INT Program INT-16-2b "The Phases of Dense Matter"
July 11 - August 12, 2016

Microscopic Calculation of Vortex-Nucleus Interaction for Neutron Star Glitches

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in collaboration with

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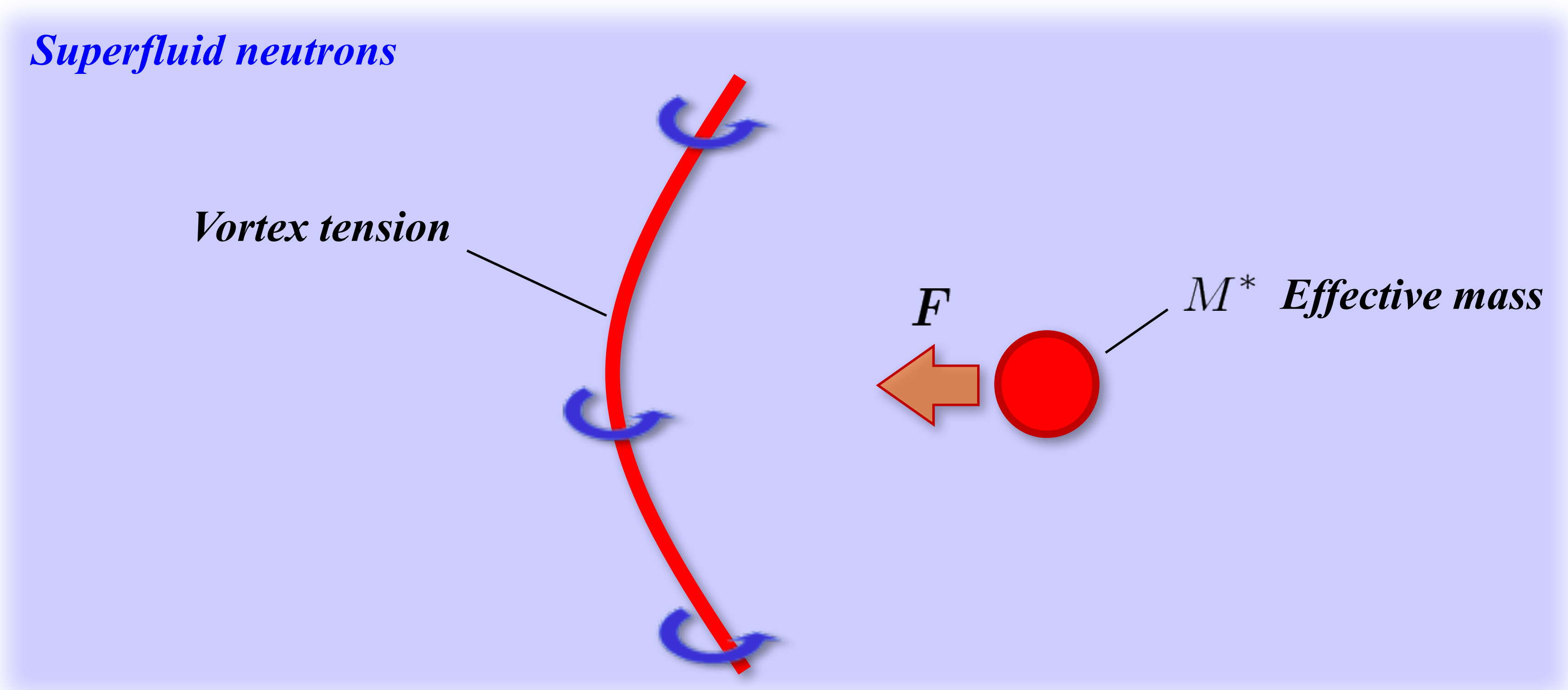
Goal: To clarify the mechanism of glitches

Need to describe **pinning/unpinning** dynamics of a huge number of **vortices**



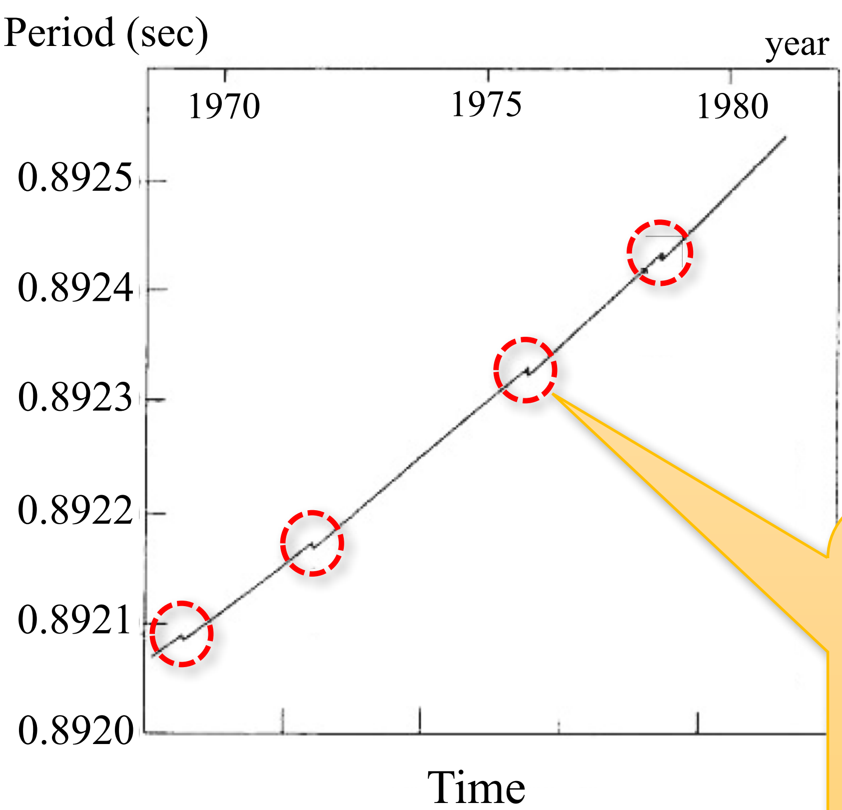
We can extract these ingredients from microscopic, dynamical simulations

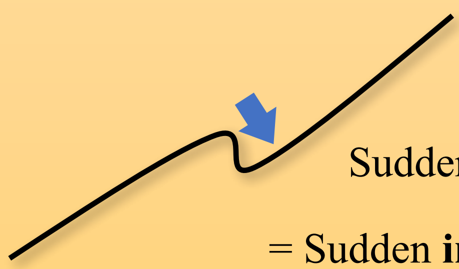
1. Introduction

What is the "glitch"?

Glitch: a sudden increase of the rotational frequency

Glitches in the Vela pulsar

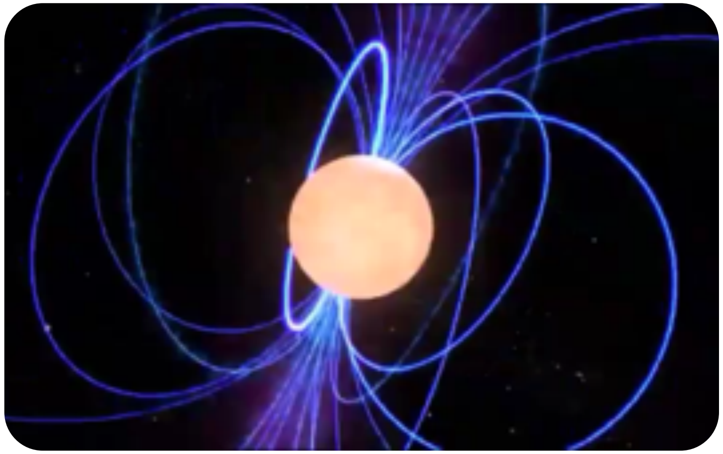




Sudden **decrease** of the period

= Sudden **increase** of the frequency

- Pulsar: a rotating neutron star



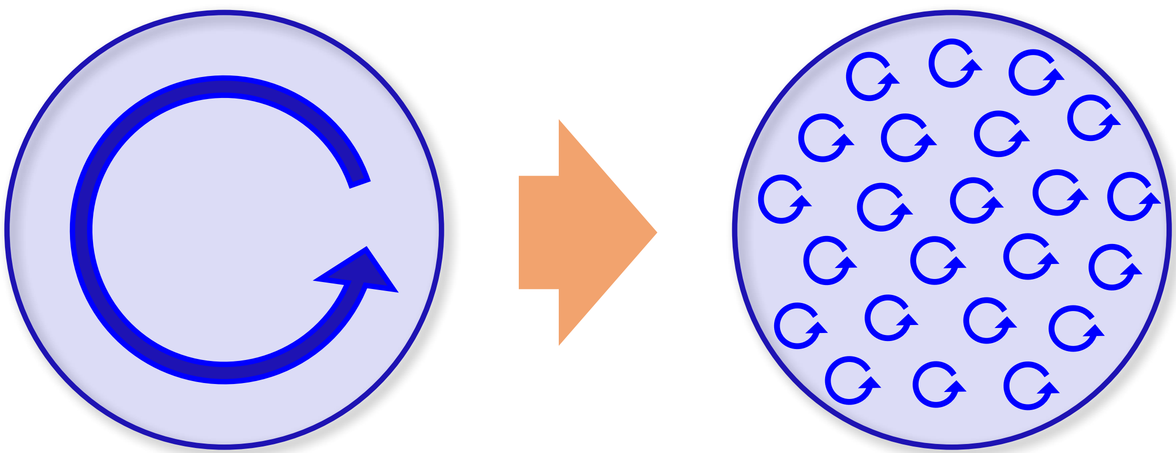
http://svs.gsfc.nasa.gov/cgi-bin/details.cgi?aid=10205

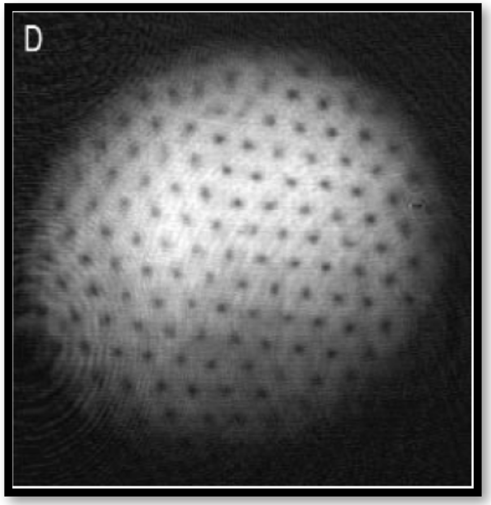
V.B. Bhatia, A Textbook of Astronomy and Astrophysics with Elements of Cosmology, Alpha Science, 2001.

Vortices and glitches

In rotating superfluid, an array of quantum vortices is generated

- Observation in ultra-cold atomic gases





W. Ketterle, MIT Physics Annual. 2001

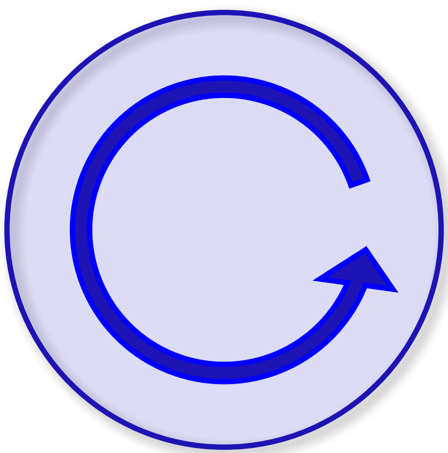
Vortices and glitches

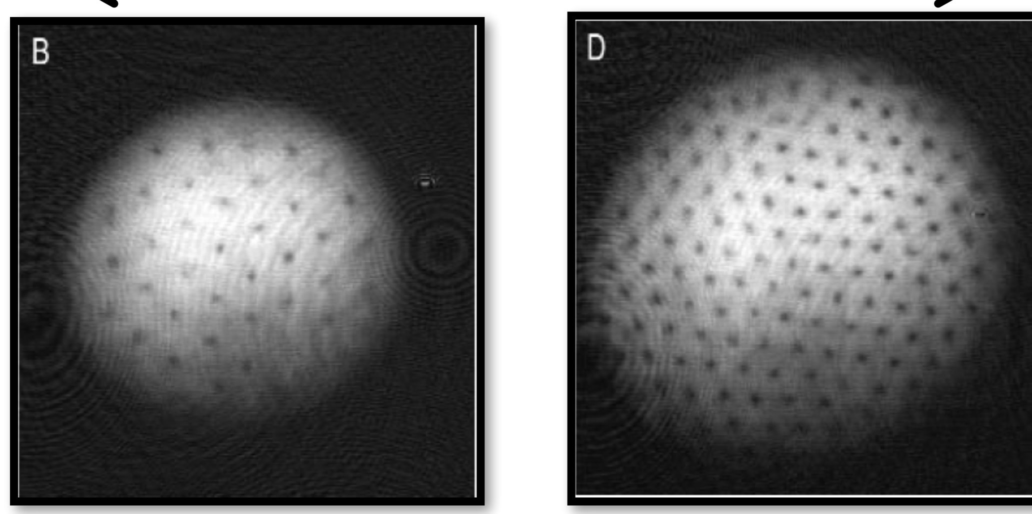
In rotating superfluid, an array of quantum vortices is generated

- Observation in ultra-cold atomic gases

Vortex-number density $\propto$ Rotation speed

Low ⟷ High





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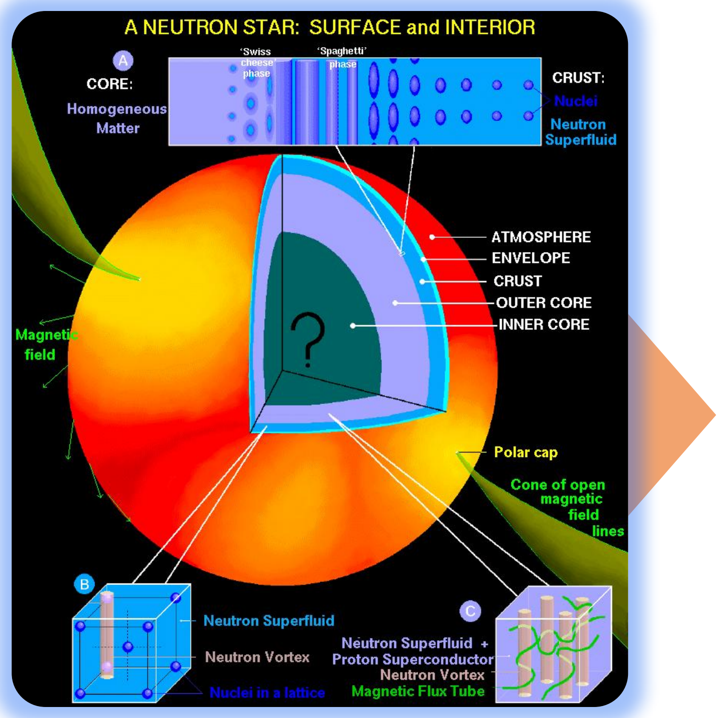
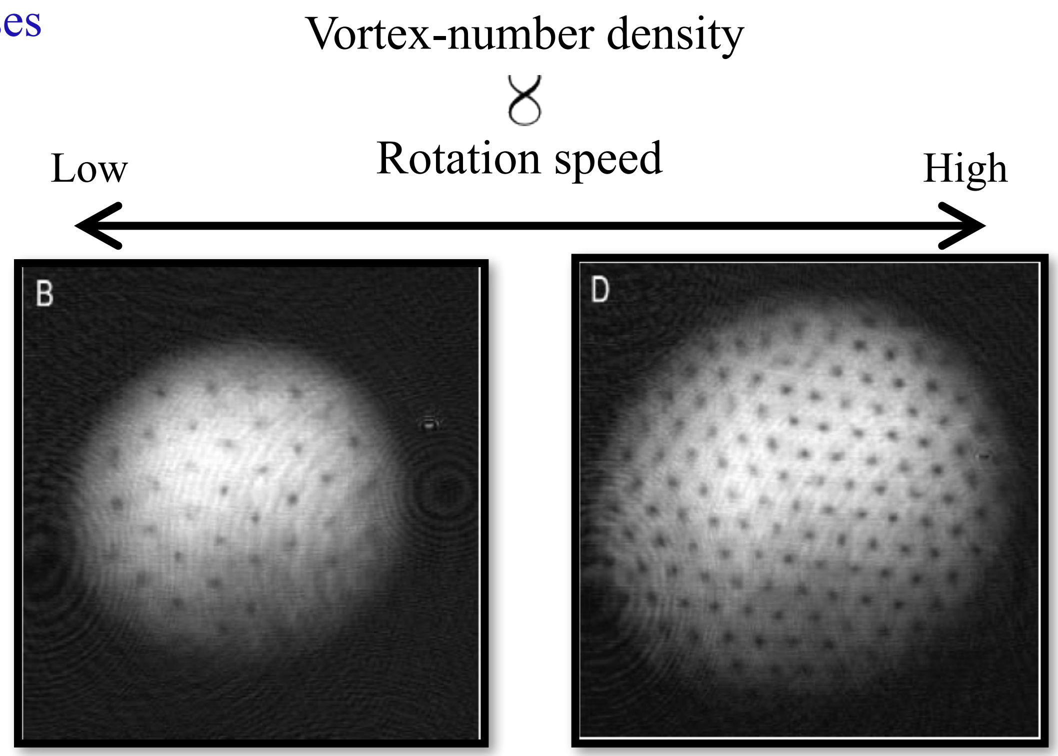




W. Ketterle, MIT Physics Annual. 2001

Studies of the pinning force

Representative studies of the pinning force

- Hartree-Fock-Bogoliubov theory

  P. Avogadro, F. Barranco, R.A. Broglia, and E. Vigezzi,
  PRC**75**(2007)012805(R); NPA**788**(2007)130; NPA**811**(2008)378

- Thomas-Fermi + LDA

  P.M. Pizzochero, L. Viverit, and R. A. Broglia, PRL**79**(1997)3347
  P. Donati and P.M. Pizzochero, PRL**90**(2003)211101; NPA**742**(2004)363; PLB**640**(2006)74
  S. Seveso, P.M. Pizzochero, F. Grill, and B. Haskell, MNRAS**455**(2016)3952

- Hydrodynamics + Ginzburg-Landau (for pairing)

  M.A. Alpar et al. Astrophys. J. **213**(1977)527; **276**(1984)325
  R.I. Epstein, G. Baym, Astrophys. J. **328**(1988)680
  R.K. Link, R.I. Epstein, Astrophys. J. **373**(1991)592

Superfluid hydrodynamics

Density dependence and asymptotic behavior of the force are predicted

$$E = E_{\text{tension}} + \frac{1}{2}M^* u^2 + 2\pi R^3 \frac{\rho_{\text{out}}(\rho_{\text{in}} - \rho_{\text{out}})}{2\rho_{\text{out}} + \rho_{\text{in}}}\left(\frac{\kappa}{2\pi r}\right)^2 + \mathcal{O}(1/r^3) \qquad (r \gg \xi)$$

Interaction energy between a vortex line and an impurity

$\rho_{\text{in}} < \rho_{\text{out}}$: attraction

$\rho_{\text{in}} > \rho_{\text{out}}$: repulsion

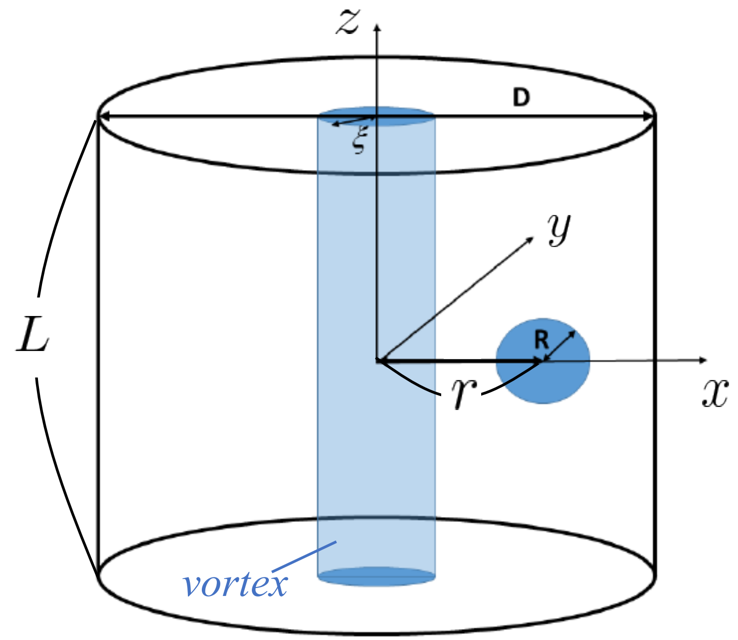
*$\rho_{\text{in/out}}$: superfluid density inside/outside a nucleus

$$F = -\frac{\mathrm{d}E}{\mathrm{d}r} \propto \frac{1}{r^3}$$

$$E_{\text{tension}} = \frac{1}{4\pi}\rho_{\text{out}}\kappa^2 L \ln\left(\frac{D}{2\xi}\right)$$

$$M^* = \frac{4\pi}{3}R^3 \frac{(\rho_{\text{out}} - \rho_{\text{in}})^2}{2\rho_{\text{out}} + \rho_{\text{in}}}$$

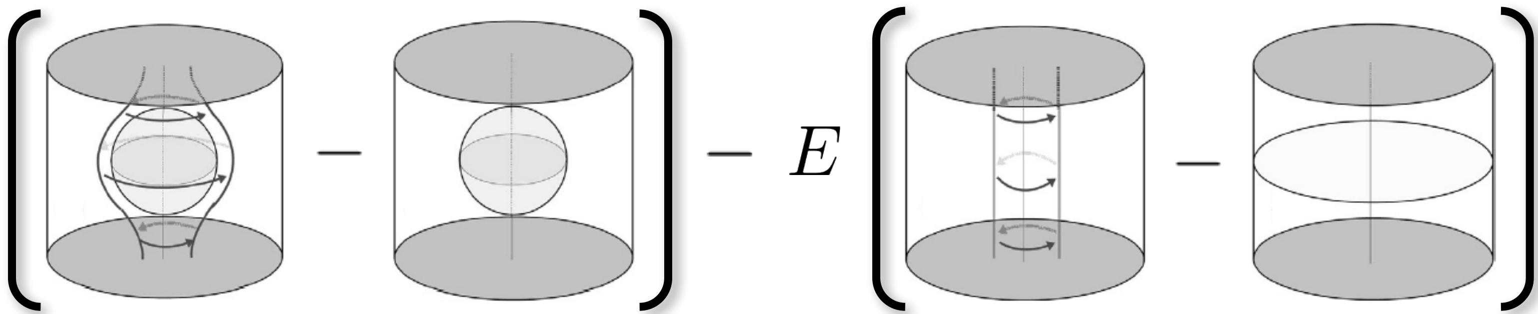
$$\kappa = \frac{2\pi\hbar}{2m_n}$$



What was the state-of-the-art?

Microscopic, static HFB calculations were performed assuming axial symmetry

$E_{\text{pin}} = E$



Energy to create a vortex line on a nuclear impurity

Energy to create a vortex line in a uniform matter

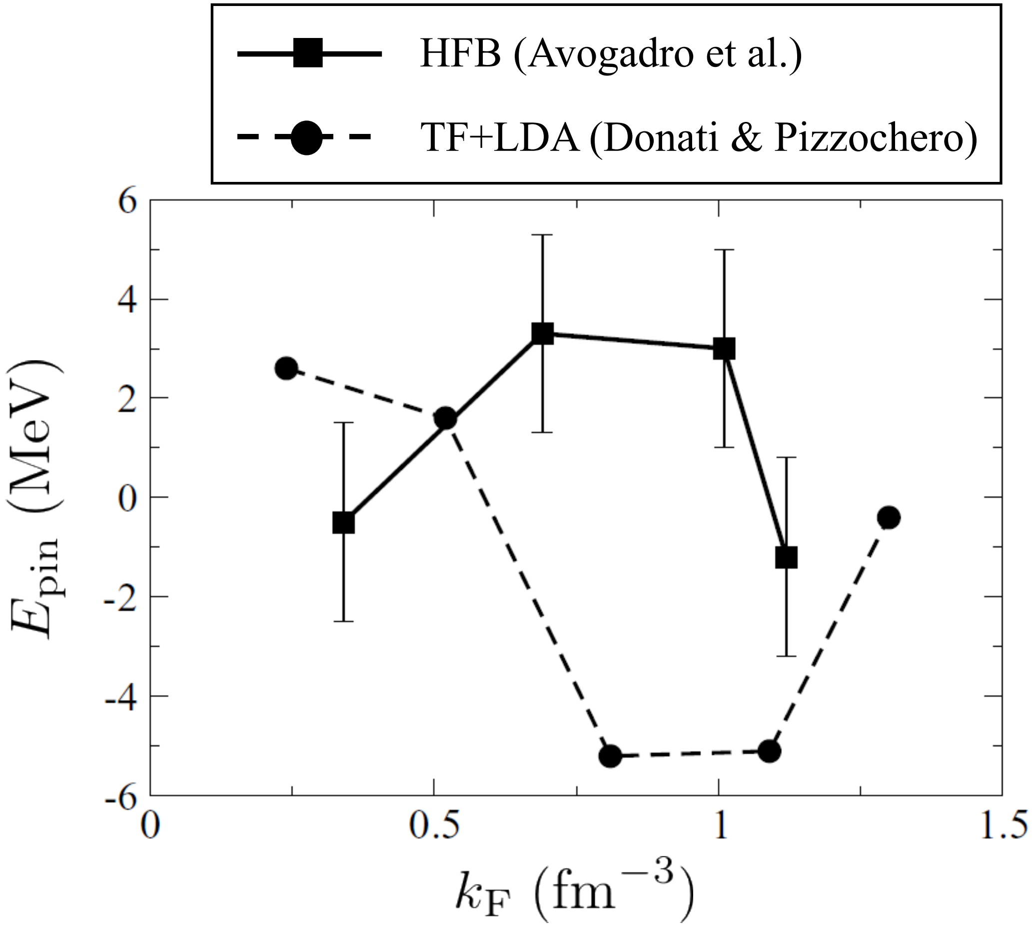
E.g.) 0.026 fm^{-3} (SLy4)

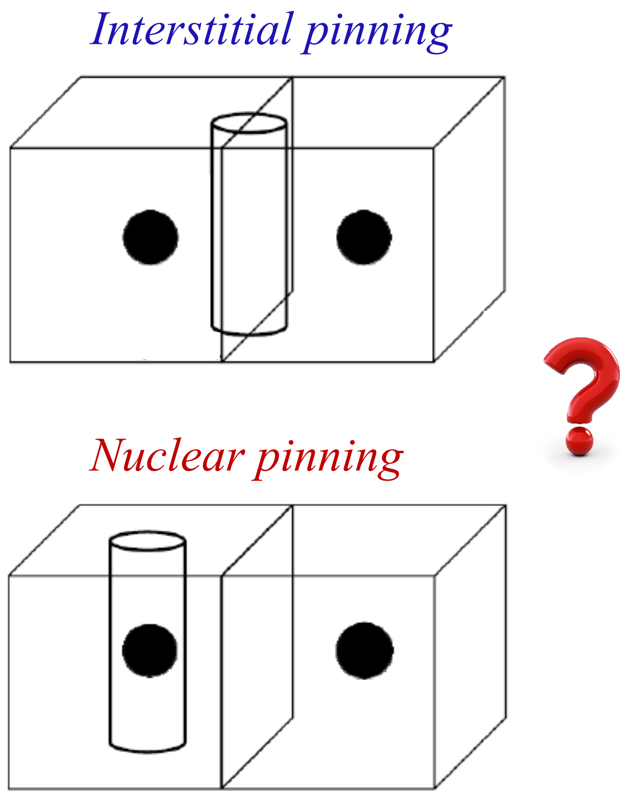
6.19 MeV	13058.04	12954.02	13714.88	13617.05

P. Avogadro, F. Barranco, R.A. Broglia, and E. Vigezzi, PRC**75**(2007)012805(R); NPA**788**(2007)130; NP**A811**(2008)378

What was the state-of-the-art?

Property of the pinning force is still unclear





P. Avogadro, F. Barranco, R.A. Broglia, and E. Vigezzi, PRC**75**(2007)012805(R); NP**A811**(2008)378

2. Methods

What we have performed

We performed 3D, dynamical simulations by TDDFT with superfluidity

TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix} = \begin{pmatrix}h(\mathbf{r}) & \Delta(\mathbf{r})\\ \Delta^*(\mathbf{r}) & -h(\mathbf{r})\end{pmatrix}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix}$$

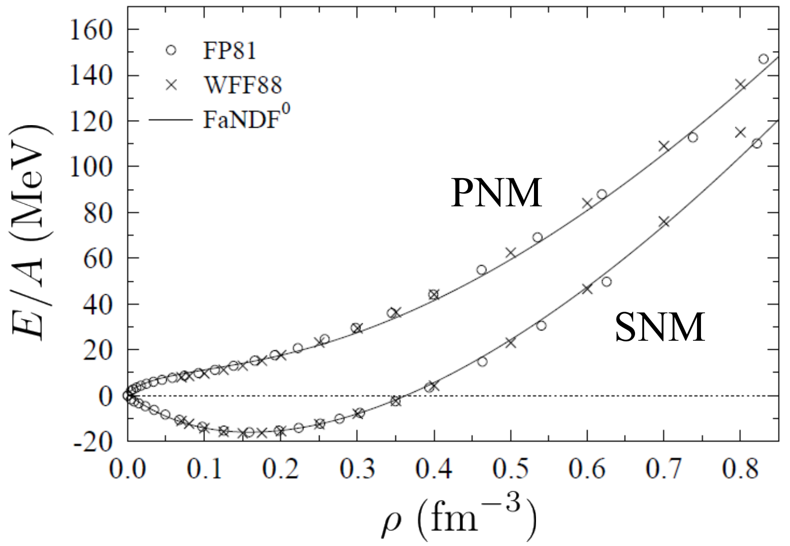
Energy density functional (EDF)

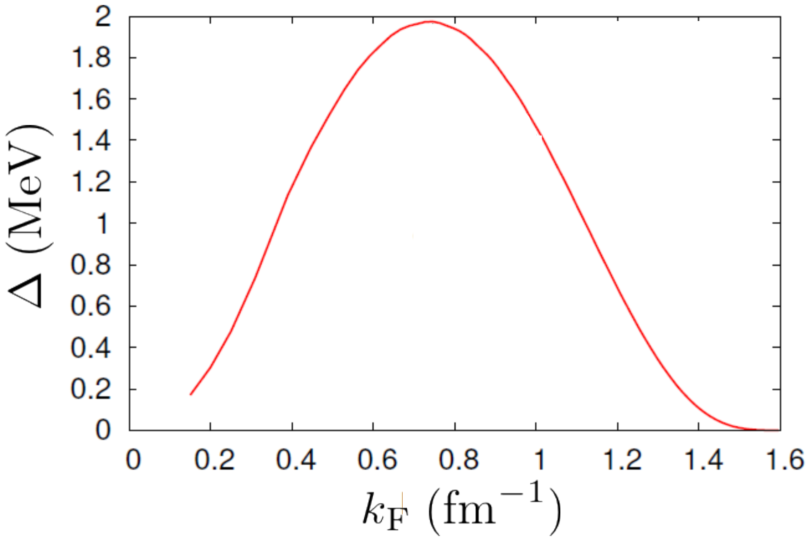
$$\mathcal{E}(\mathbf{r}) = \mathcal{E}_0(\mathbf{r}) + \mathcal{E}_{\rm pair}(\mathbf{r})$$

$\mathcal{E}_0(\mathbf{r})$: Fayans EDF (FaNDF^0) w/o LS

$$\mathcal{E}_{\rm pair}(\mathbf{r}) = g[\rho(\mathbf{r})]\big(|\nu_n(\mathbf{r})|^2 + |\nu_p(\mathbf{r})|^2\big)$$

S.A. Fayans and D. Zawischa, arXiv:nucl-th/0009034





What we have performed

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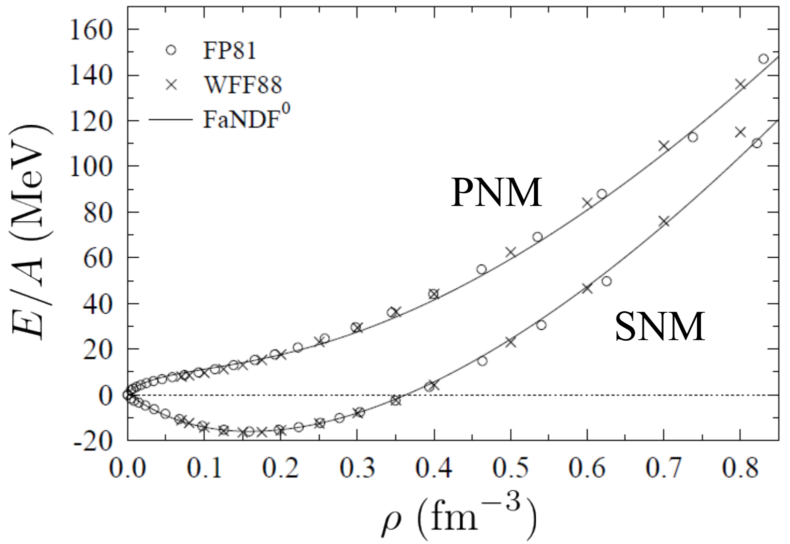

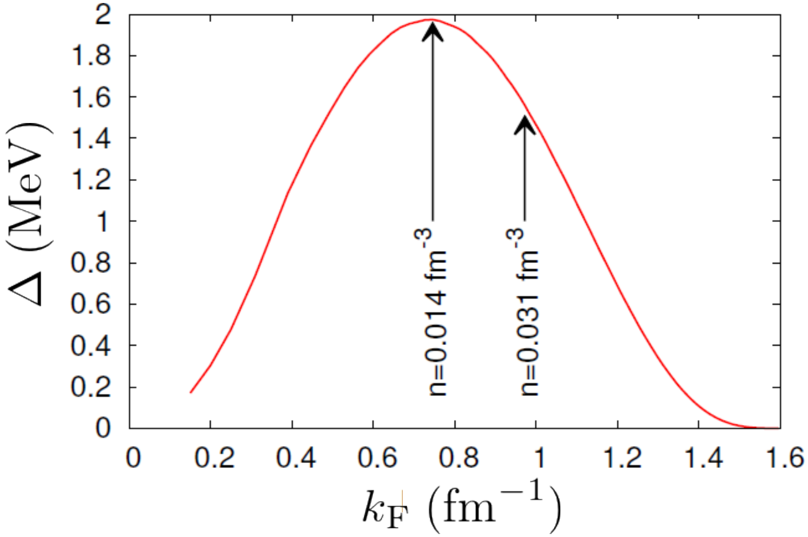

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☐ Computational details

$75\ \mathrm{fm} \times 75\ \mathrm{fm} \times 60\ \mathrm{fm}$

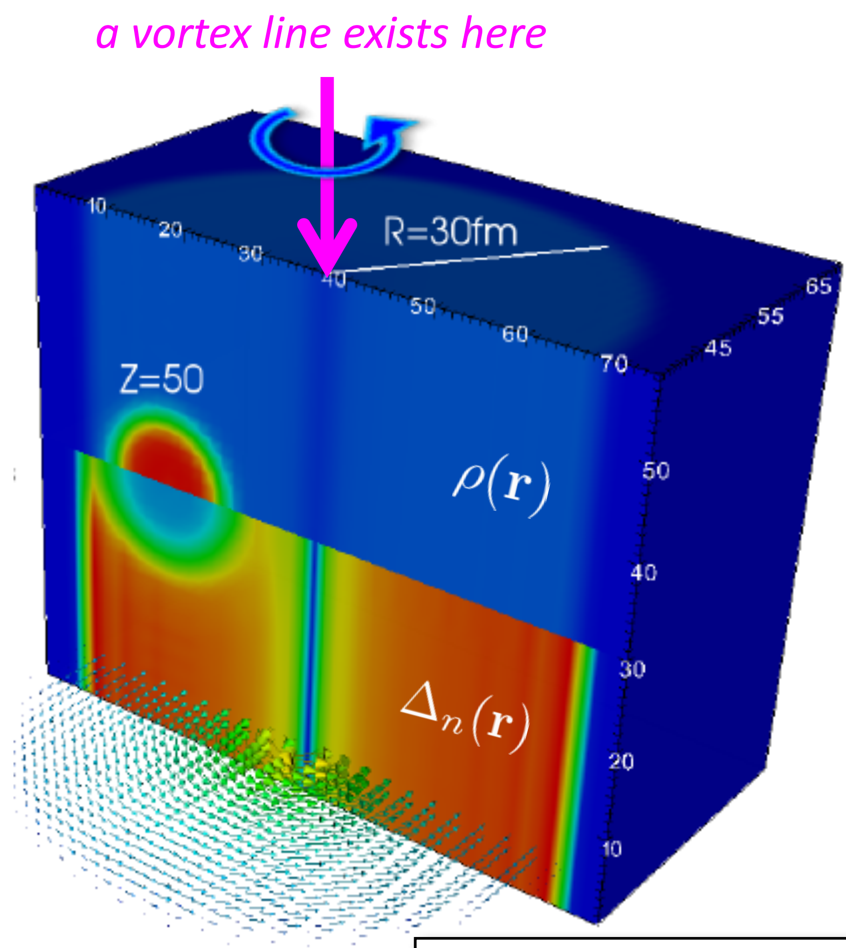
$(50 \times 50 \times 40,\ \Delta x = 1.5\ \mathrm{fm})$

Nuclear impurity: $Z = 50$

$\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$ $(N \simeq 2{,}530)$

$\rho_n \simeq 0.031\ \mathrm{fm}^{-3}$ $(N \simeq 5{,}714)$

# of quasi-particle w.f. $\approx 50{,}000$



E.g. $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$

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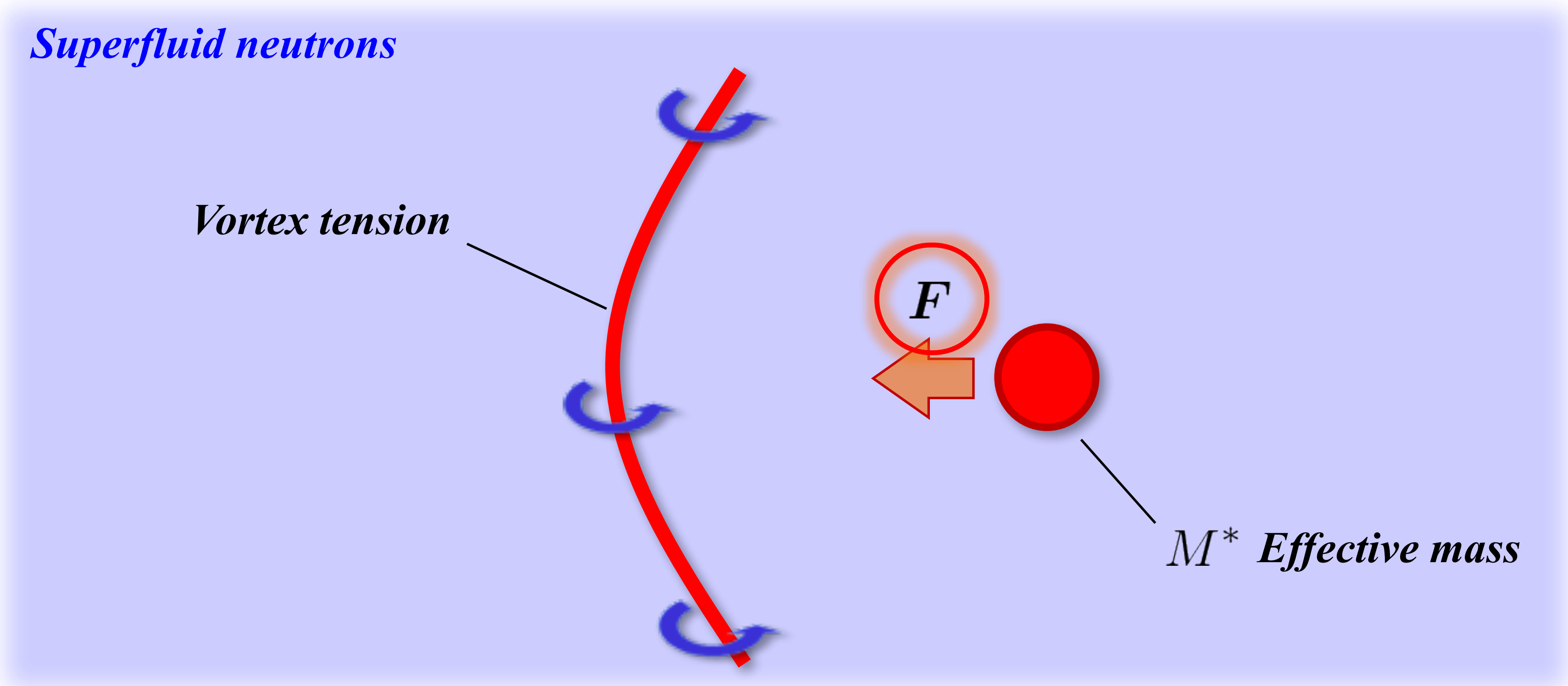
MPI+GPU
→ 48h w/ 200GPUs
for 10,000 fm/c

TITAN, Oak Ridge

NERSC Edison, Berkeley

HA-PACS, Tsukuba

3. Results



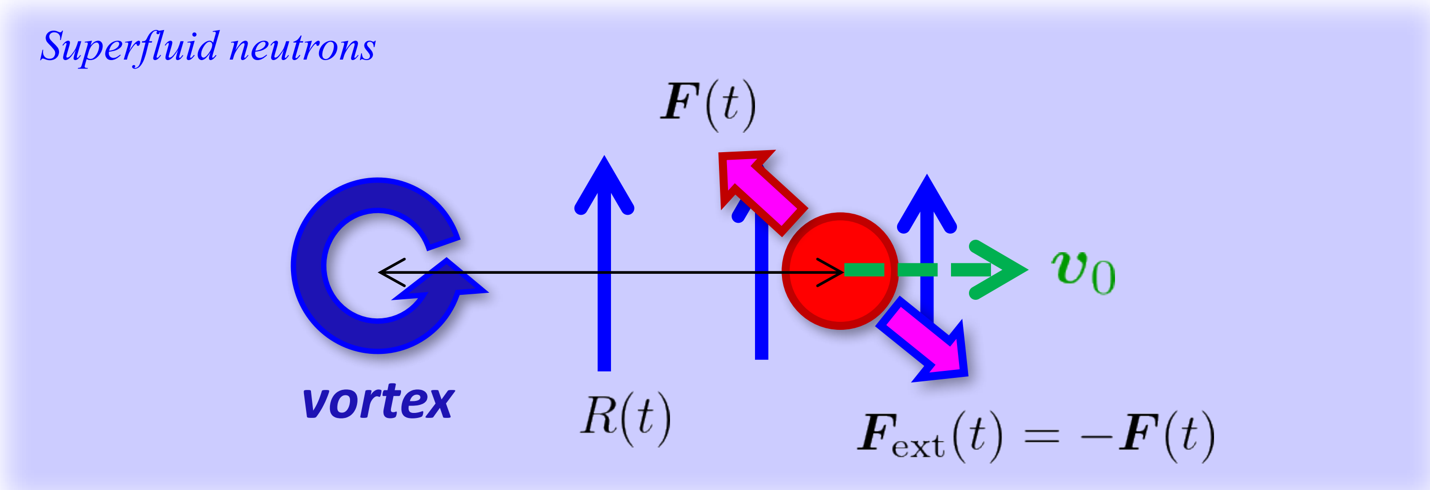
How to extract the force

We directly measure the force $\boldsymbol{F}(R)$ in dynamical simulation

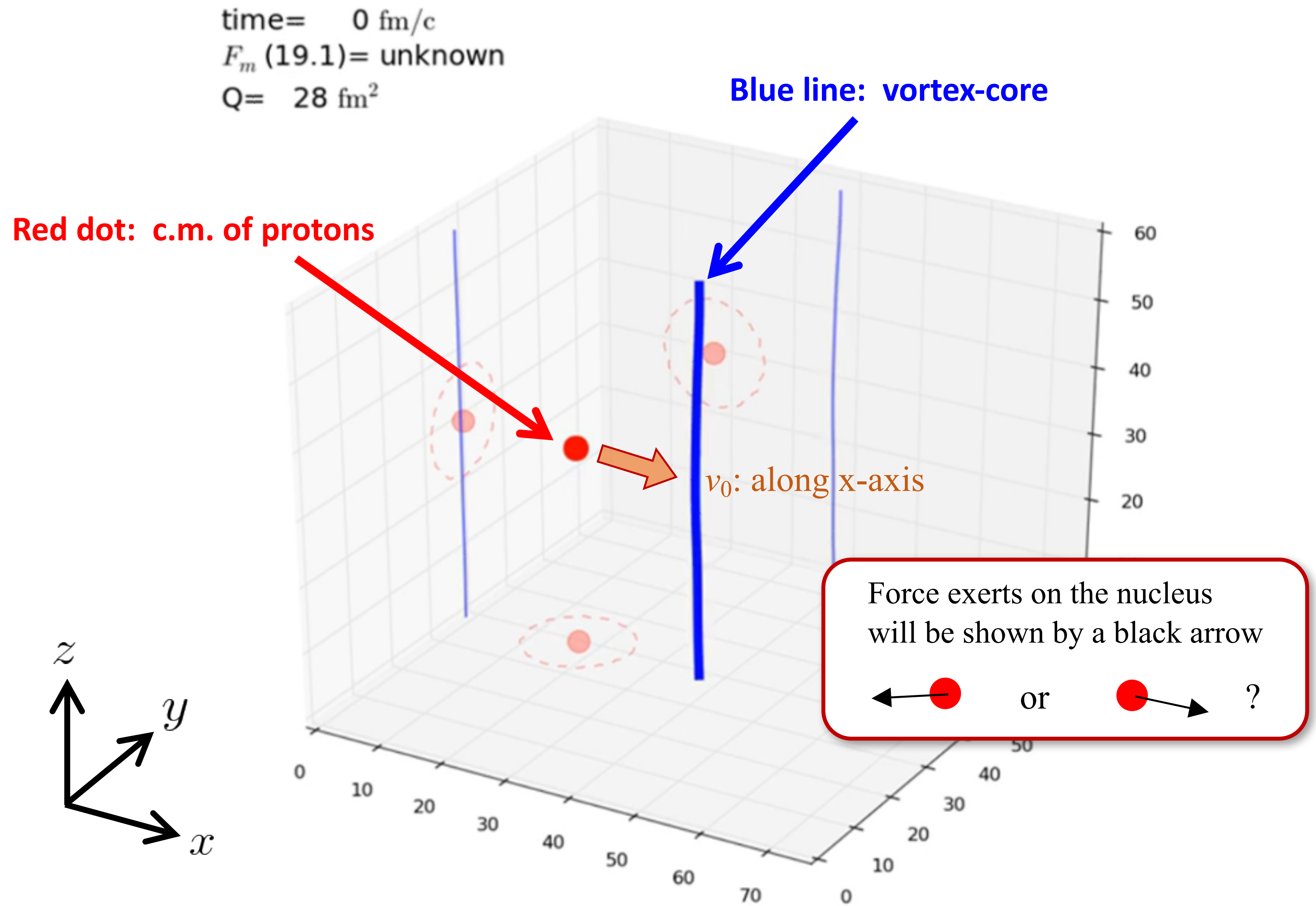
- Newton's law

$$\boldsymbol{F} = M\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \quad \Rightarrow \quad \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = 0 \quad \text{if} \quad \boldsymbol{F} = 0$$

- We keep a nuclear motion in a constant velocity $v_0 \; (\ll v_{\rm crit})$



Results of TDSLDA calculation: $n_n \simeq 0.014\ \mathrm{fm}^{-3}$



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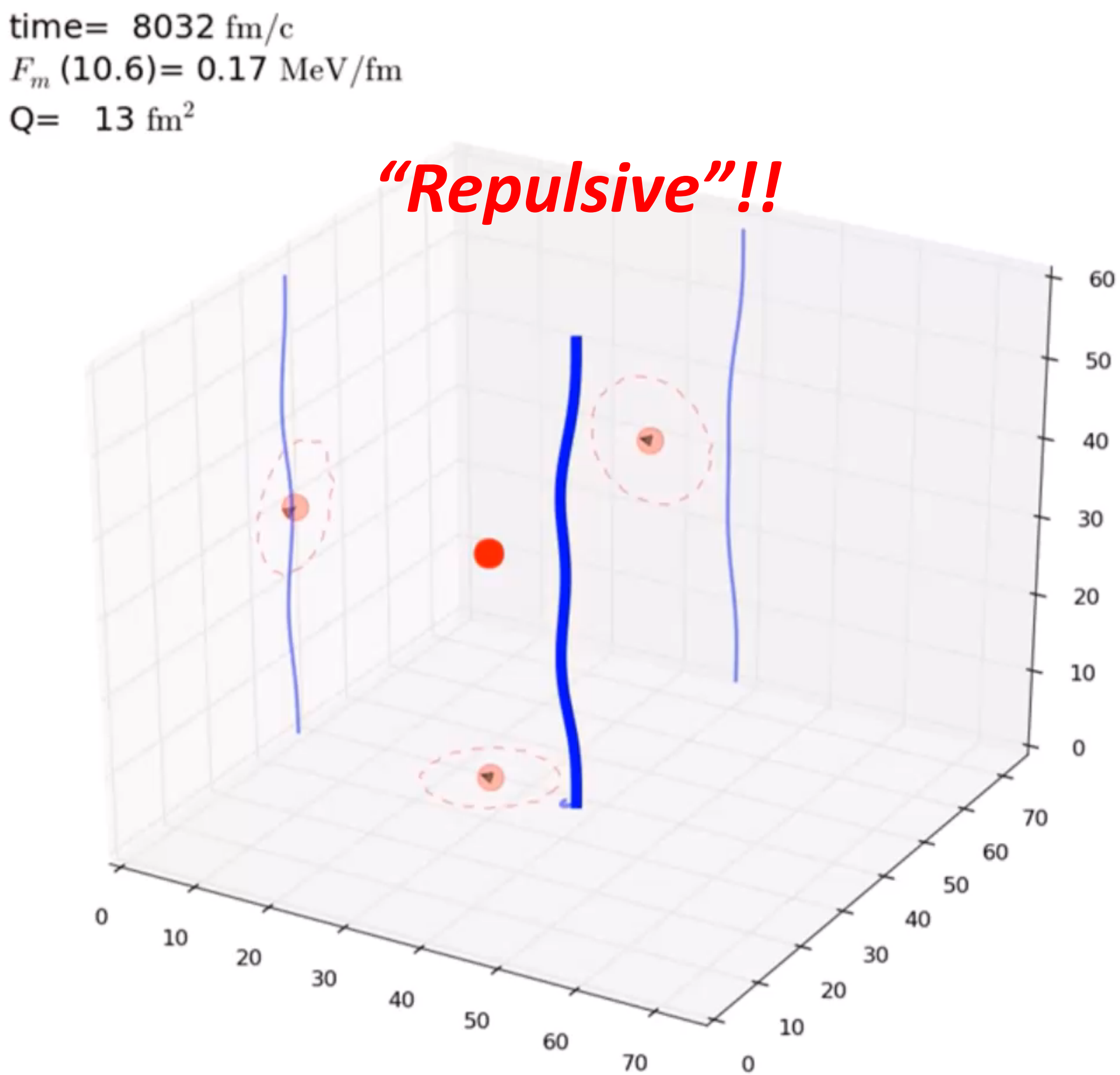
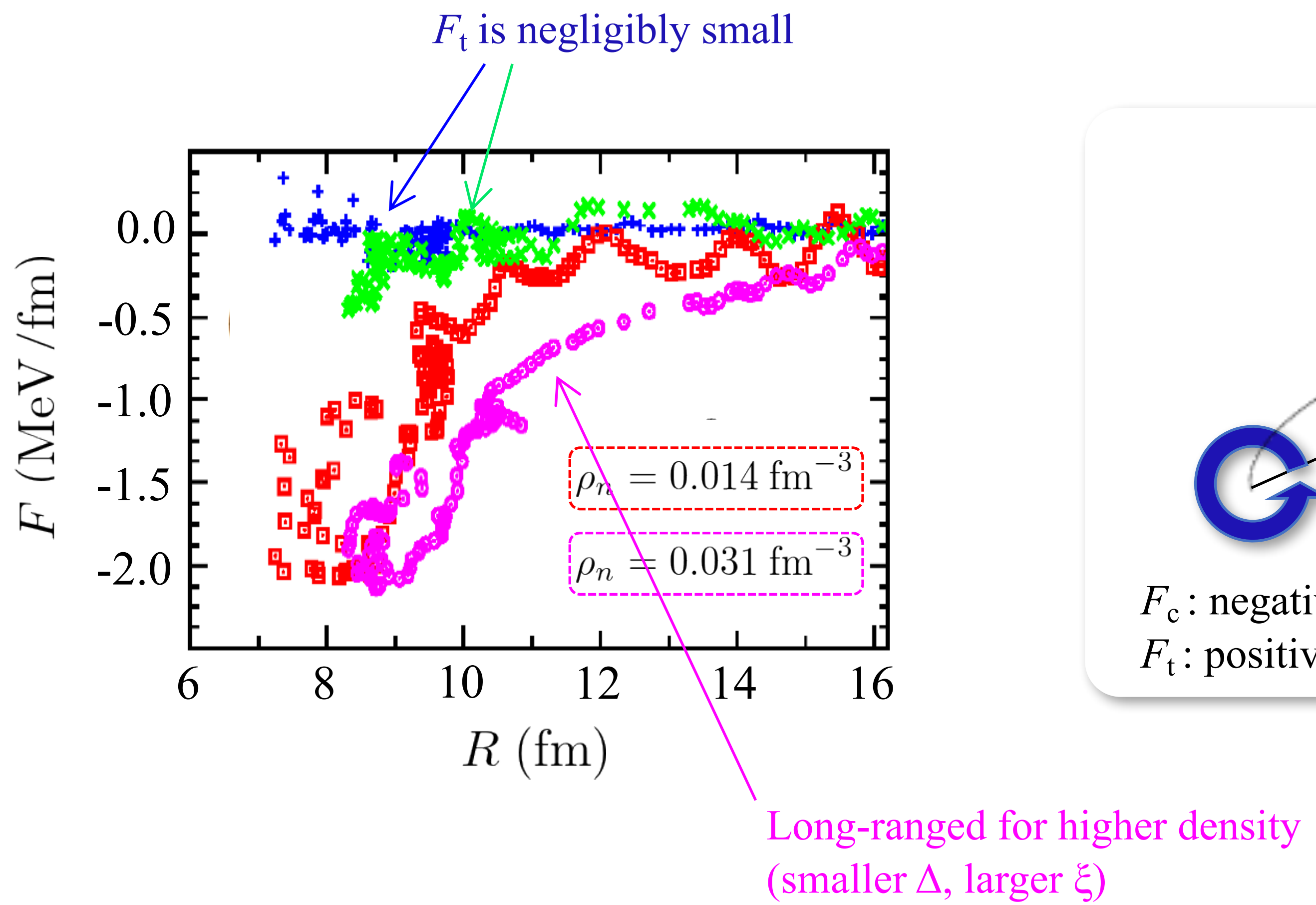
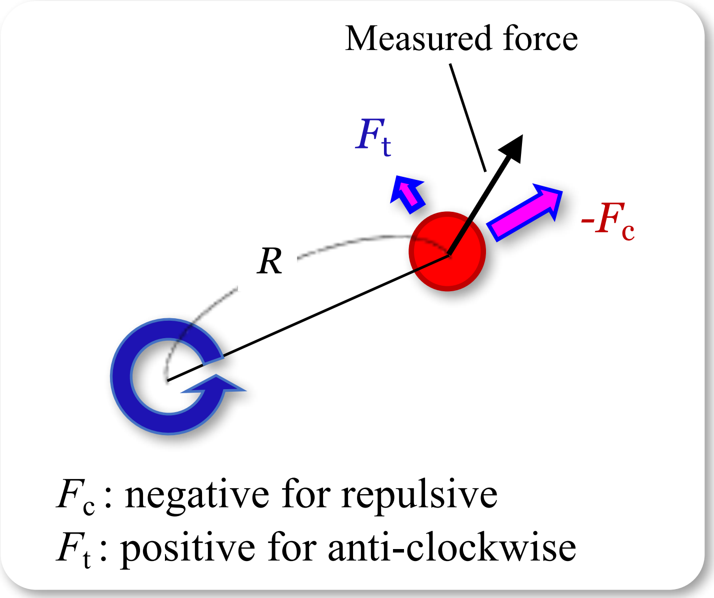


Measured force

The force is essentially central, not a simple function of *R*





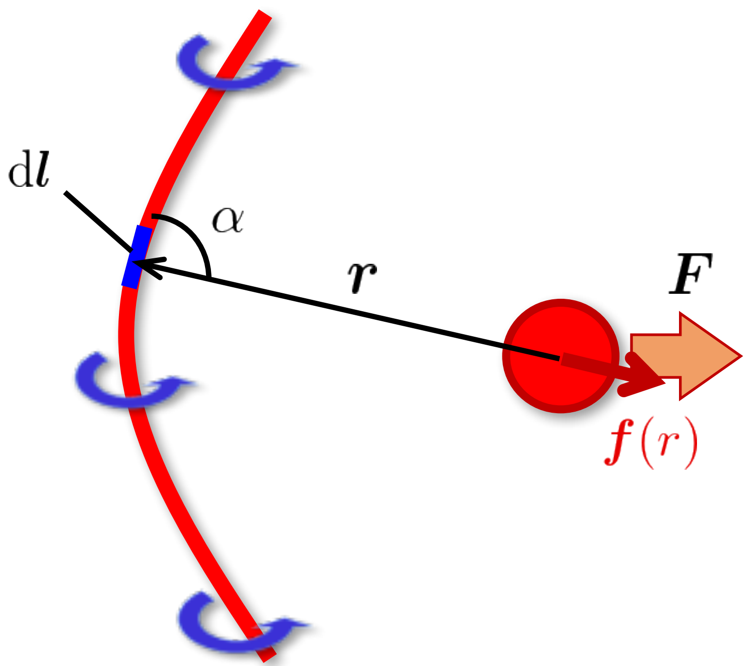
Force per unit length

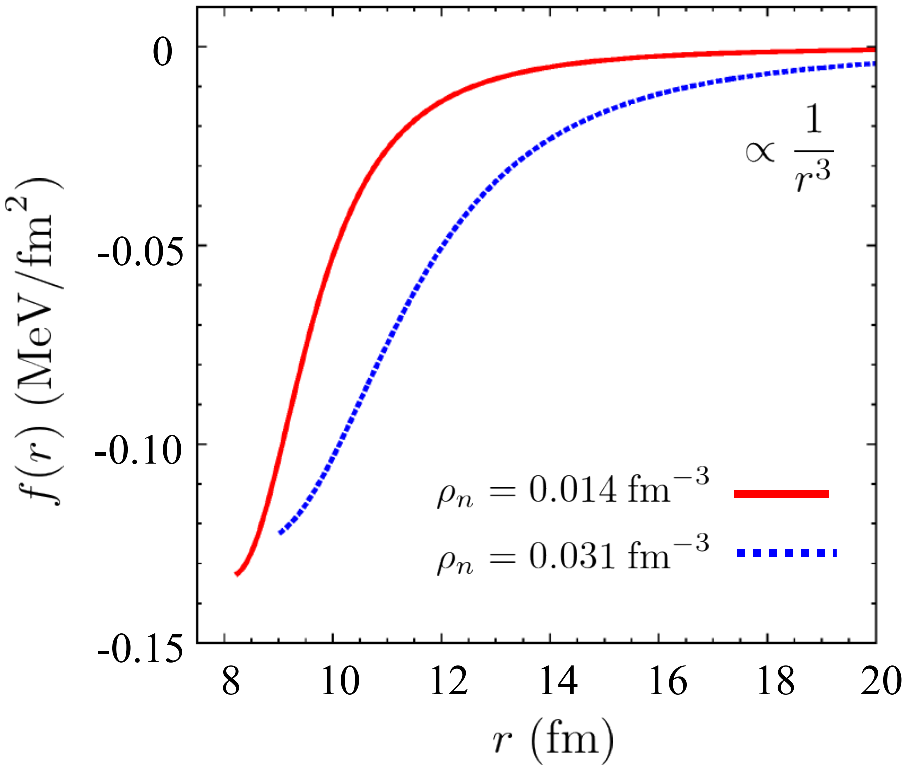
We can predict the force for any vortex-nucleus configuration

➢ Force per unit length

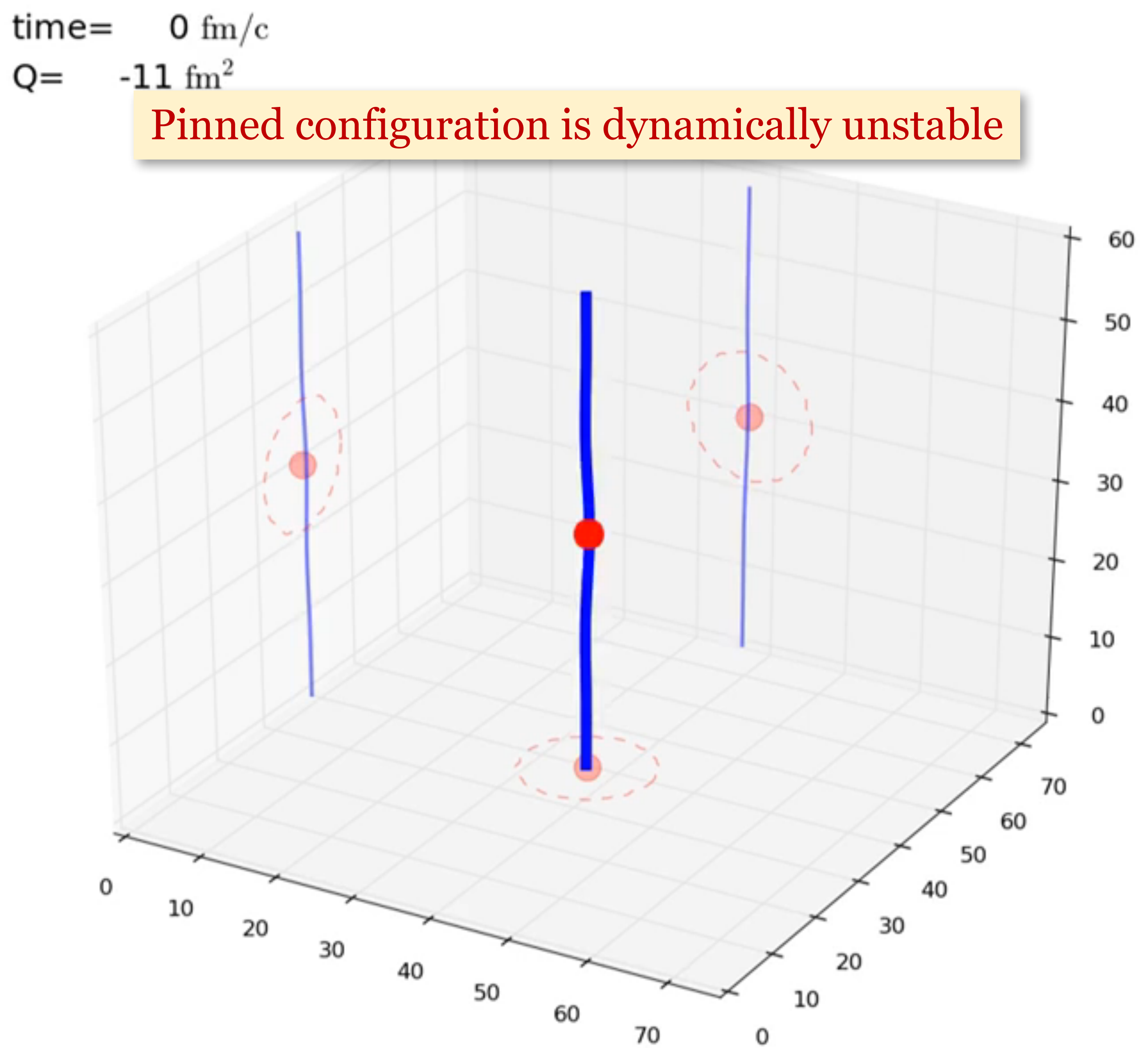
$$\boldsymbol{F} = \int_L f(r) \sin\alpha \, \boldsymbol{e}_r \, \mathrm{d}l \qquad f(r) = \frac{\sum_{k=0}^{n} a_k r^k}{1 + \sum_{k=1}^{n+3} b_k r^k}$$

Padé approximant
(n=2 was used)

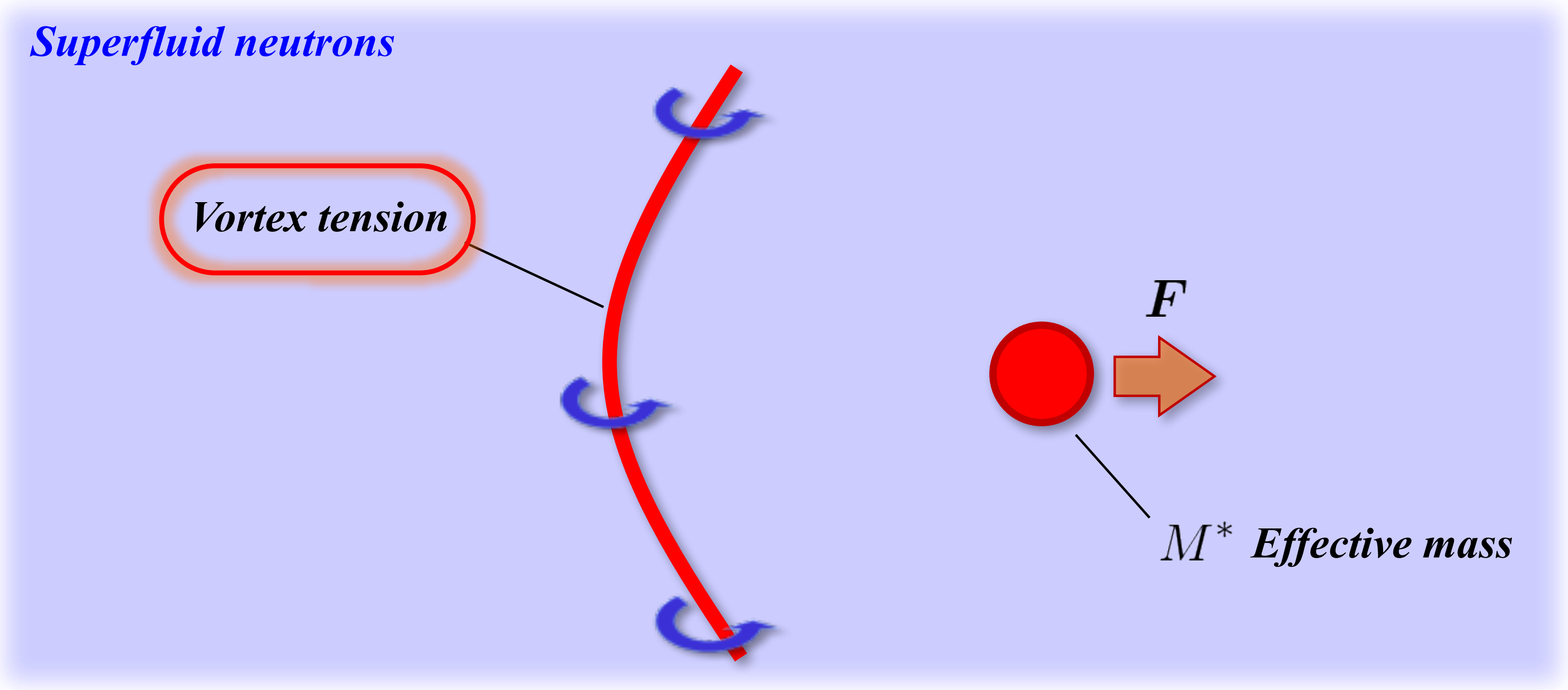




Results of TDSLDA calculation: $n \simeq 0.014\ \mathrm{fm}^{-3}$



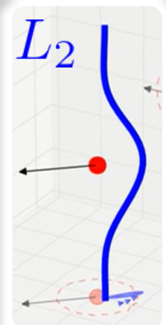
Pinned configuration is dynamically unstable


Superfluid neutrons
Vortex tension
F
M^* Effective mass

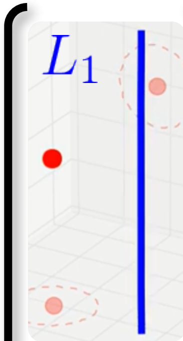
Vortex tension

We can evaluate the vortex tension from the dynamical simulations

$$T \lesssim \frac{E^*}{\Delta L} = \left(E\left[\right. \right.$$



$$\left. \left. \right] - E\left[\right. \right.$$



$$\left. \left. \right] \right) \Big/ \Delta L$$

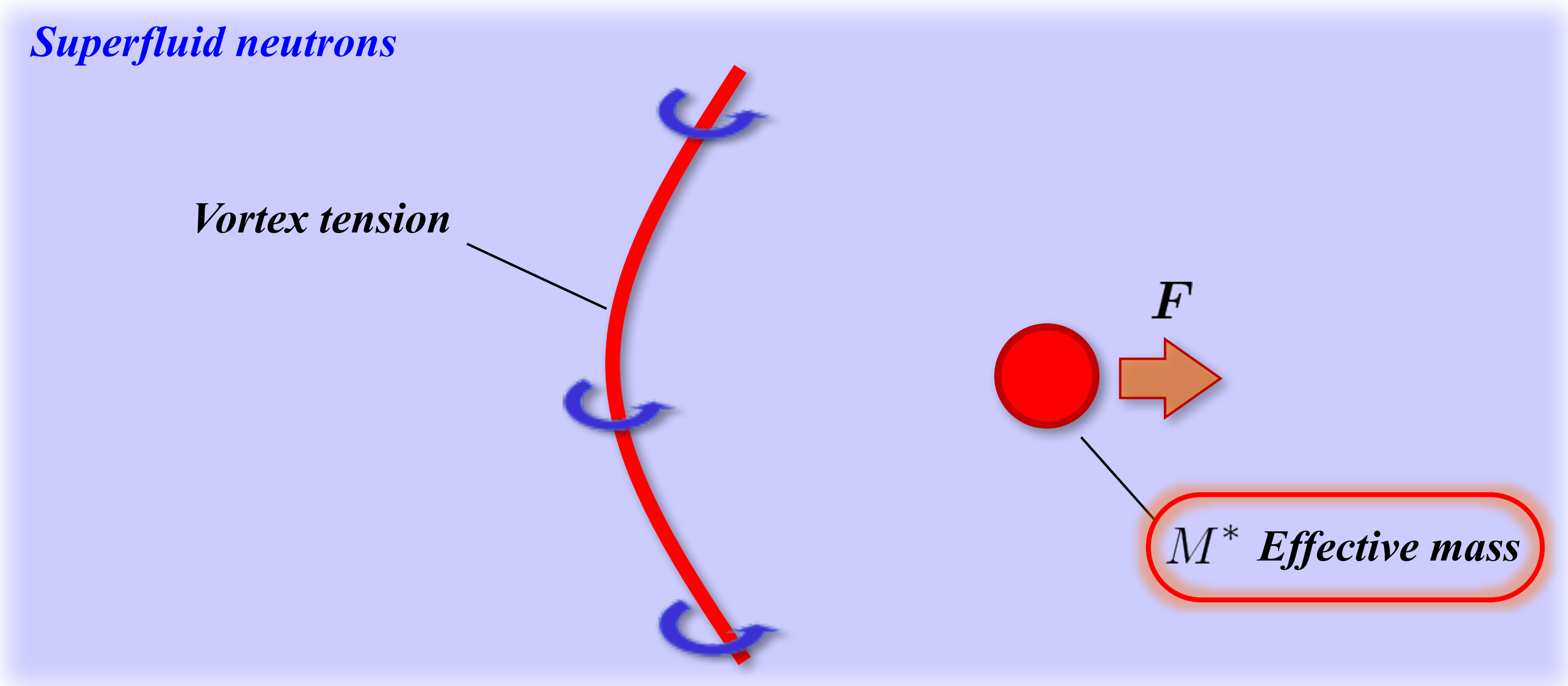
Work done by $\boldsymbol{F}_{\rm ext}$

$$\int_{t_0}^{t_1} \boldsymbol{F}(t) \cdot \boldsymbol{v}(t) dt$$

$$T \lesssim \frac{5}{3.5} = 1.4\,\mathrm{MeV/fm} \qquad n = 0.014\,\mathrm{fm}^{-3}$$

$$T \lesssim \frac{11}{1.5} = 7.3\,\mathrm{MeV/fm} \qquad n = 0.031\,\mathrm{fm}^{-3}$$

$$\frac{1.4}{7.3} = 0.19$$ cf. hydrodynamic approx.: 0.77


Superfluid neutrons
Vortex tension
F
M^* Effective mass

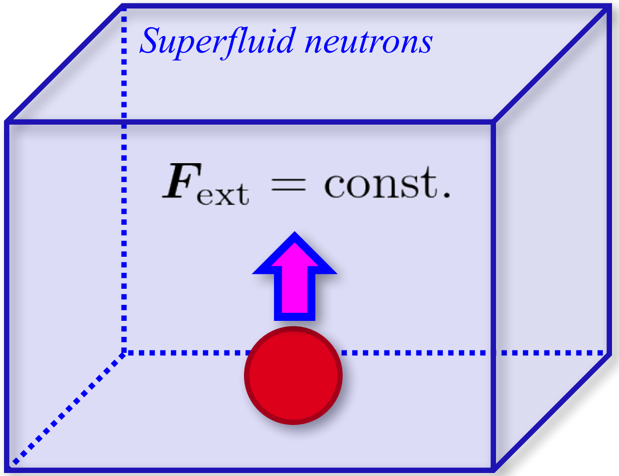
How to extract the effective mass

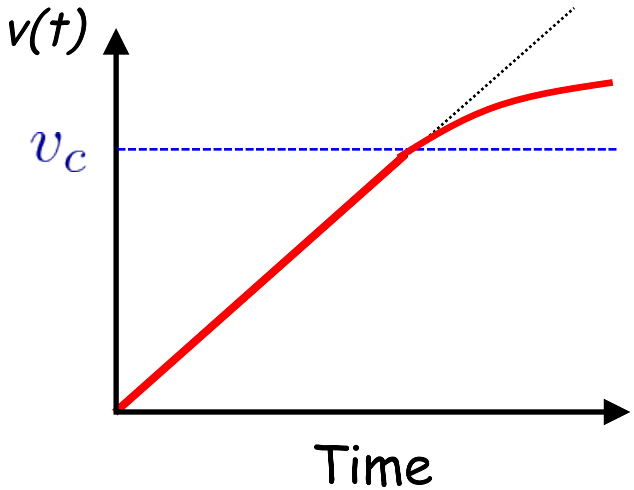
Dragging by a constant force provides the effective mass

- Newton's law

$$\boldsymbol{F} = M\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \quad \Rightarrow \quad M = F\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)^{-1}$$

- We accelerate a nuclear impurity by a constant force

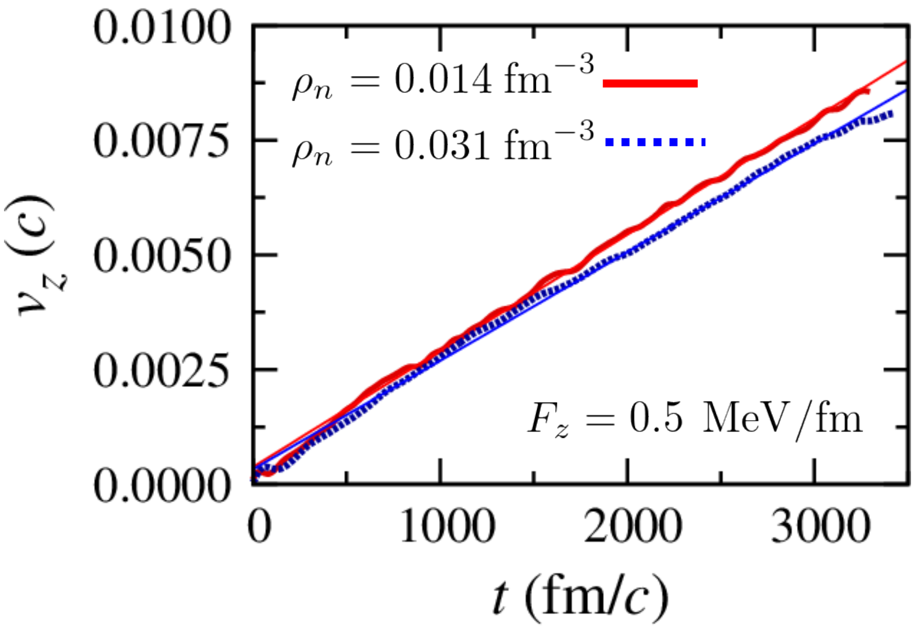




Effective mass

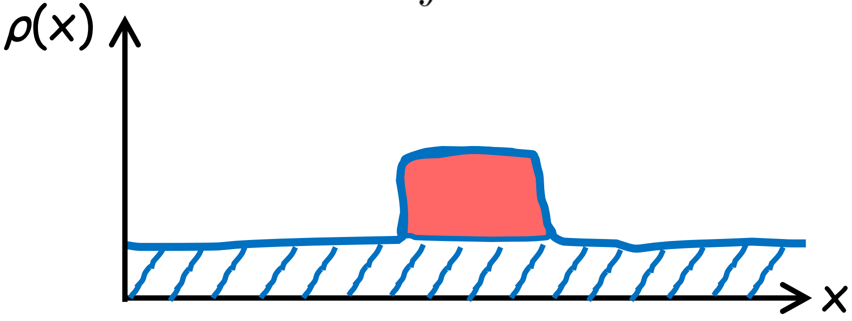
Dynamical effects may reduce the effective mass

Preliminary



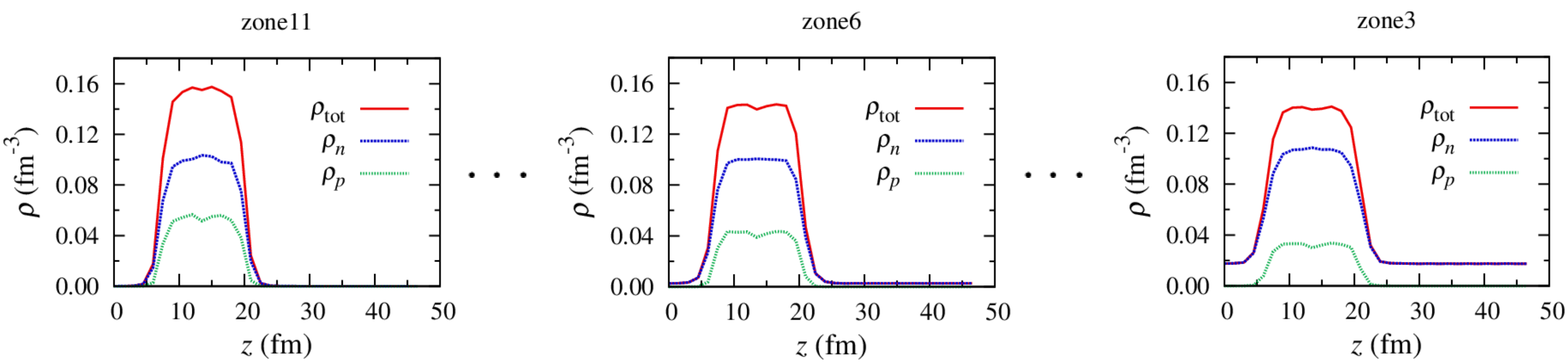
$$\frac{M^*}{M_{\mathrm{nucl}}} = \begin{cases} 0.96 & n = 0.014\ \mathrm{fm}^{-3} \\ 0.87 & n = 0.031\ \mathrm{fm}^{-3} \end{cases}$$

$$M_{\mathrm{nucl}} = m \int \left(\rho(\boldsymbol{r}) - \rho_n^{\mathrm{b.g.}}\right) d\boldsymbol{r}$$



We are going to calculate M^* and v_c through out the inner crust

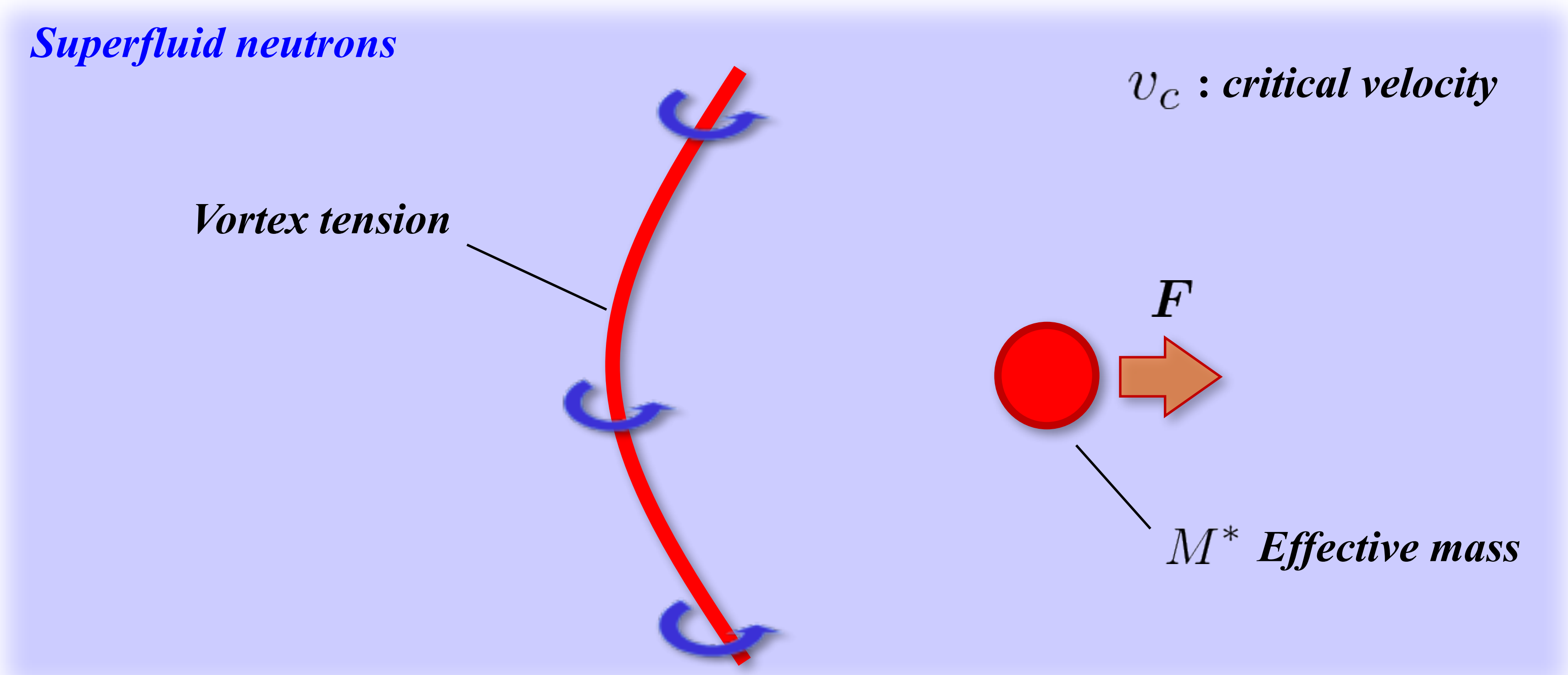
✓ We have prepared initial states for dynamical simulations



4. Conclusion

Conclusion

We can compute various ingredients of the inner crust by microscopic, dynamical simulations!



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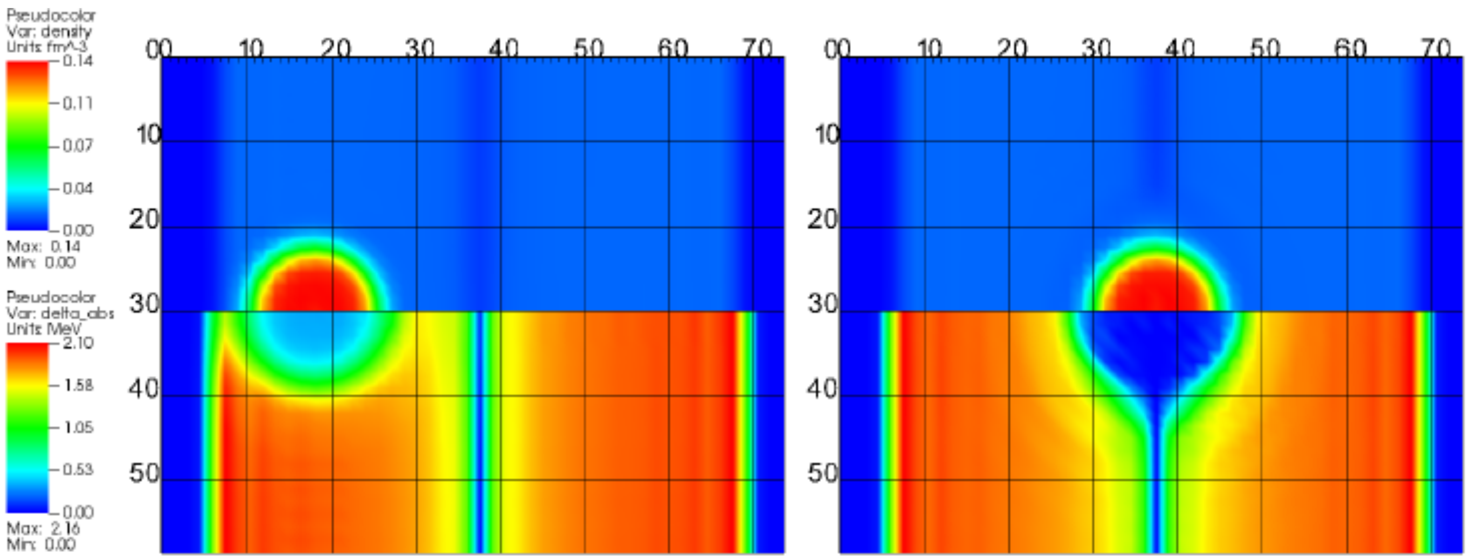
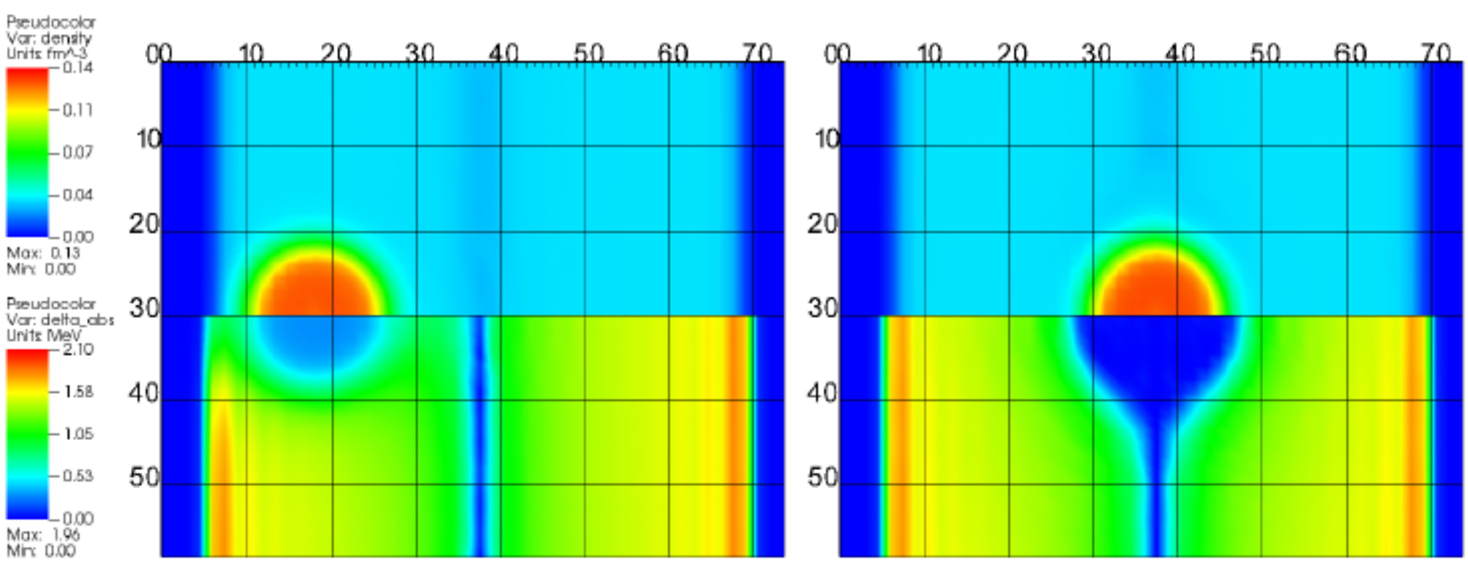
Backup

Initial states

$\rho_n = 0.014\ \mathrm{fm}^{-3}$

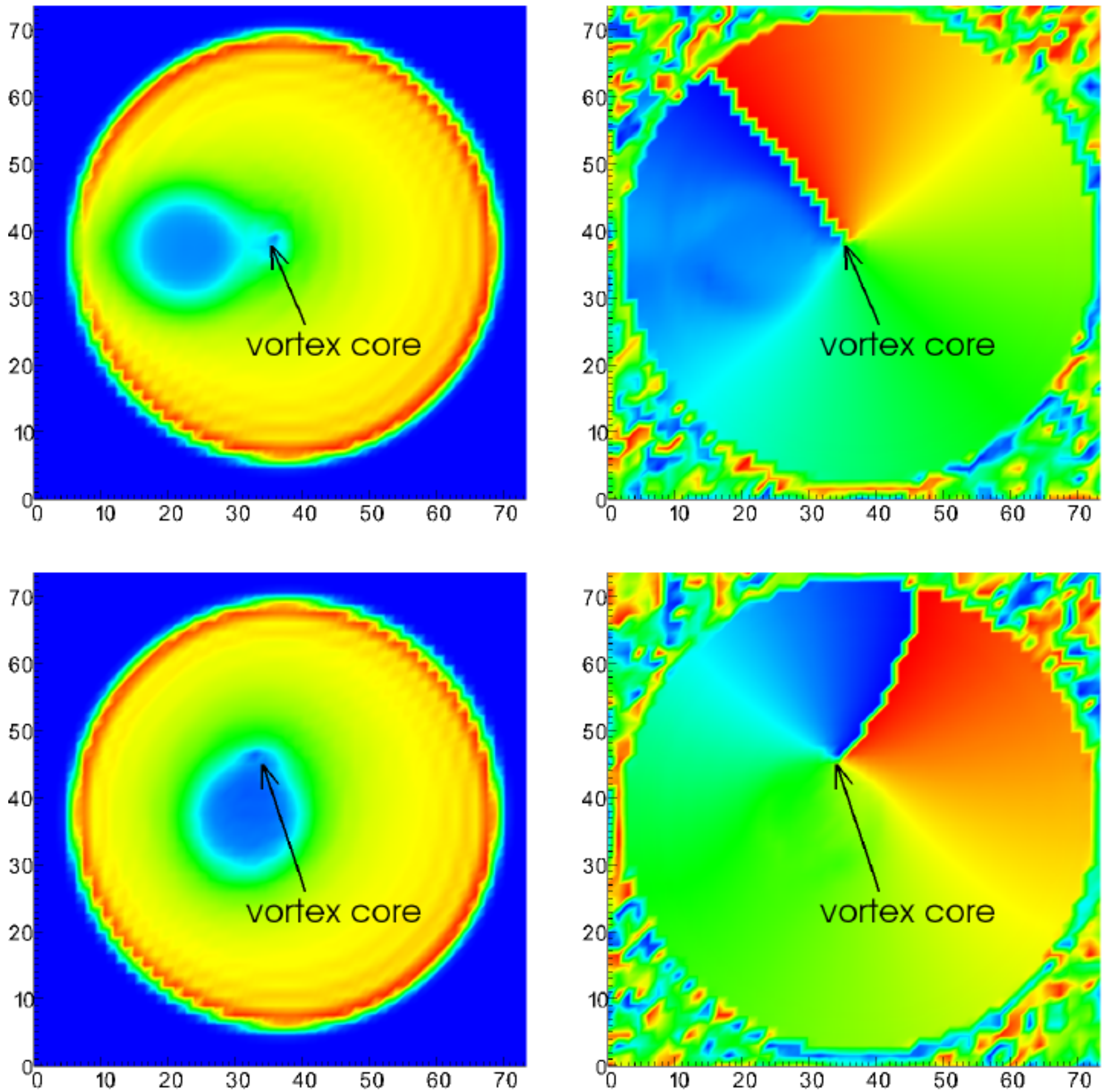


$\rho_n = 0.031\ \mathrm{fm}^{-3}$



Vortex detection

$$\rho_n = 0.031\ \mathrm{fm}^{-3}$$



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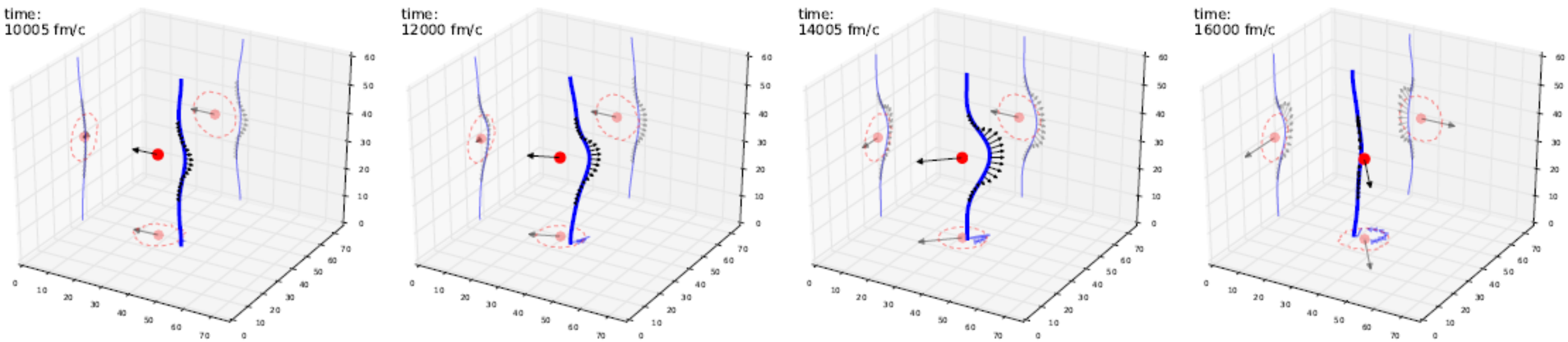
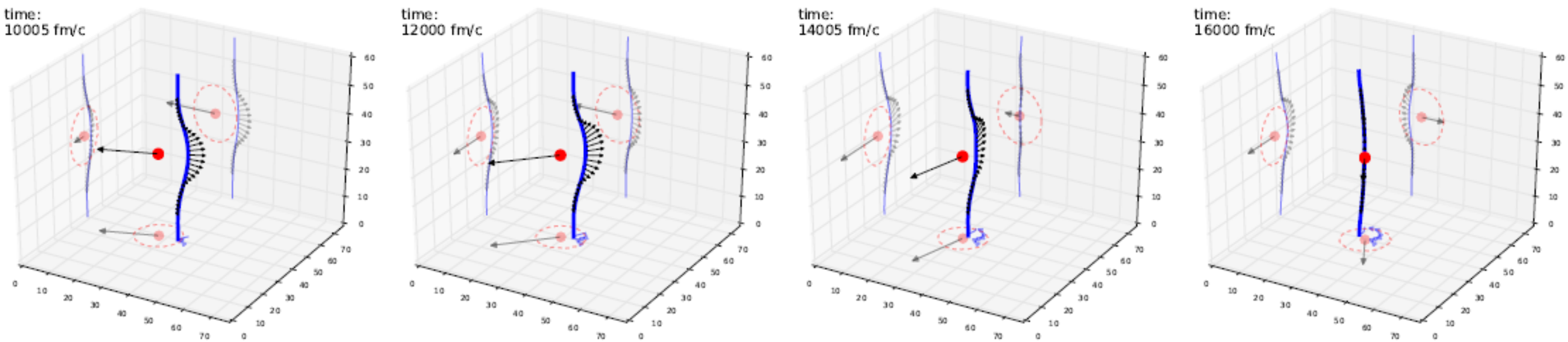
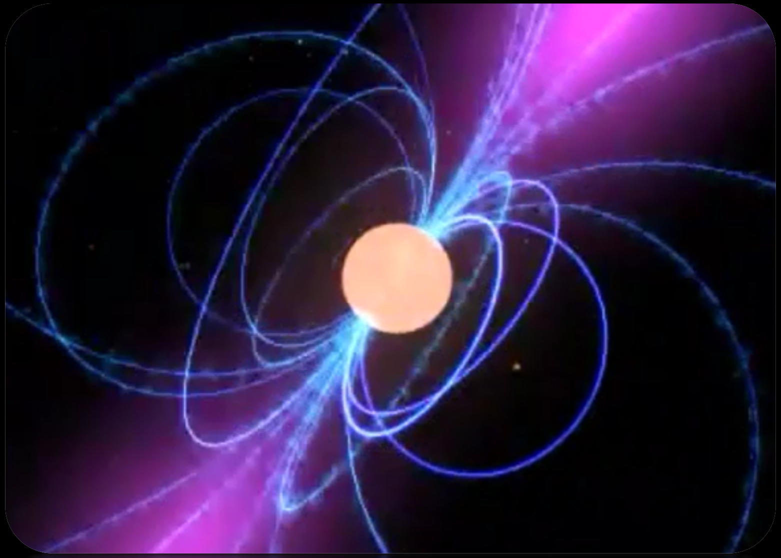


$$\rho_n = 0.031\ \mathrm{fm}^{-3}$$



Pulsar: a rotating neutron star

Pulsar is one of the most accurate atomic clock



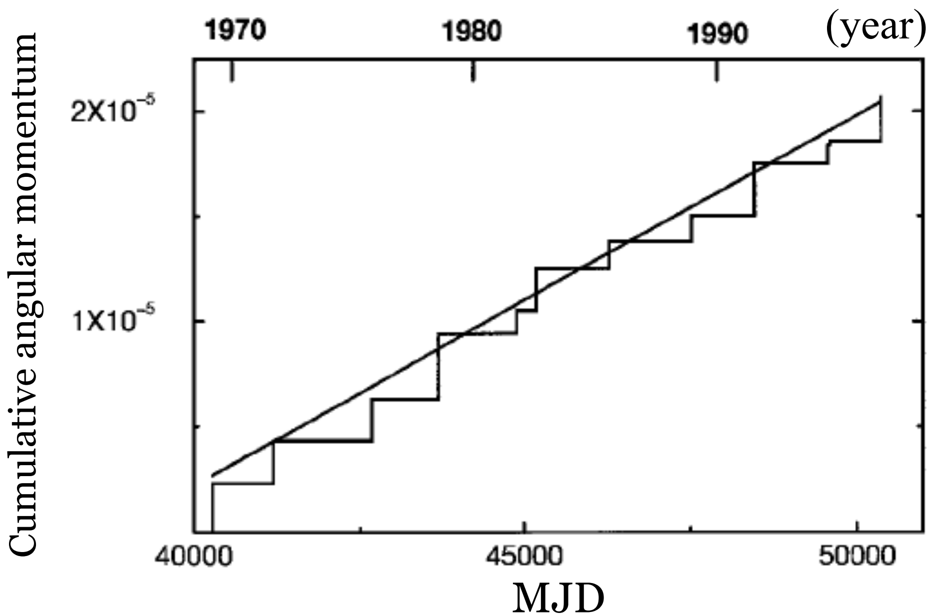
- First observation in 1968 (Crab pulsar)
- More than 2000 pulsars have been found
- Rotation period: a few ms - several seconds
- Spin-down: at most a few tens of ms per year

Irregularities in their rotational frequency have been observed: the *"glitches"*

What is the “glitch”?

Glitch is a sudden spin-up of the rotational frequency

Ex.) The Vela pulsar (PSR B0833-45)



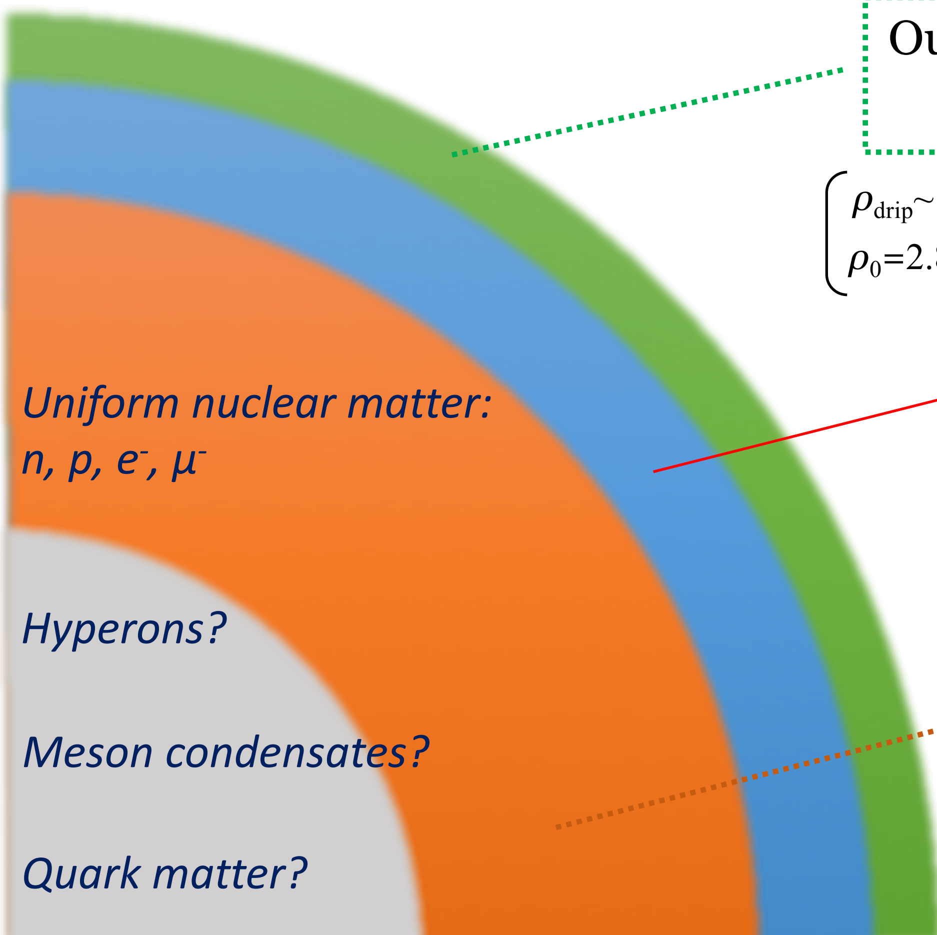
- One of the most active glitching pulsars
- Period of pulsation: 89 ms
- Time between glitches: a few years
- $\Delta\Omega/\Omega \sim 10^{-6}$
- It repeats regularly

*MJD: Modified Julian Date

Something must happen inside the neutron star!

Where are glitches originated from?

The "inner crust" of a neutron star is relevant to the glitches



Outer crust Nuclei (Fe), electrons

0.3-0.6 km $\rho < \rho_{drip}$

$\rho_{drip} \sim 0.0014\rho_0$; Above ρ_{drip} unbound neutrons exist outside nuclei

$\rho_0 = 2.8 \times 10^{14}$ g/cm^3 = 0.16 fm^{-3} ; Nuclear saturation density

Inner crust Nuclei, electrons, "dripped" neutrons

0.5-0.8 km $\rho_{drip} \leq \rho \leq 0.6\rho_0$

Outer/inner core

9-12 km $0.6\rho_0 < \rho \leq 3\text{-}5\rho_0$

Structure of the inner crust

A lattice of neutron-rich nuclei are immersed in a neutron superfluid

Quantum vortices can exist!

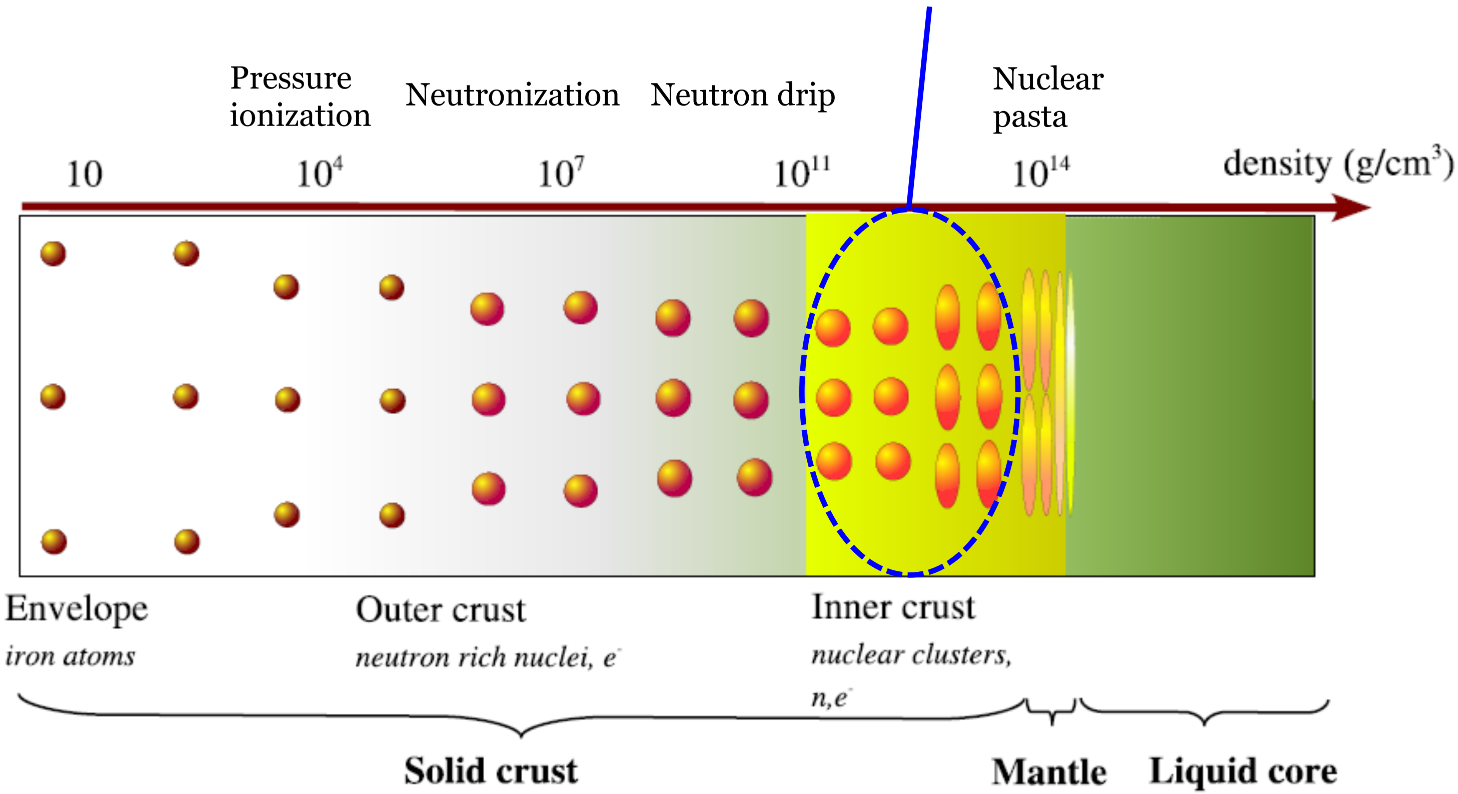
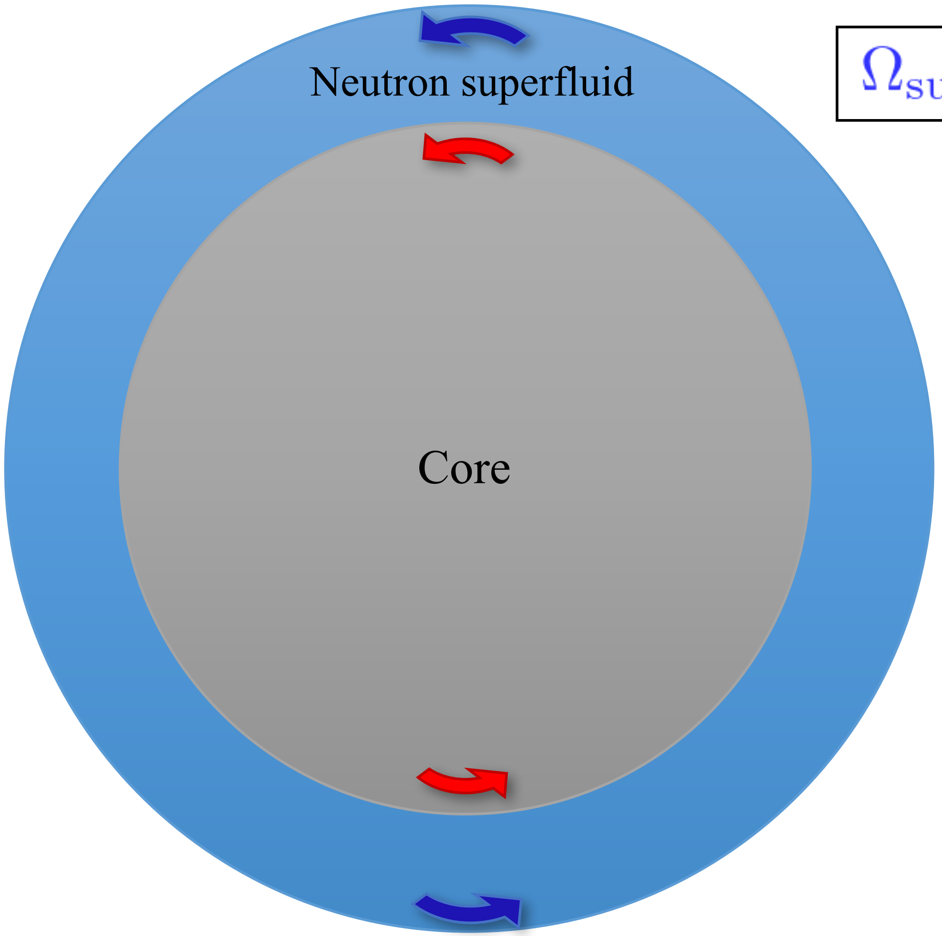


Fig.4 in N. Chamel and P. Haensel, Living Rev. Relativity **11**, 10

Scenario of the glitch

Scenario of the glitch

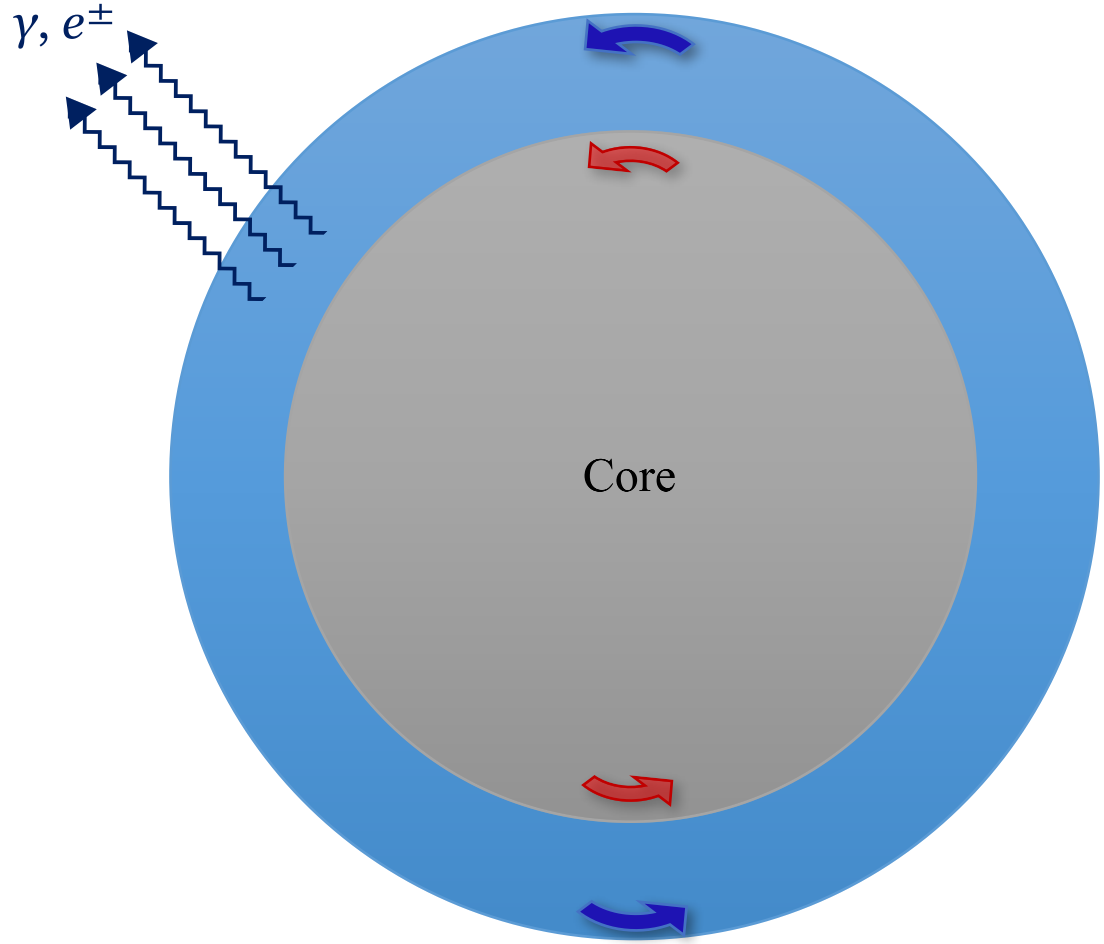
✓ Superfluid component is decoupled from normal one



$$\Omega_{\mathrm{super}} > \Omega_{\mathrm{core}}$$

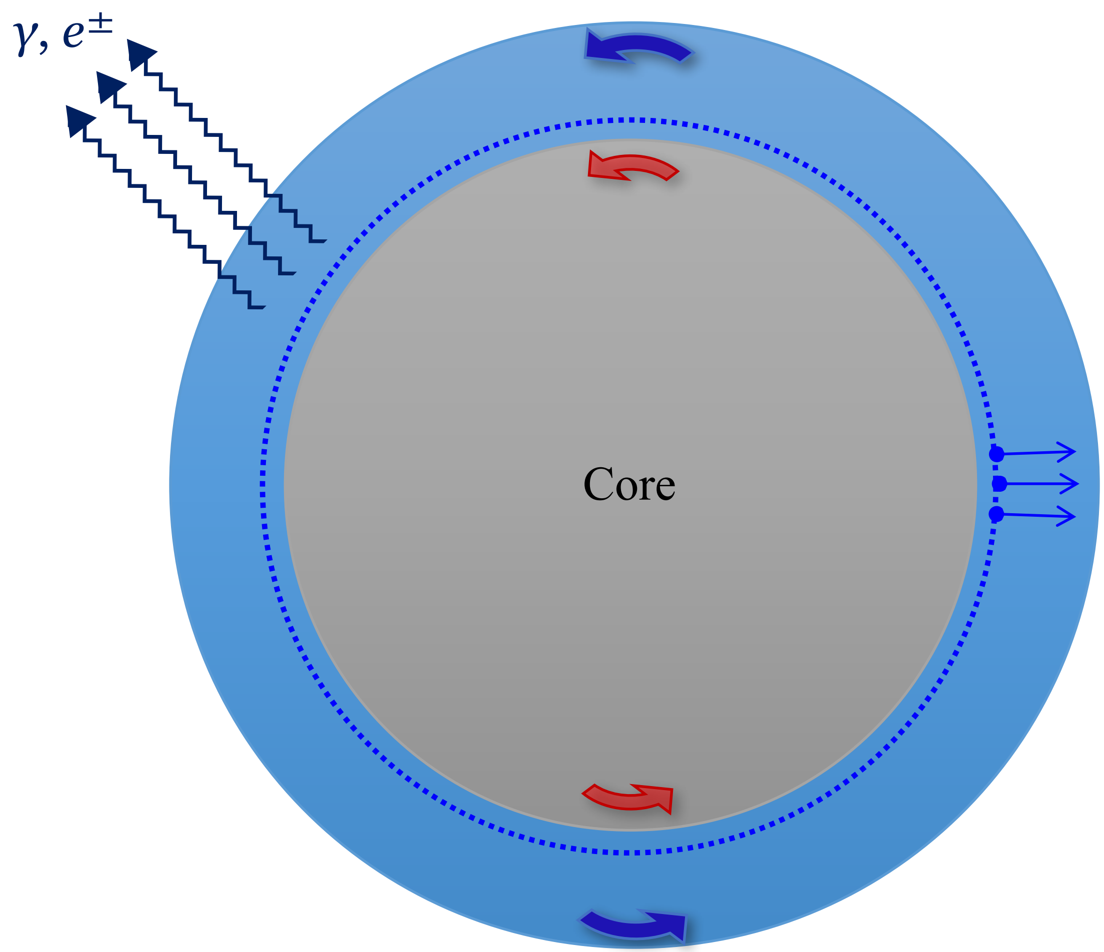
Scenario of the glitch

✓ Core must spin down due to the radiation processes



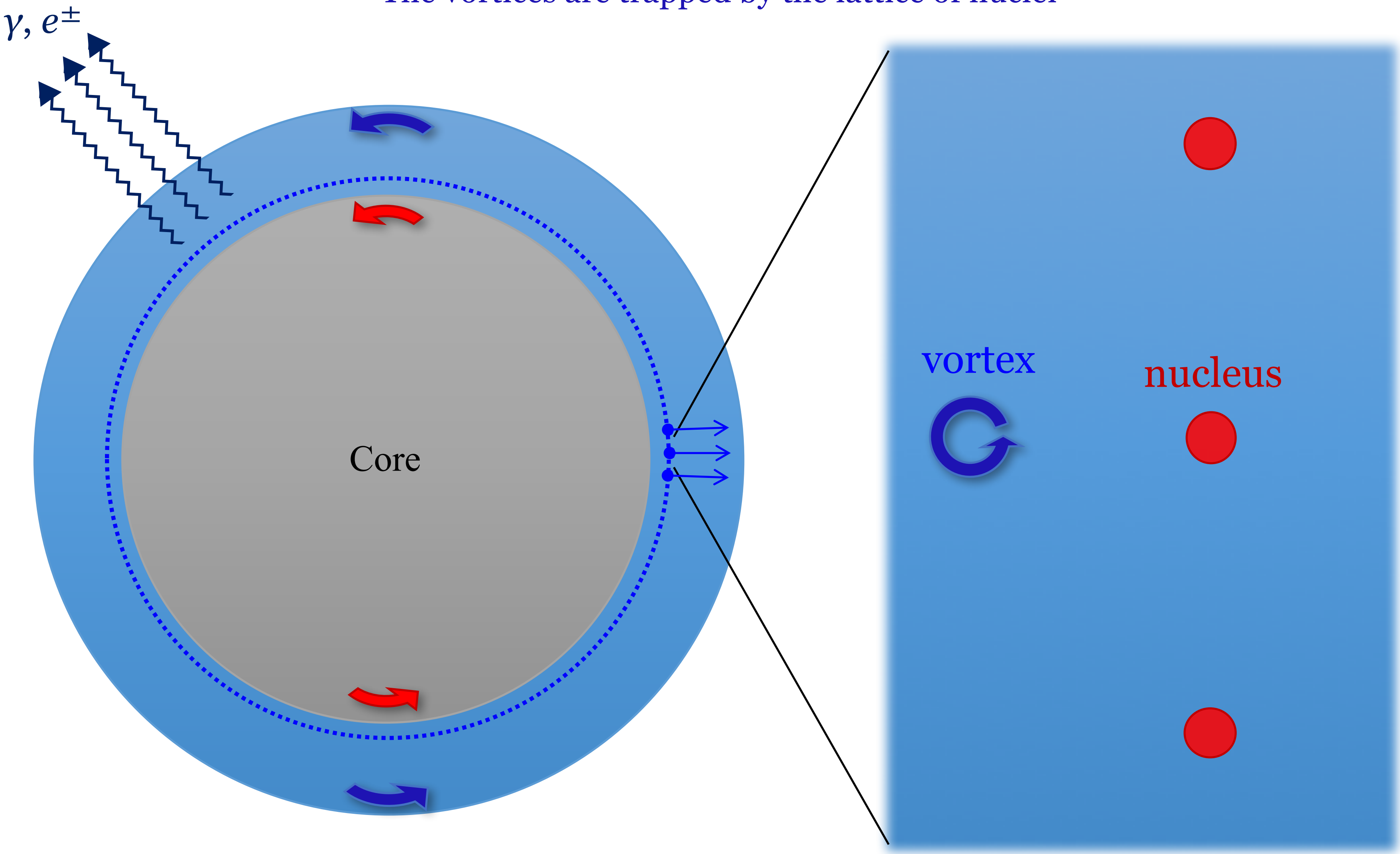
Scenario of the glitch

✓ Neutron superfluid follows the spin-down by expelling vortices outward



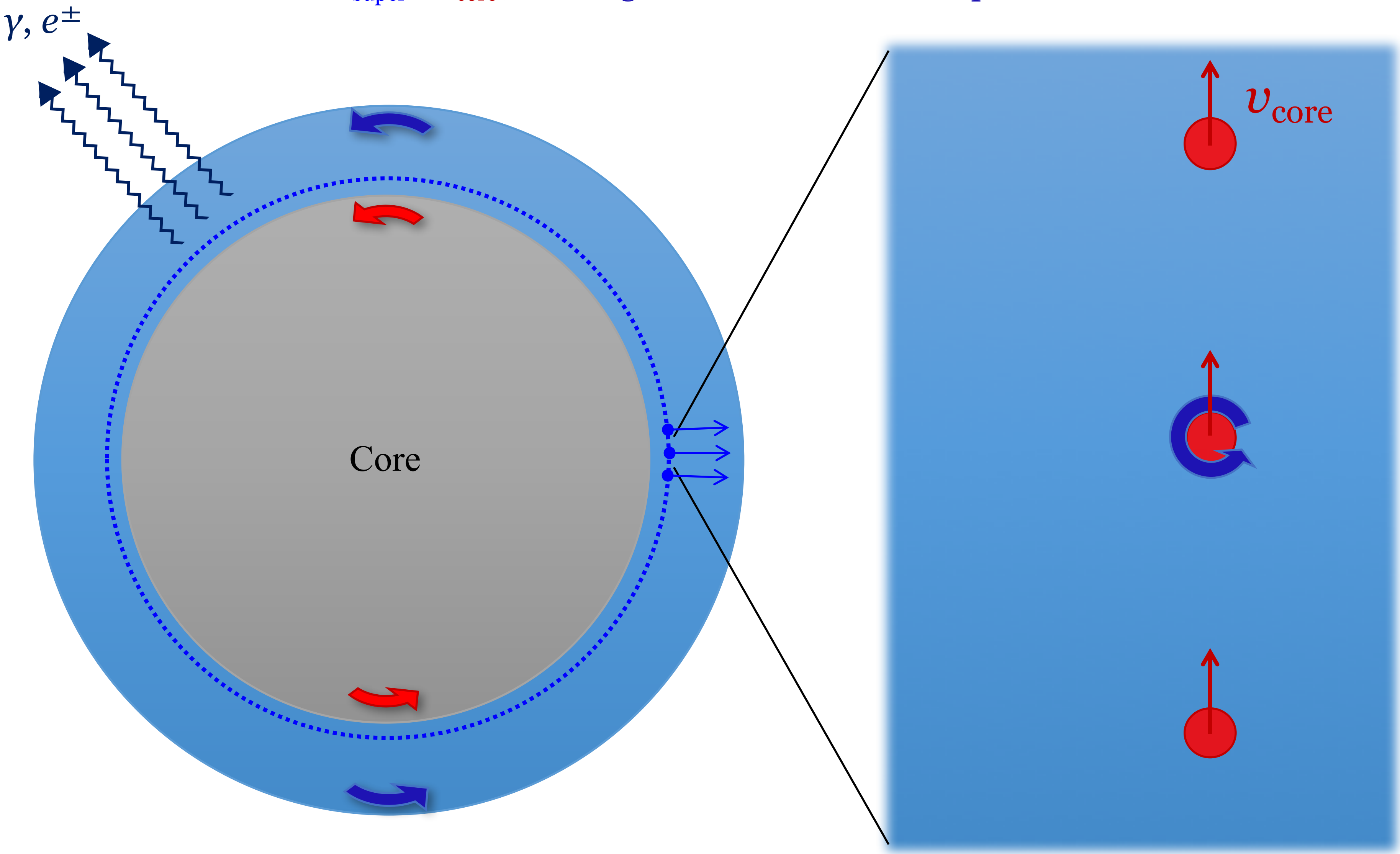
Scenario of the glitch

✓ The vortices are trapped by the lattice of nuclei



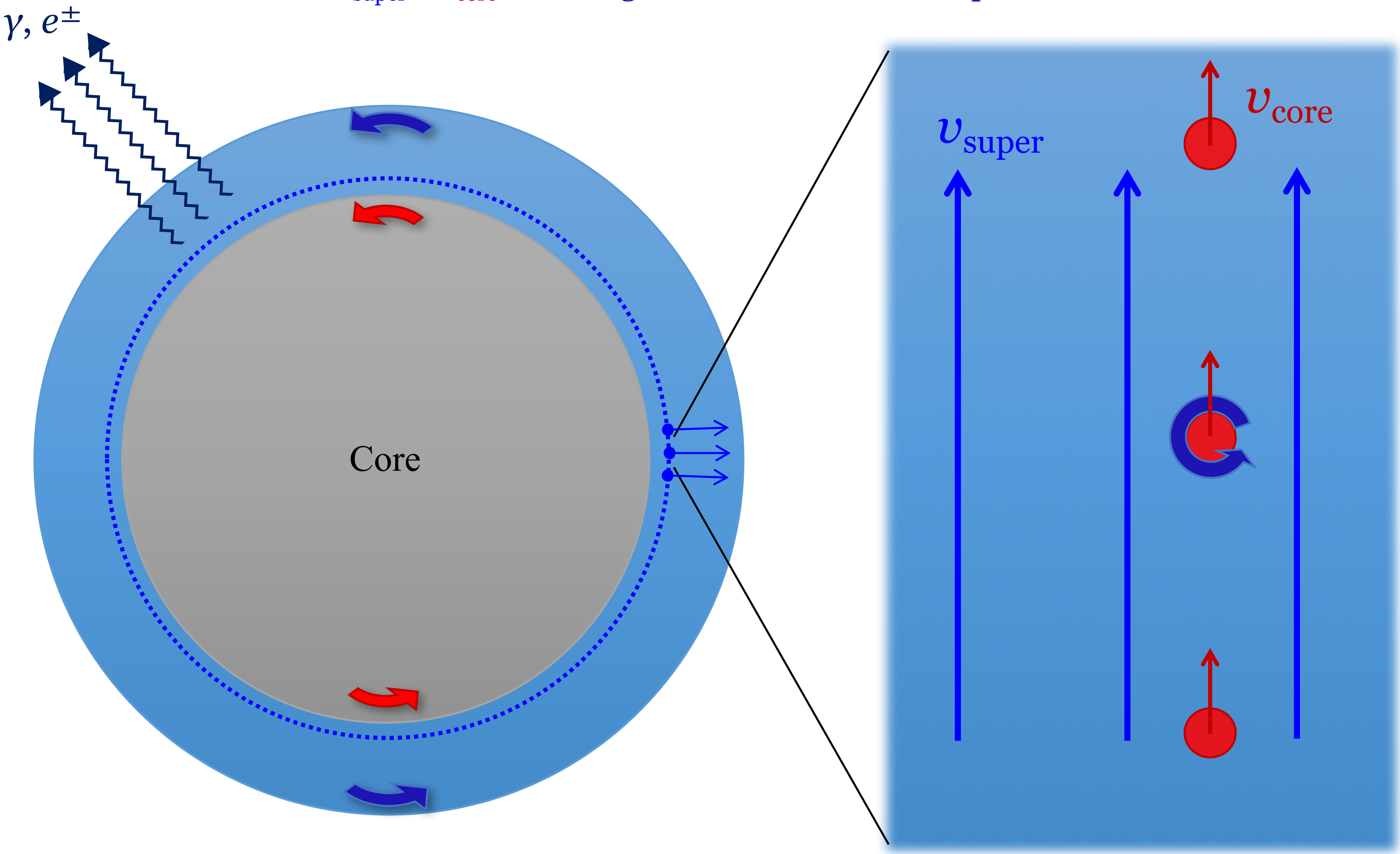
Scenario of the glitch

✓ Since $v_{\text{super}} > v_{\text{core}}$, the Magnus force exerts on the pinned vortices



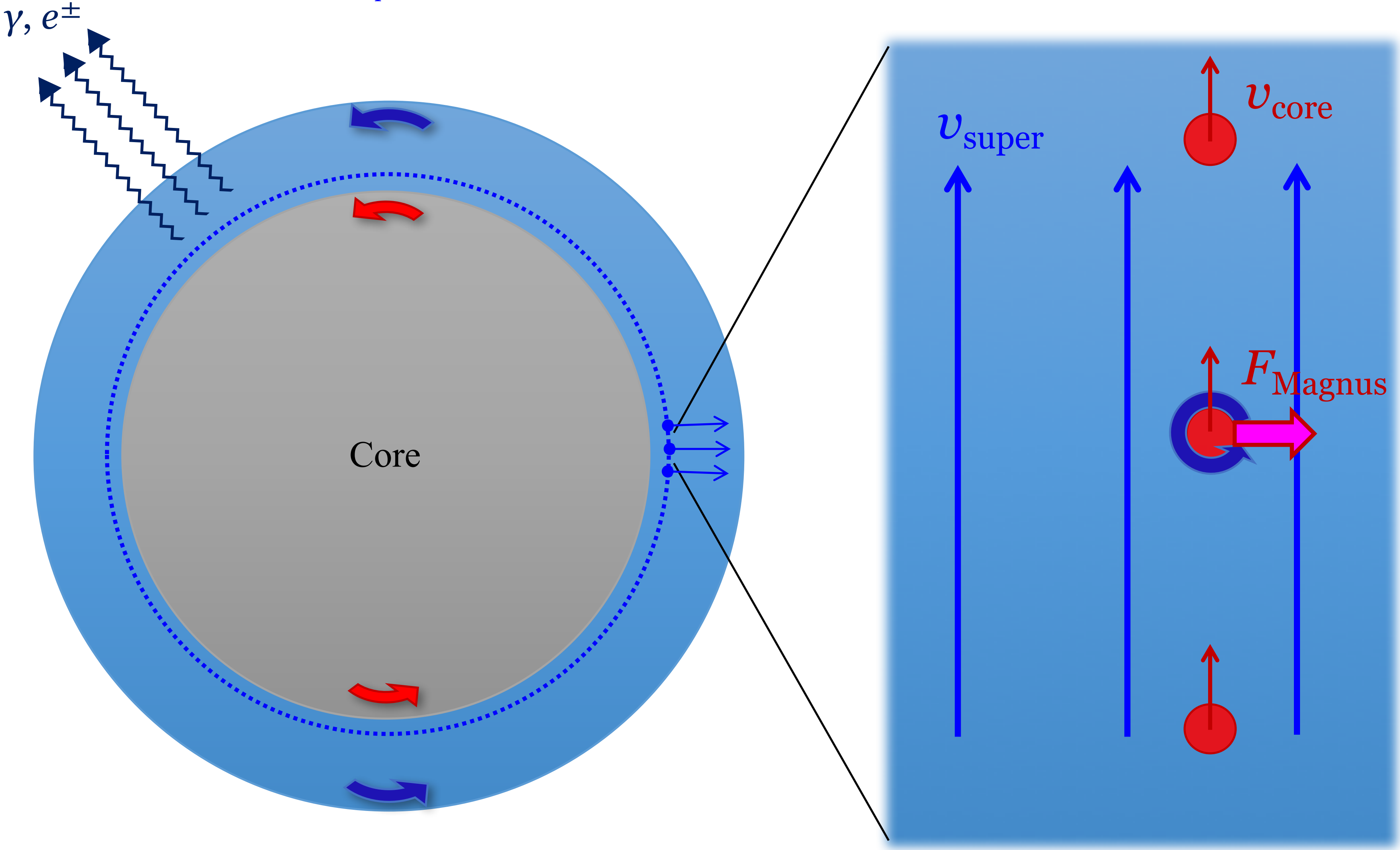
Scenario of the glitch

✓ Since $v_{\text{super}} > v_{\text{core}}$, the Magnus force exerts on the pinned vortices



Scenario of the glitch

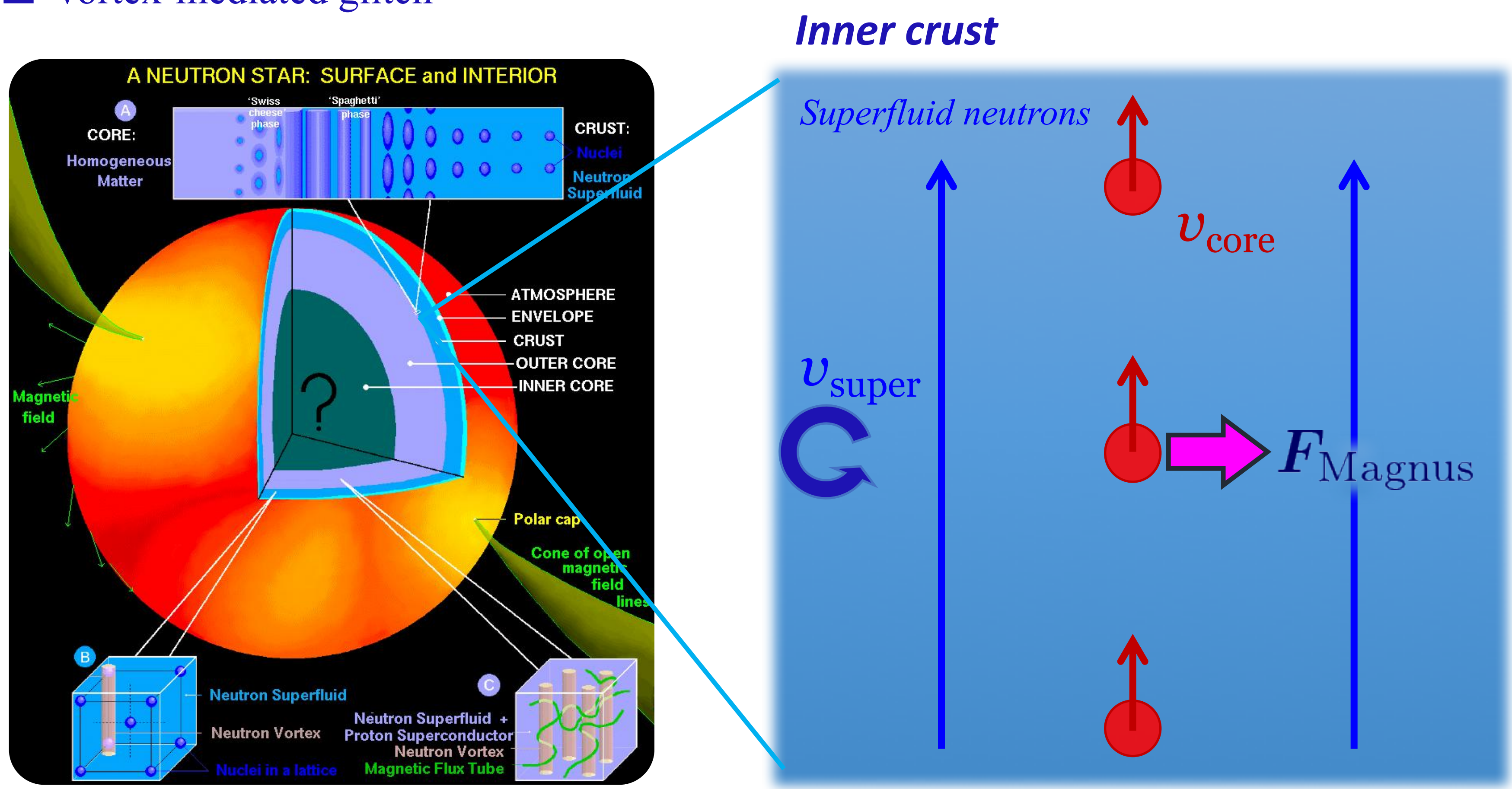
✓ Since $v_{\mathrm{super}} > v_{\mathrm{core}}$, the Magnus force exerts on the pinned vortices



P.W. Anderson and N. Itoh, Nature **256**, 25 (1975)

Pinning and unpinning of vortices may cause the glitches

- Vortex-mediated glitch

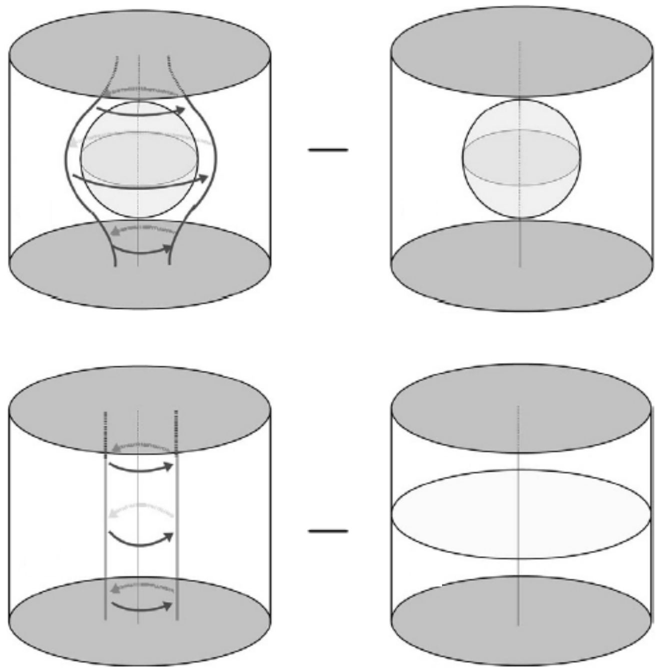


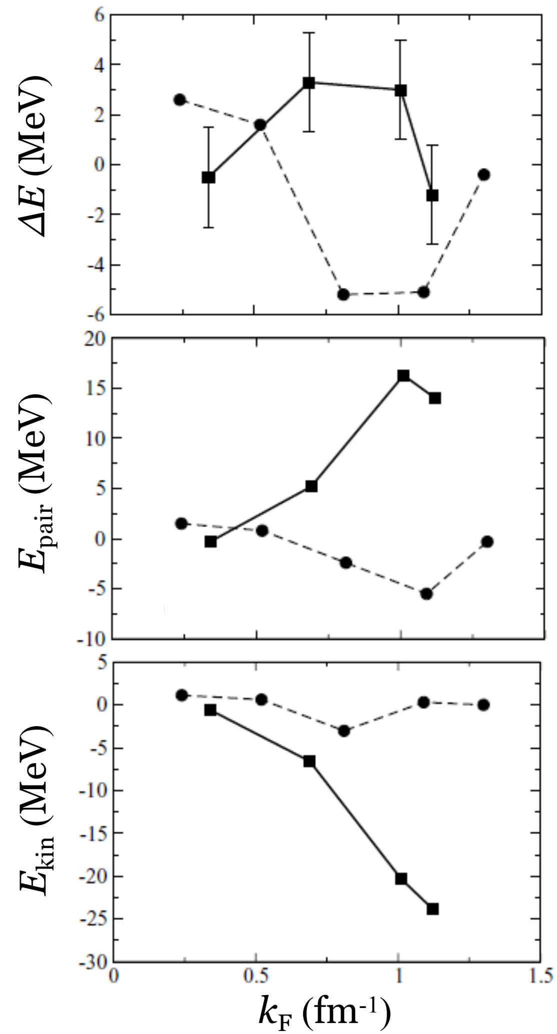
Binding energy was evaluated by axially symmetric HFB calculation

$$\Delta E = E^{\rm vor}_{\rm nucl} - E^{\rm vor}_{\rm unif}$$

$E^{\rm vor}_{\rm nucl} =$ —

$E^{\rm vor}_{\rm unif} =$ —





- Cylinder: R=30 fm, h=40 fm
- Mesh: 0.25 fm
- Skyrme SLy4 & SkM*
- Pairing: DDDI (Ec=60 MeV)

P. Avogadro, F. Barranco, R.A. Broglia, and E. Vigezzi, PRC**75**(2007)012805(R); NP**A811**(2008)378

TDSLDA: Time-Dependent Superfluid Local Density Approximation

We assume a local form of the Kohn-Sham EDF in TDDFT

TDSLDA equations:

$$i\hbar\frac{du_i(\mathbf{r})}{dt} = [h(\mathbf{r})-\mu]u_i(\mathbf{r}) + \Delta(\mathbf{r})v_i(\mathbf{r})$$

$$i\hbar\frac{dv_i(\mathbf{r})}{dt} = \Delta^*(\mathbf{r})u_i(\mathbf{r}) - [h(\mathbf{r})-\mu]v_i(\mathbf{r})$$

$u_i(\mathbf{r})$, $v_i(\mathbf{r})$: quasi-particle wave functions
$h(\mathbf{r})$: single-particle Hamiltonian
μ: chemical potential

Local energy density functional:

$$\mathcal{E}(\mathbf{r}) = \mathcal{E}_0(\mathbf{r}) + g|\nu(\mathbf{r})|^2$$

Fayans EDF (FaNDF[0]) w/o LS — S.A. Fayans and D. Zawischa, arXiv:nucl-th/0009034

$$\mathcal{E}_0 = \mathcal{E}_{\text{kin}} + \mathcal{E}_{\text{vol}} + \mathcal{E}_{\text{surf}} + \mathcal{E}_{\text{Coul}}$$

$$\rho_\pm = \rho_n \pm \rho_p \qquad x_+ = \rho_+/\rho_0$$

$$\mathcal{E}_{\text{vol}} = C_0\left[a_+^{\text{v}}\frac{\rho_+^2}{4}\frac{1-h_{1+}^{\text{v}}x_+^\sigma}{1+h_{2+}^{\text{v}}x_+^\sigma} + a_-^{\text{v}}\frac{\rho_-^2}{4}\frac{1-h_{1-}^{\text{v}}x_+}{1-h_{2+}^{\text{v}}x_+}\right]$$

$$\mathcal{E}_{\text{surf}} = \frac{C_0}{4}\frac{a_+^{\text{s}}r_0^2(\nabla\rho_+)^2}{1+h_+^{\text{s}}x^\sigma + h_\nabla^{\text{s}}r_0^2(\nabla x_+)^2}$$

$$\Delta(\mathbf{r}) = -\frac{d\mathcal{E}(\mathbf{r})}{d\nu^*(\mathbf{r})} = -g\,\nu(\mathbf{r})$$

$\Delta(\mathbf{r})$: *local* pairing field
$\nu(\mathbf{r})$: anomalous density

We can efficiently work with the local pairing field

Problem: $\nu(\mathbf{r}_1, \mathbf{r}_2)$ and thus $\Delta(\mathbf{r}_1, \mathbf{r}_2)$ diverge when $\mathbf{r}_1=\mathbf{r}_2$

$$\nu(\mathbf{r}_1,\mathbf{r}_2) = \sum_i v_i^*(\mathbf{r}_1) u_i(\mathbf{r}_2) \propto \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|}$$

Prescription:

$$\Delta(\mathbf{r}) = -g\,\nu_{\mathrm{reg}}(\mathbf{r}) = -g_{\mathrm{eff}}(\mathbf{r})\nu_{\mathrm{c}}(\mathbf{r})$$

$$\frac{1}{g_{\mathrm{eff}}(\mathbf{r})} = \frac{1}{g} - \frac{m k_{\mathrm{c}}(\mathbf{r})}{2\pi^2\hbar^2}\left[1 - \frac{k_{\mathrm{F}}(\mathbf{r})}{2k_{\mathrm{c}}(\mathbf{r})}\ln\frac{k_{\mathrm{c}}(\mathbf{r}) + k_{\mathrm{F}}(\mathbf{r})}{k_{\mathrm{c}}(\mathbf{r}) - k_{\mathrm{F}}(\mathbf{r})}\right] - \frac{m\, l_{\mathrm{c}}(\mathbf{r})}{2\pi^2\hbar^2}\left[1 - \frac{k_{\mathrm{F}}(\mathbf{r})}{2\, l_{\mathrm{c}}(\mathbf{r})}\ln\frac{k_{\mathrm{F}}(\mathbf{r}) + l_{\mathrm{c}}(\mathbf{r})}{k_{\mathrm{F}}(\mathbf{r}) - l_{\mathrm{c}}(\mathbf{r})}\right]$$

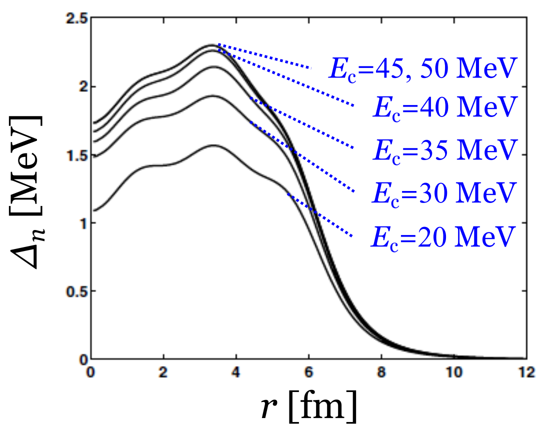
$(l_c \le k_F \le k_c)$

$$\nu_{\mathrm{c}}(\mathbf{r}) = \sum_{E_i \le E_{\mathrm{c}}} v_i^*(\mathbf{r}) u_i(\mathbf{r})$$

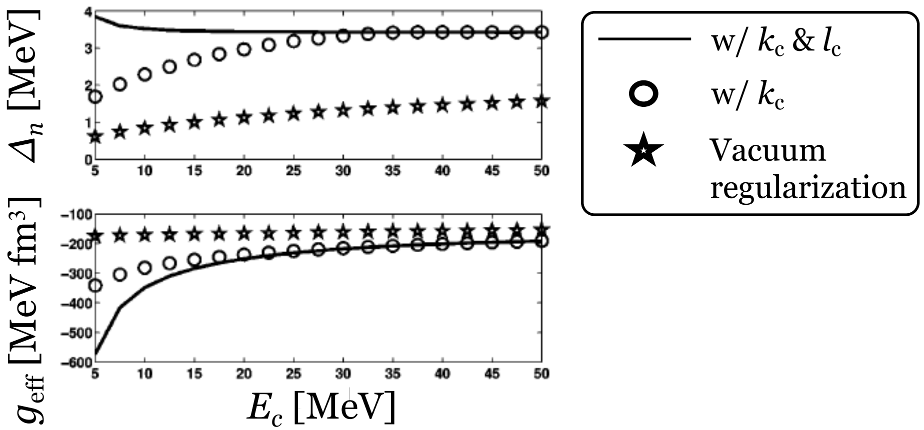
E_c : a cutoff energy

Example:

^{110}Sn, Woods-Saxon



Homogeneous neutron matter



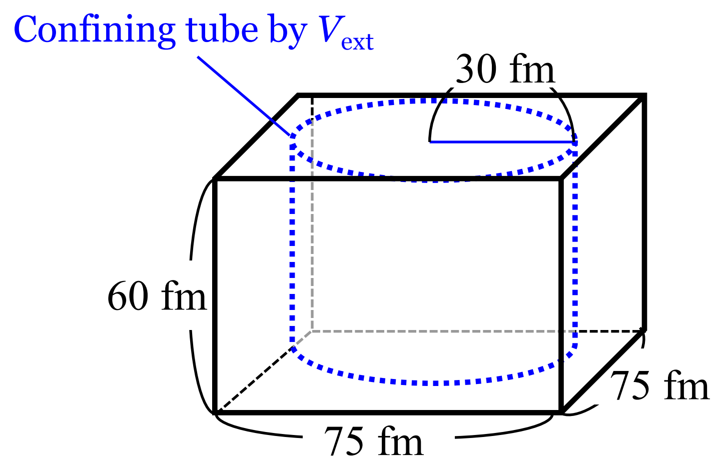
See A. Bulgac and Y. Yu, PRL**88**(2002)042504; PRC**65**(2002)051305(R); arXiv:nucl-th/0109083, and references therein

Computational settings

We use our own 3D TDSLDA code written in CUDA C with MPI

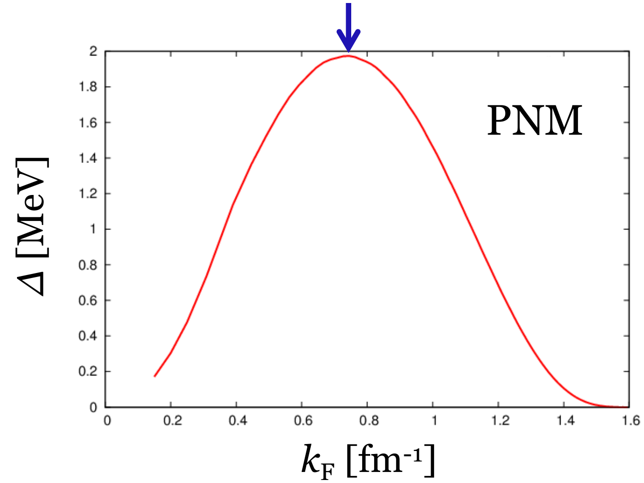
Some details

- EDF: Fayans EDF (FaNDF^0) w/o LS
- 3D uniform lattice: 50x50x40
- Mesh spacing: 1.5 fm
- dt~0.054 fm/c
- E_c=75 MeV (Nwf_n: 32,665, Nwf_p: 13,967)
- Time-evolution: split-operator w/ predictor corrector
- Derivatives: Fourier transformation
- Periodic boundary condition
- Each CUDA core is responsible for each grid point



Physical situation

- N: 2633.4 → ρ_n~0.016 fm^{-3}, k_F~0.78 fm^{-1}
- Z: 50 (Sn)



Performance

Ex: 48 nodes (192 GPUs) on HA-PACS
-> 28 hours for 10,000 fm/c time-evolution

Initial state generation

We dynamically generate an initial configuration starting from a uniform system

- Adiabatic switching

$$H(t) = s(t)H_1 + [1 - s(t)]H_0$$

$s(t)$: a smooth switch function [0, 1]

- Quantum friction

$$i\hbar\dot{\Psi}(t) = (H(t) + U_{\rm qf}(t))\Psi(t)$$

$$\dot{E} = \langle\Psi(t)|\dot{H}(t)|\Psi(t)\rangle + \underbrace{\frac{2}{\hbar}\mathrm{Im}\Big[\langle\Psi(t)|H(t)U_{\rm qf}(t)|\Psi(t)\rangle\Big]}_{\leq 0}$$

$$U_{\rm qf}(t) \propto -2\mathrm{Im}\langle\Psi(t)|H(t)|\Psi(t)\rangle = -\hbar\nabla\cdot\mathbf{j}(t) = \hbar\dot{\rho}(t)$$

*$U_{\rm qf}$ removes any irrotational currents

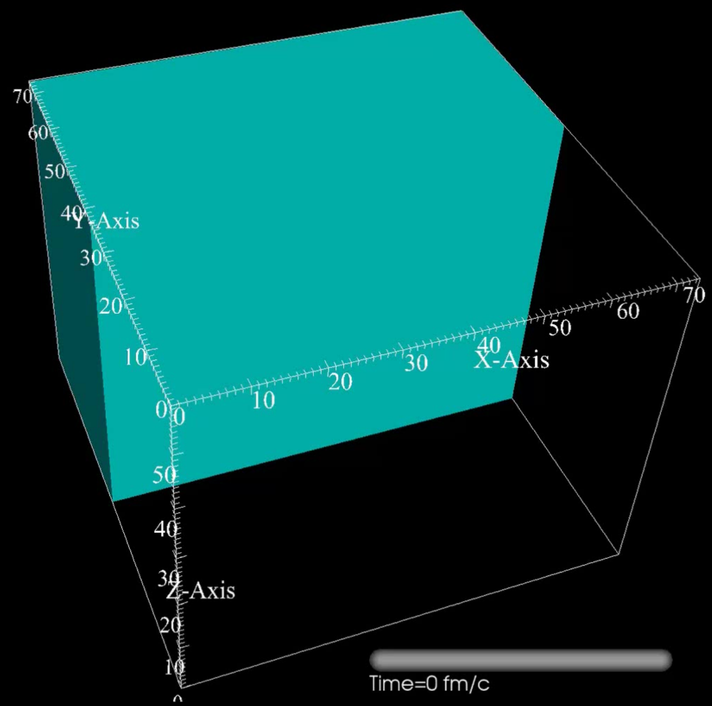
- What we do in practice:

Uniform system → +Tube → +HO → +Coulomb → -HO ⇒ Put it to a static solver w/o Coulomb

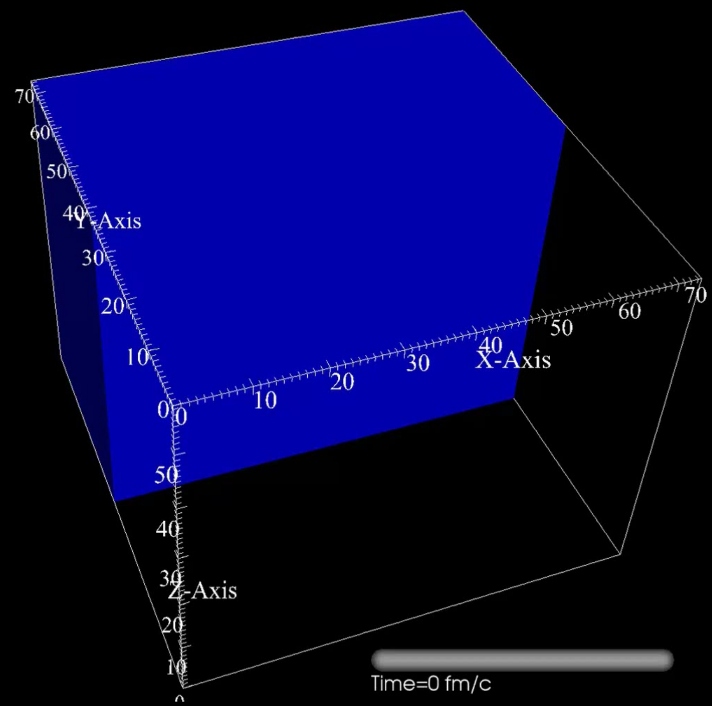
A. Bulgac, M.M. Forbes, K.J. Roche, and G. Wlazłowski, arXiv:1305.6891 [nucl-th]

Initial state generation: Impurity at the center

Neutron density

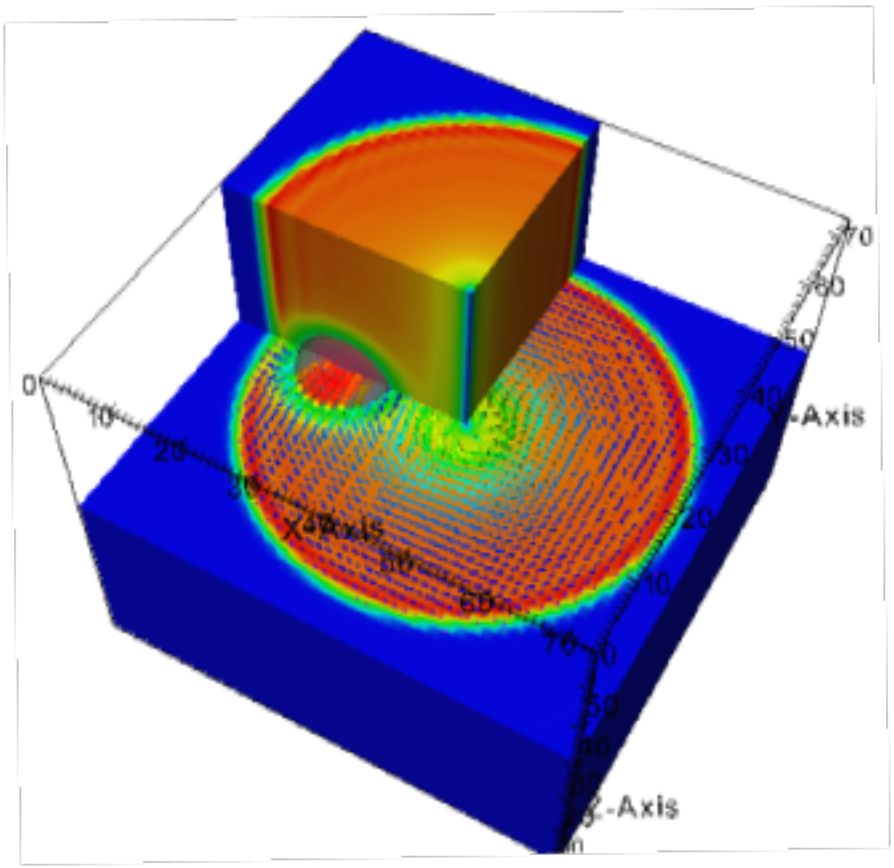


Proton density

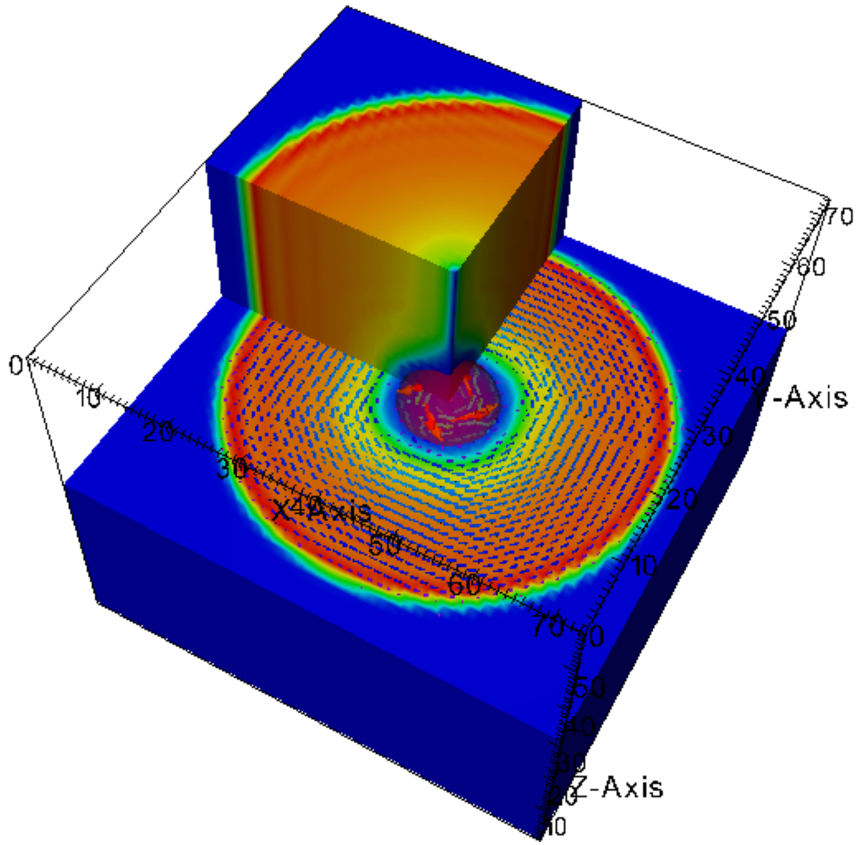


The prepared initial states

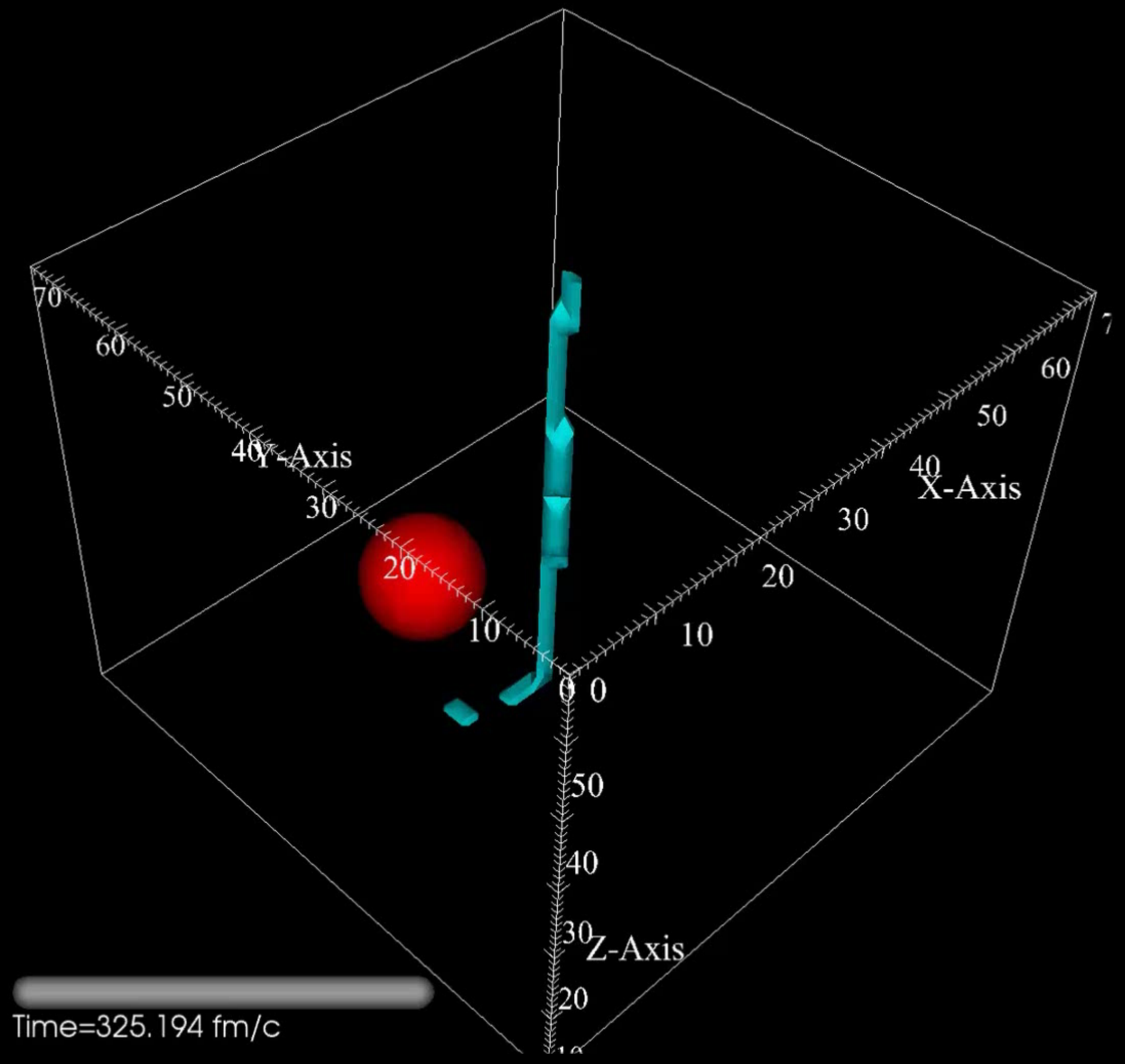
I. “separated” configuration



II. “pinned” configuration

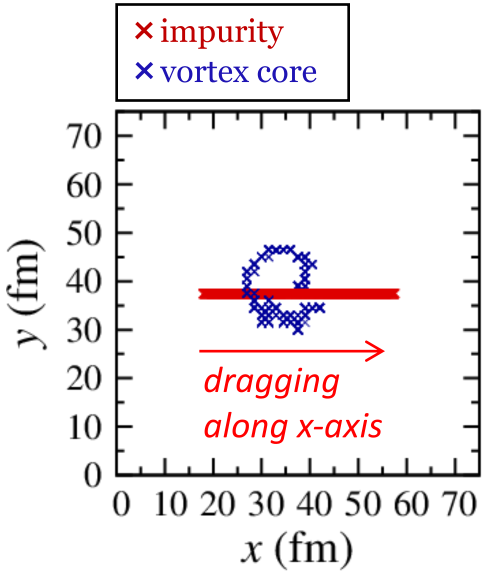


Vortex-nucleus dynamics I: from "separated" configuration

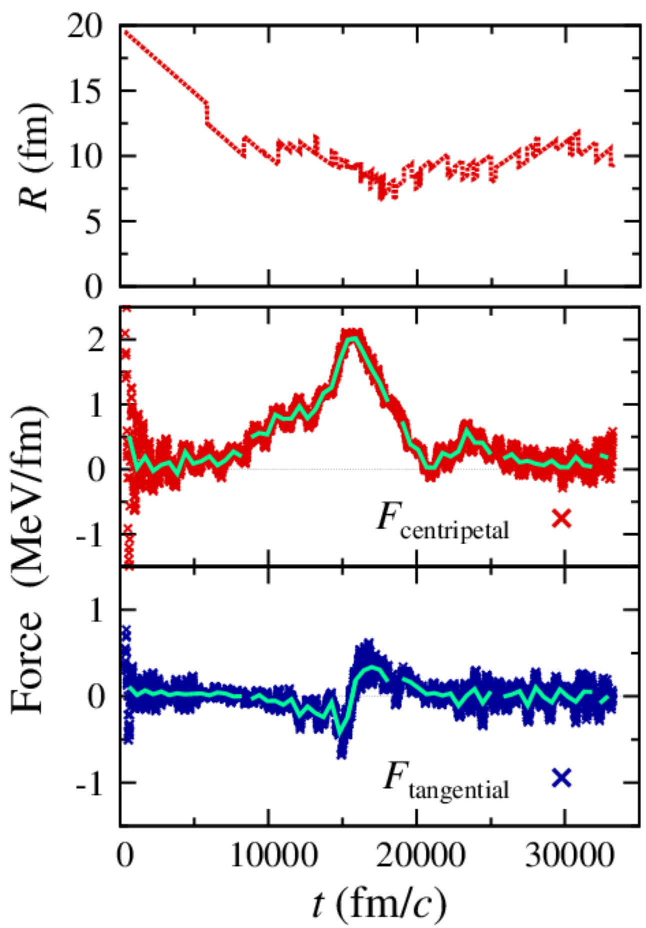


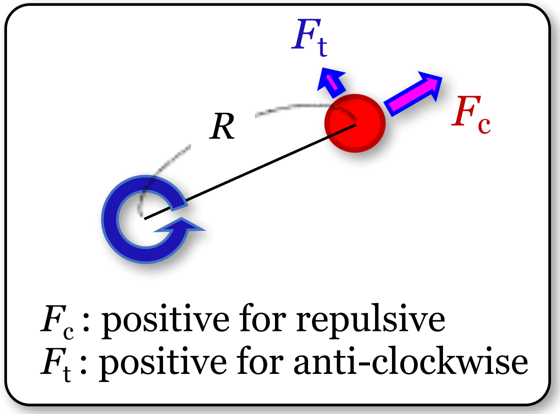
The extracted force

We find "repulsive" nature of the vortex-nucleus interaction



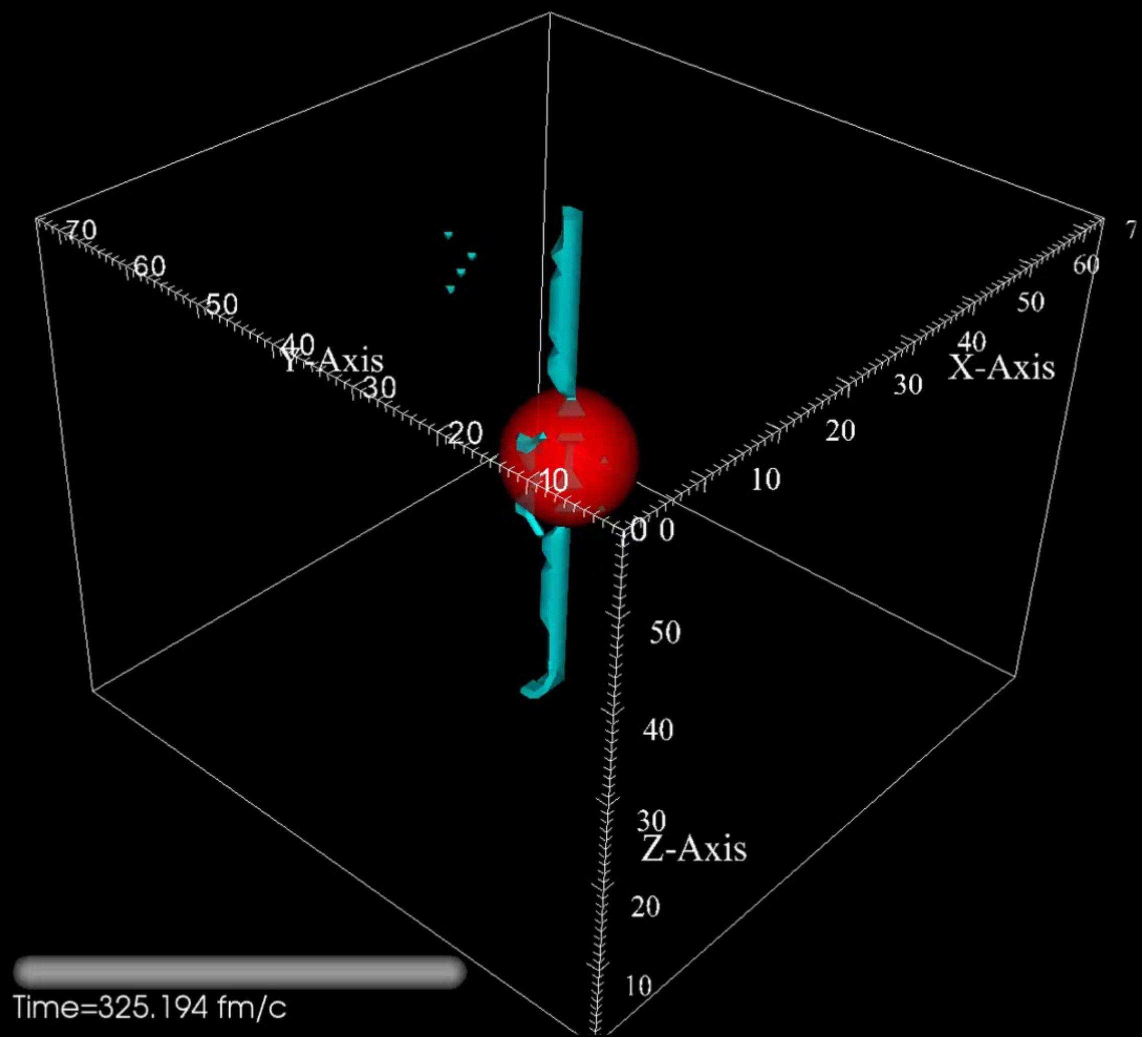
Force and R vs. time





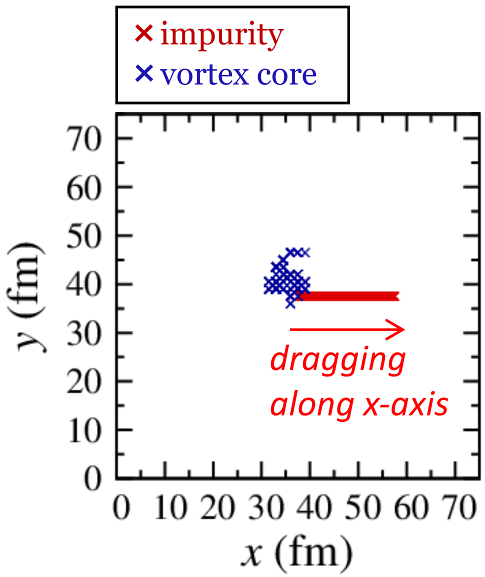
*Green line: averaged over 50 measurements (540 fm/c)

Vortex-nucleus dynamics II: from "pinned" configuration

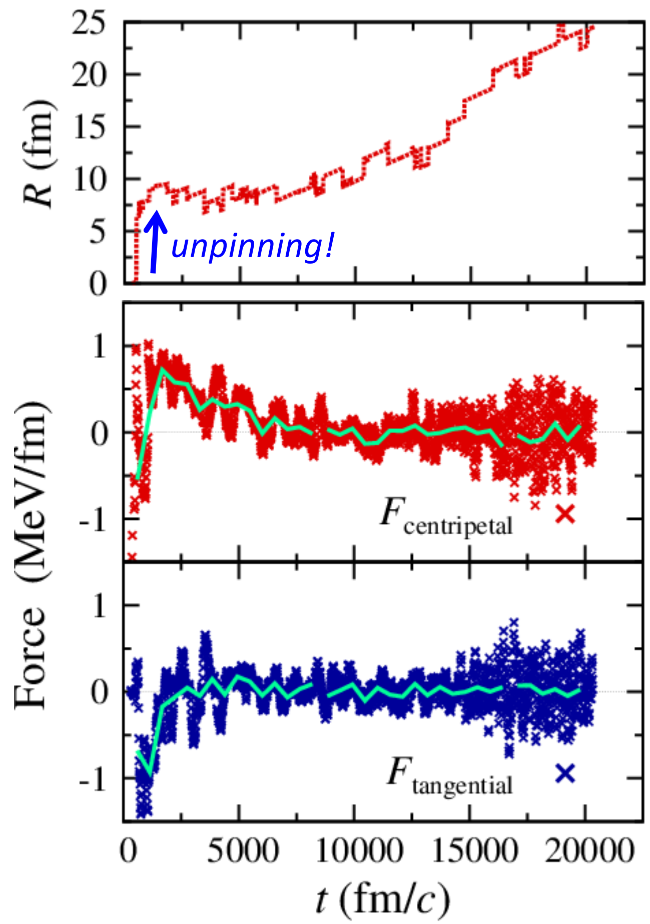


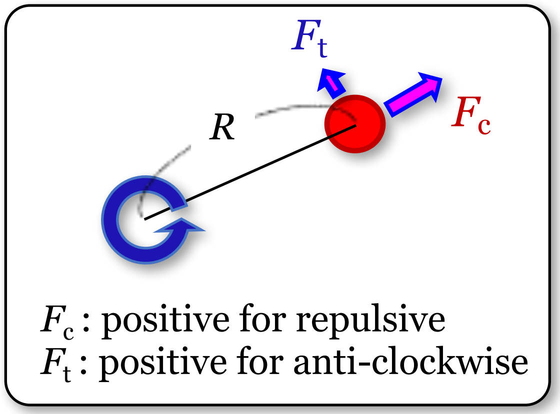
The extracted force

We find “repulsive” nature of the vortex-nucleus interaction



Force and R vs. time

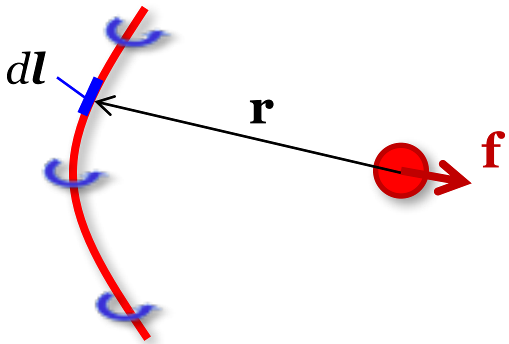




*Green line: averaged over 50 measurements (540 fm/c)

Remaining tasks

- To determine the force *per unit length* when the vortex line bends



The total force may take a form:

$$\mathbf{F} \propto \int \mathbf{f}(r)\, \mathbf{r} \times d\mathbf{l}$$

- To examine density dependence of the interaction

Summary and Conclusion

Our simulation will provide significant impact on glitch studies!

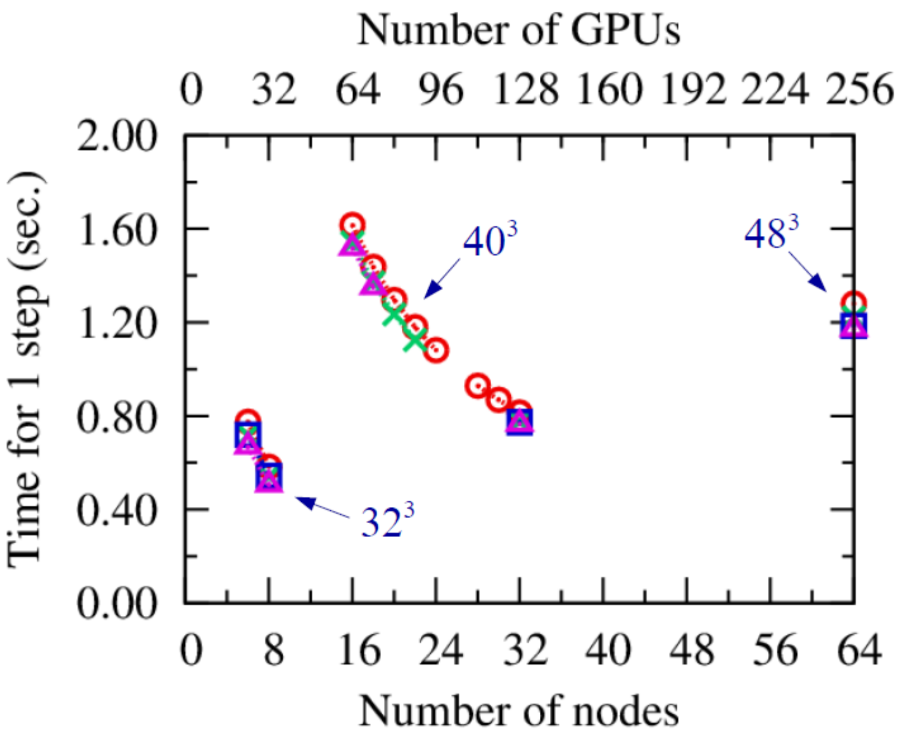
Summary

- ✓ The vortex-nucleus interaction is the essential quantity to understand the glitches.
- ✓ We are conducting microscopic, dynamical simulations with TDSLDA.
- ✓ Our simulation is providing qualitatively new things:
  - ➢ The first, three-dimensional, microscopic, dynamical simulation for the vortex-nucleus interaction with a new force extraction technique
  - ➢ The "bending" mode of the vortex line
  - ➢ The "repulsive" nature of the interaction (at least for $\rho \sim 0.1\rho_0$)

Summary of the timing results on HA-PACS (NVIDIA Tesla M2090)

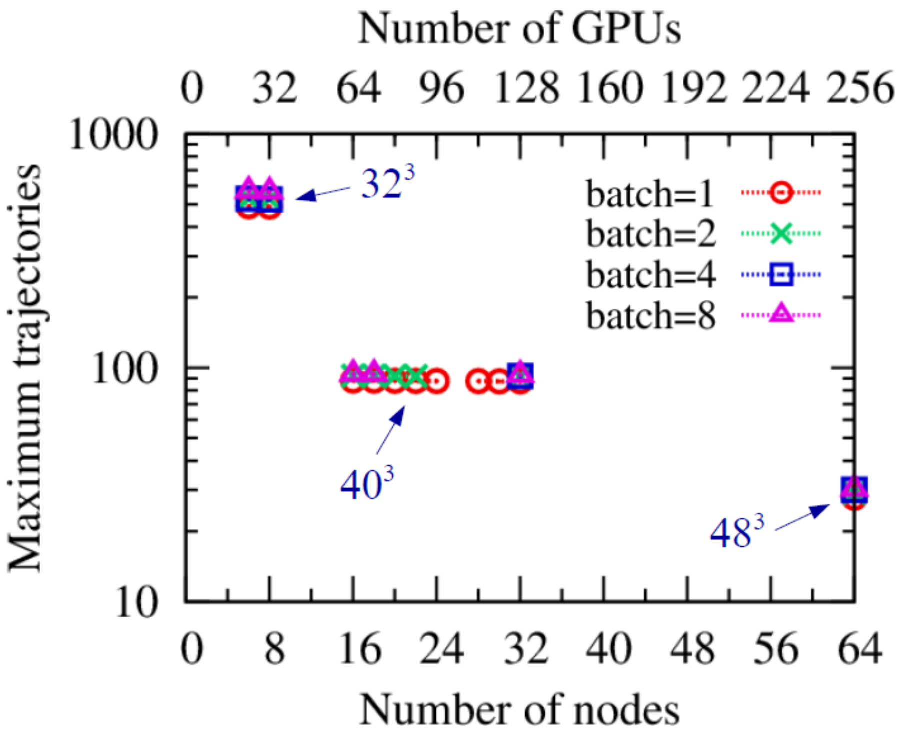
ρ~0.01, dt=0.02, dx=1.0, timesteps=10, measurements=2, uniform symmetric nuclear matter w/o Coulomb

[Left]: Computation time per time step



[Right]: Maximum number of trajectories we can simulate

*1 trajectory means 100,000 time steps; We have 63,000 node hour



- Use of larger "batch" value slightly reduces computation time, but unsignificant.
- Use of larger resource reduces computation time, but resulting maximum number of trajectories is similar.
- Conclusion: ~500 trajectories (32^3 lattice), ~100 trajectories (40^3 lattice), ~30 trajectories (48^3 lattice)

Initial state generation: Impurity at the center

Summary

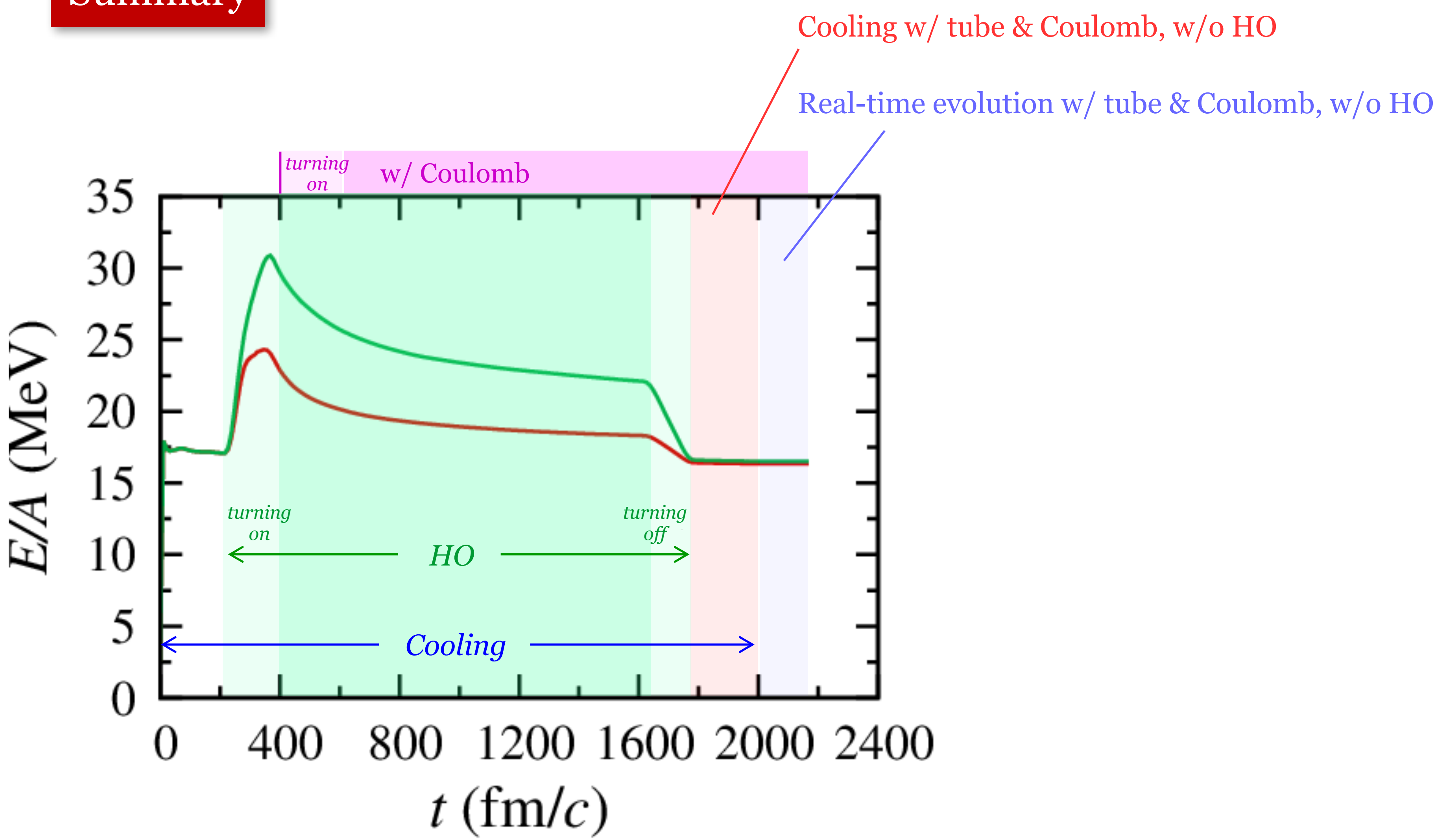
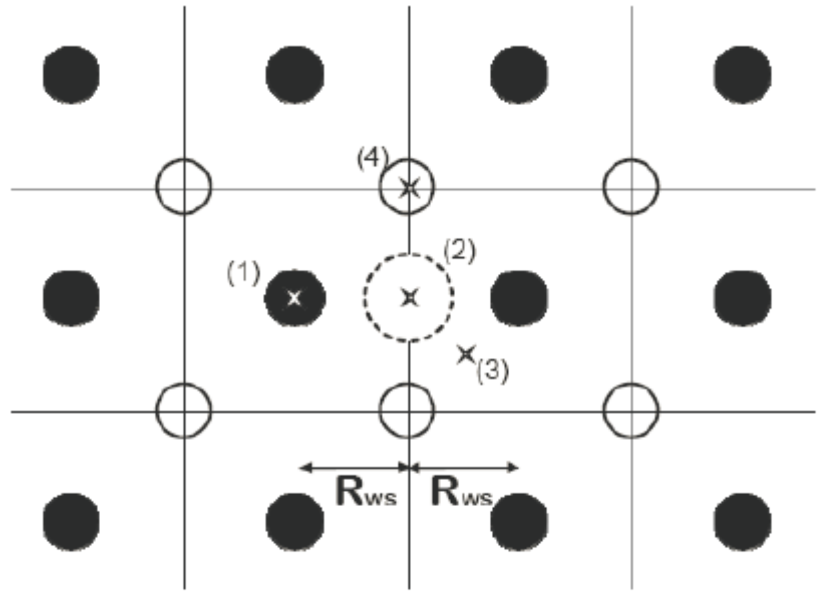
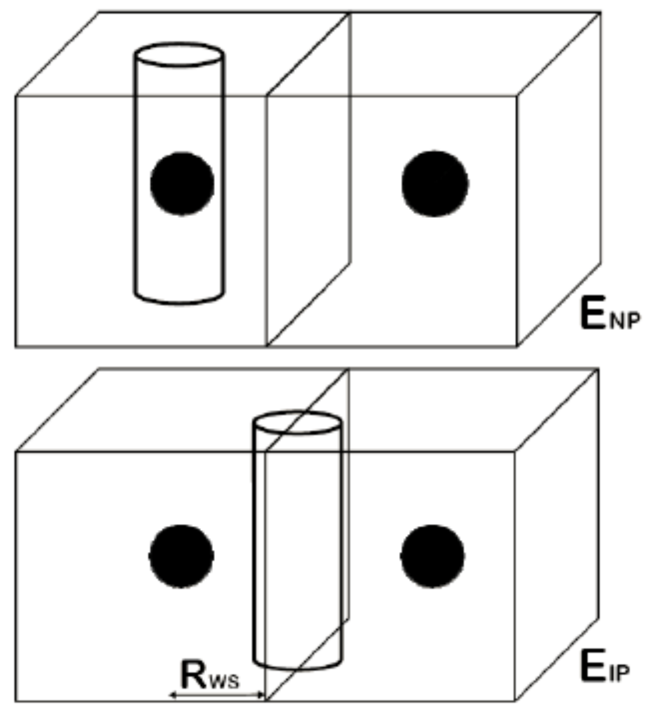


TABLE I. The WS cells representing different density regions of the inner crust. The particle numbers Z,N, the WS-cell radii R_{WS} and the baryonic density ρ_b have been taken from previous calculations [4]. $k_{F,n}$ is the Fermi momentum corresponding to the density of the outer neutron gas, as computed in this work.

Zone	Element	Z	N	R_{WS} (fm)	ρ_b (g/cm^3)	$k_{F,n}$ (fm^{-1})
11	^{180}Zr	40	140	53.6	4.67×10^{11}	0.12
10	^{200}Zr	40	160	49.2	6.69×10^{11}	0.15
9	^{250}Zr	40	210	46.4	1.00×10^{12}	0.19
8	^{320}Zr	40	280	44.4	1.47×10^{12}	0.23
7	^{500}Zr	40	460	42.2	2.66×10^{12}	0.31
6	^{950}Sn	50	900	39.3	6.24×10^{12}	0.43
5	^{1100}Sn	50	1050	35.7	9.65×10^{12}	0.51
4	^{1350}Sn	50	1300	33.0	1.49×10^{13}	0.60
3	^{1800}Sn	50	1750	27.6	3.41×10^{13}	0.80
2	^{1500}Zr	40	1460	19.6	7.94×10^{13}	1.08
1	^{982}Ge	32	950	14.4	1.32×10^{14}	1.33

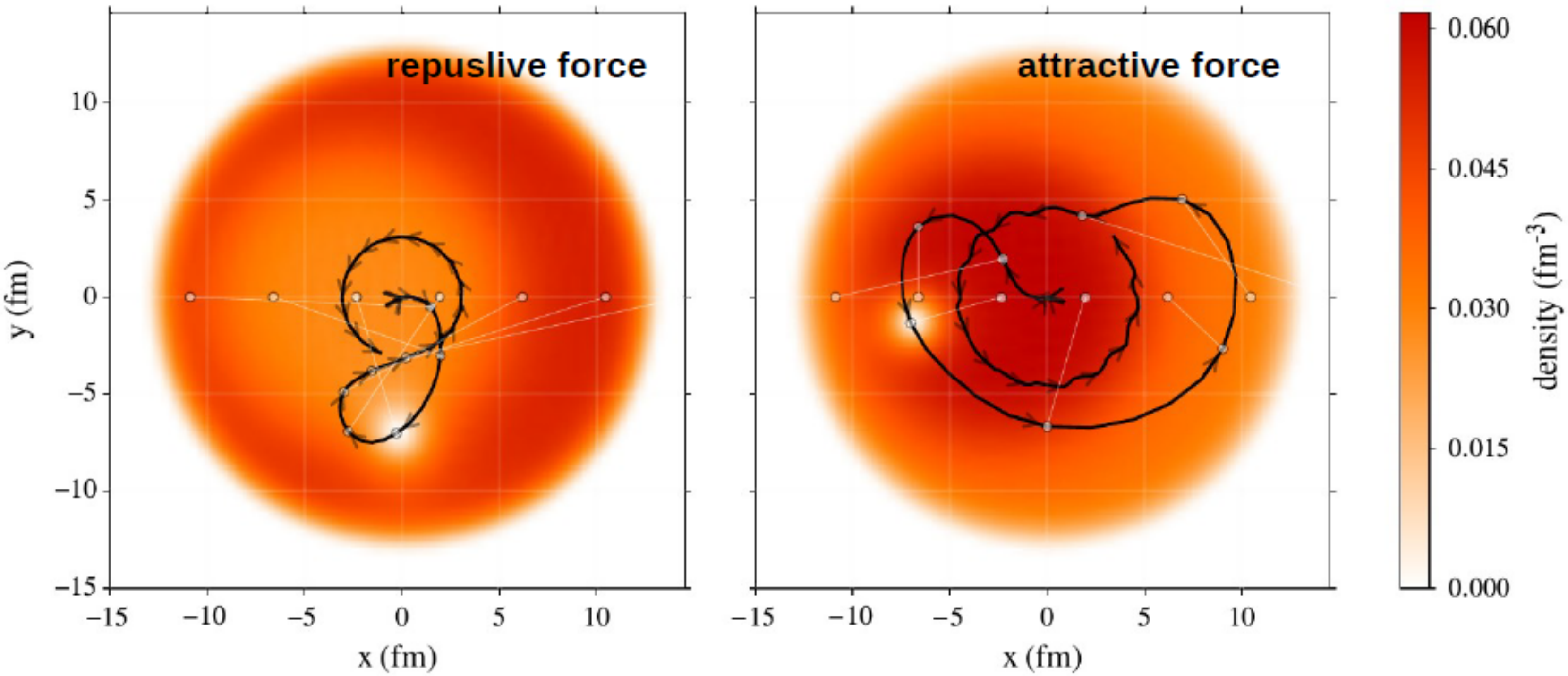




*LEFT: PRC**84**(2011)065807*
RIGHT: arXiv:0711.3393 [astro-ph]

Idea introduced in:
Aurel Bulgac, Michael McNeil Forbes, and Rishi Sharma
Phys. Rev. Lett. 110, 241102 (2013)

2D simulations with GP



Equation of motion:

$$\underbrace{M\ddot{\vec{r}}_v - \vec{f}_{qp}}_{\text{negligible}} = \rho_s \vec{\kappa} \times (\dot{\vec{r}}_v - \vec{v}_s) + \vec{F}_v$$