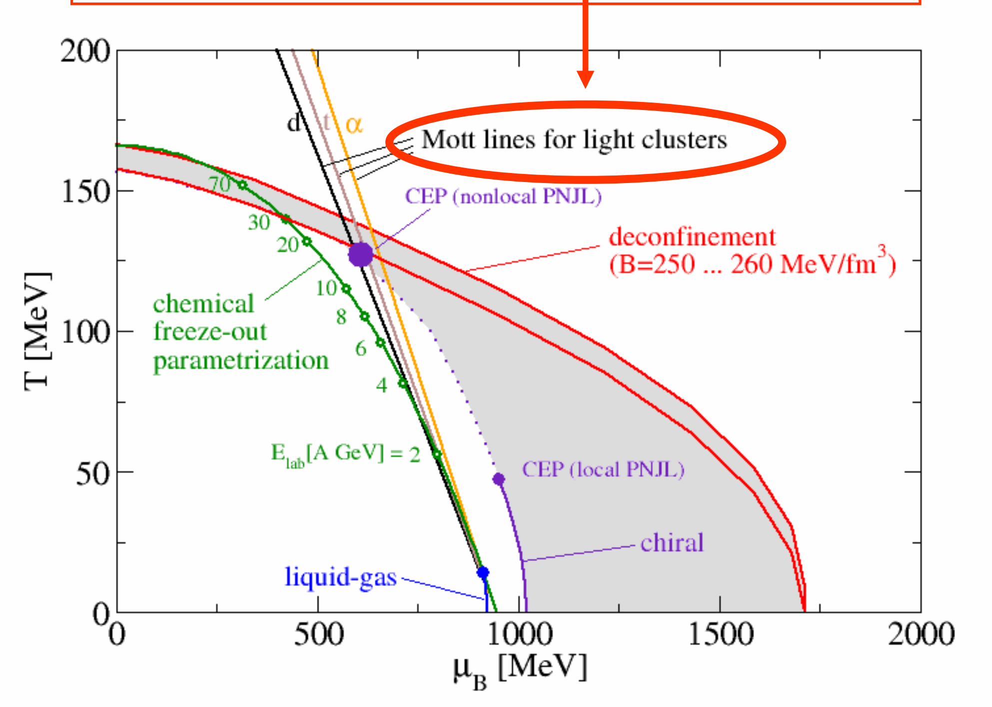
Cluster production can also be important at high energies/densities, e.g. NICA White paper,just appearing at EPJA:



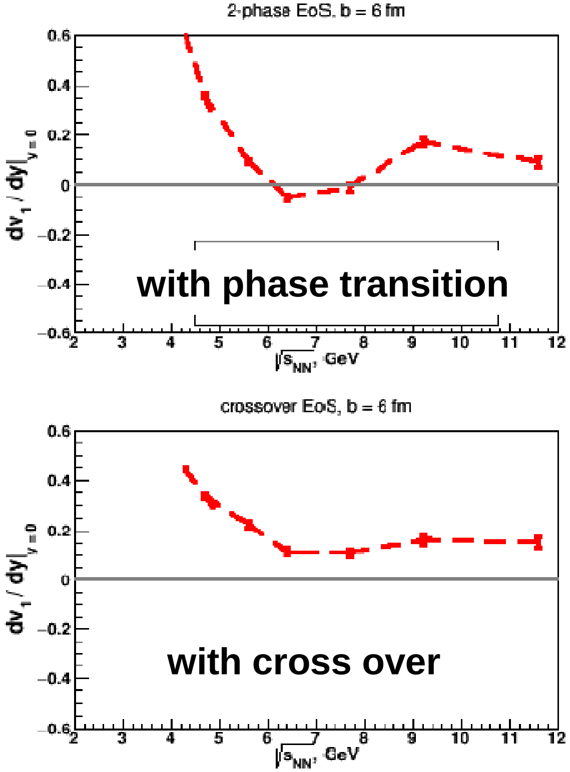
#34 Light cluster production at NICA

N.-U. Bastian[1], P. Batyuk[2], D. Blaschke[1,2,3], P. Danielewicz[4], Yu. B. Ivanov[3,5], Iu. Karpenko[6,7], G. Röpke[3,8,a], O. Rogachevsky[2], H. H. Wolter[9]

light clusters can be present at high densities near the deconfinment phase transition



3-fluid hydro, Au+Au, deuteron flow

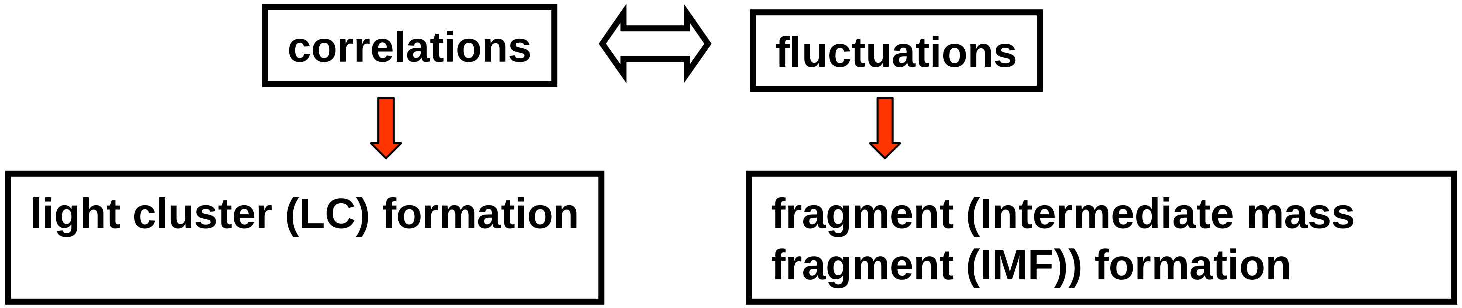


with phase transition

with cross over

Flow of light custers traces early pressure distribution, not affected by rescattering on pions and light hadrons

The issue in fragment and cluster formation in HIC collisions:



correlations ⟺ fluctuations

light cluster (LC) formation

fragment (Intermediate mass fragment (IMF)) formation

How correlations got lost:
BBGKY hierarchy of coupled
Green fcts. is truncated (formally)
by introduction of self energy Σ

$$D(1)G^{(1)}(1,2) = \delta(1-1') + (12\,|\,V\,|\,1'2')\boxed{G^{(2)}(12,1'2')}$$
$$=: \delta(1-1') + \boxed{\Sigma(1,1'')}G^{(1)}(1'',2)$$

This neglects higher order correlation effects

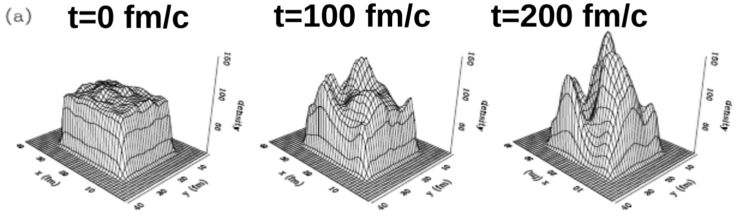
They have to reintroduced
- in the form of fluctuations (for fragments, IMF)
- explicitely (for light clusters, LC)

discuss next , how this can be handled in BV and MD approaches

Both BUU and QMD do not naturally have the correct fluctuations and correlations

way out???
answer perhaps is different
for Light clusters (LC) $(A\leq 4)$
and Int. Mass fragments (IMF) $(5\leq A\leq\approx 30)$

BUU calculation in a box (i.e. periodic boundary conditions) with initial conditions inside the instability region: $\rho=\rho_0/3$, T=5 MeV, $\delta=0$


(a) t=0 fm/c t=100 fm/c t=200 fm/c

→ Formation of „clusters (fragments)“, from small (physical) fluctuations in the density. (V.Baran, et al., Phys.Rep.410,335(05))

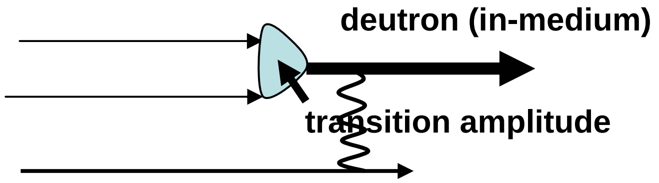
⇨ IMF: develop from fluctuation as seeds which are amplified by the mean field

issue: correct amplitude and spectrum of fluctuations

⇨ LC: correlation dominated

(common density functionals are not sophisticated enough to describe LC properly)

Issue: Introduce LCs as explicit degrees of freedom formed in 3-body colisions

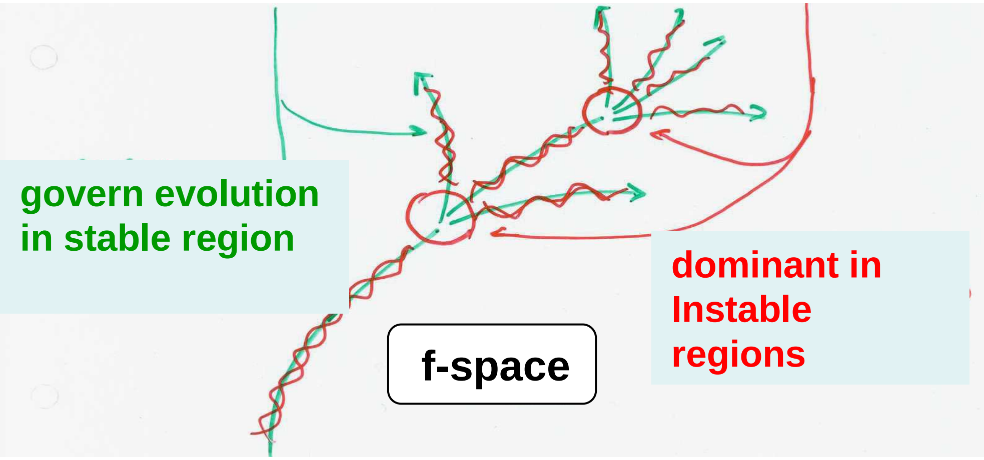


present solutions in BUU and QMD

	LC	IMF
BUU	pBUU	SMF/BLOB
QMD	clustAMD	wp width

Methods to introduce fluctuations for IMF production

BUU: statistical fluctuation of the mean field distribution function f in a Fermi system is $\sigma_f^2(r,p) = \bar{f}(r,p)(1-\bar{f}(r,p))$



govern evolution in stable region

f-space

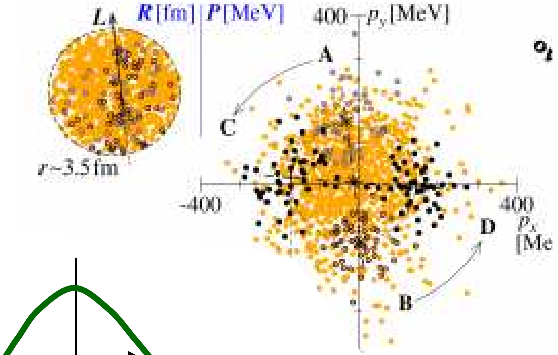
dominant in Instable regions

fuctuations around dissipative solution

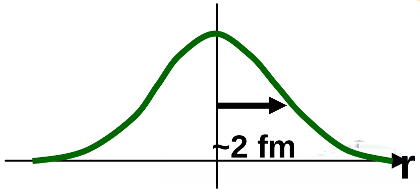
$$\boxed{\frac{df}{dt} = I_{coll} + I_{fluc}}$$

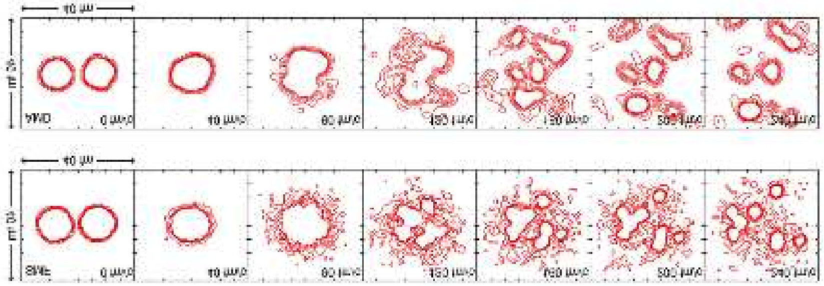
Boltzmann-Langevin eq.

SMF (stochastic mean field): project on density fluctuations
BLOB (Boltzmann-Langevin One-Body dynamics) Move N_{TP} testparticles simultaneously (in p-space) to simulate fluctuation connected to NN collisions



QMD: fluctuations controlled by wave packet width L:
limits:
L→ 0 classical point partucles, nuclei not bound
L→ ∞ complete smoothing, no fluctuations



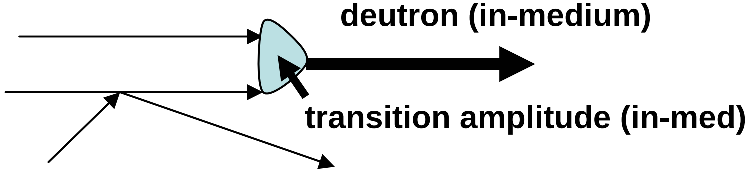


Comparison of simulations:
BUU(SMF)-AMD:
(Rizzo, Colonna,Ono, PRC82 (2010))

Methods to introduce LC correlations:

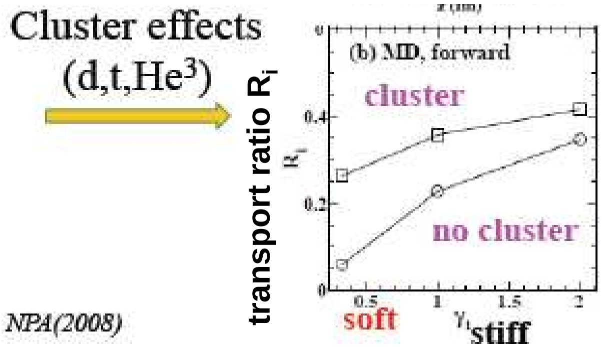
pBUU (Danielewicz)

LC as explicit degrees of freedom



→ coupled transport equations for LC

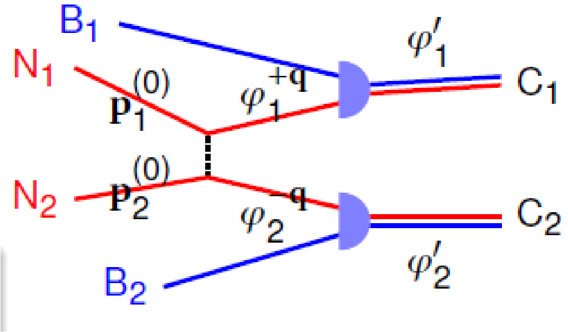
Medium modification of properties and transition amplitudes of light clusters in heavy ion reactions C. Kuhrts, et al,..PRC63 (2001)
Calculated in nuclear matter and static nuclei in gen. RMF approach by Typel, Röpke,et al., PRC81 (2010)



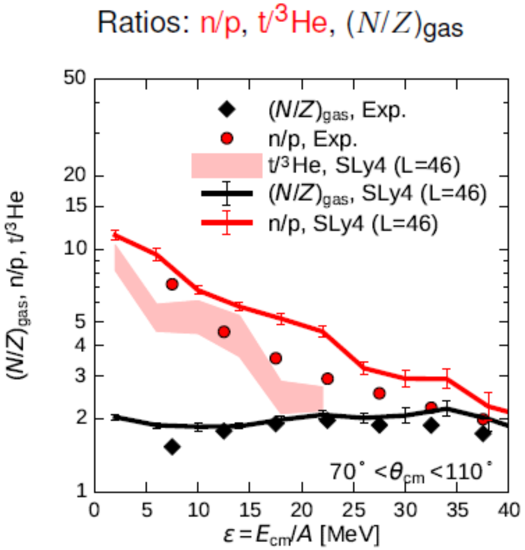
R_i: isospin transport ratio for charge equilibration in HIC between nuclei with different isospin content
e.g. ^{112,124}Sn+^{112,124}Sn (MSU experiment)

AMD (Ono)

1. **formation of clusters in terms of overlap with cluster wave function**
2. **manipulate phase space: put wave packets in same place and satisfy Pauli principle fully**
3. **include also cluster-cluster collisions to form bigger clusters**



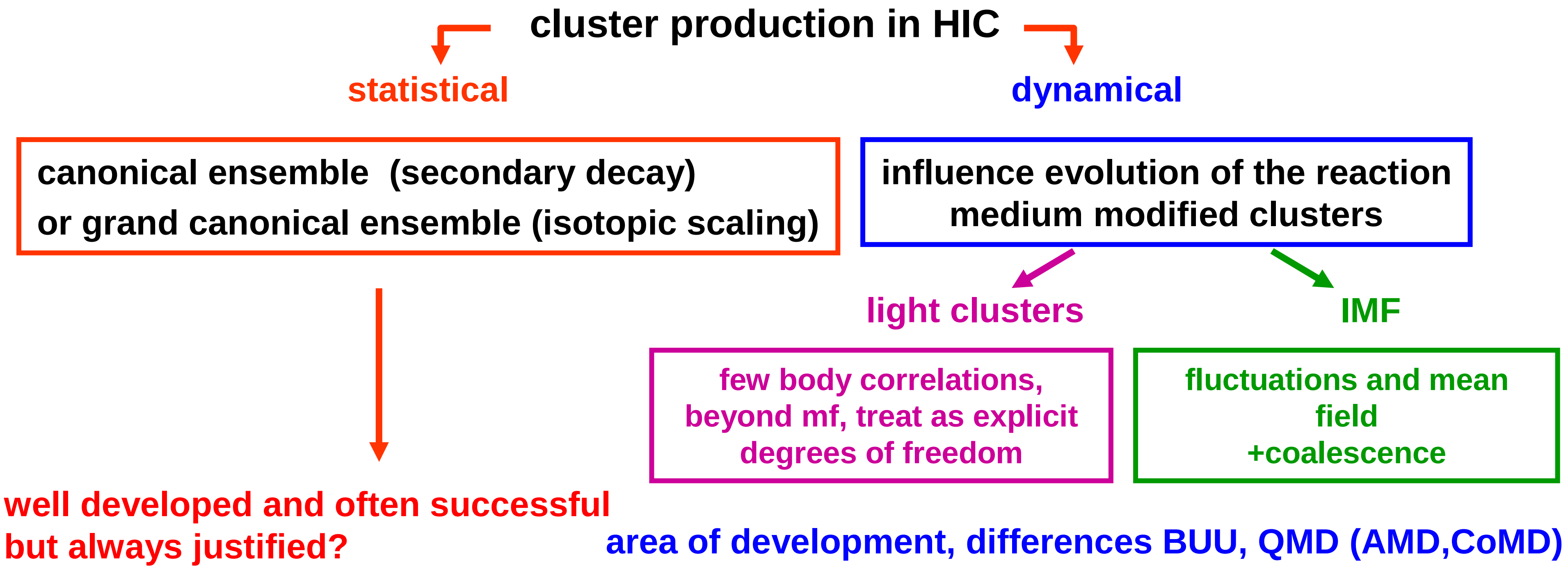
AMD with clusters
(A. Ono, NuSYM2015):
n/p (and t/h) ratios only reproduced if α-clusters included



Intermediate summary: Clusters and Fragments in Heavy Ion Collisions:

→ Clusters are ubiquitous in HIC (at low and intermediate energies)
important for analysis (observables depend on treatment of clustering)

→ contain important information on the state of the system
(e.g. equilibration, temperature, density, symmetry energy, etc)



cluster production in HIC

statistical

dynamical

canonical ensemble (secondary decay)
or grand canonical ensemble (isotopic scaling)

influence evolution of the reaction
medium modified clusters

light clusters

IMF

few body correlations,
beyond mf, treat as explicit
degrees of freedom

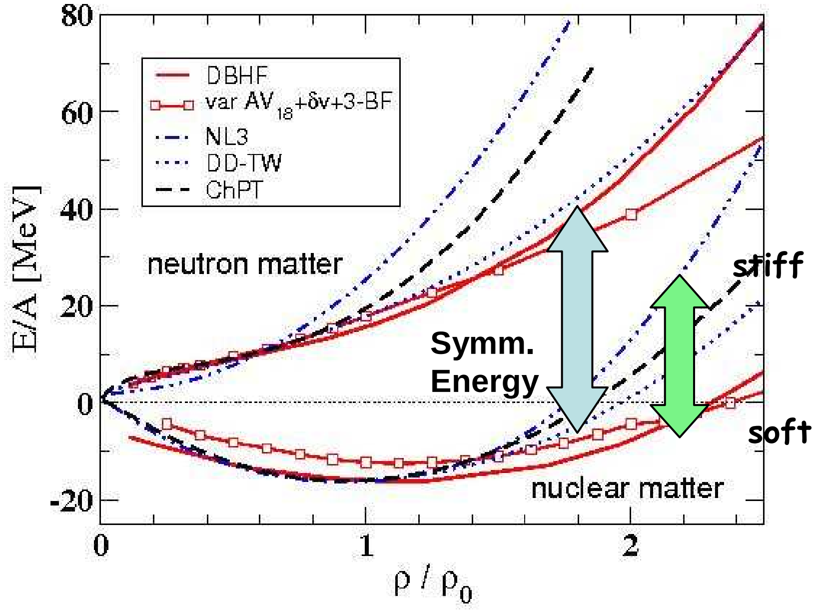
fluctuations and mean
field
+coalescence

well developed and often successful
but always justified?

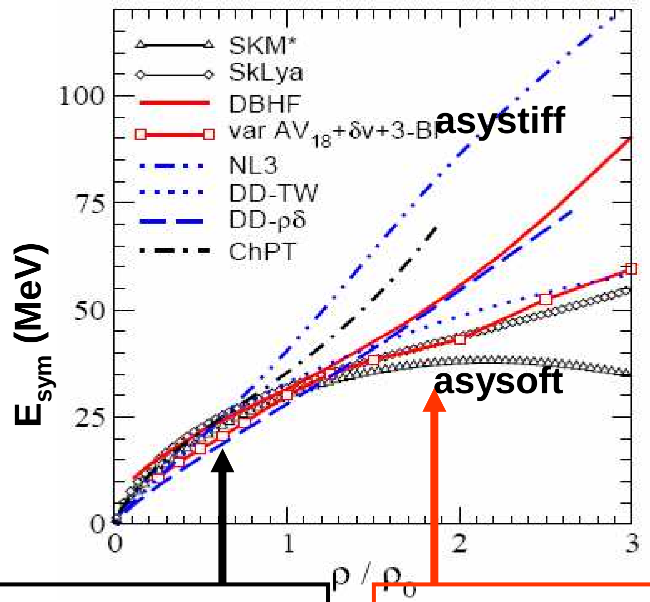
area of development, differences BUU, QMD (AMD,CoMD)

Back to the determination of the equation-of-state of nuclear matter

equation-of-state



symmetry energy



C. Fuchs, H.H. Wolter, EPJA 30(2006)5

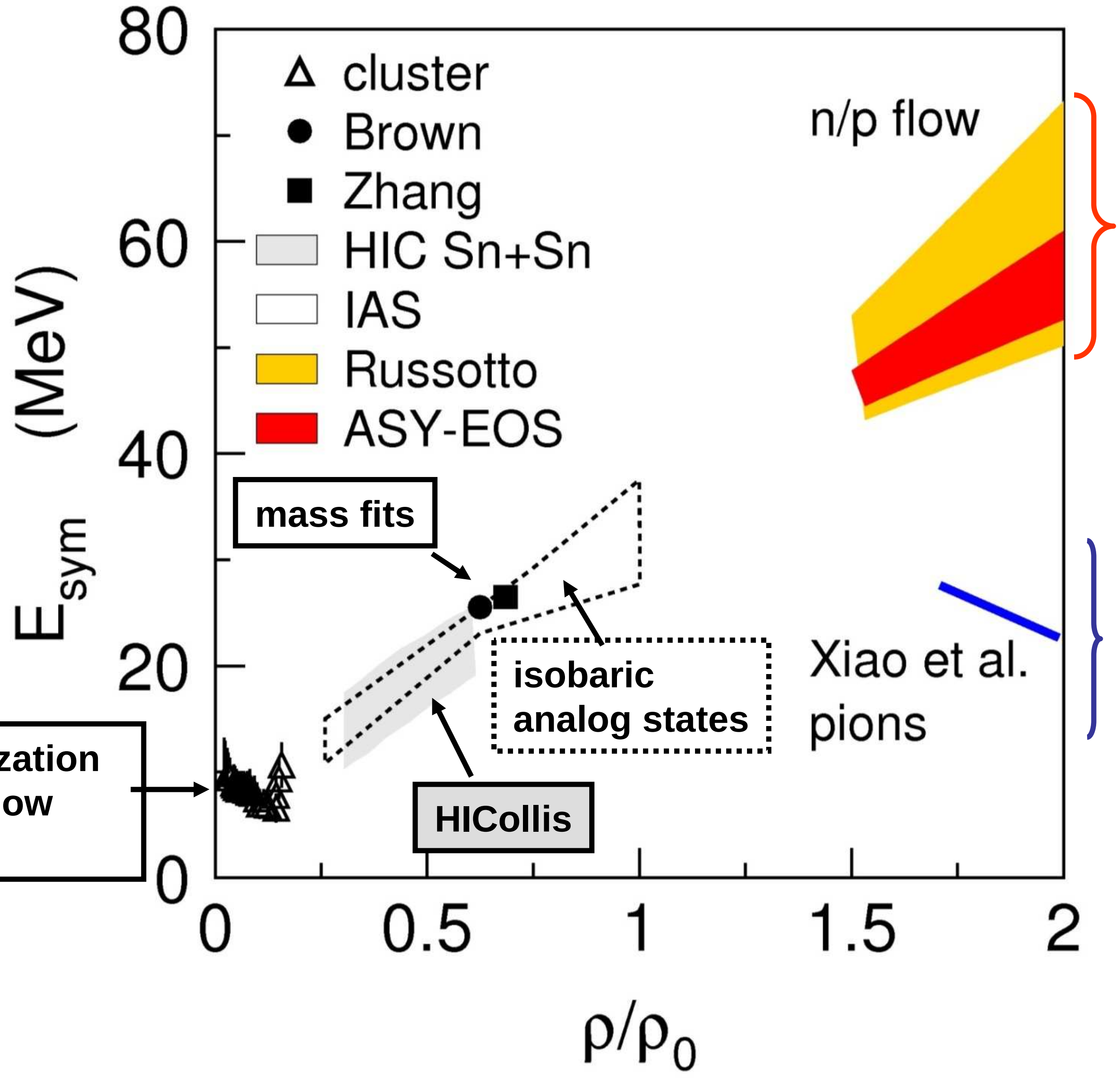
for $\rho \leq \rho_0$converging rather well

main uncertainity for $\rho \leq \rho_0$

Clusters and fragments also important for the symmetry energy

1. clusters properties are driven by the symmetry energy, i.e. the N/Z ratio
2. isospin fractionation between clusters and gas
3. clusterization gives a direct contribution to the symmetry energy: correlation depends on asymmetry of system; stronger in symmetric system

Present Constraints on the Symmetry Energy (shown as $E_{sym}(\rho/\rho_0)$)



mass fits

isobaric analog states

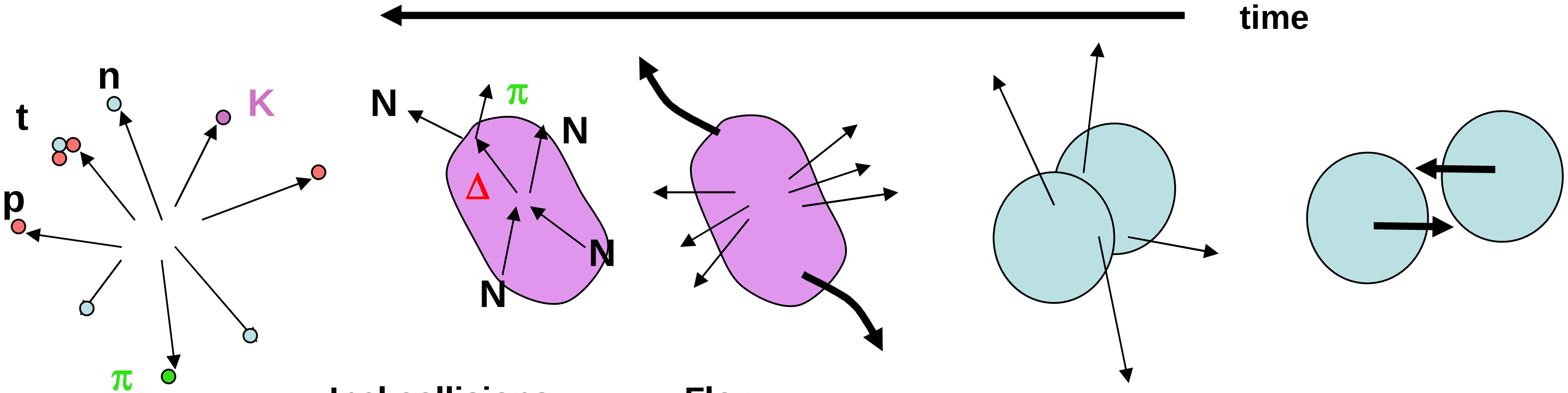
HICollis

clusterization at very low density

n/p flow at 400 – 800 MeV/A → symmetry energy rather stiff

π^-/π^+ ratio, but also other results. still a difficulty!

Sketch of reaction mechanism at intermediate energies and observables



disintegration

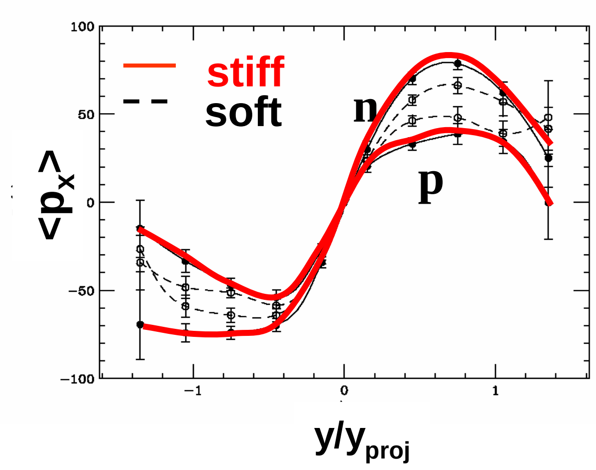
Inel.collisions
Particle product.
NN->NΔ–>NΛK
$\hookrightarrow$ Nπ

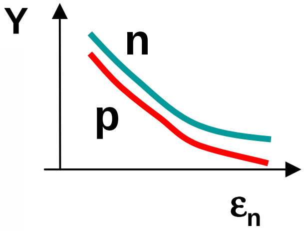
Flow,
In-plane, transverse
Squeeze-out, elliptic

Pre-equilibr emiss.
(first chance,
high momenta)

nn -> Δ^- p
$\hookrightarrow$ $n\pi^-$
pp -> Δ^{++} n
$\hookrightarrow$ $p\pi^+$
π^-/π^+ ↘ for asystiff

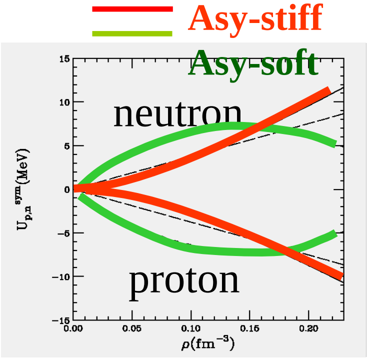
^{132}Sn + ^{132}Sn, 1.5 AGeV





Differential p/n
flow (or t/3He)

e.g.asy-stiff
n preferential
emmision
n/p ↘

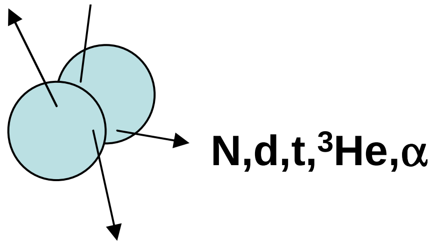


diff # p,n
(asymmetry of
system)

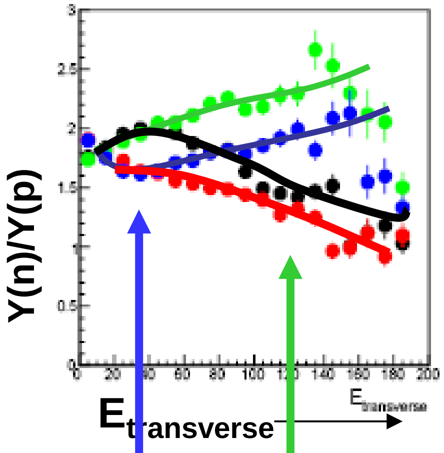
diff. force on n,p

Reaction mechanism can be tested with several observables: Consistency required!

Pre-Equilibrium Emission of Nucleons or Light Clusters

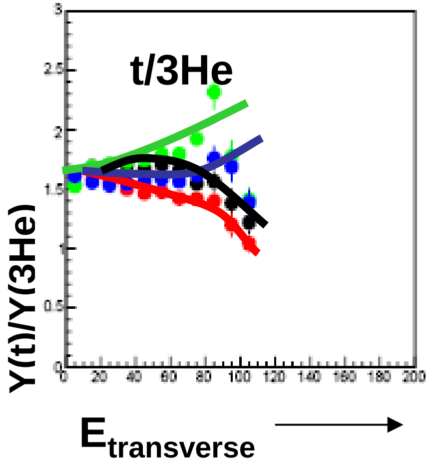


136Xe+124Sn, 150 MeV



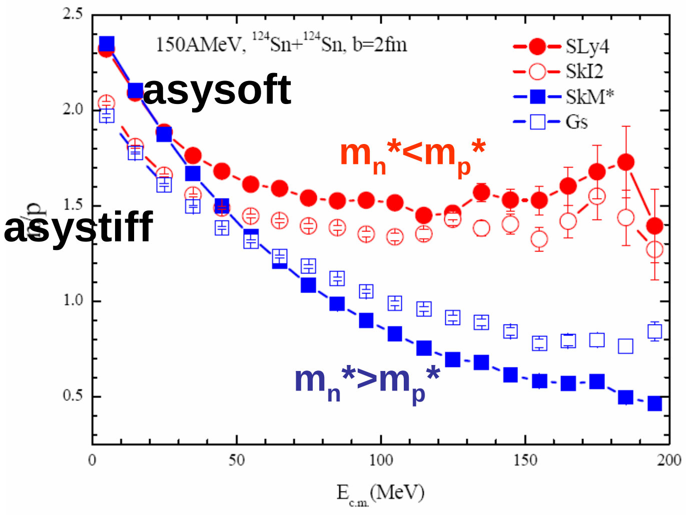
son: asysoft, $m_n^*>m_p^*$
stn: asystiff, $m_n^*>m_p^*$
sop: asysoft, $m_n^*<m_p^*$
stp: asystiff, $m_n^*<m_p^*$

density dep. dominates for slow particles;
mom.dep. (effective mass) for fast particles,
→separate density and momentum dependence



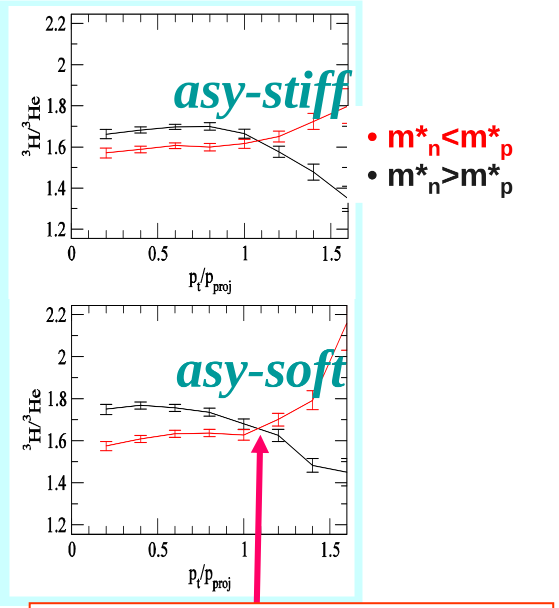
similar findings for Sn+Sn collisions (MSU)

^{124}Sn+^{124}Sn, 150 MeV



Y. Zhang,et al., PLB 732, 186 (2014)

^{197}Au+^{197}Au
600 AMeV b=5 fm, $|y_0|\leq 0.3$



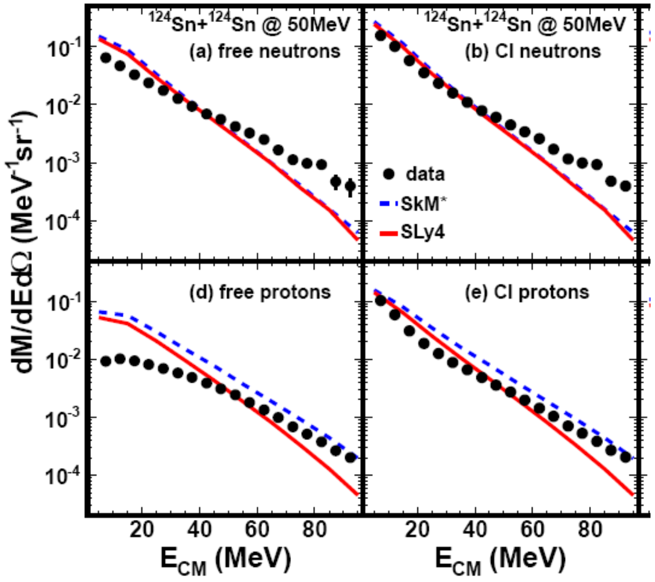
crossing connected to crossing of Lane potentials

effect of effective mass more prominent than that of asystiffness

(V.Giordano, et al., PRC 81(2010))

Comparison with data:
problem of light cluster description in transport approaches

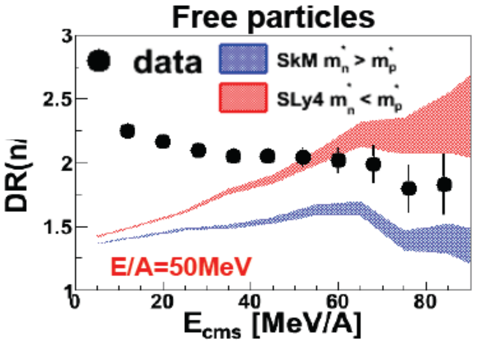
p,n spectra: free coalescence invariant (CI)

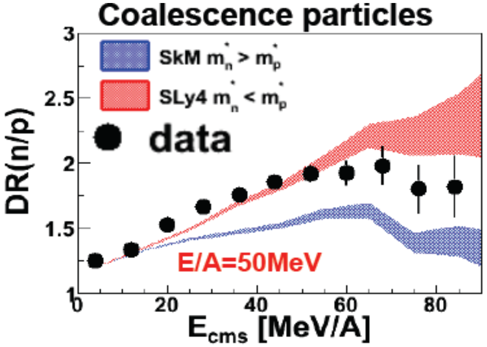


Double Ratios

$$\frac{^{124}\text{Sn} + {}^{124}\text{Sn}}{^{112}\text{Sn} + {}^{112}\text{Sn}}$$

agree only for CI spectra

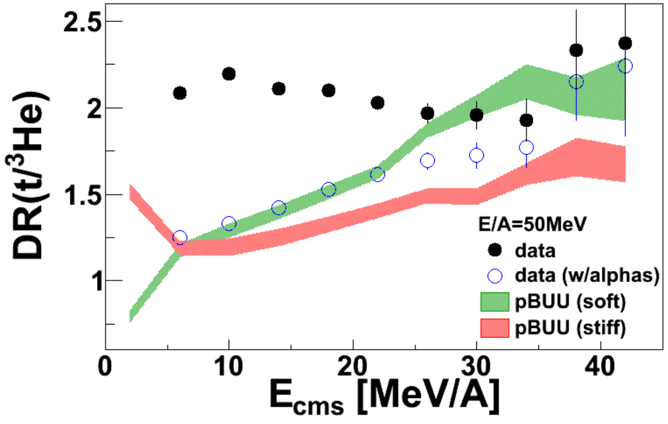




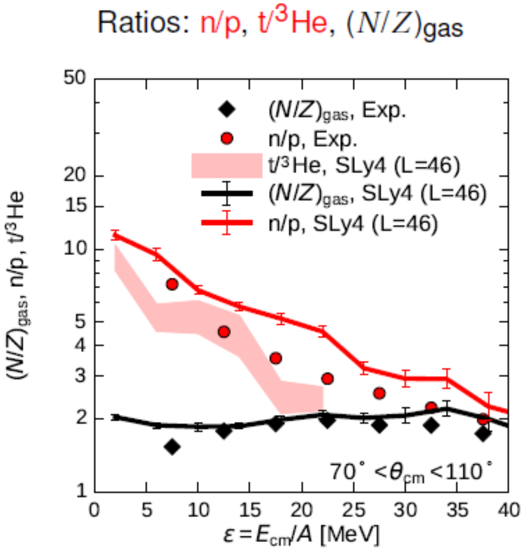
Y.X. Zhang, M.B.Tsang,et al., PLB 732, 186 (2014)
D.D.S.Coupland, arXiv 1406.4546

..or with calculation, where clusters are included explicitely

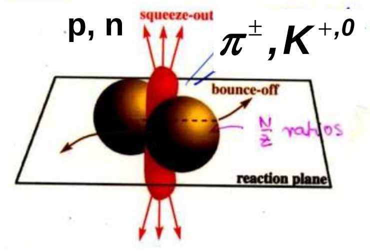
1. pBUU, when exp. α-particles are counted as t and 3He
   (Z. Chajecki, NuSYM 13)



2. AMD with clusters
   (A. Ono, NuSYM2015):
   n/p (and t/h) ratios only reproduced if α-clusters included



The Symmetry Energy at High Density

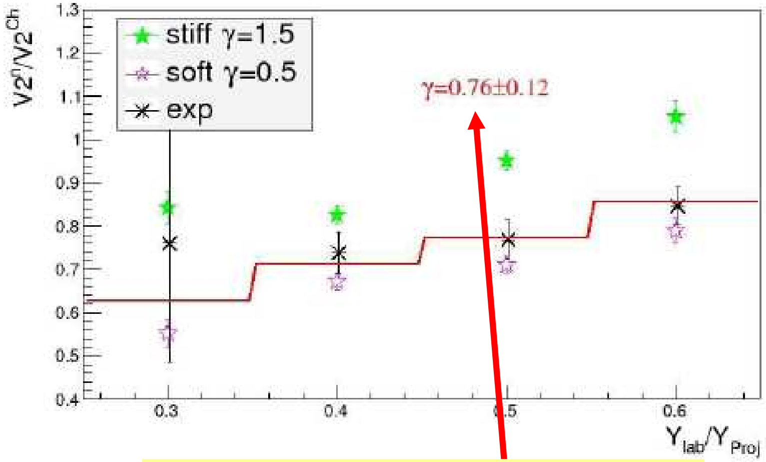


Au+Au @ 400 AMeV
new experiment ASY-EOS
(Russotto,NuSYM 2015, Krakow;
submitted PRC)

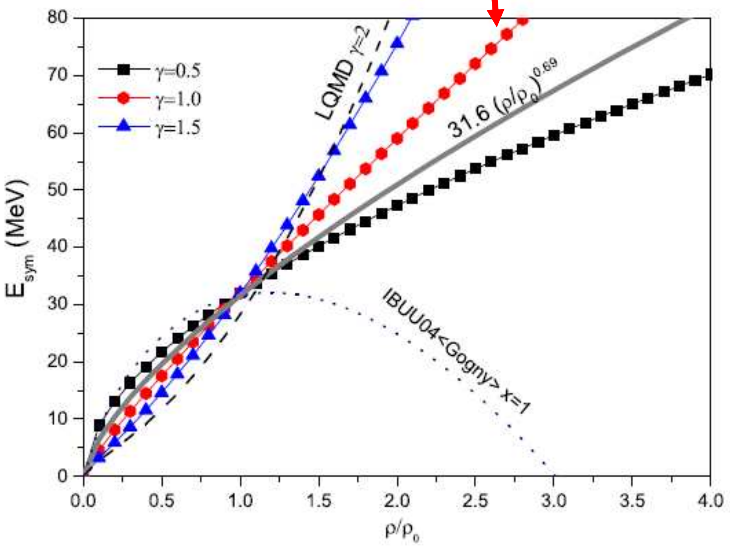
$$N(\Theta; y, p_t) = N_0(1 + \boxed{v_1}\cos\Theta + \boxed{v_2}\cos 2\Theta + \ldots)$$

ratio of neutron to proton (Z=1) flow
- Elliptic flow v_2 in this energy region good probe of high density

$$E_{sym}(\rho) = \tfrac{1}{3}\varepsilon_F\left(\frac{\rho}{\rho_0}\right)^{2/3} + C\left(\frac{\rho}{\rho_0}\right)^{\boxed{\gamma}}$$

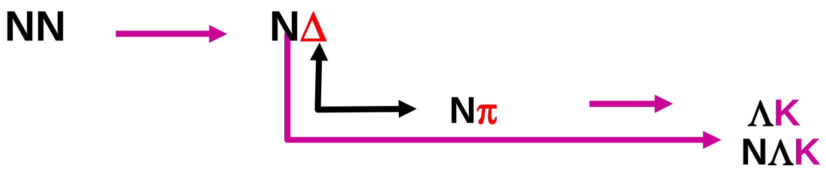


not very precise (yet) but indicates rather stiff SE, γ~1



Particle Production

Inelastic collisions: Production of particles and resonances: Coupled transport equations



NN → NΔ

Nπ → ΛK

NΛK

Many new potentials, elastic and inelastic cross sections needed, Δ dynamics in medium

symmetry energy effects on π,Δ production

1. Mean field effect: U_{sym} more repulsive for neutrons, and more for asystiff

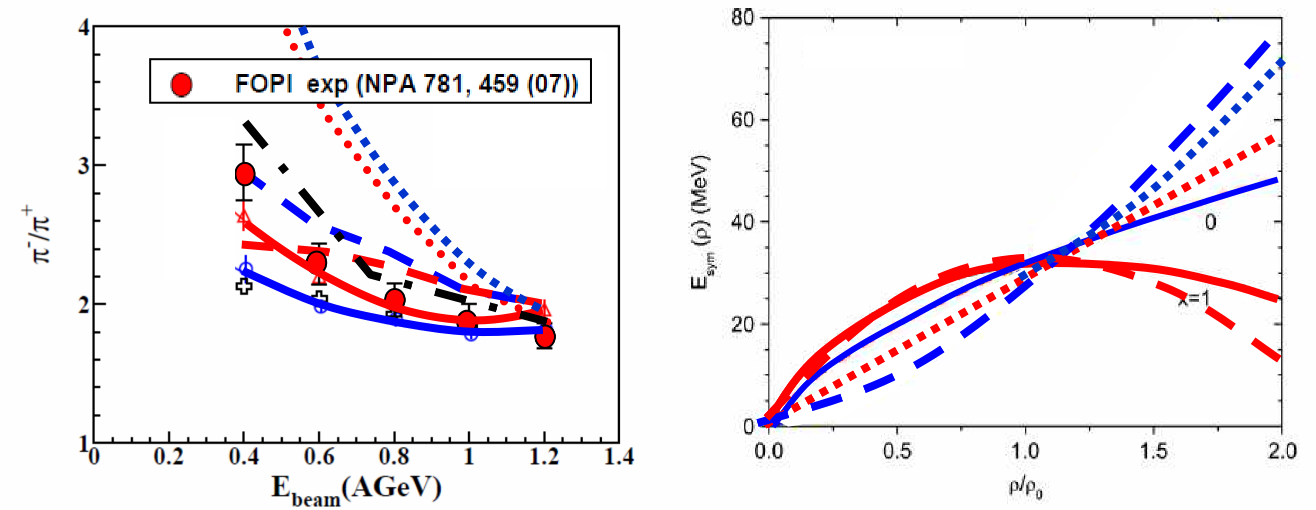
$$\frac{n}{p}\downarrow\Rightarrow\frac{Y(\Delta^{0,-})}{Y(\Delta^{+,++})}\downarrow\Rightarrow\frac{\pi^-}{\pi^+}\downarrow$$

decrease with asy – stiffness

2. Threshold effect, in medium effective masses:

$$\frac{\pi^-}{\pi^+}\uparrow \textit{ increase with asy – stiffness}$$

FOPI exp, Au+Au, 0.4-1.2 GeV/A



the result that fits best (Xiao, et al.) disagrees strongly with flow result.

more detailed measurements (π spectra) very important (SπrIt)

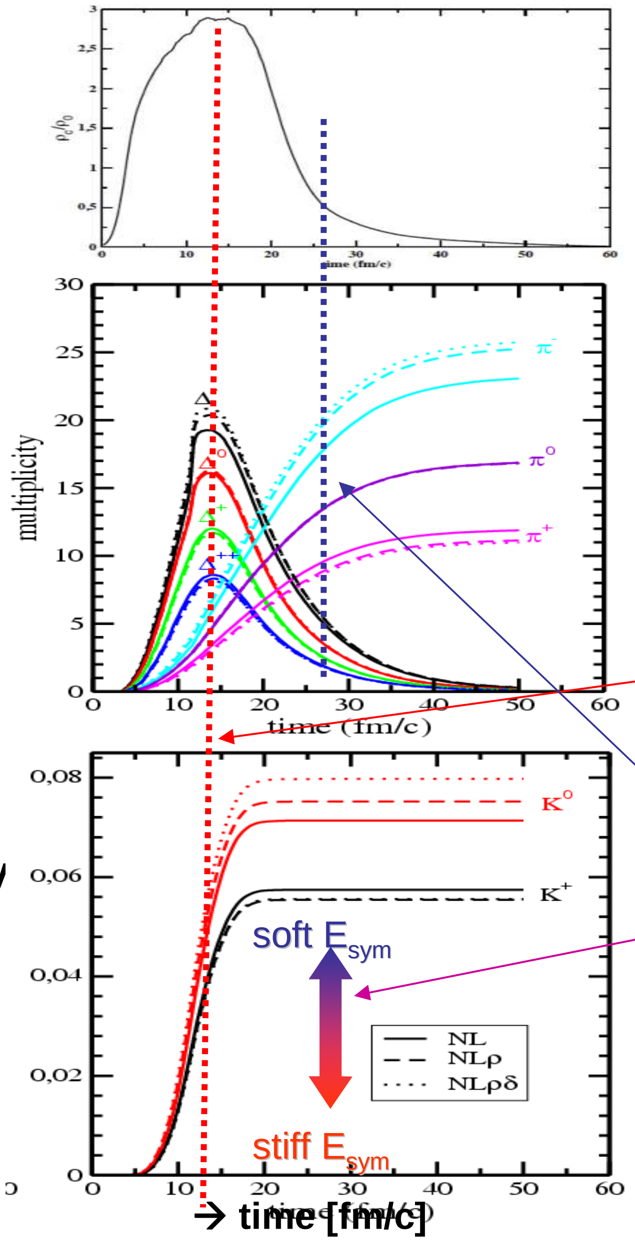
Dynamics of particle production (Δ,π,K) in heavy ion collisions

Au+Au@1AGeV

Central density

π and Δ multiplicity

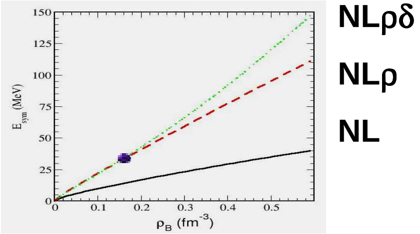
$K^{0,+}$ multiplicity



Δ and K: production in high density phase

Pions: low and high density phase

Sensitivity to asy-stiffness



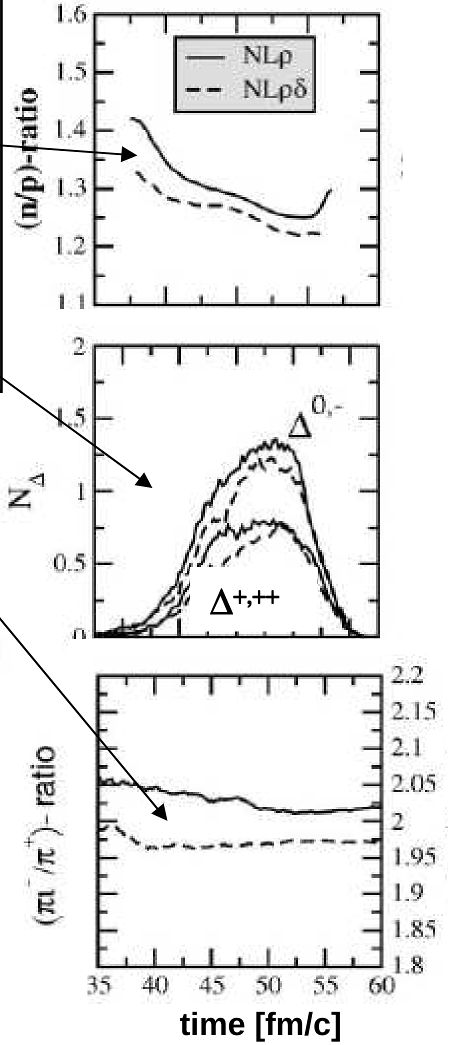
Dependence of ratios on asy-stiffness

n/p

→ $\Delta^{0,-}/\Delta^{+,++}$

→ π^-/π^+

Au+Au, 0.6AMeV

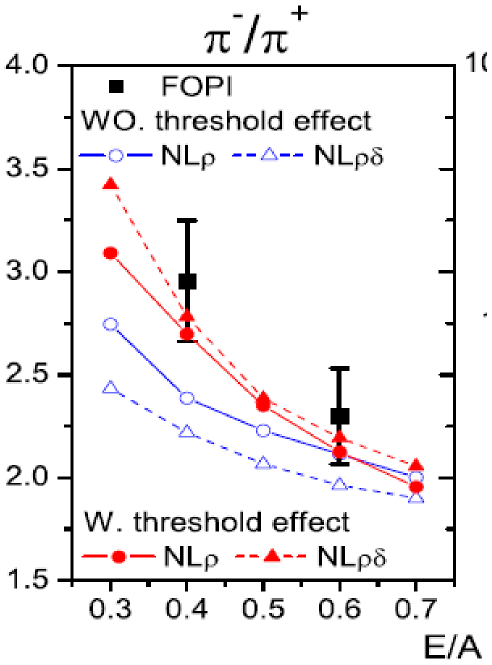


G.Ferini et al.,PRL 97 (2006) 202301

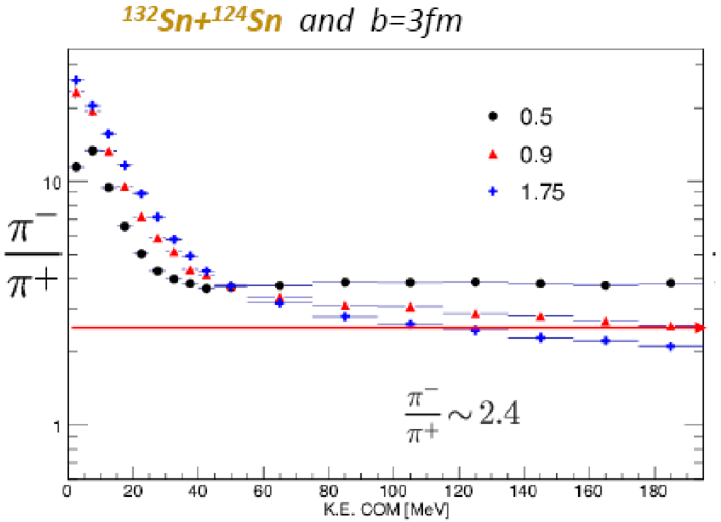
Possible reasons: πΔ dynamics, medium effects: potentials, effective cross sections, spectral fcts

(C.M. Ko)

in-medium threshold effect

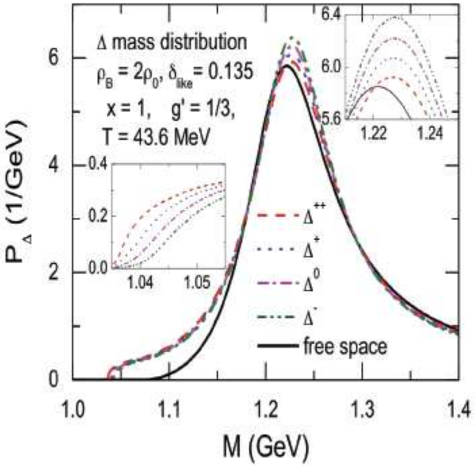


Planned experiment at Sπrit (MSU, Riken) 300 MeV: calculations with pBUU (P. Danielewicz)



high energy pions are more sensitive and less affected by rescattering

Δ Spectral function

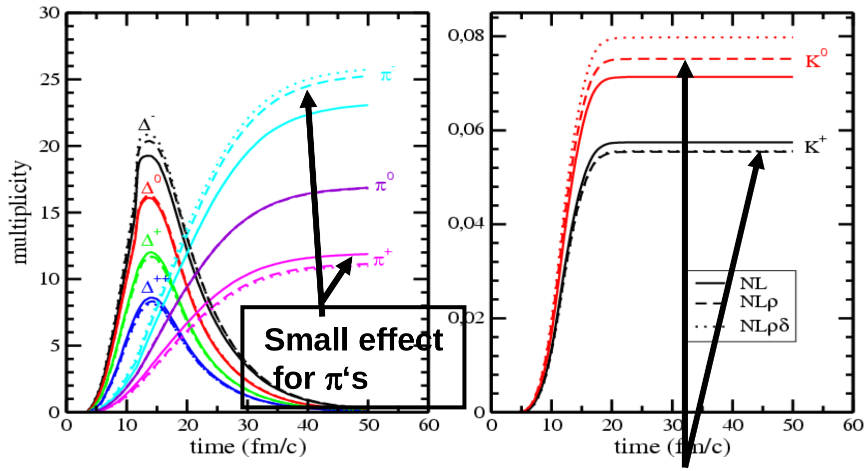


more fundamentally:
transport theory of particles with finite width,
„off-shell“ transport,
(Mosel (GiBUU) and Cassing (HSE) groups
but not systematically investigated)

Strangeness production in HIC: Kaons

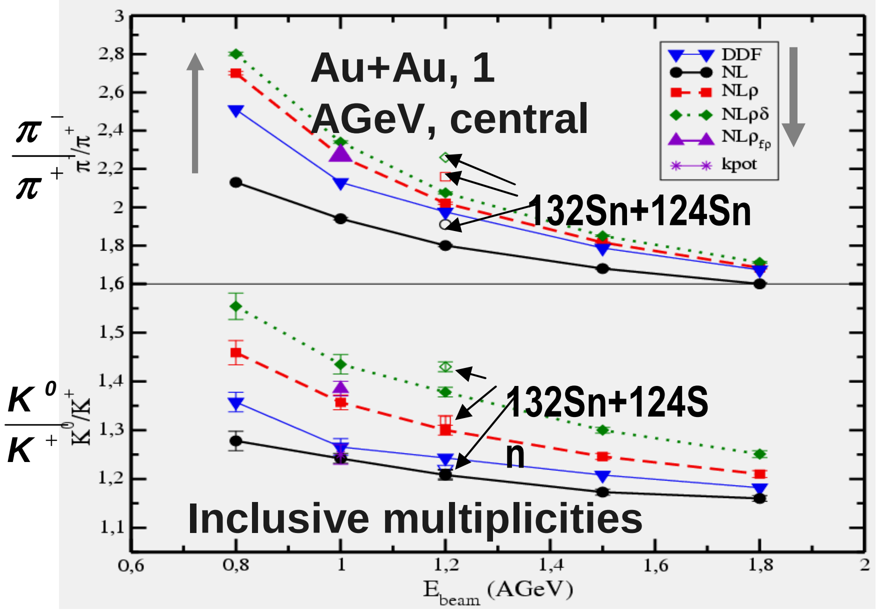
Kaons were a decisive observable to determine the symmetric EOS;

perhaps also useful for SE?
Kaons are closer to threshold, come only from high density, K^0 and K^+ have large mean free path, small width:



Small effect for π's

Larger (or equally large) effect for kaons, which come directly from high density region



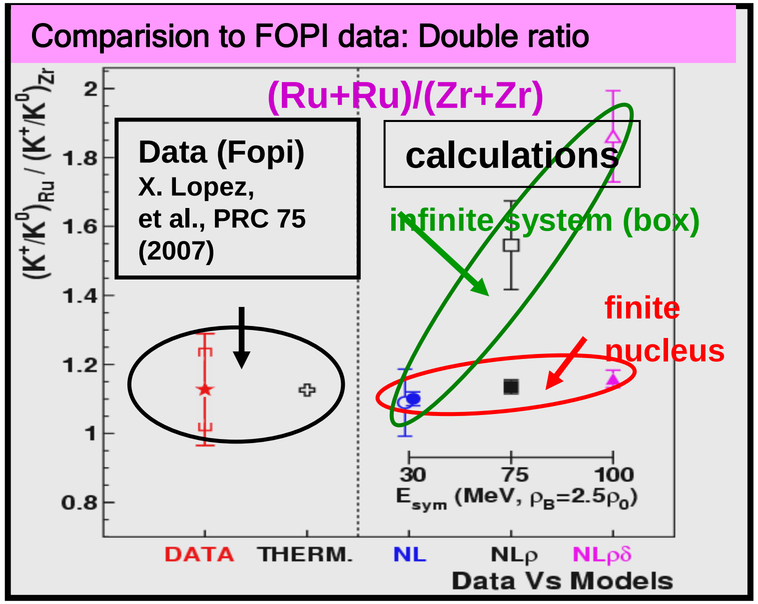
Au+Au, 1 AGeV, central

132Sn+124Sn

132Sn+124Sn

Inclusive multiplicities

G.Ferini et al.,PRL 97 (2006) 202301



Comparision to FOPI data: Double ratio

(Ru+Ru)/(Zr+Zr)

Data (Fopi)
X. Lopez,
et al., PRC 75
(2007)

calculations

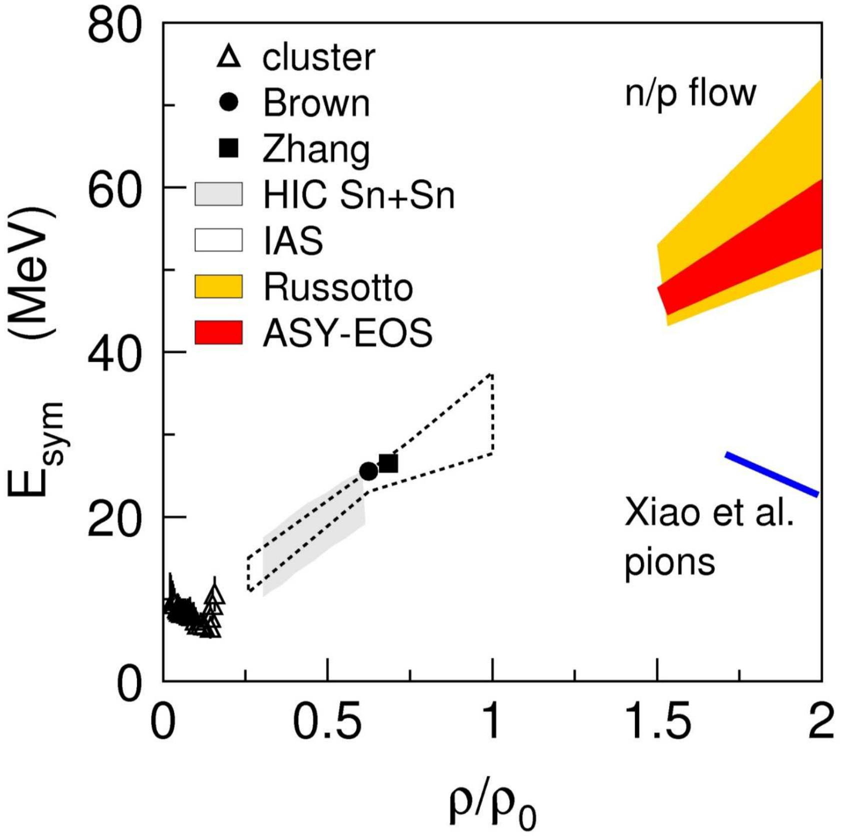
infinite system (box)

finite nucleus

G. Ferini, et al., NPA762(2005) 147

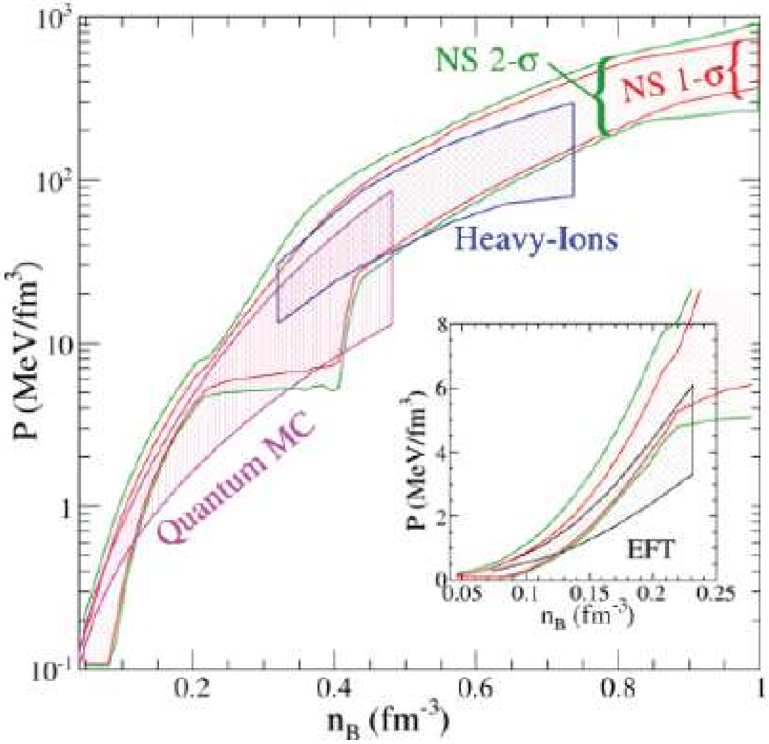
Single ratios are more sensitive!

Constraints from nuclear struture and heavy ion collisions



Synopsis of constraints from neutron stars, HIC and microscopic calculations

(for neutron star matter, i.e. β-equilibrium)



SUMMARY:

Equation-of-State (EoS) of nuclear matter of interest in itself
and important input for astrophysics:
Core Collapse Supernova, Neutron star structure, nucleosynthesis)

Investigation of EoS in the laboratory in Heavy Ion Collisions
Interpretation in complex transport models: open problems
- treatment of fluctuation and correlations to account for cluster and fragment production
- treatment of short range correlations in the kinetic energy
- consistency of transport approaches has been checked, but more work necessary
- treatment of instable particles (e.g. Δ)

EoS of symmetric nuclear matter (ρ_n=ρ_p) fairly well determined,
but symmetry energy is area of very active investigations experimentally
(new facilities) and theoretically:

→ constraints around and below ρ_0 rather stringent
→ clustering effects at very low densities
→ few experiments for high density, but new ones forthcoming,
biggest uncertainty
→ compatibility between HIC and neutron star constraints should be
checked more in detail

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(Lab. Naz. del Sud, INFN, Catania),
Malgorzata Zielinska-Pfabe (Smith College, USA)
Theo Gaitanos (Univ. Thesaloniki), et al.,

Clustering in dilute Matter, SN and NS observables:
Stefan Typel (Navi, GSI)
Gerd Röpke (Univ. of Rostock)
David Blaschke, Thomas Klähn (Univ. of Wroclaw)

Thank you for the interest