Shear viscosity in crystalline superconductors

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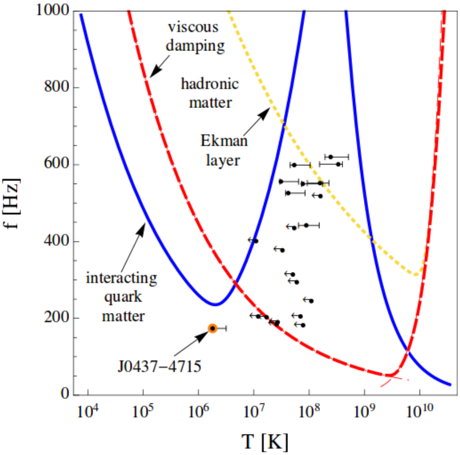
Introduction

- Transport properties are sensitive to the low energy degrees of freedom and provide information about the phases of matter
- Complementary to the EoS
- Will present a calculation of the shear viscosity in a color superconducting phase of quark matter (work in preparation with Sreemoyee Sarkar at TIFR)

Motivation

- From *Alford, Schwenzer (2013)*



Alford, Andersson, Degenaar, Haskell, Ho, Jaikumar, Mahmoodifar, Rupak, Schwenzer, Steiner, Strohmayer..

Motivation

- Some neutron stars lie in the linear instability region for typical hadronic matter
- Not necessarily a problem by itself:
  - Damping at the core-crust interface
  - Hyperons
  - Saturation of r-mode amplitudes
  - Mutual friction: phonon scattering with vortices
  - ...
- All (so far) known neutron stars may lie in the linear stability region for unpaired quark matter

Color superconductivity

- But quark matter is likely to be in a paired phase because the interaction between quarks is attractive in the color antisymmetric channel *Alford, Rajagopal, Wilczek and Shuryak, Schäfer, Rapp (1998)*
- At asymptotically high densities where the strange quark mass can be ignored, quark matter is in the CFL phase
- The diquark condensate is antisymmetric in color and in spin, and therefore also in flavor

$$\langle \psi_{\alpha i}(p)(C\gamma^5)\psi_{\beta j}(-p)\rangle = \sum_I \Delta \epsilon_{I\alpha\beta}\epsilon_{Iij}$$

- $U(1)_B \times SU_c(3) \times SU_L(3) \times SU_R(3) \to Z_2 \times SU_{c+L+R}(3)$. Goldstone bosons associated with the broken global symmetries dominate low energy dynamical properties

Constraints on CFL

- In the CFL phase all fermionic quasi-particle excitations are gapped due to pairing
- Energy scales $\mu \sim 500\text{MeV}, \gg \Delta \sim 10 - 100\text{MeV}, \gg T \sim 0.001 - 1\text{MeV}$
- Therefore the fermionic contribution to the viscosity (which was shown in the unpaired quark matter curves) is exponentially suppressed: $e^{-\Delta/T}$

Constraints on CFL

- Transport dominated by goldstone bosons *(Schäfer, Rupak and Manuel et. al.)*
- These have long mean free paths at low T and hence can give large viscosities at low T/μ, $\eta \sim 1/T^5$
- However, phonon transport is viscous only if mean free path comparable to the size of the star
- Naiive estimate, for $T < 0.01$MeV, phonons don't give viscous damping
- Including mutual friction, can't damp stars with $\Omega > 1$Hz *Manuel, Mannarelli, S'ad (2008)*

Constraints on CFL

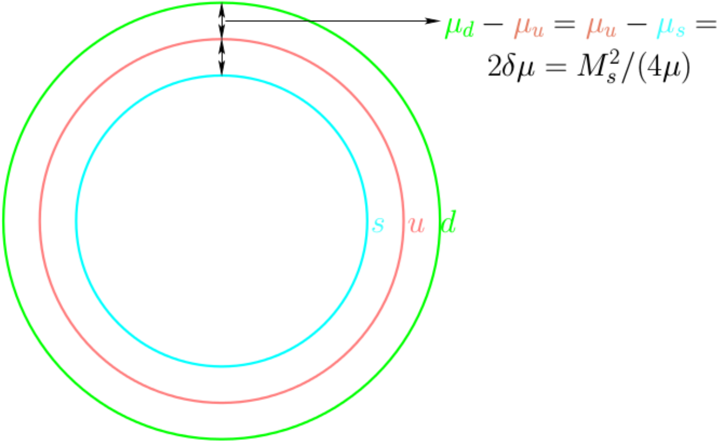
- A pure CFL star is inconsistent with r-mode stability constraints *Manuel, Mannarelli, S'ad (2008), Jaikumar, Rupak (2010) Alford, Braby, Mahmoodifar (2010)*
- Not necessarily a problem by itself:
  - Damping at the core-crust interface
  - ...
- Can there be paired phases with gaps in the fermionic excitation spectrum?

Strange quark mass and neutrality

- Ignoring M_s is not a good approximation at neutron star densities
- $\sqrt{M_s^2 + (p_s^F)^2} = \mu \implies p_s^F \approx \mu - M_s^2/(2\mu)$, but this leaves an unbalanced positive charge
- Need to introduce a chemical potential, μ_e, to restore neutrality.
- Weak equilibrium implies $\mu_d - \mu_s = 0$, $\mu_d - \mu_u = \mu_e$
- Electrical neutrality is imposed by demanding $\frac{\partial \Omega}{\partial \mu_e} = 0$
- Similarly, color neutrality by desiring $\frac{\partial \Omega}{\partial \mu_{3,8}} = 0$

Neutral unpaired quark matter

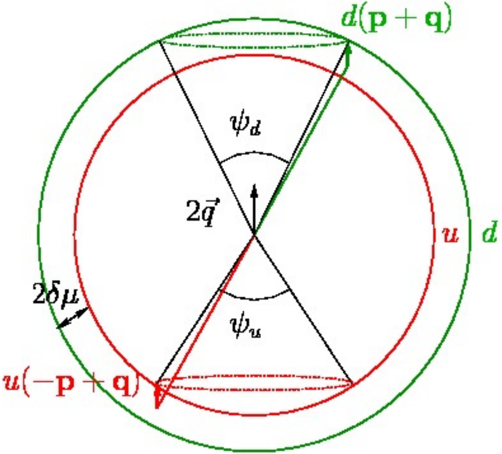
- For unpaired quark matter we obtain $\mu_e = M_s^2/(4\mu)$, $\mu_3 = \mu_8 = 0$



Gapless phases

- BCS like pairing is stressed in asymmetric or imbalanced Fermi gases with $(\mu_1 - \mu_2) = 2\delta\mu \neq 0$
- A simple argument tells us that $E = -\delta\mu + \sqrt{(|p| - \mu)^2 + \Delta^2}$ is gapless if $\delta\mu > \Delta$
- A more careful analysis shows that a gapless-CFL phase has a lower free energy than unpaired quark matter *(Alford, Kouvaris, Rajagopal)* for some M_s, but this is unstable to formation of position dependent condensates
- An alternate possibility is that M_s in the medium is so large that the strange quark is completely suppressed and we have 2 flavor quark matter which features gapless "blue" quarks and electrons (difficult)

Introduction to FF phases



Alford, Bowers, Rajagopal

Introduction to FF phases

- $\Delta(x) = \Delta \sum_{\{\mathbf{q}^a\}} e^{i2\mathbf{q}^a \cdot \mathbf{r}}$
- FF phases (only single plane wave) is thermodynamically preferred state compared to isotropic states for $\delta\mu \sim [0.707, 0.754]\Delta_0$, where Δ_0 is the gap for $\delta\mu = 0$
- The free energy depends on the set of momentum vectors $\{\mathbf{q}^a\}$ or equivalently the lattice structure
- $|\mathbf{q}^a|$ is chosen to minimize the free energy. $|\mathbf{q}^a| = \eta\delta\mu$ with $\eta \sim 1.2$
- For simple lattice structures (one or two plane waves) there is a second order phase transition from the normal phase to the LOFF phase at $\delta\mu = 0.754\Delta_0$
- This has motivated Ginzburg-Landau (GL) analyses
- For multiple waves a first order transition expected and GL analyses break down but give insight
- More complex lattice structures not considered in this first study and we don't make a GL approximation

Three flavor crystalline superconductivity

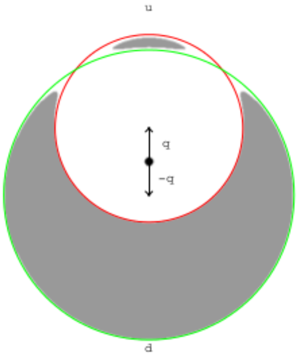
- Consider three flavor condensates that at antisymmetric in color, spin and flavor
$\langle\psi_{i\alpha}(\vec{r})(C\gamma^5)\psi_{j\beta}(\vec{r})\rangle \propto \sum_I \Delta_I \epsilon_{Iij}\epsilon_{I\alpha\beta} \sum_{\{\vec{q}_I\}} e^{2i\vec{q}_I\cdot\vec{r}}$
- Pairing between $u-d$, $u-s$ quarks dominant. $d-s$ pairing can be ignored due to larger splitting
- In the Ginzburg Landau approximation (which is not quite reliable near the first order phase transition it predicts) the LOFF phase is favoured in the region $\mu \sim 450-500$MeV *(Rajagopal, RS and Ippolito, Nardulli, Ruggieri)*

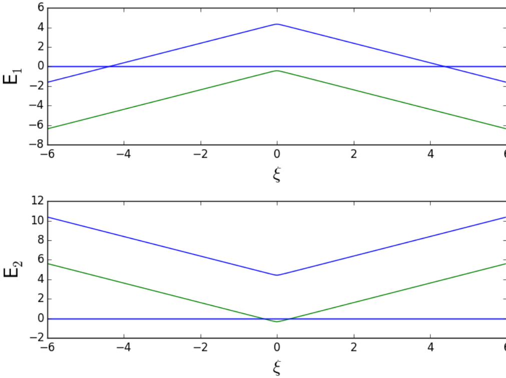
Degrees of freedom: fermions

- Consider first a simple case of only a single plane wave pairing $\Delta_3 \epsilon_{312} e^{2i\mathbf{q}\cdot\mathbf{r}} \psi_{u1} \psi_{d2}$ (FF state)
-
  - $E_1 = -\delta\mu - q\cos\theta + \sqrt{\xi^2 + \Delta^2}$
  - $E_1 = -\delta\mu - q\cos\theta - \sqrt{\xi^2 + \Delta^2}$
- These dispersion relations have gapless surfaces (if $|\delta\mu \pm q| < \Delta$)
- Quarks have a large phase space because the chemical potential is the largest scale in the problem
- Assume all goldstone modes are Landau damped due to gapless fermions and don't contribute
- The u, d, quarks near the gapless regions dominate transport
- To begin with, take a simple interaction: fermions interacting with a Debye screened gauge boson (to clarify some aspects)

$$\frac{g^2}{q^2 + m_D^2} \qquad (1)$$

Gapless fermionic modes





Shear viscosity in the FF phase

- η_{ijkl} is no longer rotationally invariant because of the special direction of q which we choose to be in the z direction. There are 5 independent η components
- For example, corresponding to the projection operator $\Pi^{(0)}_{ijkl} = \frac{3}{2}[z_i z_j - \delta_{ij}][z_k z_l - \delta_{kl}]$
- Will show the result for $\eta^{(0)}$, others similar but numerical values can differ

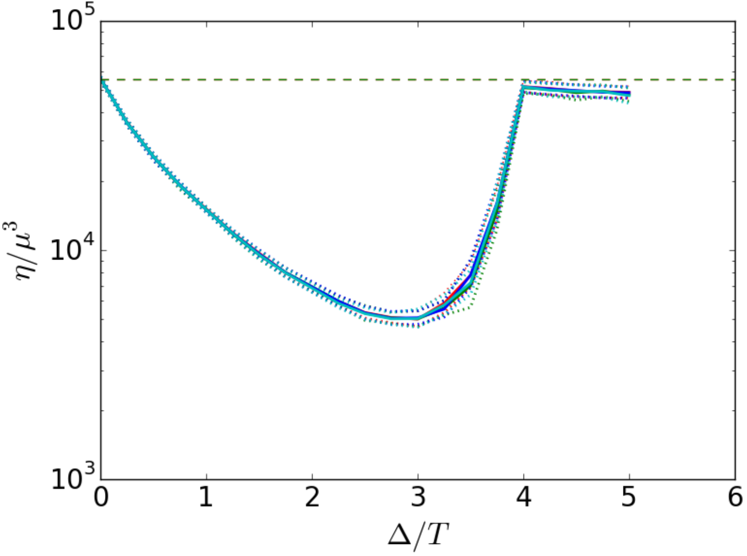
Shear viscosity in the FF phase

- The modification of the density of states is simple — geometric
- $\eta \sim np\tau$
- $\eta^{(0)} \approx \frac{\mu^4}{5\pi^2}\frac{1}{2}(1 - \frac{\Delta}{2q})\tau^{(0)}$
- $\tau^{(0)}$ is related to the collision integral

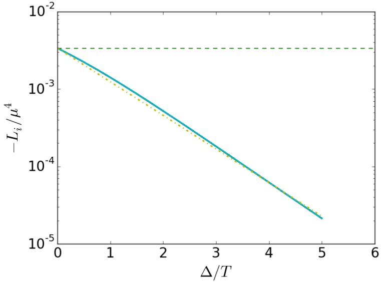
$$\frac{1}{\tau^{(0)}} \propto \frac{1}{T}\int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\frac{d^3p_4}{(2\pi)^3}$$
$$|\mathcal{M}(12 \to 34)|^2$$
$$(2\pi)^4\delta(\sum p^\mu)[f_1 f_2(1 - f_3)(1 - f_4)]$$
$$\phi_i^{ab}.\Pi^{(0)}_{abcd}.\phi_i^{cd}$$

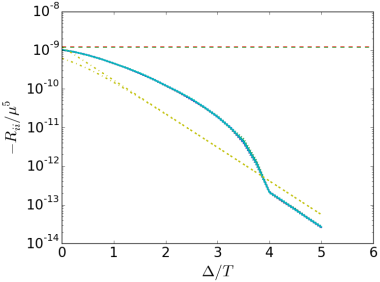
  with $\phi_i^{ab} = v^a p^b$
- Complicated because the distribution functions f depend on the angles in addition to the magnitude of the momentum. Needs to be done numerically

Check for $\delta\mu = 0$, $\mathbf{q} = 0$



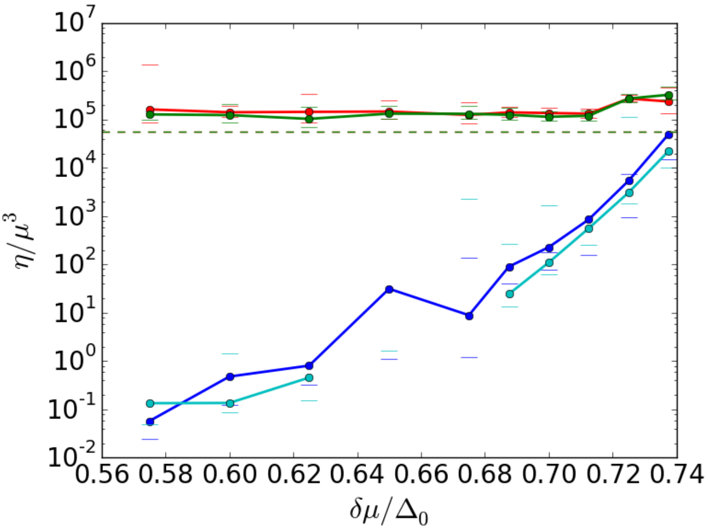
Check for $\delta\mu = 0$, $\mathbf{q} = 0$





- $\eta \sim np\tau$, $n \to ne^{-\Delta/T}$, $\tau \to \tau e^{2\Delta/T}$
- Scattering with phonons gives $\tau(\mu/T)^5 e^{\Delta/T}$ and gives the expected result: exponentially suppressed viscosity

Result for FF



- Using $\Delta(\delta\mu)$ from *Mannarelli, Rajagopal, RS*
- Keeping Δ/T fixed, $\eta^{(0)}/\eta_{\text{unpaired}} = 3 - 4$

Interactions: gluons

- Gluons t^1 to t^7 are gapped because of the Meissner effect
- The t^8 gluon mixes with the photon A_μ^Q to give one linear combination $A_\mu^{\tilde{Q}}$ that does not have a Meissner effect and one X_μ that does
- This can be understood by noting that if we define $t^8 = \frac{1}{\sqrt{3}}\mathrm{diag}(-2, 1, 1)$ in color space and $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ in flavor space, $\tilde{Q} = Q + \frac{1}{\sqrt{3}}t^8$
- The condensate is neutral under $\tilde{Q}$
- Therefore, transverse $A^{\tilde{Q}}$ is only dynamically screened and dominantly contributes
- Similarly t^1, t^2, t^3 are also dynamically screened (this has not been calculated yet) due to the gapless fermionic modes

Degrees of freedom in LOFF phases: goldstone modes

- There is a Goldstone mode associated with $U(1)_B$ breaking
- One (in general three) “lattice phonons”
- These four massless modes can also give rise to long distance interactions between quarks but these interactions are suppressed because of derivative coupling
- The four Goldstone modes can also transport momentum at low energies, but this contribution is suppressed because of scattering with gapless fermions

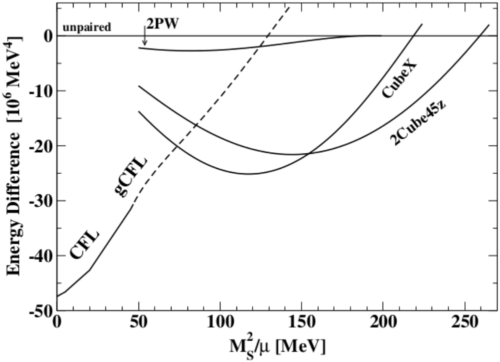
Summary

- Data on the angular velocity of neutron stars puts constraints on the viscosity of the matter the cores of neutron stars: possibly suggesting the presence of a (1) deconfined phase with (2) gapless fermionic excitations
- Crystalline color superconducting phases are natural candidates for a paired quark matter phase with gapless excitations
- Calculations with simplified quark-quark interaction suggest that the shear viscosity is $3-4$ times unpaired quark matter for the two flavor phase

Future work

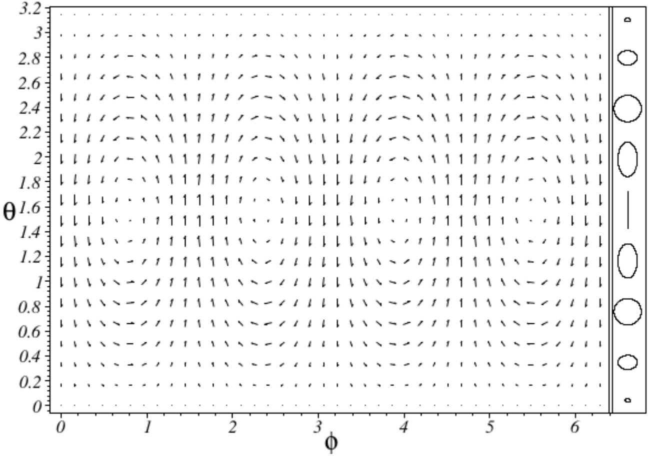
- A more controlled calculation of the free energy of the LOFF state is desirable, in particular (1) including the constraints of charge neutrality and (2) multiple plane waves more carefully
- Could anisotropic viscosities play a role in neutron star dynamics? The phase space integrals for $\eta^{(1)}$ and $\eta^{(2)}$ differ by trigonometric factors and hence expected to differ from $\eta^{(0)}$ by $\mathcal{O}(1)$
- What's the point of this calculation if we know that $\eta_{\mathrm{LOFF}} = \mathcal{O}(1) \times \eta_{\mathrm{unpaired}}$?

Three flavor Free energy

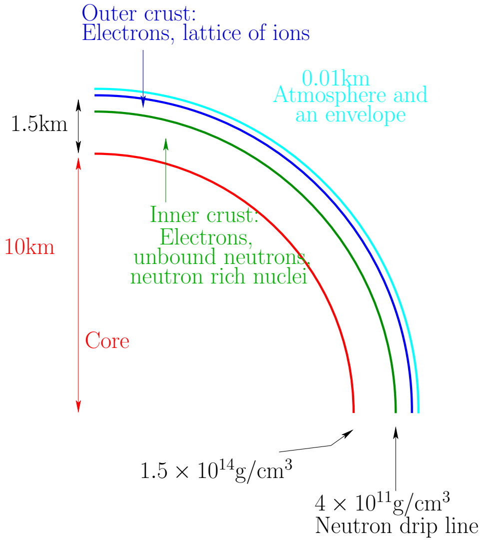


- A more detailed analysis (*Ippolito, Nardulli, Ruggieri (2007)*) suggests that for $440 \lesssim \mu \lesssim 520$MeV LOFF might be the ground state. This is the relevant region for neutron star cores. Caveats

Profile of a r-mode

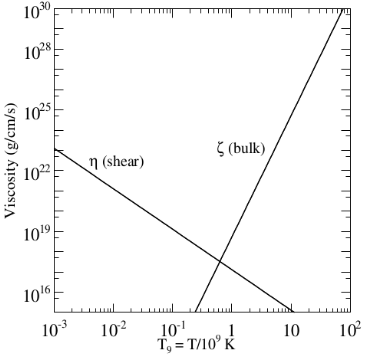


Profile of a neutron star



Hadronic matter

- For example assuming only hadronic matter in neutron stars
- $\eta \sim T^{-2}$
- Turns out that $\Gamma(\sim T^6) \ll \Omega$. Therefore $\zeta \sim T^6/\Omega^2$. *Flowers and Itoh (1979)*



- Plot at $2n_{\rm sat}$ *Jaikumar, Rupak, Steiner (2008)*