Neutral Goldstone Modes of the Color-Flavor Locked Phase in a Magnetic Field

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Outline

- QCD phase diagram and low temperature high density regime.
- Why we care about magnetic fields.
- Structure of color-flavor-locked (CFL) phase.
- CFL phase in a magnetic field and the Goldstone modes.
- Claim : The propagation of the neutral Goldstone modes gets affected in a magnetic field → demonstrated in NJL model.
- Conclusion.

QCD Phase Diagram and High Density- Low Temperature Regime

- At high temperature there is quark-gluon plasma phase.
- Low temperature and low baryon chemical potential corresponds to hadronic phase.
- High baryon chemical potential and low temperature regime is color-superconducting,

  → is the regime of interest.

  → Low temperature, high baryon chemical potential regime relevant for neutron star physics.

Why we care about high magnetic fields

- Response of color superconductors to a magnetic field is different from terrestrial metallic superconductors.

  → Interesting physics..

- Typical magnetic field in neutron stars of the order of 10^{12} Gauss. Magnetars can have 10^{16} Gauss on the surface.
- In the core magnetic fields can have larger values.
- Upper bound of magnetic field for gravitationally bound stars is ~ 10^{18} Gauss.

High Baryon Chemical Potential and Low Temperature

- Baryonic matter forms Fermi sphere at low temperature.
- Quark degree of freedom at the Fermi surface as Fermi energy is very high.

  → Rigorous perturbative calculations possible.
- Attractive interaction at the Fermi surface leads to formation of Cooper pairs giving rise to superfluidity and superconductivity.

Color Superconductivity

- Two quarks at the Fermi surface forming Cooper pair.
- These Cooper pairs are not color neutral.
- Cooper pairs being Bosonic excitation condense at low temperature.
- Gluons acquire Meissner mass due to the breaking of color and get screened.

Color-Flavor Locked Phase

- Many quark pairings at different densities predicted using QCD and model calculations.
- At very high density the favored pairing is known as color-flavor locking (CFL).
- The CFL condensate is given by (spin indices i and j)

$$\langle \Psi^{\alpha}_{Lai} \Psi^{\beta}_{Lbj} \rangle = \langle \Psi^{\alpha}_{Rai} \Psi^{\beta}_{Rbj} \rangle = \Delta \epsilon^{\alpha\beta c} \epsilon_{abc} \epsilon_{ij}$$

where,

$$\alpha = (s, d, u) = (1, 2, 3)$$

$$a = (b, g, r) = (1, 2, 3)$$

Symmetry Properties of CFL Phase

- Below the weak scale, standard model Lagrangian with three massless flavors of quark has the approximate symmetry

$$[SU(3)]_C \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_B \otimes U(1)_A$$

- CFL breaks it down to $SU(3)_{R+L+C}$

  7 of the 8 gluons acquire Meissner mass. A linear combo of the 8th gluon and the electromagnetic U(1) acquires mass and while the orthogonal combo remains massless → called rotated EM.

Rotated U(1) EM :

- Denoted as $[\widetilde{U}(1)]_{em}$, is not screened by the condensate.
- Hence, a magnetic field of $[\widetilde{U}(1)]_{em}$ can penetrate the condensate.
- This feature distinguishes CFL from terrestrial superconductors
- Opens up exciting new possibilities for interesting new physics for superconductors in magnetic field.

Magnetic CFL Phase (MCFL)

- Charges of quarks under rotated EM

s_b	s_g	s_r	d_b	d_g	d_r	u_b	u_g	u_r
0	0	-	0	0	-	+	+	0

- In the presence of a magnetic field the symmetry of the Lagrangian gets explicitly broken to

$$[SU(3)]_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_B \otimes U(1)_A \otimes U(1)_{\bar{A}}$$

- For magnetic fields that are of the order of the square of the gap, the diquark pairing of the condensate itself is altered.

Form of the MCFL Condensate and Goldstone Modes

- For CFL the Goldstone modes

$$\pi^{\pm}, \pi^0, \ K^{\pm}, \ K^0, \ \bar{K}^0, \ \eta, \ \text{and} \ \eta'$$

  and φ due to baryon symmetry breaking.

- The MCFL condensate is given by

$$\langle \Psi^{\alpha}_{Lai} \Psi^{\beta}_{Lbj} \rangle = \langle \Psi^{\alpha}_{Rai} \Psi^{\beta}_{Rbj} \rangle = \Delta^{C}_{\epsilon}^{\alpha\beta c} \epsilon_{abc} \epsilon_{ij}$$

  → invariant under $SU(2)_{R+L+C}$

- The Goldstone modes are

$$\pi^0, K^0, \ \bar{K}^0, \ \eta, \ \eta', \ \text{and} \ \varphi$$

Goldstone Modes of the MCFL phase

- In the presence of a magnetic field the charged Goldstone modes(GB) of CFL acquire mass as expected but even the neutral GB s get affected.

- The speed of the neutral GB s become anisotropic: although expected from symmetry arguments, have never been quantified at high density.

  → I quantify this anisotropy here.

Why Anisotropy ?

- The neutral Goldstone modes are made of excitations of both two oppositely charged quarks and two neutral quarks.

- Small magnetic fields cannot resolve the internal structure of these neutral GB s, but a high enough magnetic fields can.

- Since these GB s are still GB s of the theory in the presence of a magnetic field, the only manifestation of the magnetic field would be through anisotropy in speed.

Demonstration with NJL Model

- Model Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + e\tilde{Q}\tilde{A\!\!\!/} + \mu\gamma_0)\psi + \sum_{\eta=1}^{3} \frac{G}{4}(\bar{\psi}P_\eta\psi_c)(\bar{\psi}_c P_\eta\psi)$$

where $\tilde{A}_\mu$ = the rotated electromagnetic field.

ψ = quark field, $(P_\eta)_{ia-jb} = i\gamma_5\epsilon_{ij\eta}\epsilon_{ab\eta}$

$\psi_c = C\bar{\psi}^T$ where $C = i\gamma^2\gamma^0$

$\tilde{Q}$= charge of the quarks under rotated EM

- Introduce auxiliary field to rewrite the Lagrangian.

Lagrangian with Auxiliary Field

- To rewrite the Lagrangian we need to use the Gorkov basis for quarks,

$$\Psi = \begin{pmatrix} \psi \\ \psi_c \end{pmatrix}$$

where the Lagrangian is,

$$\mathcal{S} = -\frac{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}{G} + \frac{1}{2}\mathrm{Tr}\left(\mathrm{Log}\left(S_{\mathrm{quark}}^{-1}\right)\right)$$

Lagrangian with Auxiliary Field

$$S^{-1}_{\text{quark}} = \begin{pmatrix} i\partial\!\!\!/ + \tilde{e}\tilde{Q}\tilde{A\!\!\!/} + \mu\gamma_0 & i\gamma^5\Delta^- \\ -i(\gamma_0\gamma^5(\Delta^-)^\dagger\gamma_0) & i\partial\!\!\!/ + \tilde{e}\tilde{Q}\tilde{A\!\!\!/} - \mu\gamma_0 \end{pmatrix}$$

and

$$\Delta^- = \begin{pmatrix} 0 & 0 & 0 & 0 & \Delta_3 & 0 & 0 & 0 & \Delta_2 \\ 0 & 0 & 0 & -\Delta_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\Delta_2 & 0 & 0 \\ 0 & -\Delta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Delta_1 & 0 \\ 0 & 0 & -\Delta_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\Delta_1 & 0 & 0 & 0 \\ \Delta_2 & 0 & 0 & 0 & \Delta_1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

NJL Model Continued

- We introduce an external electromagnetic field

$$\tilde{A}^{\mu} = (0, 0, \tilde{B}x, 0)$$

- The strategy is to first minimize the Lagrangian w.r.t Δ^{-} get the gap equation.
- To show how the calculation proceeds, solve the gap equation and choose a specific μ and $\tilde{B}$ where all the gaps are equal.
- Then expand the Lagrangian about this point.

Gap Equation

- Before we write the gap equation, we can say more from the symmetries of the Lagrangian : $\Delta_1 = \Delta_2$
- The two gap equations : (where S_{quark} is the quark propagator)

$$\frac{d\mathcal{S}}{d\Delta_1^\dagger}\Big|_{\Delta_1=\Delta_3=\Delta} = \frac{8\Delta_1}{4G}\Big|_{\Delta_1=\Delta_3=\Delta} - \frac{1}{2}\mathrm{Tr}\left(S_{\text{quark}}\cdot\frac{dS^{-1}_{\text{quark}}}{d\Delta_1^\dagger}\right)\Big|_{\Delta_1=\Delta_3=\Delta} = 0$$

$$\frac{d\mathcal{S}}{d\Delta_3^\dagger}\Big|_{\Delta_1=\Delta_3=\Delta} = \frac{4\Delta_3}{4}\Big|_{\Delta_1=\Delta_3=\Delta} - \frac{1}{2}\mathrm{Tr}\left(S_{\text{quark}}\cdot\frac{dS^{-1}_{\text{quark}}}{d\Delta_3^\dagger}\right)\Big|_{\Delta_1=\Delta_3=\Delta} = 0.$$

The Quark Propagator

- But S_{quark} is not diagonal in position space.
- Moreover the propagator is not diagonal in the free particle momentum space basis either, as the position space propagator is not translation invariant.
- So we use the Landau level basis in which the propagator is diagonal.

Gap Equation

- Setting $\Delta_1 = \Delta_3 = \Delta$ in the gap equation would yield following equations : (Here Λ is a sharp cutoff)

$$\frac{2}{G} = \frac{2\mu^2}{\pi^2}\log\left(\frac{2\Lambda}{\Delta}\right) + \frac{\tilde{e}\,\tilde{B}}{\pi^2}\mathrm{Log}\left(\frac{2\Lambda}{\Delta}\right) \quad \frac{1}{G} = 2\frac{\mu^2}{\pi^2}\log\left(\frac{2\Lambda}{\Delta}\right)$$

→ solving these we get $\tilde{e}\tilde{B} = 2\mu^2$.

- $\tilde{e}\tilde{B} = 2\mu^2$ is the only point in the high magnetic field regime where the gaps are equal. High magnetic field regime in this context means that only the lowest landau level is filled by the quarks.

The Goldstone Modes

- Under color chiral rotation the gap transforms as

$$\begin{aligned}\Delta' &= e^{i\gamma^5 G^a \alpha^a}\bar{\Delta}e^{i\gamma^5 (G^a)^T\alpha^a} \\ &= \left(1 + i\gamma^5 G^a\alpha^a\right)\bar{\Delta}\left(1 + i\gamma^5 (G^a)^T\alpha^a\right) + .. \\ &= \bar{\Delta} + i\alpha^a\gamma^5 M^a + ..., \text{ for } a = 1, .., 8\end{aligned}$$

- Under axial U(1) the gap transforms as

$$\begin{aligned}\Delta' &= e^{i\gamma^5\alpha^9}\bar{\Delta}e^{i\gamma^5\alpha^9} \\ &= \bar{\Delta} + 2i\alpha^9\gamma^5\bar{\Delta} + ... \equiv \bar{\Delta} + i\alpha^9\gamma^5 M^9 + ..\end{aligned}$$

- Under ordinary U(1) the gap transforms as

$$\begin{aligned}\Delta' &= e^{i\alpha^{10}}\bar{\Delta}e^{i\alpha^{10}} \\ &= \bar{\Delta} + 2i\alpha^{10}\bar{\Delta} + .. \equiv \bar{\Delta} + i\alpha^{10}M^{10} + ...\end{aligned}$$

The Neutral Goldstone Modes

- The neutral modes have $\tilde{Q}.M^a + M^a.\tilde{Q} = 0$ where $\tilde{Q}$ is the 9 dimensional charge matrix with only diagonal entries that denote quark charges under rotated EM.
- The neutral modes are :

  $M^1, M^2, M^3,\ M^8,\ M^9$ and M^{10}
- For demonstration we choose M^3

Propagator for M^3

- Expand the action

$$\mathcal{S} = -\frac{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}{G} + \frac{1}{2}\mathrm{Tr}\left(\mathrm{Log}\left(S_{\mathrm{quark}}^{-1}\right)\right)$$

to second order in Δ^-

$$\begin{aligned}\mathcal{S}_3[\Delta] = \mathcal{S}[\bar{\Delta}] - \frac{1}{4G}3\Delta^2(\alpha^3)^2 \\ -\frac{\Delta^2(\alpha^3)^2}{4}\left[\int \frac{d^4p}{(2\pi)^4}\left(1+\frac{\mathrm{p}.(\mathrm{p}-\mathrm{k})}{|\mathrm{p}||\mathrm{p}-\mathrm{k}|}\right)\frac{4(p_0(p_0-k_0)-4(|\mathrm{p}-\mathrm{k}|-\mu)(|\mathrm{p}|-\mu)-4\Delta^2)}{((p_0-k_0)^2-(|\mathrm{p}-\mathrm{k}|-\mu)^2-\Delta^2)\left(p_0^2-(|\mathrm{p}|-\mu)^2-\Delta^2\right)}\right] \\ -\frac{\Delta^2(\alpha^3)^2}{4}\left[\int \frac{dp_0dp_3}{(2\pi)^4}\frac{1}{2}\frac{4\pi^2\tilde{e}\tilde{B}}{4}\frac{16}{2\pi}\frac{p_0(p_0-k_0)-(|p_3-k_3|-\mu)(|p_3|-\mu)-\Delta^2}{((p_0-k_0)^2-(|p_3-k_3|-\mu)^2-\Delta^2)\left(p_0^2-(|p_3|-\mu)^2-\Delta^2\right)}+\mathcal{O}\left(\frac{k_\perp^2}{\tilde{e}\tilde{B}}\right)\right]+\ldots\end{aligned}$$

Propagator for M^3

- Using the gap equation

$$\frac{3}{4G} = -i\left(\int \frac{d^4p}{(2\pi)^4}\frac{2}{(p_0)^2-(|\mathbf{p}|-\mu)^2-\Delta^2} + \frac{|\tilde{e}\tilde{B}|}{4\pi}\int \frac{dp_0dp_3}{(2\pi)^2}\frac{1}{2}\frac{2}{(p_0)^2-(|p_3|-\mu)^2-\Delta^2}\right)$$

we find the action with only convergent terms which we expand in $\frac{k_0^2}{\Delta^2}, \frac{k_3^2}{\Delta^2}$ and $\frac{|\mathbf{k}|^2}{\Delta^2}$ to get the effective action for the mode:

$$\mathcal{S}_3[\Delta] = \mathcal{S}[\bar{\Delta}] - \frac{i((\alpha^3)^2)\mu^2}{4\pi^2}\left(k_0^2 - \frac{|\mathbf{k}|^2}{3}\right) - \frac{i((\alpha^3)^2)|\tilde{e}\tilde{B}|}{16\pi^2}(k_0^2 - k_3^2) + \ldots$$

$$(v^i)_\perp^2 = \frac{2}{9}, (v^i)_\parallel^2 = \frac{4}{9}$$

for i = 3

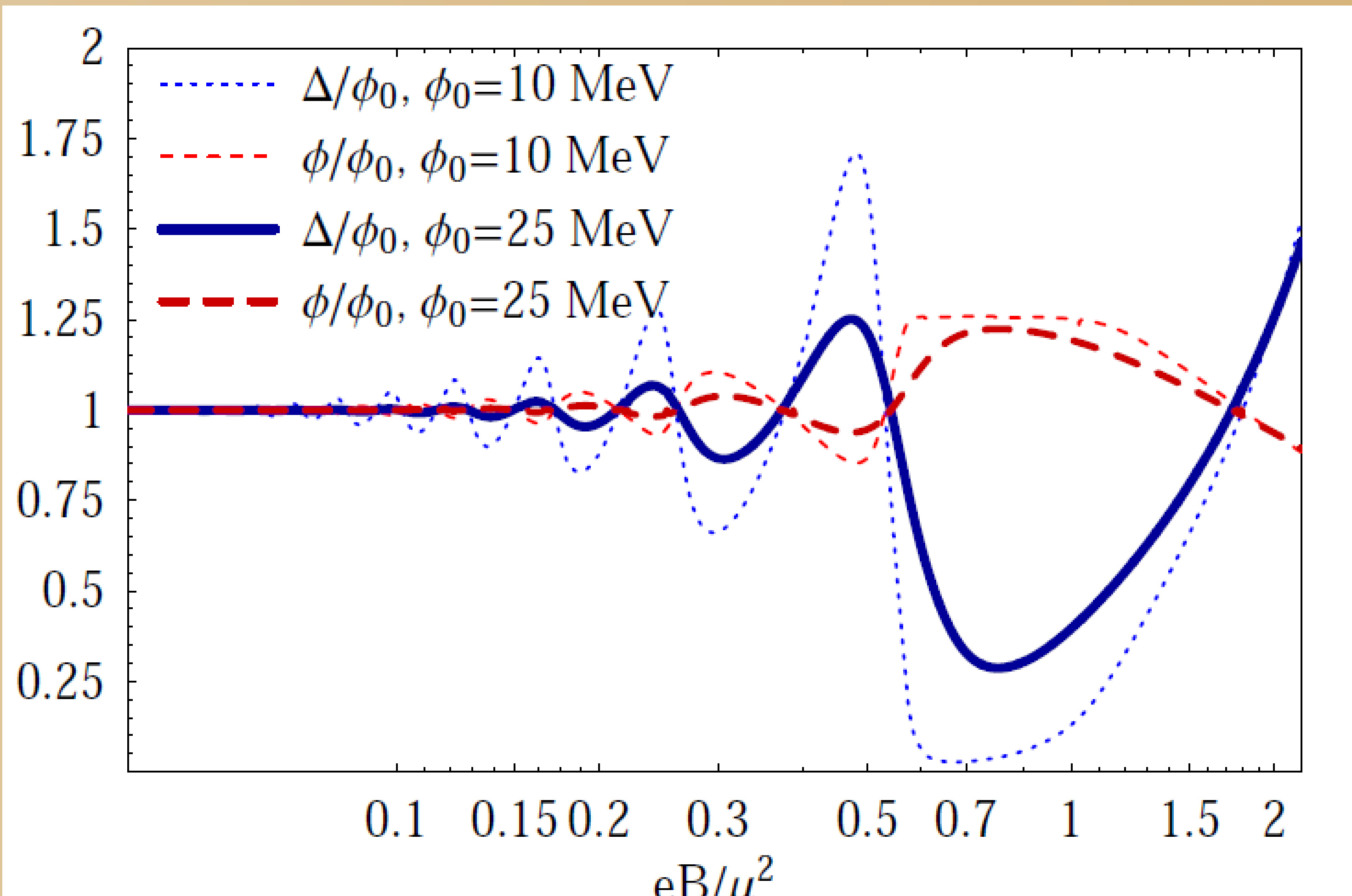
Lower Magnetic Fields

- The regime we are interested in

$$|\mathbf{k}|^2 < \Delta^2 \leq \tilde{e}\,\tilde{B} < 0.3\mu^2$$

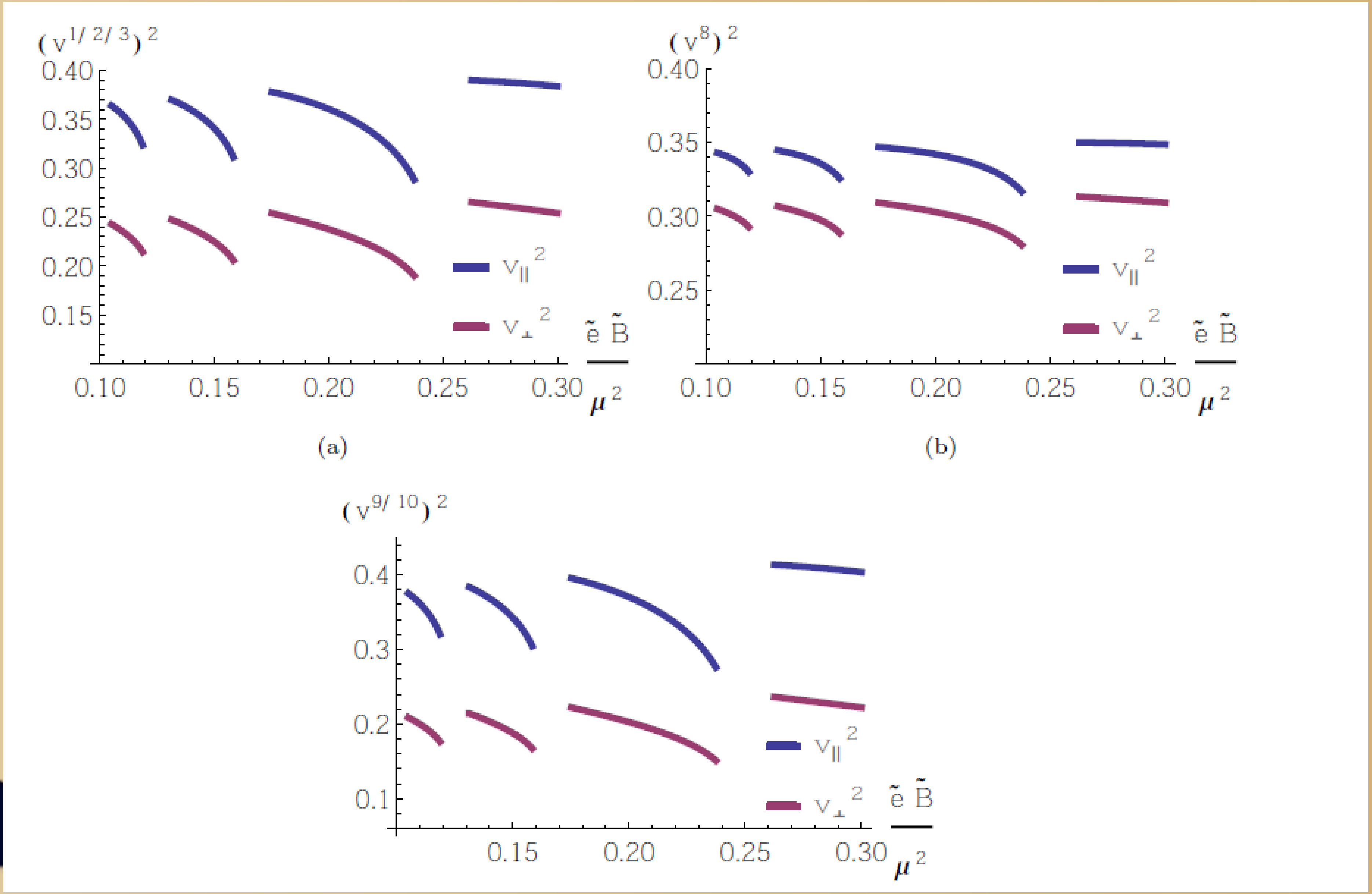
- Multiple Landau levels are filled.
- Effect of multiple Landau levels : De Haas van Alphen oscillation.
- Gaps oscillate appreciably only when $\tilde{e}\tilde{B} \geq 0.5\mu^2$
- The gaps are almost equal for $\tilde{e}\,\tilde{B} \leq 0.3\mu^2$

Oscillating Gaps.



Courtesy: J. L. Noronha and I. Shovkovy, Phys. Rev. D 76, 105030 (2007).

Speed of the Modes



Conclusion

- Speed of GB s considered in a magnetic field. Appreciable anisotropy was found for magnetic fields of the order of the gap.
- Calculation can be improved by taking into account unequal gaps, which will require numerical calculation. Also, chiral condensation needs to be taken into account.
- Implications for neutrino emission and transport properties for magnetic CFL phase in neutron stars.