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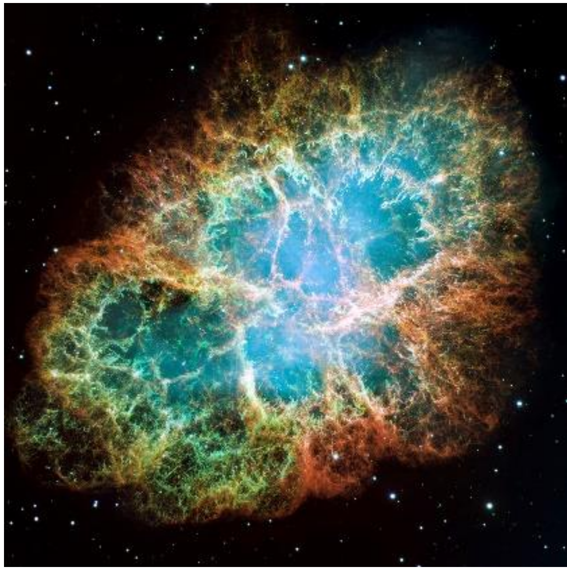
Excitations in relativistic superfluids

M.G. Alford, A. Schmitt, S.K. Mallavarapu, A. Haber

[A. Haber, A. Schmitt, S. Stetina, PRD93, 025011 (2016)]
[S. Stetina, arXiv: 1502.00122 hep-ph]
[M.G. Alford, S. K. Mallavarapu, A. Schmitt, S. Stetina, PRD89, 085005 (2014)]
[M.G. Alford, S. K. Mallavarapu, A. Schmitt, S. Stetina, PRD87, 065001 (2013)]

Superfluidity in dense matter

Microscopic mechanism: Spontaneous Symmetry Breaking (SSB)



- Quark matter at asymptotically high densities:

  → colour superconductors break Baryon conservation $U(1)_B$

  [M. Alford, K. Rajagopal, F. Wilczek, NPB 537, 443 (1999)]

- Quark matter at intermediate densities:

  → meson condensate breaks conservation of strangeness $U(1)_S$

  [T. Schäfer, P. Bedaque, NPA, 697 (2002)]

- nuclear matter:

  → SSB of $U(1)_B$ (exact symmetry at any density)

Goal: translation between field theory and hydrodynamics

SSB in U(1) x U(1) invariant model at T=0 → 2 coupled superfluids (see talk by A. Schmitt)

SSB in U(1) invariant model at finite T → superfluid coupled to normal fluid

Superfluidity from Quantum Field Theory

start from simple microscopic complex scalar field theory:

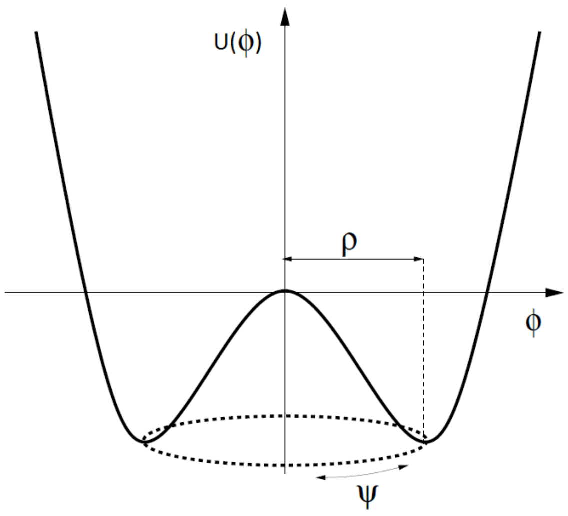
$$\mathcal{L} = \partial_\mu \varphi \partial^\mu \varphi^* - m^2 |\varphi|^2 - \lambda |\varphi|^4$$

- separate condensate\fluctuations:

$$\varphi \to \varphi + \phi \qquad \phi = \rho\, e^{i\psi}$$

→ superfluid related to condensate
[L. Tisza, Nature 141, 913 (1938)]

→ normal-fluid related to quasiparticles
[L. Landau, Phys. Rev. 60, 356 (1941)]



- static ansatz for condensate: (infinite uniform superflow) $\rho\,, \partial_\mu \psi = \mathrm{const.}$

- Fluctuations $\delta\rho(\boldsymbol{x}, t)$ and $\delta\psi(\boldsymbol{x}, t)$ around the static solution determined by classical EOM, can be thermally populated

$$\Box \rho = \rho\left(\partial_\mu \psi^2 - m^2 - \lambda \rho^2\right) \qquad \partial_\mu(\rho \partial^\mu \psi) = 0$$

→ Goldstone mode + massive mode

Hydrodynamics vs. Field Theory

Relativistic two fluid formalism (none dissipative)

[B. Carter, M. Khalatnikov, PRD 45, 4536 (1992)]

- Based on conserved currents $\partial_\mu j^\mu = 0$, $\partial_\mu s^\mu = 0$ and their conjugate momenta

stress-energy tensor $T^{\mu\nu} = -g^{\mu\nu}\Psi + j^\mu \partial^\nu \psi + s^\mu \Theta^\nu$

TD relation $\Psi + \Lambda = \partial\psi \cdot j + \Theta \cdot s$

$$j^\mu = \frac{\partial \Psi}{\partial(\partial_\mu \psi)} = \bar{\mathcal{B}}\partial^\mu \psi + \bar{\mathcal{A}}\Theta^\mu$$

$$s^\mu = \frac{\partial \Psi}{\partial \Theta^\mu} = \bar{\mathcal{A}}\partial^\mu \psi + \bar{\mathcal{C}}\Theta^\mu$$

$$\Psi = \Psi[\partial\psi^2, \Theta^2, \partial\psi \cdot \theta]$$

$$\Lambda = \Lambda[j^2, s^2, j \cdot s]$$

connection to field theory at T=0:

$$v_s^\mu = \partial^\mu \psi / \sigma \qquad \sigma^2 = \partial_\mu \psi \partial^\mu \psi = \mu(1 - \boldsymbol{v}_s^2) \qquad \mu = \partial_0 \psi \qquad \boldsymbol{v}_s = -\nabla\psi/\mu$$

→ Hydrodynamic quantities can be calculated from microscopic physics

finite temperature calculation

Microscopic calculation introduces preferred rest frame ("heat bath")

[M.G. Alford, S. K. Mallavarapu, A. Schmitt, S. Stetina, PRD87, 065001 (2013)]

→ calculations in the rest frame of the normal fluid defined by $s^\mu = (s^0, \vec{0})$ (depend on $\boldsymbol{v}_s$)

→ in this frame we can identify $\boldsymbol{\Psi} = \boldsymbol{\Gamma_{eff}}$ and $\boldsymbol{\Theta_0} = \boldsymbol{T}$!

Calculate self consistently for any temperature T<T$_c$

[M.G. Alford, S. K. Mallavarapu, A. Schmitt, S. Stetina, PRD89, 085005 (2014)]

→ 2PI effective action (2-loop Hartree approx.):

$$\Gamma[\rho, S] = -U(\rho) - \frac{1}{2}\frac{T}{V}\sum_k \mathrm{Tr}\ \ln\frac{S^{-1}(k)}{T^2} - \frac{1}{2}\frac{T}{V}\sum_k \mathrm{Tr}\left[S_0^{-1}(k,\rho)S(k) - 1\right] - V_2[\rho, S]$$

→ ρ and S are determined self. cons. by stat. equations:

$$\delta\Gamma[\rho, S]/\delta\rho = 0\,, \quad \delta\Gamma[\rho, S]/\delta S = 0$$

→ find solutions which fulfil the Goldstone theorem

[M. Alford, M. Braby, A. Schmitt J.Phys.G35:025002 (2008)]

classification of excitations

elementary excitations

- **poles of the quasi particle propagator**

collective modes

- **fluctuations in the *density* of elementary excitations**

→ solutions to a given set of (linearized) hydro equations

$$\boldsymbol{\partial_\mu j^\mu = 0}\ , \quad \boldsymbol{\partial_\mu s^\mu = 0} \text{ and } \boldsymbol{\partial_\mu T^{\mu\nu} = 0}$$

→ introduce fluctuations for all thermo – and hydrodynamic quantities

$$x \to x_0 + \delta x(\boldsymbol{x}, t) \qquad x = \{\Psi\ , s\ , n\ , \ \mu,\ \theta\ ,\ \vec{v}_s\ \}$$

→ use TD relation $\delta\psi = j_\mu \delta(\partial^\mu \psi) + s_\mu \delta\theta^\mu$

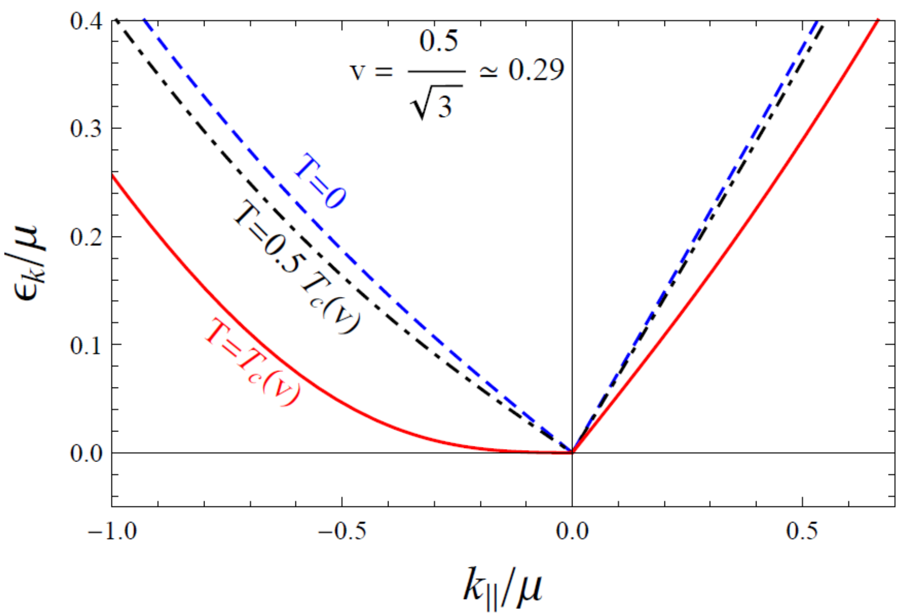
→ solve $\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} \delta T \\ \delta\mu \end{pmatrix} = 0$

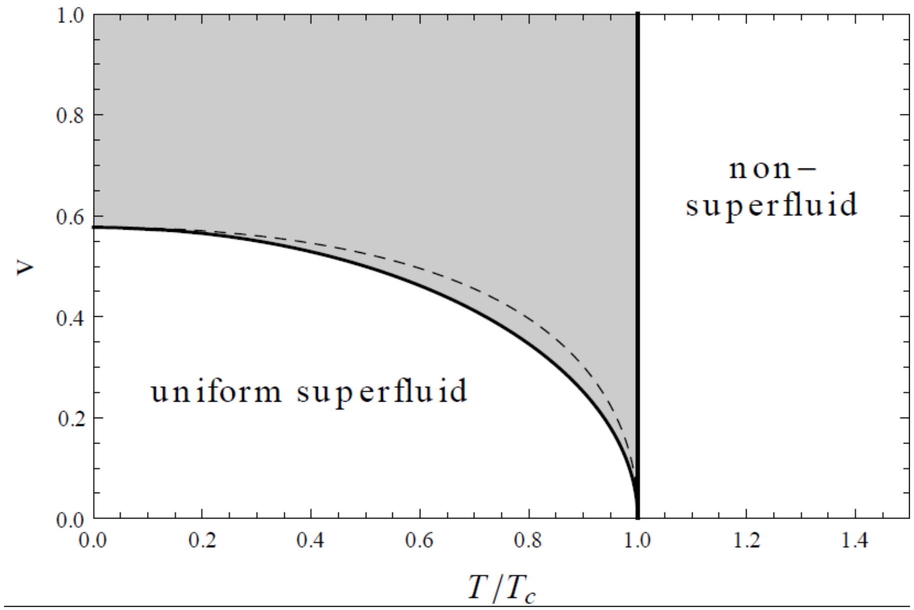
C_{ij} (equilibrium quantities) are second order partial derivatives of Ψ

elementary excitations

→ critical temperature: condensate has "melted" completely

→ critical velocity: negative Goldstone dispersion relation





Generalization of Landau critical velocity

- normal and super frame connected by Lorentz boost
- back reaction of condensate on Goldstone dispersion

sound excitations

- **Scale invariant limit**
  - → pressure can be written as $\Psi = T^4\, h(T/\mu)$
    [C. Herzog, P. Kovtun, and D. Son, Phys.Rev.D79, 066002 (2009)]
  - → second sound still complicated! Compare e.g. to ^{4}He:

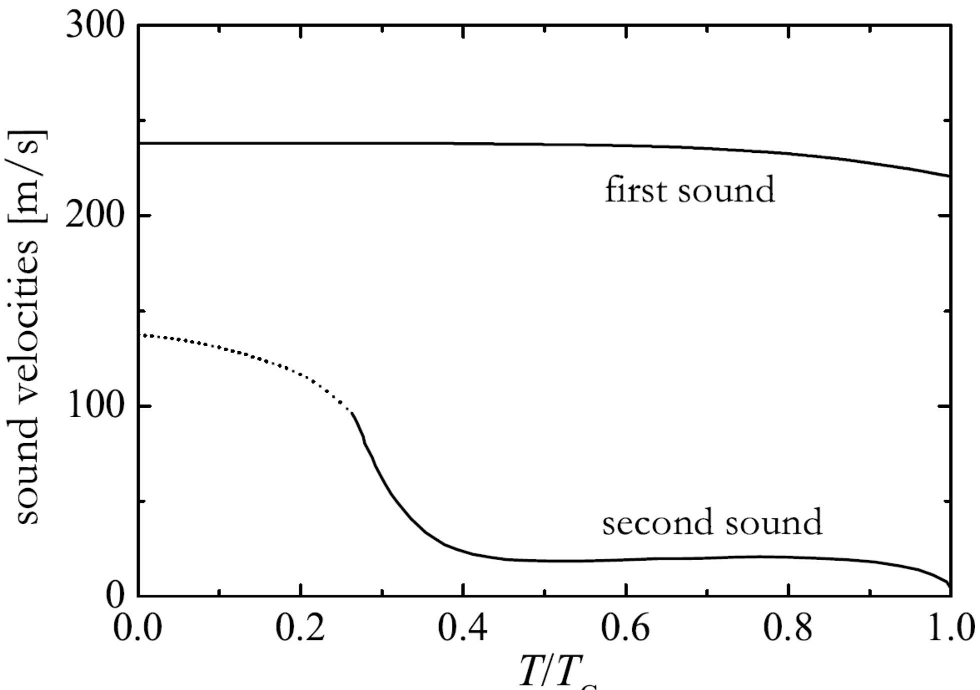
$$u_1^2 = \frac{1}{3}$$

$$u_2^2 = \frac{n_s s^2}{\mu n_n + Ts}\left(n\,\frac{\partial s}{\partial T} - s\,\frac{\partial n}{\partial \mu}\right)^{-1}$$

→ ratios of amplitudes

$$\left.\frac{\delta T}{\delta \mu}\right|_{u_1} = \frac{T}{\mu} \quad \text{(in phase)}$$

$$\left.\frac{\delta T}{\delta \mu}\right|_{u_2} = -\frac{n}{s} \quad \text{(out of phase)}$$

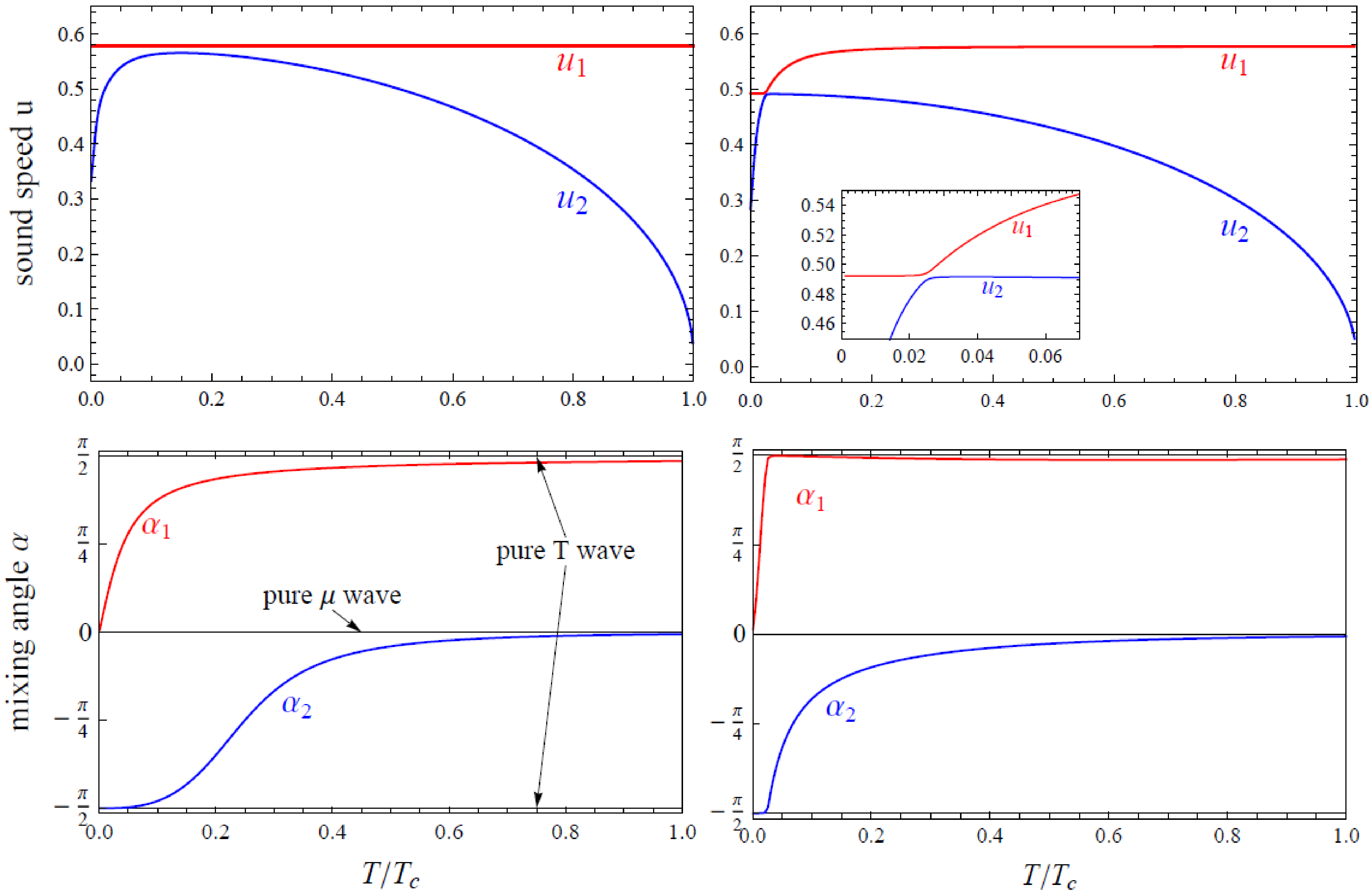


[E. Taylor, H. Hu, X. Liu, L. Pitaevskii, A. Griffin, S. Stringari, Phys. Rev. A 80, 053601 (2009)]

Role reversal, no superflow

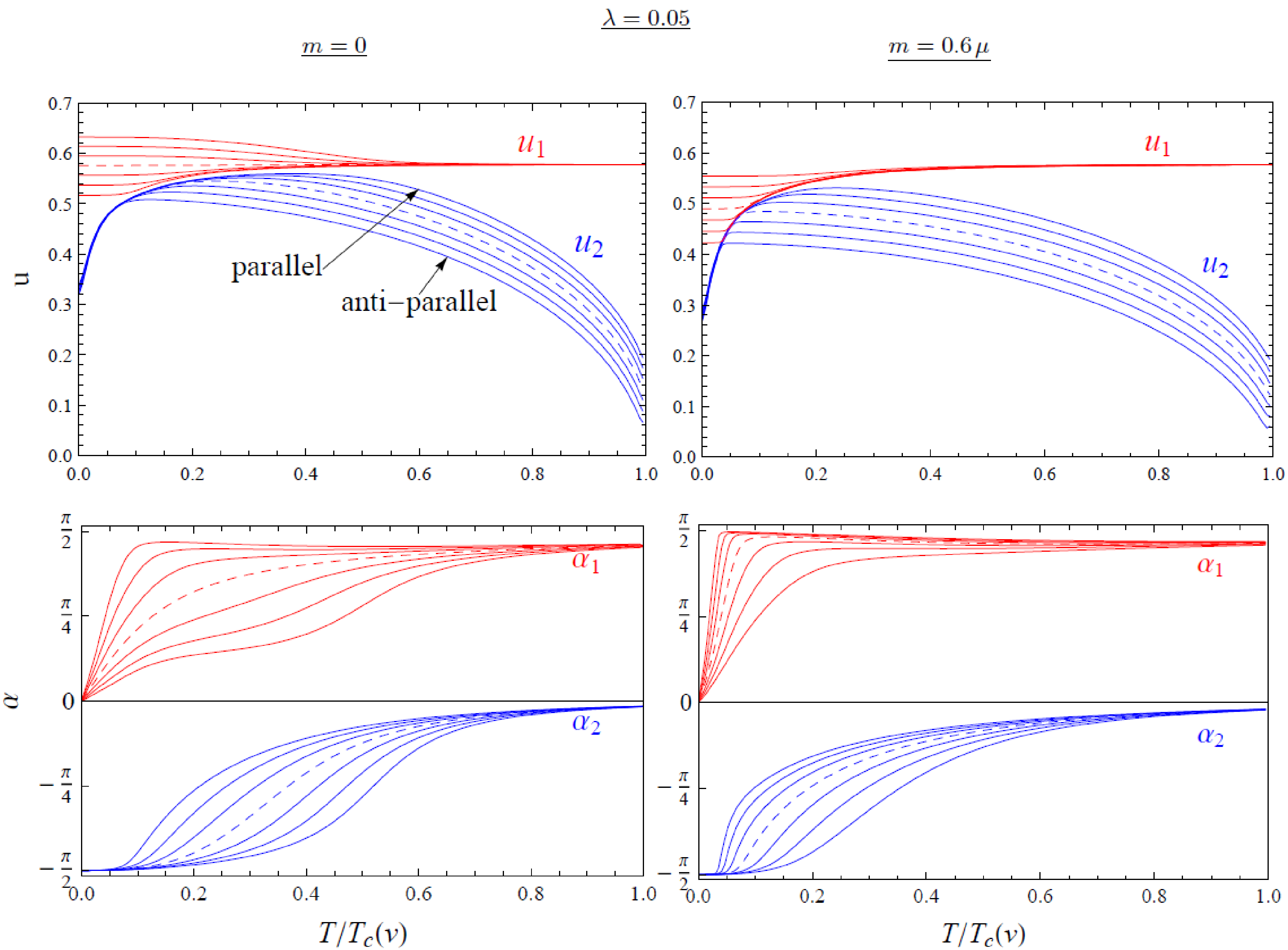
m={0 , 0.6 μ}

$$\alpha := \arctan\frac{\delta T}{\delta\mu}$$



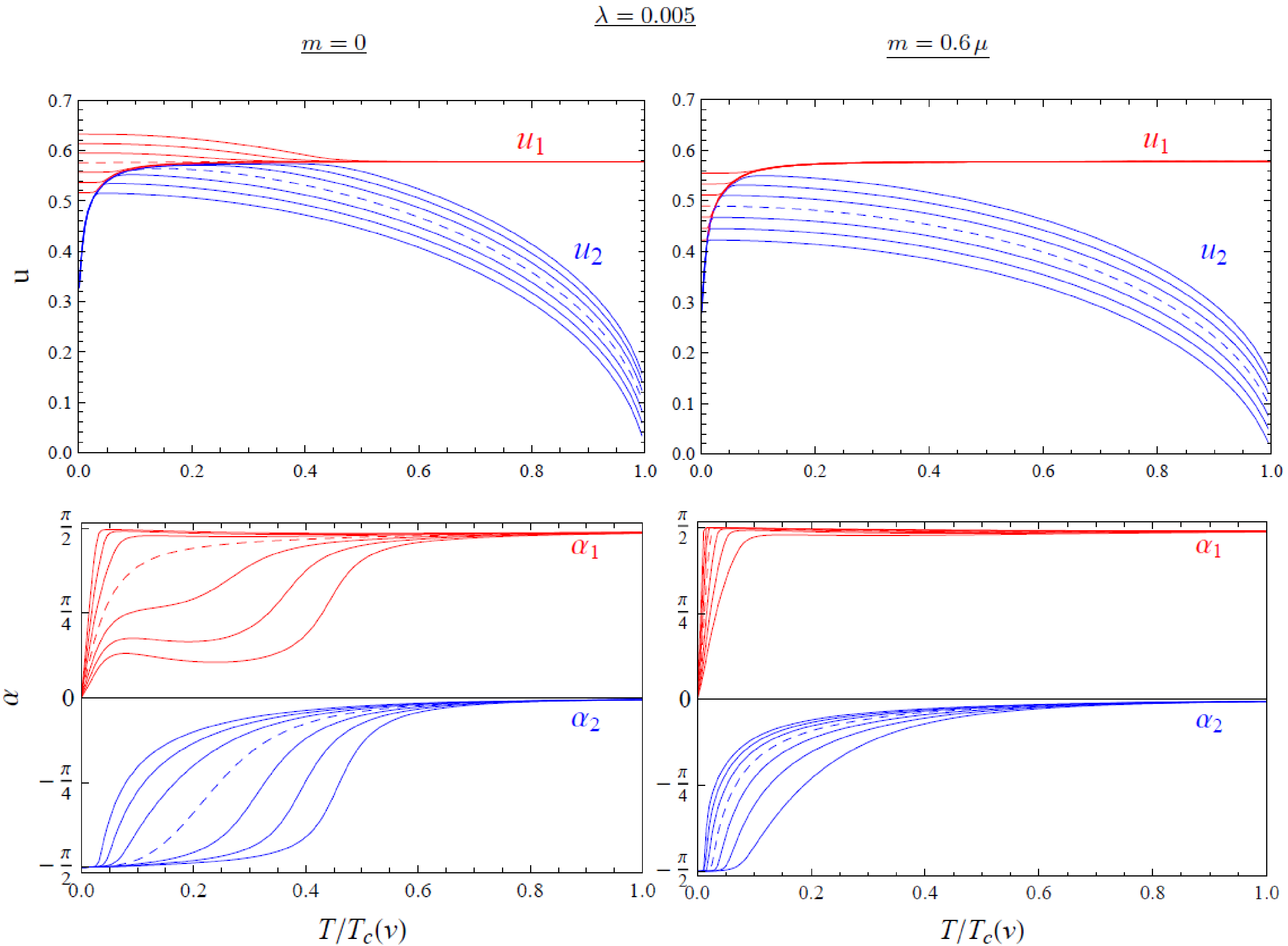
Role reversal including superflow (1)

$$\alpha := \arctan \frac{\delta T}{\delta \mu}$$

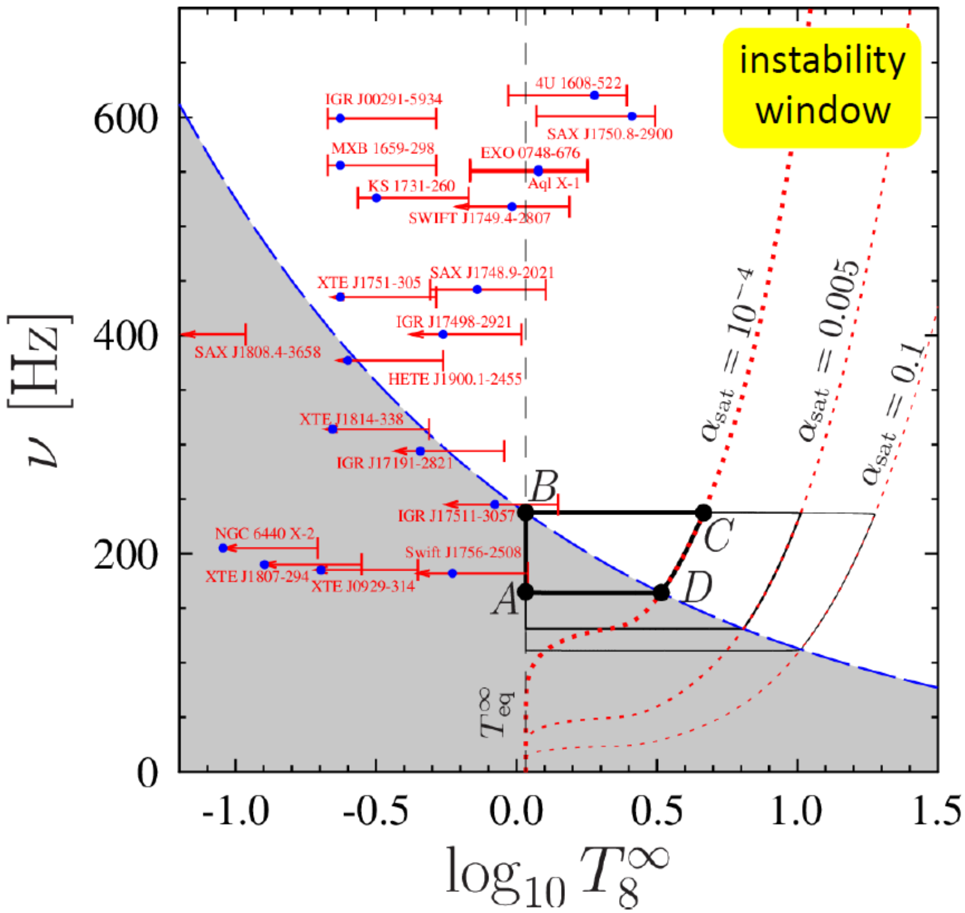


Role reversal including superflow (2)

$$\alpha := \arctan\frac{\delta T}{\delta \mu}$$



Role reversal - comparison to r-modes



Conventional picture:

Amplitude of r-modes:

$$\partial_t \alpha = -\alpha\left(\tau_{grav}^{-1} + \tau_{diss}^{-1}\right)$$

τ_{grav} time scale of gravitational radiation

τ_{visc} time scale of viscous diss. (damping)

A→ B: - star spins up (accretion)

- T increase is balanced by ν cooling

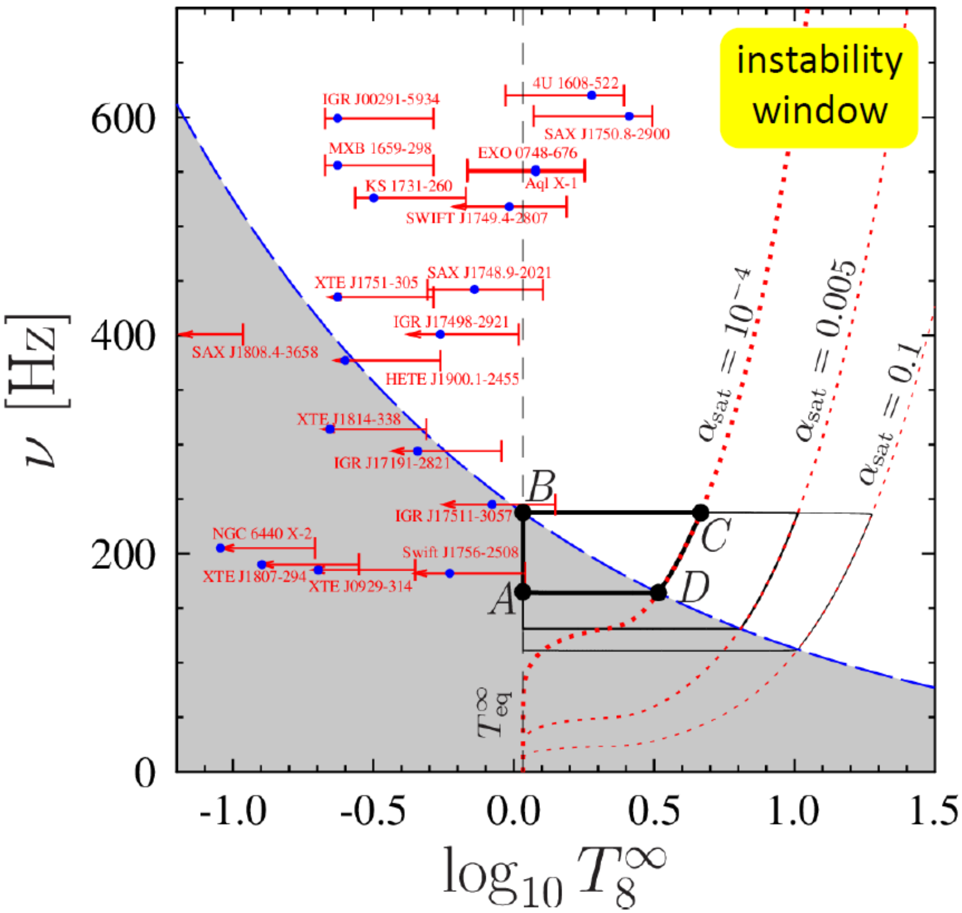
B→ C: - unstable r-modes are excited

- r modes radiate gravitational waves (spin up stops)

- star heats up (viscous dissipation of r-modes)

[**images**: M. Gusakov, talk at "the structure and signals of neutron stars" , 24. – 28.3. 2014, Florence, Italy]

Role reversal - comparison to r-modes



Conventional picture:

Amplitude of r-modes:

$$\partial_t \alpha = -\alpha\left(\tau_{grav}^{-1} + \tau_{diss}^{-1}\right)$$

τ_{grav} time scale of gravitational radiation

τ_{visc} time scale of viscous diss. (damping)

C→ D: - ν emission balances heating

- r-modes are saturated

- spin down (grav. wave radiation)

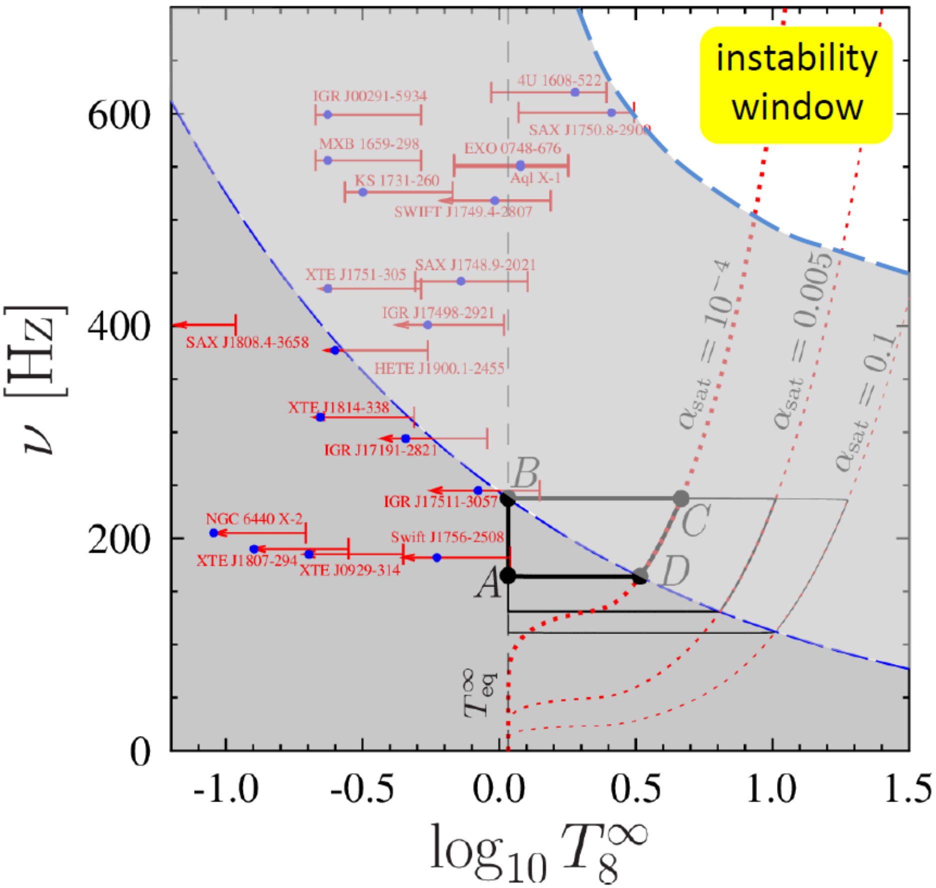
D→ A: - r-modes decay (stable region)

- star cools down to equil. temp.

[**images**: M. Gusakov, talk at “the structure and signals of neutron stars“ , 24. – 28.3. 2014, Florence, Italy]

Role reversal - comparison to r-modes

→ why are fast spinning stars observed in nature?



possible resolutions:

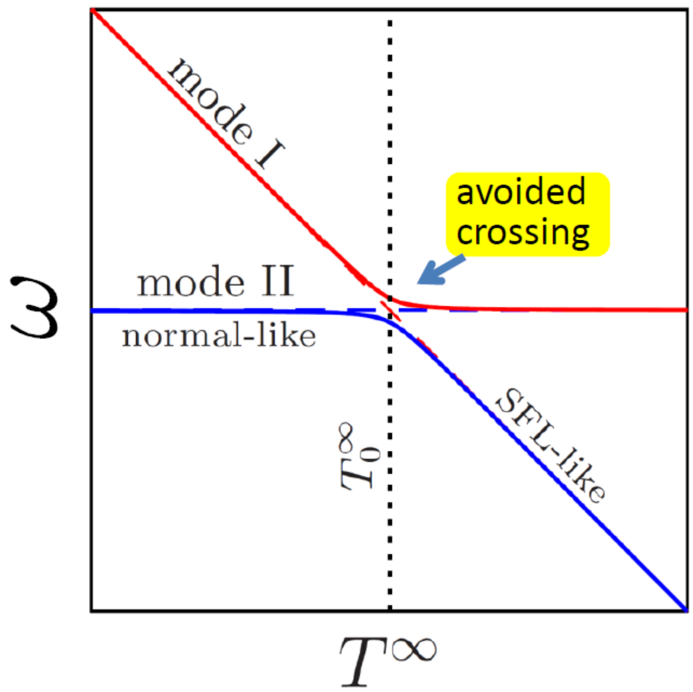
- Increase viscosity by a factor of 1000
  - all stars are in stable region (unrealistic for p, n, e^-, μ^-)
- Consider more exotic matter with high bulk viscosity (hyperons, quark matter)

→ impact of superfluidity on r-modes?

[M. Gusakov, A. Chugunov, E. Kantor Phys.Rev.Lett. 112 (2014) no.15, 151101]

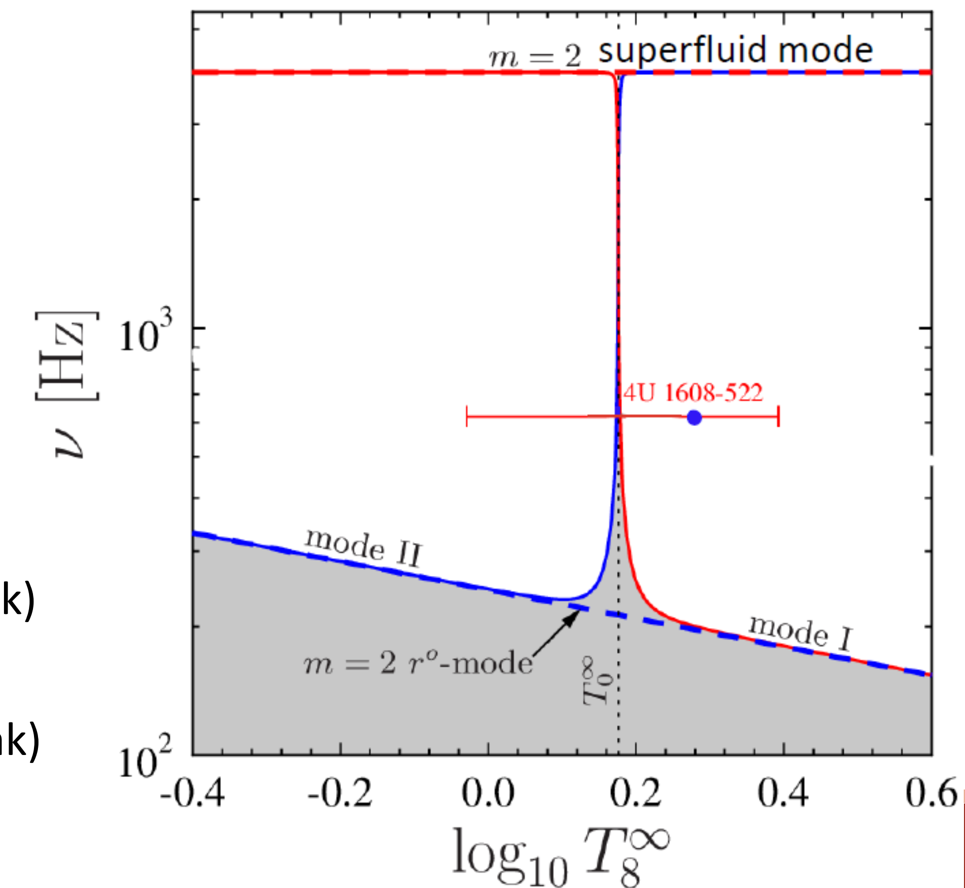
[**images**: M. Gusakov, talk at “the structure and signals of neutron stars“ , 24. – 28.3. 2014, Florence, Italy]

Role reversal - comparison to r-modes



Excitation of normal fluid and superfluid modes

- **avoided crossing** if modes are coupled
- superfluid modes: **faster damping** $\tau_{diss}^{SFL} \ll \tau_{diss}^{normal}$



- **Close to avoided crossing:**

normal mode → SFL mode
(enhanced dissipation, left edge of stability peak)

SFL mode → normal mode
(reduced dissipation, right edge of stability peak)

Outlook

- Study excitations of coupled superfluids at finite temperature.
  → in particular instabilities (see also talk by A. Schmitt)

- Study mixture of superconductor\superfluid (i.e. gauge one U(1) symmetry).
  [A. Schmitt, A. Haber, work in progress] (see also talk by A. Haber)

- Consider fermions and Cooper pairing.

- Add dissipative terms. [A. Schmitt, work in progress]

- Consider explicit symmetry breaking: what happens to superfluidity?