Neutron matter equation of state from Chiral Effective Field Theory

Ingo Tews,

In collaboration with S. Gandolfi, A. Gezerlis, K. Hebeler, T. Krüger, J. Lynn, A. Schwenk, …

Talk, INT program: *"Phases of dense matter"*, July 19, 2016, Seattle

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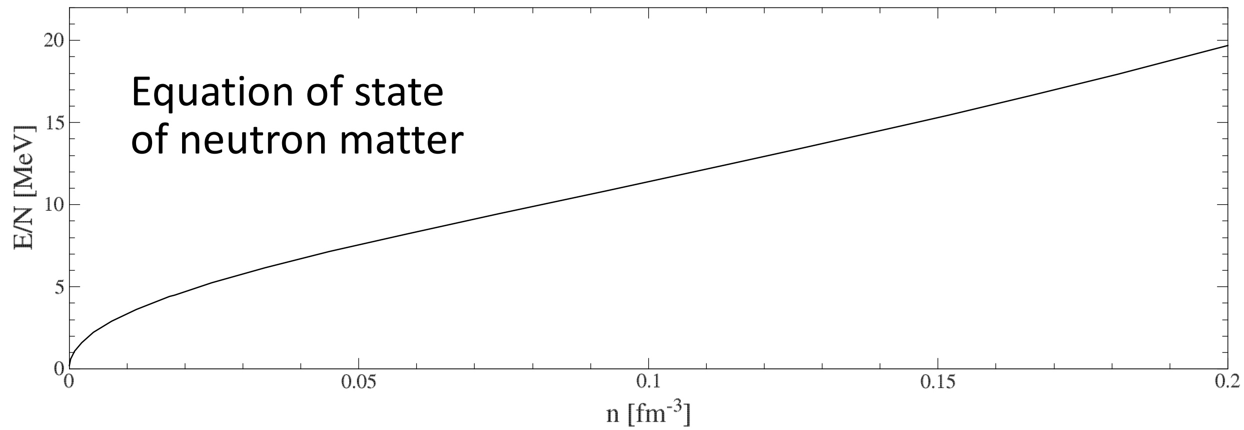
Motivation

- The neutron-matter equation of state connects several physical systems over a wide density range.
- An accurate description of the neutron-matter equation of state is therefore crucial.



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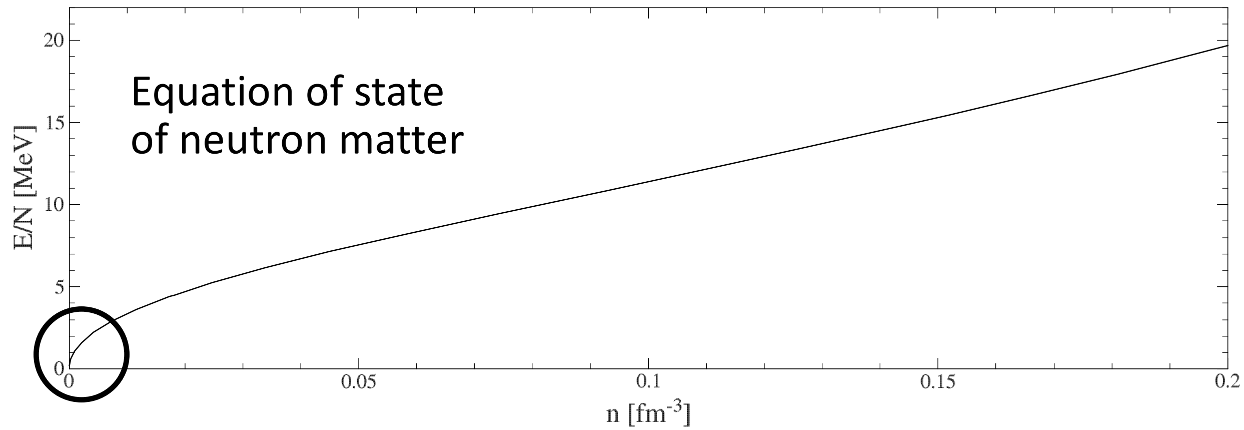

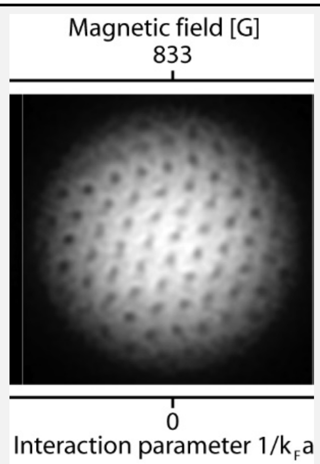

Zwierlein et al., Nature (2005)

Ultracold atoms

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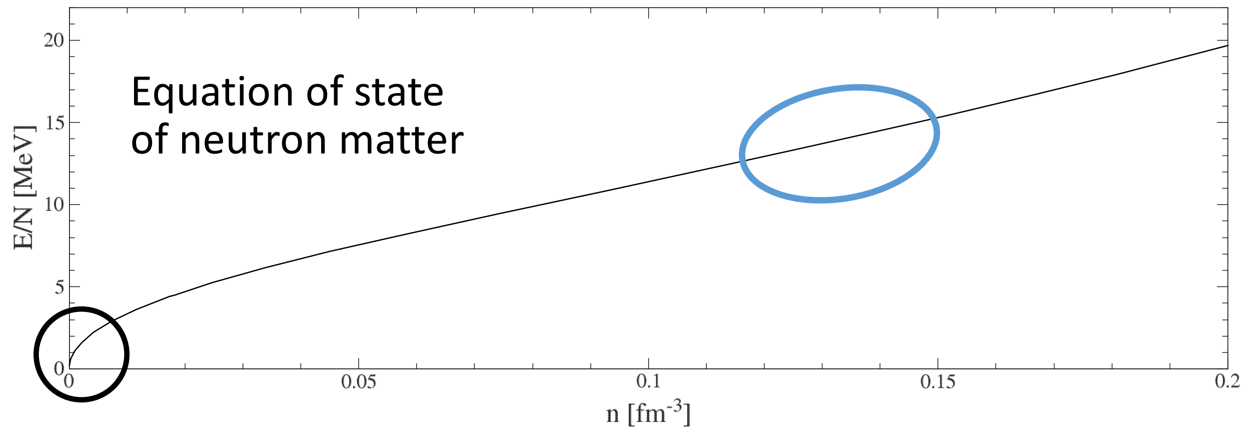

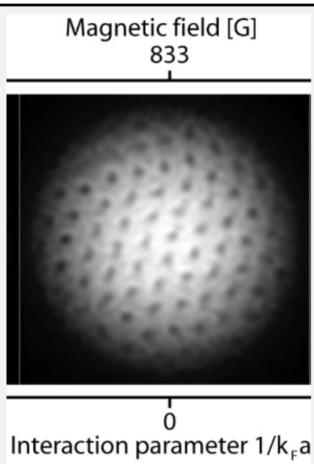

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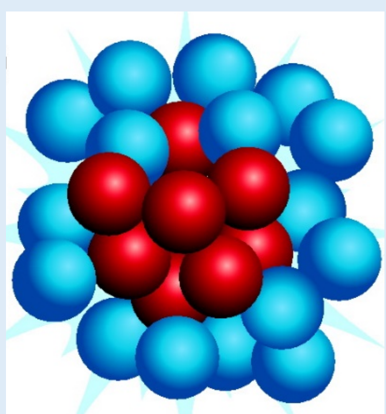
Ultracold atoms



Credit: B.A. Brown

Neutron-rich nuclei

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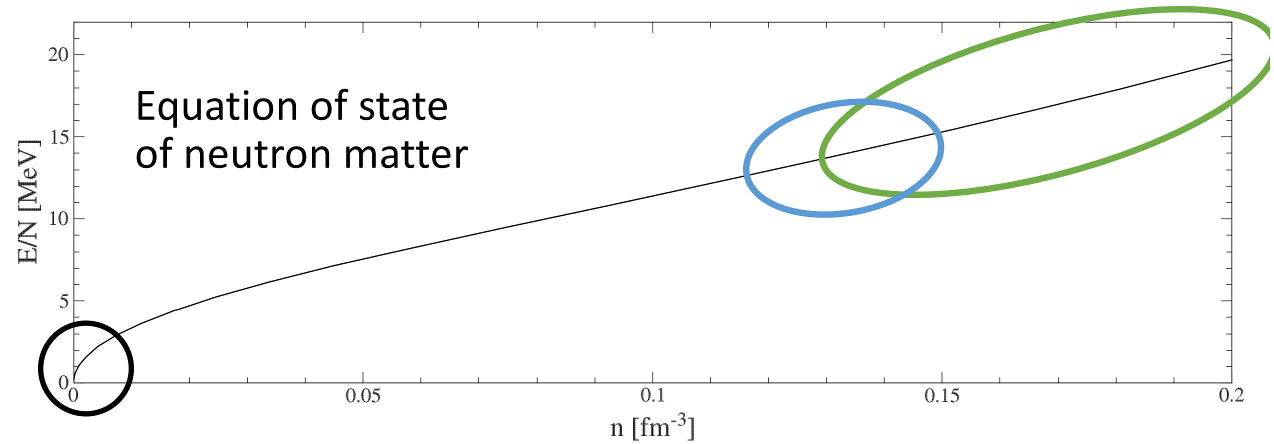

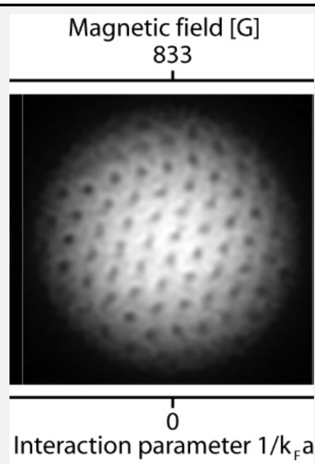

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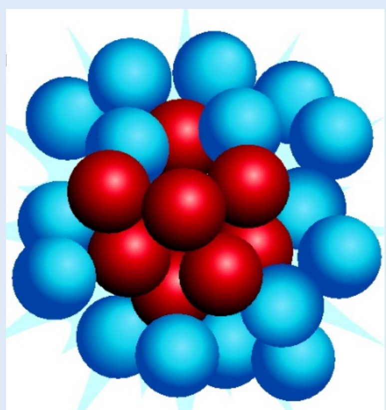
Ultracold atoms



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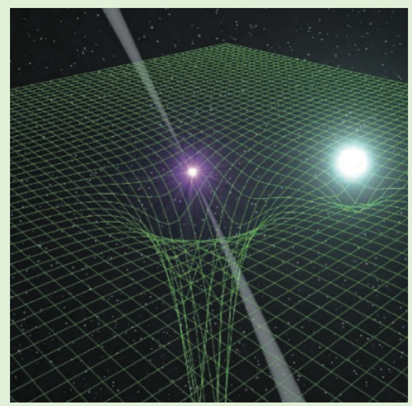
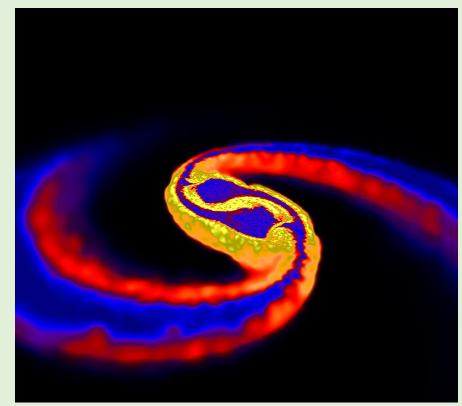
Neutron-rich nuclei



Antoniadis et al., Science (2013)

Neutron stars



Credit: S. Rosswog

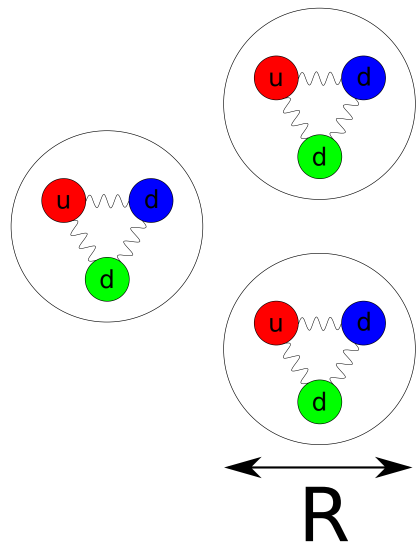
Gravitational waves from neutron star mergers

Outline

- Chiral effective field theory: Epelbaum *et al.*, PPNP (2006) and RMP (2009)
  - Systematic basis for low-energy nuclear forces, connected to QCD
  - naturally includes many-body forces
  - Very successful in calculations of nuclei and nuclear matter
- Ab-initio calculations using chiral EFT can be used to constrain equation of state of neutron matter
- Neutron-matter applications: IT, Krüger, Hebeler, Schwenk, PRL & PRC & PLB (2013)
  - Symmetry energy
  - Chiral condensate
  - Neutron-star mass-radius relation
- Improving neutron-matter results using Quantum Monte Carlo methods
  Gezerlis, IT, et al., PRL & PRC (2013, 2014, 2016)
- Summary

Chiral effective field theory for nuclear forces

Basic principle of effective field theory:



Quantum Chromodynamics

At low energies (long wavelength) details not resolved!

- Choose relevant degrees of freedom for low-energy processes
- Systematic expansion of interactions constrained by symmetries

Chiral effective field theory for nuclear forces

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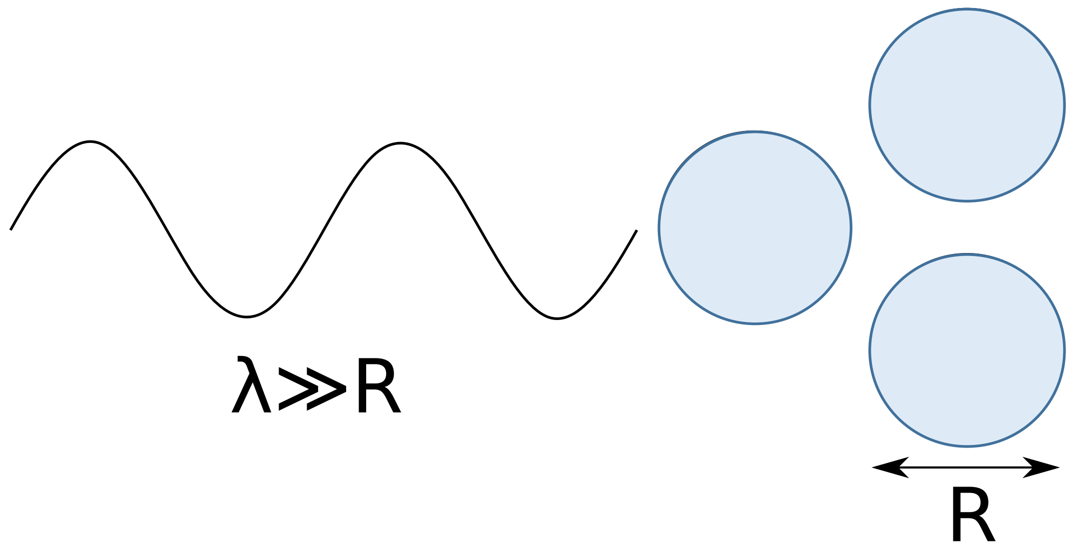

Effective field theory for nuclear forces

At low energies (long wavelength) details not resolved!

- Choose relevant degrees of freedom for low-energy processes
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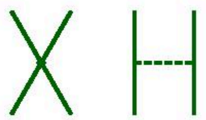
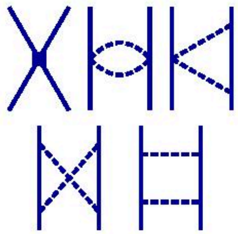
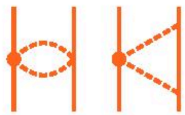
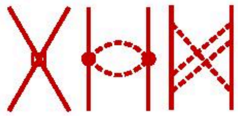
Chiral effective field theory for nuclear forces

	NN
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$	
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$	
N²LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$	
N³LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...

Explicit degrees of freedom:

- Pions and nucleons

Write most general Lagrangian consistent with the symmetries of QCD

Separation of scales:

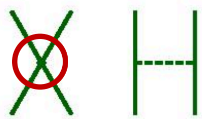
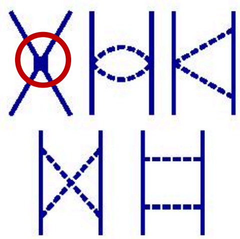
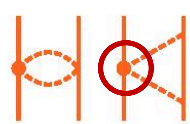
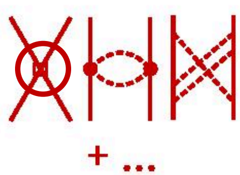
- Low momenta $Q \ll$ breakdown scale Λ_b
- Expand in powers of $\left(\frac{Q}{\Lambda_b}\right)^\nu \sim \left(\frac{1}{3}\right)^\nu$

Power counting:

- $\nu = 0$: leading order (LO),
- $\nu = 2$: next-to-leading order (NLO), ...

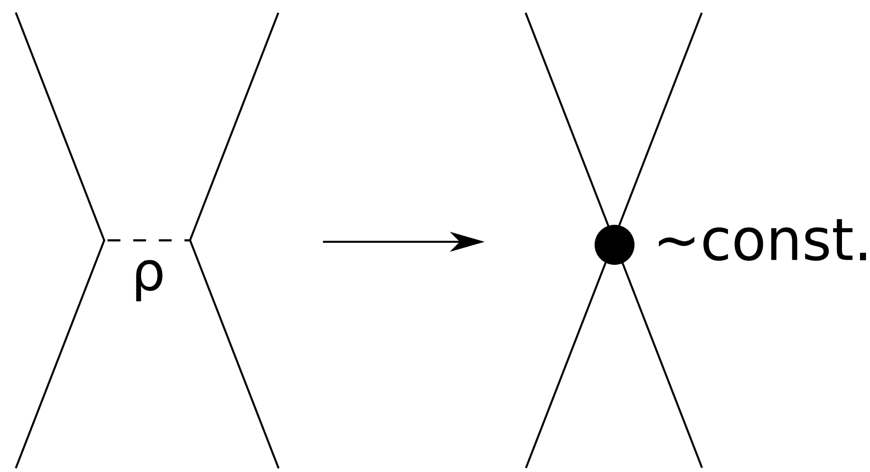
Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Chiral effective field theory for nuclear forces

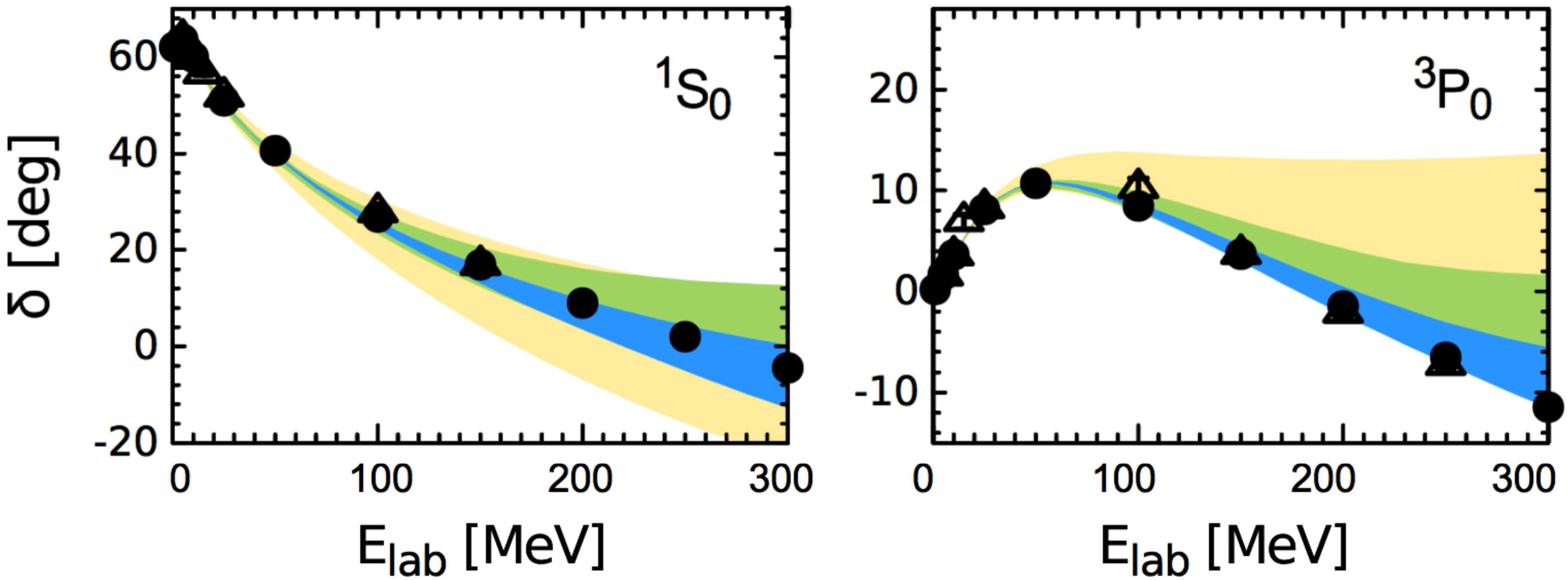
	NN	
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$		2 LECs
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$		7 LECs
N²LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$		
N³LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$		15 LECs

Explicit degrees of freedom:

- Pions and nucleons
- Long-range physics explicit
- Short-range physics expanded in general operator basis
- High-momentum physics absorbed into short-range couplings, fit to experiment (phase shifts)



Chiral effective field theory for nuclear forces

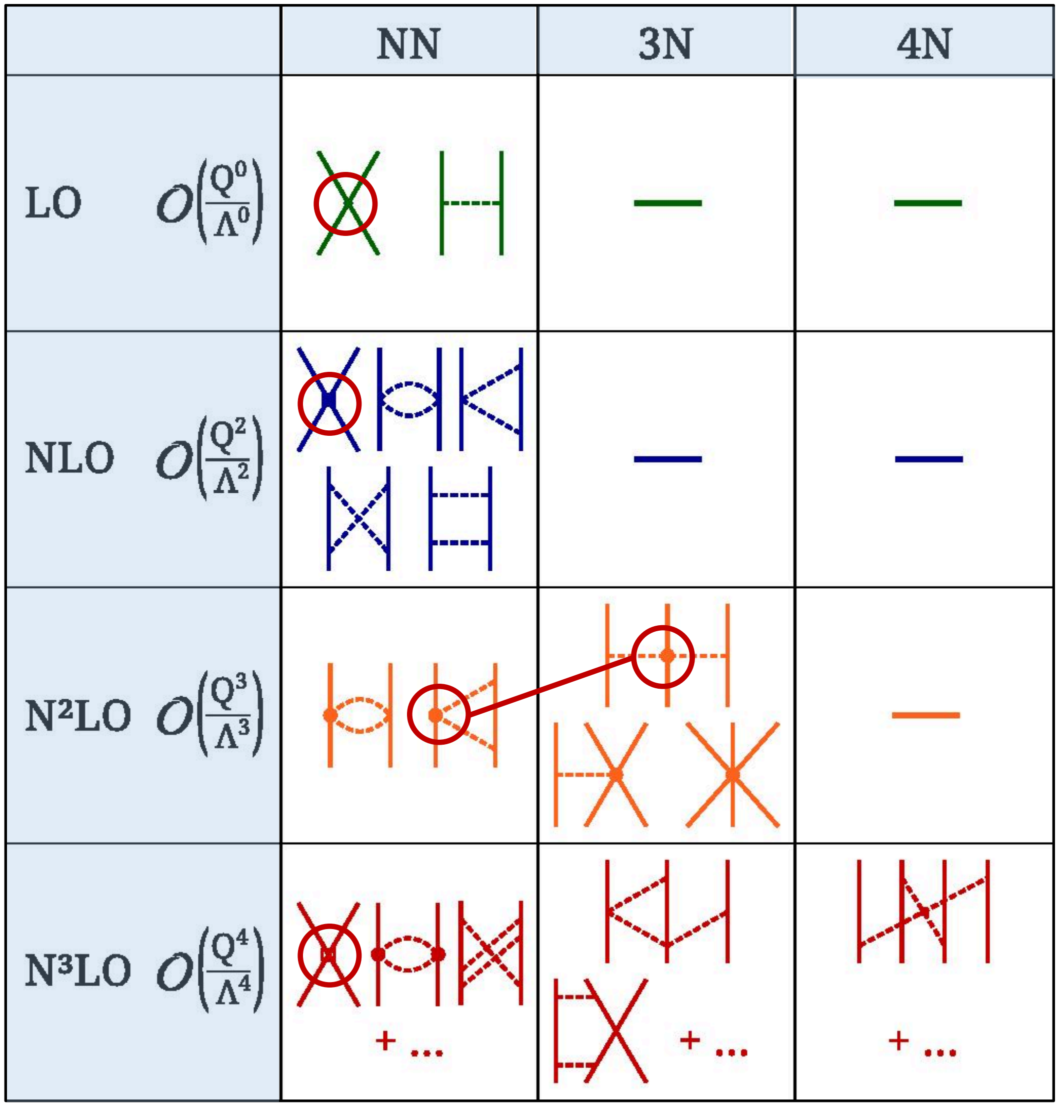


Epelbaum *et al.*, Eur. Phys. J (2015)

Systematic expansion of the nuclear forces:

- Can work to desired accuracy
- Can obtain systematic error estimates

Chiral effective field theory for nuclear forces



Many-body forces:

- Have been found to be crucial ingredient to describe nuclear physics

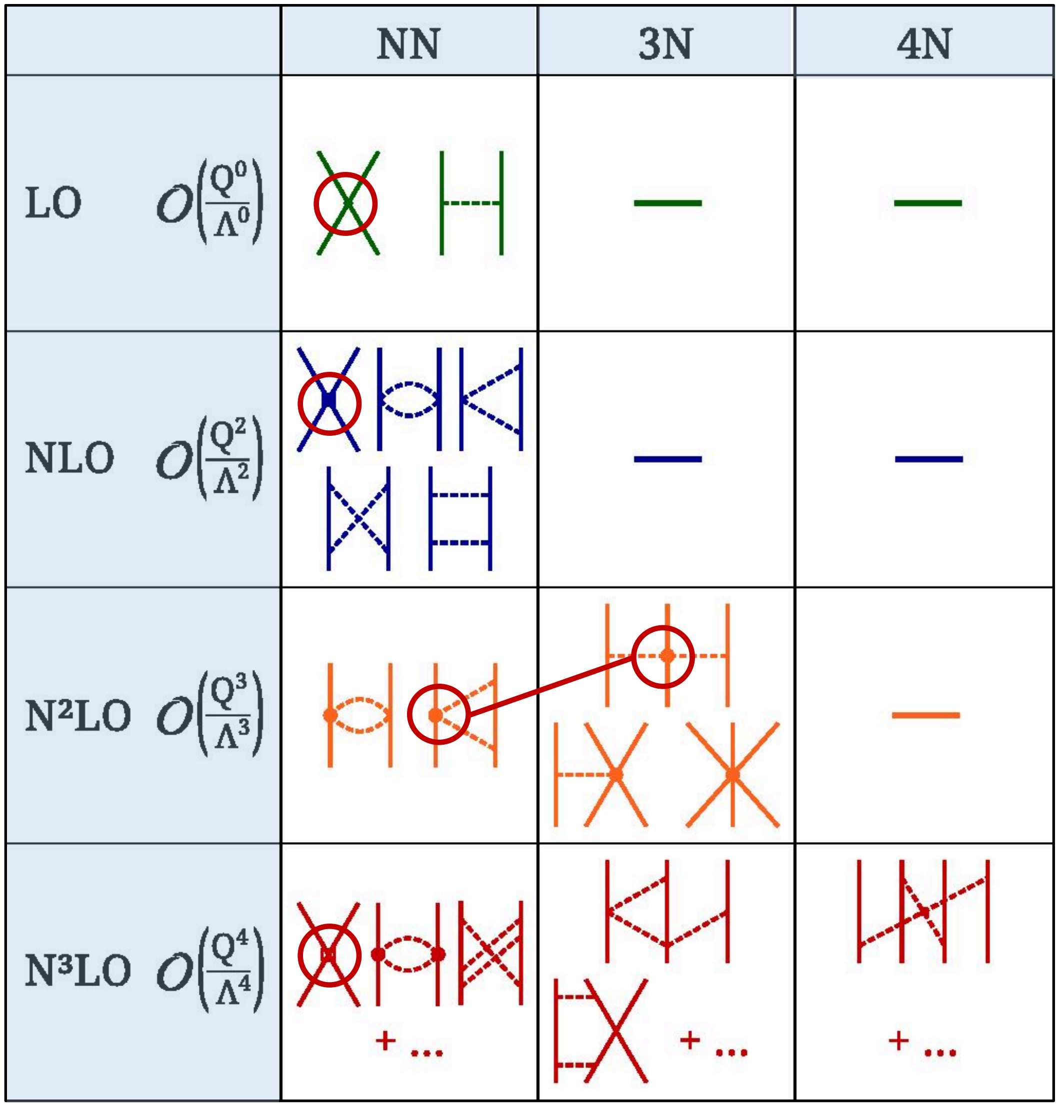
Natural hierarchy of nuclear forces:

- Two-body (NN) forces start at first order
- Three-body (3N) forces start at third order (2 LECs)

Fitting:

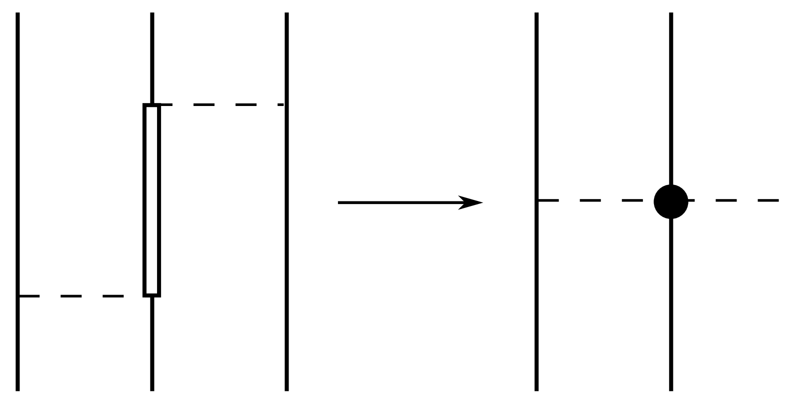
- NN forces in NN system (NN phase shifts, …)
- 3N forces in 3N/4N system (Binding energies, radii, …)

Chiral effective field theory for nuclear forces



Consistent interactions:

- Same couplings for two-nucleon and many-body sector
- In contrast to phenomenological interactions



Chiral effective field theory for nuclear forces

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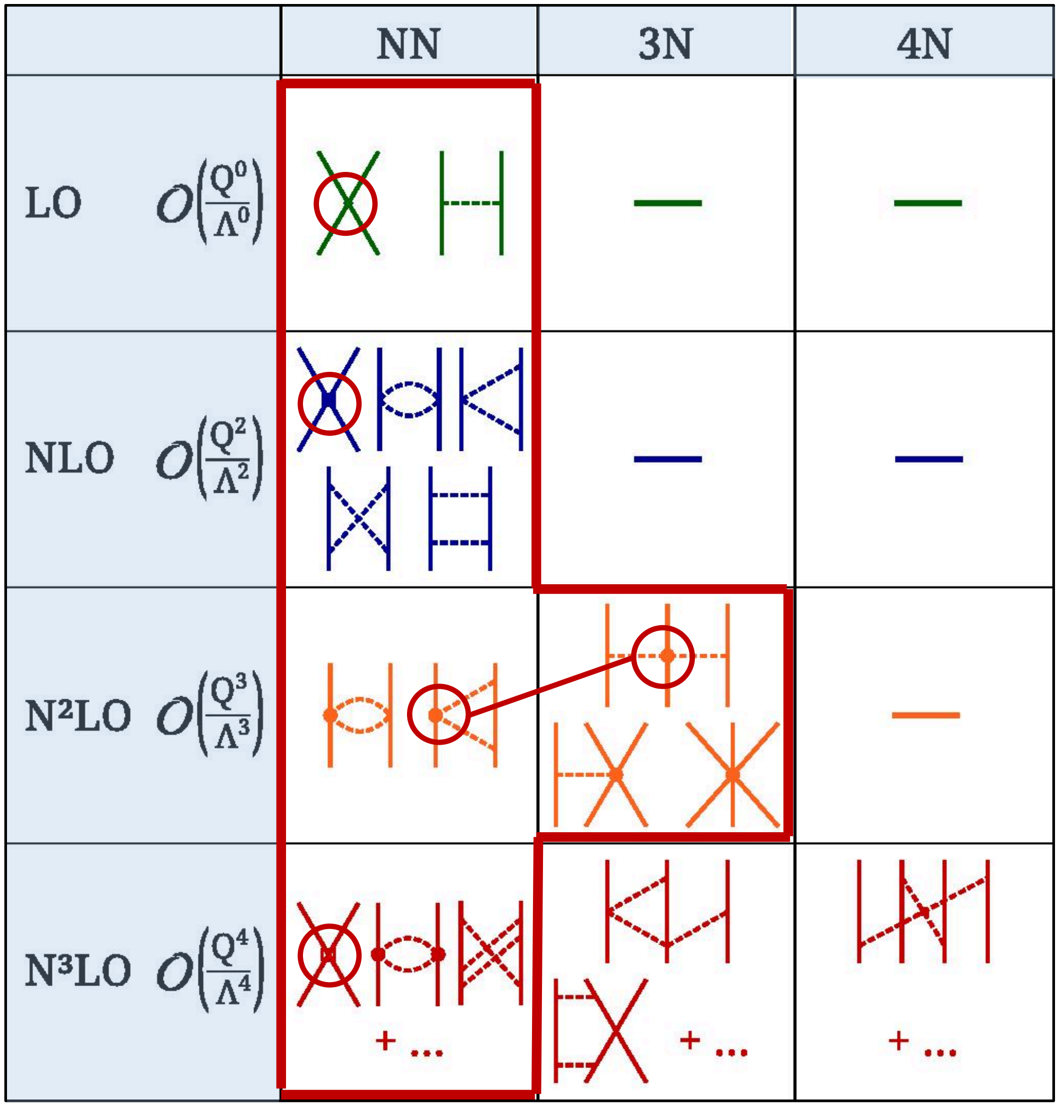

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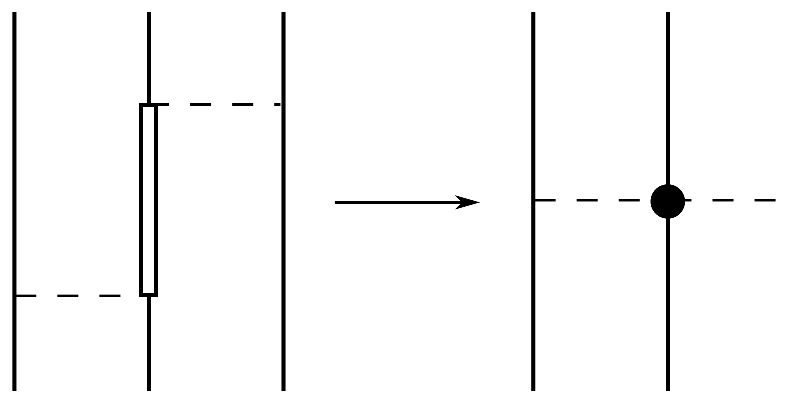
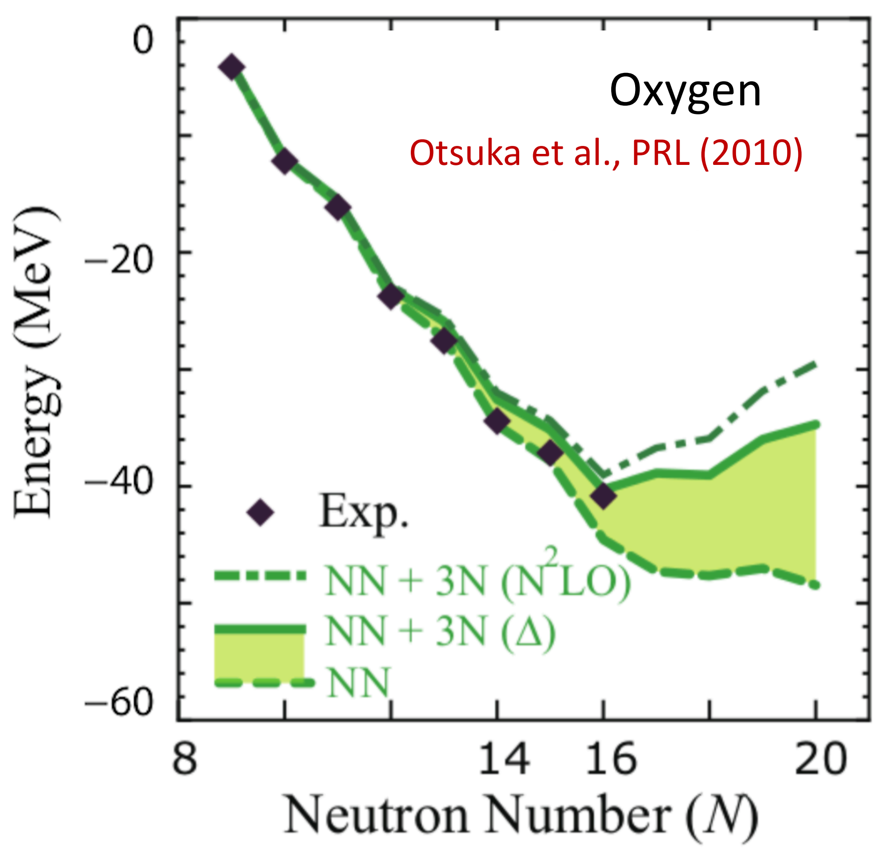


Chiral effective field theory for nuclear forces

Many-body forces are crucial:



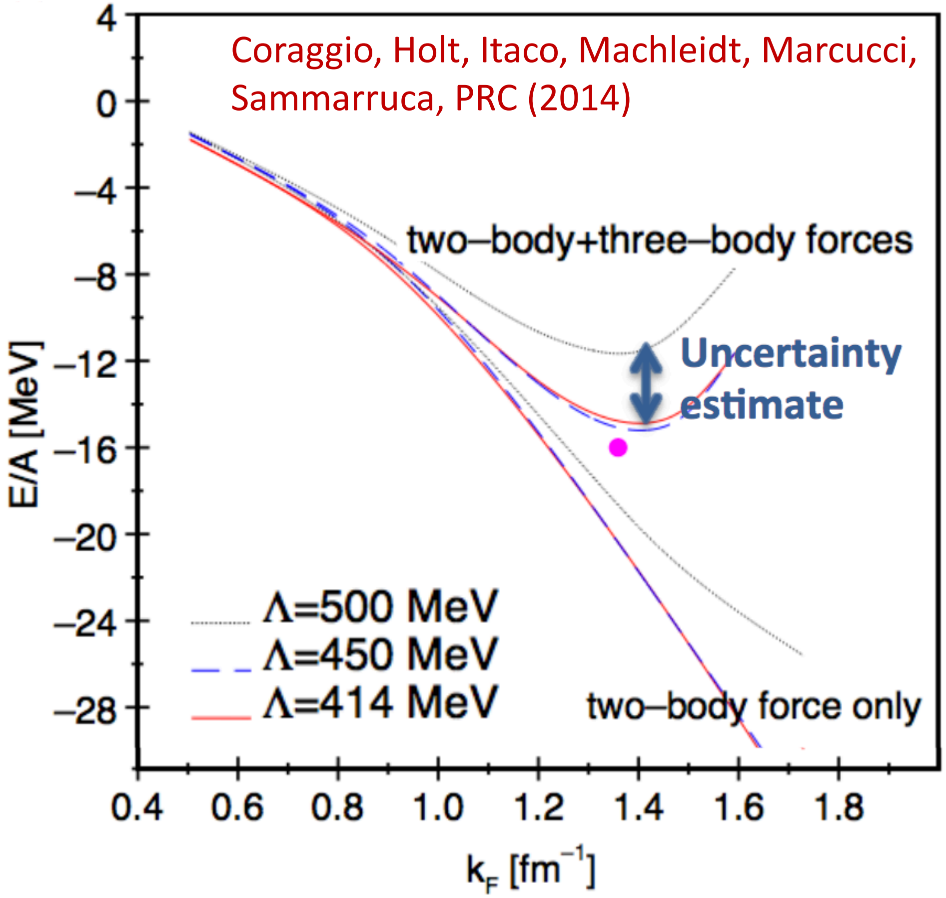
NN + 3N forces:

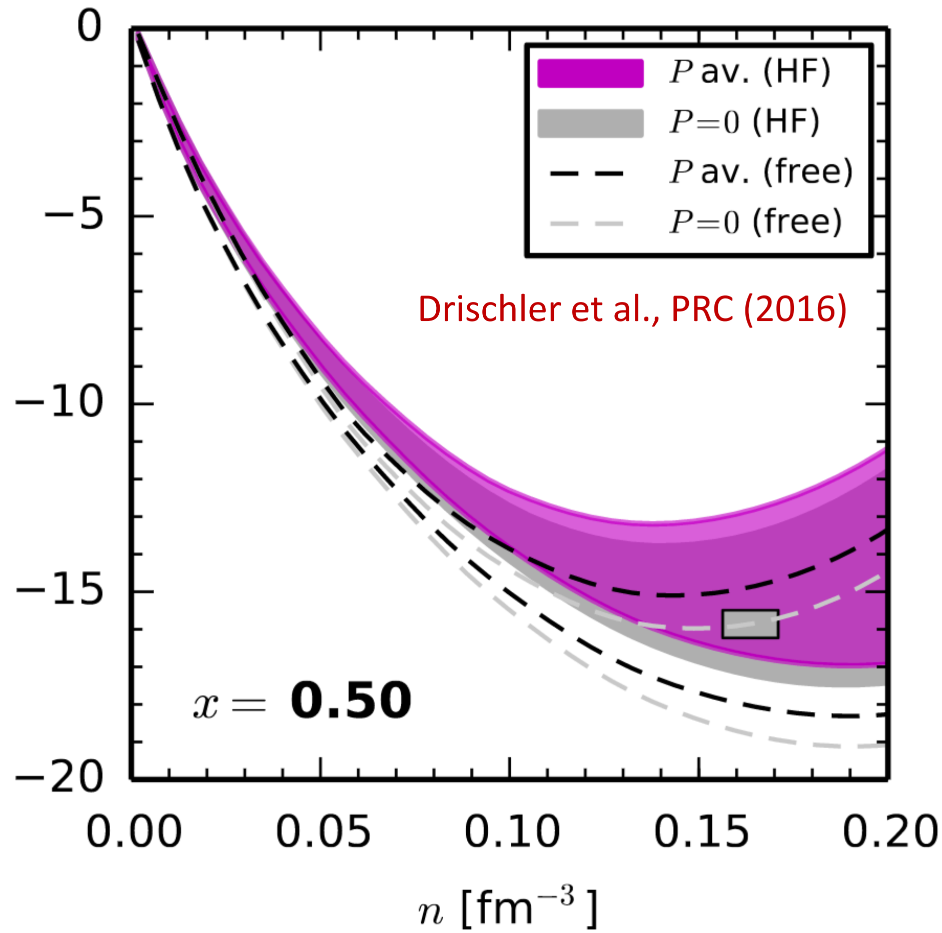
- Give correct physics of neutron-rich nuclei

See also Hebeler et al., ARNPS (2015)

Chiral effective field theory for nuclear forces

Many-body forces are crucial:





NN + 3N forces:

- Give correct saturation with theoretical uncertainties in nuclear matter
  Drischler et al., PRC (2016)

Chiral effective field theory for nuclear forces

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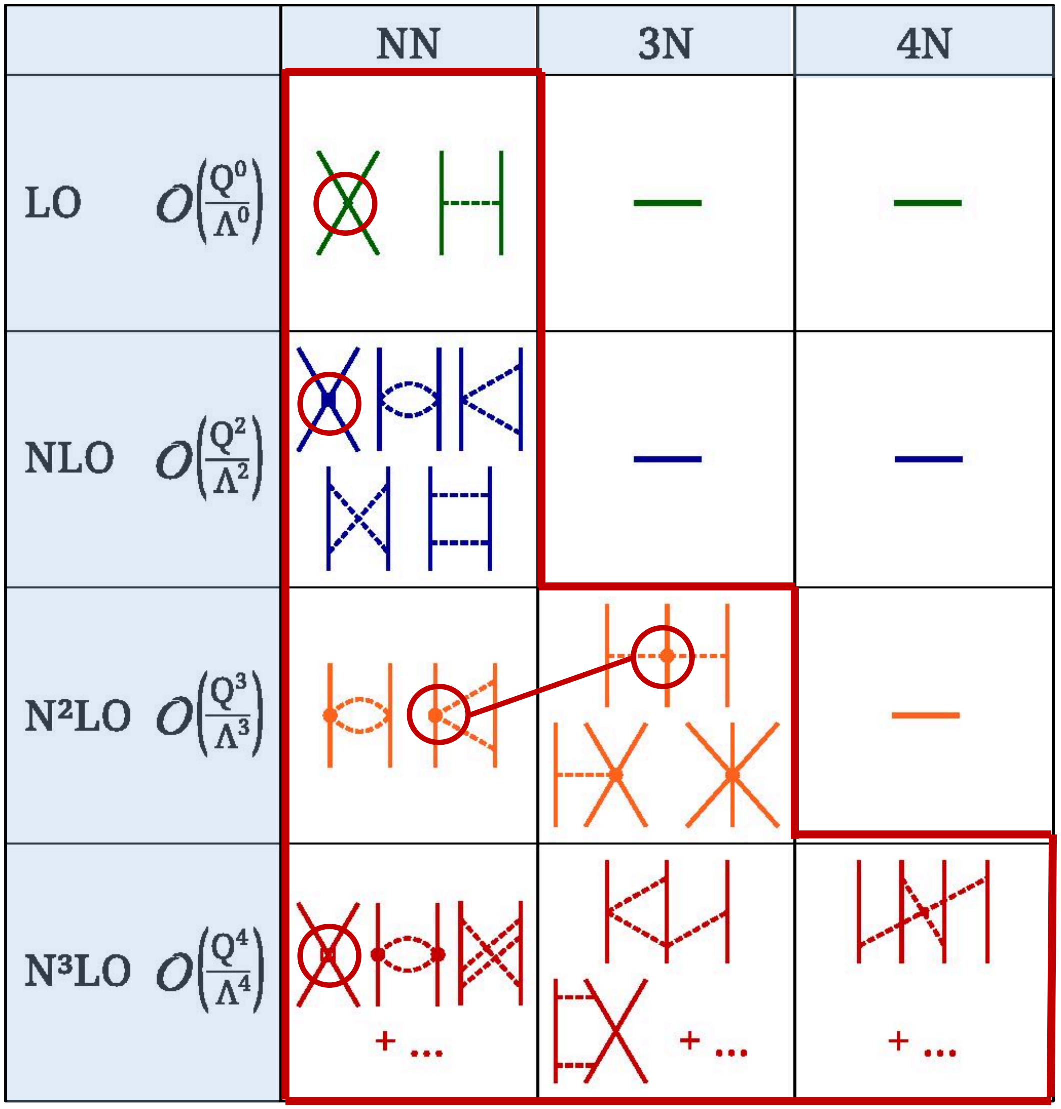


Neutron matter:

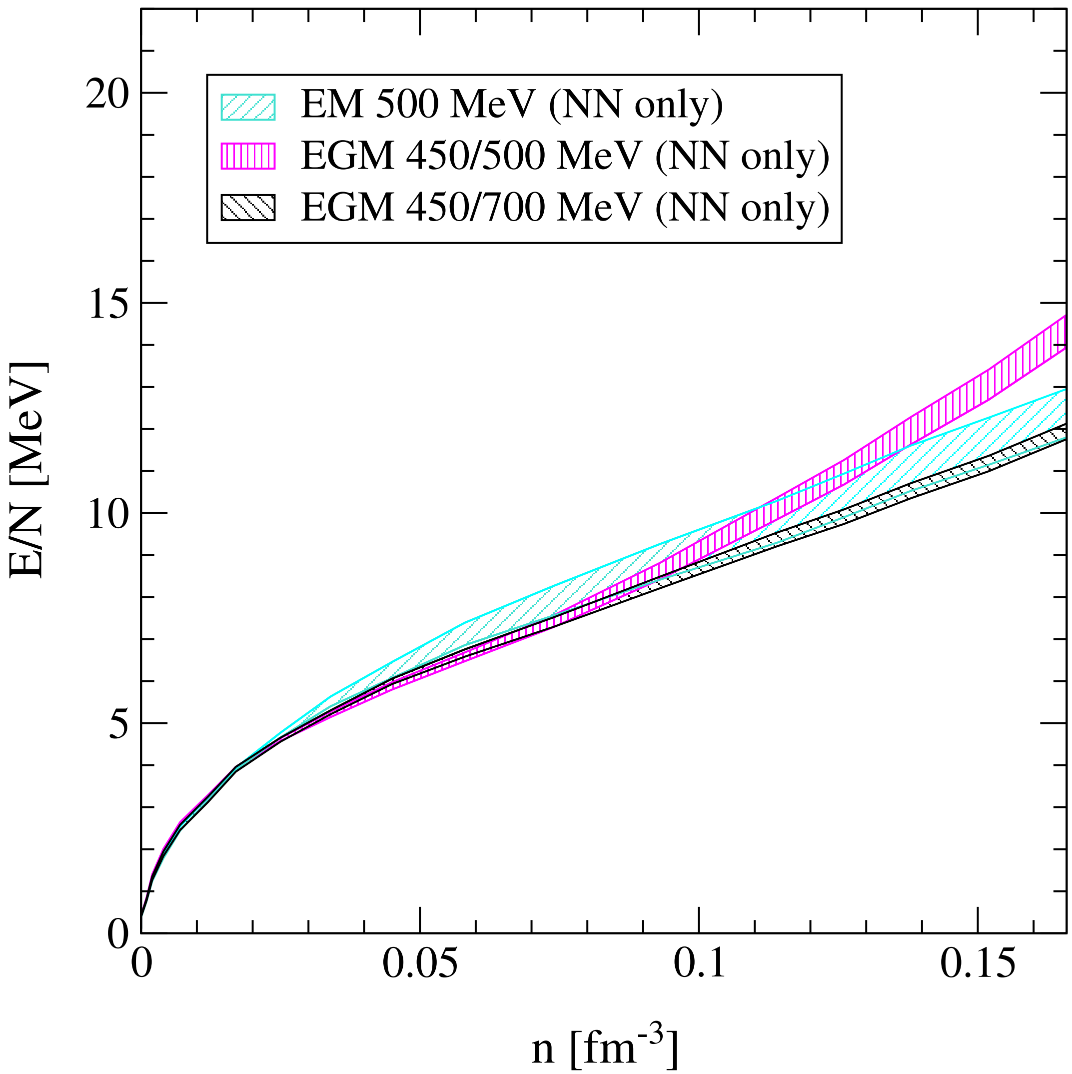
- Complete calculation at N^{3}LO using many-body perturbation theory (MBPT)

  IT, Krüger, Hebeler, Schwenk, PRL (2013)

Calculation is simpler in neutron matter:

- Only certain parts of the many-body forces contribute
- Chiral many-body forces completely predicted from NN sector

Neutron matter



IT, Krüger, Hebeler, Schwenk, PRL (2013)

Bands:

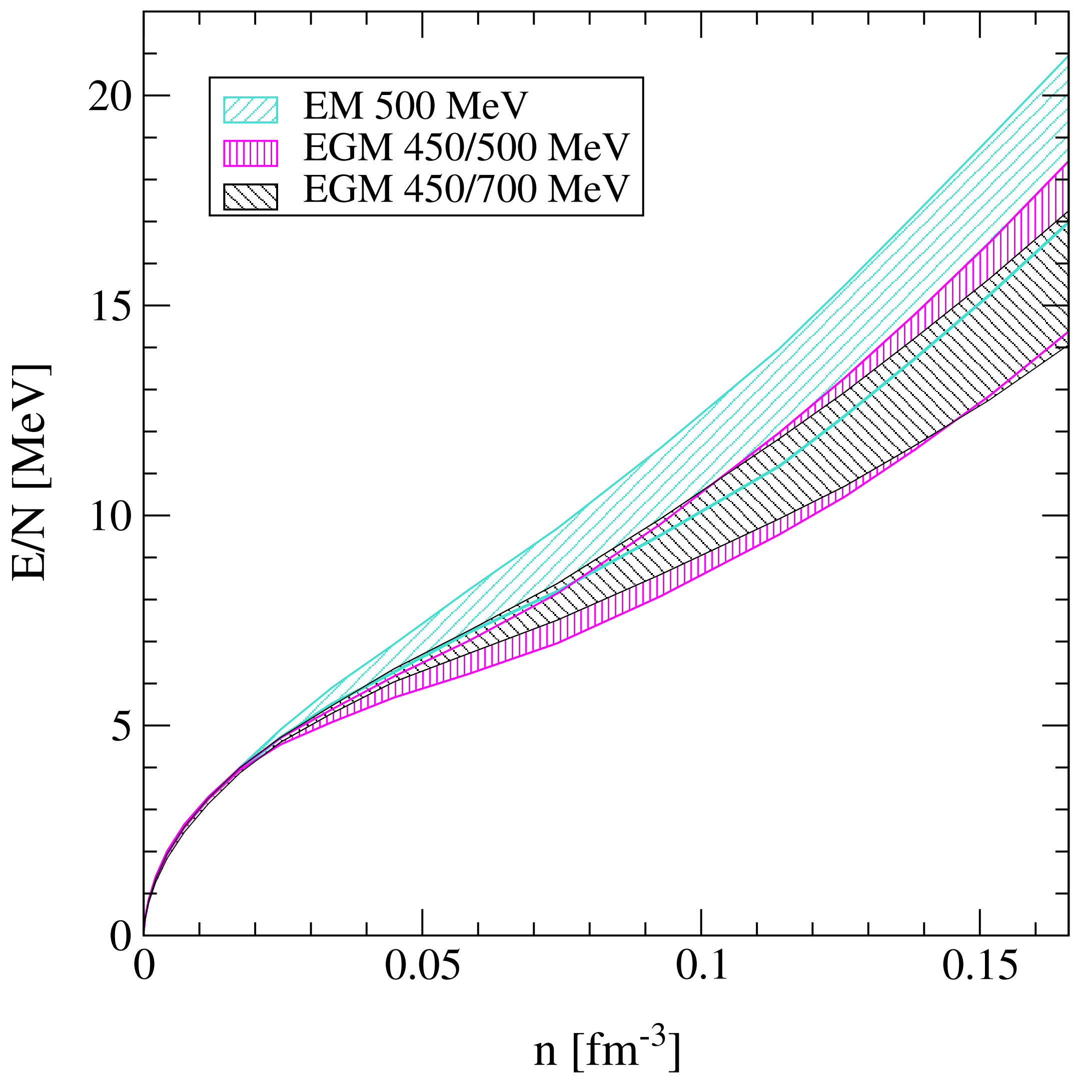
- Include several sources of uncertainty:
  - Chiral Hamiltonians (cutoff)
  - Many-body method
- Uncertainties due to many-body calculation small

NN interactions:

- E/N at saturation density:

12-15 MeV

Neutron matter



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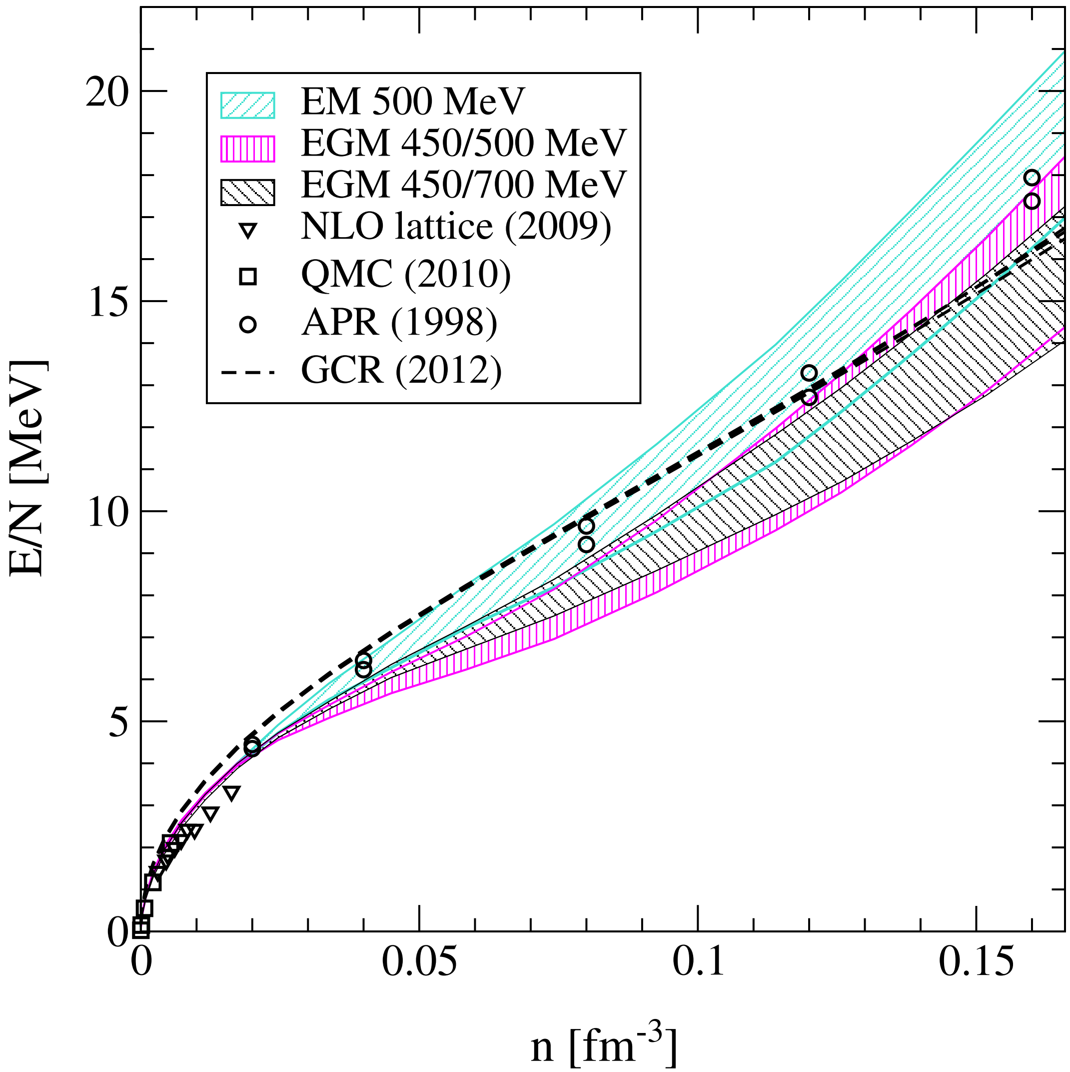
3N interactions:

- Have large impact on energy and uncertainty:

  14-21 MeV

Neutron matter



IT, Krüger, Hebeler, Schwenk, PRL (2013)

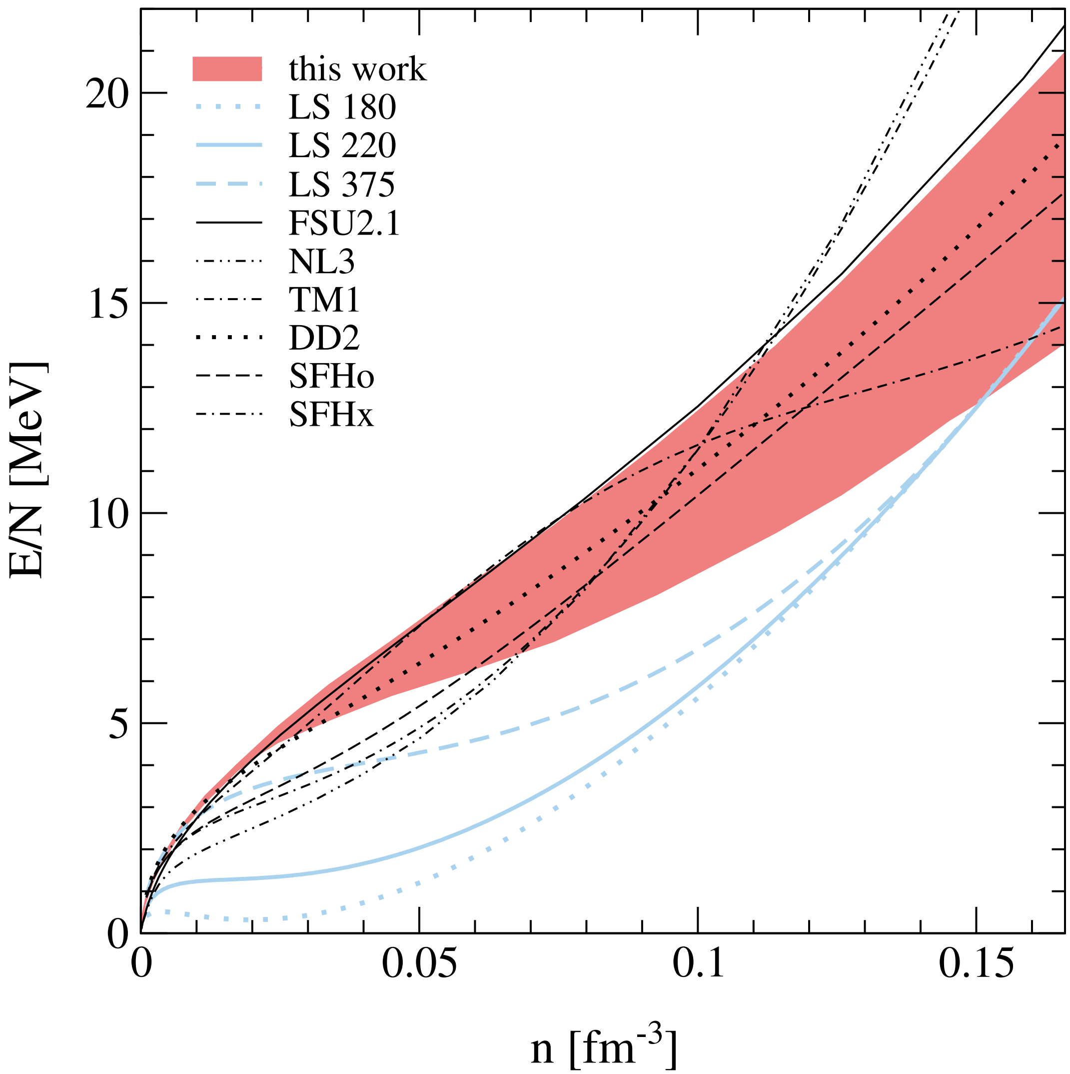
Good agreement with other calculations

- but in those no theoretical uncertainties

Akmal *et al.*, PRC (1998)
Gandolfi *et al.*, PRC (2012)

Chiral EFT puts constraints on neutron matter EOS

Neutron matter



Lines from Hempel, Lattimer, G. Shen

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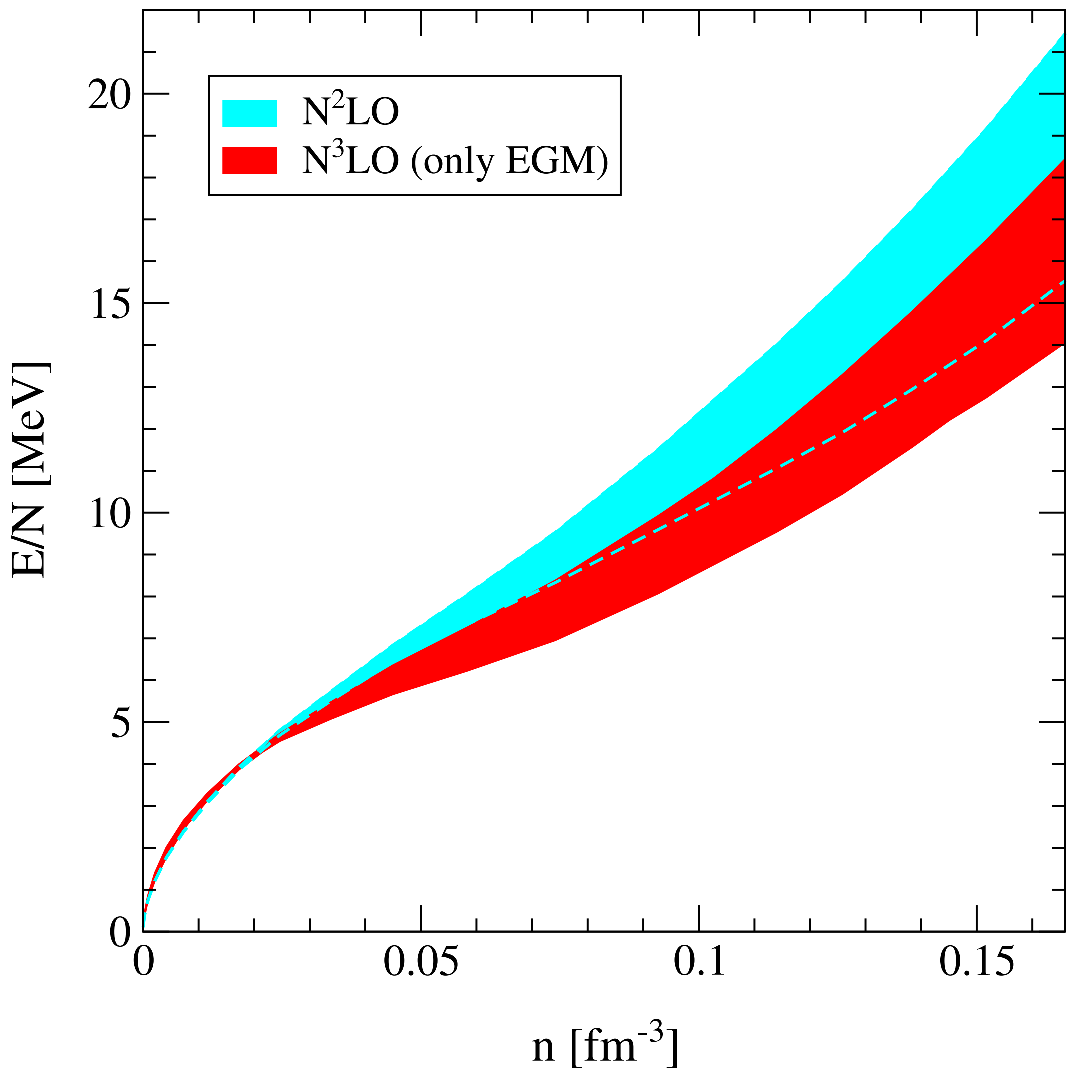

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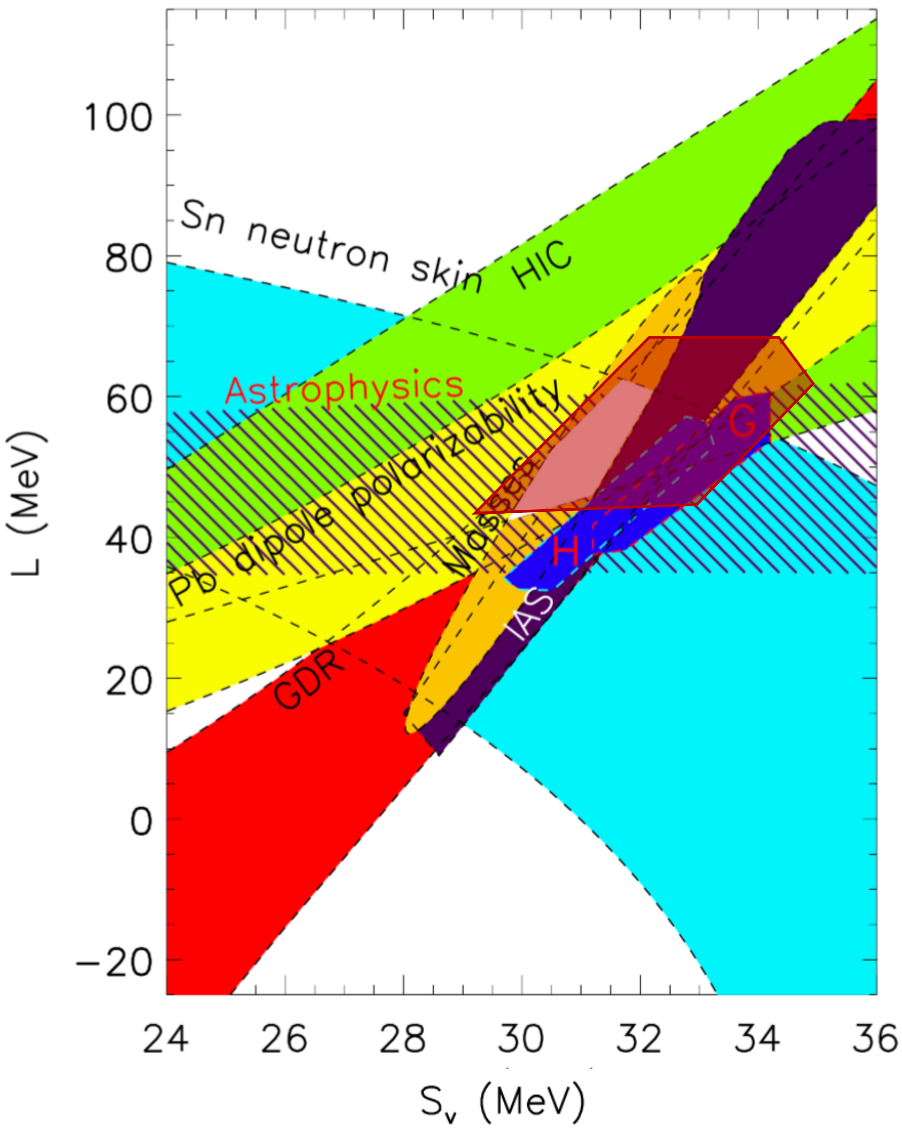
Chiral EFT puts constraints on neutron matter EOS

N^2LO to N^3LO:

- Theoretical uncertainty reduced
- E/N reduced

Symmetry energy and L parameter



Lattimer, Lim, ApJ (2013)

Put constraints on symmetry energy and its density dependence L:

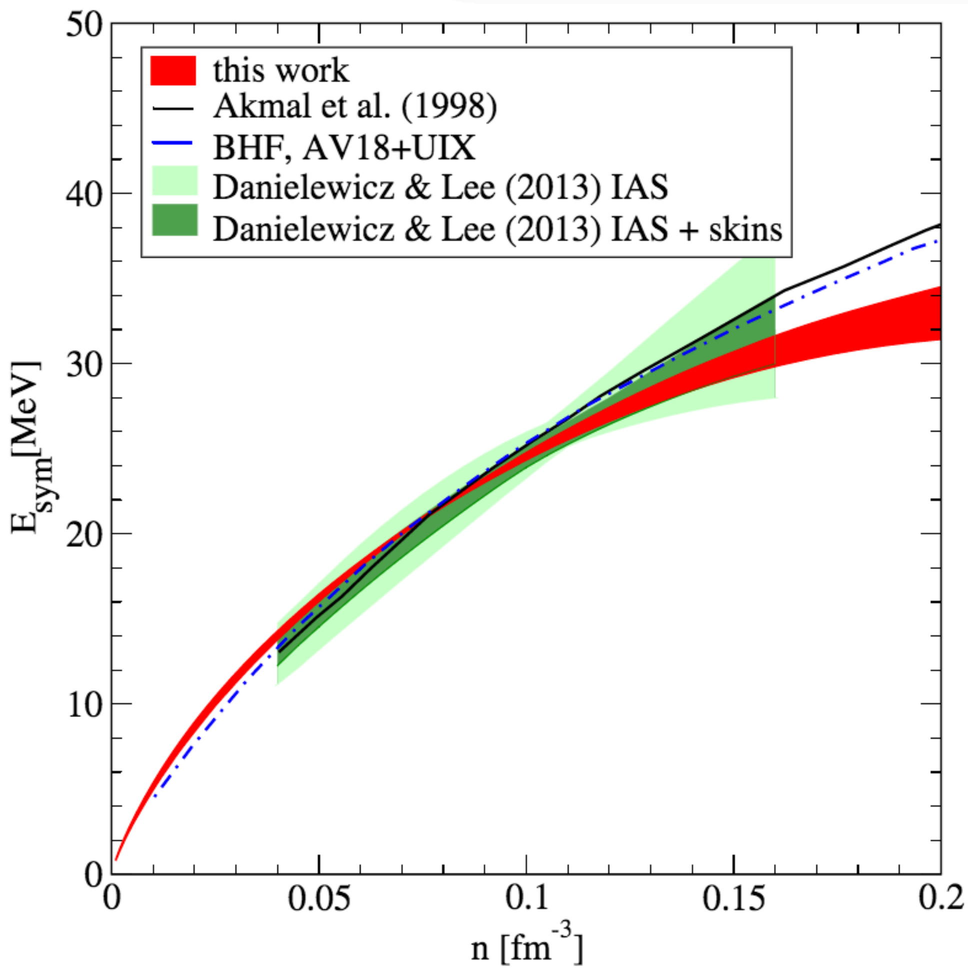
$$S_v(n) = \frac{1}{8}\frac{\partial^2}{\partial x^2}\frac{E}{A}(n,x)\Big|_{x=1/2},$$

$$L(n_0) = 3n_0\frac{\partial}{\partial n}S_V(n)\Big|_{n_0},$$

- $S_V = 28.9 - 34.9$ MeV
- $L = 43.0 - 66.6$ MeV

Good agreement with experimental constraints

Symmetry energy and L parameter



Drischler, Soma, Schwenk, PRC (2014)

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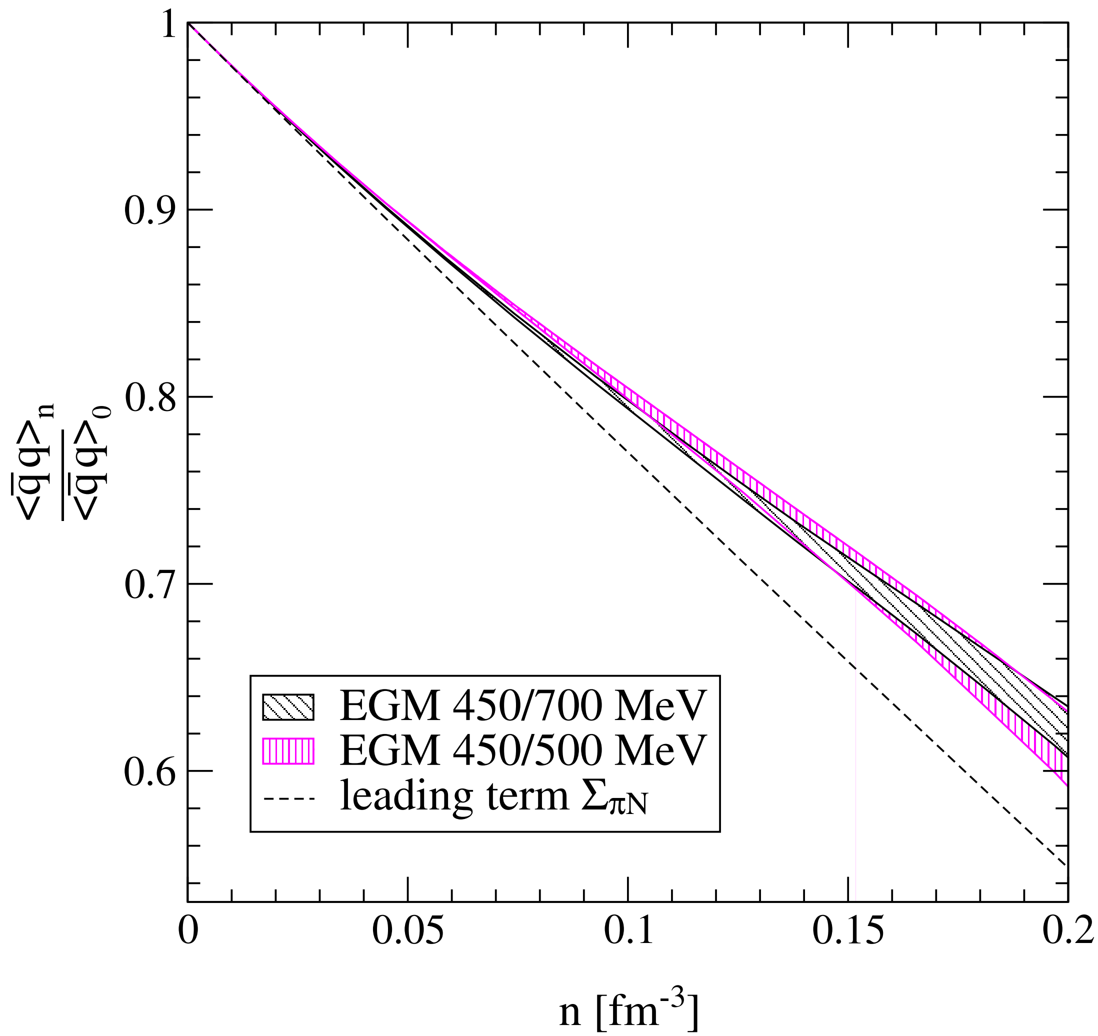
Chiral condensate



Krüger, IT, Hebeler, Friman, Schwenk, PLB (2013)

$$\langle \bar{q}q \rangle_n - \langle \bar{q}q \rangle_0 = n \frac{\partial}{\partial m_q} \left[\frac{E_{\text{free}}(m_q, k_F)}{N} + \frac{E_{\text{int}}(m_q, k_F)}{N} \right]$$

Includes:

- Explicit m_π dependence
- No m_π dependence of couplings

Constraints on chiral condensate:

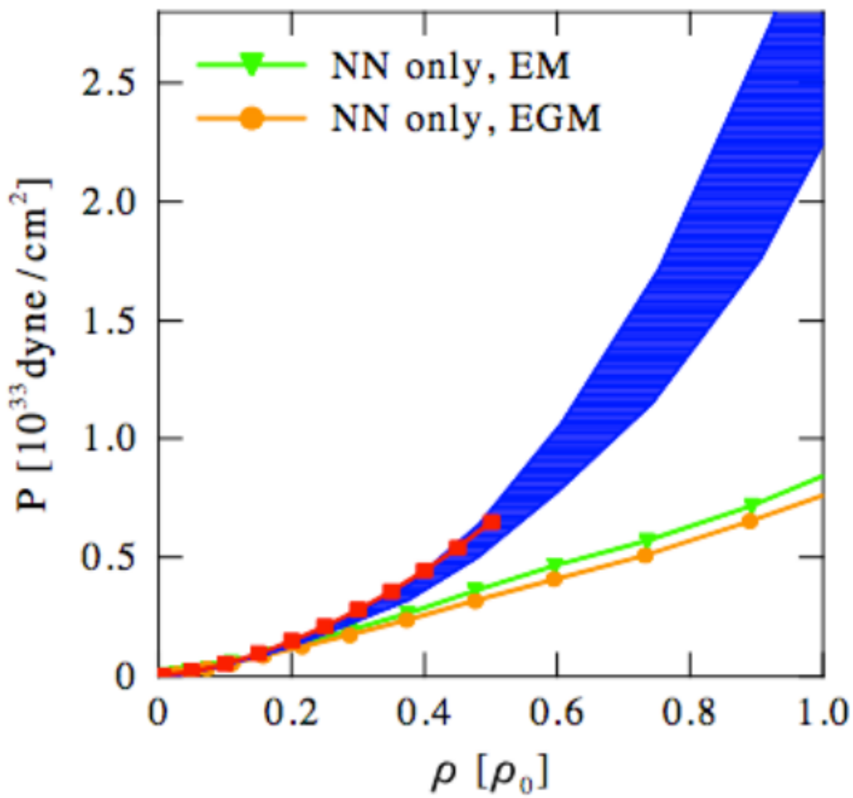
- Interactions increase the chiral condensate
- Leading σ term: 0.62 at n_0
- Chiral band: 0.67-0.69 at n_0
- Good agreement with calculation in symmetric matter

Weise, PPNP (2012)

Neutron Stars

Equation of state for neutron star matter: extend results to small $Y_{e,p}$

Hebeler, Lattimer, Pethick, Schwenk, PRL (2010) and APJ (2013)



Agrees with standard crust EOS
after inclusion of many-body forces

Neutron Stars

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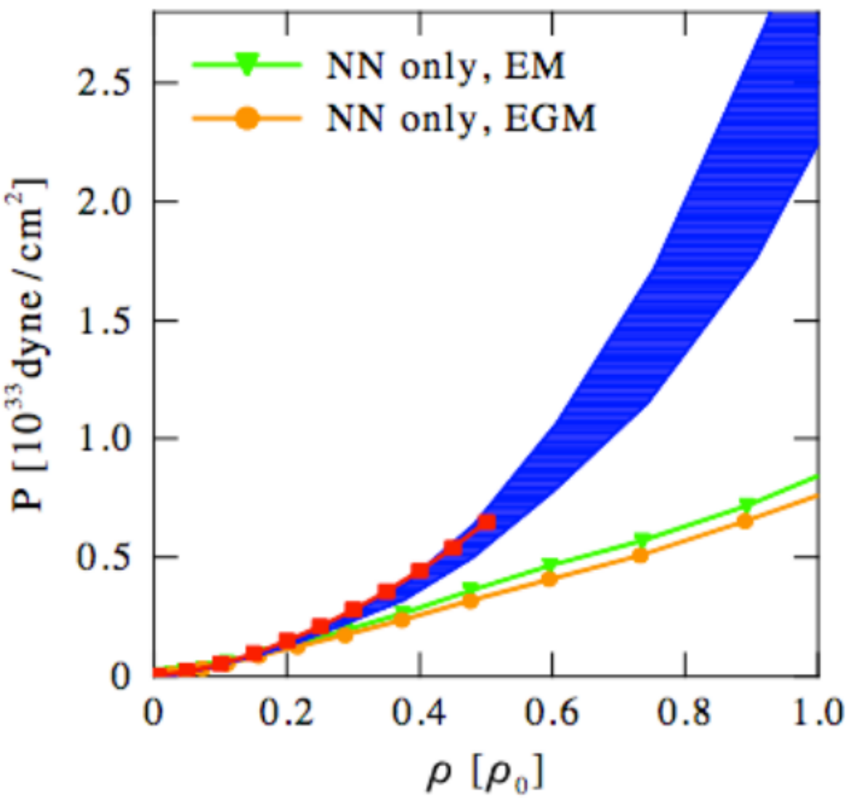
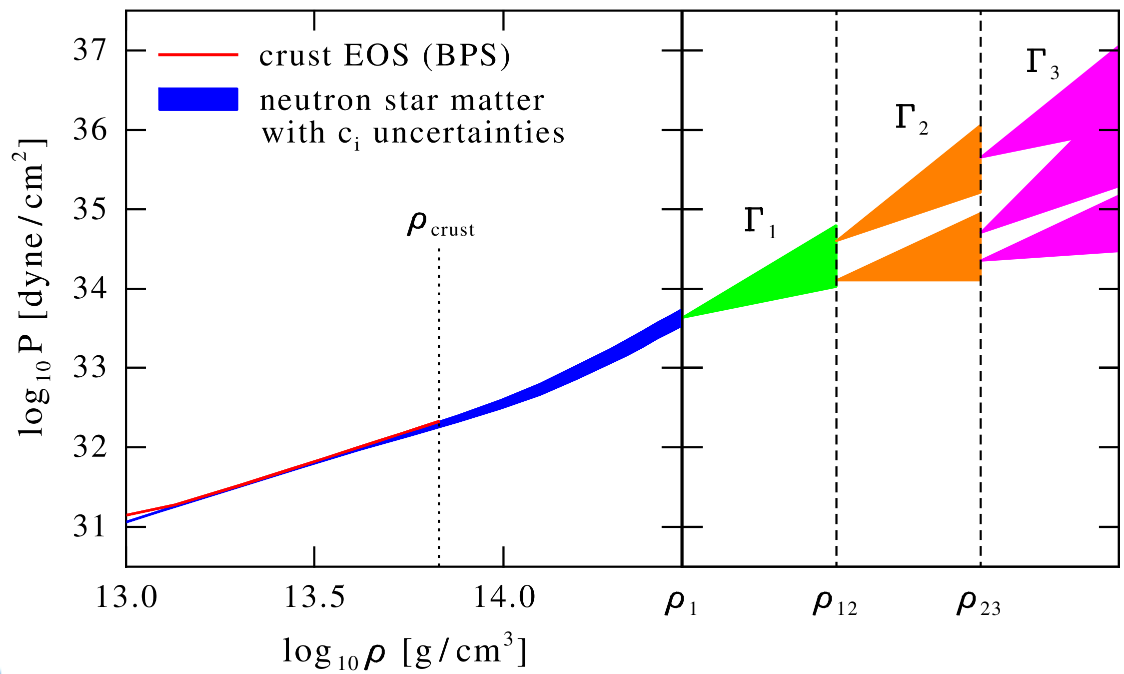
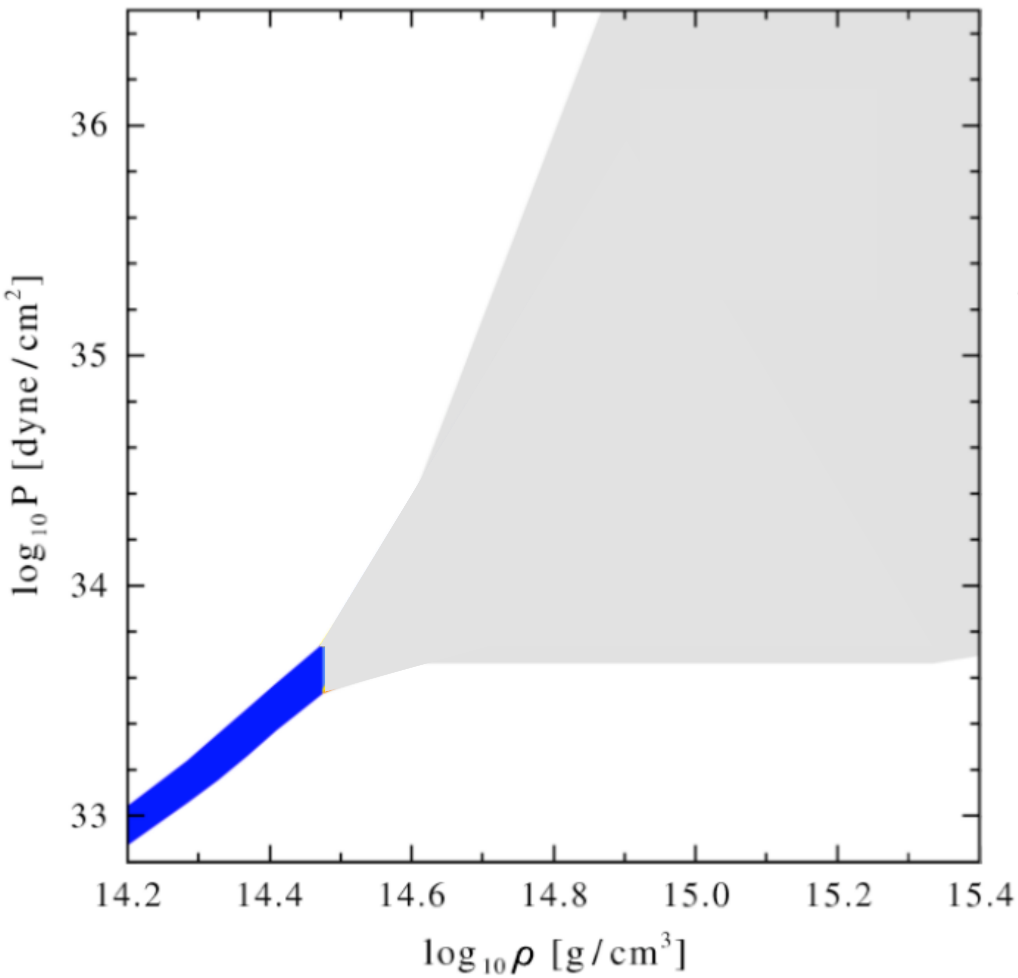




Agrees with standard crust EOS after inclusion of many-body forces

Extend to higher densities using polytropic expansion

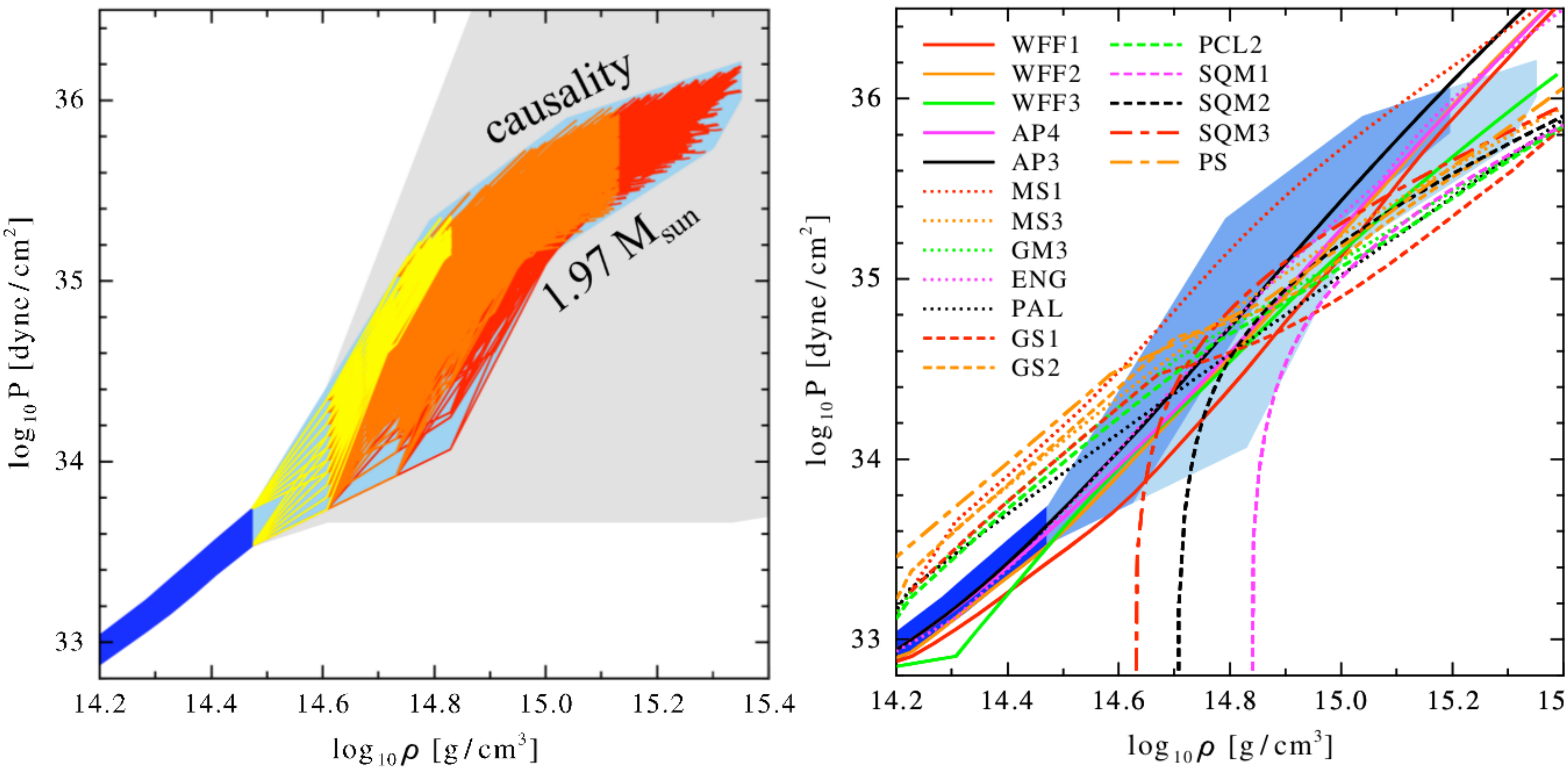
Neutron Stars



Hebeler, Lattimer, Pethick, Schwenk, PRL (2010) and APJ (2013)

Neutron Stars

Constrain resulting EOS: causality and observed 1.97 $M_\odot$ neutron star



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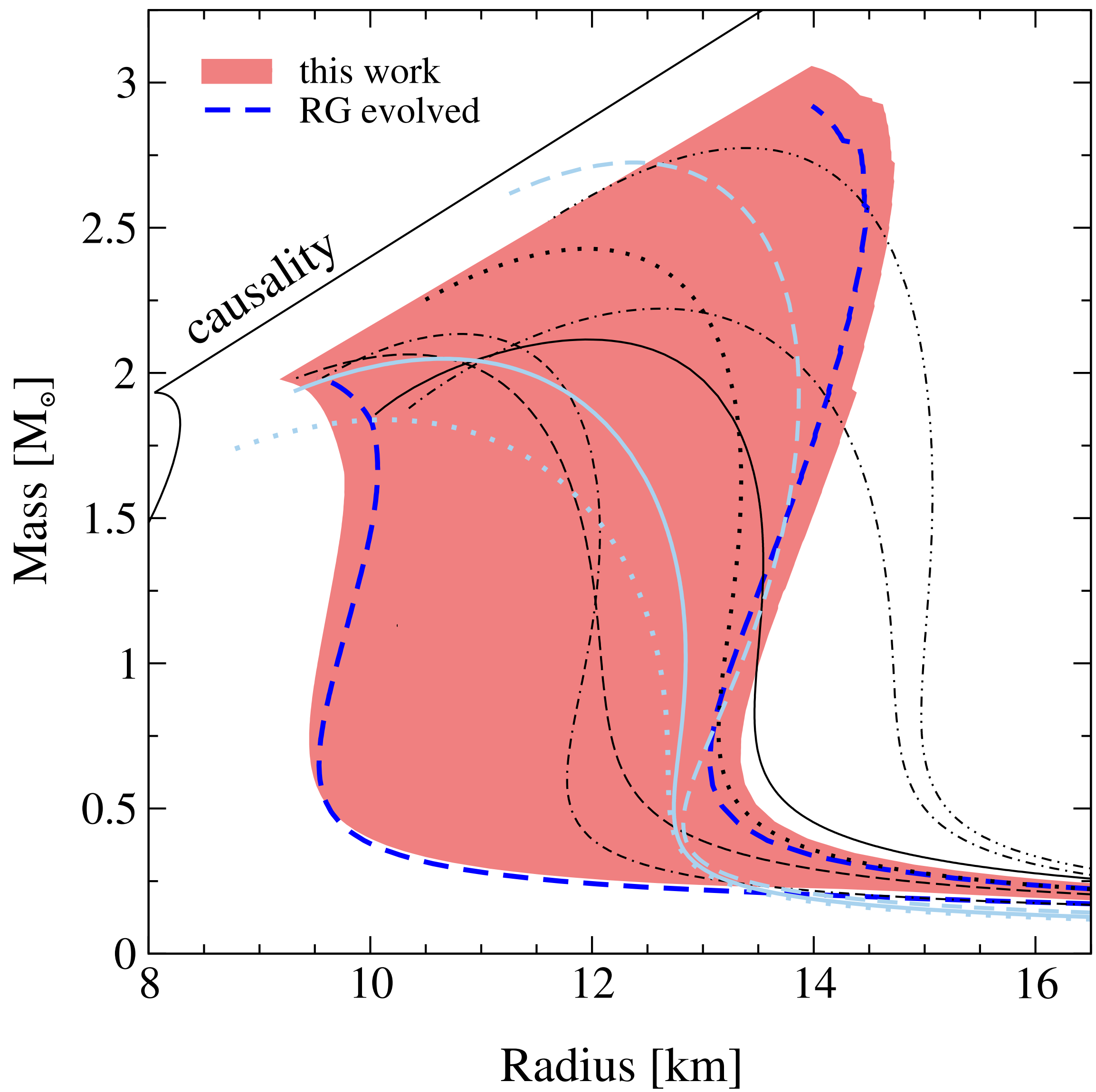
Neutron Stars



Radius for 1.4 $M_\odot$ neutron star:

- $R = 9.7 - 13.9$ km

Maximum mass neutron star:

- $M_{max} \leq 3.05 M_\odot$ (14 km)

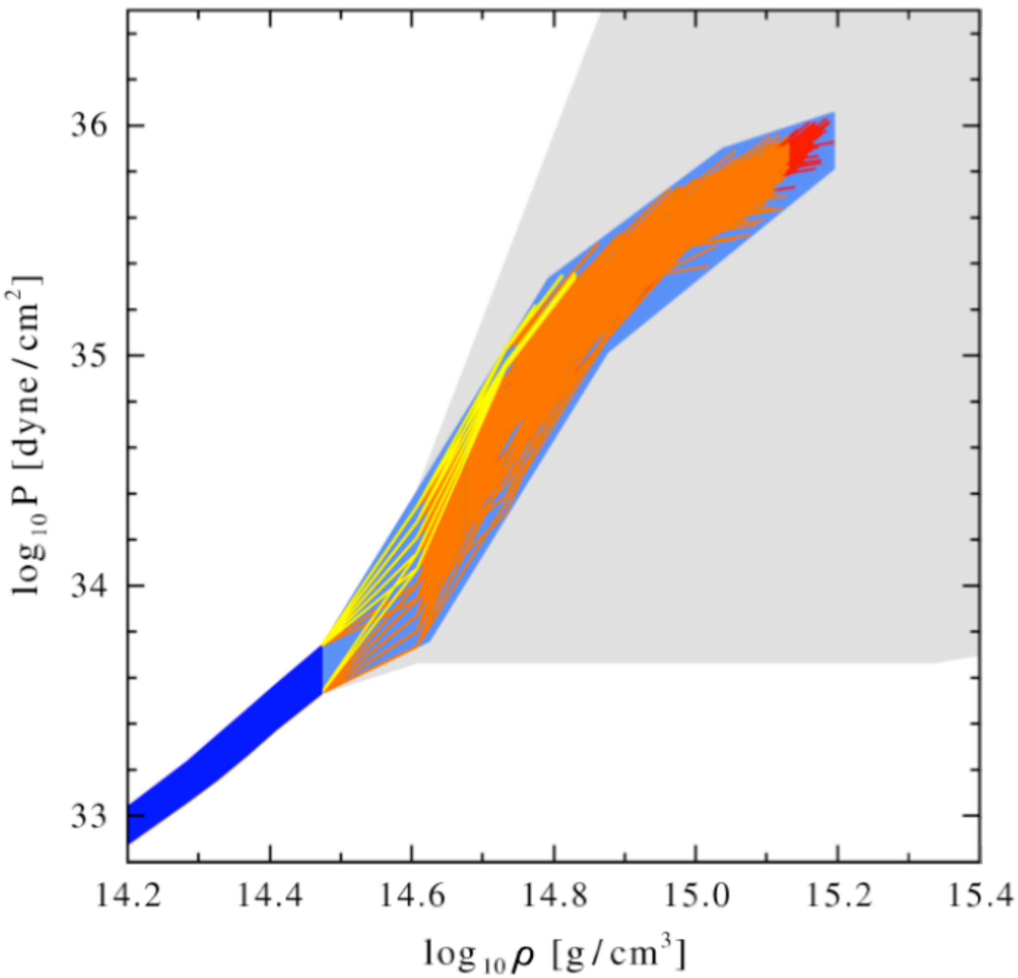
Uncertainties from many-body forces and polytropic expansion

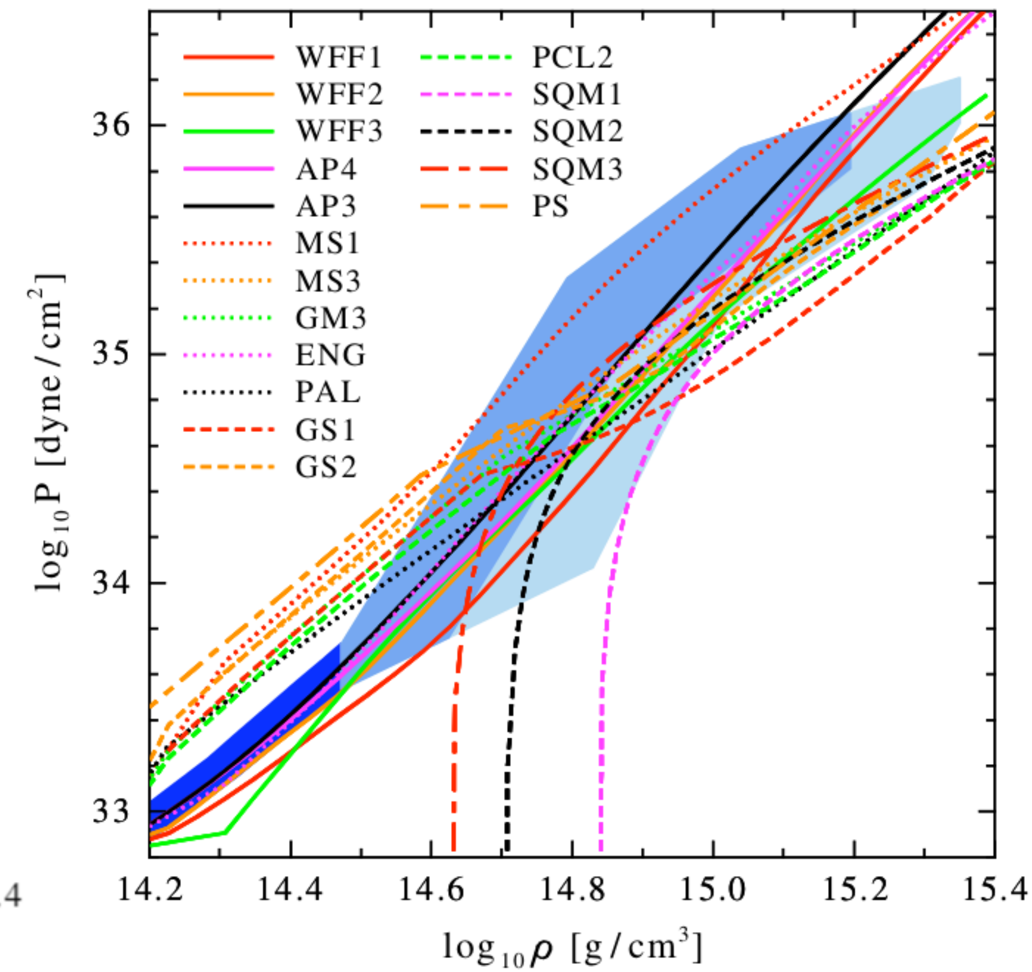
- How to reduce uncertainties?

IT, Krüger, Hebeler, Schwenk, PRL (2013)

Neutron Stars

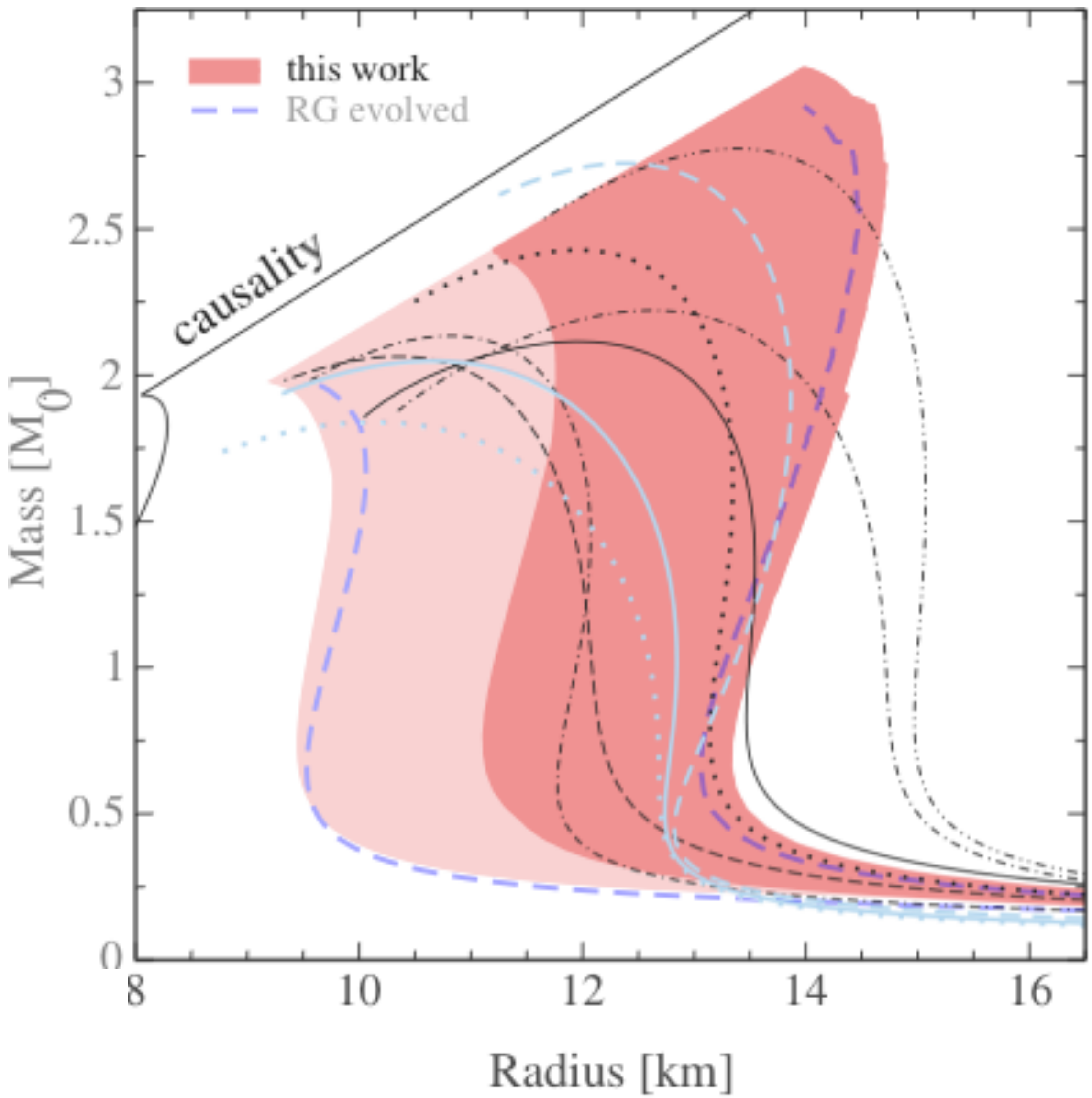
If a 2.4 $M_\odot$ neutron star was observed:





Hebeler et al., PRL (2010) and APJ (2013)

Neutron Stars



Radius for 1.4 $M_\odot$ neutron star:

- $R = 11.5 - 13.9$ km

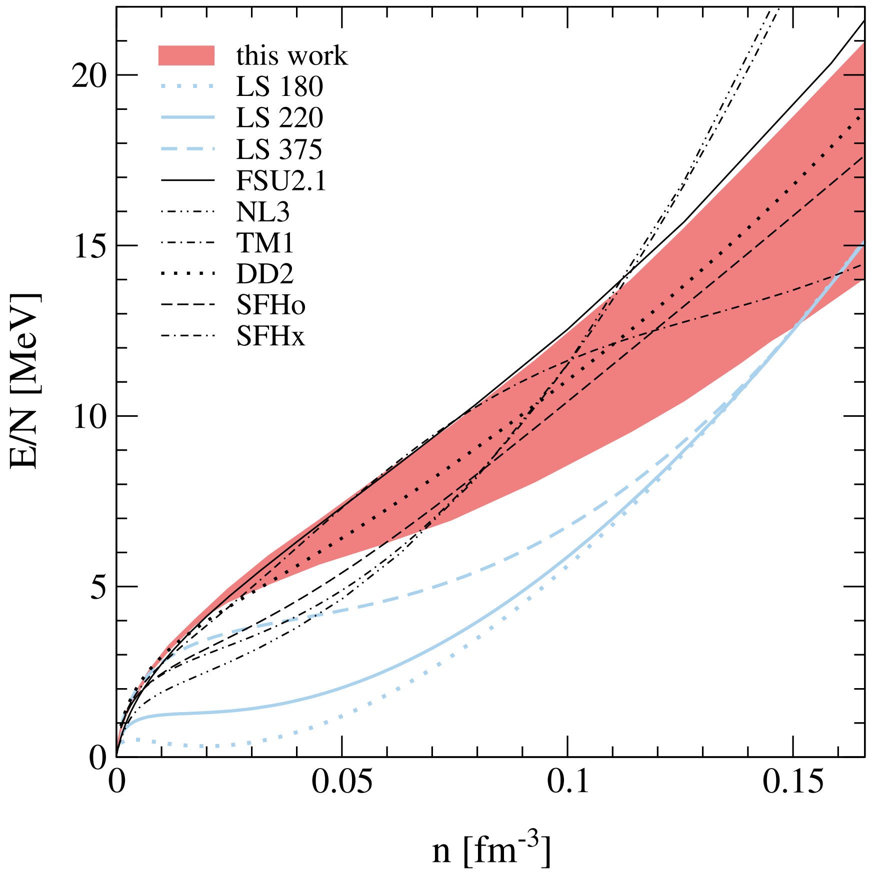
Maximum mass neutron star:

- $M_{max} \leq 3.05 M_\odot$ (14 km)

Uncertainties from many-body forces and polytropic expansion

IT, Krüger, Gezerlis, Hebeler, Schwenk (2013)

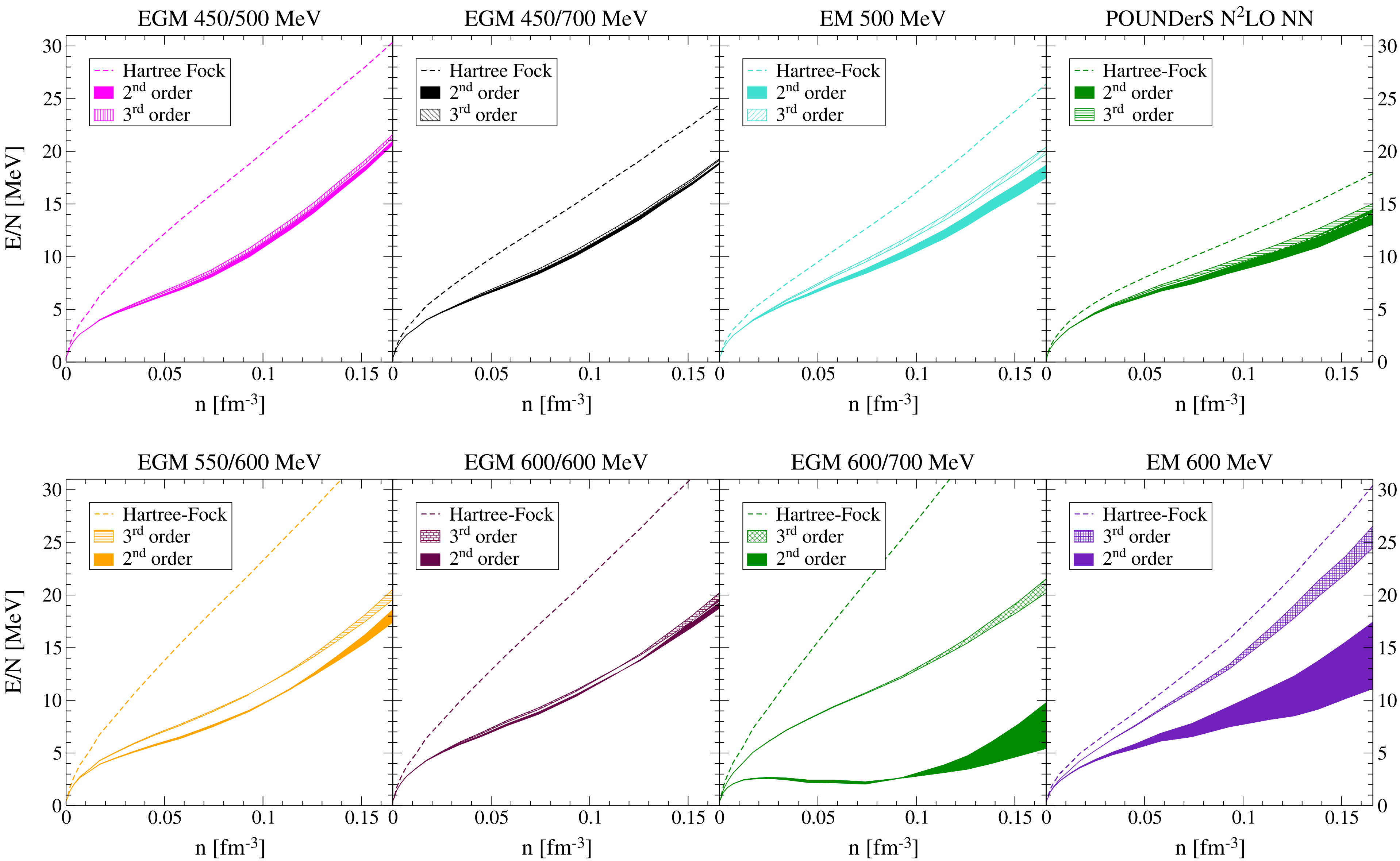
Improving neutron-matter band



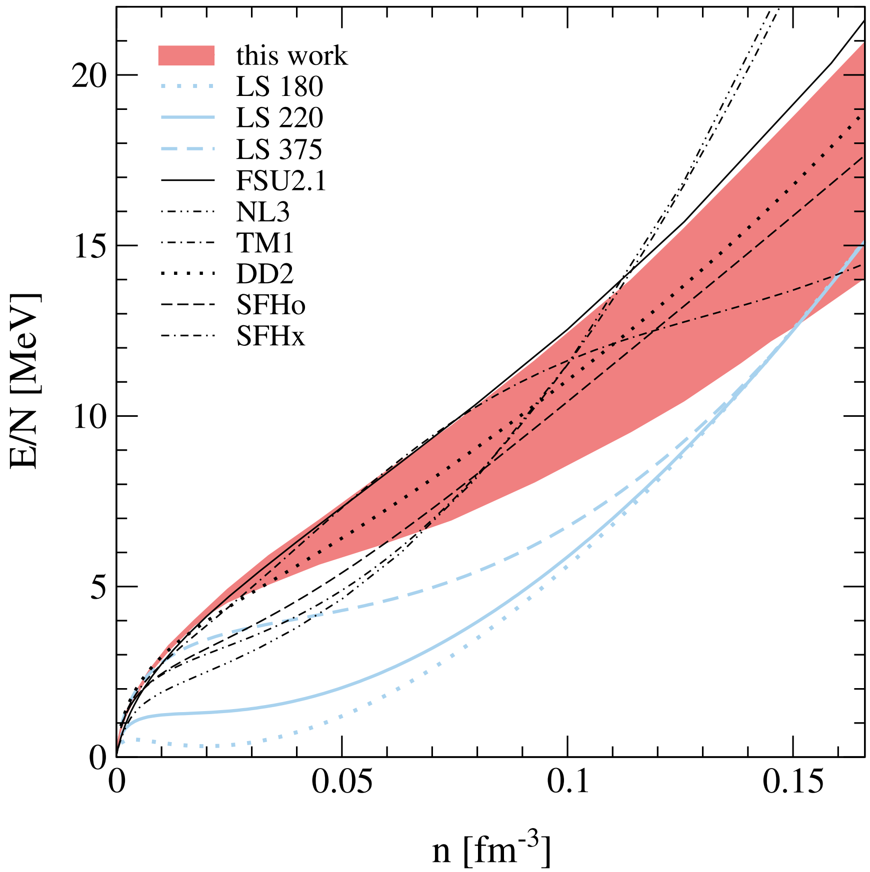
IT, Krüger, Hebeler, Schwenk (2013)

- Chiral EFT constrains neutron matter equation of state
- So far used in perturbative calculations (MBPT)
- need for nonperturbative benchmark

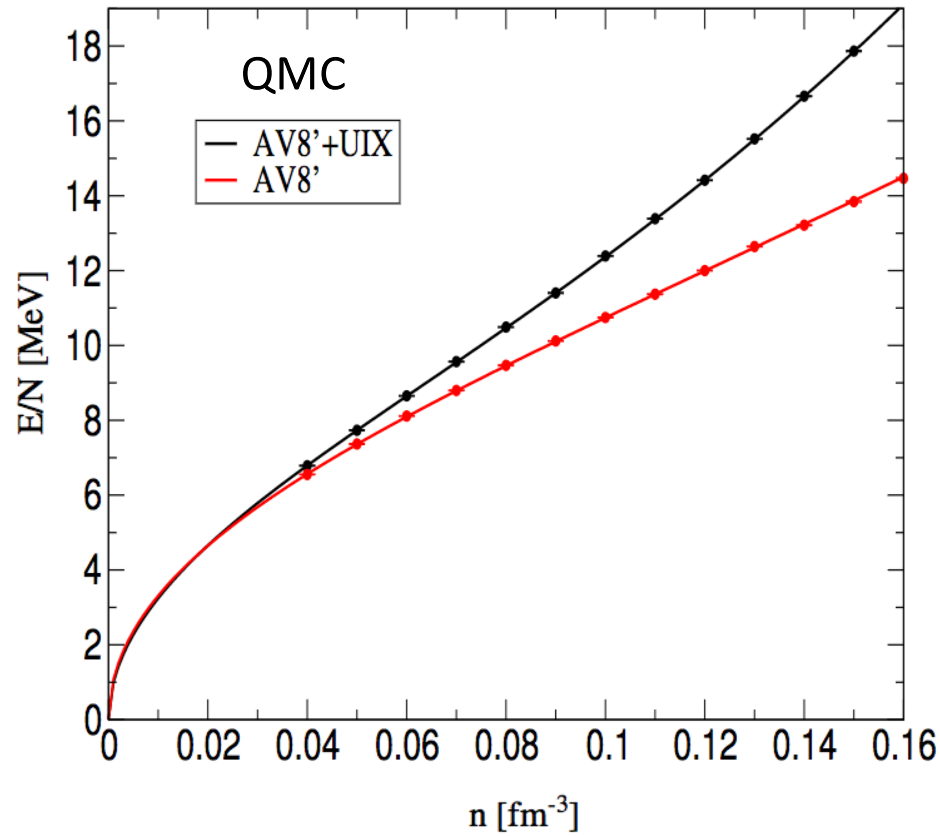
Improving neutron-matter band



Improving neutron-matter band



IT, Krüger, Hebeler, Schwenk (2013)



Credit: Stefano Gandolfi

Fourth order in chiral EFT (nonlocal interactions), Many-body perturbation theory

Phenomenological forces, Quantum Monte Carlo (needs local interaction)

Quantum Monte Carlo method

Solve the many-body Schrödinger equation

$$H|\psi\rangle = -\frac{\partial}{\partial\tau}|\psi\rangle\,, \qquad \tau = it$$

$$\psi(R,\tau) = \int dR'^{3N} \langle R| e^{-(T+V)\tau} |R'\rangle \psi(R',0)$$

Basic steps:

- Choose trial wavefunction which overlaps with the ground state

$$|\psi(R,0)\rangle = |\psi_T(R,0)\rangle = \sum_i c_i |\phi_i\rangle \rightarrow \sum_i c_i e^{-(E_i - E_0)\tau} |\phi_i\rangle$$

- Evaluate propagator for small timestep $\Delta\tau$, feasible only for local potentials
- Make consecutive small time steps using Monte Carlo techniques to project out ground state

$$|\psi(R,\tau)\rangle \rightarrow |\phi_0\rangle \quad \text{for} \quad \tau \rightarrow \infty$$

More details:
Carlson, Gandolfi, Pederiva, Pieper, Schiavilla, Schmidt, Wiringa, RMP (2015)

Quantum Monte Carlo method

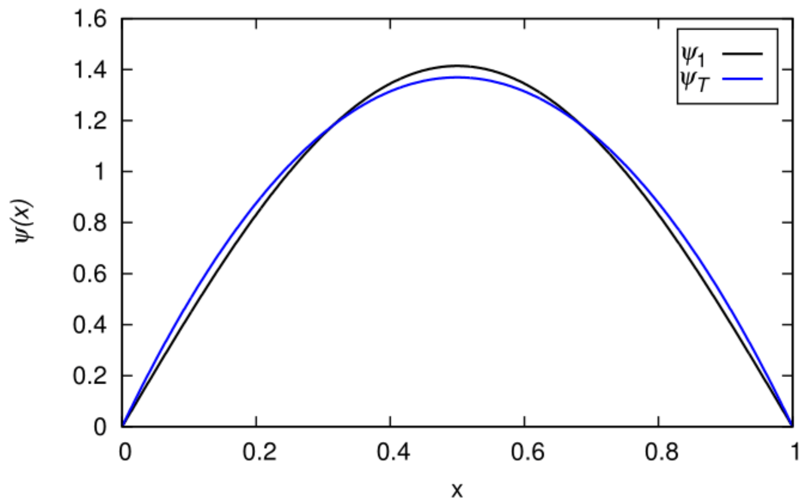
Particle in a 1D box, solution:

$$\psi_n(x) = \sqrt{2}\sin(n\pi x), \quad E_n = \frac{n^2\pi^2}{2}$$

Basic steps:

- Choose parabolic trial wavefunction which overlaps with the ground state

Animation by Joel Lynn, TU Darmstadt



Quantum Monte Carlo method

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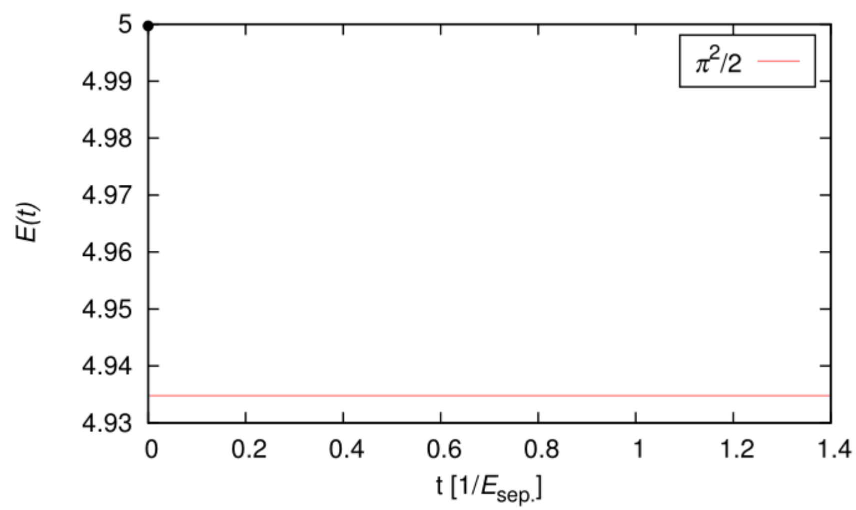
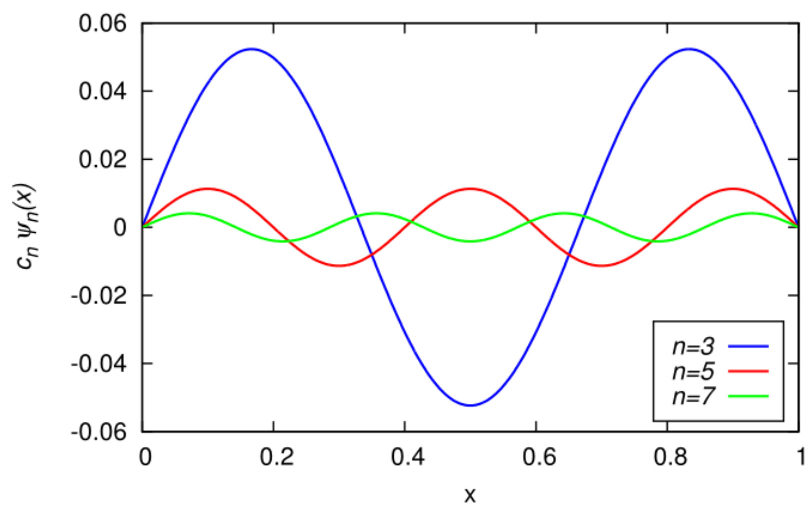
Particle in a 1D box, solution:

$$\psi_n(x) = \sqrt{2}\sin(n\pi x), \quad E_n = \frac{n^2\pi^2}{2}$$

- Make consecutive small timesteps, $\tau = 0.0 \left(\frac{1}{E_{\text{sep}}}\right)$

Animation by Joel Lynn, TU Darmstadt





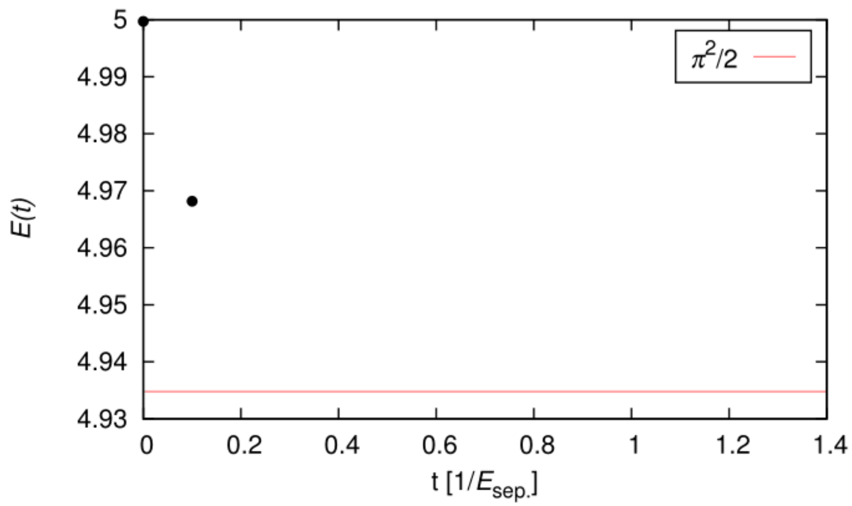
Quantum Monte Carlo method

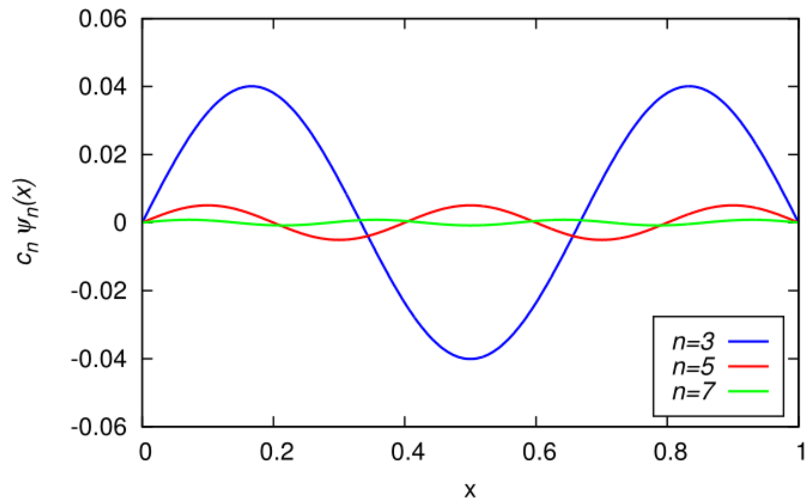
Particle in a 1D box, solution:

$$\psi_n(x) = \sqrt{2}\sin(n\pi x), \quad E_n = \frac{n^2\pi^2}{2}$$

- Make consecutive small timesteps, $\tau = 0.1\left(\frac{1}{E_{\text{sep}}}\right)$

Animation by Joel Lynn, TU Darmstadt





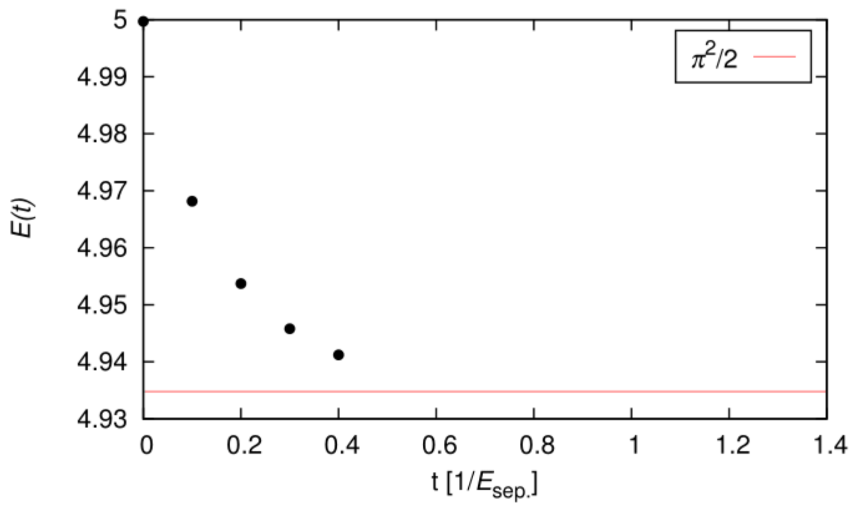
Quantum Monte Carlo method

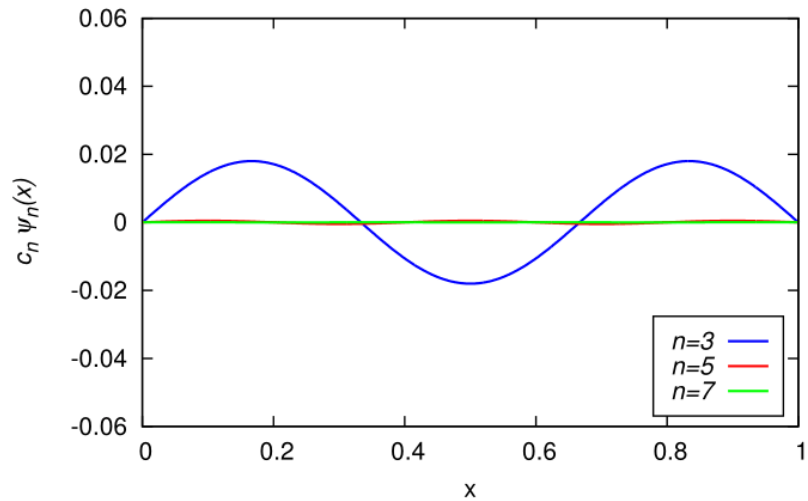
Particle in a 1D box, solution:

$$\psi_n(x) = \sqrt{2}\sin(n\pi x), \quad E_n = \frac{n^2\pi^2}{2}$$

- Make consecutive small timesteps, $\tau = 0.4\left(\frac{1}{E_{\text{sep}}}\right)$

Animation by Joel Lynn, TU Darmstadt





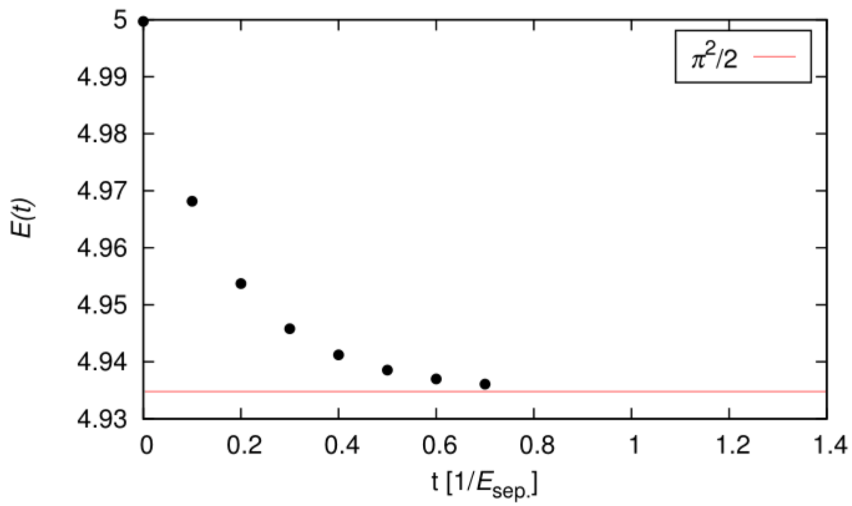
Quantum Monte Carlo method

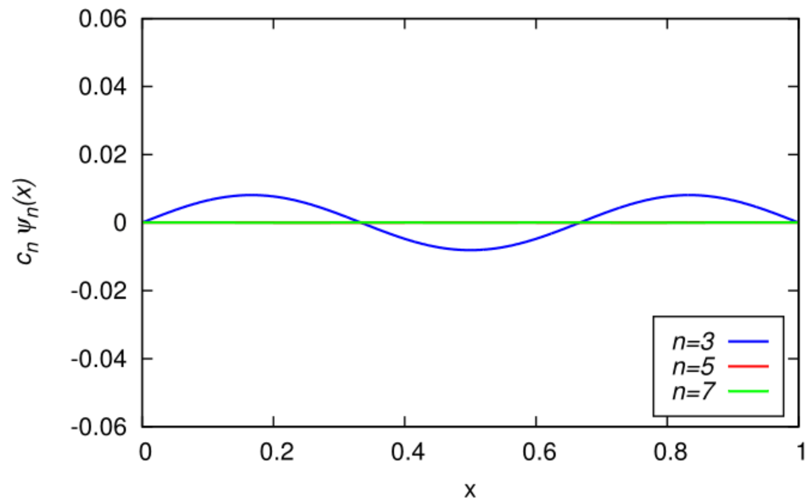
Particle in a 1D box, solution:

$$\psi_n(x) = \sqrt{2}\sin(n\pi x), \quad E_n = \frac{n^2\pi^2}{2}$$

- Make consecutive small timesteps, $\tau = 0.7\left(\frac{1}{E_{\mathrm{sep}}}\right)$

  Animation by Joel Lynn, TU Darmstadt





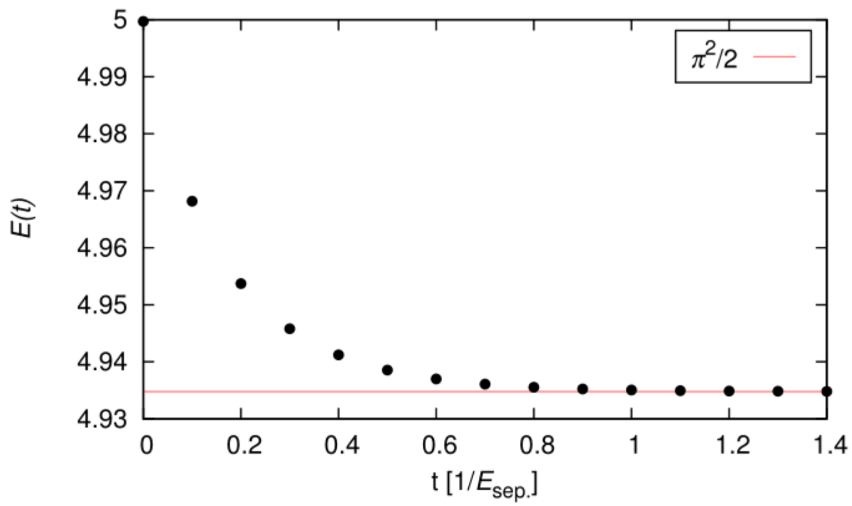
Quantum Monte Carlo method

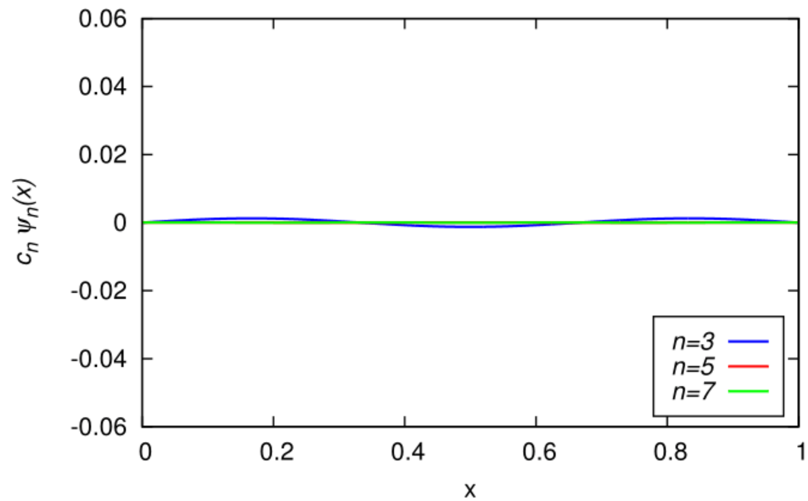
Particle in a 1D box, solution:

$$\psi_n(x) = \sqrt{2}\sin(n\pi x), \quad E_n = \frac{n^2\pi^2}{2}$$

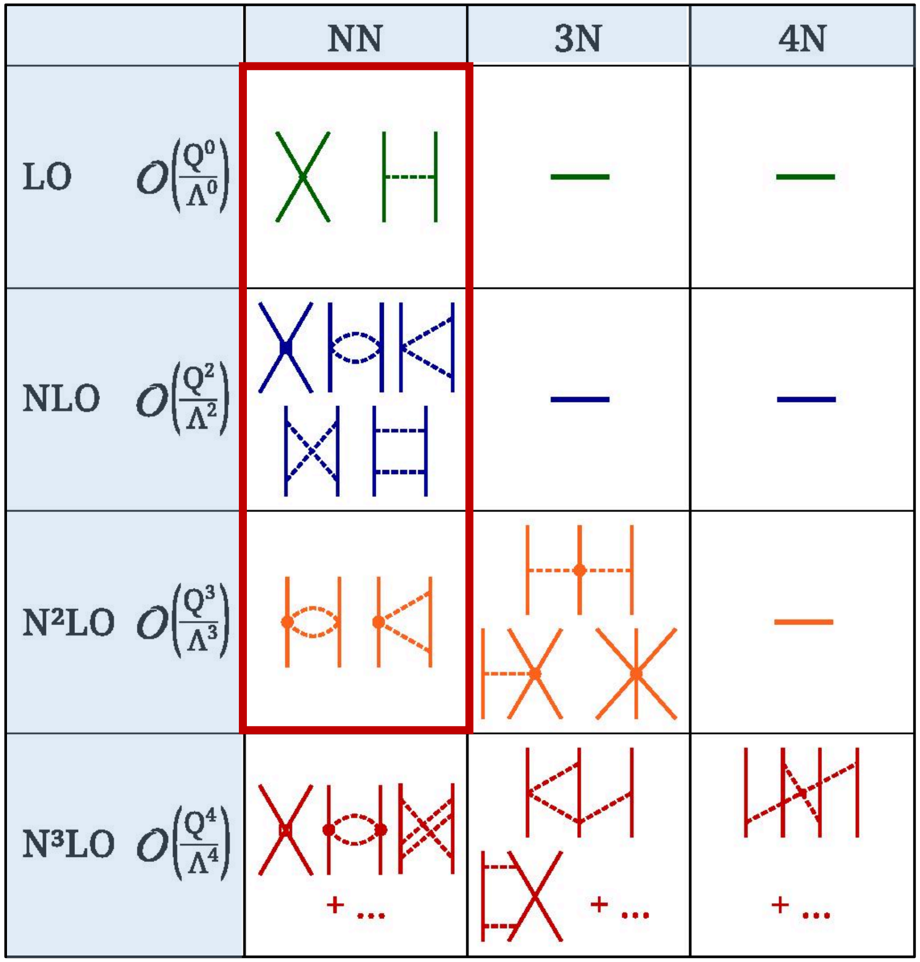
➢ Make consecutive small timesteps, $\tau = 1.4\left(\frac{1}{E_{\mathrm{sep}}}\right)$

Animation by Joel Lynn, TU Darmstadt





Local chiral interactions



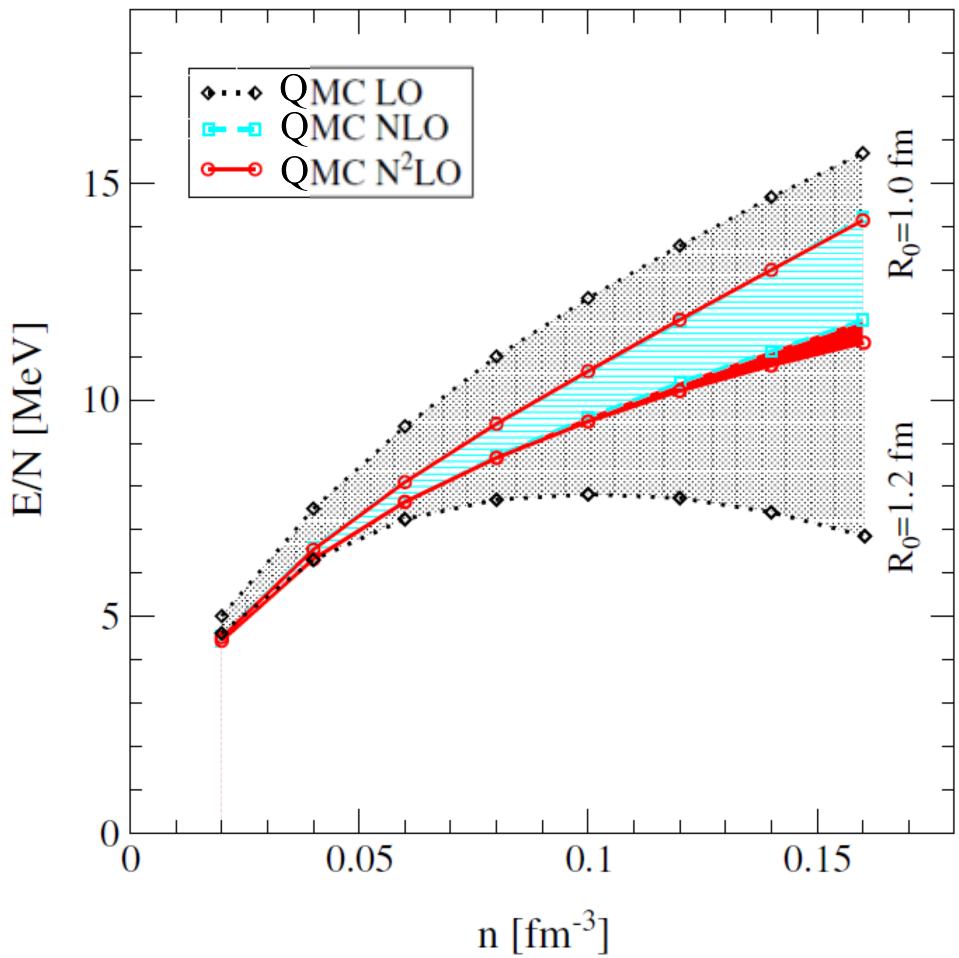
- Choose local set of short-range operators at NLO (7 out of 14)
- Pion exchanges up to N^2LO are local
- This freedom can be used to remove all nonlocal operators up to N^2LO

  Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013)

  Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)
- LECs fit to phase shifts

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

QMC results for NN forces

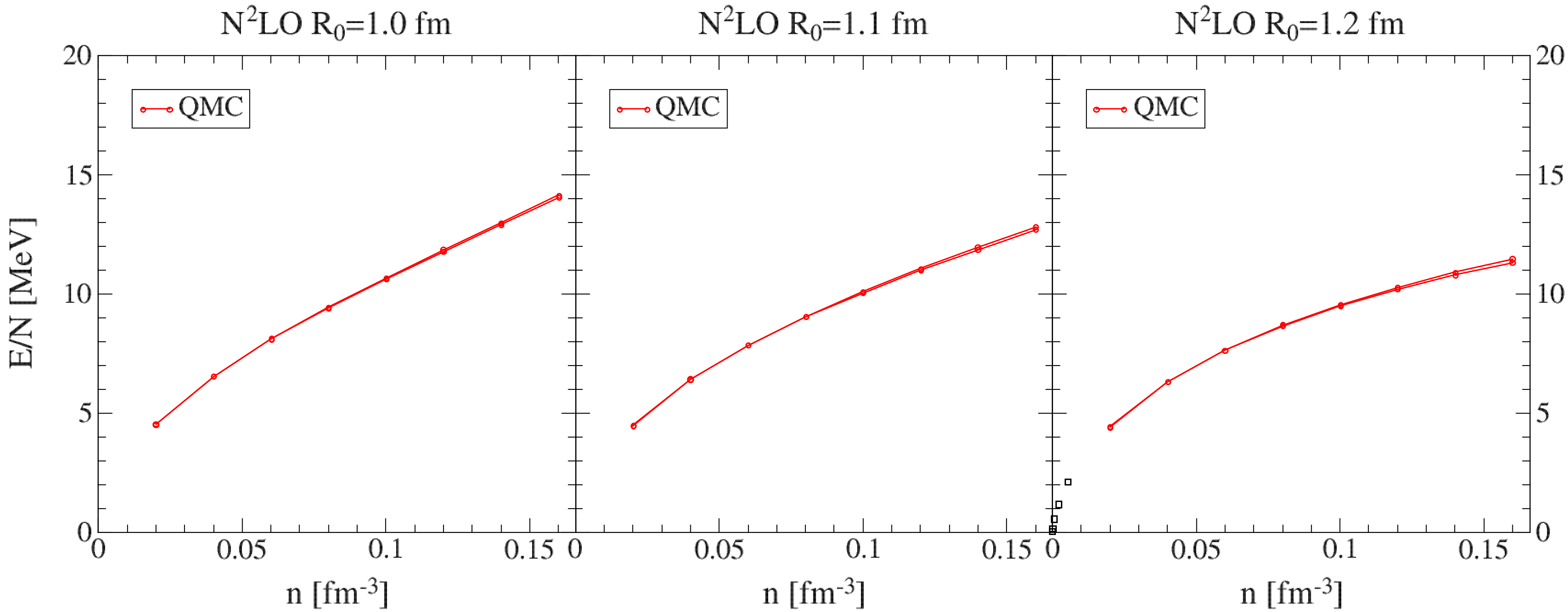


Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013) and PRC (2014)

NN-only calculation:

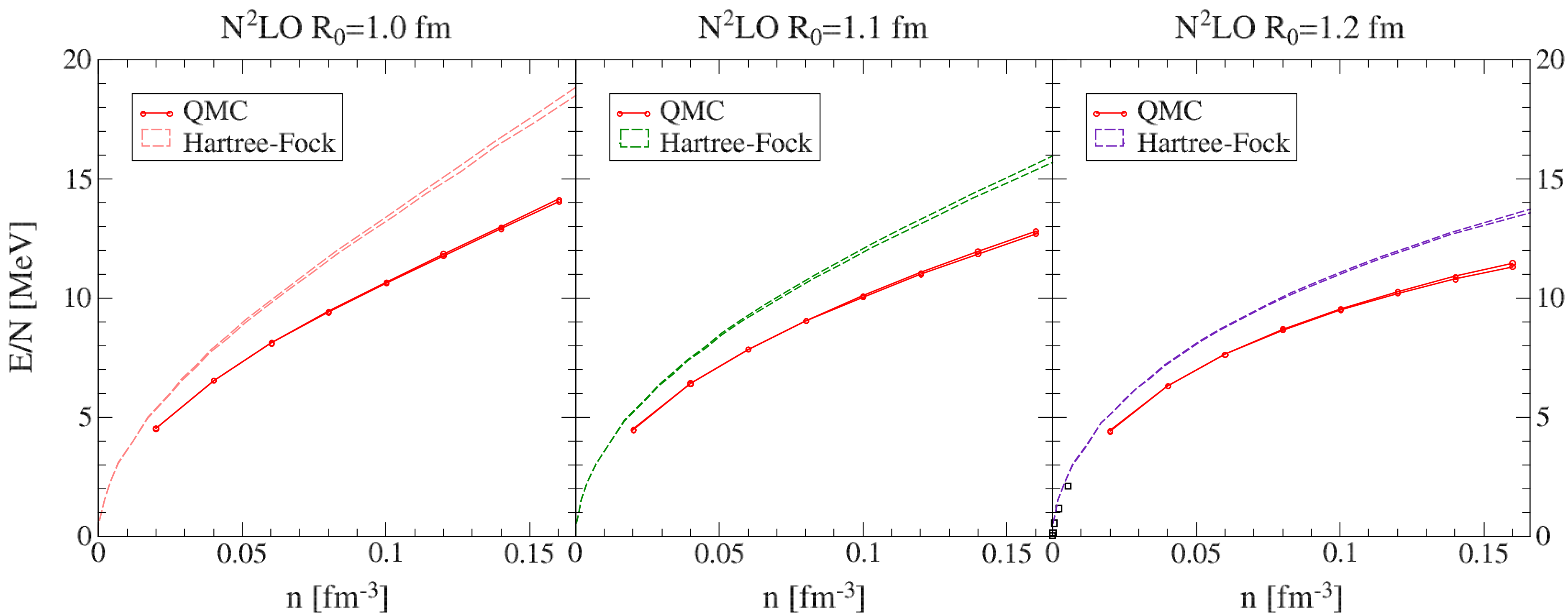
- QMC:
  Statistical uncertainty of points negligible
- Bands include NN cutoff variation
  $R_0 = 1.0 - 1.2$ fm
- Order-by-order convergence up to saturation density

Benchmark of MBPT



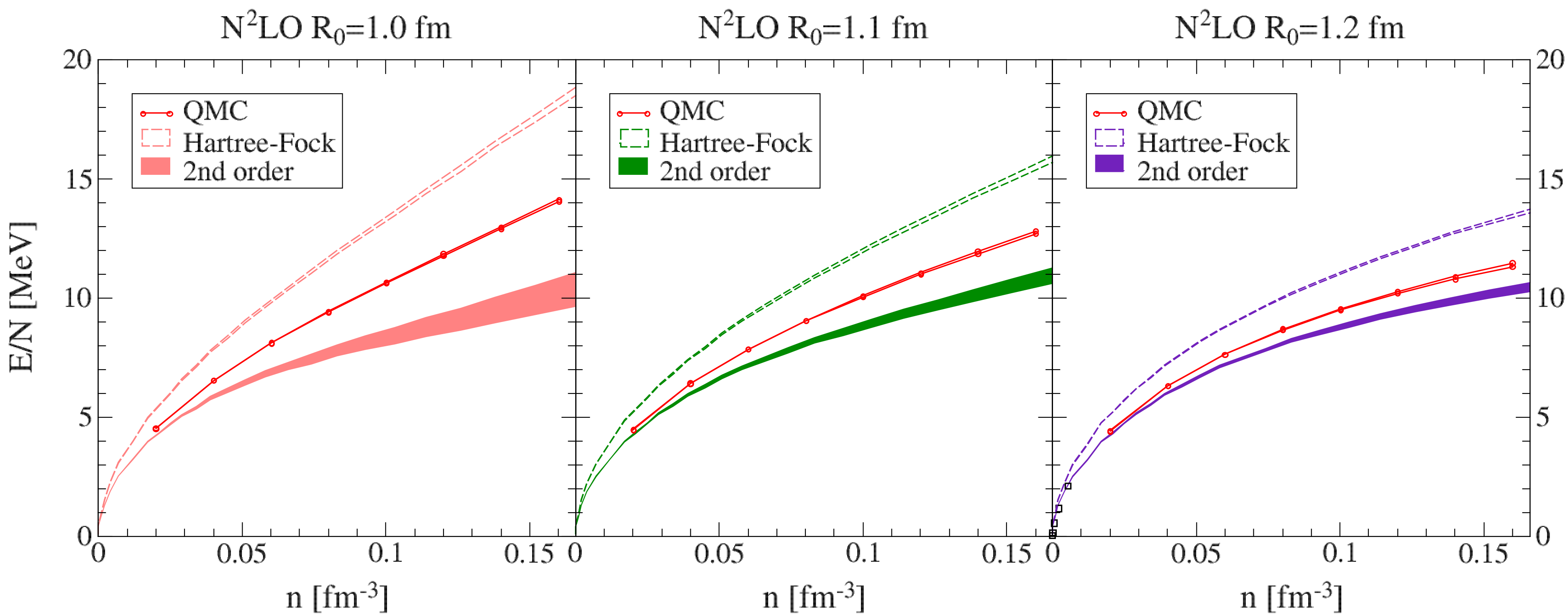
Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

Benchmark of MBPT



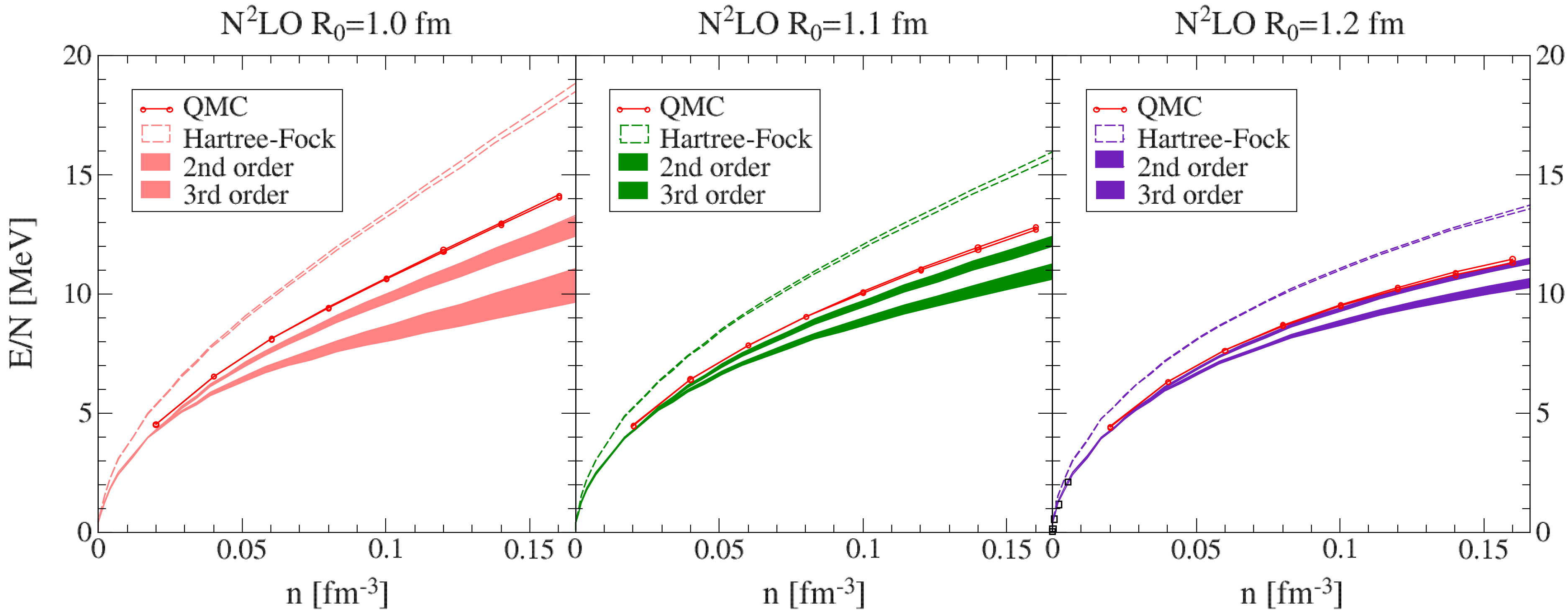
Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

Benchmark of MBPT



Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

Benchmark of MBPT

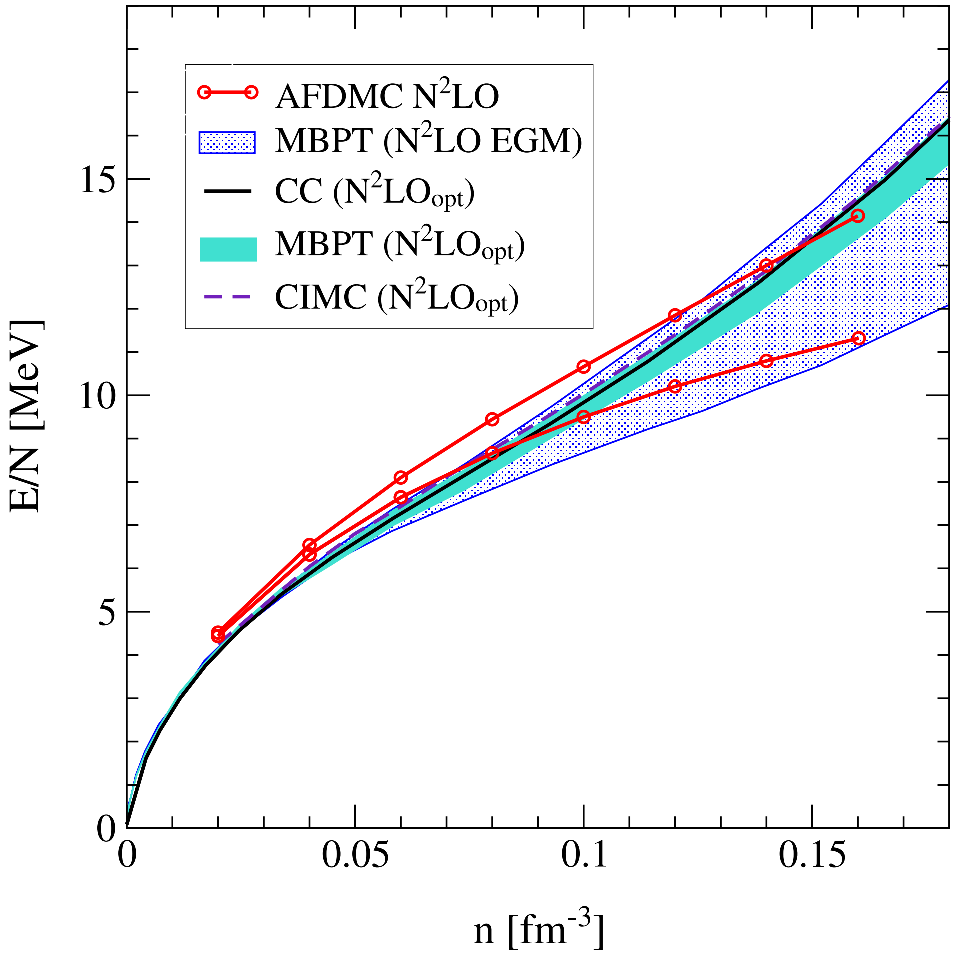


Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

Many-body perturbation theory:

- Excellent agreement with QMC for soft potentials ($R_0 = 1.2$ fm)
- Validates perturbative calculations for those interactions

QMC results for NN forces



Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

NN-only calculation

- Good agreement with other approaches:

MBPT with N^2LO EGM

IT, Krüger, Hebeler, Schwenk, PRL (2013)

CC with N^2LO_{opt}

Hagen, Papenbrock, Ekström, Wendt, Baardsen, Gandolfi, Hjorth-Jensen, Horowitz, PRC (2013)

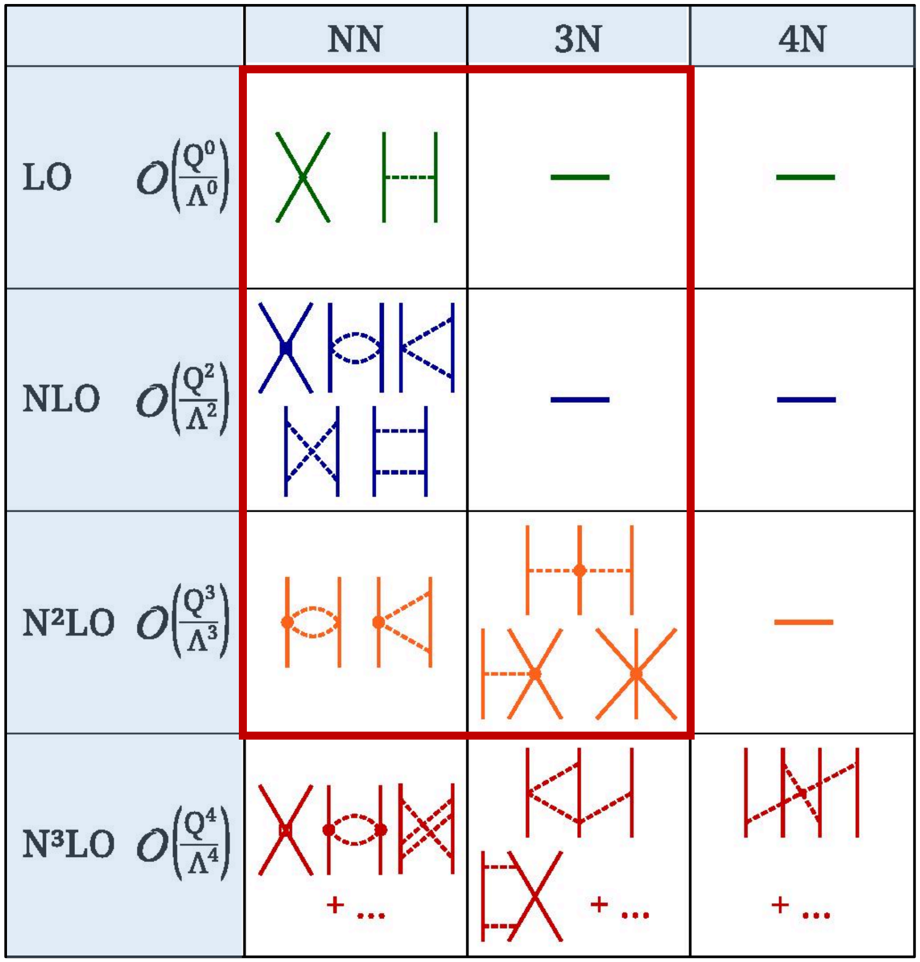
MBPT with N^2LO_{opt}

IT, Krüger, Gezerlis, Hebeler, Schwenk, NTSE (2013)

CIMC with N^2LO_{opt}

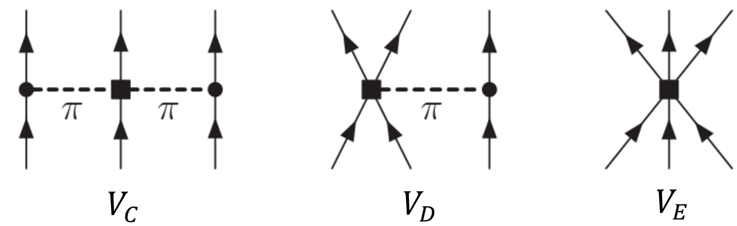
Roggero, Mukherjee, Pederiva, PRL (2014)

QMC with chiral 3N forces



Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Next: inclusion of leading 3N forces



Three topologies:

- Two-pion exchange V_C
- One-pion-exchange contact V_D
- Three-nucleon contact V_E

Only two new couplings: c_D and c_E

Two-pion-exchange most important in PNM:
usually V_D and V_E vanish in neutron matter
(only for regulator symmetric in particle labels)

Fits of 3N LECs

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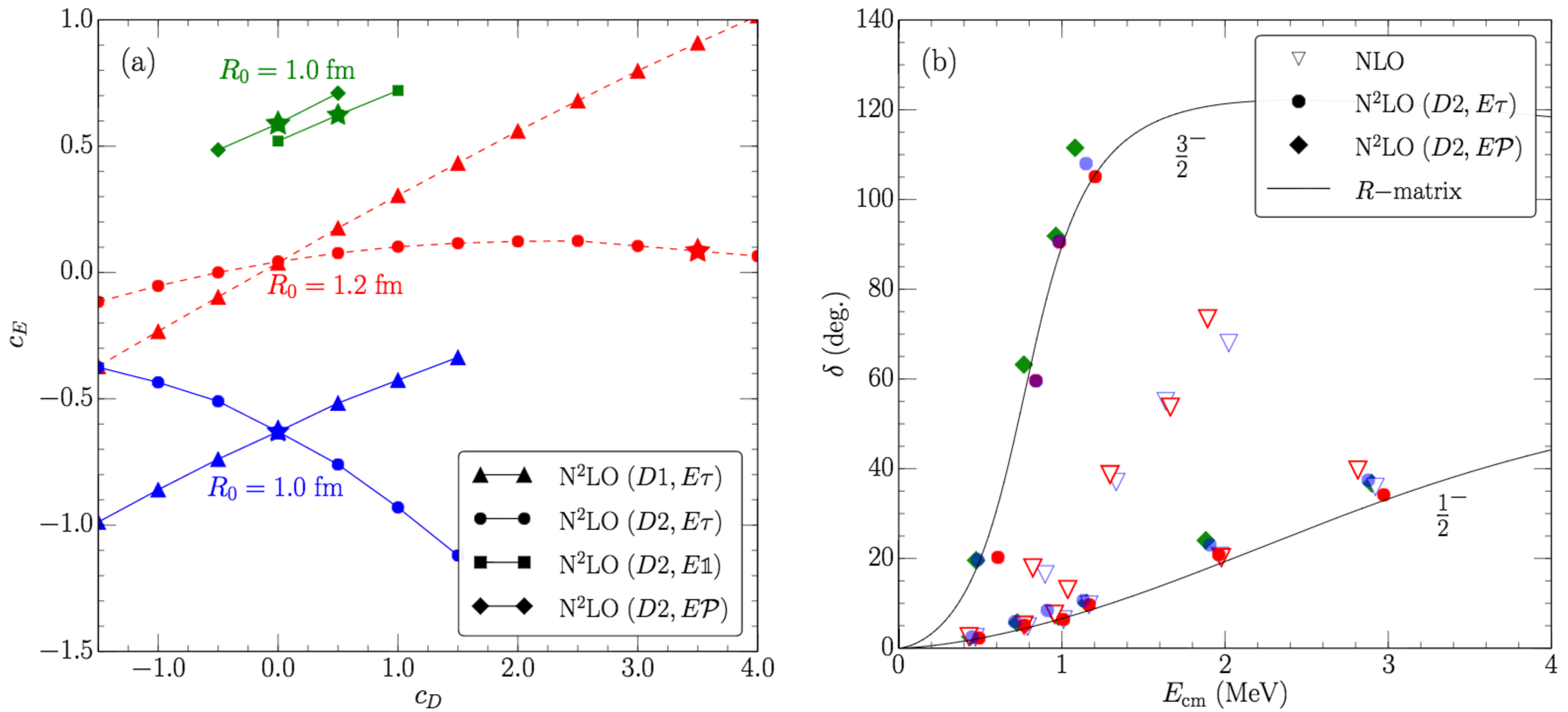
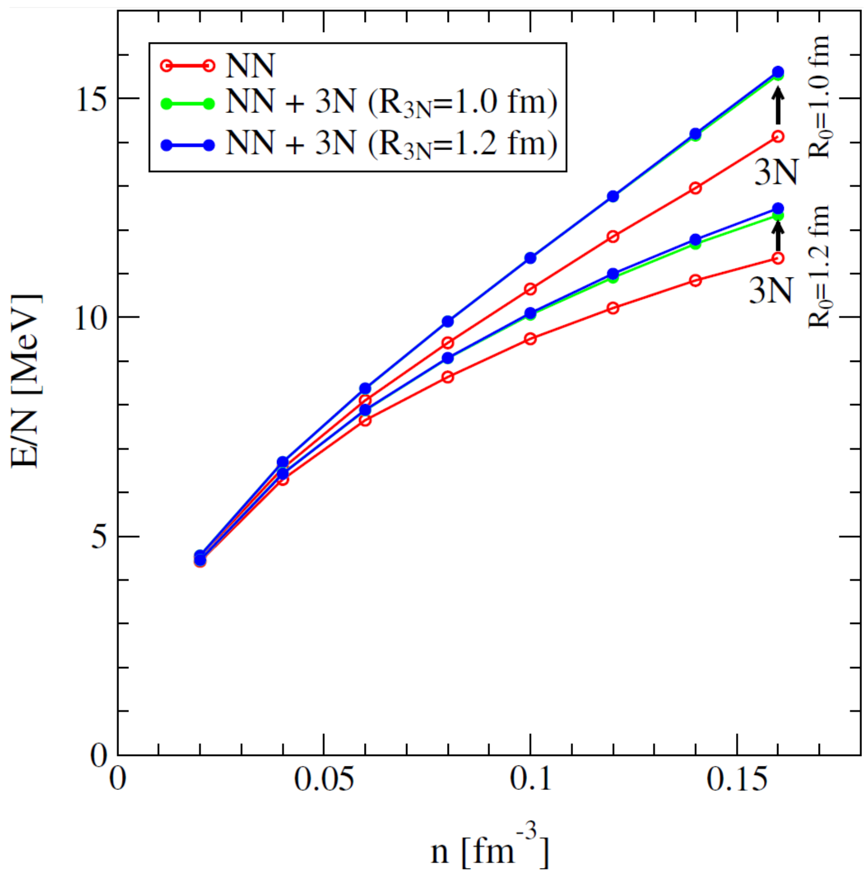
- Fit c_E and c_D to ^{4}He binding energy and n-α scattering



Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

QMC results with 3N TPE



IT, Gandolfi, Gezerlis, Schwenk, PRC (2016)

- Only three-nucleon two-pion exchange $\sim c_1$ and c_3
- Auxiliary-field diffusion Monte Carlo:
  - NN + 3N forces
  - $R_0 = 1.0 - 1.2$ fm
  - $R_{3N} = 1.0 - 1.2$ fm
- TPE 3N contributions ≈ 1 – 2 MeV
- 3N cutoff dependence small
- Variation with $c_1 = -(0.37 - 0.81)$ and $c_3 = -(2.71 - 3.40)$ smaller 0.3 MeV

Results

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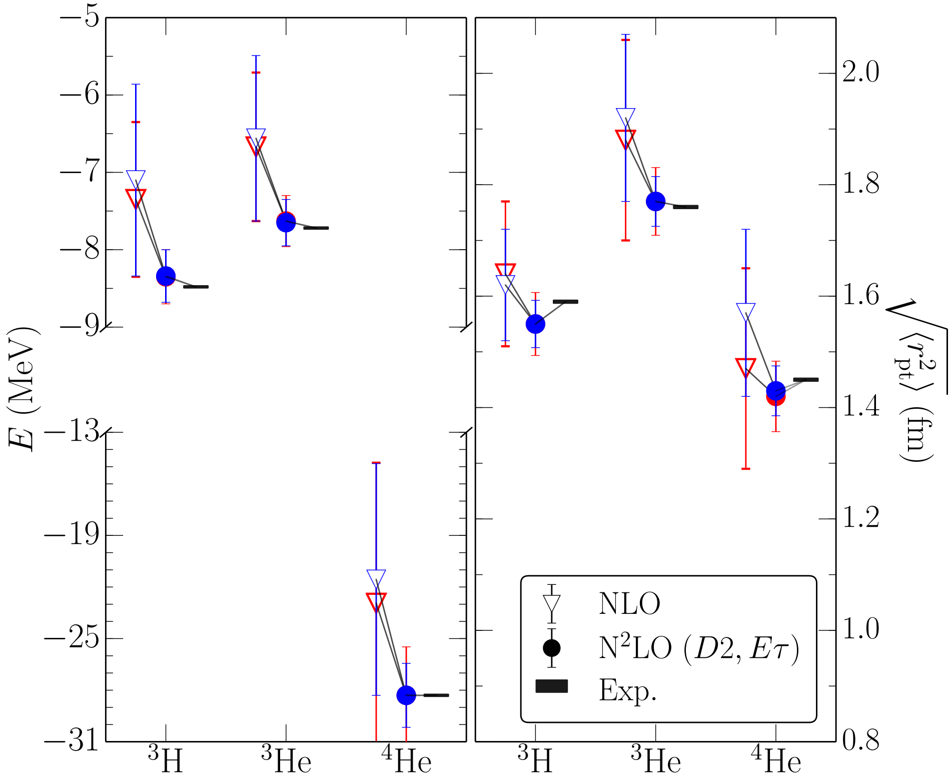

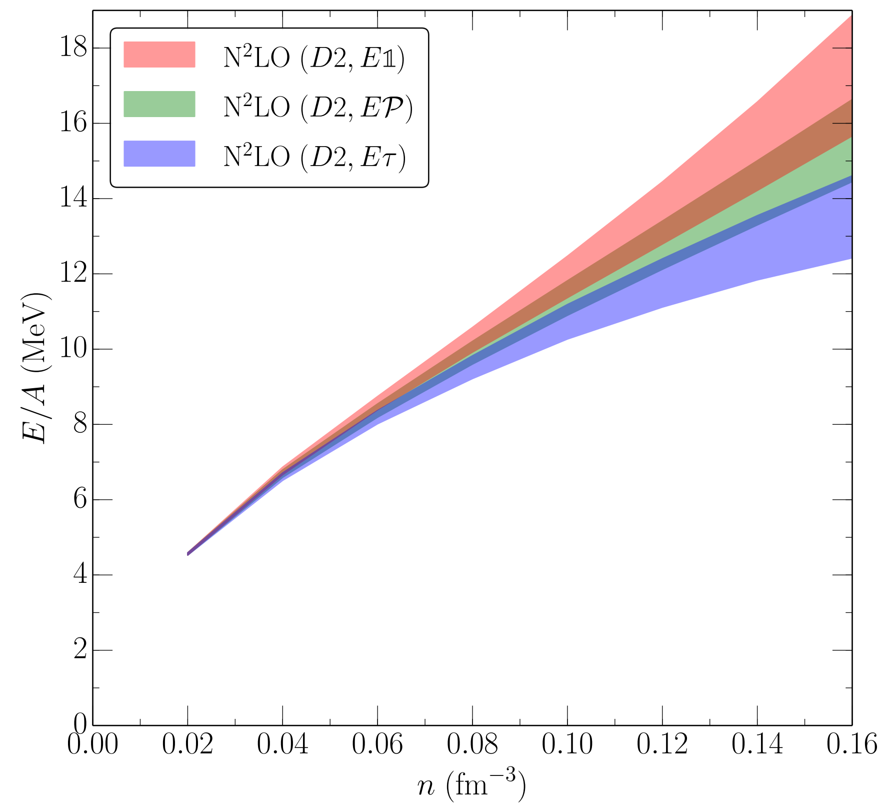

Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

- Chiral interactions at N^{2}LO simultaneously reproduce the properties of A=3, 4, 5 systems and of neutron matter
- Commonly used phenomenological 3N interactions fail for neutron matter
  Sarsa, Fantoni, Schmidt, Pederiva, PRC (2003)

Results

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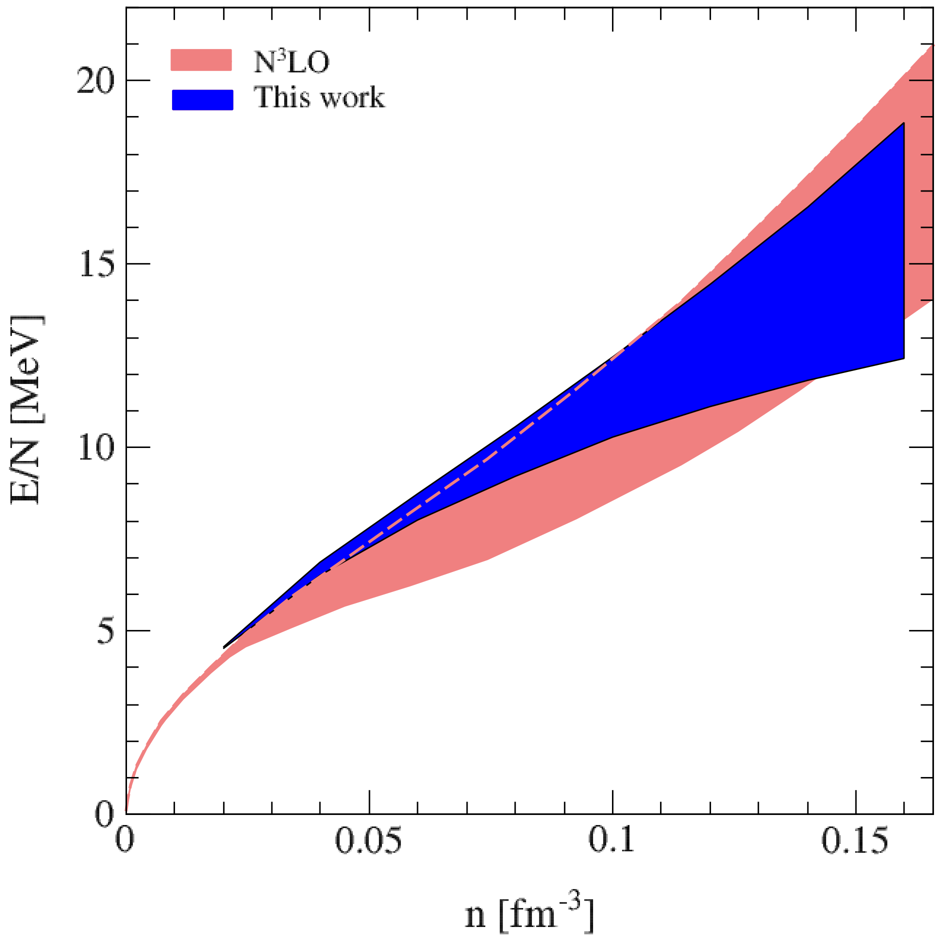
Comparing to N^3LO calculation:



IT, Krüger, Hebeler, Schwenk, PRL (2013)
Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

Chiral EFT forces with the Quantum Monte Carlo method:

- Energies agree well with MBPT result within uncertainty bands
- Many-body uncertainty negligible
- uncertainties comparable but QMC band only at N^2LO

- Improve local chiral interactions:
  - Maximally local N^3LO potentials

Summary

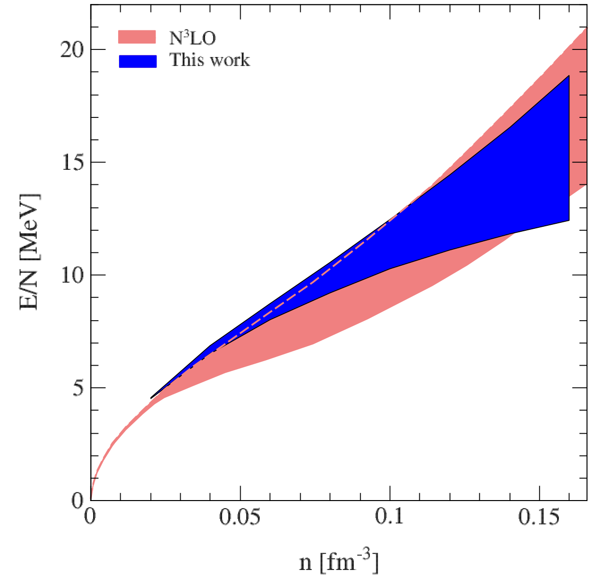
Chiral effective field theory:

- Provides constraints on symmetry energy, neutron star EOS
- Improvement of neutron-matter EOS work in progress
- Using QMC methods with higher order interactions expected to reduce theoretical uncertainties by a factor of two

Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013) & PRC (2014)

IT, Gandolfi, Gezerlis, Schwenk, PRC (2016)

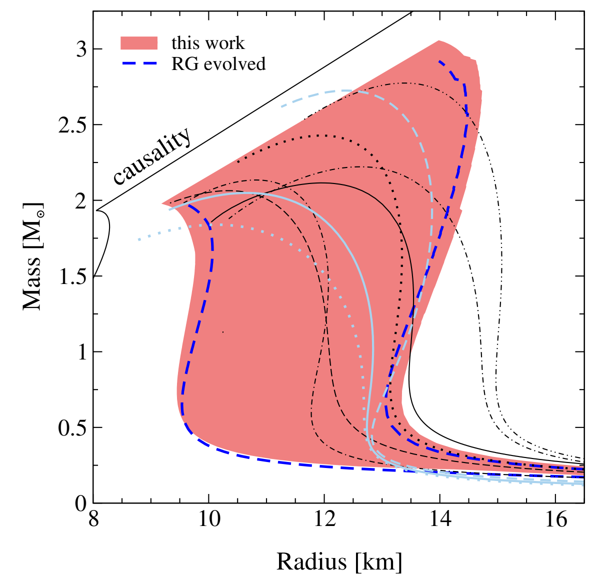
Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)



Constraints on symmetry energy and neutron stars:

- $S_V = 28.9 - 34.9$ MeV
- $L = 43.0 - 66.6$ MeV
- Radius for 1.4 $M_\odot$ neutron star: 9.7 – 13.9 km

IT, Krüger, Hebeler, Schwenk, PRL & PRC (2013)



Thanks

Thanks to my collaborators:

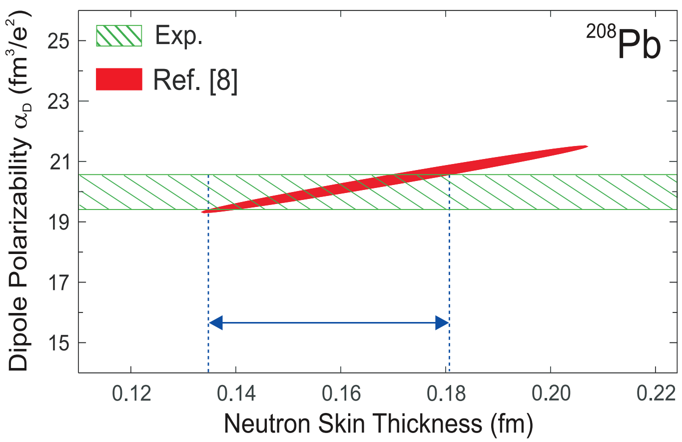
- Technische Universität Darmstadt:
  K. Hebeler, J. Lynn, A. Schwenk
- Universität Bochum: E. Epelbaum
- Los Alamos National Laboratory: J. Carlson, S. Gandolfi
- University of Guelph: A. Gezerlis
- Forschungszentrum Jülich: A. Nogga

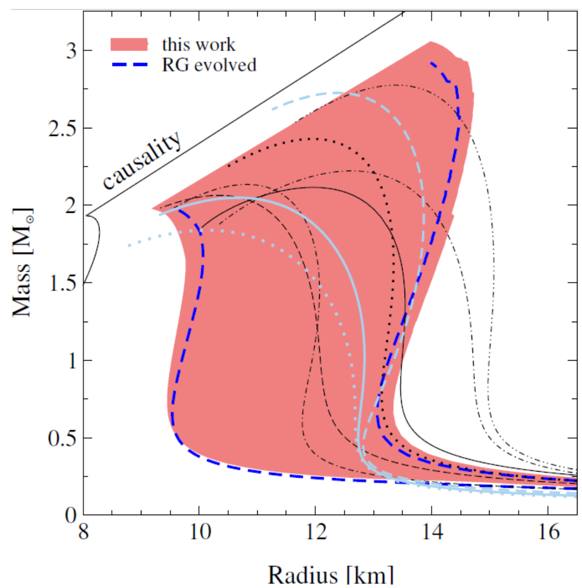
Thanks to FZ Jülich for computing time and
NIC excellence project.

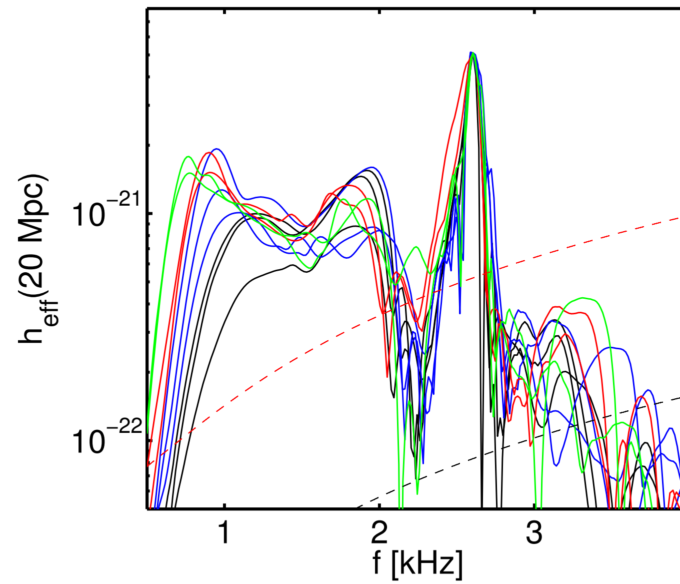
Thank you!

Outlook

- Improve local chiral interactions to nuclei and nuclear matter:
  - Maximally local N^3LO potentials
  - Inclusion of Delta degree of freedom
  - Try different power counting (?)

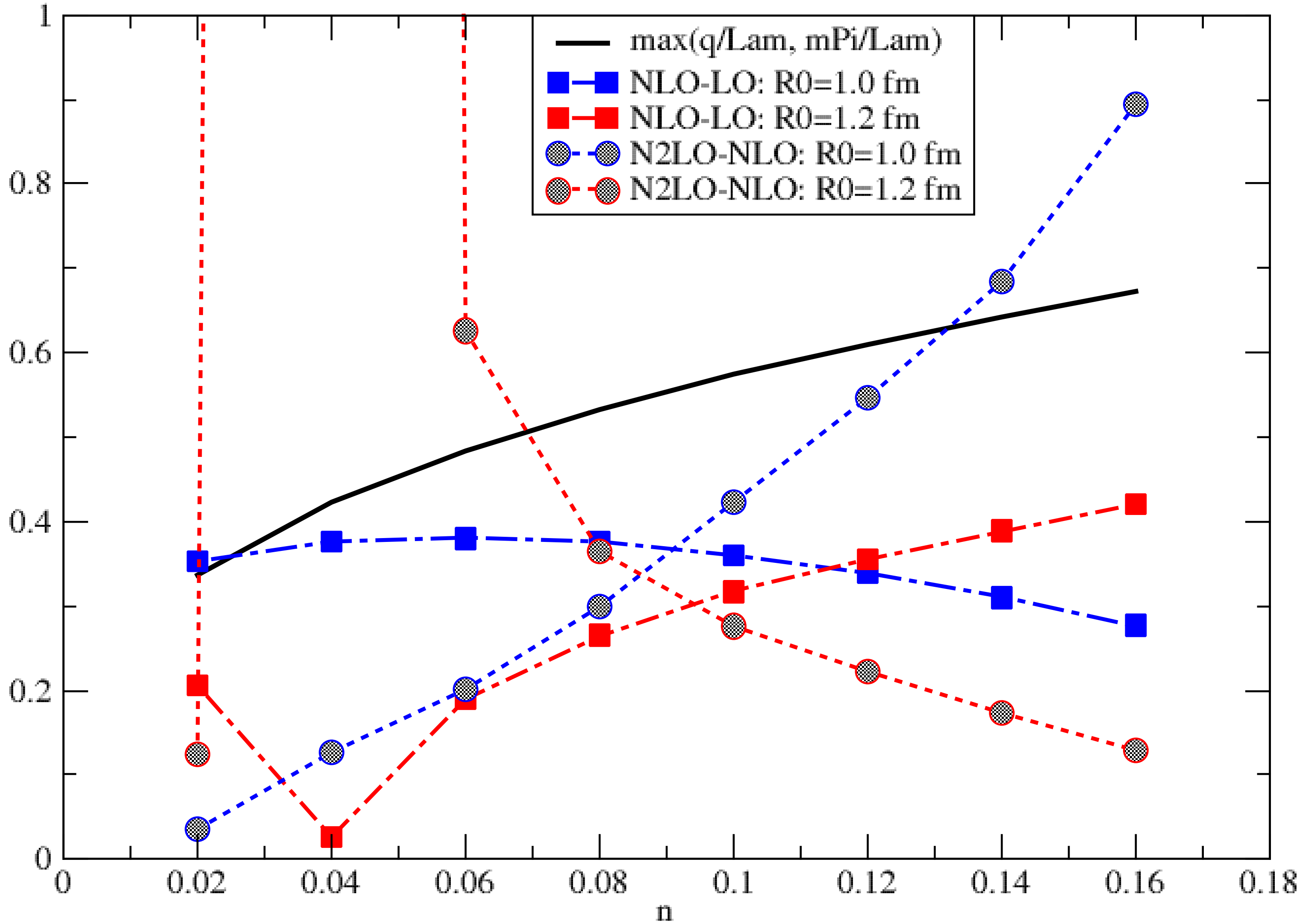






- Study perturbativeness of different chiral orders and power-counting scheme
- Connect nuclear physics to the underlying theory of QCD
- Long-term goal: enable chiral EFT predictions from first principles

BACKUP


max(q/Lam, mPi/Lam)
NLO-LO: R0=1.0 fm
NLO-LO: R0=1.2 fm
N2LO-NLO: R0=1.0 fm
N2LO-NLO: R0=1.2 fm
n
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.4
0.6
0.8
1

Chiral effective field theory for nuclear forces

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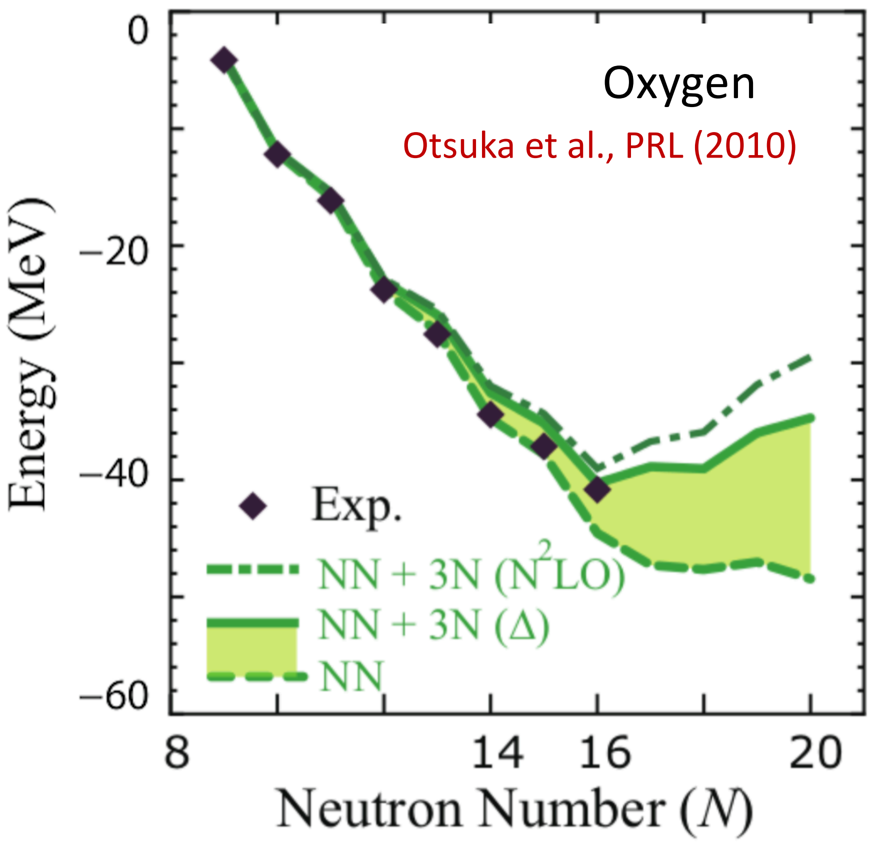
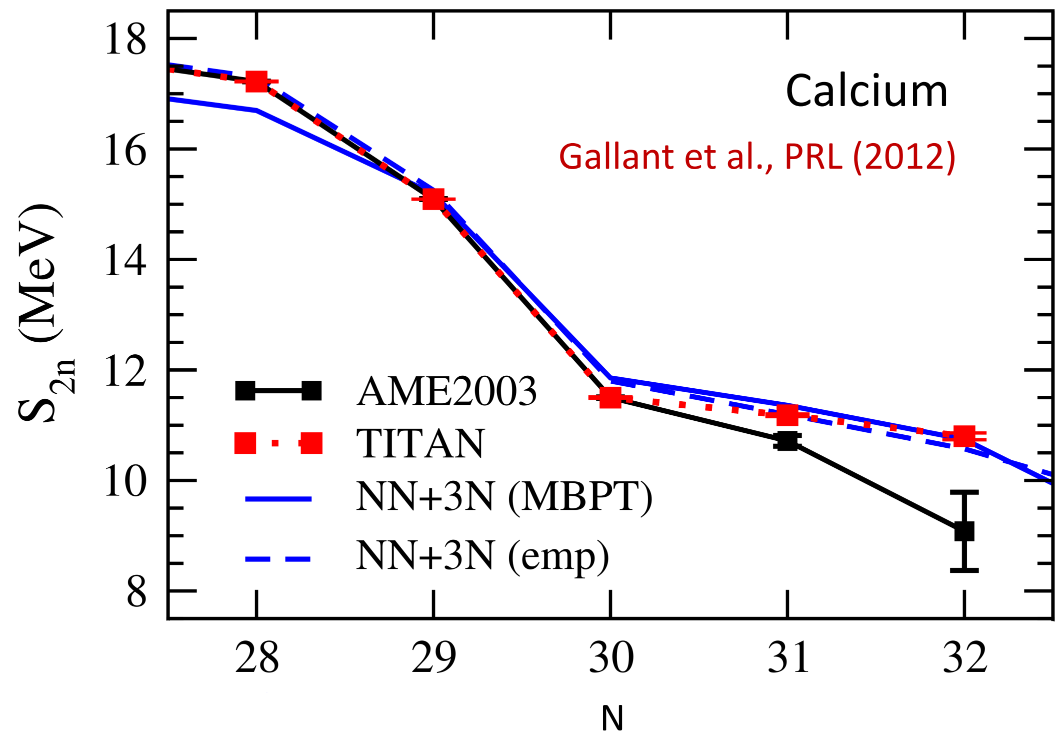
Many-body forces are crucial:



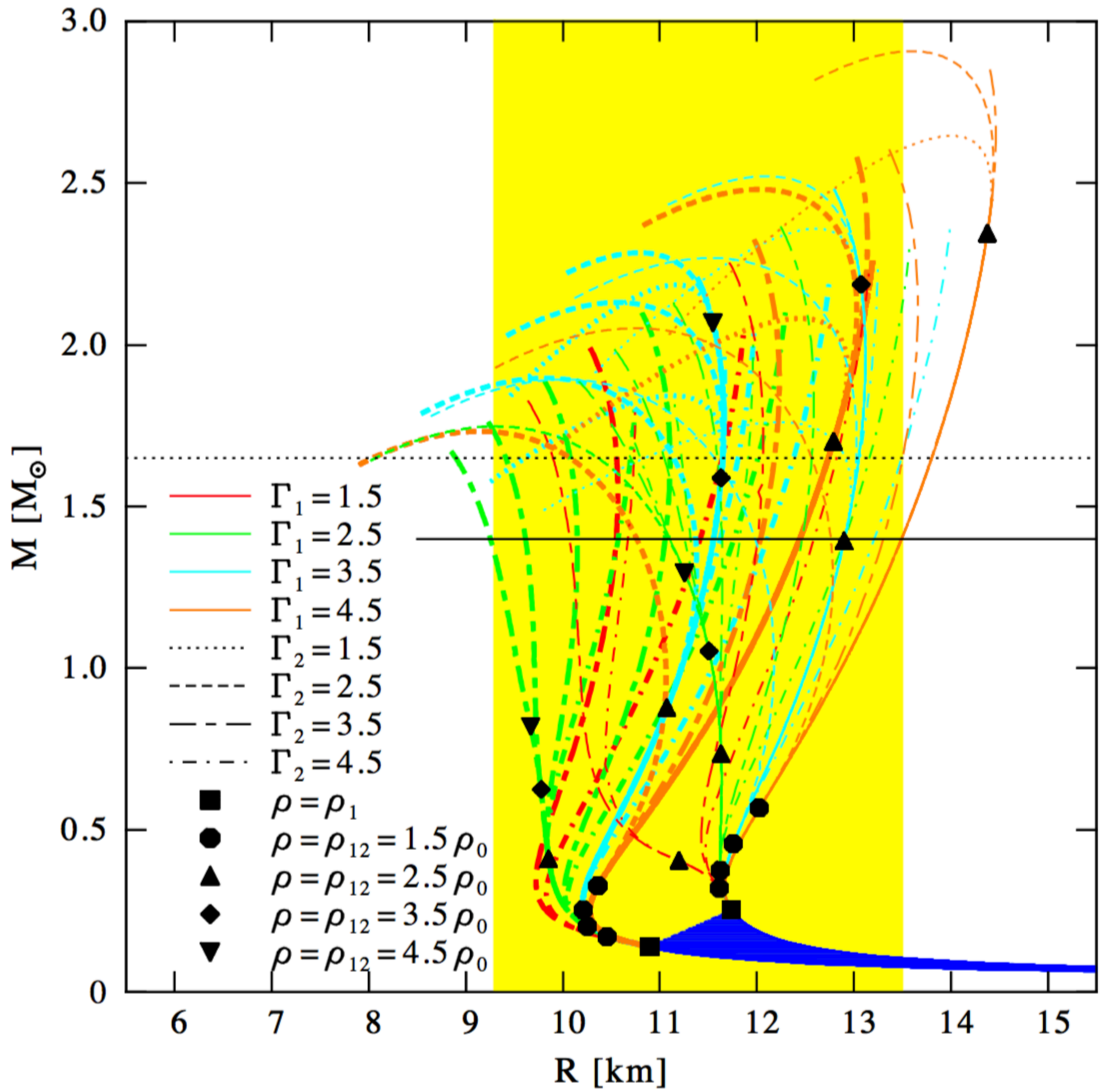


NN + 3N forces:

- ➢ Give correct physics of neutron-rich nuclei

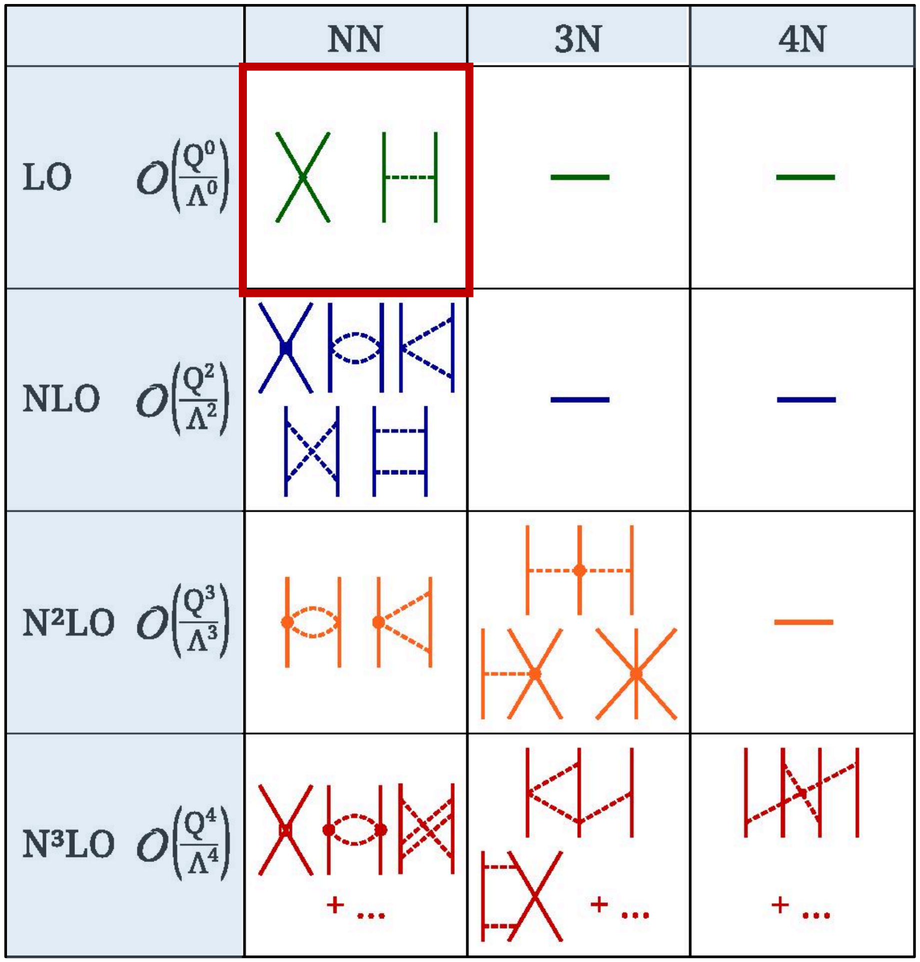
See also Hebeler et al., ARNPS (2015)

Neutron Stars



Hebeler, Lattimer, Pethick, Schwenk, PRL (2010) and APJ (2013)

Local chiral interactions



Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

- Leading order $V^{(0)} = V^{(0)}_{\text{cont}} + V^{\text{OPE}}$
- Pion exchange local → local regulator

$$f_{\text{long}}(r) = 1 - \exp(-r^4/R_0^4)$$

- Contact potential:

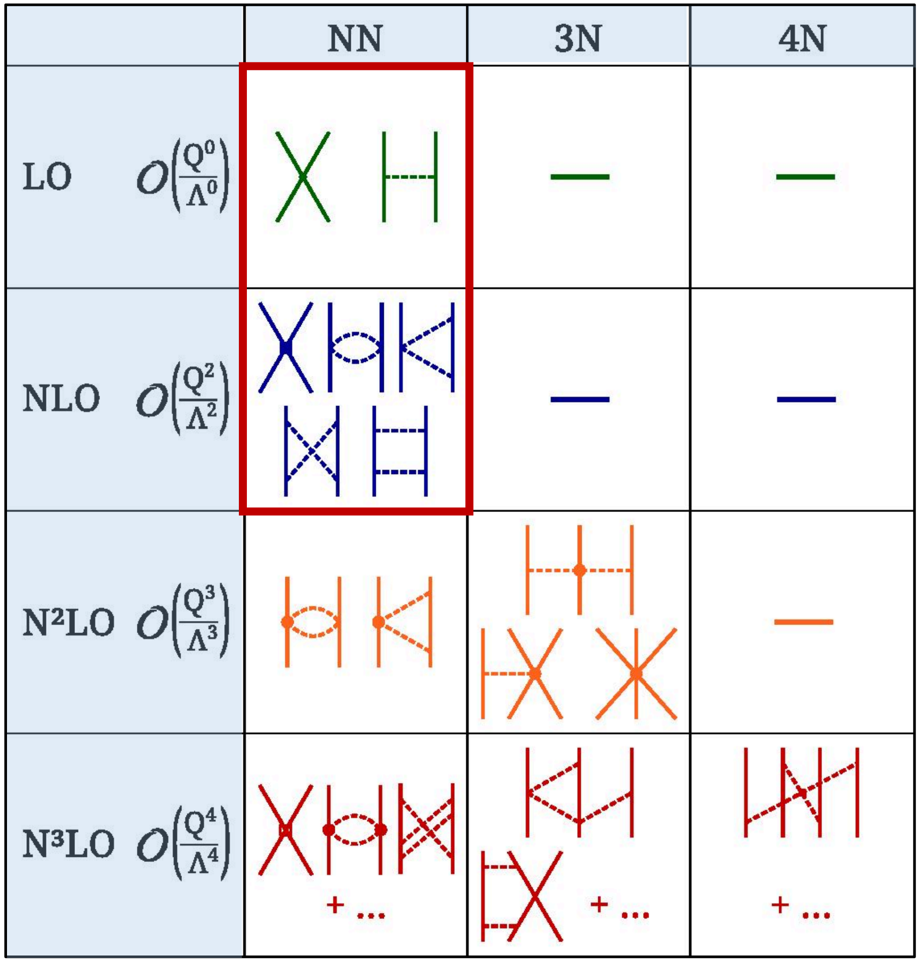
$$V^{(0)}_{\text{cont}} = \alpha_1 \mathbf{1} + \alpha_2 \sigma_1 \cdot \sigma_2 + \alpha_3 \tau_1 \cdot \tau_2 + \alpha_4 \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$$

→ Only two independent (Pauli principle)

$$V^{(0)}_{\text{cont}} = C_S \mathbf{1} + C_T \sigma_1 \cdot \sigma_2$$

$$f_{\text{short}}(r) = \alpha \exp(-r^4/R_0^4)$$

Local chiral interactions

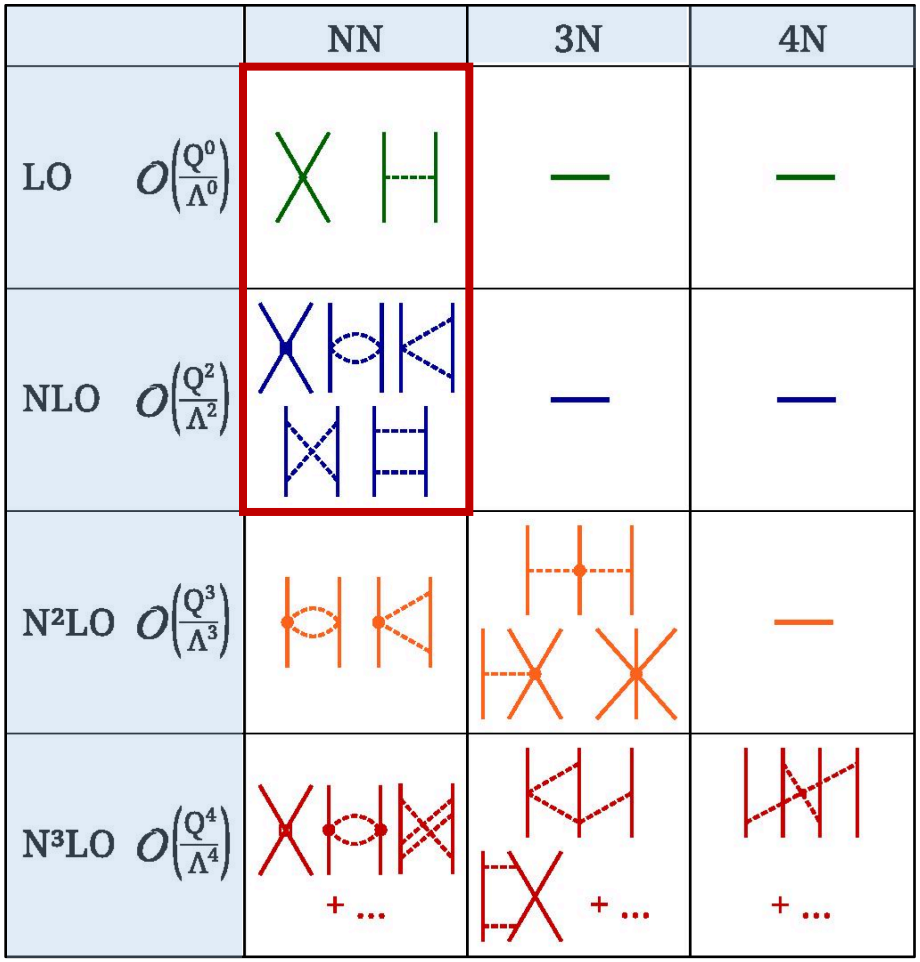


Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

- Choose local set of short-range operators at NLO (7 out of 14)

$$
\begin{aligned}
V_{\rm cont}^{(2)} = {} & \gamma_1\, q^2 + \gamma_2\, q^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \gamma_3\, q^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_4\, q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_5\, k^2 + \gamma_6\, k^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \gamma_7\, k^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_8\, k^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_9\, (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\mathbf{q} \times \mathbf{k}) \\
& + \gamma_{10}\, (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\mathbf{q} \times \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_{11} (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \\
& + \gamma_{12} (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_{13} (\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\
& + \gamma_{14} (\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \,.
\end{aligned}
$$

Local chiral interactions

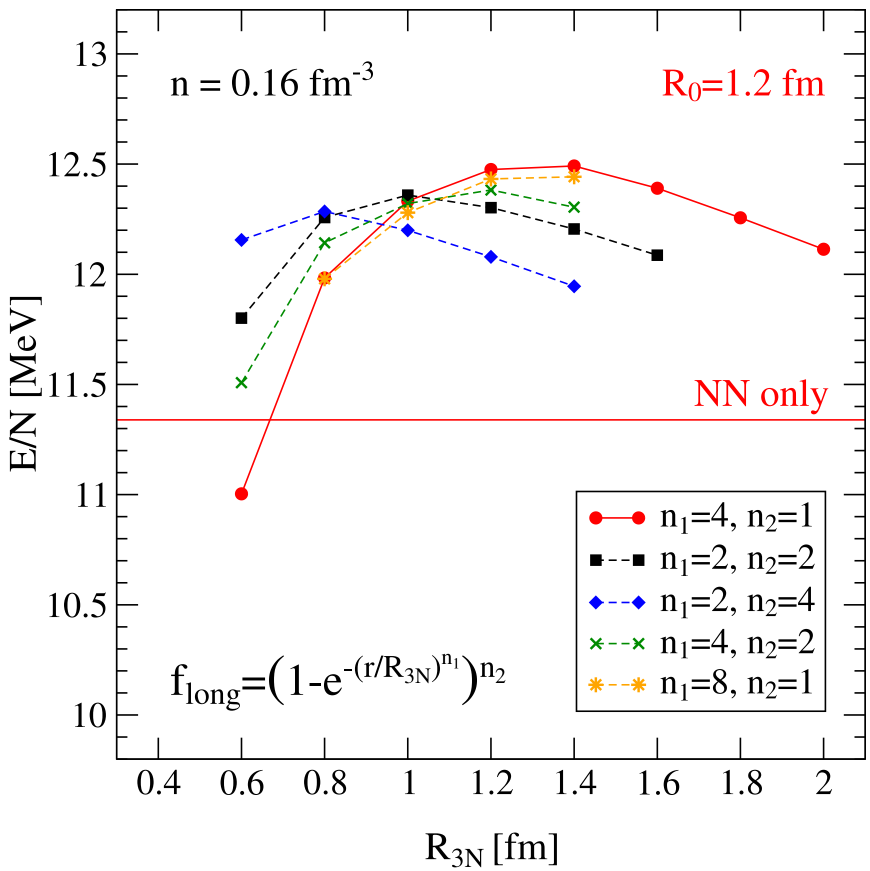


- Choose local set of short-range operators at NLO (7 out of 14)

$$
\begin{aligned}
V_{\text{cont}}^{(2)} = \;& \gamma_1 q^2 + \gamma_2 q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \gamma_3 q^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_4 q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_5 k^2 + \gamma_6 k^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \gamma_7 k^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_8 k^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_9 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\mathbf{q} \times \mathbf{k}) \\
& + \gamma_{10} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\mathbf{q} \times \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_{11} (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \\
& + \gamma_{12} (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
& + \gamma_{13} (\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\
& + \gamma_{14} (\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \,.
\end{aligned}
$$

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

QMC results with 3N TPE



IT, Gandolfi, Gezerlis, Schwenk, PRC (2016)

- Only three-nucleon two-pion exchange $\sim c_1$ and c_3
- Auxiliary-field diffusion Monte Carlo:
  - NN + 3N forces
  - $R_0 = 1.0 - 1.2$ fm
  - $R_{3N} = 1.0 - 1.2$ fm
- TPE 3N contributions ≈ 1 – 2 MeV, smaller than for nonlocal regulators
- 3N cutoff dependence small
- Variation with $c_1 = -(0.37 - 0.81)$ and $c_3 = -(2.71 - 3.40)$ smaller 0.3 MeV
- Independent of exact regulator form