

# **QUARK DECONFINEMENT IN NEUTRON STARS**

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The Phases of Dense Matter, July 11 — August 12, 2016

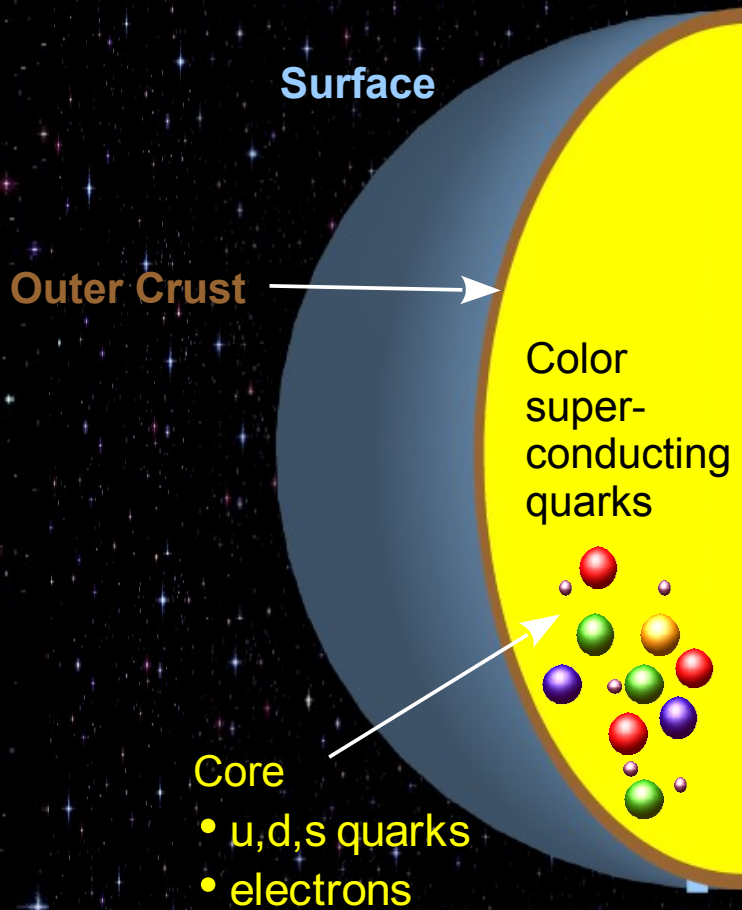


# OUTLINE

- **Strange Stars versus Neutron Stars**
  - Distinguishing features
- **Quark-Hadron Phase Transition inside Pulsars**
  - Current status
  - Possible signal of quark deconfinement
- **$\Delta$  populations**
- **Rotating Neutron Stars**
  - Rotation in GR
  - Rotation-driven compositional changes
- **Quark-hadron Lattices**
- **Non-rotating but nevertheless deformed?**
- **Summary**

# Strange Quark Stars

# Strange Quark Star



- **Made entirely of deconfined quarks**
- Self-bound ( $M \sim R^3$ )
- Electron dipole layer at surface (ultra-strong electric fields)
- Either bare or “dressed”
- Only outer crusts
- No inner crusts
- Two-parameter stellar sequence

Alcock, Farhi, Olinto, ApJ 310 (1986) 261;  
Alcock & Olinto, Ann. Rev. Nucl. Part. Sci. 38 (1988) 161;  
Madsen, Lecture Notes Phys. 516 (1999) 162;  
“Proc. of Strange Quark Matter in Physics and Astrophysics”,  
J. Madsen & P. Haensel, NPB (Proc. Suppl. 24B (1991);  
V. Usov, ApJ 559 (2001), 550 (2001)

Schematic illustration  
of a quark star carrying  
a nuclear crust

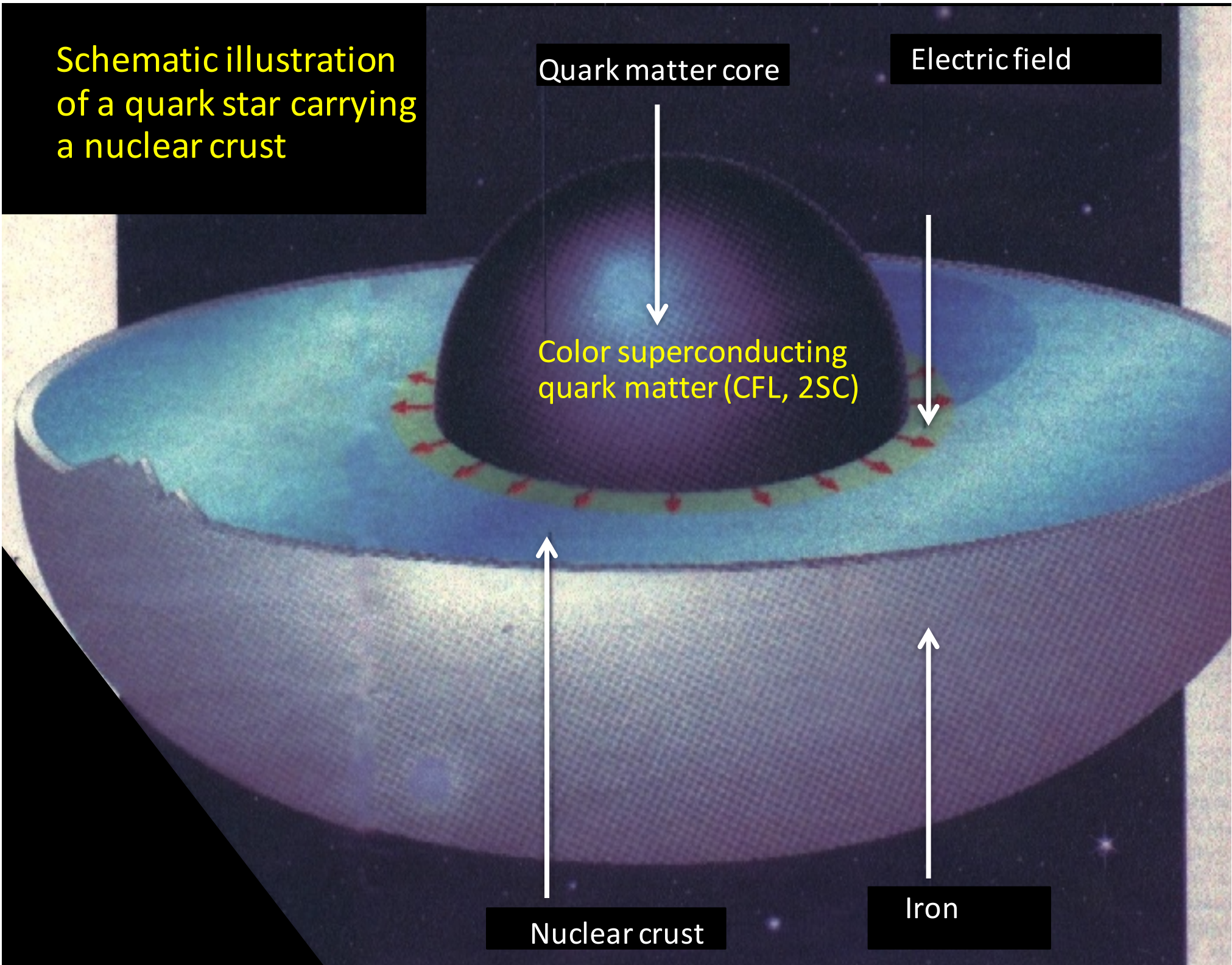
Quark matter core

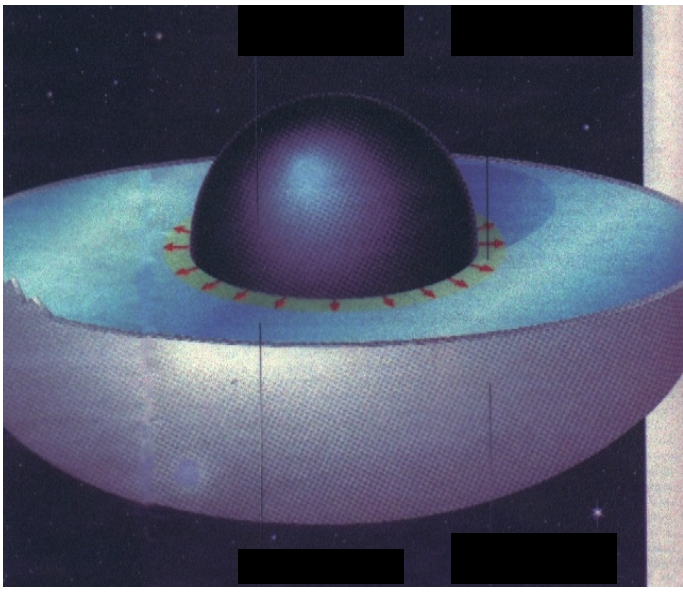
Electric field

Color superconducting  
quark matter (CFL, 2SC)

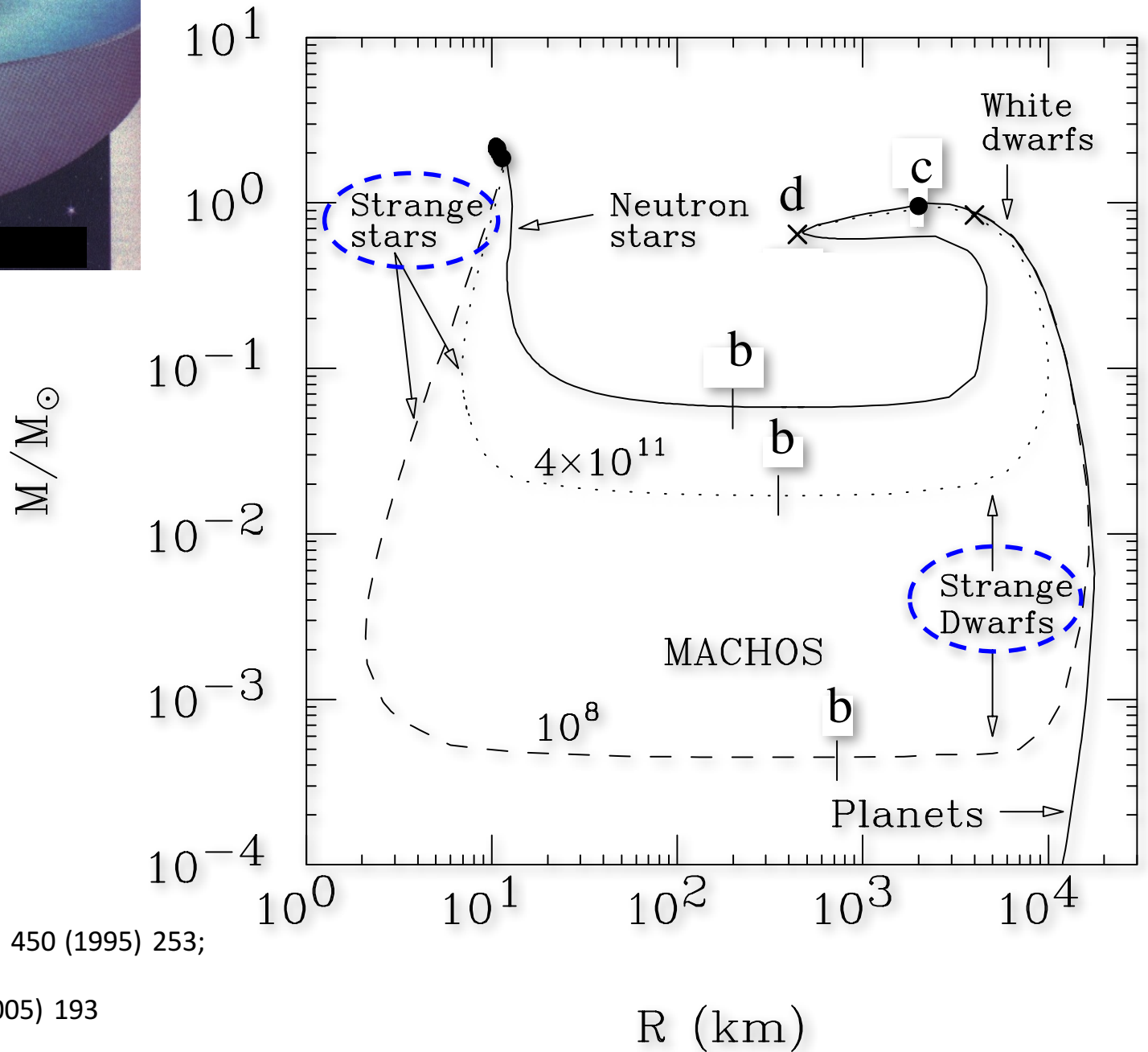
Nuclear crust

Iron



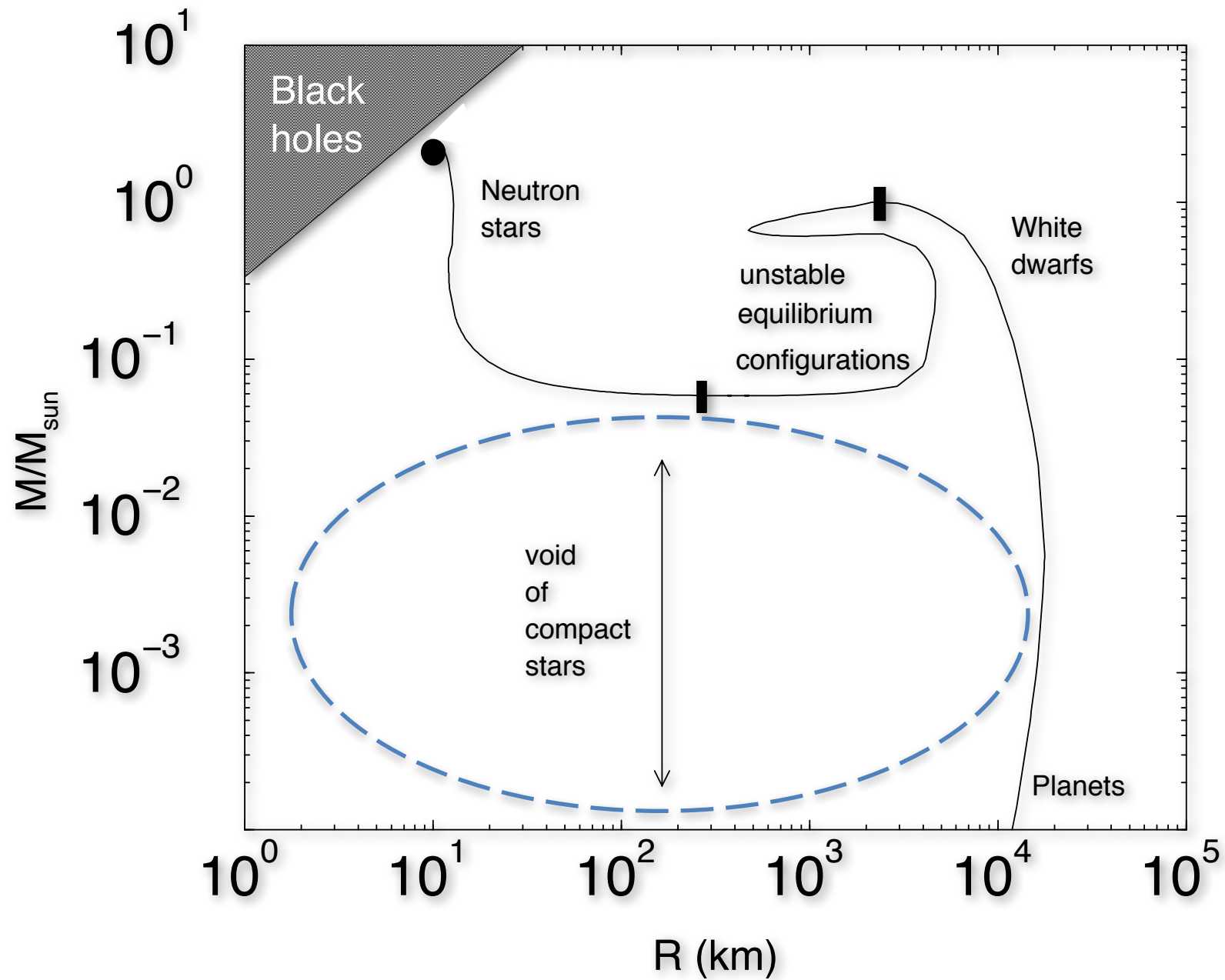


# Mass-radius relationship of neutron stars and **strange quark stars**



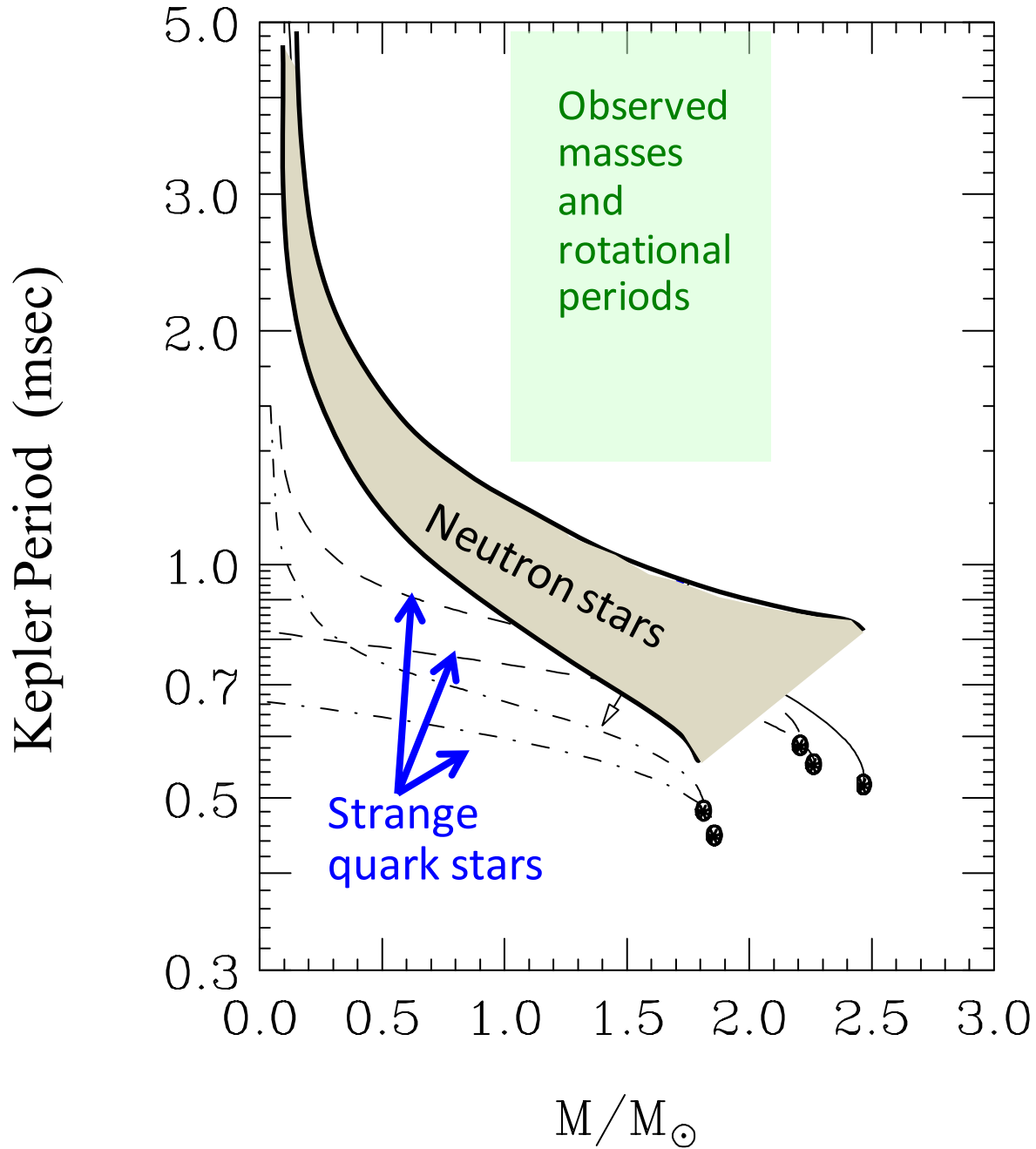
Glendenning, Kettner, FW, ApJ 450 (1995) 253;  
 PRL 74 (1995) 3519  
 FW, Prog Part Nucl Phys 54 (2005) 193

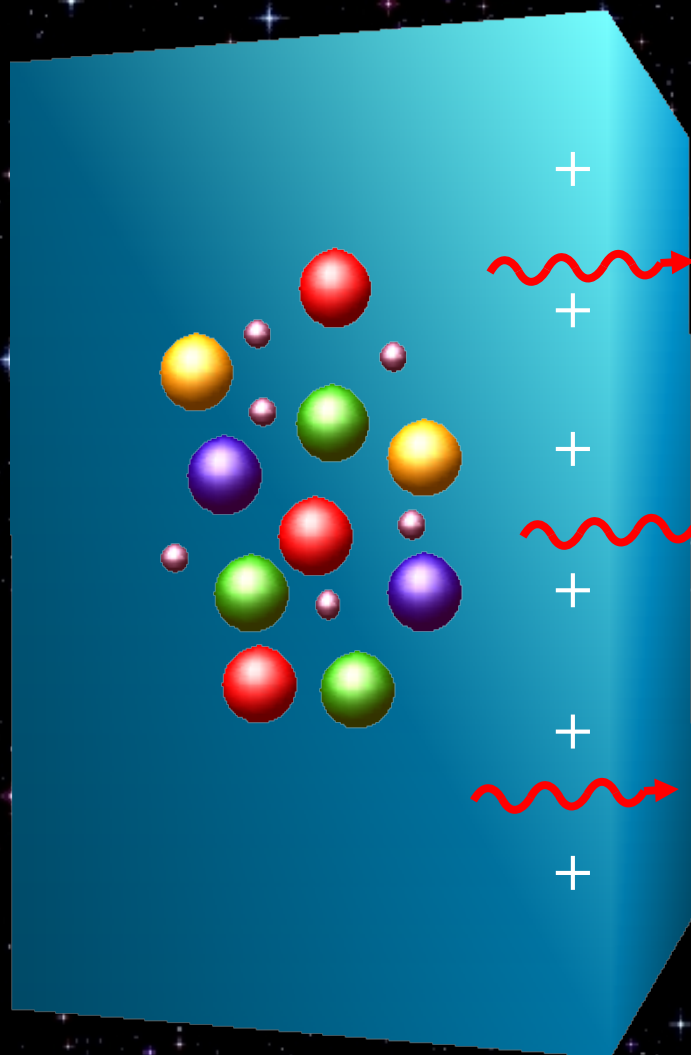
# Mass-radius relationship of **neutron stars**



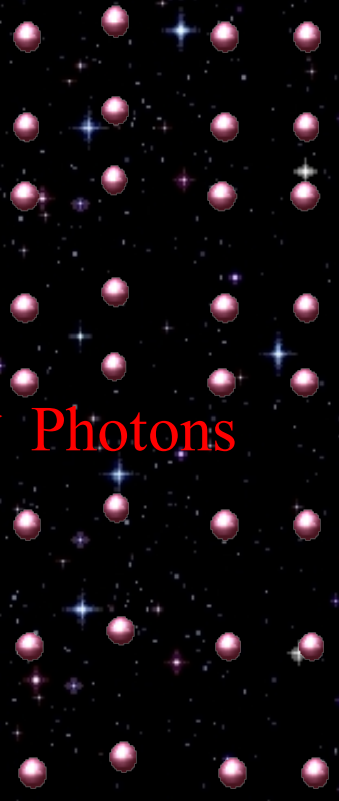


# Rotation at Sub-Millisecond Periods





Photons



Electron sea

Electron sea  
may perform  
global  
oscillations

Oscillation frequencies  
calculated by R. Xu et al.  
(PRD 85 (2012) 023008).

Related to spectral  
absorption lines  
observed for CCOs and  
XDINs?

### CCOs and XDINs with observed spectral absorption lines

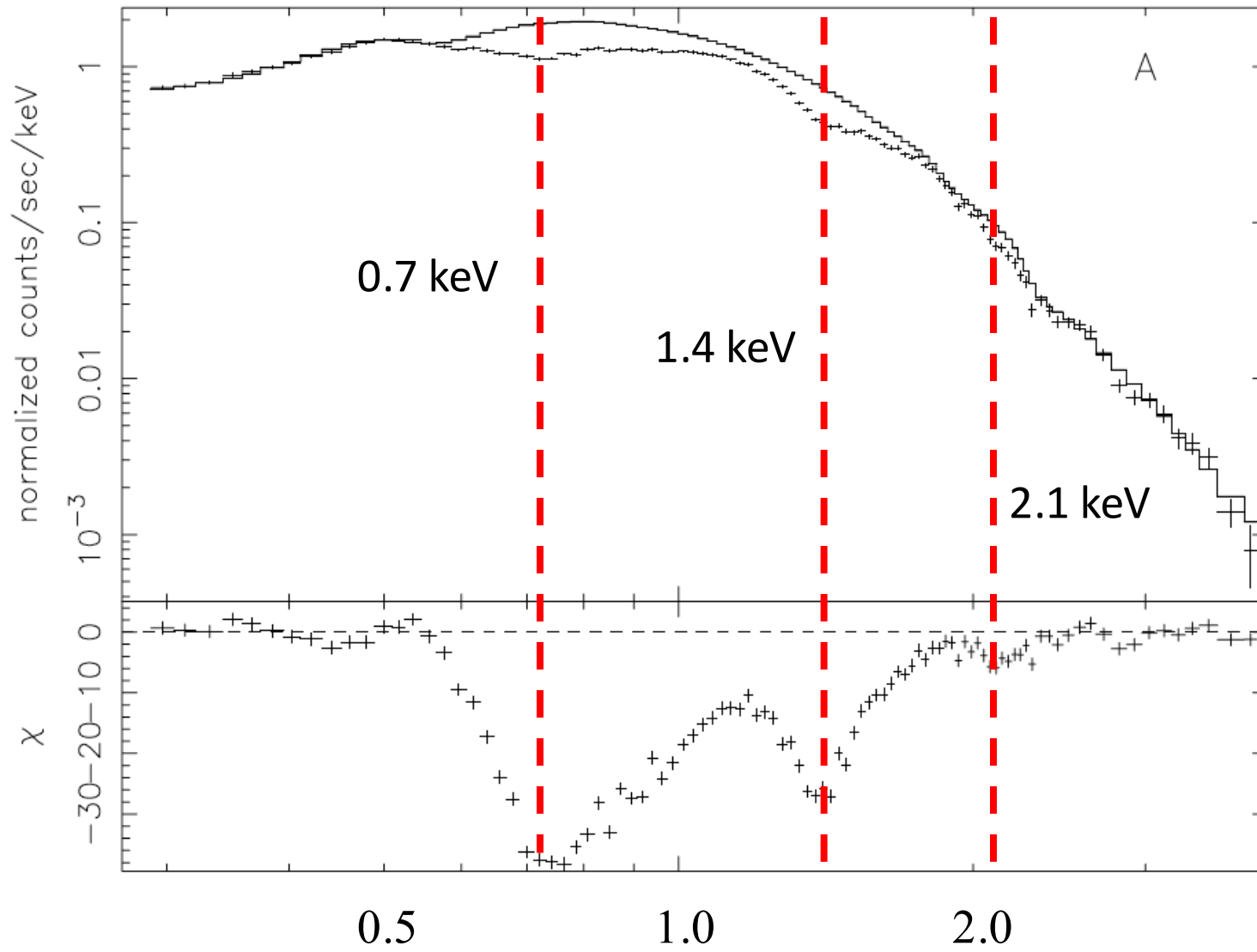
Source	$P/s$	$B_{10}$	$kT/\text{keV}$	$E_a/\text{keV}$
RX J0822.0 – 4300	0.112	<98	0.4	...
1E 1207.4 – 5209	0.424	<33	0.22	0.7, 1.4
CXOU J185238.6 + 004020	0.105	3.1	0.3	...
RX J0720.4 – 3125	8.39		0.085	0.27
RX J0806.4 – 4123	11.37		0.096	0.46
RX J0420.0 – 5022	3.45		0.045	0.33
RX J1308.6 + 2127	10.31		0.086	0.3
RX J1605.3 + 3249	...		0.096	0.45
RX J2143.0 + 0654	9.43		0.104	0.7

Pavlov, Sanwal & Teter (2004); Halpern & Gotthelf (2010)

### Oscillation frequencies (R. Xu et al. (2012))

$\ell$	1	2	3	4	5	6
$\omega(\ell)/\text{keV}$	4.2	1.4	0.7	0.4	0.3	0.2

Absorption features detected in the thermal X-ray spectrum of  
1E 1207.4-5209 at 0.7, 1.4 and 2.1 keV\*



\*G. F. Bignami, P. A. Caraveo, A. De Luca, & S. Mereghetti, Nature 423 (2003) 725

# Quark-Hadron Phase Transition

# Modeling the Quark-Hadron Phase Transition in the Cores of Neutron Stars

- ❑ Baryonic matter (Schroedinger-based, RMF, RHF, RBHF, . . .)
- ❑ Quark matter (MIT bag model, NJL)

$$P_h(\mu, \mu^e, \chi) = P_q(\mu, \mu^e, \chi) \quad \text{Gibbs or Maxwell?}$$

- ❑ Electric charge neutrality (global versus local)
- ❑ Chemical equilibrium

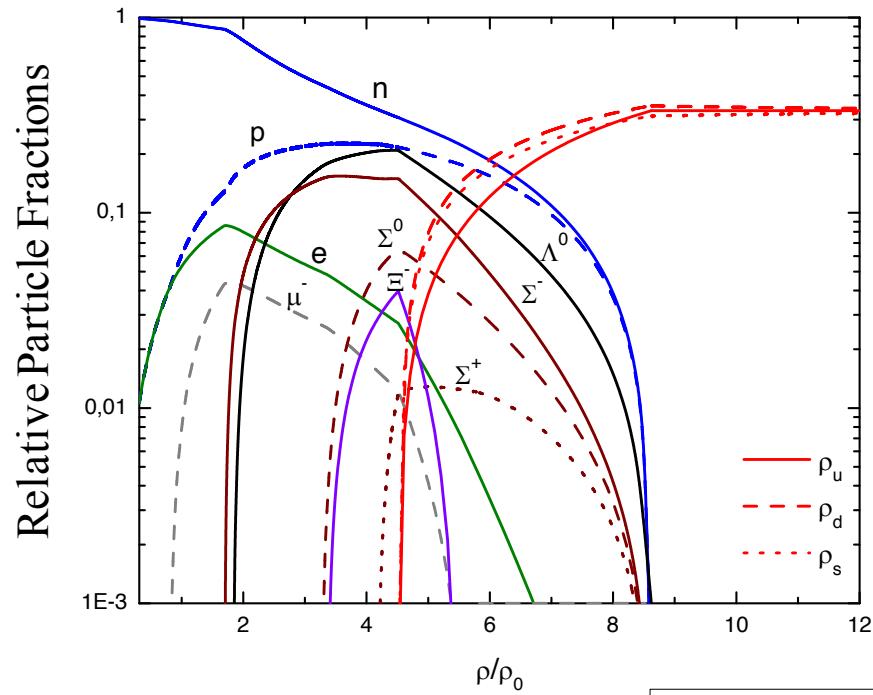
$$\begin{aligned}
\mathcal{L} = & \sum_{B=n,p,\Lambda,\Sigma,\Xi} \bar{\psi}_B [\gamma_\mu (i\partial^\mu - g_\omega \omega^\mu - g_\rho \vec{\rho}^\mu) \\
& - (m_N - g_\sigma \sigma)] \psi_B + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) \\
& - \frac{1}{3} b_\sigma m_N (g_\sigma \sigma)^3 - \frac{1}{4} c_\sigma (g_\sigma \sigma)^4 - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} \\
& + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu \\
& - \frac{1}{4} \vec{\rho}_{\mu\nu} \vec{\rho}^{\mu\nu} + \sum_{\lambda=e^-, \mu^-} \bar{\psi}_\lambda (i\gamma_\mu \partial^\mu - m_\lambda) \psi_\lambda,
\end{aligned}$$

RMF

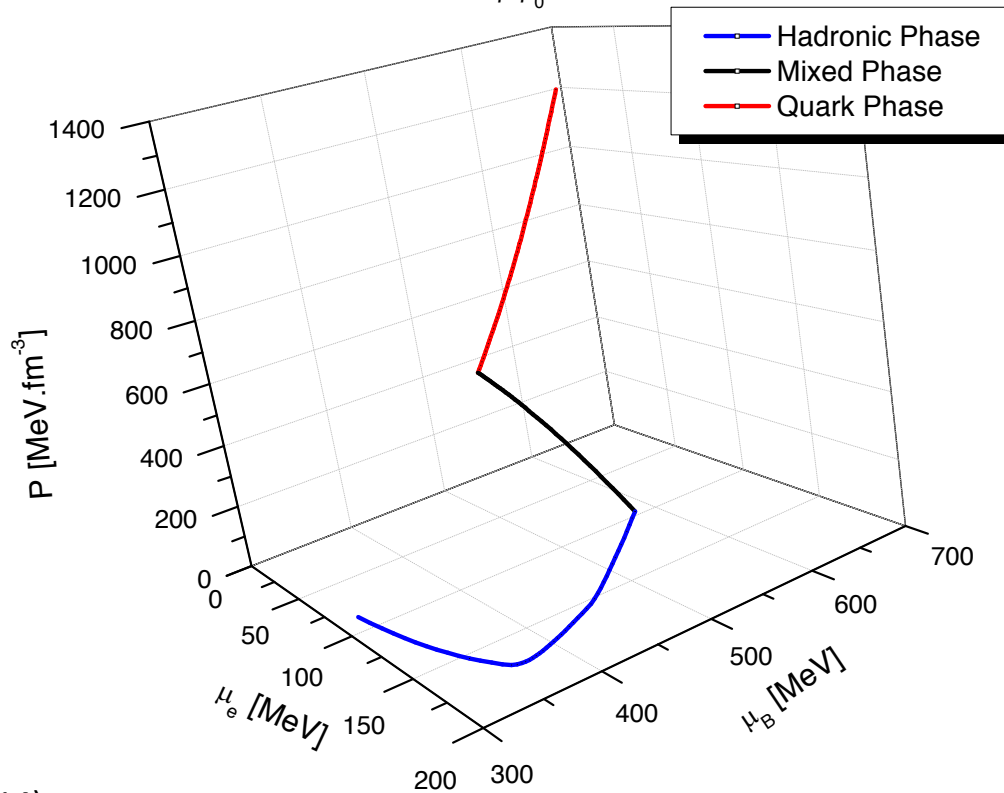
$$\begin{aligned}
S_E = & \int d^4x \{ \bar{\psi}(x) [-i\gamma_\mu \partial_\mu + \hat{m}] \psi(x) \\
& - \frac{G_s}{2} [j_a^S(x) j_a^S(x) + j_a^P(x) j_a^P(x)] \\
& - \frac{H}{4} T_{abc} [j_a^S(x) j_b^S(x) j_c^S(x) - 3 j_a^S(x) j_b^P(x) j_c^P(x)] \\
& - \frac{G_V}{2} j_{V,f}^\mu(x) j_{V,f}^\mu(x),
\end{aligned}$$

Local/  
non-local  
NJL models

# Model neutron star matter composition



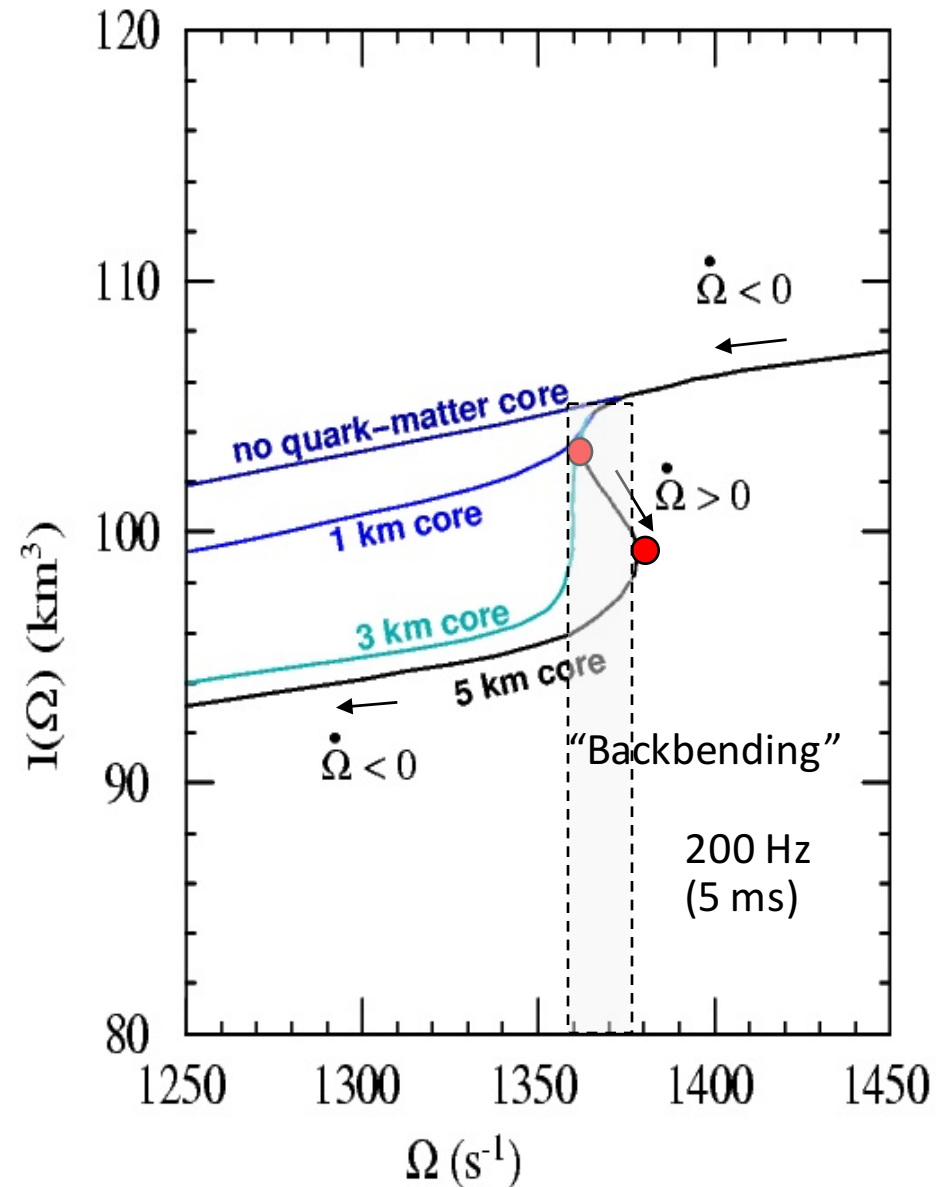
# Associated equation of state





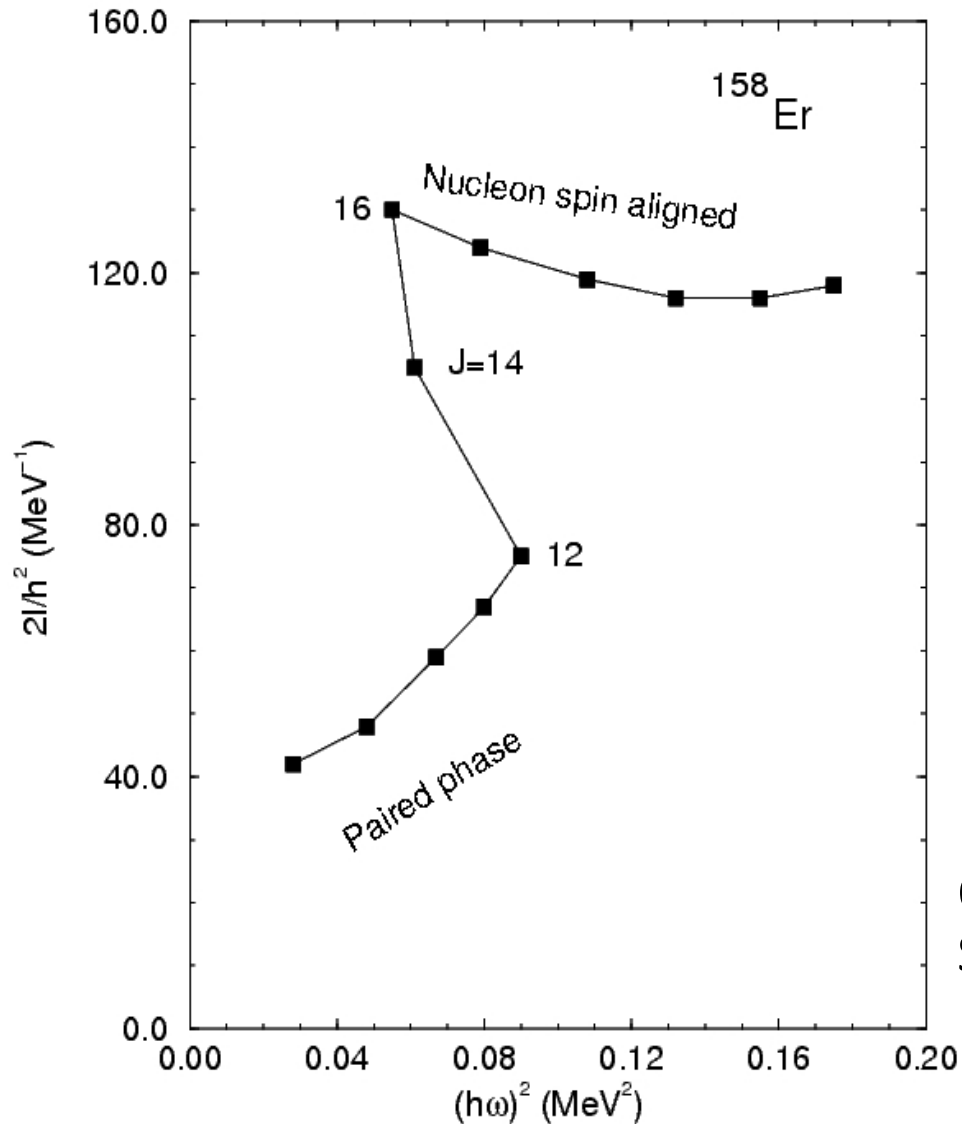
# A Possible Signal of Quark Deconfinement in isolated Pulsars during spin-down

$$J = \int d^3x T_3^0 \sqrt{-g}$$



# Backbending - well known in Nuclear Physics ...

and from the  
Olympics

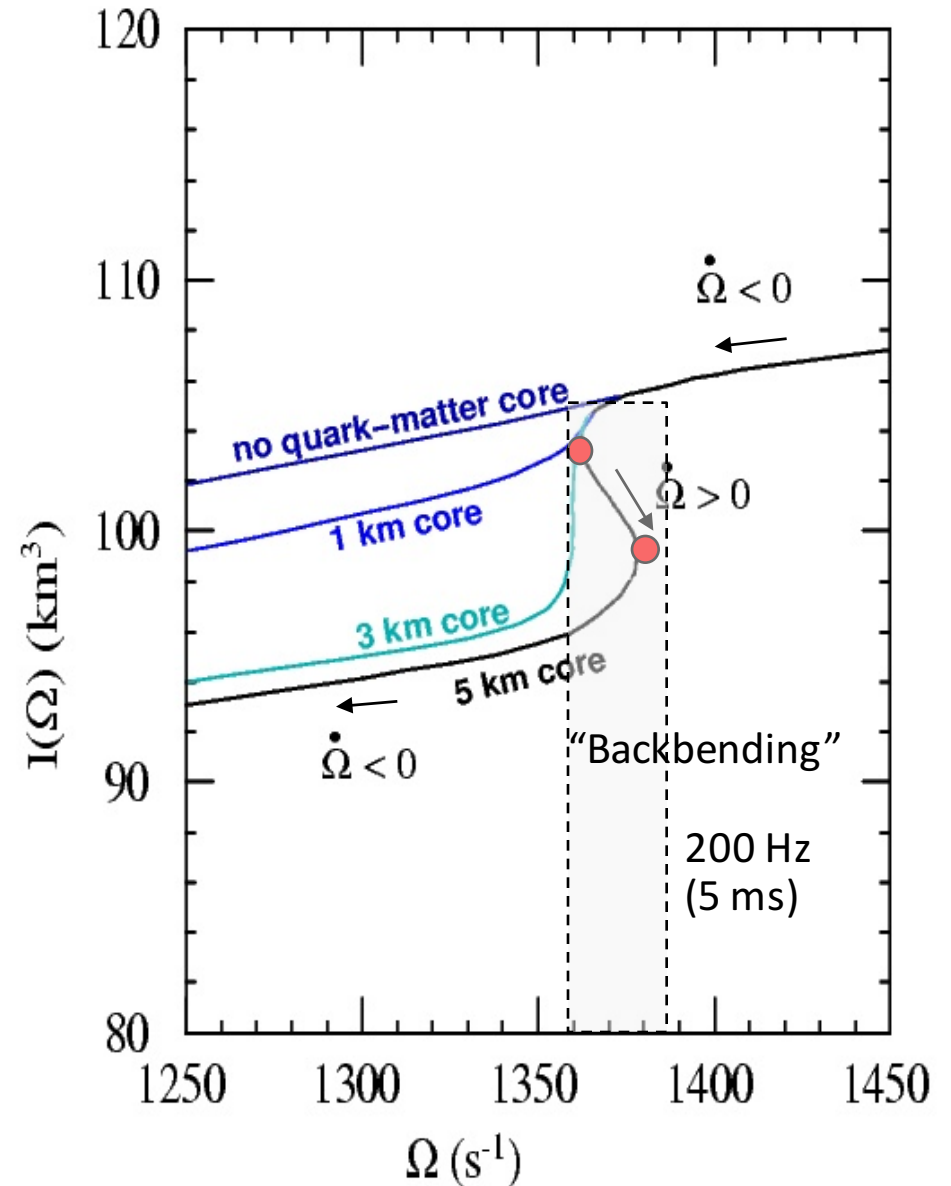


Backbending predicted in  
1960 by Mottelson & Valatin

Observed in 1972 by  
Stephens & Simon

# Braking index of a pulsar

$$n(\Omega) \equiv \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2} = 3 - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}$$

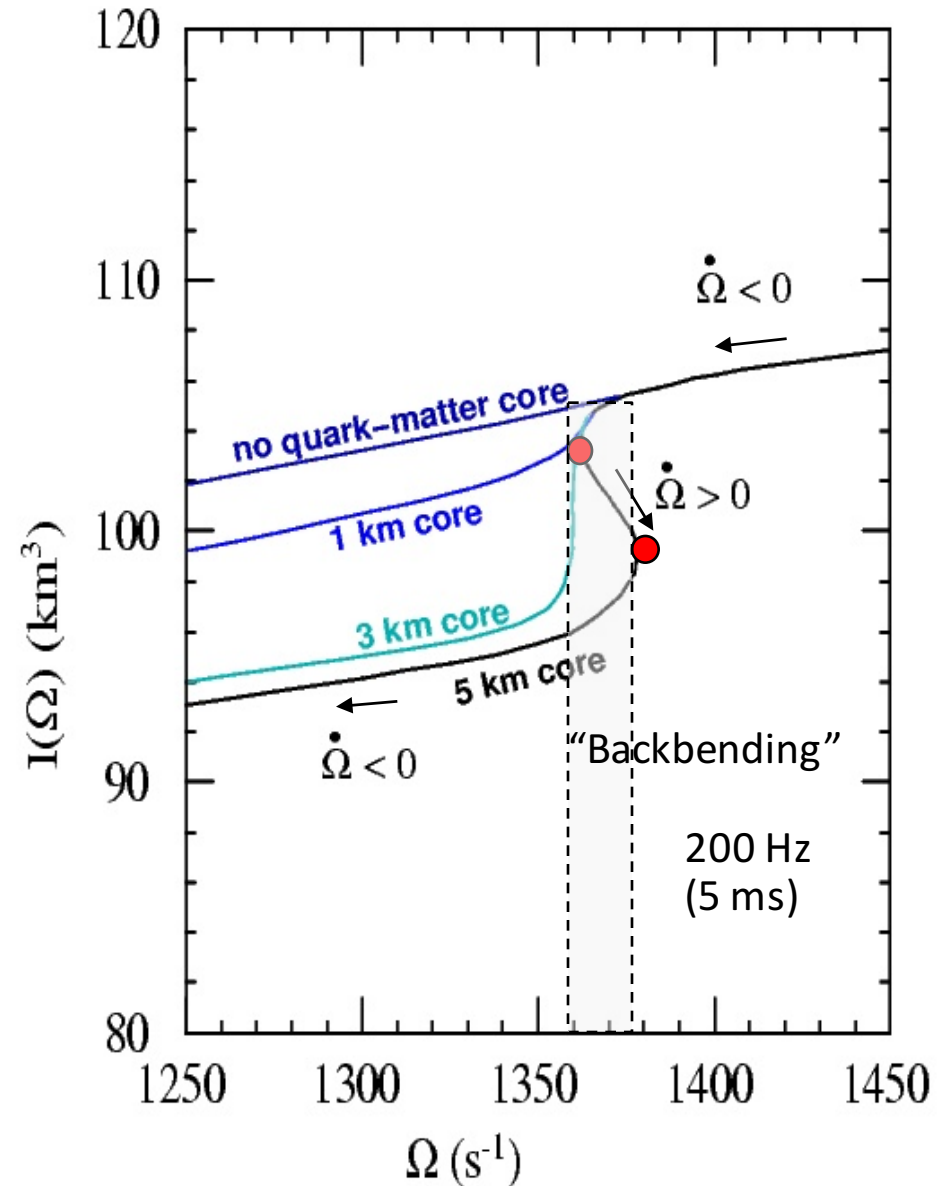


# Braking index of a pulsar

$$n(\Omega) \equiv \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2} = 3 - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}$$

## Signals of quark deconfinement

- Braking indices of pulsars  $-\infty < n < +\infty$
- Spin-up of isolated MSPs



**$\Delta$ 's populations**

# M-R relationship of non-rotating neutron stars

EOS: DD2 (Typel, 2010)

**Saturation parameters:**  $n_0 = 0.149 \text{ fm}^{-3}$ :

$E/A = -16.03 \text{ MeV}$ ,  $K_0 = 242.7 \text{ MeV}$ ,

$a_{\text{sym}} = 31.64 \text{ MeV}$ ,  $L_0 = 56.7 \text{ MeV}$ ,  $m^* = 0.56$

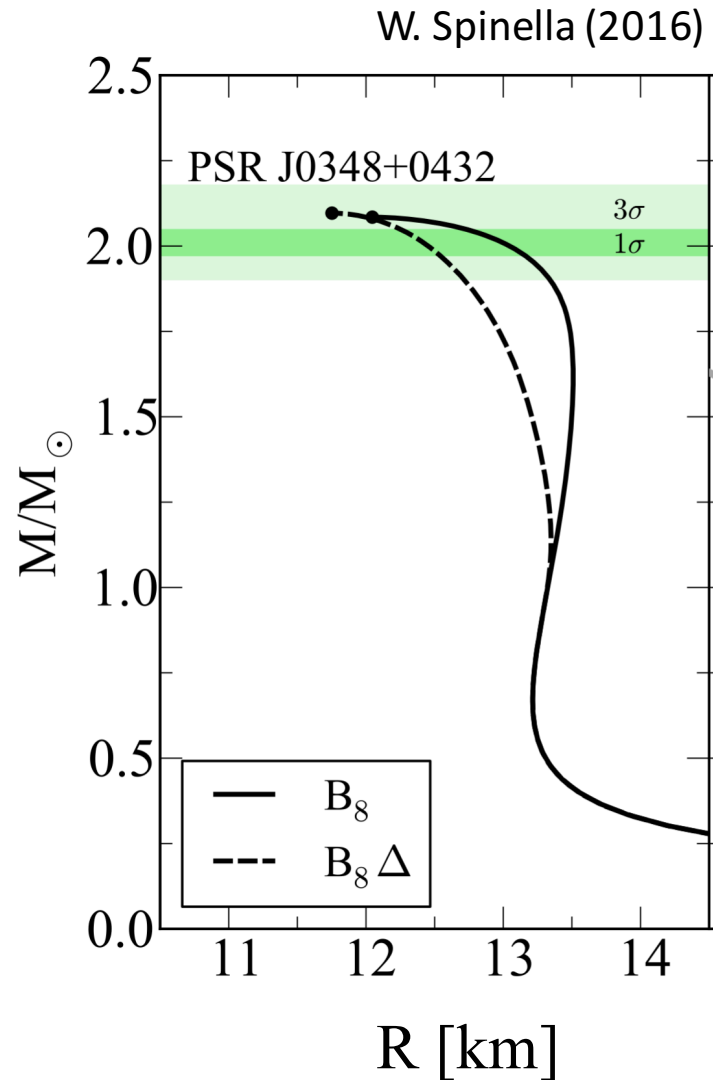
**Couplin constants:**

$g_{\sigma Y}$  fixed to hypernuclear potentials at  $n_0$ :

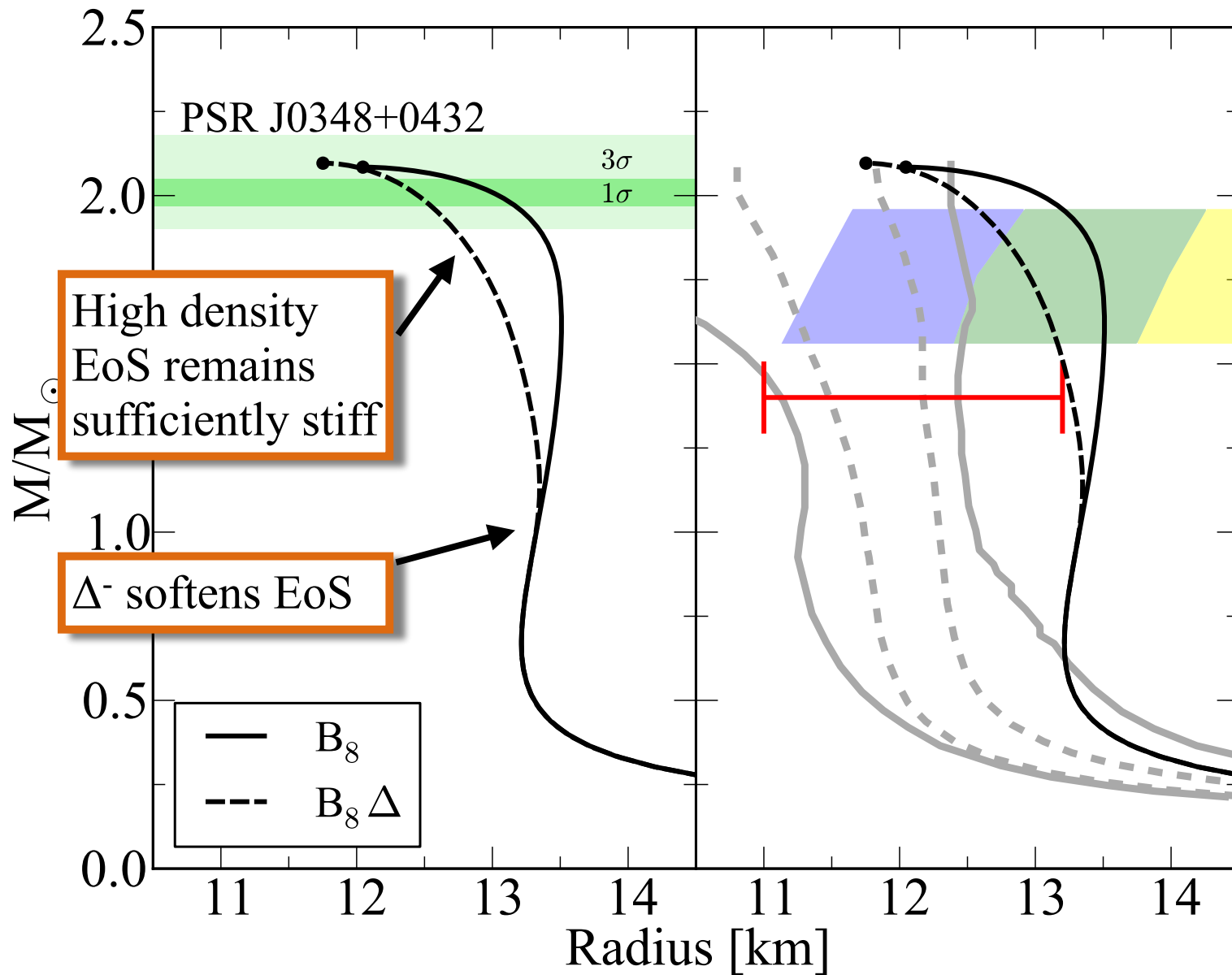
$U_{\Lambda}^{(N)} = -28 \text{ MeV}$ ,  $U_{\Sigma}^{(N)} = +30 \text{ MeV}$ ,  $U_{\Xi}^{(N)} = -18 \text{ MeV}$

$g_{\omega Y}$  at  $n_0$ :  $x_{\omega\Lambda} = x_{\omega\Sigma} = 0.79$ ,  $x_{\omega\Xi} = 0.59$

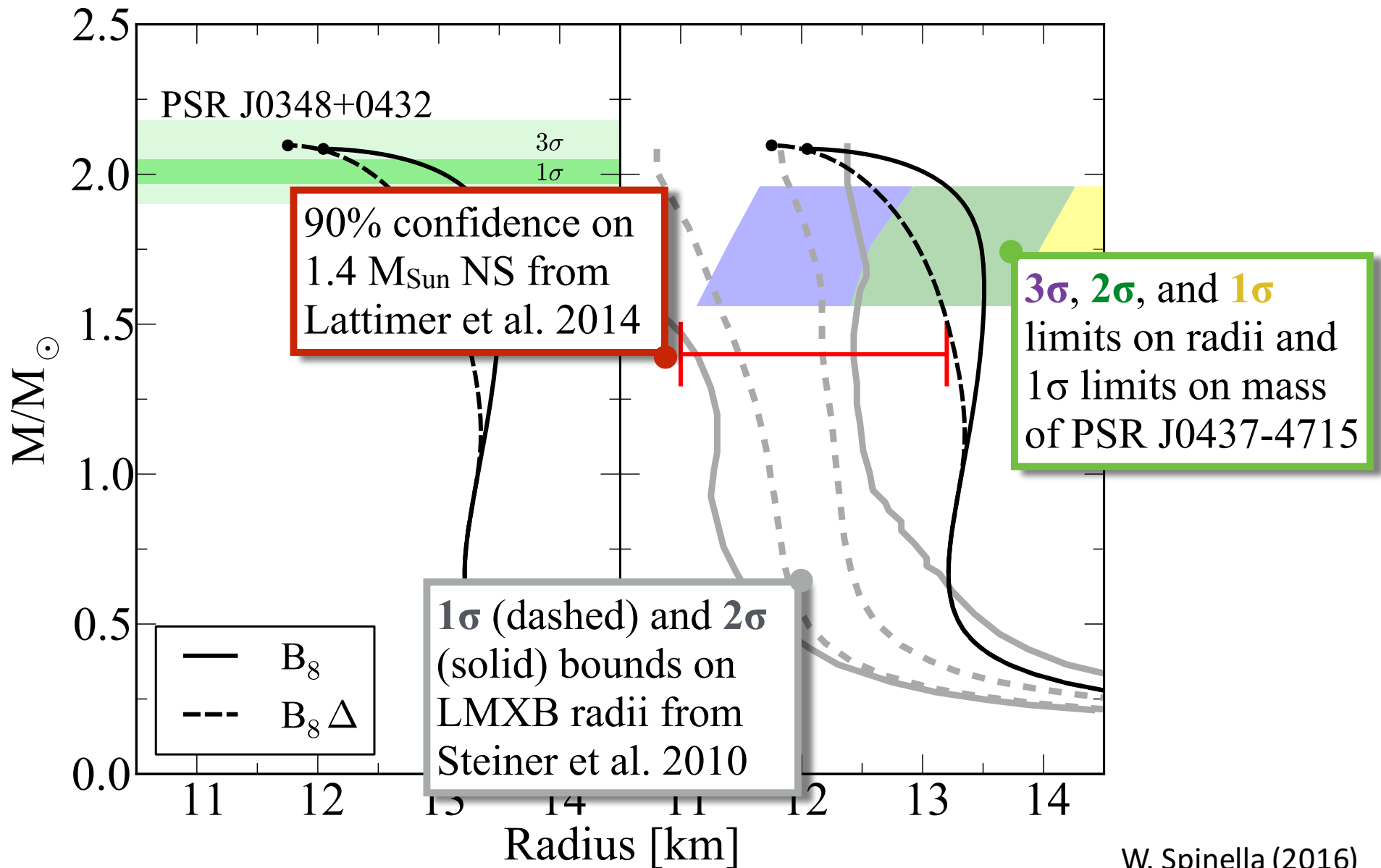
(Rijken et al., 2010, Mayatsu et al., 2013)



# Mass-Radius+Constraints with Hyperons+Deltas

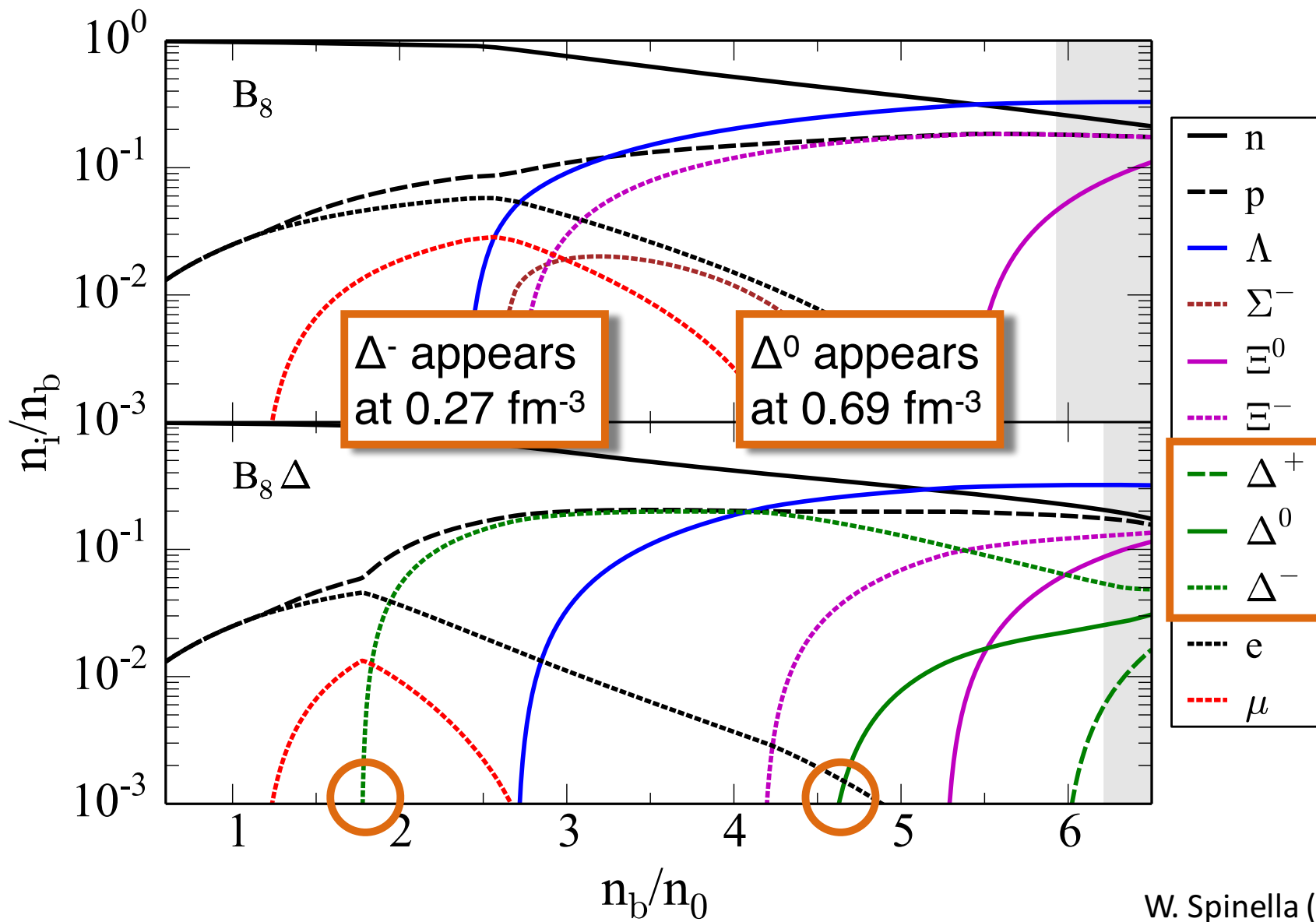


# Mass-Radius+Constraints with Hyperons+Deltas

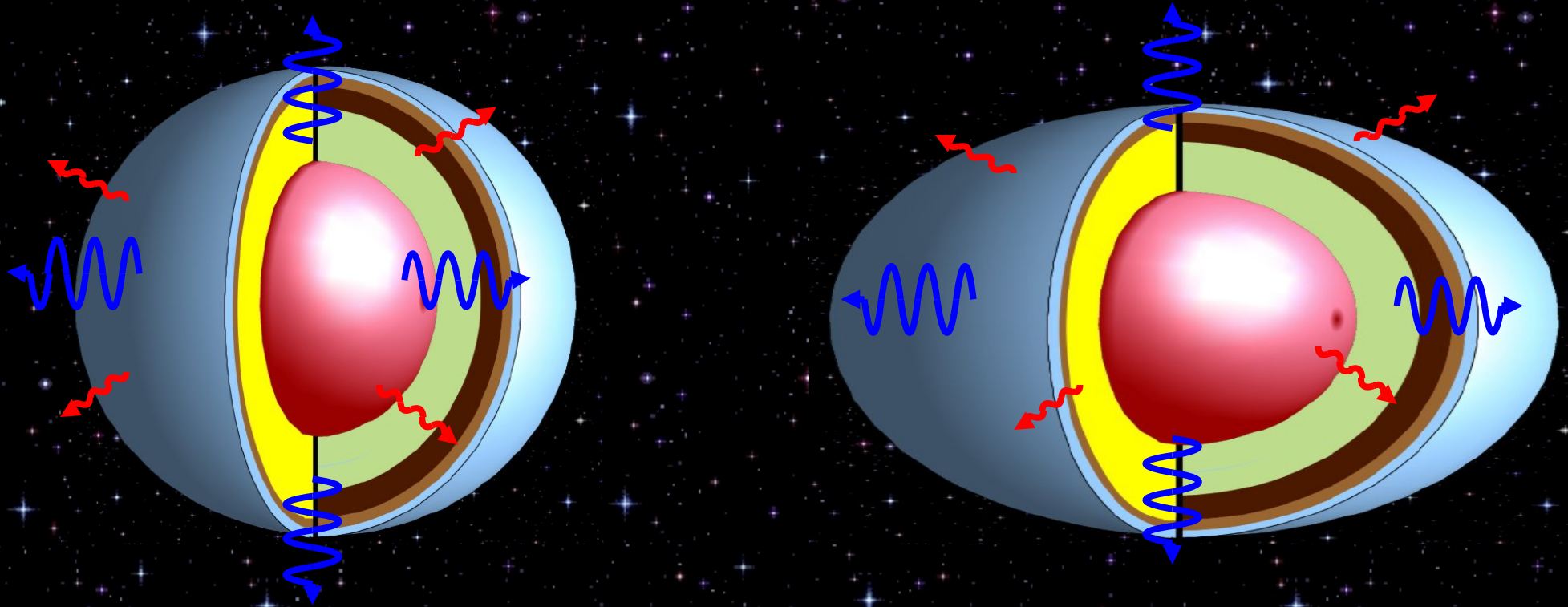


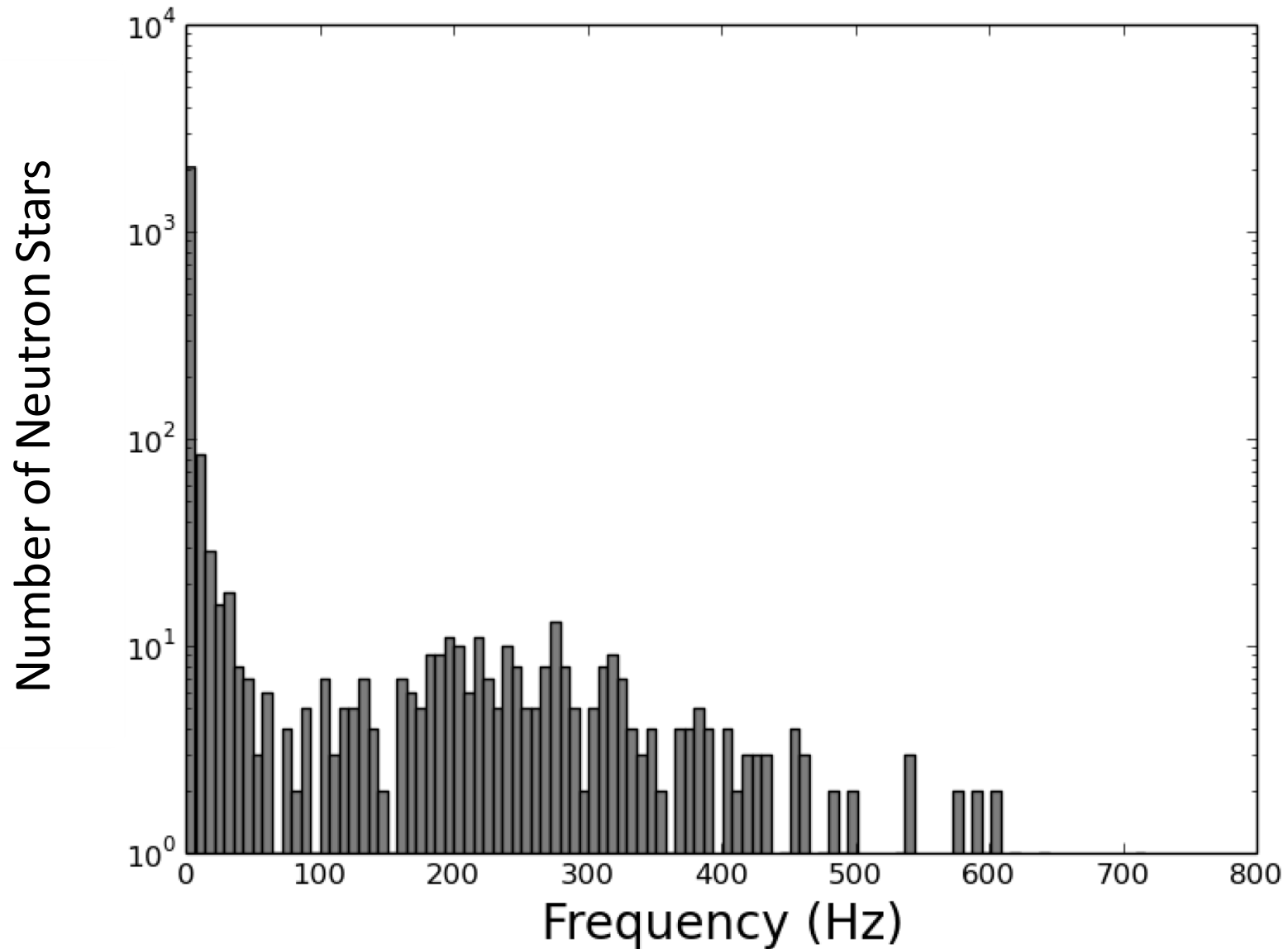


# $\Delta$ 's in Neutron Star Matter

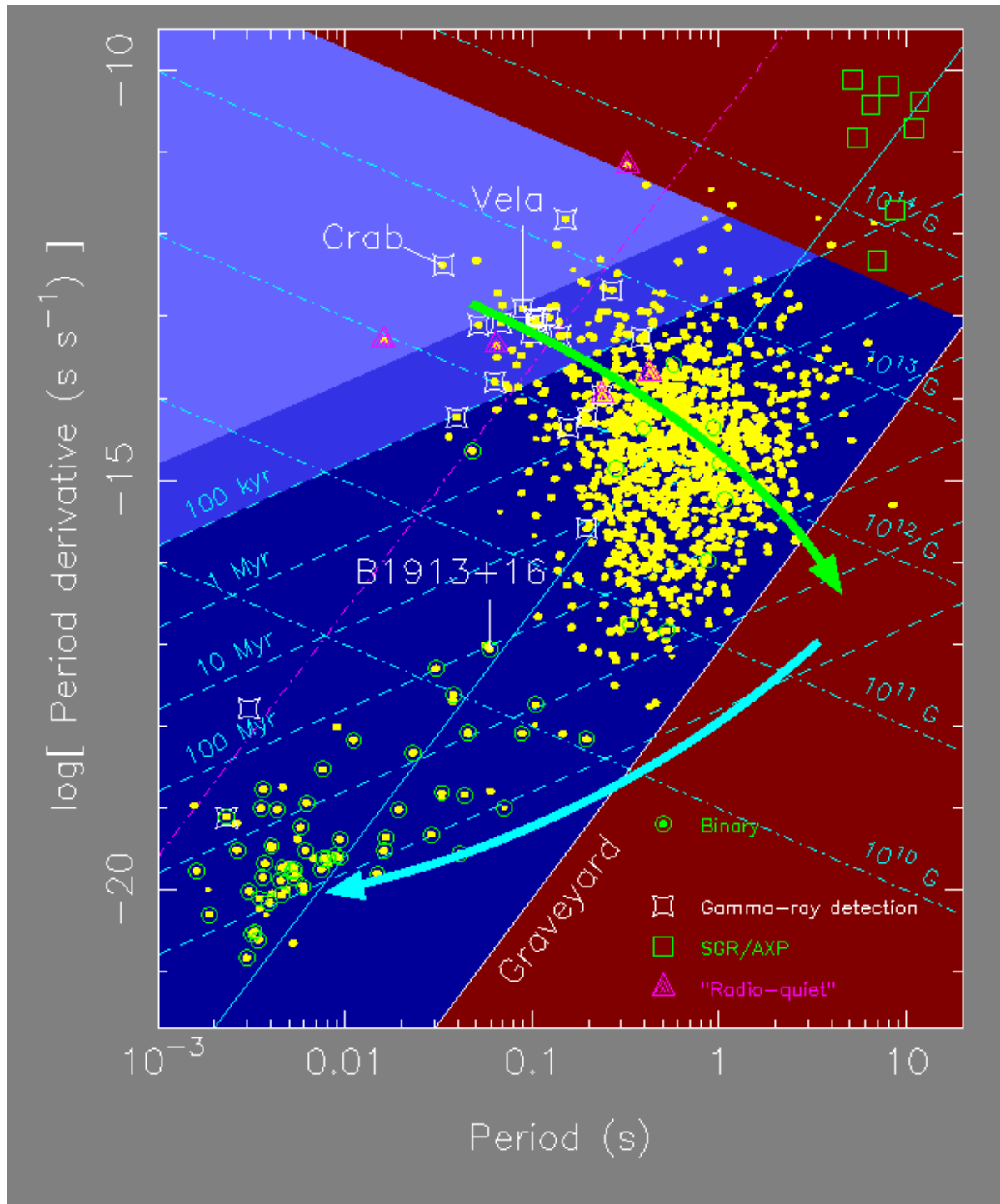


# Rotating Neutron Stars



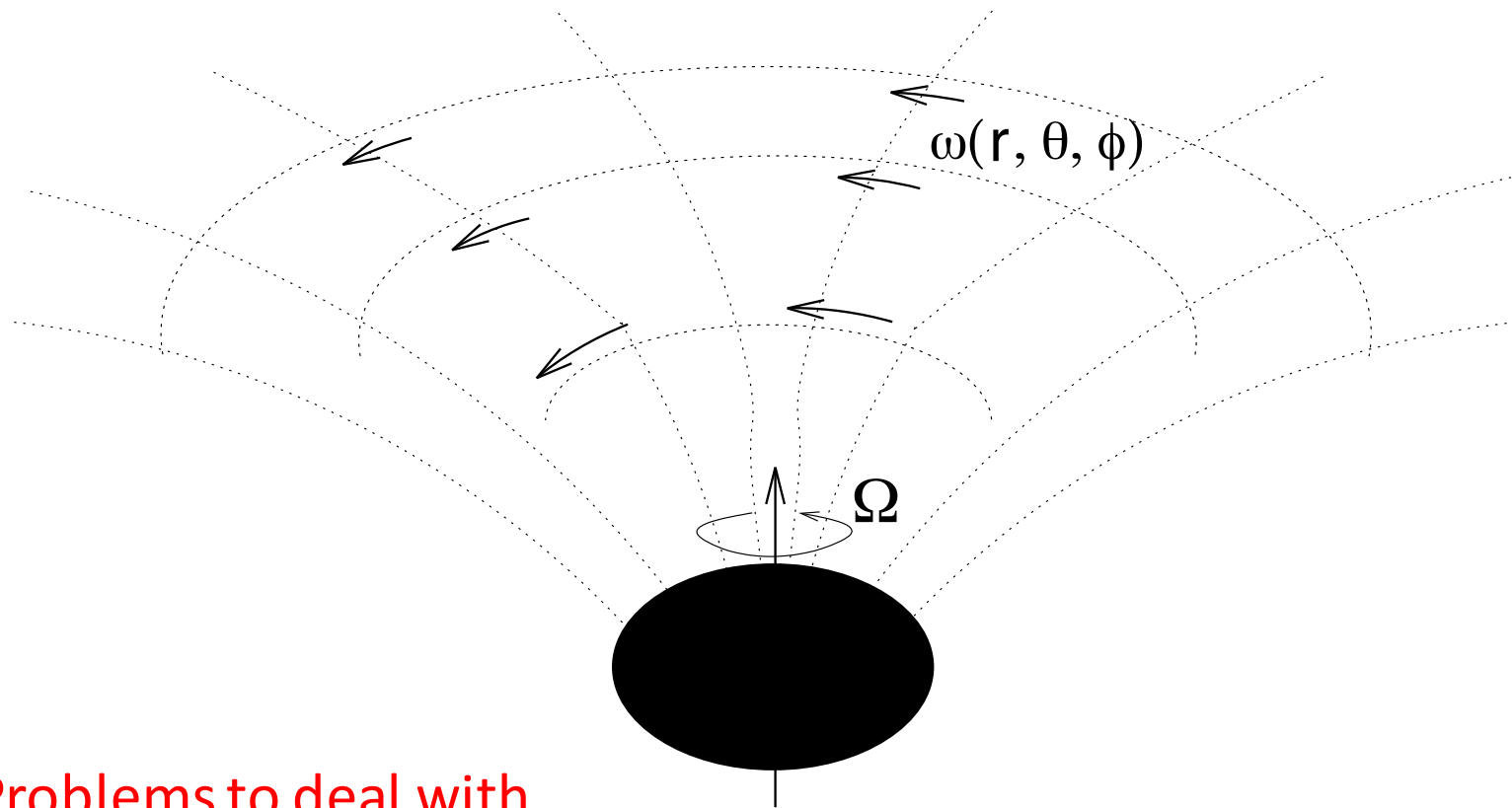


**Figure 1. Histogram for the frequencies of all 2,510 pulsars which have frequency data in version 1.53 of The Australia Telescope National Facility Pulsar Catalogue.**



Source:  
 Jodrell Bank Centre  
 for Astrophysics  
<http://www.jb.man.ac.uk>

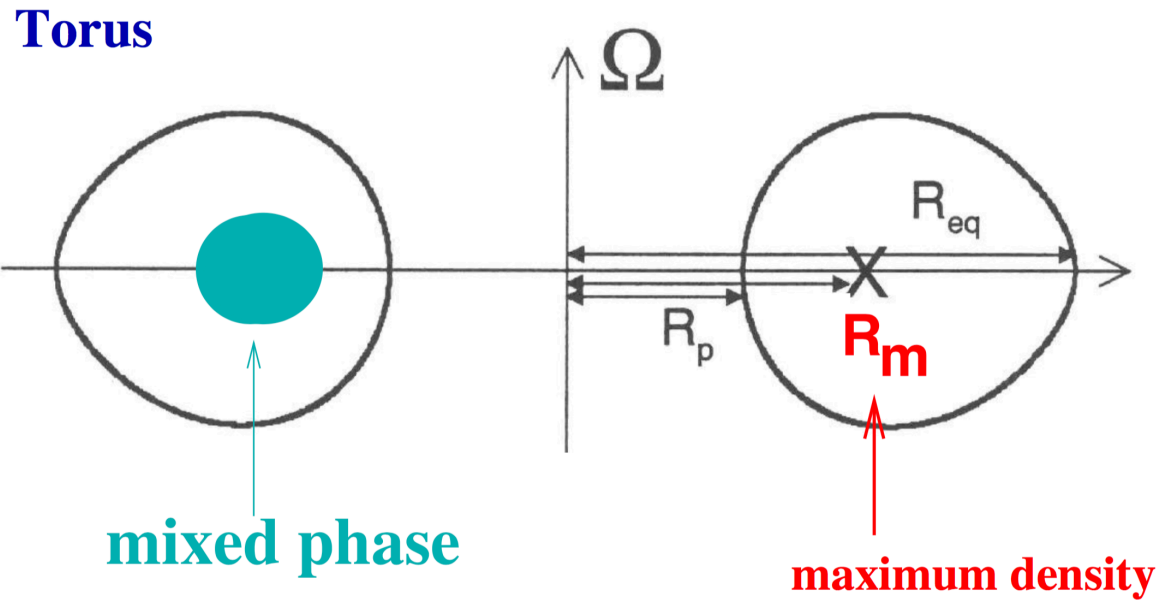
# Rotation in General Relativity



Problems to deal with

- 2-D system
- Dragging of local inertial frames (Lense-Thirring effect)
- Maximum rotational frequency

# Differential rotation



$$R_{eq} = 14.3 \text{ km}$$

$$R_p = 1.9 \text{ km}$$

$$R_m = 5.5 \text{ km}$$

# Einstein's Field Equations for Rotating Compact Objects

□ Metric:  $ds^2 = - e^{-2\nu} dt^2 + e^{2(\alpha+\beta)} r^2 \sin^2\theta (d\phi - N^\phi dt)^2 + e^{2(\alpha-\beta)} (dr^2 + r^2 d\theta^2)$

Friedman, Ipser & Parker (1986)

□ Christoffel symbols:

$$\Gamma^\sigma_{\mu\nu} = g^{\sigma\lambda} (\partial_\nu g_{\mu\lambda} + \partial_\mu g_{\nu\lambda} - \partial_\lambda g_{\mu\nu}) / 2$$

□ Riemann tensor:

$$R^\tau_{\mu\nu\sigma} = \partial_\nu \Gamma^\tau_{\mu\sigma} - \partial_\sigma \Gamma^\tau_{\mu\nu} + \Gamma^\kappa_{\mu\sigma} \Gamma^\tau_{\kappa\nu} - \Gamma^\kappa_{\mu\nu} \Gamma^\tau_{\kappa\sigma}$$

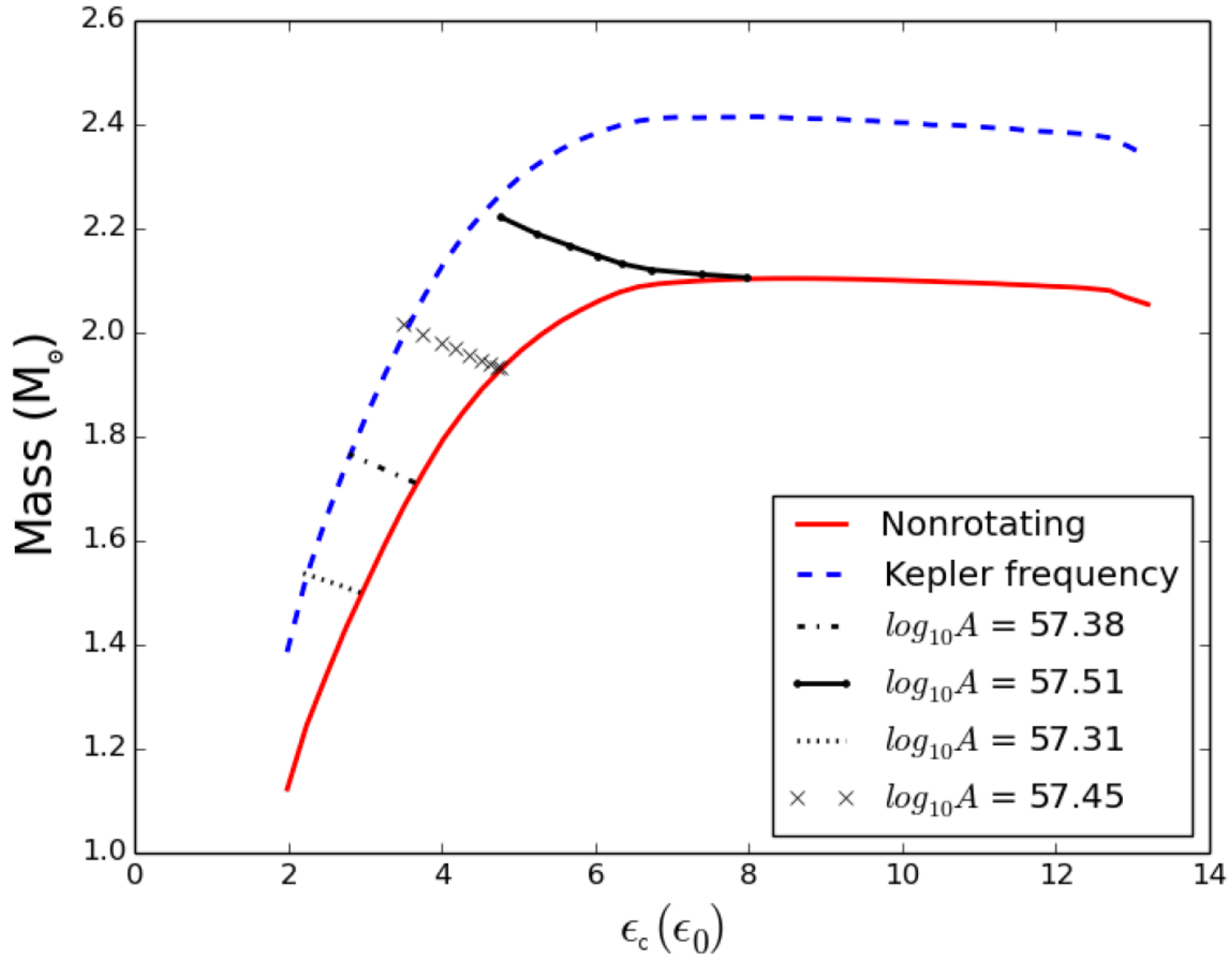
□ Ricci tensor:  $R_{\mu\nu} = R^\tau_{\mu\tau\nu}$

□ Scalar curvature:  $R = R_{\mu\nu} g^{\mu\nu}$

□ Kepler frequency:  $\Omega_K = r^{-1} e^{\nu-\alpha-\beta} U_K + N^\phi \xrightarrow{\text{Newtonian limit}} \sqrt{M/R^3}$

□ Differential rotation/uniform rotation at  $0 < \Omega < \Omega_K$

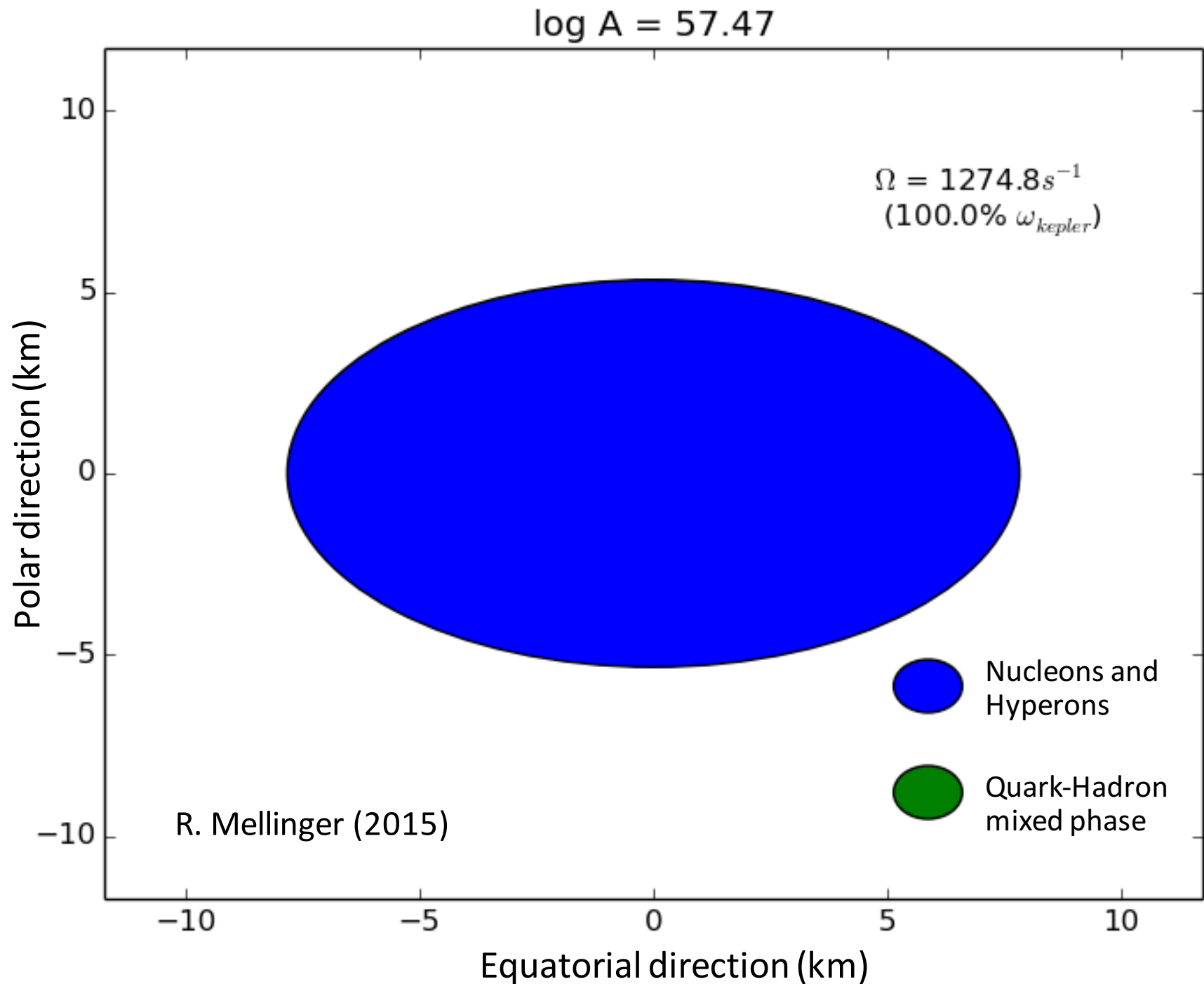
**Stellar properties:  $M, R_p, R_{eq}, I, z, \Omega_K, \omega, P(r), \varepsilon(r), \rho(r)$**



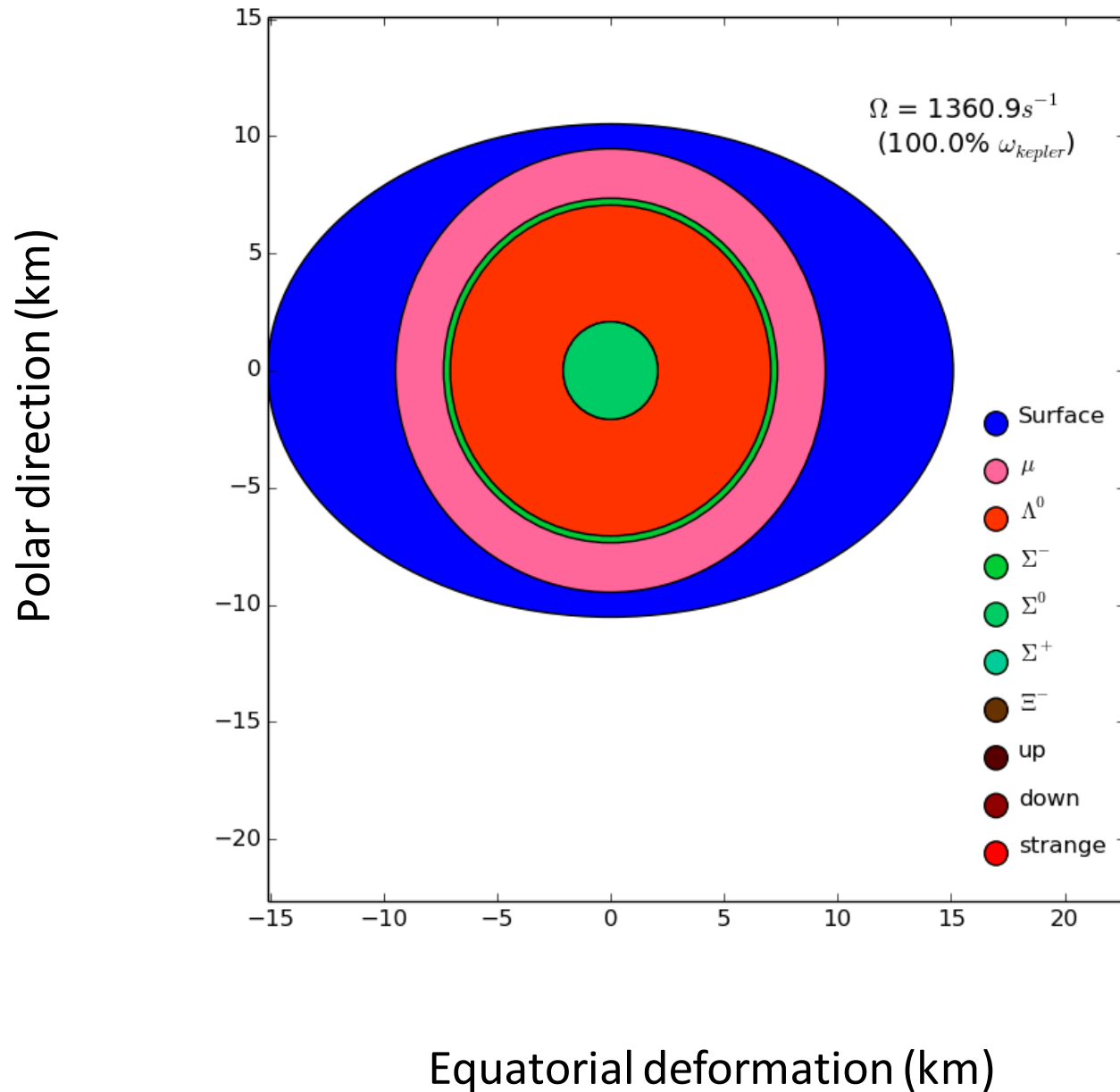
Central stellar density



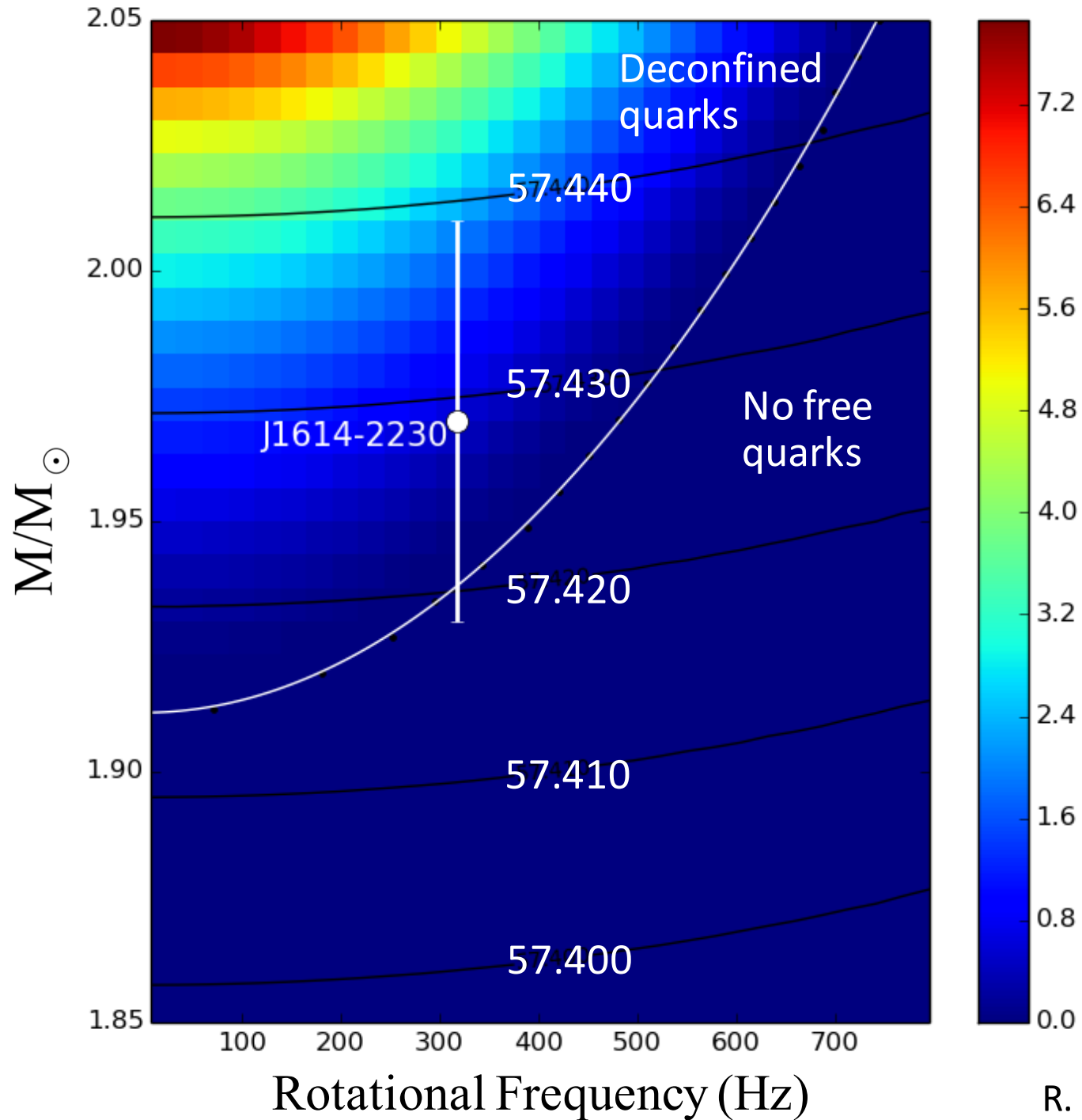
# Rotation-driven compositional changes inside of neutron stars



# Rotation-driven compositional changes inside of neutron stars



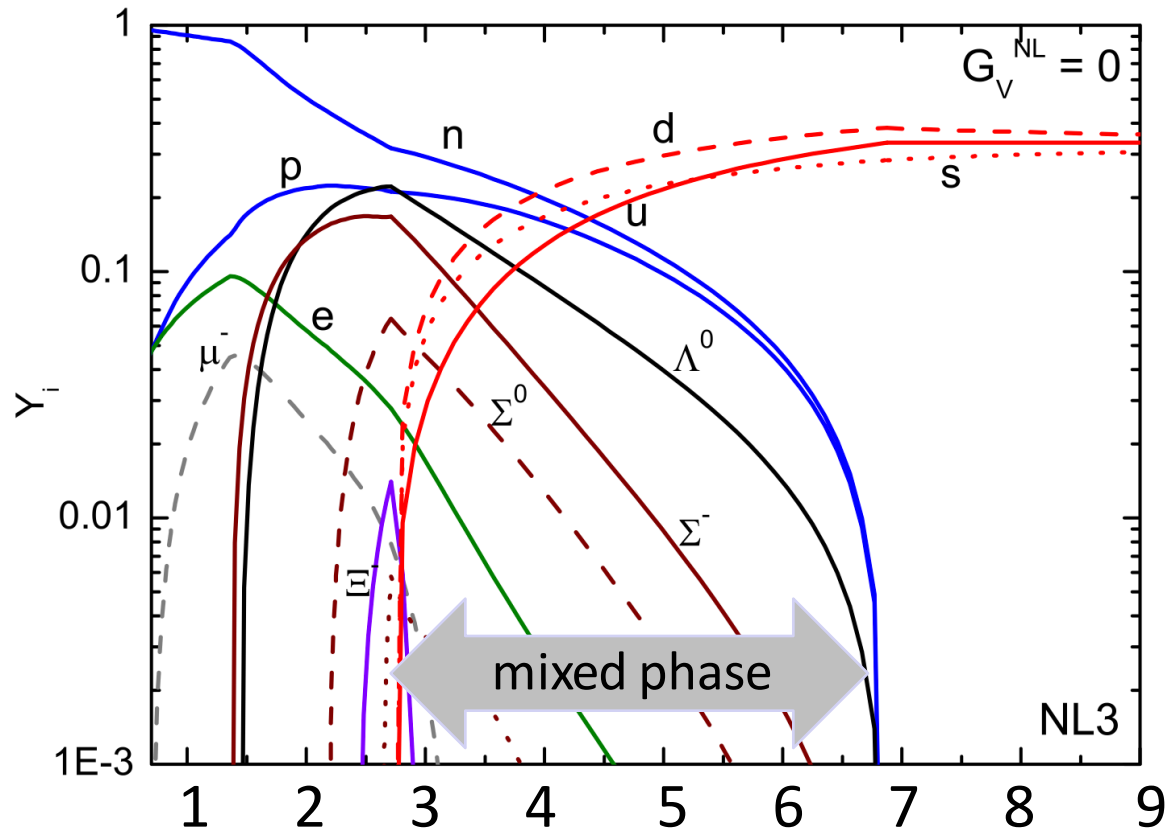
# Rotation-driven compositional changes inside of neutron stars



# Quark-Hadron Lattices

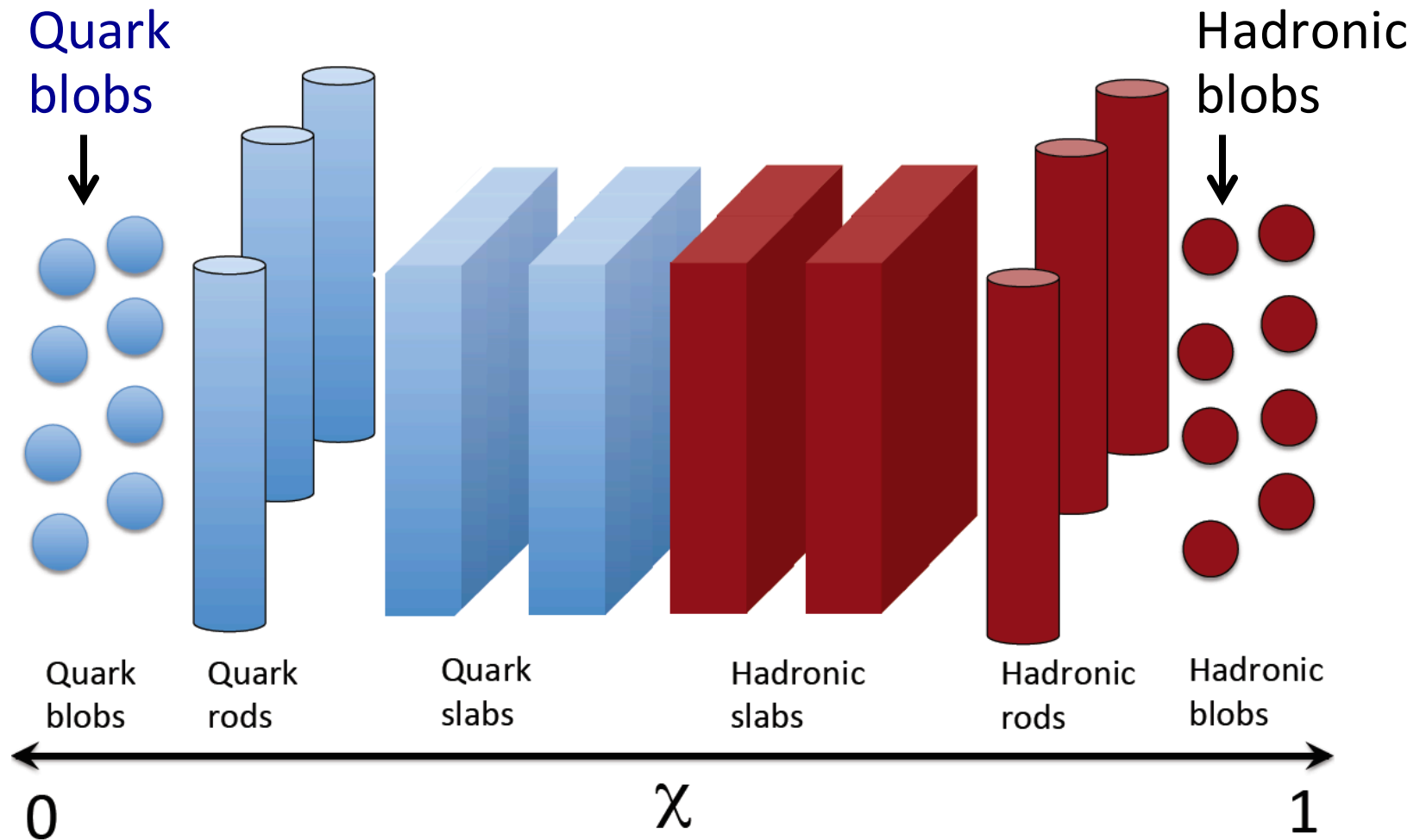
# Geometrical Structures in the mixed Quark-Hadron Phase

N. K. Glendenning, PRD 46 (1992) 1274

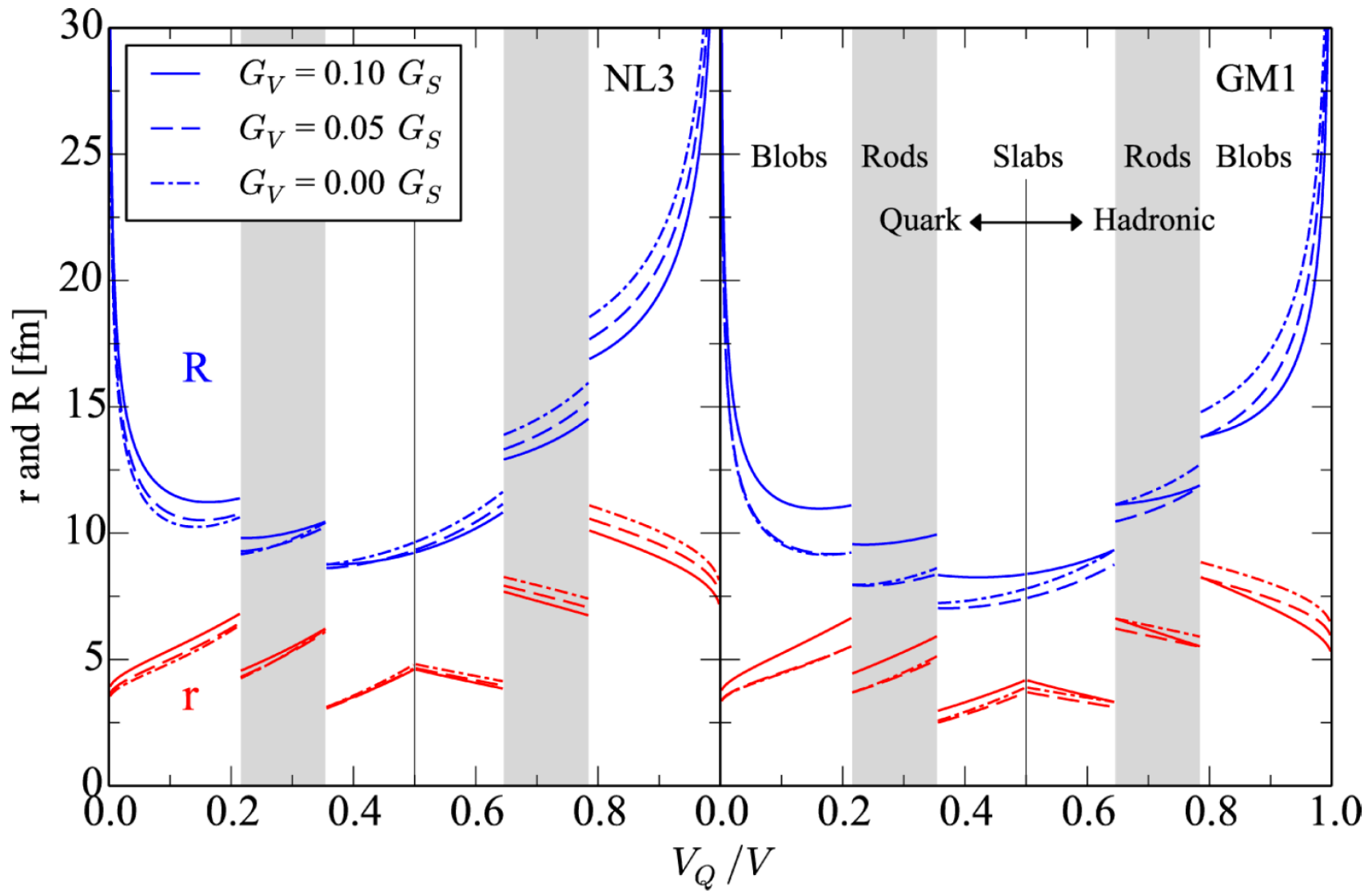


- Competition between Coulomb and surface energies in the mixed phase
- Mixed quark-hadron phase may develop [geometrical structures](#)

# Quark-Hadron Mixed Phase in Cores of Neutron Stars?



Leads to changes the heat capacity, thermal conductivity, neutrino emissivity!



W. Spinella et al. (EPJA 52 (2016) 61)

See also Glendenning Phys Rep 342 (2001) 393; X. Na et al., PRD 86 (2012) 123016

# Electron-Quark Scattering leads to Bremsstrahlung

$$e + (Z,A) \rightarrow e + (Z,A) + \nu + \bar{\nu}$$



Quark blobs/rods/slabs

For the scattering of neutrinos from quark droplets, see S. Reddy, G. Bertsch, M. Prakash, PLB 475 (2000) 1



# Electron-Quark Scattering leads to Bremsstrahlung

$$e + (Z,A) \rightarrow e + (Z,A) + \nu + \bar{\nu}$$

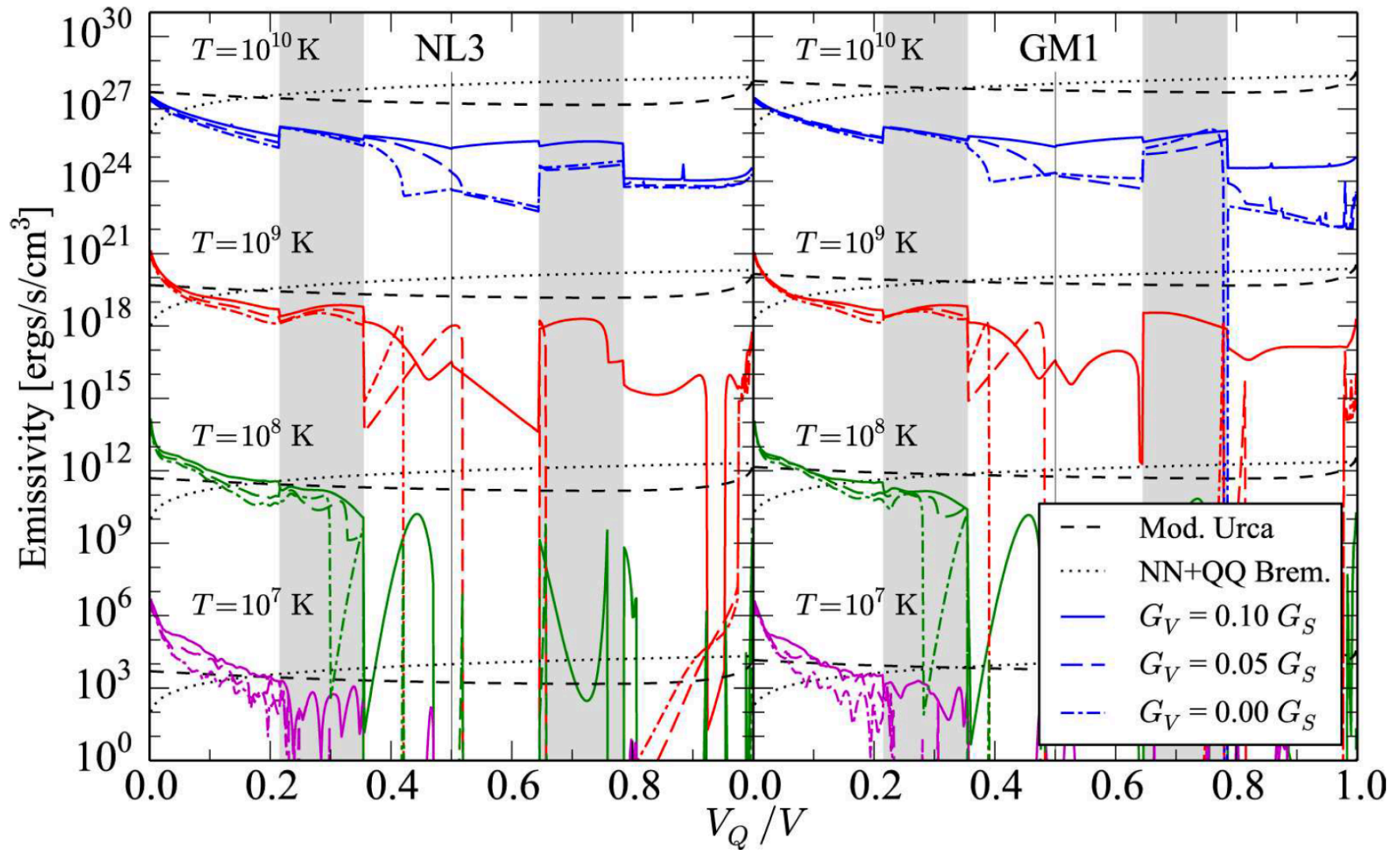
Sub-nuclear NS matter (electrons + heavy atomic nuclei):

- Haensel Kaminker, Yakovlev (1996)
- Yakovlev, Kaminker (1996)
- Kaminker, Pethick, Potekhin, Thorsson, Yakovlev (1999)

Modified URCA:  $n+n \rightarrow n+p+e+\bar{\nu}$

Nucleon Bremsstrahlung:  $n+n \rightarrow n+n+\nu+\bar{\nu}$

Electron-quark blob Bremsstrahlung:  $e+(Z,A) \rightarrow e+(Z,A)+\nu+\bar{\nu}$



# **Non-rotating but nevertheless Deformed?**

Anisotropic equations of state (see E. J. Ferrer et al., PRC 82 (2010) 065802)

Metric of spherically symmetric mass distributions  
(Schwarzschild metric)

$$ds^2 = -e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

isotropic case

$$(T^{\mu\nu}) = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P_{xx} & P_{xy} & P_{xz} \\ 0 & P_{yx} & P_{yy} & P_{yz} \\ 0 & P_{zx} & P_{zy} & P_{zz} \end{pmatrix} \begin{matrix} \downarrow \\ \\ \\ \end{matrix} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

More on the isotropic case

$$(T^{\mu\nu}) = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P_{xx} & P_{xy} & P_{xz} \\ 0 & P_{yx} & P_{yy} & P_{yz} \\ 0 & P_{zx} & P_{zy} & P_{zz} \end{pmatrix} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Rank-2 tensor transformation

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + g^{\mu\nu} P$$

## Tolman-Oppenheimer-Volkoff equation

$$\frac{dP}{dr} = - \frac{\epsilon \left(1 + \frac{P}{\epsilon}\right) m \left(1 + \frac{4\pi P r^3}{m}\right)}{r^2 \left(1 - \frac{2m}{r}\right)}$$

$$m(r) = 4\pi \int_0^r r'^2 \epsilon(r') dr'$$

# Energy-momentum tensor of non-isotropic matter

$$T^{\mu}_{\nu} = \begin{matrix} & t & r & z & \phi \\ \begin{matrix} t \\ r \\ z \\ \phi \end{matrix} & \left( \begin{array}{cccc} \epsilon & 0 & 0 & 0 \\ 0 & P_{rr} & P_{rz} & 0 \\ 0 & P_{zr} & P_{zz} & 0 \\ 0 & 0 & 0 & P_{\phi\phi} \end{array} \right) \end{matrix}$$

$$T^t_t = \epsilon$$

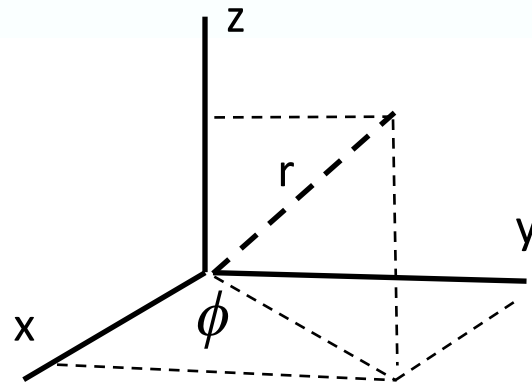
$$T^r_r = P_{\parallel}$$

$$T^z_r = \tilde{P}$$

$$T^r_z = \tilde{P}$$

$$T^z_z = P_{\perp}$$

$$T^{\phi}_{\phi} = \tilde{P}$$



# Energy-momentum tensor of non-isotropic matter

$$T^{\mu}_{\nu} = \begin{matrix} & t & r & z & \phi \\ \begin{matrix} t \\ r \\ z \\ \phi \end{matrix} & \left( \begin{array}{cccc} \epsilon & 0 & 0 & 0 \\ 0 & P_{rr} & P_{rz} & 0 \\ 0 & P_{zr} & P_{zz} & 0 \\ 0 & 0 & 0 & P_{\phi\phi} \end{array} \right) \end{matrix}$$

$$T^t_t = \epsilon$$

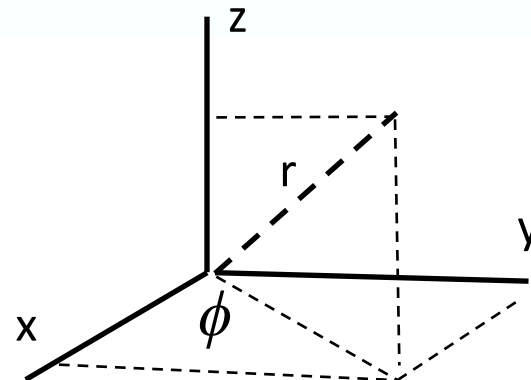
$$T^r_r = P_{\parallel}$$

~~$$T^z_r = \tilde{P}$$~~

~~$$T^r_z = \tilde{P}$$~~

$$T^z_z = P_{\perp}$$

~~$$T^{\phi}_{\phi} = \tilde{P}$$~~





# Full Stellar Structure Equations of Deformed NSs

$$\frac{\partial P_{\parallel}}{\partial r} = - \frac{(\epsilon + P_{\parallel}) \left[ \frac{1}{2}r + 4\pi r^3 P_{\parallel} - \frac{1}{2}r \left( 1 - \frac{2\mathcal{M}(r,z)}{r} \right) \right]}{r^2 \left( 1 - \frac{2\mathcal{M}(r,z)}{r} \right)},$$

$$\frac{dP_{\perp}}{dz} = - \frac{(\epsilon + P_{\perp}) \left[ \frac{z}{2} + 4\pi z^3 P_{\perp} - \frac{z}{2} \left( 1 - \frac{2\mathcal{M}(r,z)}{z} \right) \right]}{z^2 \left( 1 - \frac{2\mathcal{M}(r,z)}{z} \right)}$$

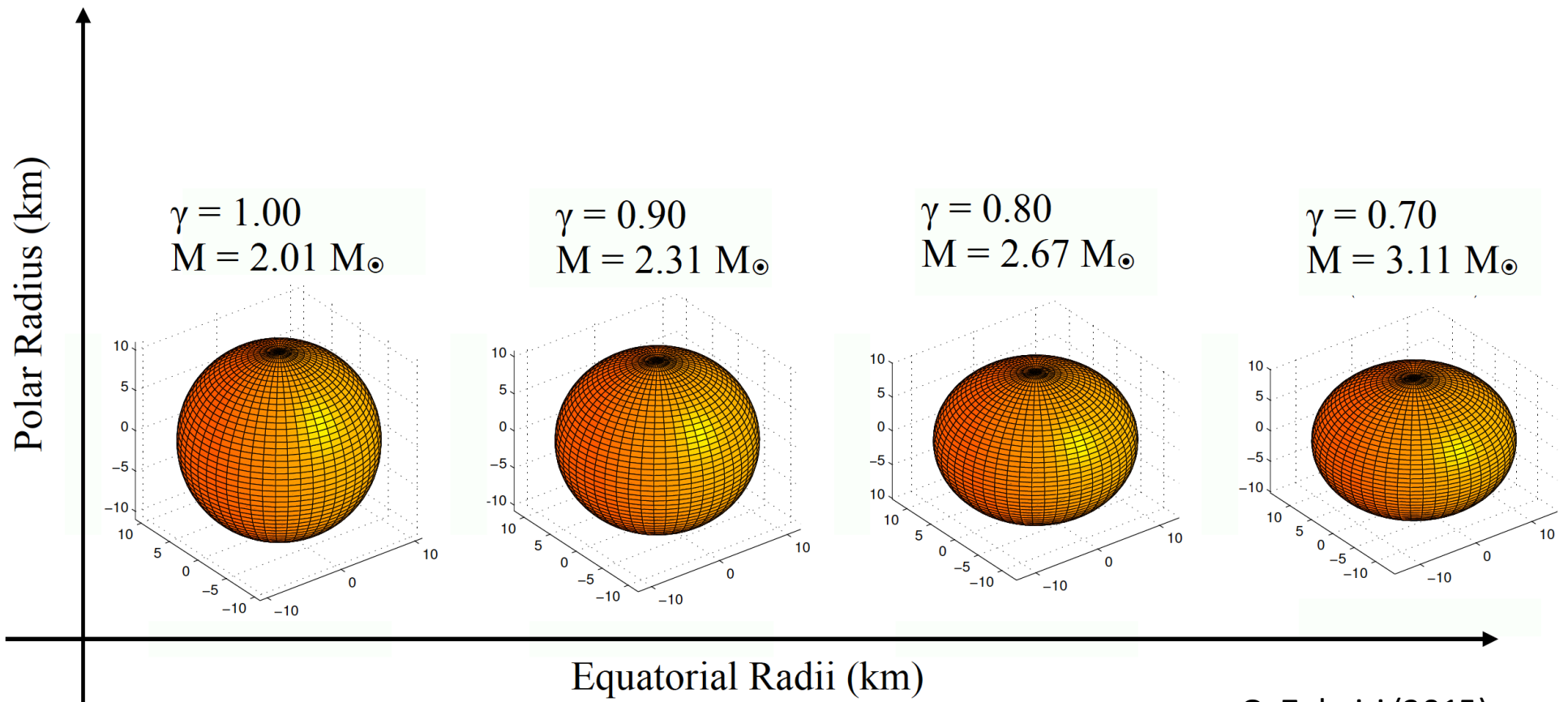
O. Zubairi (2015)

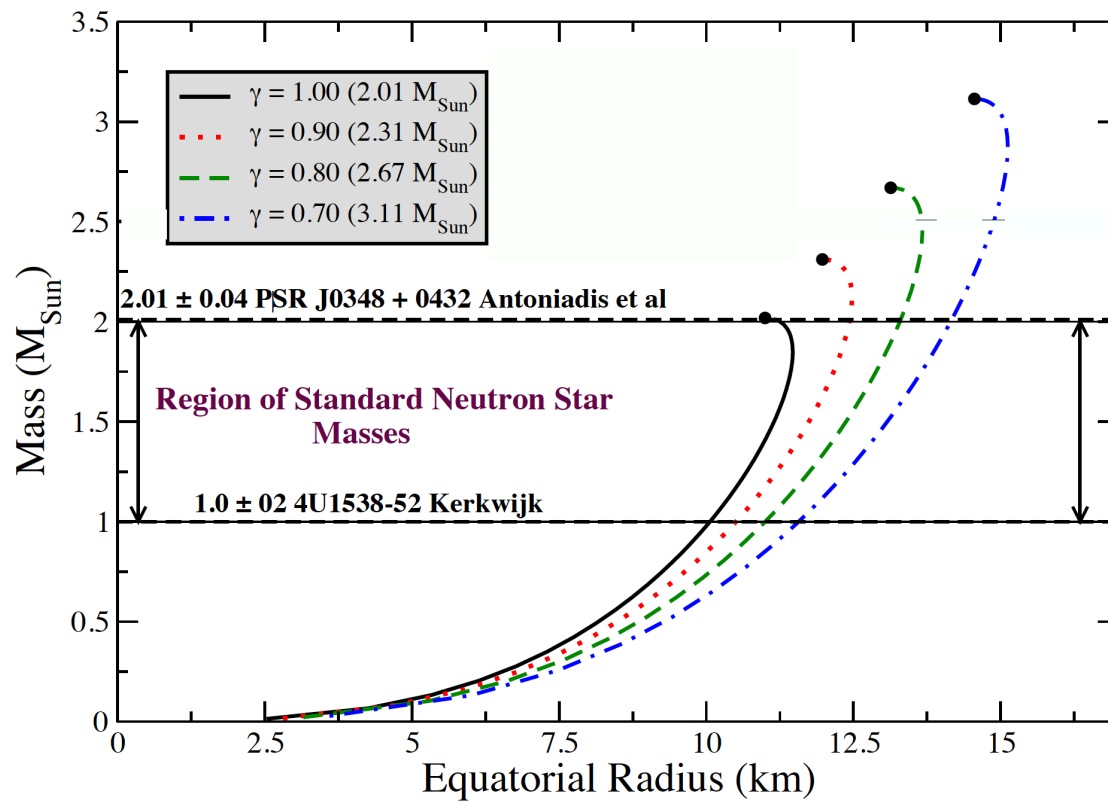
$$M(r, z) = \frac{\partial m(r, z)}{\partial r} + \frac{\partial m(r, z)}{\partial z} - \frac{1}{3} \pi \epsilon(r, z) r^2 z$$

$$P_{\parallel}(r = R) = 0$$

$$P_{\perp}(r = Z) = 0$$

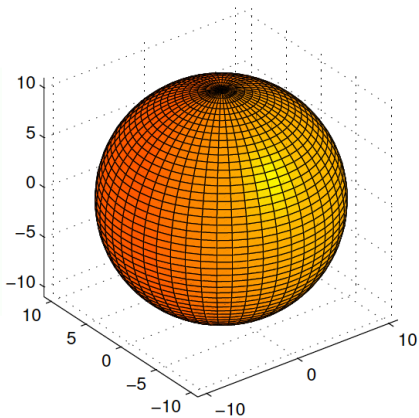
# Oblate deformation



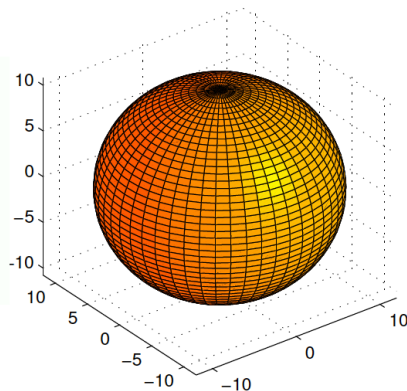


Polar Radius (km)

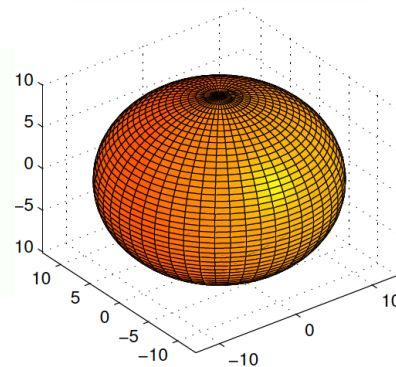
$\gamma = 1.00$   
 $M = 2.01 M_{\odot}$



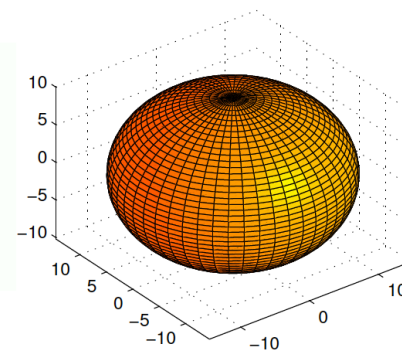
$\gamma = 0.90$   
 $M = 2.31 M_{\odot}$



$\gamma = 0.80$   
 $M = 2.67 M_{\odot}$

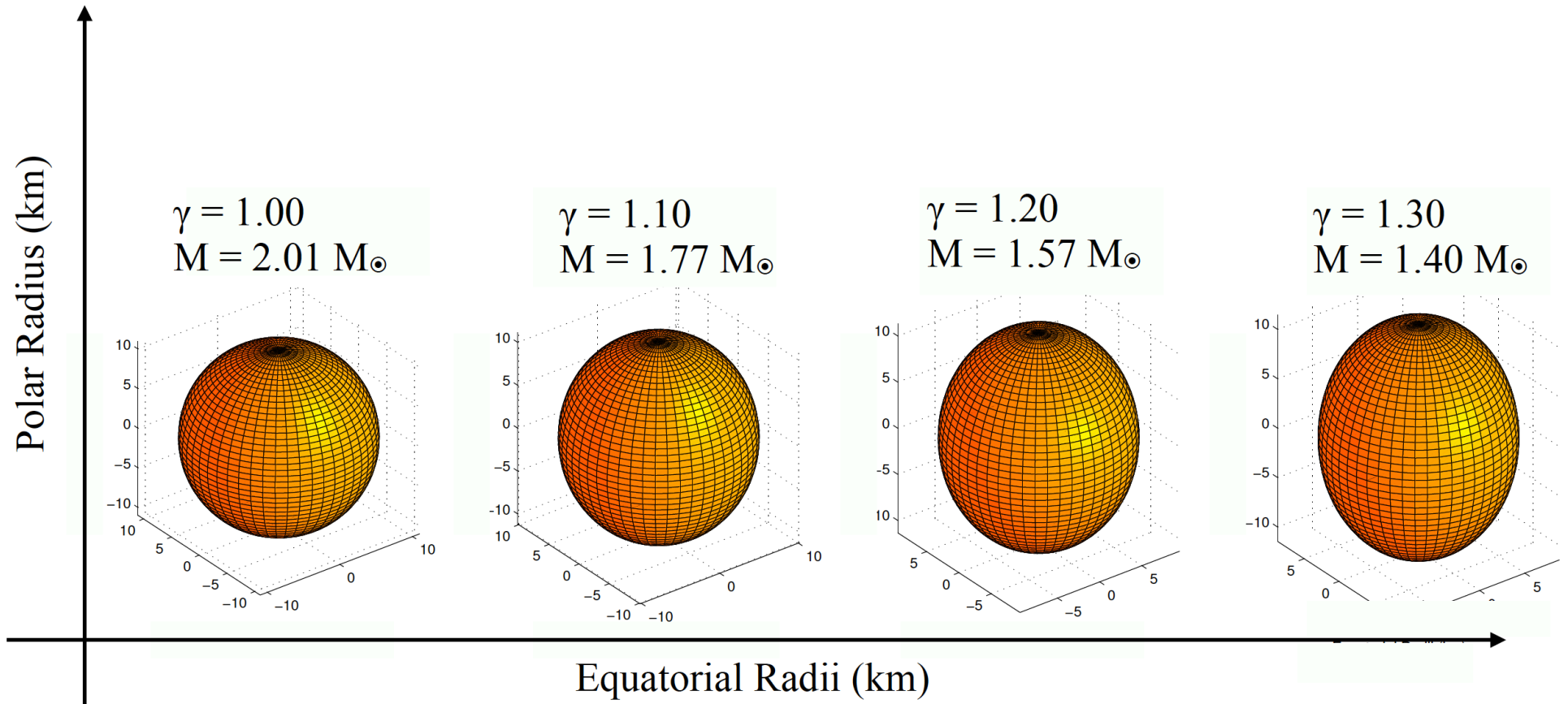


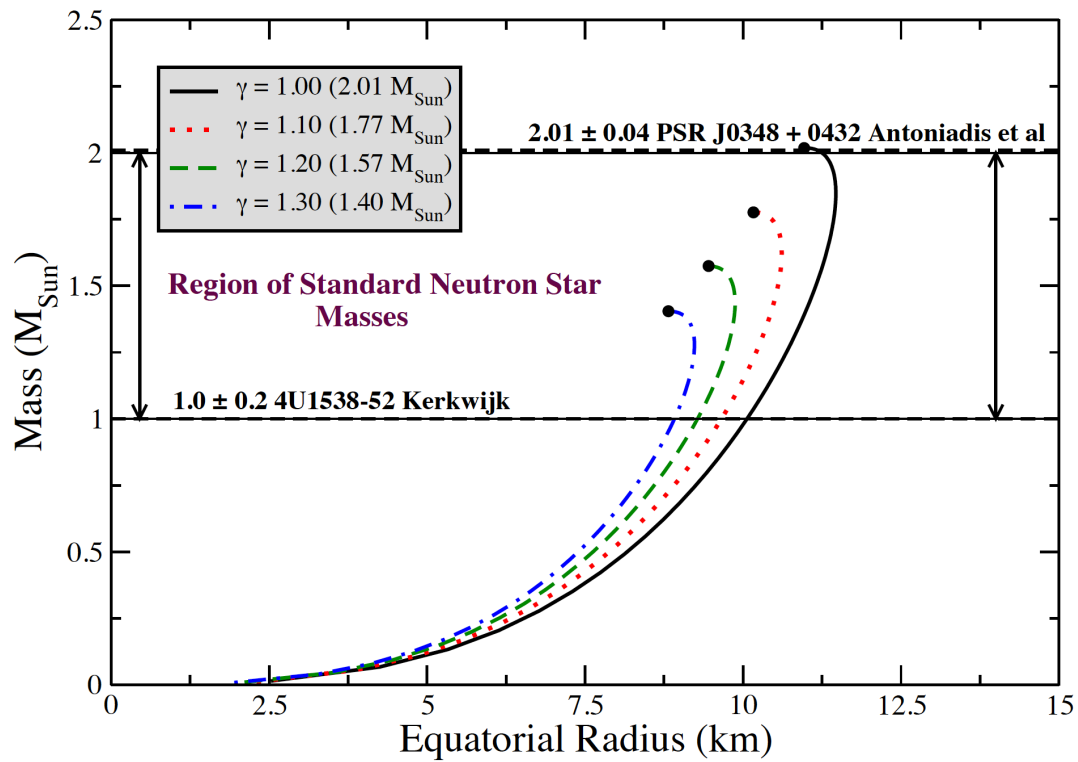
$\gamma = 0.70$   
 $M = 3.11 M_{\odot}$



Equatorial Radii (km)

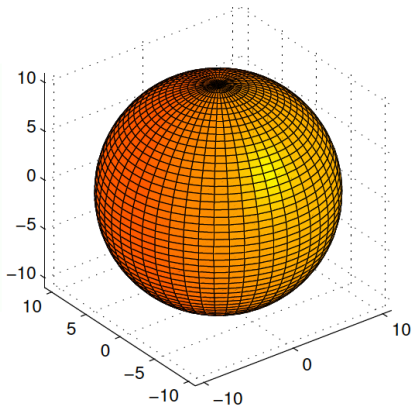
# Prolate deformation



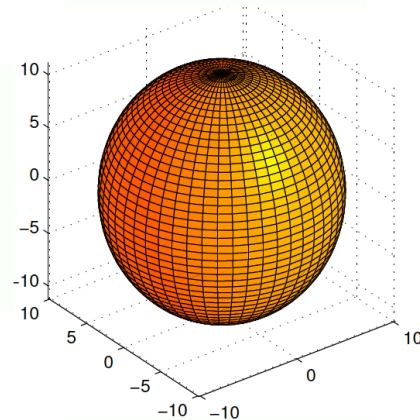


Polar Radius (km)

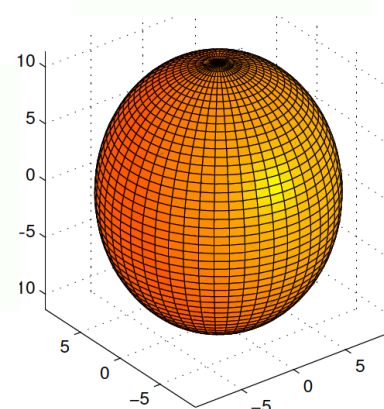
$\gamma = 1.00$   
 $M = 2.01 M_{\odot}$



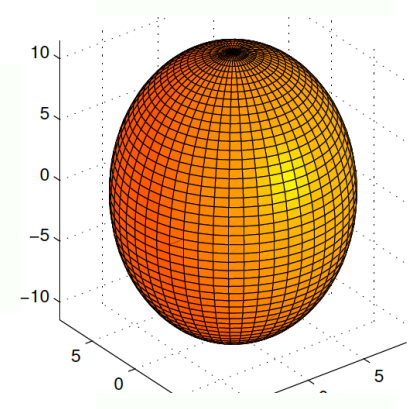
$\gamma = 1.10$   
 $M = 1.77 M_{\odot}$



$\gamma = 1.20$   
 $M = 1.57 M_{\odot}$



$\gamma = 1.30$   
 $M = 1.40 M_{\odot}$



Equatorial Radii (km)

# SUMMARY

- Composition and structure of rotating neutron stars depend on rotational frequency (neutron-to-proton ratio, hyperon population, boson condensates, quark-hadron composition)
- MSPs & NSs in LMXBs may be ideal objects to look for phase transitions (e.g., stellar backbending)
- $\Delta$ 's in NS matter? (open issue)
- Broad collection of quark-hybrid EOSs -> all predict a mixed phase
- Quark-hadron lattices in NSs may lead to enhanced cooling of older NSs
- Anisotropic EOSs may impact maximum mass.