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Transport coefficients of npeμ matter in NS cores / Neutron star cooling

INT Phases of Dense Matter
Seattle July 18—22 2016

Transport coefficients of npeμ matter in NS cores / Neutron star cooling

Part I. Thermal conductivity and shear viscosity in non-superfluid npeμ matter

Part II. “Enhancement” of the modified Urca cooling in beta-stable nuclear matter

Collaboration

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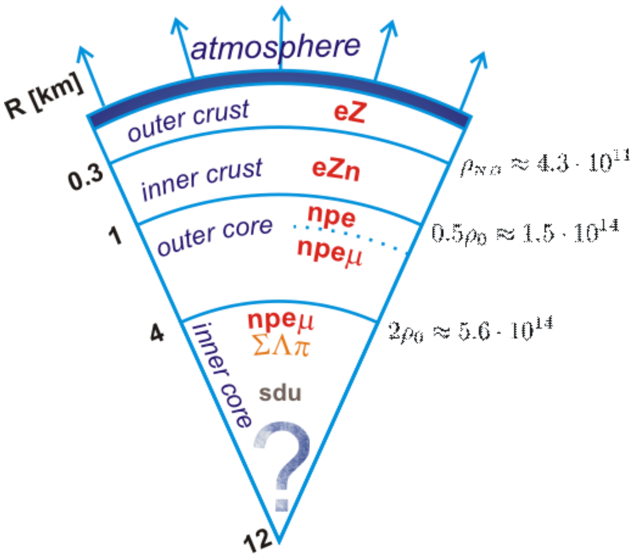
Part I. Transport coefficients of npeμ matter



Non-superfluid beta-stable npeμ matter

No magnetic fields

Kinetic coefficients due to particle collisions

$$\kappa = \kappa_{e\mu}[ee, e\mu, ep] + \kappa_n[nn, np]$$

$$\kappa_\mathrm{p} \ll \kappa_\mathrm{n}$$

Electromagnetic part: $\kappa_{e\mu}, \; \eta_{e\mu}$

PS, Yakovlev, 2007,2008

Nuclear part: $\kappa_\mathrm{n}, \; \eta_\mathrm{n}$

PS, Baldo, Haensel, 2013

Kinetic coefficients in multi-component Fermi-liquid: Formalism

$$\kappa = \sum_c \frac{\pi^2 k_B^2 T n_c \tau_c^\kappa}{3 m_c^*}; \quad \eta = \sum_c \frac{n_c p_{\mathrm{F}c}^2 \tau_c^\eta}{5 m_c^*}$$

Perturbation $\nabla T,\ V_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial V_\alpha}{\partial x_\beta} + \frac{\partial V_\beta}{\partial x_\alpha}\right)$ ⇨ **Deviation of the distribution function** $F = f_{eq} - \Phi \frac{\partial f_{eq}}{\partial \epsilon}$

Kinetic equation ***(linearized)***

$$\left.\begin{array}{ll} \kappa: & (\epsilon_1 - \mu_1)\mathbf{v}_1 \frac{\nabla T}{T} \\ \eta: & \left(v_{1\alpha} p_{1\beta} - \frac{1}{3}\delta_{\alpha\beta} v_1 p_1\right) V_{\alpha\beta} \end{array}\right\} \frac{\partial f_1}{\partial \epsilon_1} = \sum_i I_{ci}(12; 1'2'),$$

Boltzmann collision integral

$$I_{ci} = \frac{1}{(1+\delta_{ci}) k_B T} \sum_{\sigma_{1'}\sigma_2\sigma_{2'}} \int\int\int \frac{\mathrm{d}\boldsymbol{p}_{1'}\mathrm{d}\boldsymbol{p}_2\mathrm{d}\boldsymbol{p}_{2'}}{(2\pi\hbar)^9} w_{ci}(12;1'2') f_1 f_2 (1-f_{1'})(1-f_{2'}) \left(\Phi_{1'} + \Phi_{2'} - \Phi_1 - \Phi_2\right)$$

Transition probability $\sum_{\mathrm{spins}} w_{ci}(12|1'2') = 4\frac{(2\pi)^4}{\hbar}\delta(\epsilon_1 + \epsilon_2 - \epsilon_1' - \epsilon_2')\delta(\boldsymbol{P} - \boldsymbol{P}')\mathcal{Q}_{ci}$

Solution: $\Phi^{\kappa,\eta} \to \tau_c^{\kappa,\eta} \to \kappa,\ \eta$

Input: $m^*,\ \mathcal{Q}$ **on the Fermi surface**

Simplest variational solution

Simplest form of the trial functions

$$\Phi^\kappa \propto (\varepsilon - \mu)\boldsymbol{v}\frac{\nabla T}{T} \qquad \Phi^\eta \propto (v_\alpha p_\beta - \frac{1}{3}\delta_{\alpha\beta} v p) V_{\alpha\beta}$$

Leads to the linear system of equations for the relaxation times

$$\sum_i \nu_{ci}\tau_i = 1$$

Collision frequencies are given by averaging sq. matrix element over the allowed phasespace

$$\nu_{ci} \propto \langle Q_{ci} W(\Omega) \rangle_\Omega$$

Exact solutions can be obtained

Sykes and Brooker 1970, Flowers, Itoh, 1979, Anderson et al., 1987

Correction is usually small (less than 20%)

Electromagnetic part

Electrons and muons collide with themselves and with protons

$$\nu_e = \nu_{ee} + \nu_{e\mu} + \nu_{ep}$$

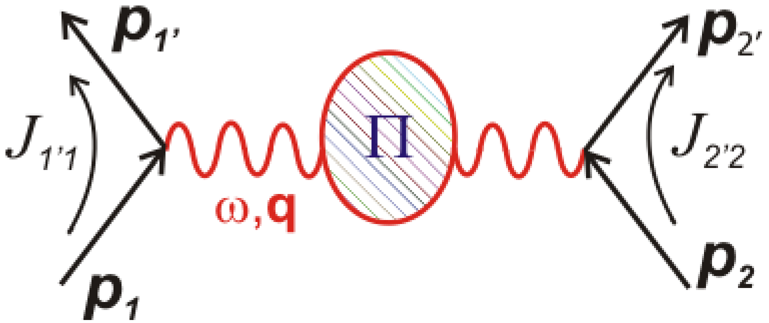
Collisions are mediated by electromagnetic interaction

Need to consider screening

Transverse plasmon exchange

Matrix element

$$|M_{12}|^2 \propto \left| \frac{J^{(0)}_{1'1} J^{(0)}_{2'2}}{q^2 + \Pi_l} - \frac{(\boldsymbol{J}_{t1'1} \cdot \boldsymbol{J}_{t2'2})}{q^2 - \omega^2/c^2 + \Pi_t} \right|^2$$



$$|\boldsymbol{J}|/J^{(0)} \propto v/c$$

Classical limit $q \ll p_{\mathrm{F}i}$, $\omega \ll p_{\mathrm{F}i} v_{\mathrm{F}i}$

Strong degeneracy $\omega \ll q v_{Fi}$

Tomas-Fermi screening of the longitudinal plasmons

$$\Pi_l = q_0^2 = \frac{4\alpha}{\pi\hbar^2} \sum_i m_i^* c p_{Fi}$$

$$q_{\min,l} \sim q_0$$

Landau damping of the transverse plasmons

$$\Pi_t = i \frac{\pi}{4} \frac{\omega}{qc} q_t^2, \quad q_t^2 = \frac{4\alpha}{\pi\hbar^2} \sum_i p_{Fi}^2$$

$$q_{\min,t} \sim \left(q_t^2 \omega / c\right)^{1/3} < \left(q_t^2 k_B T / \hbar c\right)^{1/3}$$

Heiselberg & Pethick, 1993

Transverse part of the matrix element dominates

$$q_{\min,\ell} \gg q_{\min,t}$$

Results. Non-standard temperature behavior

P.S., Yakovlev, 2007,2008

Transverse part dominates

$$\nu_{ci} = \nu_{ci}^{(t)} + \nu_{ci}^{(l)} + \nu_{ci}^{(tl)}$$

$$\nu_{ci}^{(t)} \gg \nu_{ci}^{(l,tl)}$$

Leading term
Non-standard temperature dependence

$$\kappa_c = \kappa_c^{(t)} = \frac{\pi^2}{54\zeta(3)} \frac{k_B c p_{Fc}^2}{\hbar^2 \alpha} = C p_{Fc}^2$$

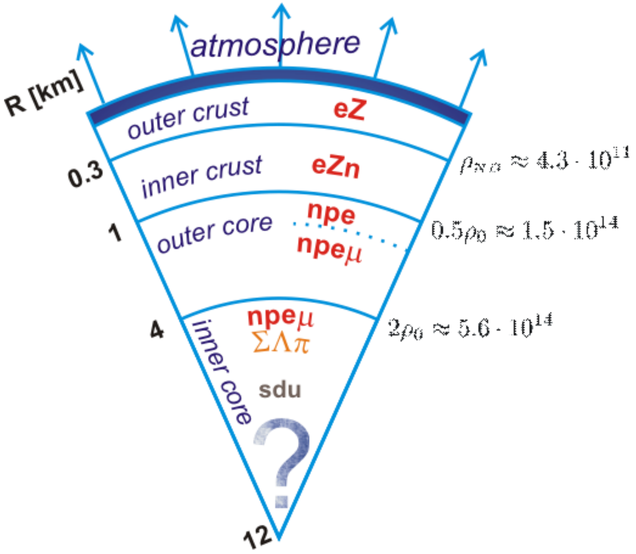
Instead of T^{-1}

$$\eta_c = \eta_c^{(t)} = \frac{\pi^2 c^2 \hbar^3}{5\xi_\eta^t \alpha} \frac{n_c^2}{q_t (\hbar c q_t)^{1/3}} (k_B T)^{-5/3}$$

Instead of T^{-2}

Can be used for any EOS if particles fractions and effective masses are known

Nuclear part



Non-superfluid beta-stable npeμ matter

Kinetic coefficients due to particle collisions

$$\kappa = \kappa_{e\mu}[ee, e\mu, ep] + \kappa_n[nn, np]$$

$$\kappa_{\rm p} \ll \kappa_{\rm n}$$

Electromagnetic part: $\kappa_{e\mu},\ \eta_{e\mu}$

PS, Yakovlev, 2007,2008

Nuclear part: $\kappa_{\rm n},\ \eta_{\rm n}$

PS, Baldo, Haensel, 2013

Strongly interacting multicomponent Fermi-liquid

Variational solution

2x2 algebraic system

$$\sum_{i=n,p} \nu_{ci}\tau_i = 1$$

$$\kappa = \sum_c \frac{\pi^2 k_B^2 T n_c \tau_c^\kappa}{3m_c^*}; \quad \eta = \sum_c \frac{n_c p_{\mathrm{F}c}^2 \tau_c^\eta}{5m_c^*}$$

$$\nu_{ci}^{(\kappa)} = \frac{64 m_c^* m_i^{*2}(k_B T)^2}{5 m_N^2 \hbar^3} S_{\kappa ci}, \quad \nu_{ci}^{(\eta)} = \frac{16 m_c^* m_i^{*2}(k_B T)^2}{3 m_N^2 \hbar^3} S_{\eta ci}.$$

Effective transport cross-sections

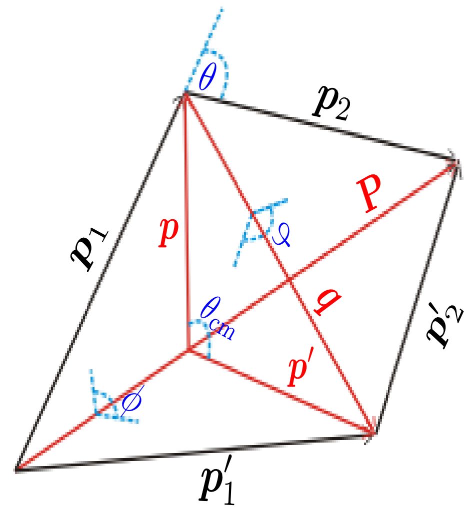
$$S_{\kappa cc} = \frac{m_N^2}{128\pi^2\hbar^4 p_c^3}\int_0^{2p_c} \mathrm{d}P \int_0^{q_m(P)} \mathrm{d}q \frac{(4p_c^2 - P^2)}{\sqrt{q_m^2 - q^2}} \mathcal{Q}_{cc},$$

$$S_{\kappa ci} = \frac{m_N^2}{128\pi^2\hbar^4 p_c^3}\int_{|p_c-p_i|}^{p_c+p_i} \mathrm{d}P \int_0^{q_m(P)} \mathrm{d}q \frac{(4p_c^2 + q^2)}{\sqrt{q_m^2 - q^2}} \mathcal{Q}_{ci}, \quad c \neq i,$$

$$S_{\eta cc} = \frac{3m_N^2}{128\pi^2\hbar^4 p_c^5}\int_0^{2p_c} \mathrm{d}P \int_0^{q_m(P)} \mathrm{d}q \frac{q^2(4p_c^2 - P^2 - q^2)}{\sqrt{q_m^2 - q^2}} \mathcal{Q}_{cc},$$

$$S_{\eta ci} = \frac{3m_N^2}{128\pi^2\hbar^4 p_c^5}\int_{|p_c-p_i|}^{p_c+p_i} \mathrm{d}P \int_0^{q_m(P)} \mathrm{d}q \frac{q^2(4p_c^2 - q^2)}{\sqrt{q_m^2 - q^2}} \mathcal{Q}_{ci}, \quad c \neq i,$$

All particles on Fermi surface



Two angles fix all momenta

$$\frac{1}{\tau_i} = \frac{1}{\tau_i}\left[\langle w(12|1'2')\beta(\theta,\phi)\rangle\right]$$

Compare:
Scattering cross-sections:

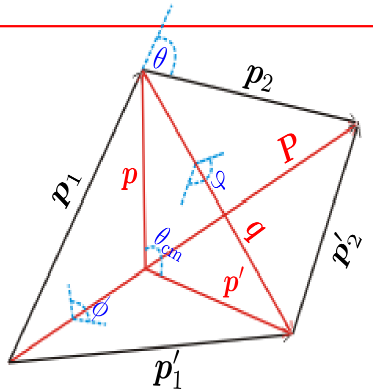
$$\frac{d\sigma_{\alpha\beta}}{d\Omega} = \frac{m_{\alpha\beta}^{*2}}{16\pi^2\hbar^4}\mathcal{Q}_{\alpha\beta}$$

Accuracy (*aposteriori*)

$$\kappa_{exact}/\kappa_{var} \approx 1.2$$
$$\eta_{exact}/\eta_{var} \approx 1.05$$

Approximate estimates

Pure neutron matter: $\tau_{\rm n} = \dfrac{1}{\nu_{\rm nn}}$

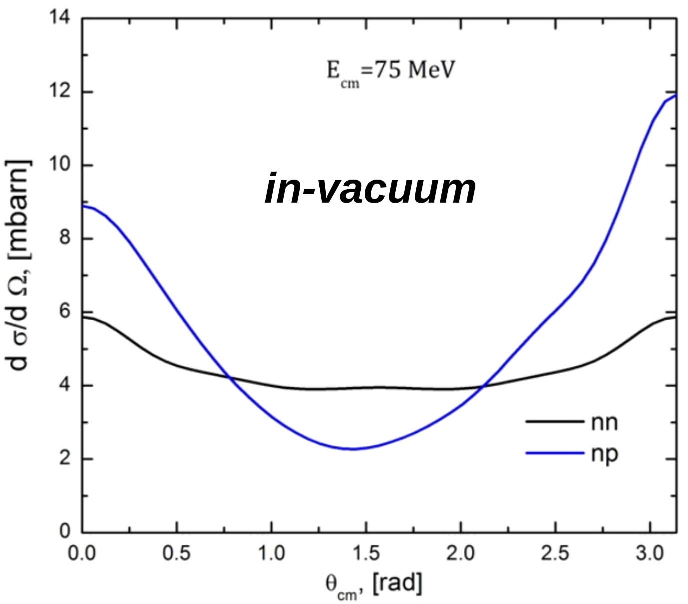


$$\frac{1}{\tau_n} = \frac{1}{\tau_{\rm n}}\left[\langle \mathcal{Q}_{\rm nn}\beta(\theta,\phi)\rangle\right]$$

1. kinematics: $\theta \sim \pi \; (P \approx 0)$

2. nn cross-section is flat $\dfrac{{\rm d}\sigma_{\rm nn}}{{\rm d}\Omega} \to \dfrac{\sigma_{\rm tot}}{2\pi}$

$$\frac{d\sigma_{\rm nn}}{d\Omega} = \frac{m_{\rm n}^{*2}}{16\pi^2\hbar^4}\mathcal{Q}_{\rm nn}$$



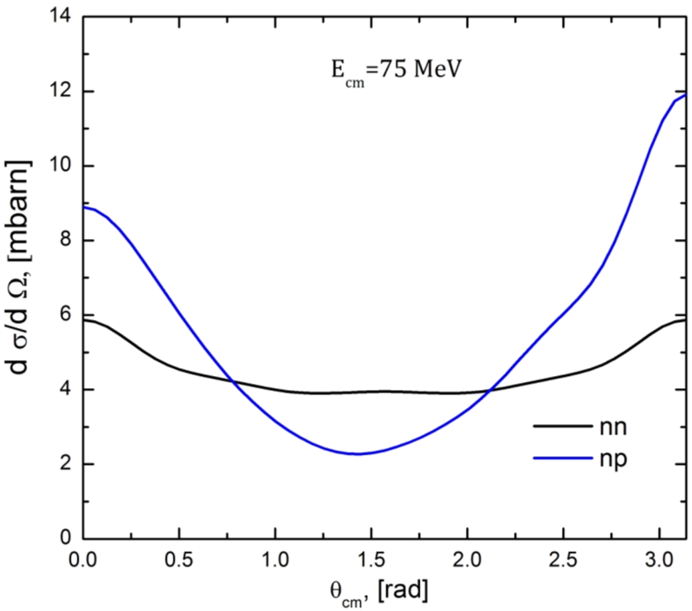
Finally: simple relations

$$\kappa_{\rm Var} = \frac{5}{3}\frac{p_F^3}{m^2}\frac{1}{T}\left[32\sigma_{\rm tot}\left(E_{\rm cm} = \frac{p_F^2}{m}\right)\right]^{-1}$$

$$\eta_{\rm Var} = \frac{p_F^5}{16\pi^2 m^2}\frac{1}{(k_B T)^2}\left[\sigma_{\rm tot}\left(E_{\rm cm} = \frac{p_F^2}{m}\right)\right]^{-1}$$

Effects of the proton fraction

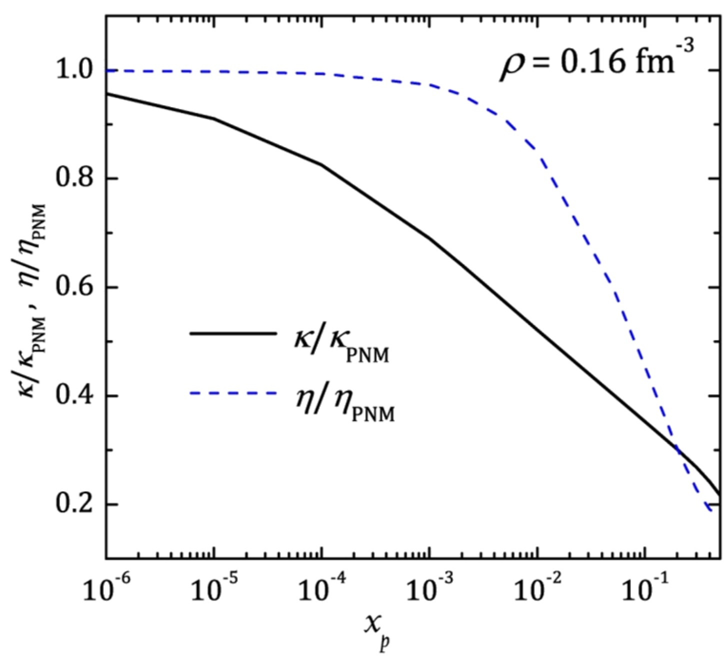
PNM result is inaccurate even at small proton fraction



$$\nu_{\rm np} \sim \nu_{\rm nn}$$

$$\kappa = \kappa_{\rm n} + \kappa_{\rm p}$$

$$\eta = \eta_{\rm n} + \eta_{\rm p}$$



Reasons

small-angle scattering
smaller c.m. energy

$$p_{F\rm p} \ll p_{F\rm n}$$

Higher cross-section
(T_z=0 isospin channel)

In-medium

$$\kappa = \sum_c \frac{\pi^2 k_B^2 T n_c \tau_c^\kappa}{3 m_c^*}; \qquad \eta = \sum_c \frac{n_c p_{\mathrm{F}c}^2 \tau_c^\eta}{5 m_c^*}$$

$$\sum_{i=n,p} \nu_{ci}\tau_i = 1$$

$$\nu_{ci}^{(\kappa)} = \frac{64 m_c^* m_i^{*2} (k_B T)^2}{5 m_N^2 \hbar^3} S_{\kappa ci}, \qquad \nu_{ci}^{(\eta)} = \frac{16 m_c^* m_i^{*2} (k_B T)^2}{3 m_N^2 \hbar^3} S_{\eta ci}.$$

From in-medium theory we need

$$m^*, \ \mathcal{Q}$$

4th power of m* – the main effect?

BHF calculations

Interaction is described via the G-matrix

$$V \to G$$

Brueckner-Bethe-Salpeter equation with the self-consistent potential

$$\langle p_1 p_2 | G^{\alpha\beta}(\omega) | p_3 p_4 \rangle = \langle p_1 p_2 | V^{\alpha\beta} | p_3 p_4 \rangle + \sum_{k_1, k_2} \langle p_1 p_2 | V^{\alpha\beta} | k_1 k_2 \rangle \frac{Q^{\alpha\beta}(k_1, k_2)}{\omega - \epsilon_\alpha(k_1) - \epsilon_\beta(k_2)} \langle k_1 k_2 | G^{\alpha\beta} | p_3 p_4 \rangle$$

$$\epsilon_\alpha(p) = \frac{p^2}{2m_\alpha} + U_\alpha(p) \qquad U_\alpha(p_1) = \sum_{\beta; p_2 < p_{F\beta}} \langle p_1 p_2 | G^{\alpha\beta}(\epsilon_1(p_1) + \epsilon_2(p_2)) | p_1 p_2 \rangle_\mathrm{A}$$

BBS equation is solved in the partial wave basis up to J=12 with Argonne v18 potential and Urbana IX three-nucleon forces

UIX parameters are adjusted to give the correct saturation point of SNM

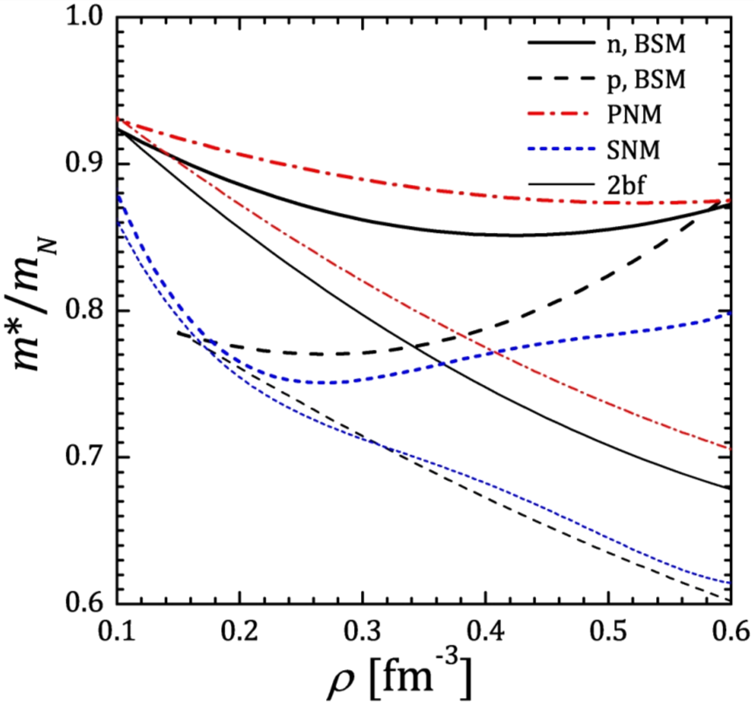
Av18+UIX. Effective mass

$$m^* = \left(\frac{1}{p} \frac{\mathrm{d}\epsilon(p)}{\mathrm{d}p} \right)^{-1}_{p=p_F}$$

2bf decrease effective masses
UIX 3bf increase

$$\eta,\ \kappa \propto (m^*)^{-4}$$

Effective masses at Fermi surface



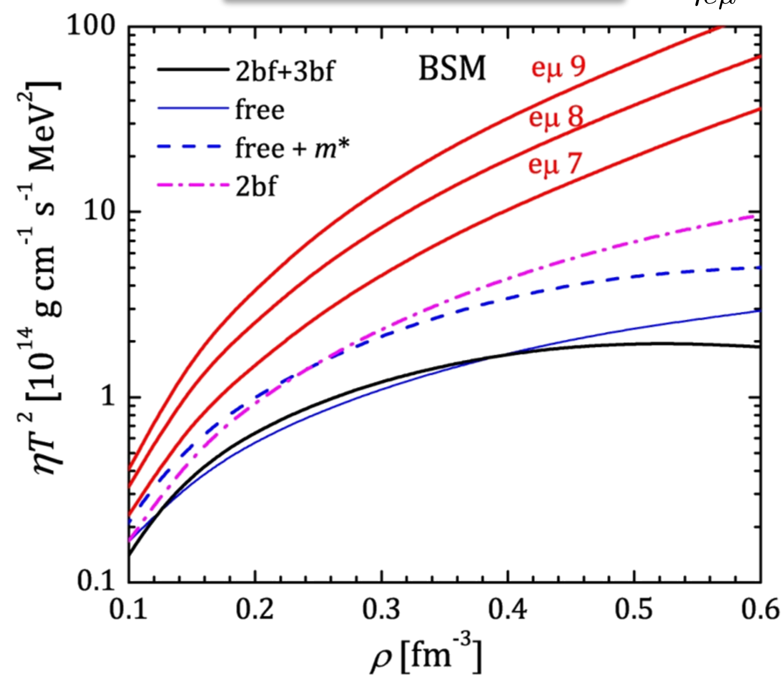
Results. Kinetic coefficients.

Exact solutions are shown

PS, Baldo,Haensel, 2013

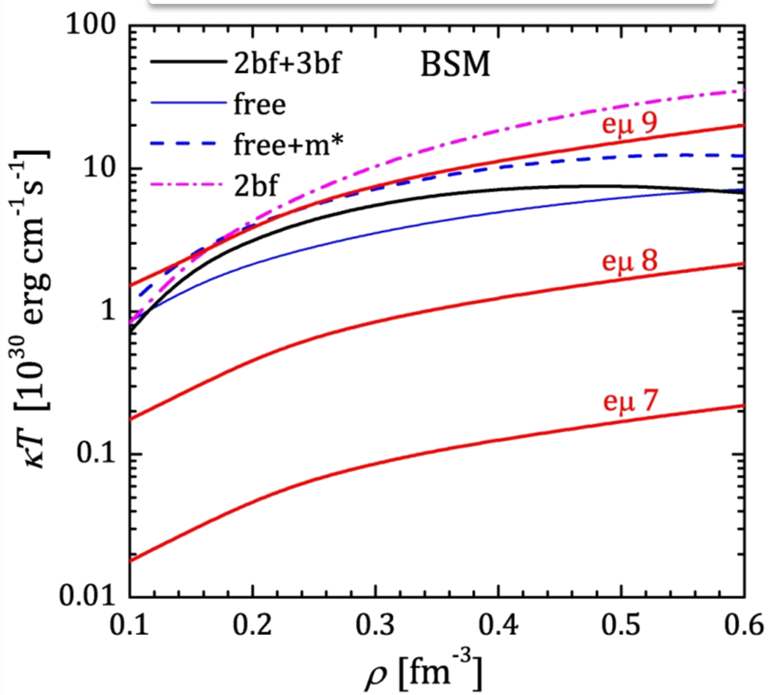
Shear viscosity

$\eta_{e\mu} \propto T^{-5/3}$



Thermal conductivity

$\kappa_{e\mu} \propto T^0$



Electron viscosity dominates

Nuclear thermal conductivity dominates

Av18+UIX results are comparable with 'free-scattering'

Different nuclear potentials

Following *Baldo et al. 2014*

Effective masses:

Argonne v18

Wiringa et al., 1995

CDBonn

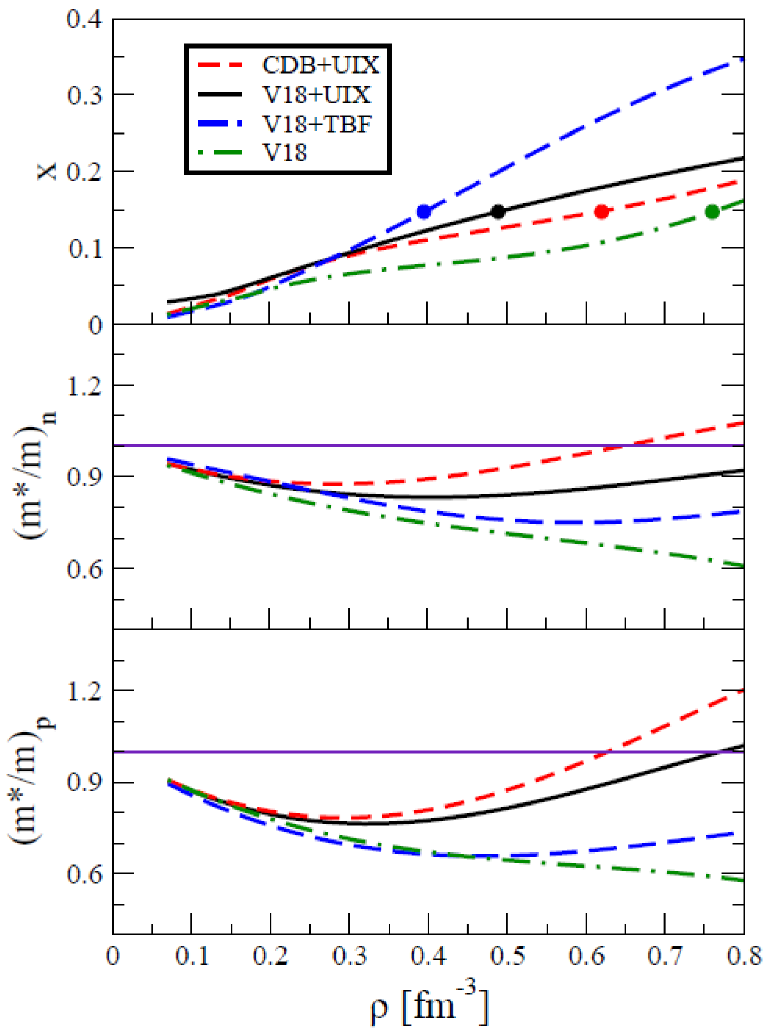
Machleidt, 2001

+UIX (adjusted)

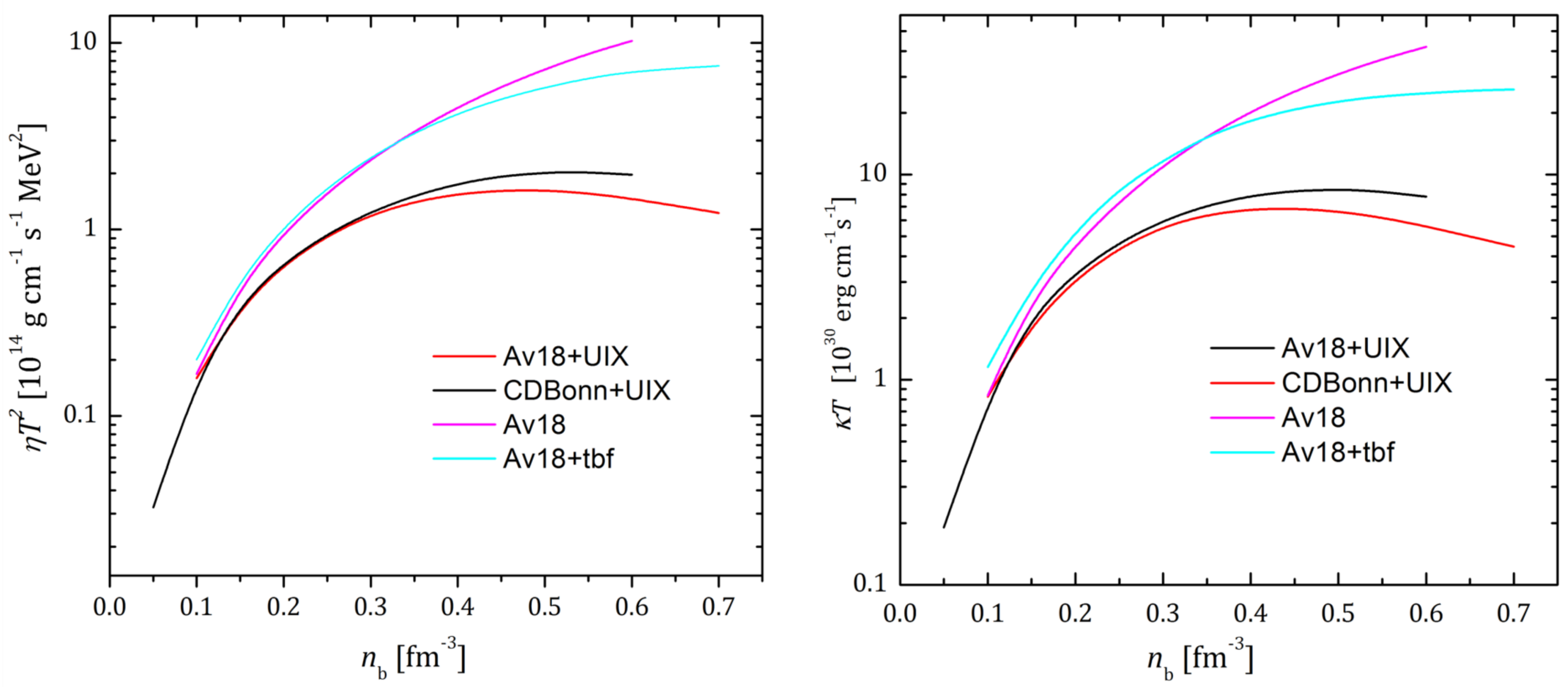
Different three-body force:
Microscopic meson-exchange

Grange et al., 1989, Li&Schulze 2008,2012,..

Av18+tbf(mic)



Different nuclear potentials



Electron viscosity still dominates despite large uncertainty in nuclear one

Superfluidity

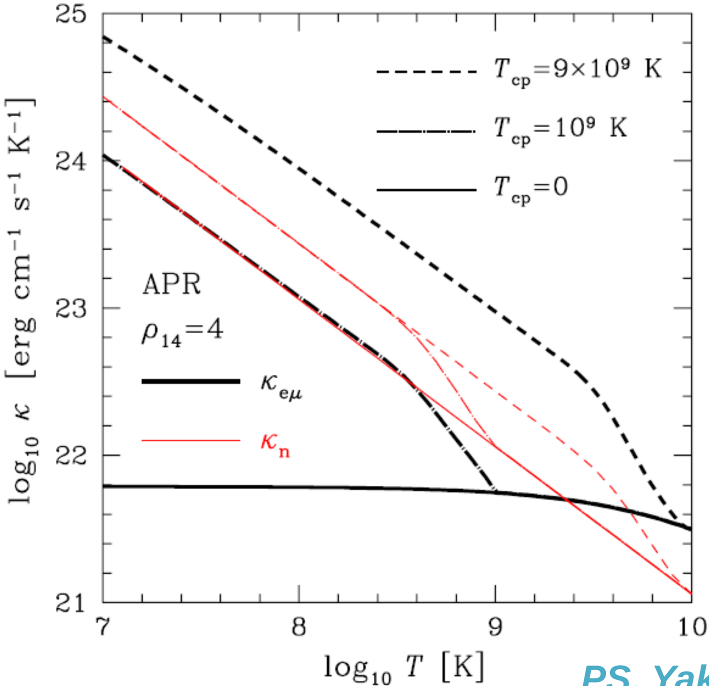
$\kappa_{e\mu},\ \eta_{e\mu}$ **in presence of proton superfluidity (superconductivity)**

Screening changes to static

$$\Pi_t^{(p)} = \frac{\pi^2}{4}\frac{\Delta}{qc}q_{t,p}^2, \quad \Delta/T \gg 1$$

Collision frequencies temperature dependence restores

$$\nu_{ci} \propto T^2$$



PS, Yakovlev, 2007

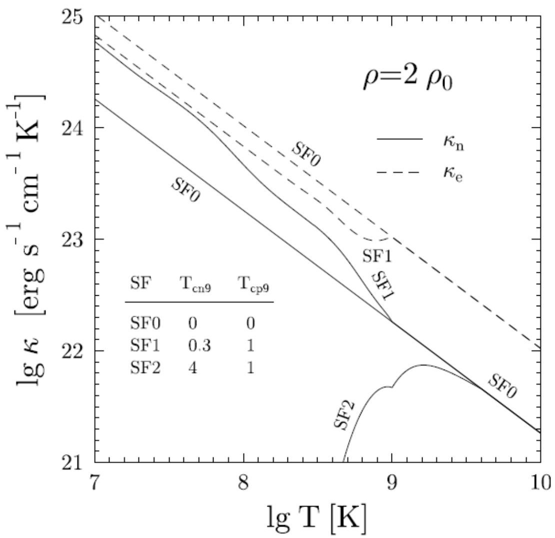
Superfluidity

$\kappa_n, \; \eta_n$ **in presence of proton and/or neutron superfluidity**

Single-particle (Bogoliubov) excitations

e.g., Baiko, Haensel, Yakovlev, 2001
(only effect of gaps)

$$\nu \propto \exp(-\Delta/T)$$



Baiko et al., 2001

superfluid phonons

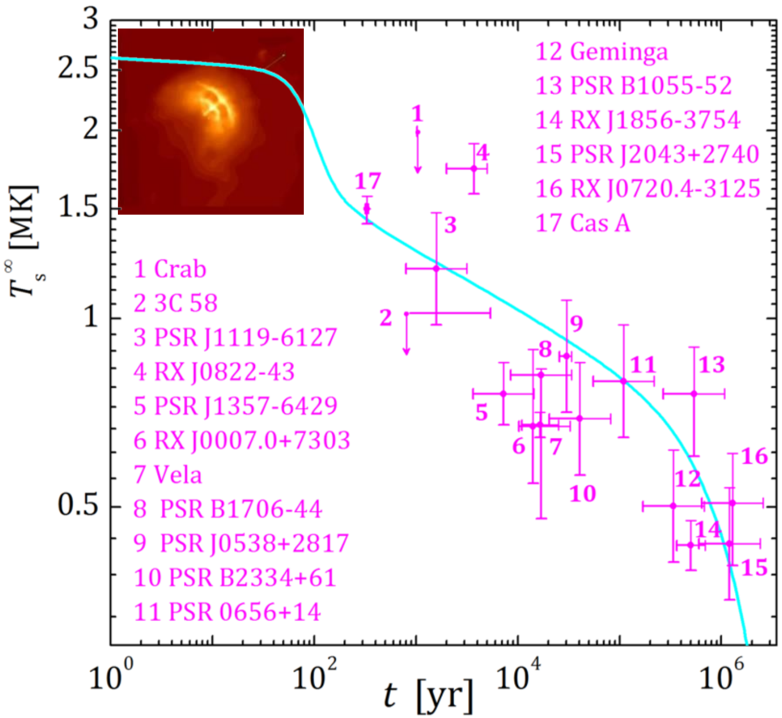
Manuel & Tolos 2011,2013; Kolomeitsev & Voskresensky (2015)

Part II. NS cooling

“Enhancement” of the modified Urca cooling in beta-stable nuclear matter

Introduction. Neutron star cooling and neutrino emission

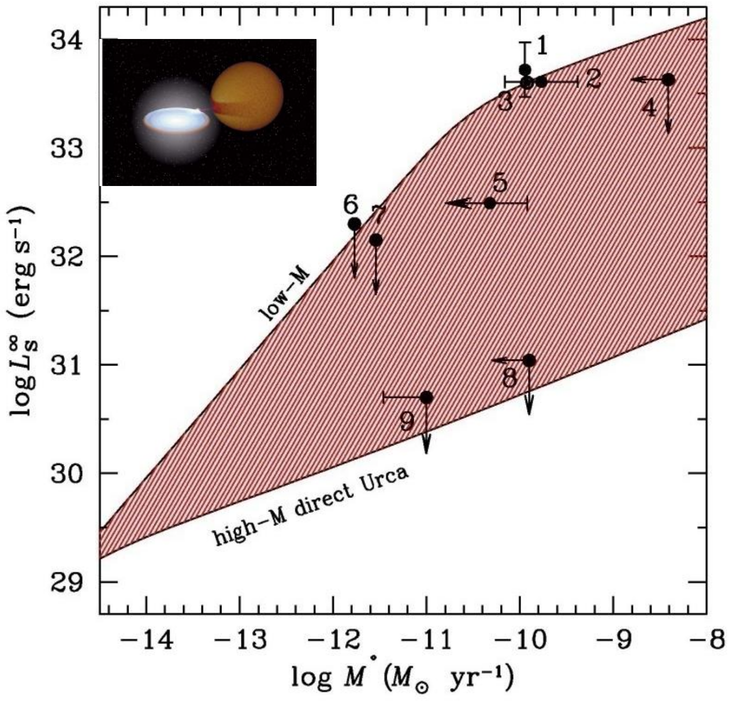
Cooling isolated neutron stars



Thermal evolution

$$C\frac{\mathrm{d}\tilde{T}_{\rm int}}{\mathrm{d}t} = -L_s - L_\nu + H$$

NSs in X-ray transients

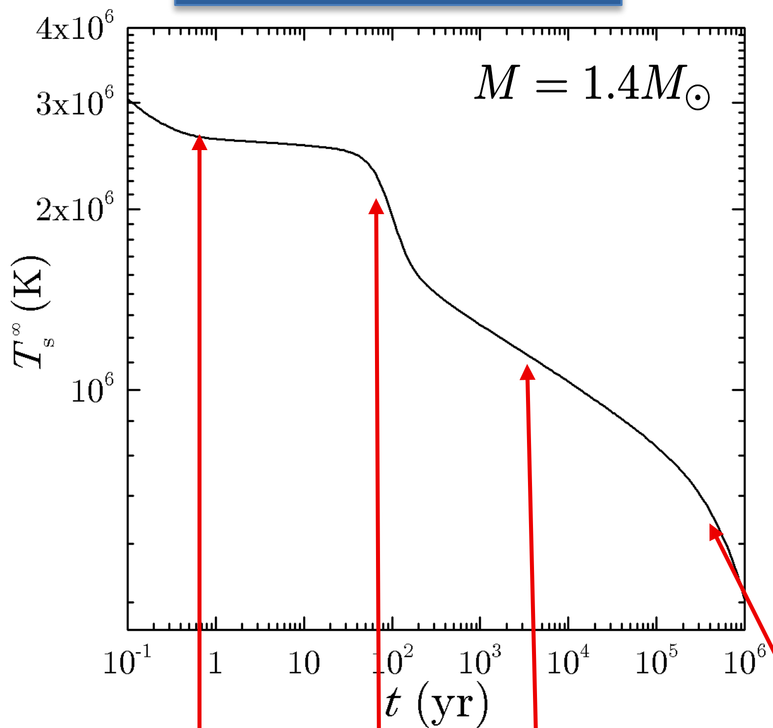


Thermal balance

$$L_\nu + L_\gamma = H_{\rm ob} = fQ\frac{\langle \dot{M} \rangle}{m_{\rm N}}, \quad f \lesssim 1$$

Basics. Cooling stages

Typical cooling curve



Crustal cooling

Neutrino stage

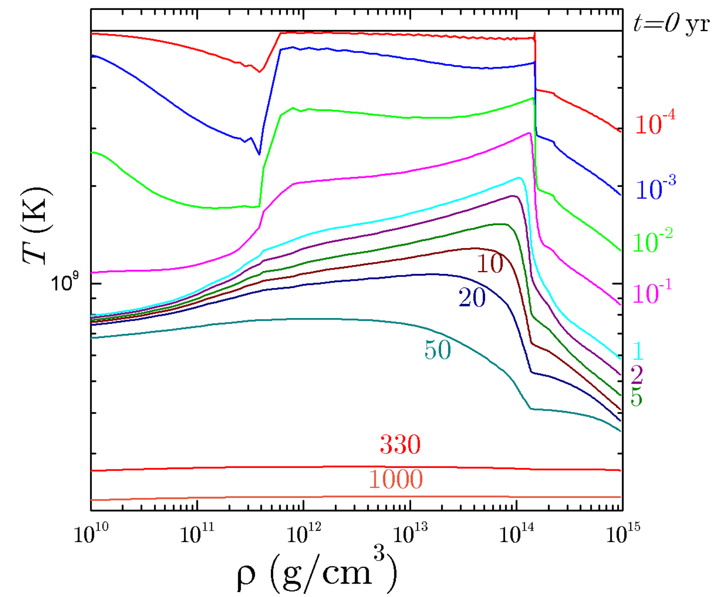
Relaxation

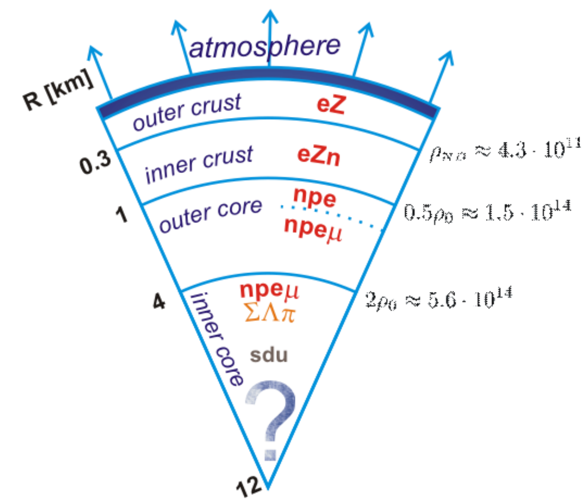
$$t_r \sim 30 - 300 \text{ yr}$$

Photon stage

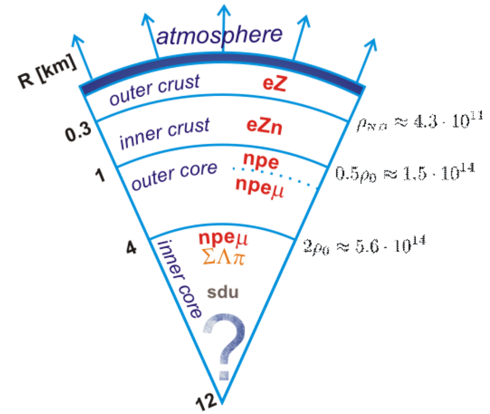
$$C\frac{\mathrm{d}\tilde{T}_{\mathrm{int}}}{\mathrm{d}t} = -L_s - L_\nu + H$$

internal temperature





Basics. Cooling regulators



Main cooling regulators

Neutrino emission mechanisms

Composition and EOS

Heat capacity

Thermal conductivity

Superfluidity

Heat sources

$$C\frac{\mathrm{d}\tilde{T}_{\mathrm{int}}}{\mathrm{d}t} = -L_s - L_\nu + H$$

Observed emission

Heat blanketing envelope $T_s(\tilde{T}_{\mathrm{int}})$

Atmosphere models $F_\nu(T_s)$

Introduction (direct) Urca processes

Urca processes (β-transformations)

$$B_1 \to B_2 + l + \overline{\nu_l}, \quad B_2 + l \to \nu_l$$

$B_{1,2}$ - baryons; l - lepton

$\overline{\nu_l}\nu_l$ **Freely escape the star**

Nucleon direct Urca $B_1 = n;\ B_2 = p;\ l = e, \mu$

Fastest neutrino cooling

$$Q \sim 10^{27} T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1}$$

$$L_\nu \sim 10^{46} T_9^6 \text{ erg s}^{-1}$$

Threshold process $p_{Fn} \leqslant p_{Fp} + p_{Fe}$

Should be enough protons $x_p \gtrsim 11\% \Rightarrow \rho \gtrsim \rho_{DU}$

Operates in inner cores of neutron stars depending on the EOS

Introduction. Modified Urca processes

Urca processes (β-transformations)

$$B_1 \to B_2 + l + \overline{\nu_l}, \quad B_2 + l \to \nu_l$$

$B_{1,2}$ - baryons; l - lepton

$\overline{\nu_l}\nu_l$ **Escape the star freely**

Nucleon modified Urca

$$n + N \to p + N + \ell + \bar{\nu}_\ell, \quad p + N + \ell \to n + N + \nu_\ell$$

Operate always

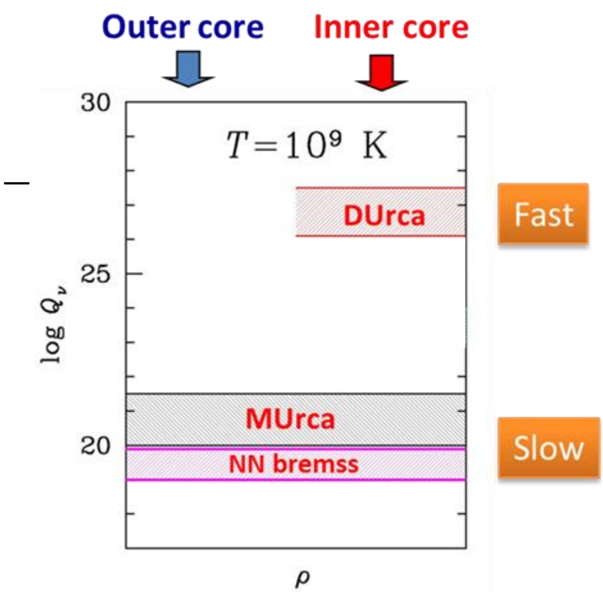
The price to pay

$$\left(\frac{k_B T}{\epsilon_F}\right)^2 \ll 1$$

Slow cooling

$$Q \sim 10^{20} - 10^{21} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-}$$

$$L_\nu \sim 10^{39} T_9^8 \text{ erg s}^{-1}$$

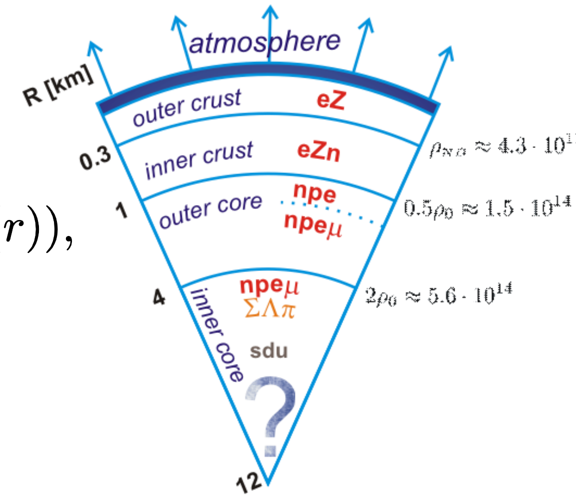


Neutrino pair bremmsstrahlung

$$N + N \to N + N + \nu_\ell + \bar{\nu}_\ell$$

Neutrino cooling stage

Isothermal interior $\tilde{T}(r) = T(r)\exp(\Phi(r)) = const$



After relaxation:

$$C(\tilde{T})\frac{\mathrm{d}\tilde{T}}{\mathrm{d}t} = -L_\nu^\infty(\tilde{T}) - \cancel{L_s^\infty(T_s)} + H$$

Global thermal balance

$$L_\nu^\infty(\tilde{T}) = \int \mathrm{d}V\, Q_\nu(T,\rho)\exp(2\Phi(r)),$$

$$C(\tilde{T}) = \int \mathrm{d}V\, C_V(T,\rho).$$

$$\frac{\mathrm{d}\tilde{T}}{\mathrm{d}t} = -\ell(\tilde{T}), \quad \ell(\tilde{T}) \equiv \frac{L_\nu^\infty(\tilde{T})}{C(\tilde{T})}$$

There exists a unique $\ell(\tilde{T})$ which depend only on mass

Cooling theory of INS can provide $\ell(\tilde{T})$ and nothing else

$$\ell(\tilde{T}) = q\tilde{T}^n \Rightarrow \tilde{T}(t) \propto (qt)^{-1/(n-1)}$$

Slow n=7
Fast n=5

Heat blanket $T_s(\tilde{T})$

$$L_\mathrm{s}^\infty(T_\mathrm{s}) = 4\pi\sigma T_\mathrm{s}^4 R^2\,(1-x)\,, \quad x = \frac{2GM}{Rc^2}$$

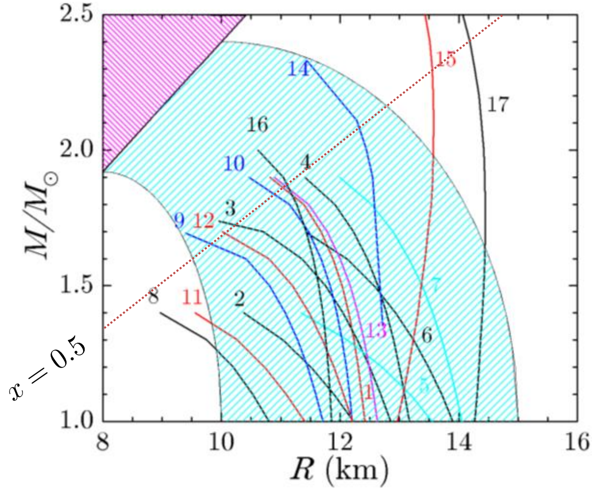
$$T_s \propto \sqrt{\tilde{T}}$$

Standard cooling

Slow cooling of nucleon NSs (MUrca+Bremss in OPE, FM79)

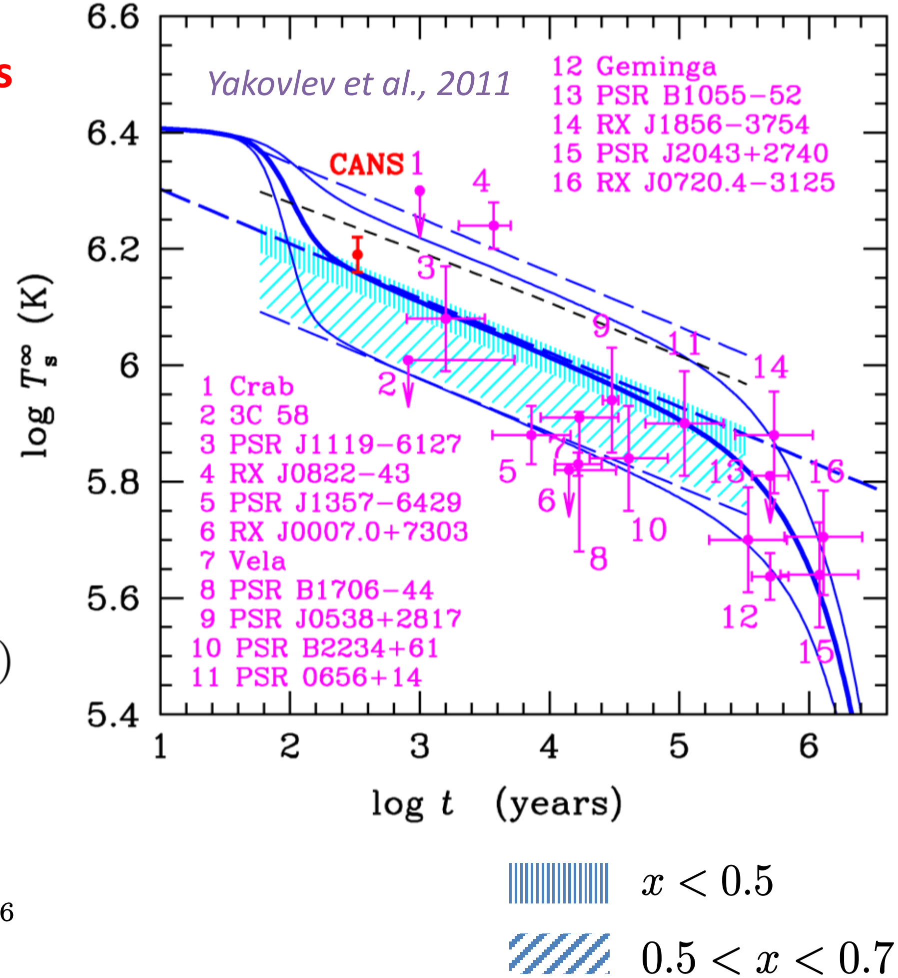
$$T_s \propto \sqrt{\tilde{T}}$$

Many EOSs



compactness

$$x = \frac{2GM}{Rc^2}$$



$$\ell(\tilde{T}) = q\tilde{T}^n \Rightarrow \tilde{T}(t) \propto (qt)^{-1/(n-1)}$$

Standard candles

$$\tilde{T}_{\rm SC}(t) = 2.87 \times 10^8 \ {\rm K}(1 - x)\left(1 + 0.12R_{\rm 10km}^2\right) t_{\rm kyr}^{-1/6}$$

Yakovlev et al., 2011

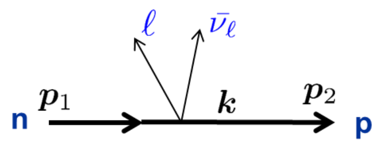
accuracy 3%

Standard cooling. mUrca emission

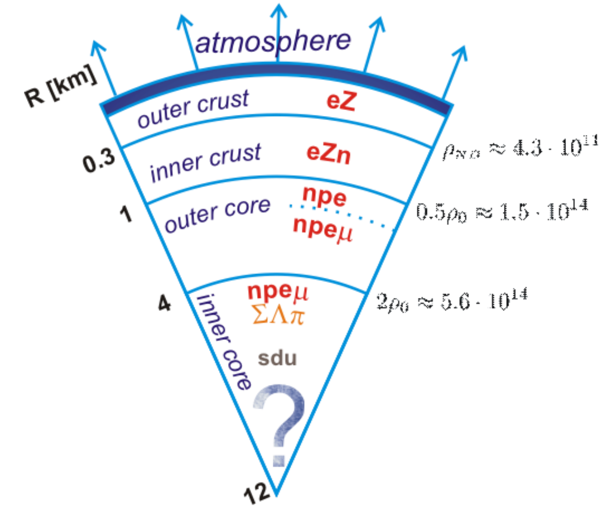
Core: Non-superfluid beta-stable npeμ matter

Direct Urca processes

$$n \to p + \ell + \bar{\nu}_\ell$$
$$p + \ell \to n + \nu_\ell$$
$$\ell = e,\ \mu$$



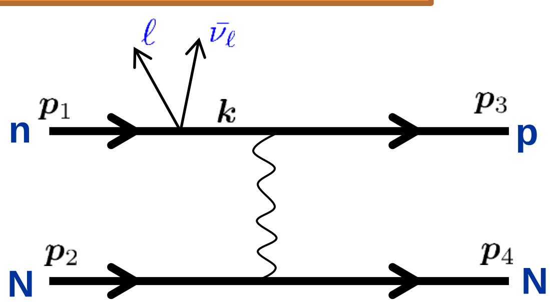
$$Q \sim 10^{27} T_9^6 \text{ erg cm}^{-3}\text{ s}^{-1}$$



Are forbidden: $p_{\mathrm{Fn}} > p_{\mathrm{Fp}} + p_{\mathrm{F}\ell}$

Modified Urca processes – the main neutrino cooling regulator

$$n + N \to p + N + \ell + \bar{\nu}_\ell$$
$$p + N + \ell \to n + N + \nu_\ell$$



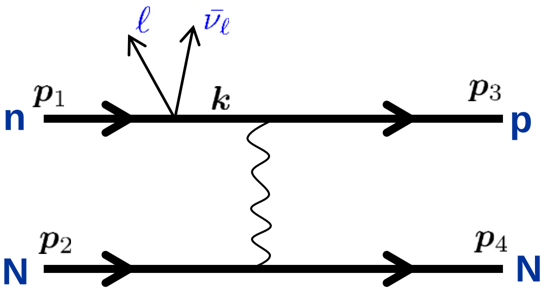
$$Q \sim 10^{20} - 10^{21} T_9^8 \text{ erg cm}^{-3}\text{ s}^{-1}$$

Neutron branch: $N = n$

Proton branch: $N = p$

Modified Urca. Basic formalism

$$n+N\to p+N+\ell+\bar\nu_\ell,\quad p+N+\ell\to n+N+\nu_\ell$$

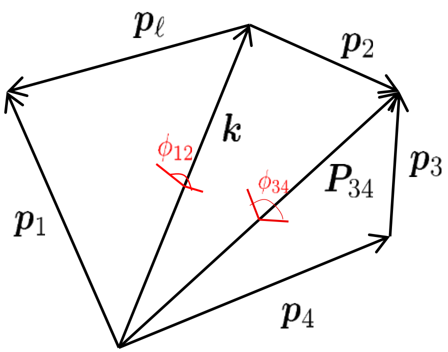


Fermi golden rule

$$Q = 2\int\prod_{j=1}^{4}\frac{\mathrm{d}\boldsymbol{p}_j}{(2\pi)^3}\int\frac{\mathrm{d}\boldsymbol{p}_\ell}{(2\pi)^3}\int\frac{\mathrm{d}\boldsymbol{p}_\nu}{(2\pi)^3}\,\varepsilon_\nu(2\pi)^4\delta(E_f-E_i)\delta(\boldsymbol{P}_f-\boldsymbol{P}_i)$$
$$\times f_1f_2(1-f_3)(1-f_4)(1-f_\ell)\frac{1}{2}\sum_{\text{spins}}|M_{fi}|^2$$

All quasiparticles on Fermi surface

$$Q=\frac{1}{(2\pi)^{14}}T^8 I\,p_{F\ell}m_\ell^*\prod_{j=1}^{4}p_{Fj}m_j^*\langle|M_{fi}|^2\rangle,\ \ I=\frac{11513\pi^8}{120960}$$



Phase space integration

$$\langle|M_{fi}|^2\rangle=4\pi\frac{8\pi^2}{p_1p_2p_3p_4p_\ell}\int\mathrm{d}k\int\mathrm{d}P_{34}\int_0^{2\pi}\mathrm{d}\phi_{12}\int_0^{2\pi}\mathrm{d}\phi_{34}\sum_{\text{spins}}|M_{fi}|^2$$

Medium effects

$$Q \approx 8.1 \times 10^{21} \left(\frac{m_N^*}{m_0}\right)^2 \left(\frac{m_p^* m_n^*}{m_o^2}\right) \left(\frac{n_p}{n_0}\right)^{1/3} \left(\frac{p_{F\ell}c}{\mu_\ell}\right) \Theta_{nN\ell} T_9^8 \, \alpha_U \text{ erg cm}^{-3} \text{ s}^{-1}$$

Voskresenskii & Senatorov (1986), Migdal et al. (1990),
Blaschke et al. (1995),
Voskresenskii (2001), Hanhart et al. (2000)

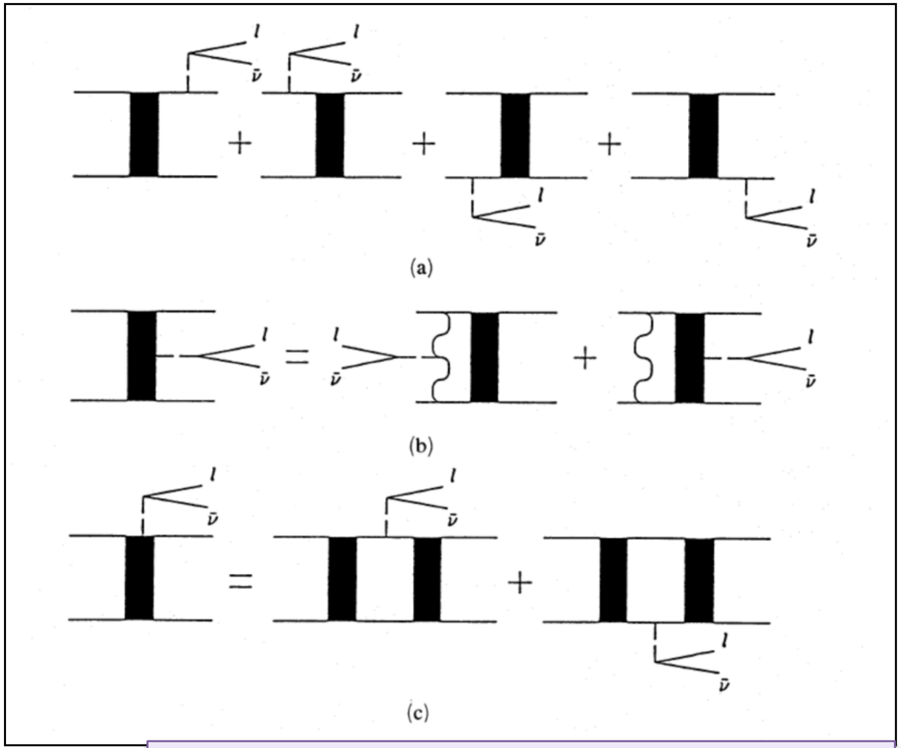
Effective masses

Baldo et al., 2014

T-matrix instead of OPE;
In-medium interaction

Blaschke et al. (1995)

$$\alpha_U^{(a)} \sim 1$$
$$\alpha_U^{(c)} \lesssim 1$$



From Blaschke et al. 1995

Additional channels

e.g., Voskresenskii (2001)

In case of soft pion mode
(b) processes dominate

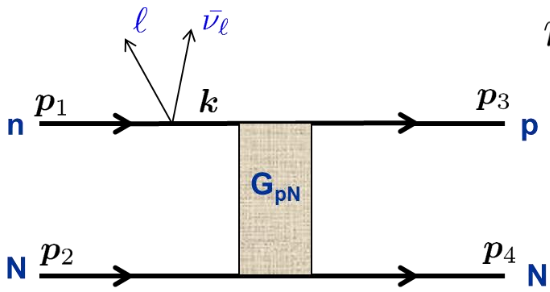
$$\alpha_U^{(b)} \gg 1$$

MMU in Blaschke et al., 2004

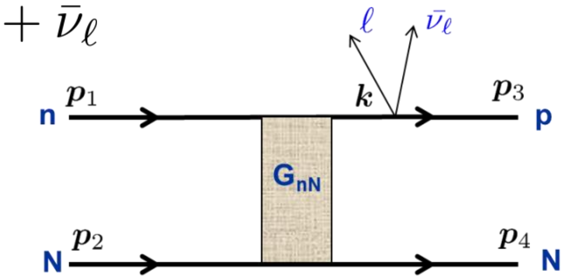
We consider (a) diagrams

Emission from the external leg

$$n + N \to p + N + \ell + \bar{\nu}_\ell$$



$+Exch.$



$$M_{fi} : \mathcal{G}(\boldsymbol{k}) \cdot \langle \boldsymbol{p}_3 \boldsymbol{p}_4 | \hat{G}_{\rm pN} | \boldsymbol{k} \boldsymbol{p}_1 \rangle$$

Propagation the intermediate nucleon line (*k*)

$$k = |\boldsymbol{p}_1 - \boldsymbol{p}_\ell| \geqslant p_{Fn} - p_{F\ell} > p_{Fp}$$

Proton propagator

$$\mathcal{G}_p = \frac{i}{E_n(p_1) - E_\ell(p_\ell) - E_p(k) - E_\nu}$$

$$p_{F\ell} + p_{Fp} \geq k \geq p_{Fn}$$

Neutron propagator

$$\mathcal{G}_p = \frac{i}{E_p(p_3) + E_\ell(p_\ell) - E_n(k) + E_\nu}$$

Beta-stability condition $E_n(p_1) - E_\ell(p_\ell) = E_{Fn} - E_{F\ell} = E_{Fp}$

$$\mathcal{G}_p = \frac{i}{\mu_p - E_p(k) - E_\nu}$$

Usual approximation (FM79) $\mathcal{G} \approx -\frac{i}{E_\ell(p_\ell)}$

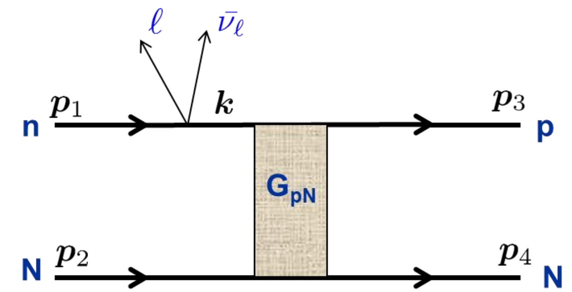
softening at $k \to p_{Fp}$

Amplification factor

Parabolic spectrum:

$$\mathcal{G} = \frac{i}{\mu_p - E_p(k) - \cancel{E_\nu}} \approx \frac{2m_{\rm p}^* i}{p_{\rm Fp}^2 - k^2}$$

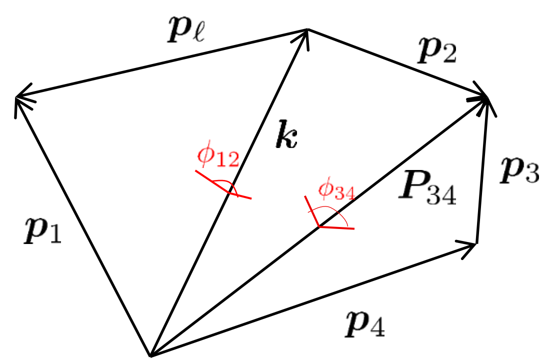
Standard approximation: $\mathcal{G} \approx -\dfrac{i}{E_\ell(p_\ell)} = -\dfrac{i}{\mu_\ell}$



$$M_{fi} \,:\, \mathcal{G}(\boldsymbol{k}) \cdot \langle \boldsymbol{p}_3 \boldsymbol{p}_4 | \hat{G}_{\rm pN} | \boldsymbol{k} \boldsymbol{p}_1 \rangle$$

$$\left(\frac{2m_{\rm p}^* \mu_\ell}{k^2 - p_{\rm Fp}^2} \right)^2 \gg 1, \quad k \to p_{\rm Fp}$$

$$\langle \cdot \rangle_{\rm ph.sp.} \to \int\!\!\int\!\!\int\!\!\int \cdot\, {\rm d}k\, {\rm d}P_{34}\, {\rm d}\phi_{12}\, {\rm d}\phi_{34}$$



Considerable enhancement in a part of phase-space (backward emission)

$$p_{\rm Fn} + p_{\rm F\ell} \geqslant k \geqslant p_{\rm Fn} - p_{\rm F\ell}$$

Amplification factor

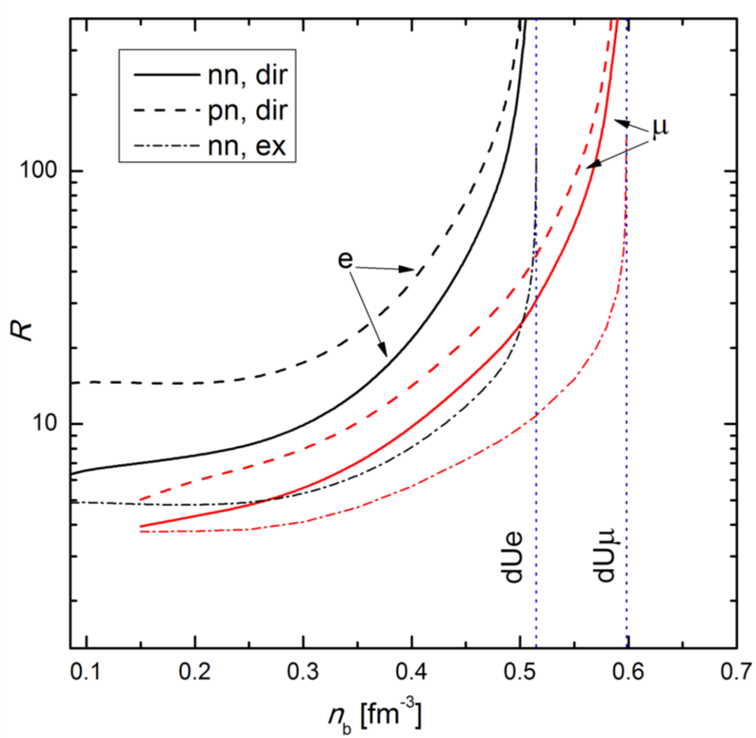
$$R = \langle \mathcal{G}^2(k) \mu_\ell^2 \rangle_{\rm ph.sp.}$$

Amplification factor

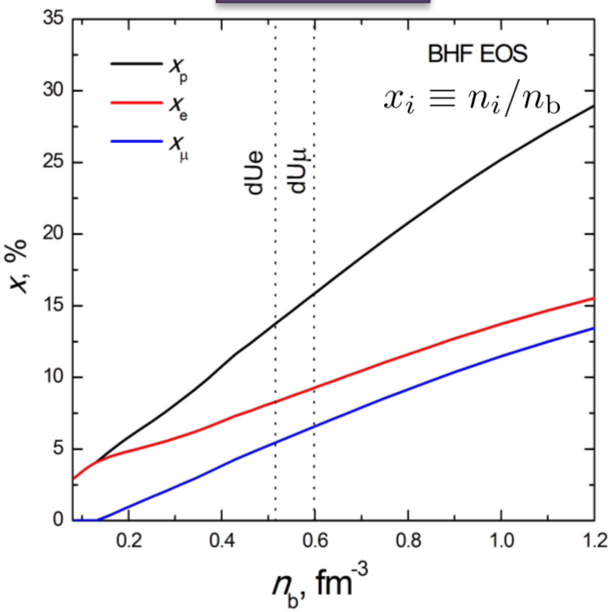
$$R^{(\mathrm{nn})}_{\mathrm{dir}} = \left(\frac{2m^*_\mathrm{p}\mu_\ell}{p^2_{\mathrm{Fn}}}\right)^2 \frac{p^4_{\mathrm{Fn}}}{2p_{\mathrm{F}\ell}} \int\limits_{p_{\mathrm{Fn}}-p_{\mathrm{F}\ell}}^{p_{\mathrm{Fn}}+p_{\mathrm{F}\ell}} \frac{\mathrm{d}k}{(p^2_{\mathrm{Fp}}-k^2)^2}$$

Near the dUrca threshold

$$R^{(\mathrm{nn})}_{\mathrm{dir}} \to \left(\frac{2m^*_\mathrm{p}\mu_\ell}{p^2_{\mathrm{Fn}}}\right)^2 \frac{p^3_{\mathrm{Fn}}}{4p_{\mathrm{Fp}}p_{\mathrm{F}\ell}(p_{\mathrm{Fn}}-p_{\mathrm{F}\ell}-p_{\mathrm{Fp}})}$$



fractions



$$m^*_\mathrm{p} \approx 0.8 m_\mathrm{N}$$

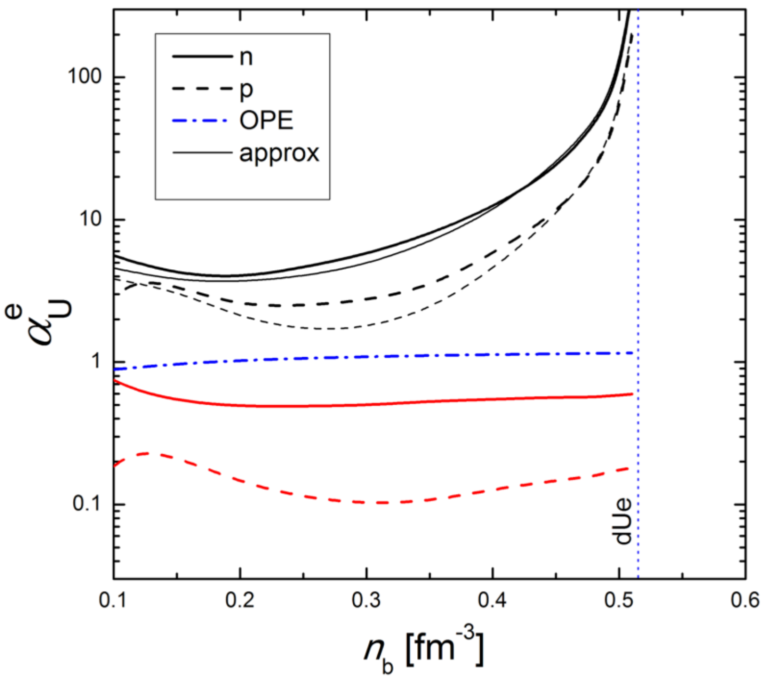
Neutrino emission is enhances everywhere in the core

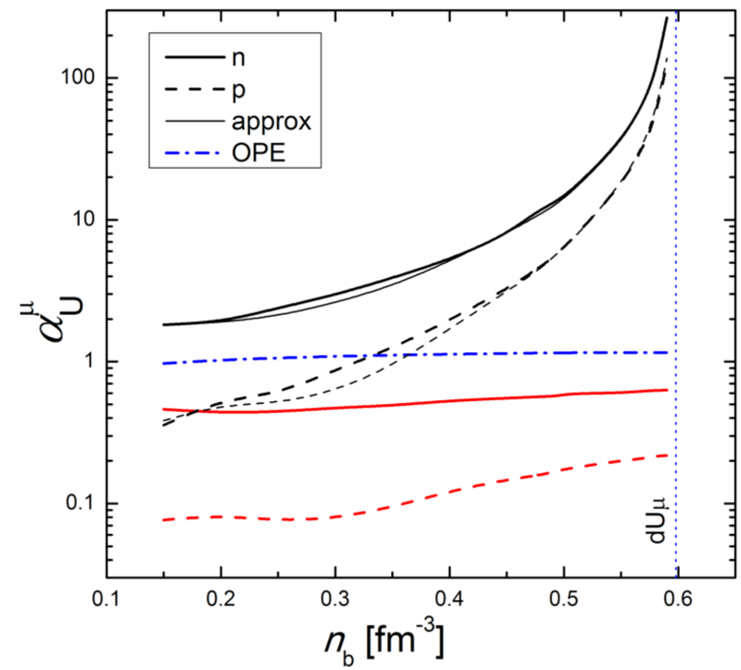
Universal effect – due to beta-equilibrium

Results. Neutrino emission. BHF

$$n + N \to p + N + \ell + \bar{\nu}_\ell, \quad p + N + \ell \to n + N + \nu_\ell$$

$$Q \approx 8.1 \times 10^{21} \left(\frac{m_N^*}{m_0}\right)^2 \left(\frac{m_p^* m_n^*}{m_0^2}\right) \left(\frac{n_p}{n_0}\right)^{1/3} \left(\frac{p_{F\ell}c}{\mu_\ell}\right) \Theta_{nN\ell} T_9^8 \, \alpha_U \text{ erg cm}^{-3} \text{ s}^{-1}$$





G-matrix reduces

Van Dalen, 2001, Hanhart et al., 2000, Blaschke et al., 1995

R-factor strongly enhances

Density dependence of mUrca

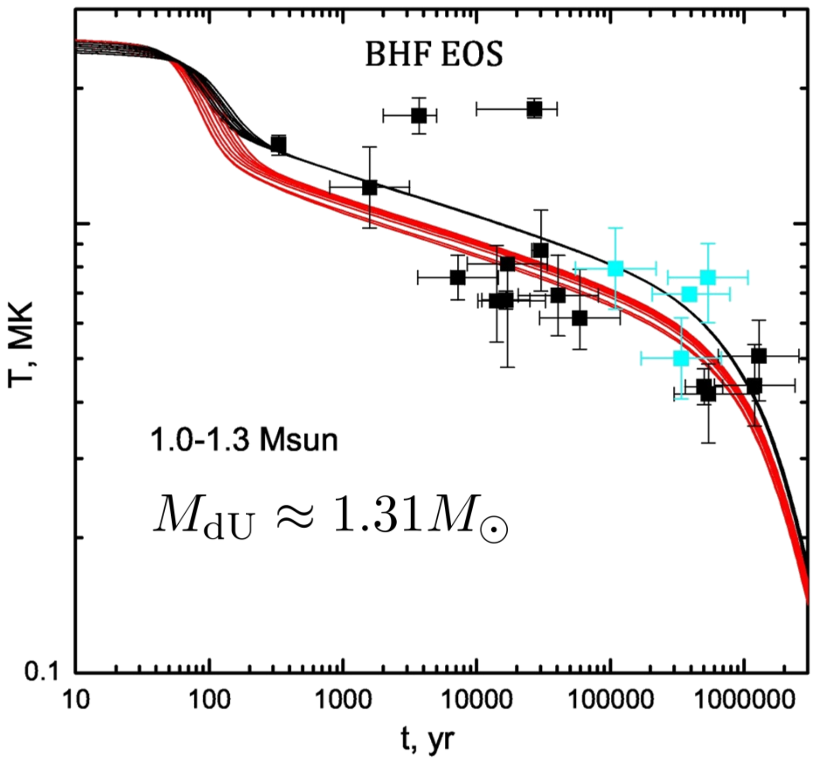
Approximation:

$$\alpha_U = \alpha_U^{(0)} R(n_b, m^*, x_i)$$

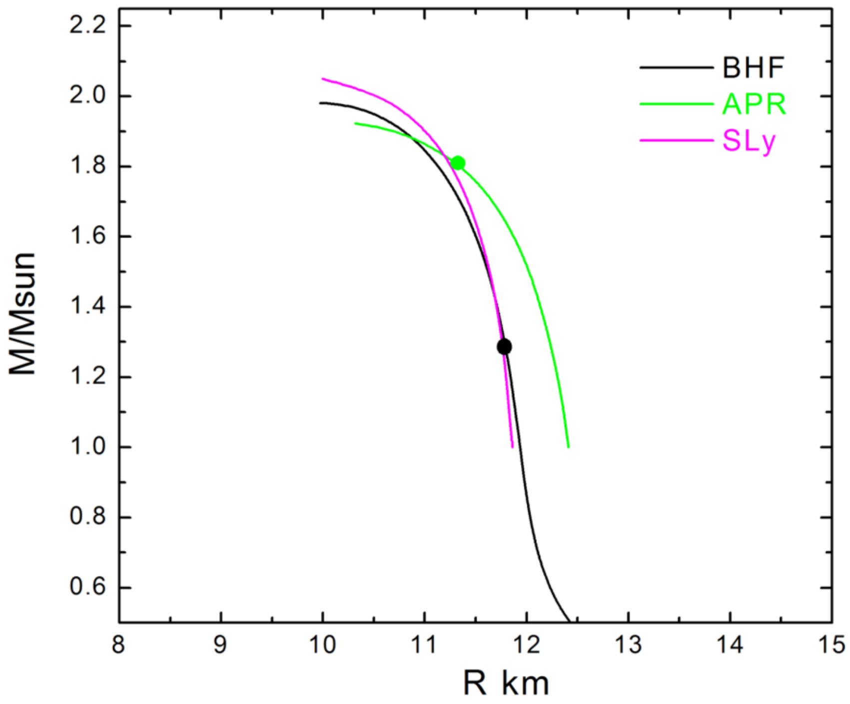
$$\langle |M_{fi}|^2 \rangle \to \langle |\mathcal{G}(k)\mu_\ell|^2 \rangle \langle |G_{fi}/\mu_\ell|^2 \rangle$$

«Standard» cooling. BHF EOS

Cooling curves



Mass-radius diagramm



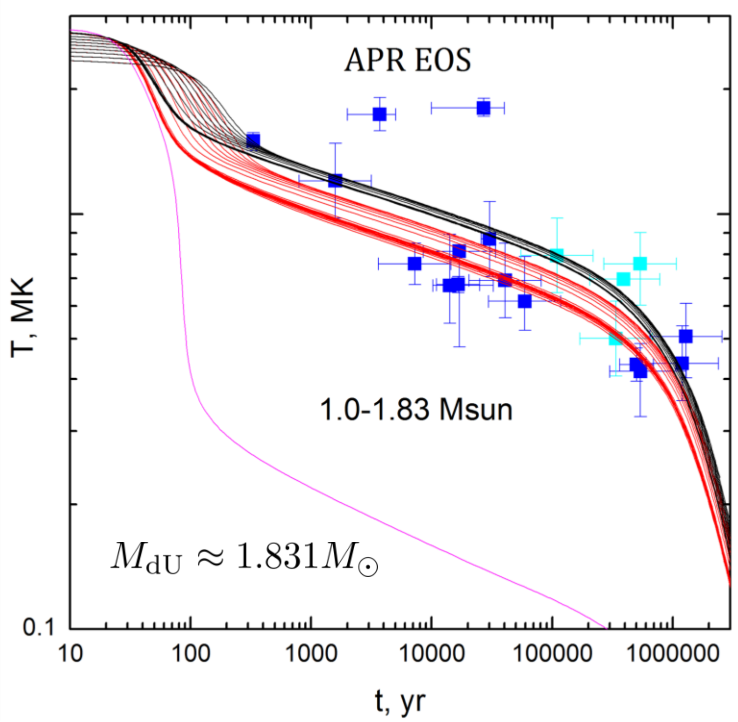
"Self-consistent" cooling and EOS calculations

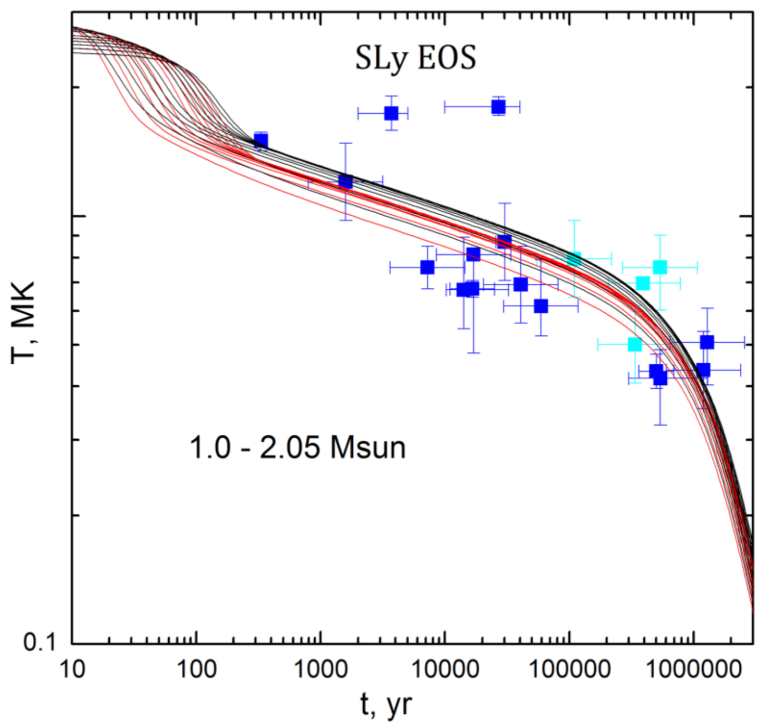
«Standard» cooling. Other EOSs

Approximate treatment

$$\alpha_U = \alpha_U^{(0)} R(n_b, m^*, x_i)$$

Quantities $\alpha_U^{(0)} = const$ **as for BHF EOS**





Standard cooling is not so standard

No direct Urca:
Cooling enhancement is weaker

Thank you

Thank you!

Bethe-Brueckner-Goldstone equation in partial waves

Basis $|J(\ell S)M\rangle$

G **– is a diagonal matrix over** J, S, M, P **m.el. :** $G^{JS}_{\ell\ell'}(P,p,p';\omega)$

$$G^{JS}_{\ell\ell'}(P,p,p';w) = V^{JS}_{\ell\ell'}(p,p') + \sum_{\tilde{\ell}} \int \mathrm{d}k\, k^2 V^{JS}_{\ell\tilde{\ell}}(p,k) \frac{\overline{Q}(P,k)}{\omega - \overline{E}(P,k)} G^{JS}_{\tilde{\ell}\ell'}(P,k,p';\omega)$$

$\overline{Q}, \overline{E}$ **– averaging over directions of** $\boldsymbol{P}$

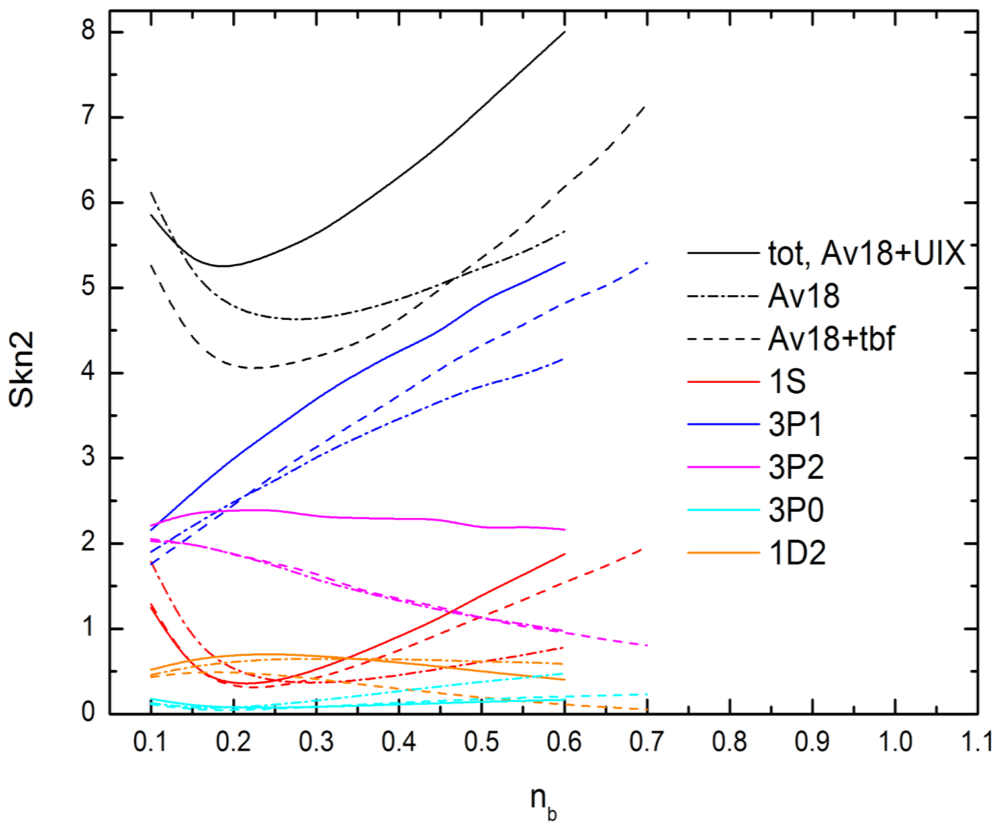
Equations for G – matrix and single-particle potential are solved self-consistently

$$\langle SM'_S|G|SM_S\rangle = \sum i^{\ell'-\ell} C^{JM}_{\ell'\lambda' SM'_S} C^{JM}_{\ell\lambda SM_S} Y_{\ell'\lambda'}(\hat{p}') Y^*_{\ell\lambda}(\hat{p}) \langle \ell' p'|G^{JS}(P)|\ell p\rangle$$

Partial waves

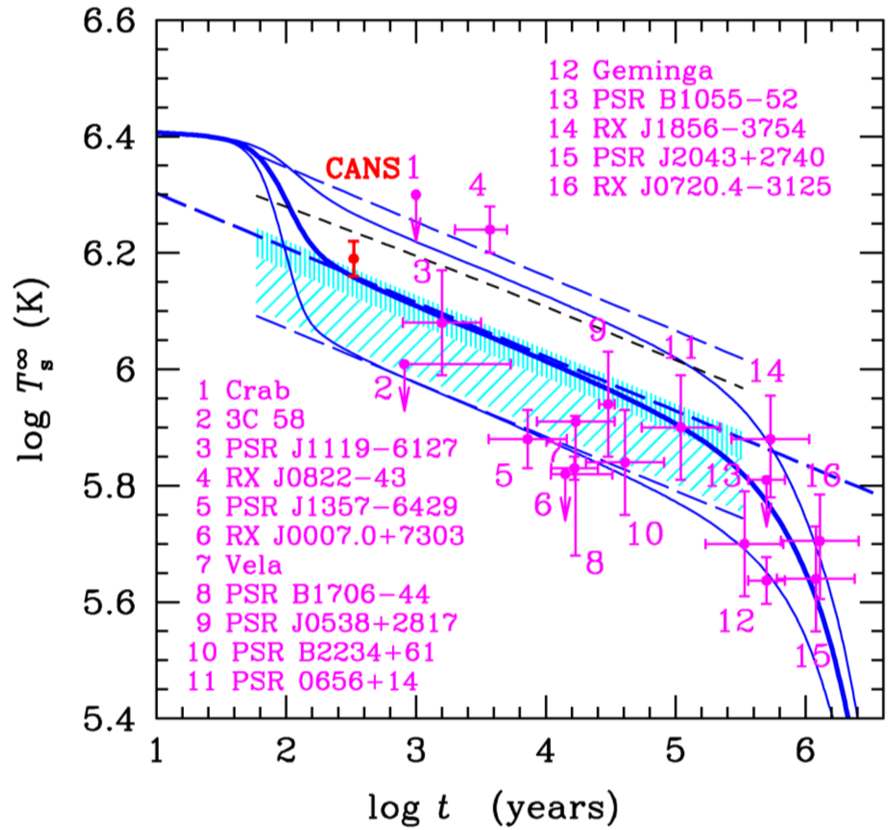
$$\nu_{ci}^{(\kappa)} = \frac{64 m_c^* m_i^{*2} (k_B T)^2}{5 m_N^2 \hbar^3} S_{\kappa ci},$$

$$\mathcal{Q} = \frac{1}{4} \sum_L \frac{1}{4\pi^2} \mathcal{P}_L(\hat{p}\hat{p}') \sum i^{\ell'-\ell+\bar{\ell}-\bar{\ell}'} \Pi_{\ell\ell'\bar{\ell}\bar{\ell}'} \Pi^2_{J\bar{J}} C^{L'0}_{\ell'0\bar{\ell}'0} C^{L0}_{\ell 0\bar{\ell}0} \left\{ \begin{array}{ccc} \bar{\ell} & S & \bar{J} \\ J & L & \ell \end{array} \right\} \left\{ \begin{array}{ccc} \bar{\ell}' & S & \bar{J} \\ J & L & \ell' \end{array} \right\} G^{JS}_{\ell\ell'} \left(G^{\bar{J}S}_{\bar{\ell}\bar{\ell}'} \right)^*$$



Standard cooling and observations

- $1.4\ M_\odot$ APR
- $x < 0.5$
- $0.5 < x < 0.7$
- $1.4\ M_\odot$ SC
- $f_\ell = 30\ \&\ f_\ell = 1/30$



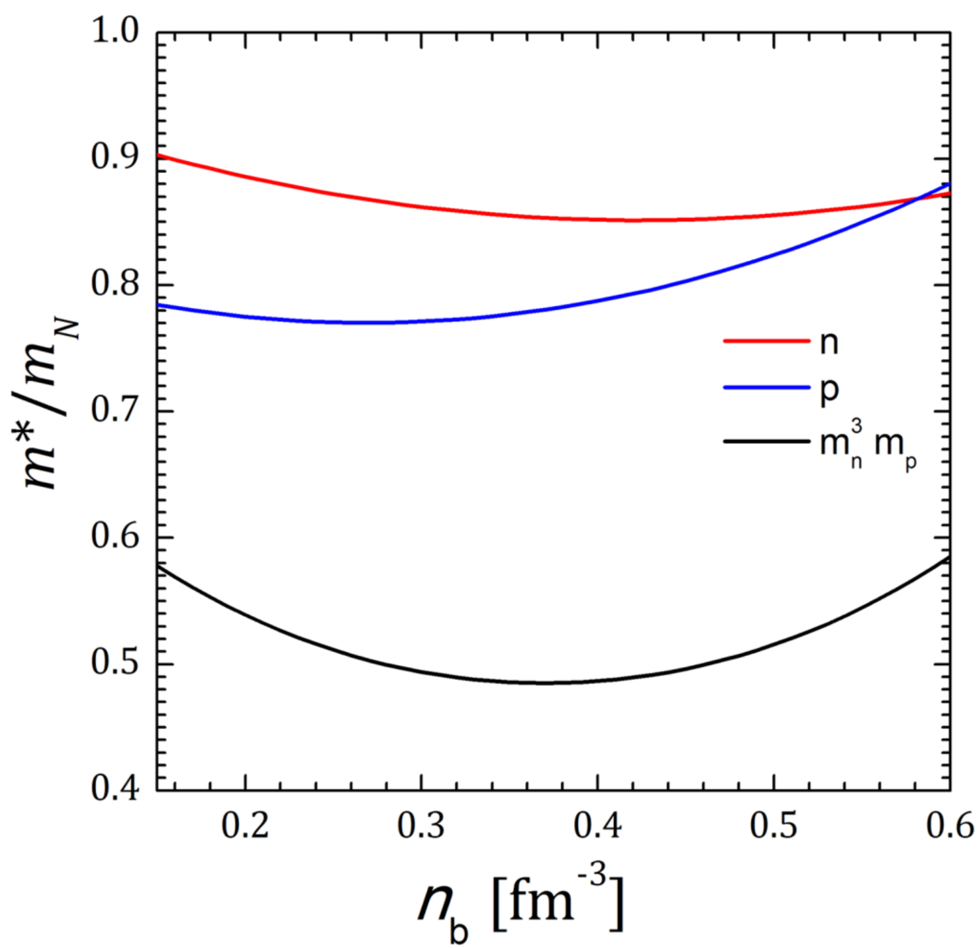
Fast neutrino emission mechanisms

ν	f_ℓ
DU	$10^6 - 10^7$
π	$10^2 - 10^3$
K	$10^4 - 10^5$

Too fast

Results. Effective masses

BHF effective masses



$$m^* = \left(\frac{1}{p}\frac{\mathrm{d}\epsilon(p)}{\mathrm{d}p}\right)^{-1}_{p=p_F}$$

Constant effective mass is a good approximation

FM79 m*=0.8

See

Baldo et al., 2014;

Results. In-medium cross-sections

"cross-section" at Fermi surface

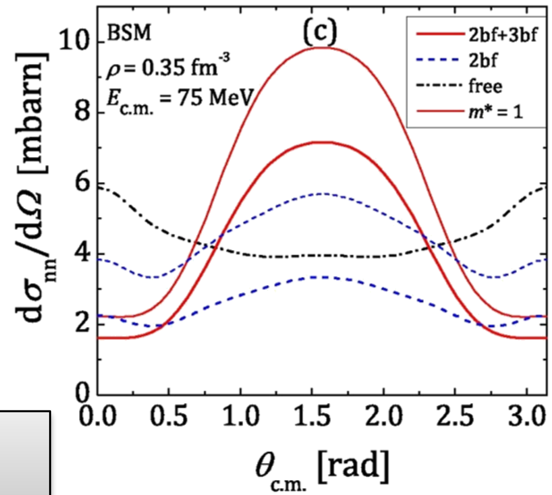
$$\frac{d\sigma_{\alpha\beta}}{d\Omega} = \frac{m^{*2}_{\alpha\beta}}{16\pi^2\hbar^4}\mathcal{Q}_{\alpha\beta}$$

2bf decrease cross-sections
3bf increase

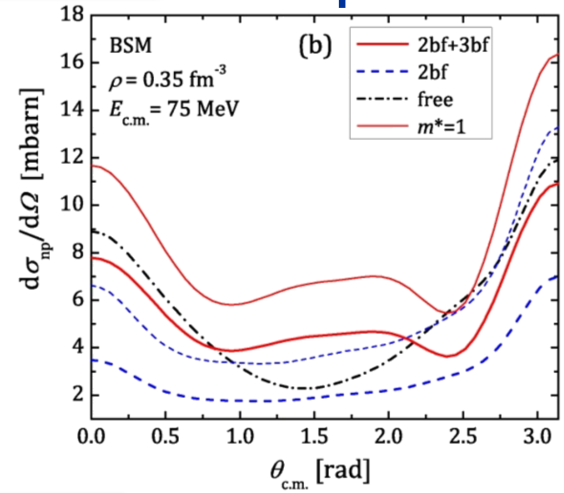
$$\sigma_{ab} = \frac{1}{1+\delta_{ab}}\int_{(4\pi)} \mathrm{d}\Omega \frac{\mathrm{d}\sigma_{ab}}{\mathrm{d}\Omega}$$

differential

nn

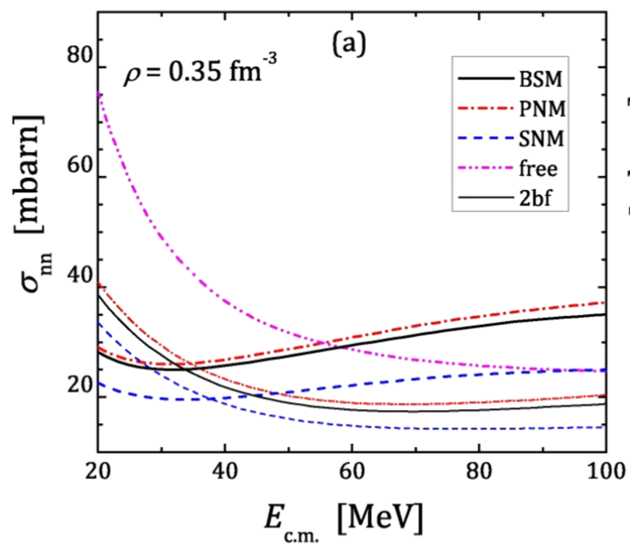


np

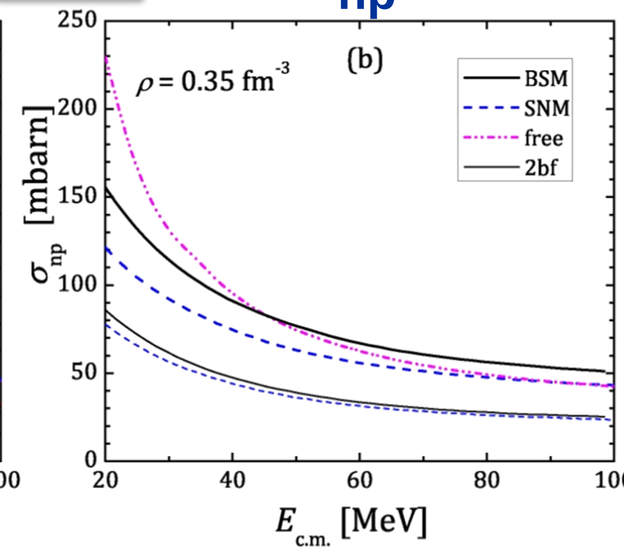


total

nn

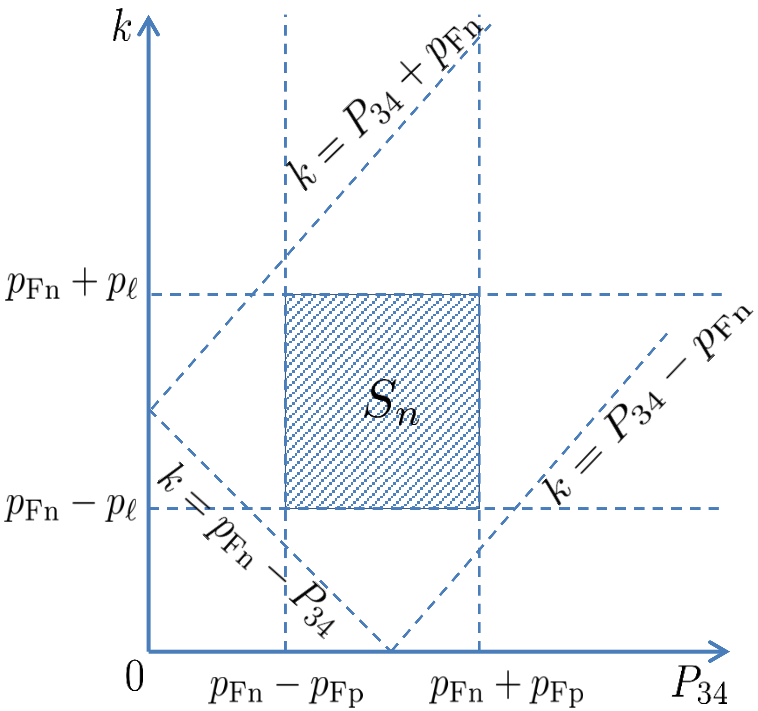


np

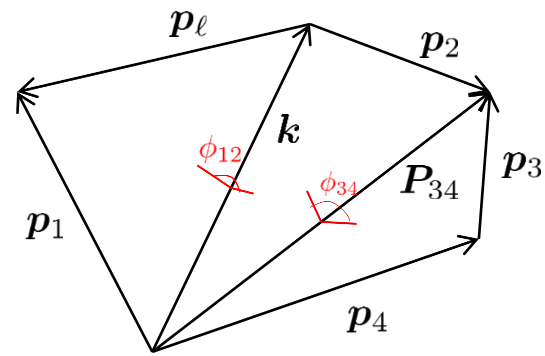


Phase space integration. Neutron branch

$$\int_{p_{\rm Fn}-p_\ell}^{p_{\rm Fn}+p_\ell} \mathrm{d}\,k \int_{p_{\rm Fn}-p_{\rm Fp}}^{p_{\rm Fn}+p_{\rm Fp}} \mathrm{d}\,P_{34}$$



$$S_n = 4 p_\ell p_{\rm Fp}$$



$$p_1 = p_2 = p_4 = p_{\rm Fn}$$

$$p_3 = p_{\rm Fp}$$

No dUrca

$$p_{\rm Fn} \geqslant p_{\rm Fp} + p_\ell$$

$$p_{\rm Fn} > p_{\rm Fp} \geqslant p_{\rm F\ell}$$

Three triangle relations

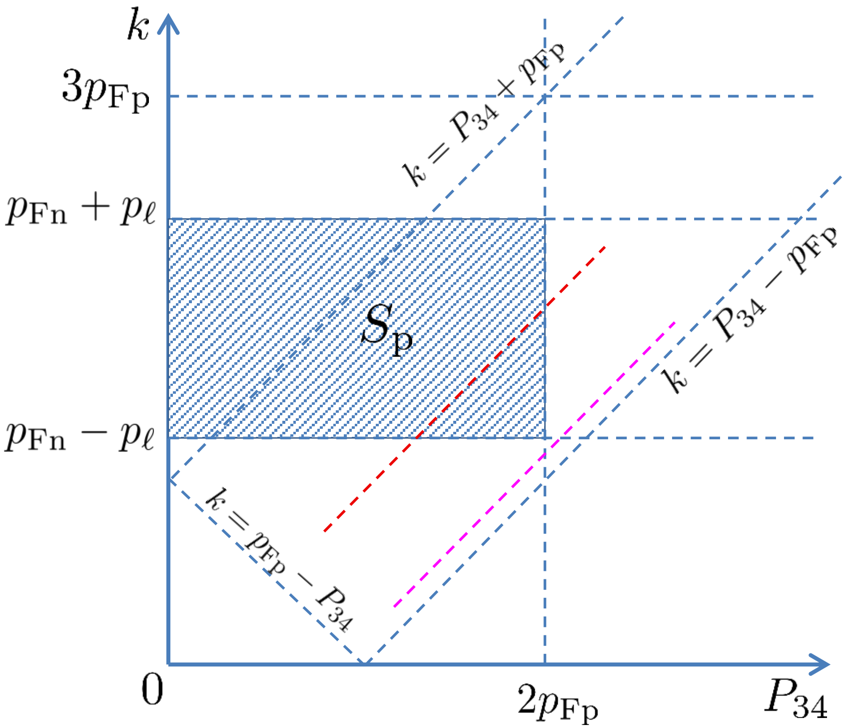
$$p_{\rm Fn} - p_\ell \leqslant k \leqslant p_{\rm Fn} + p_\ell$$

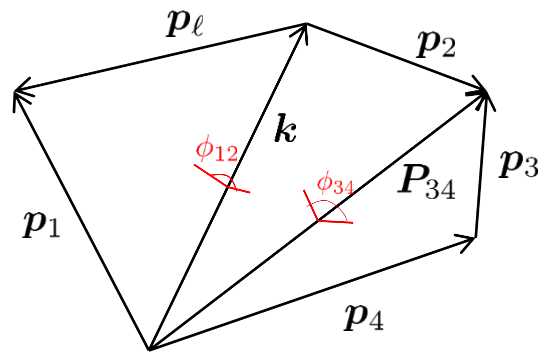
$$p_{\rm Fn} - p_{\rm Fp} \leqslant P_{34} \leqslant p_{\rm Fn} + p_{\rm Fp}$$

$$|P_{34} - p_{\rm Fn}| \leqslant k \leqslant P_{34} + p_{\rm Fn}$$

Phase space integration. Proton branch

$$\int_{p_{\rm Fn}-p_\ell}^{\min(p_{\rm Fn}+p_\ell,3p_{\rm Fp})} {\rm d}\,k \int_{k-p_{\rm Fp}}^{2p_{\rm Fp}} {\rm d}\,P_{34}$$





$$p_2 = p_3 = p_4 = p_{\rm Fp}$$
$$p_1 = p_{\rm Fn}$$

No dUrca

$$p_{\rm Fn} \geqslant p_{\rm Fp} + p_\ell$$
$$p_{\rm Fn} > p_{\rm Fp} \geqslant p_{\rm F\ell}$$

Three triangle relations

$$p_{\rm Fn} - p_\ell \leqslant k \leqslant p_{\rm Fn} + p_\ell$$
$$0 \leqslant P_{34} \leqslant 2p_{\rm Fp}$$
$$|P_{34} - p_{\rm Fp}| \leqslant k \leqslant P_{34} + p_{\rm Fp}$$

$$S_{\rm p} = \begin{cases} 0, & 3p_{\rm Fp} < p_{\rm Fn} - p_\ell \\ (3p_{\rm Fp} + p_\ell - p_{\rm Fn})^2/2, & p_{\rm Fn} + p_\ell < 3p_{\rm Fp} < p_{\rm Fn} + p_\ell \\ 2p_\ell(3p_{\rm Fp} - p_{\rm Fn}), & p_{\rm Fn} + p_\ell < 3p_{\rm Fp} \end{cases}$$

Heat Blanket

Partially accreted envelopes

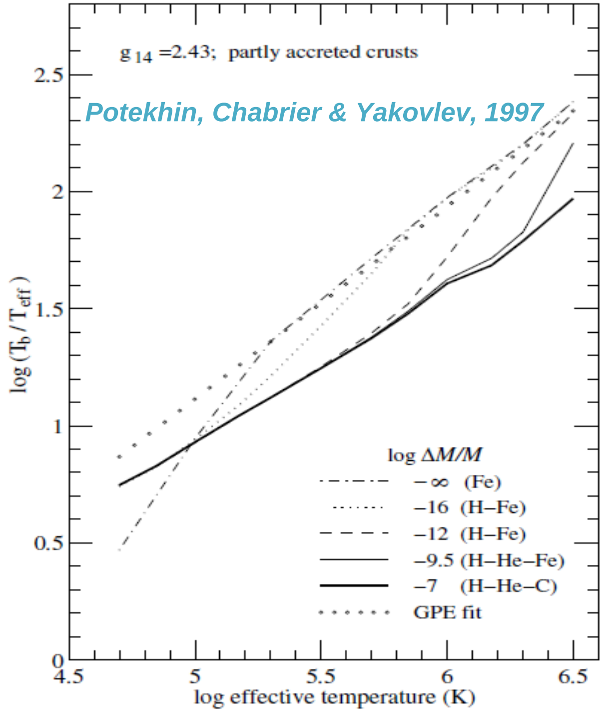


Fig. 8. Temperature increase through partly accreted NS crusts.

Accreted stars look hotter

Dipole magnetic field

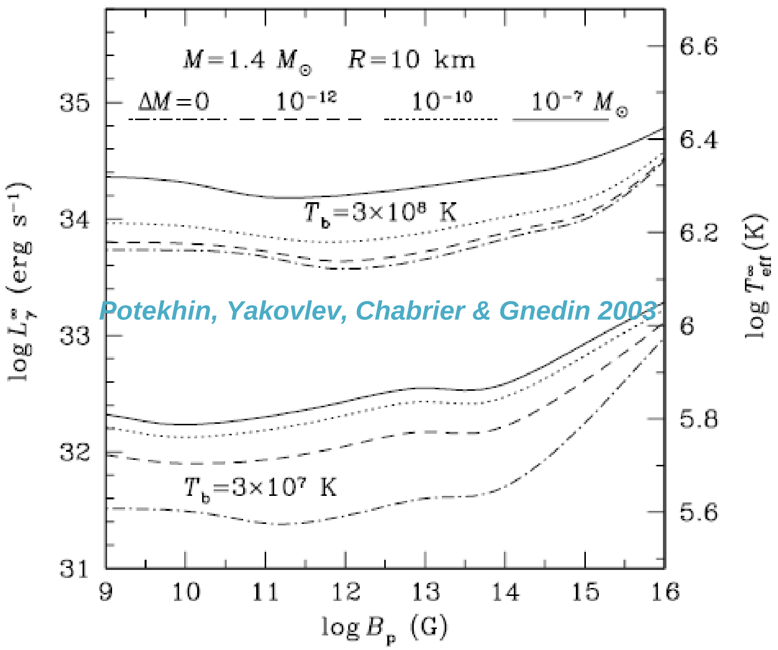
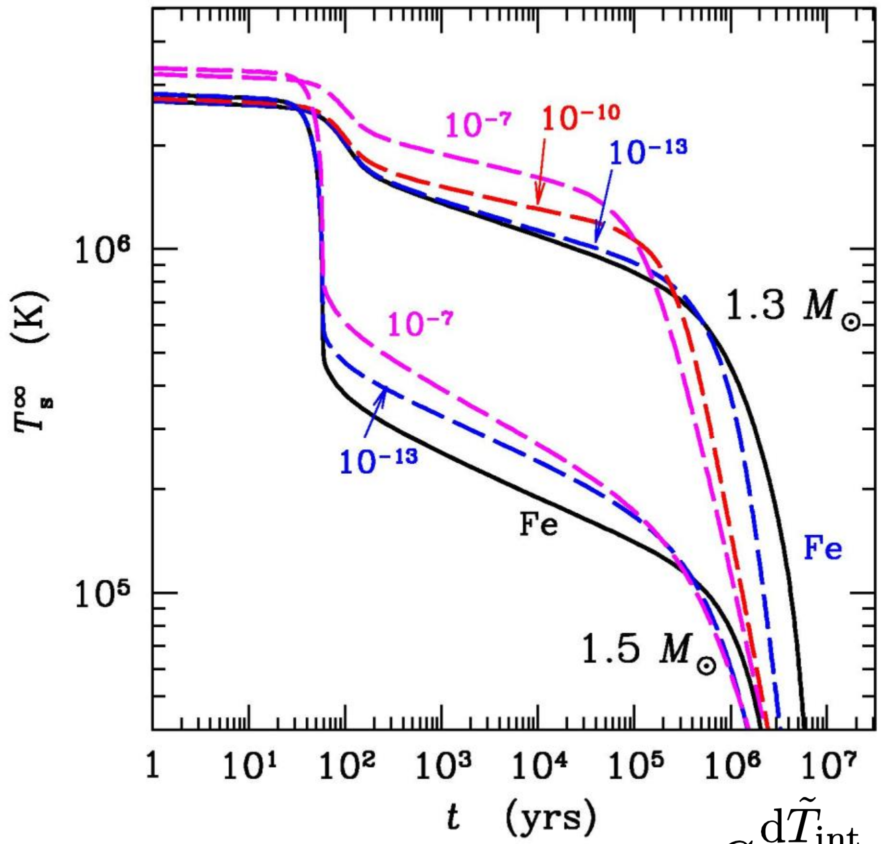


FIG. 8.—Photon surface luminosity (redshifted as detected by a distant observer: *left-hand axis*; redshifted effective surface temperature: *right-hand axis*) of a canonical NS with a dipole magnetic field, for two values of T_b and four models of the heat-blanketing envelope (accreted mass $\Delta M = 0$, 10^{-12}, 10^{-10}, or $10^{-7}\, M_\odot$) vs. magnetic field strength at the magnetic pole.

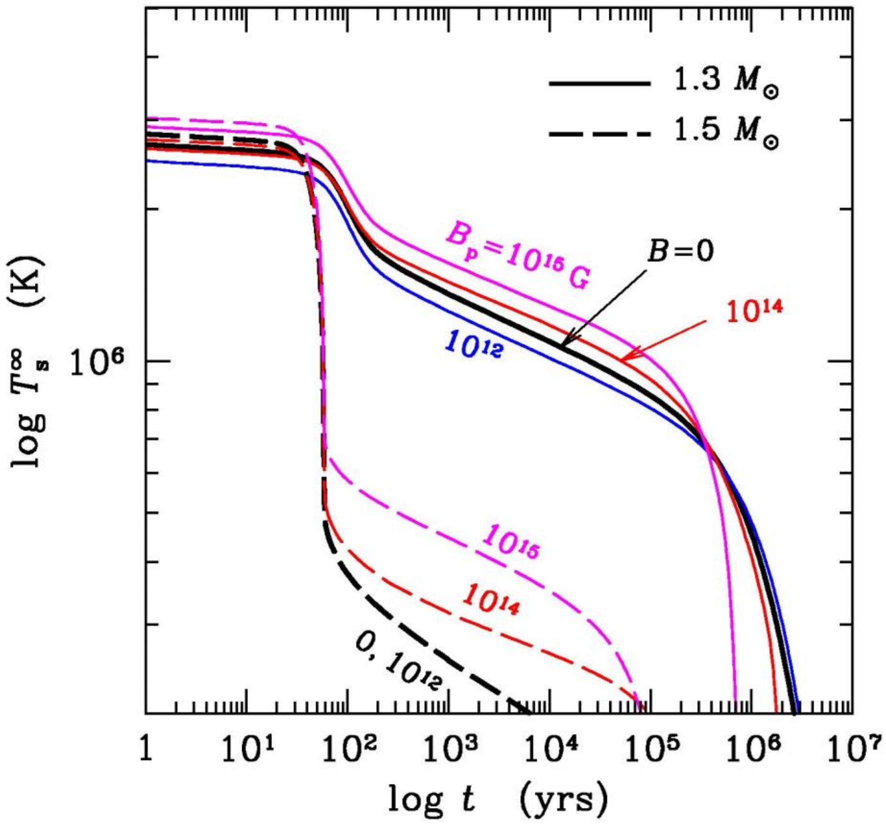
Stars look same from inside look different from outside

Effects of the heat blanket

Accreted envelopes (ΔM/M)



Magnetic field



$$C\frac{\mathrm{d}\tilde{T}_{\mathrm{int}}}{\mathrm{d}t} = -L_s - L_\nu$$

$$L_{\mathrm{s}}^{\infty}(T_{\mathrm{s}}) \quad = \quad 4\pi\sigma T_{\mathrm{s}}^{4} R^{2}\,(1-x)\,,$$

Superfluidity. Impact on cooling.

Two main effects

$$C\frac{\mathrm{d}\tilde{T}_{\rm int}}{\mathrm{d}t} = -L_s - L_\nu$$

I. Damping of the traditional neutrino processes

$$L_\nu \to L_\nu \times \exp\left(-\alpha T_c/T\right), \quad T \ll T_c$$

Modification of $C, \ \kappa$

Yakovlev&Pethick, 2004

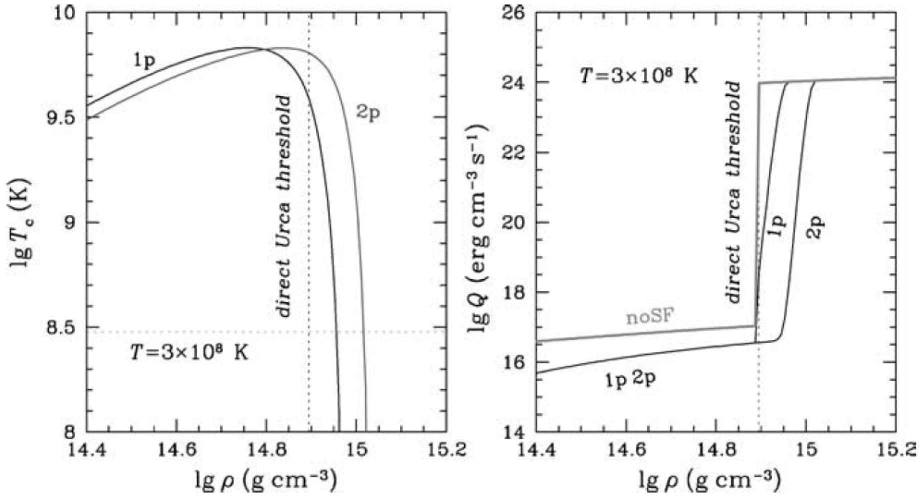
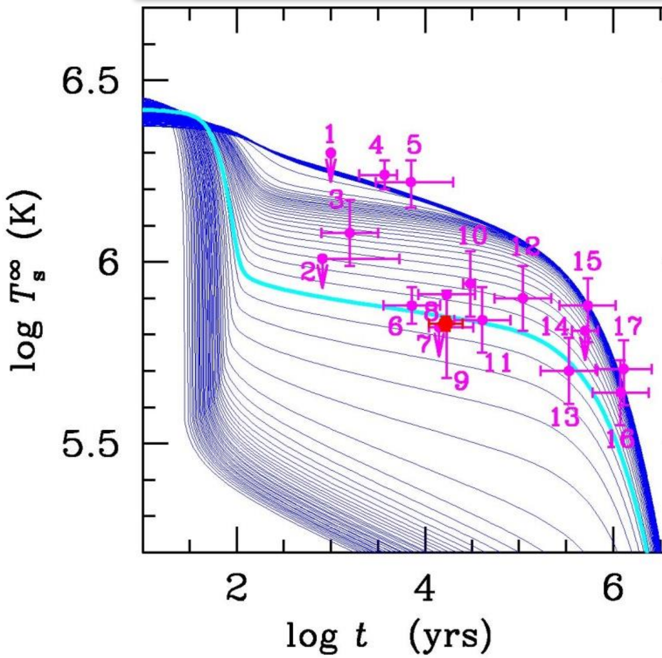


Figure 5 Left: Superfluid transition temperature versus density for two models (1p and 2p) for proton superfluidity in the neutron star core. Right: Neutrino emissivity profiles in the core at $T = 3 \times 10^8$ K for nonsuperfluid matter (noSF) and for matter with superfluid protons (models 1p or 2p).

Proton superfluidity only (for a moment)

Smooth transition to DUrca



From $1.1M_\odot$ to $1.9M_\odot$ with step $0.01M_\odot$

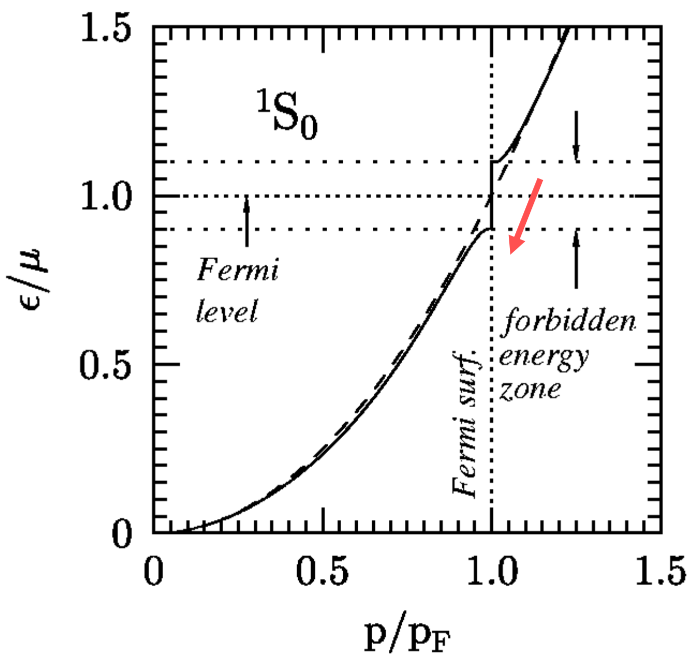
Cooper pairing formation emission

Two main effects

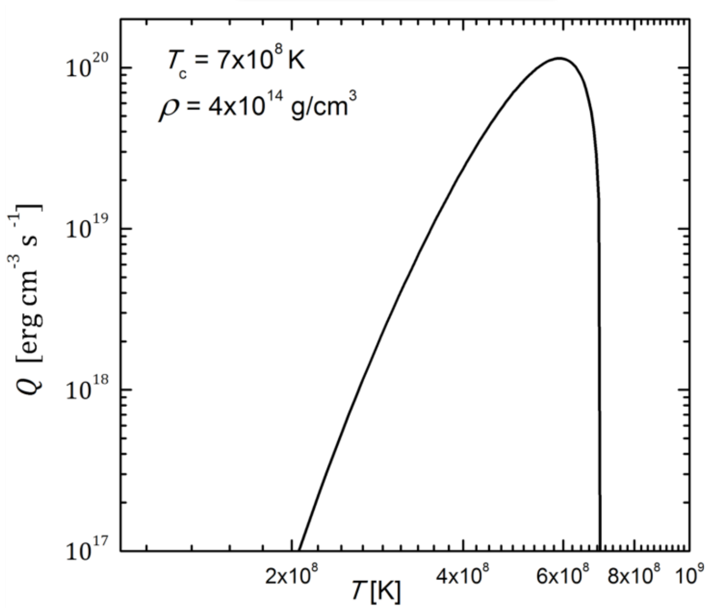
II. New channel of neutrino emission due to Cooper pairing

Flowers, Ruderman and Sutherland (1976) $\tilde{N} + \tilde{N} \rightarrow \nu + \bar{\nu}$

$$Q_0^{(\mathrm{CP})} = 1.17 \times 10^{21} \frac{m_\mathrm{n}^*}{m_\mathrm{n}} \frac{p_{\mathrm{Fn}}}{m_\mathrm{n} c} T_9^7 \mathcal{N}_\nu R\left(\frac{\Delta}{T}\right) \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{s}}$$



CPF emissivity



Maximal at $T \sim 0.7 T_c$

Cooper pairing formation emission

Suppression by collective effects

$$Q^{(\mathrm{CP})} = q\, Q_0^{(\mathrm{CP})}$$

Leinson & Perez 2006, Kolomeitsev & Voskresensky 2010, Steiner & Reddy, 2

Singlet paring

$$Q_s^{(CP)} \propto \left(\frac{4}{81} \left(\frac{v_\mathrm{F}}{c} \right)^4 C_\mathrm{V}^2 + \frac{6}{7} \left(\frac{v_\mathrm{F}}{c} \right)^2 C_\mathrm{A}^2 \right) \qquad q_s \ll 1$$

Triplet paring

$$Q_t^{(\mathrm{CP})} \propto (C_\mathrm{V}^2 + 2C_\mathrm{A}^2)$$

Page et al. 2009 $q_t = 0.76$

$$C_\mathrm{V} = 1, \quad C_\mathrm{A} = 1.26$$

Leinson 2010 $q_t = 0.19$

Divisions of responsibility

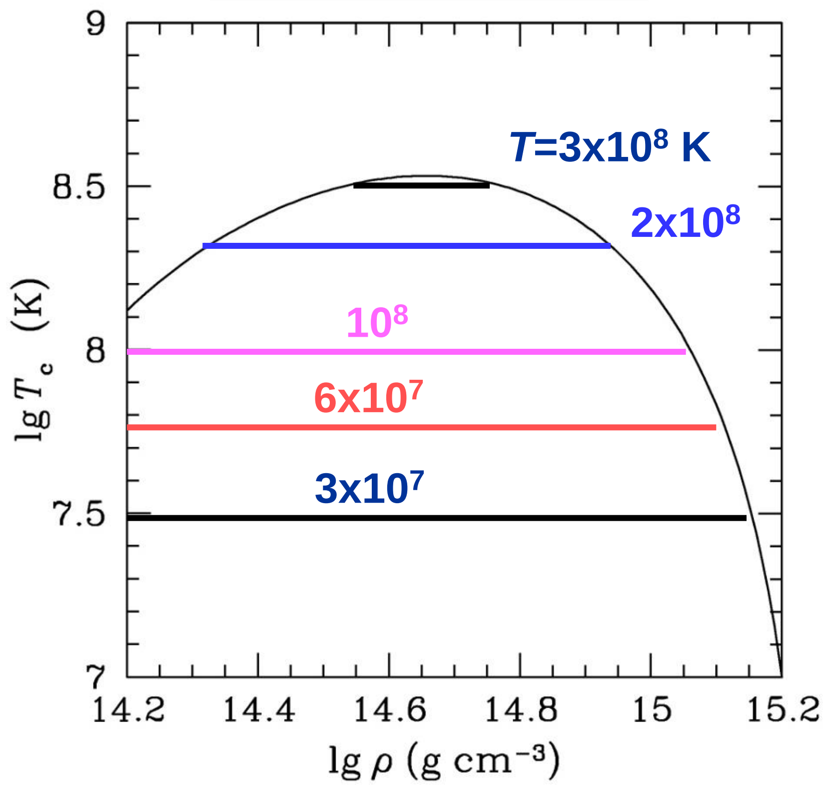
$^1S_0$ proton superfluidity: effectively damp standard neutrino reactions which involves protons: MUrca, Durca, pp and np bremsstrahlung

$^3P_J/^3F_J$ neutron superfluidity: moderately enhance neutrino emission with respect ot MUrca

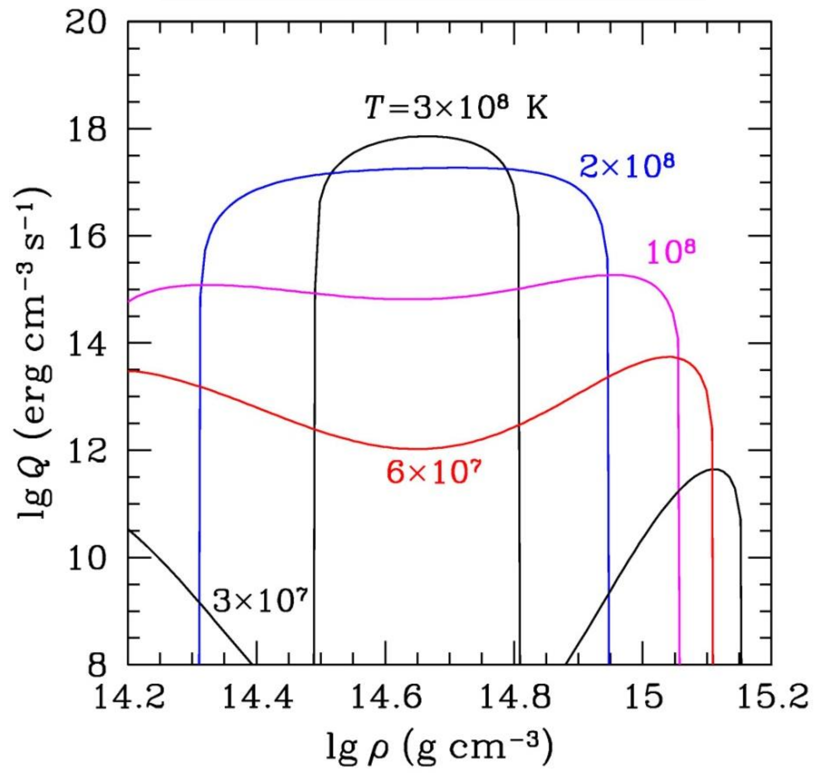
CPF emission of triplet neutron pairing

$$L_{\rm CP} = \int Q_{\rm CP}\, {\rm d}V \propto T^7 \cdot T$$ – *similar to slow cooling*

Bell-shaped profile



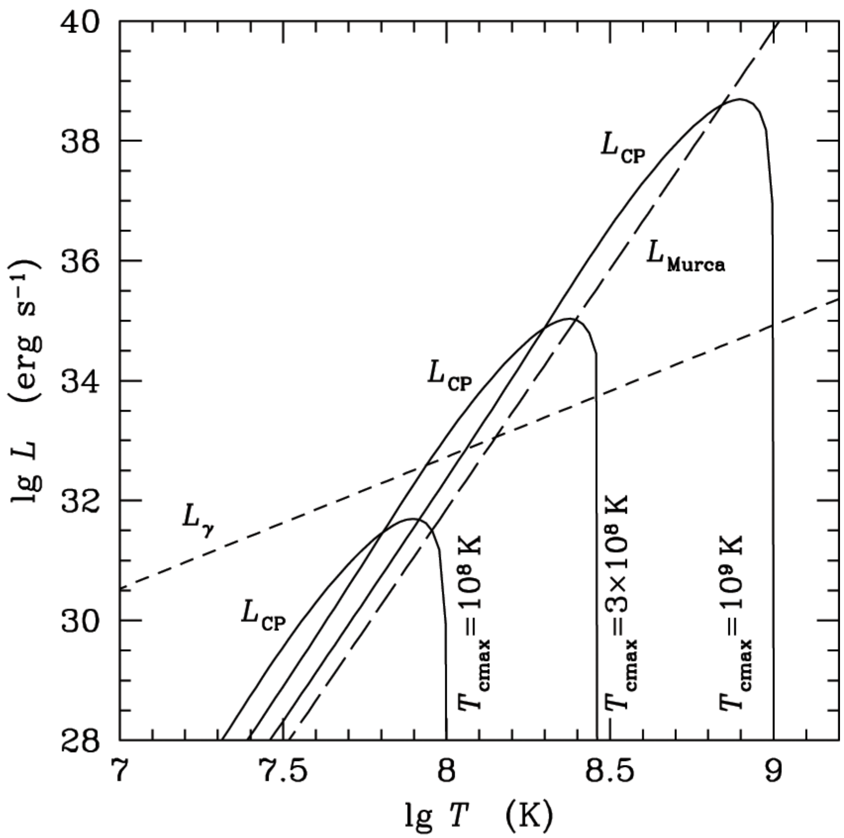
Distribution over the core



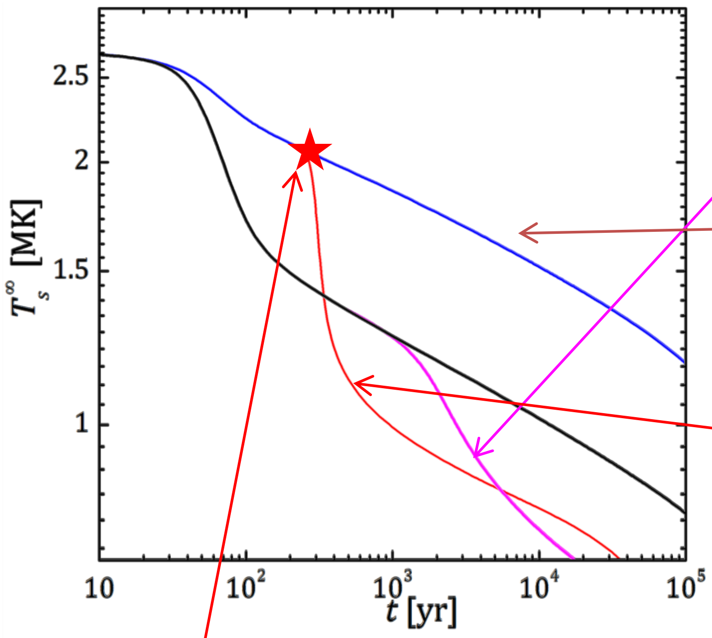
CPF neutrino emission

$$L_\nu^{Cooper} \sim (10-100)\ L_\nu^{Murca} \propto T^8$$

Neutrino emission due to Cooper pairing of neutrons can be 10—100 times stronger than Murca in non-superfluid NSs



Nucleon superfluidity and cooling



Neutron superfluidity:

accelerate cooling with CPF

Proton superfluidity:

decelerate cooling
CPF is unimportant

Together:

Sharp increase of the cooling rate

Cooling regulators

$T_{\rm cn}^{\rm max}$ **and cooling rate at** $T < T_{\rm cn}^{\rm max}$ **– turn on point**

CPF luminosity (*q parameter*)

Critical temperature profile

– cooling rate (slope)

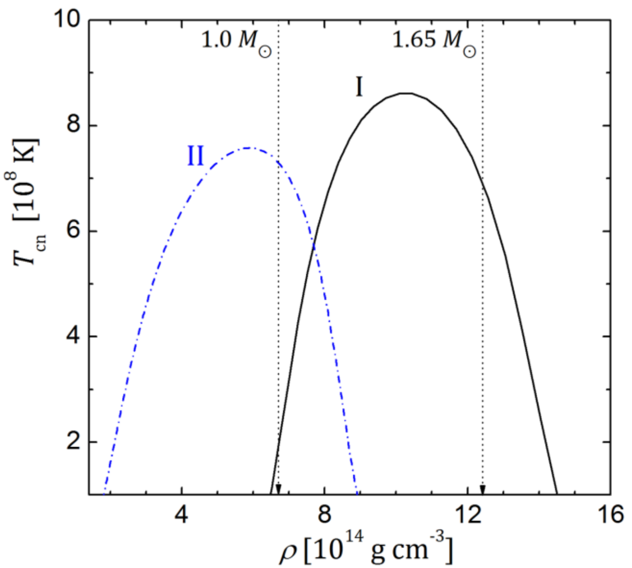
Model of the star

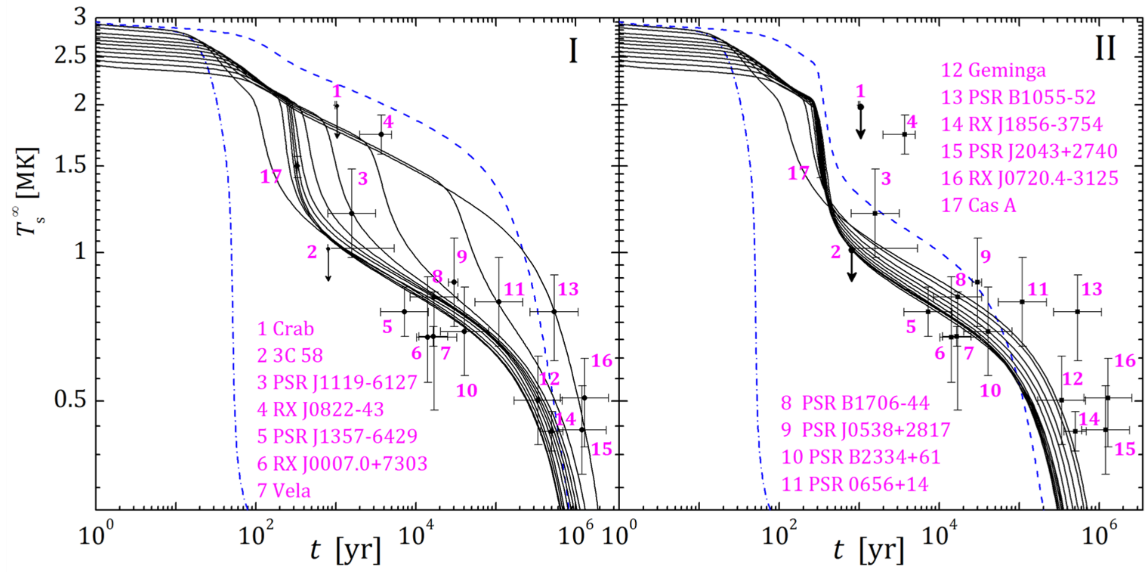
Minimal cooling

All isolated neutron stars without fast cooling

Page et al. 2004,2009
Gusakov et al. 2004

Strong proton, moderate neutron superfluidity





Hot stars: Need to shift superfluidity towards high densities

Gusakov et al. 2004