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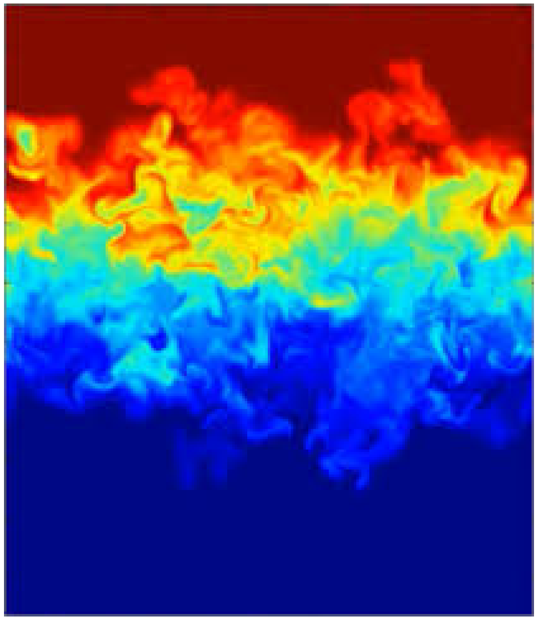
Instabilities in two-(super)fluid systems

A. Haber, A. Schmitt and S. Stetina, PRD 93, 025011 (2016)

A. Schmitt, work in progress

- two-component superfluids
- two-fluid picture of a superfluid
  - first and second sound
  - field-theoretic approach
- energetic and dynamical instabilities
  - onset of instabilities through relative flow
  - role of dissipation



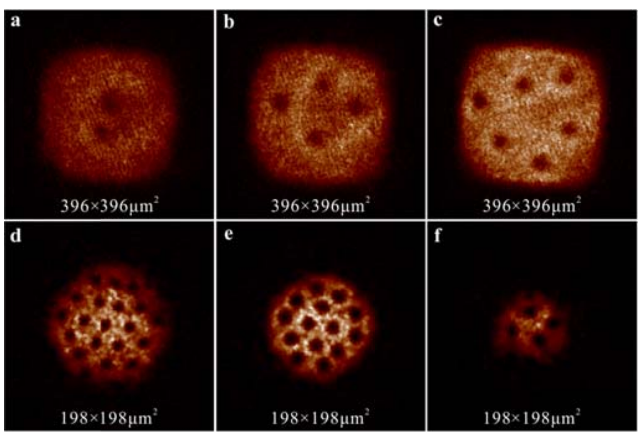
D. Livescu *et al.*, JPhCS, 318, 082007 (2011)

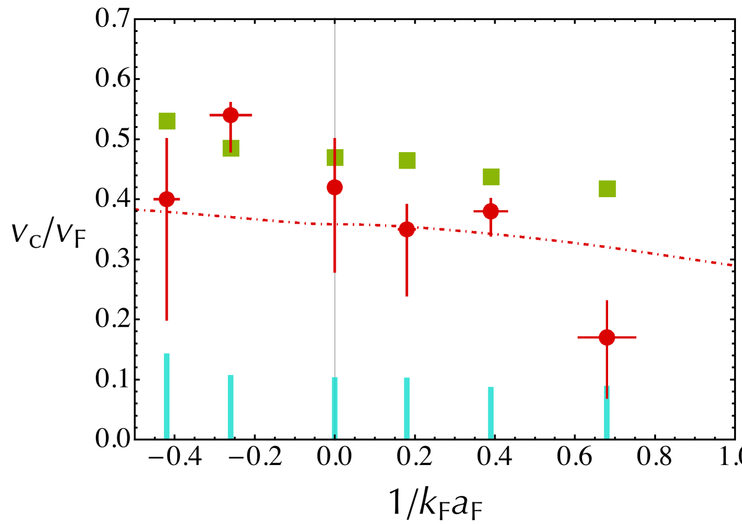
• Two-component superfluids in the laboratory

- ^{3}He-^{4}He mixtures: difficult to create experimentally
  J. Tuoriniemi, *et al.*, JLTP 129, 531 (2002)

- Bose-Fermi gas mixtures
  ^{6}Li-^{7}Li superfluid I. Ferrier-Barbut *et al.*, Science 345, 1035 (2014)

- Simultaneous vortex lattices in ^{6}Li-^{41}K
  Yao, X.-c. *et al.* arXiv:1606.01717





- Critical counterflow velocity in ^{6}Li-^{7}Li (comparing data to $v_{\text{two-stream}} = v_{L,1} + v_{L,2}$)
  Delehaye, M. *et al.* PRL 115, 265303 (2015)

• Two-component (super)fluids in compact stars

- neutron superfluid/proton superconductor
  M. A. Alpar, S. A. Langer and J. A. Sauls, Astrophys. J. 282, 533 (1984)
  M. G. Alford and G. Good, PRB 78, 024510 (2008)
  nucleon-hyperon: M.E. Gusakov, E.M. Kantor, P. Haensel, PRC 79, 055806 (2009)

- neutron superfluid in ion lattice
  – two-stream instability as trigger for collective vortex unpinning
  → pulsar glitches N. Andersson, G.L. Comer, R. Prix, PRL 90, 091101 (2003)
  – Landau and dynamical instabilities of BEC in optical lattice
  B. Wu and Q. Niu, PRA 64, 061603 (2001)

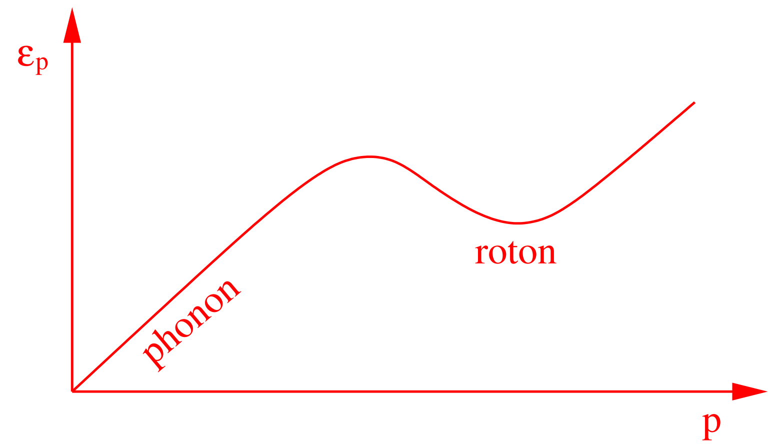
- CFL-K^0 quark matter
  P. F. Bedaque and T. Schäfer, NPA 697, 802 (2002)
  D. B. Kaplan and S. Reddy, PRD 65, 054042 (2002)

• Two-fluid picture of a superfluid (liquid helium)

London, Tisza (1938); Landau (1941)

relativistic: Khalatnikov, Lebedev (1982); Carter (1989)

- "superfluid component": condensate, carries no entropy
- "normal component": excitations (Goldstone mode), carries entropy



Hydrodynamic eqs. ⇒ **two sound modes**

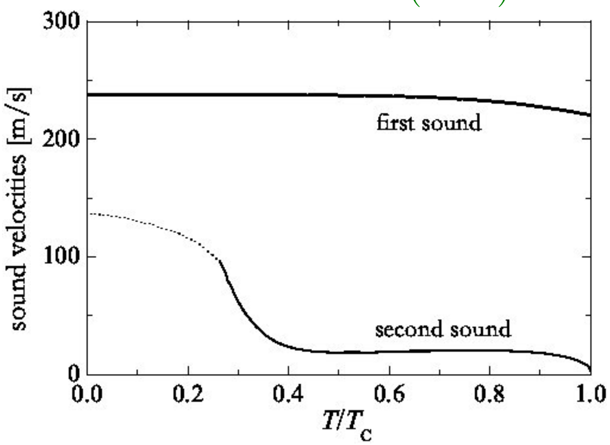
1st sound	**2nd sound**
in-phase oscillation (primarily) density wave	out-of-phase oscillation (primarily) entropy wave

(two-fluid picture also explains thermomechanical effect, "viscosity paradox", etc.)

• First and second sound in various systems

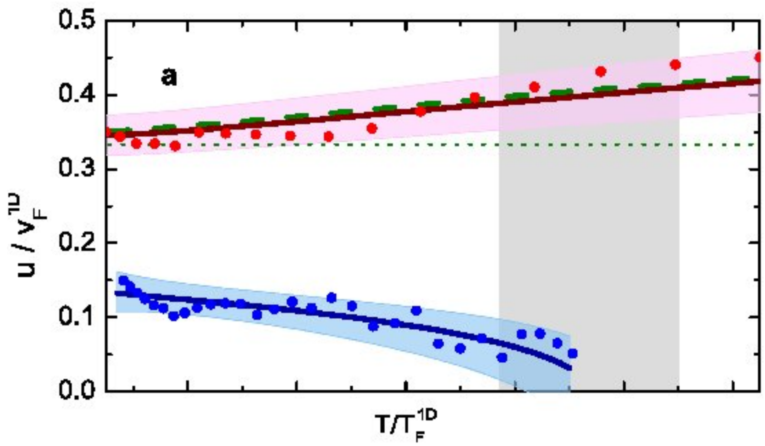
liquid helium

K.R. Atkins *et al.* (1953)



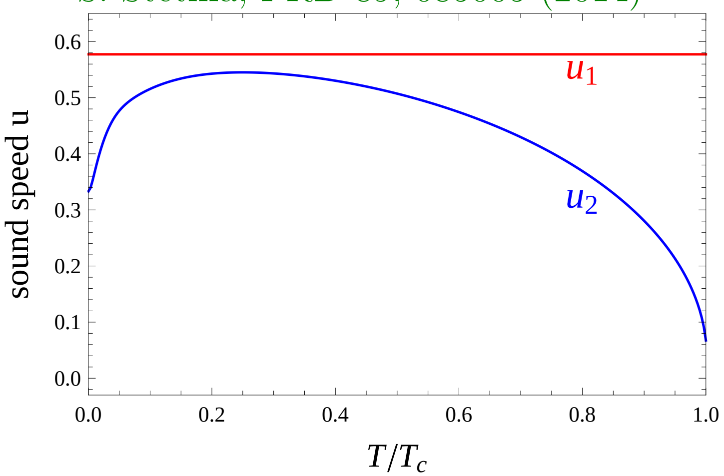
ultracold fermionic gas (exp.)

L.A. Sidorenkov *et al.*, Nature 498, 78 (2013)



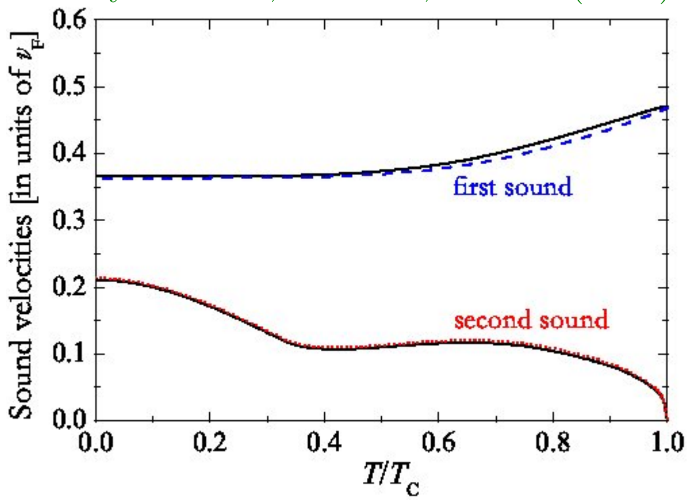
relativistic Bose superfluid

M. G. Alford, S. K. Mallavarapu, A. Schmitt, S. Stetina, PRD 89, 085005 (2014)



unitary Fermi gas

E. Taylor *et al.*, PRA 80, 053601 (2009)



• Superfluidity from field theory

- starting point: complex scalar field

$$\mathcal{L} = \partial_\mu\varphi\partial^\mu\varphi^* - m^2|\varphi|^2 - \lambda|\varphi|^4$$

- Bose condensate $\langle\varphi\rangle = \rho\, e^{i\psi}$ spontaneously breaks $U(1)$

- zero temperature: single-fluid system

	Field theory	Hydrodynamics
current j^μ	$\frac{(\partial\psi)^2}{\lambda}\partial^\mu\psi$	nv^μ
stress-energy tensor $T^{\mu\nu}$	$-g^{\mu\nu}\mathcal{L} + \frac{(\partial\psi)^2}{\lambda}\partial^\mu\psi\partial^\nu\psi$	$(\epsilon + P)v^\mu v^\nu - g^{\mu\nu}P$

- superfluid velocity

$$v^\mu = \frac{\partial^\mu\psi}{\sigma}$$

$$\sigma = \sqrt{\partial_\mu\psi\partial^\mu\psi^*}$$

$$\mu = \partial_0\psi$$

$$\vec{v} = -\nabla\psi/\mu$$

• Relativistic two-fluid formalism

- write stress-energy tensor as

$$T^{\mu\nu} = -g^{\mu\nu}\Psi + j^\mu \partial^\nu \psi + s^\mu \Theta^\nu$$

- "generalized pressure" $\Psi = \Psi[(\partial\psi)^2, \Theta^2, \partial\psi \cdot \Theta]$
- "generalized energy density" $\Lambda \equiv -\Psi + j \cdot \partial\psi + s \cdot \Theta = \Lambda[j^2, s^2, j \cdot s]$

$$j^\mu = \frac{\partial \Psi}{\partial(\partial_\mu \psi)} = \mathcal{B}\, \partial^\mu \psi + \mathcal{A}\, \Theta^\mu$$

$$s^\mu = \frac{\partial \Psi}{\partial \Theta_\mu} = \mathcal{A}\, \partial^\mu \psi + \mathcal{C}\, \Theta^\mu$$

non-relativistically:

$$\vec{j}_1 = \rho_{11}\vec{v}_1 + \rho_{12}\vec{v}_2$$

$$\vec{j}_2 = \rho_{12}\vec{v}_1 + \rho_{22}\vec{v}_2$$

- $\mathcal{A}$ "entrainment coefficient" ("Andreev-Bashkin effect")
- $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ can be computed from microscopic physics
  M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, PRD 87, 065001 (2013)

• $U(1) \times U(1)$ superfluid: setup

A. Haber, A. Schmitt, S. Stetina, PRD 93, 025011 (2016)

Lagrangian: $\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_I$

$$\mathcal{L}_i = \partial_\mu \varphi_i \partial^\mu \varphi_i^* - m_i^2 |\varphi_i|^2 - \lambda_i |\varphi_i|^4$$

entrainment coupling:

$$\mathcal{L}_I = -g(\varphi_1\, \varphi_2\, \partial_\mu \varphi_1^*\, \partial^\mu \varphi_2^* + \text{c.c.})$$

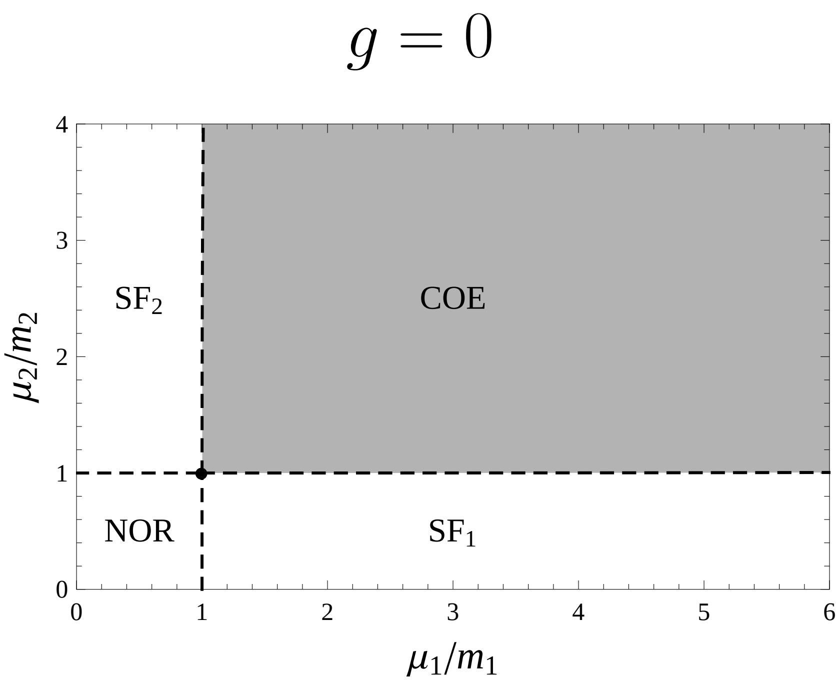
(non-entrainment coupling: $\mathcal{L}_I = -h|\varphi_1|^2|\varphi_2|^2$)

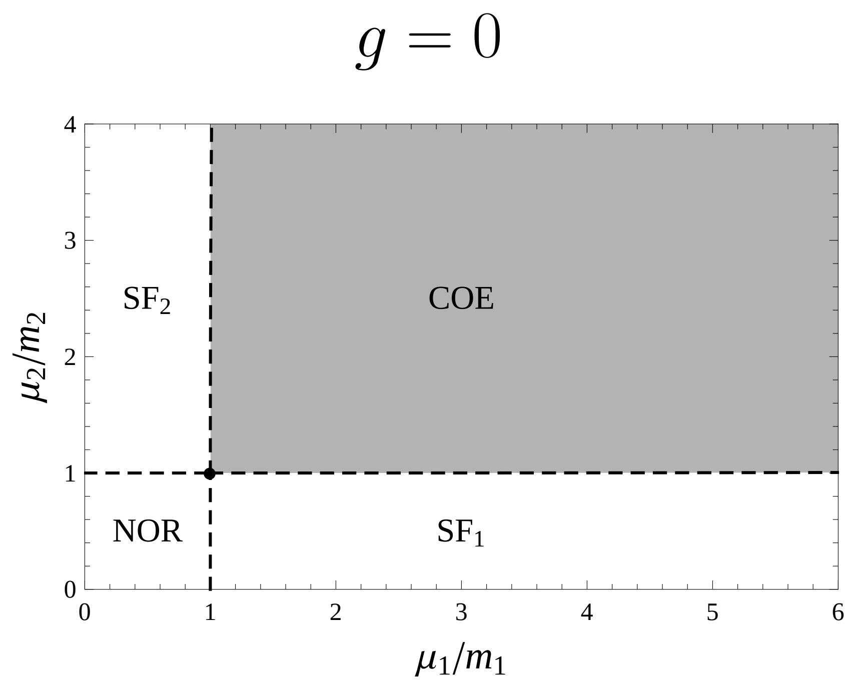
conserved currents:

$$j_1^\mu = \rho_1^2 \left(\partial^\mu \psi_1 + \frac{g}{2}\, \rho_2^2\, \partial^\mu \psi_2 \right)$$

$$j_2^\mu = \rho_2^2 \left(\partial^\mu \psi_2 + \frac{g}{2}\, \rho_1^2\, \partial^\mu \psi_1 \right)$$

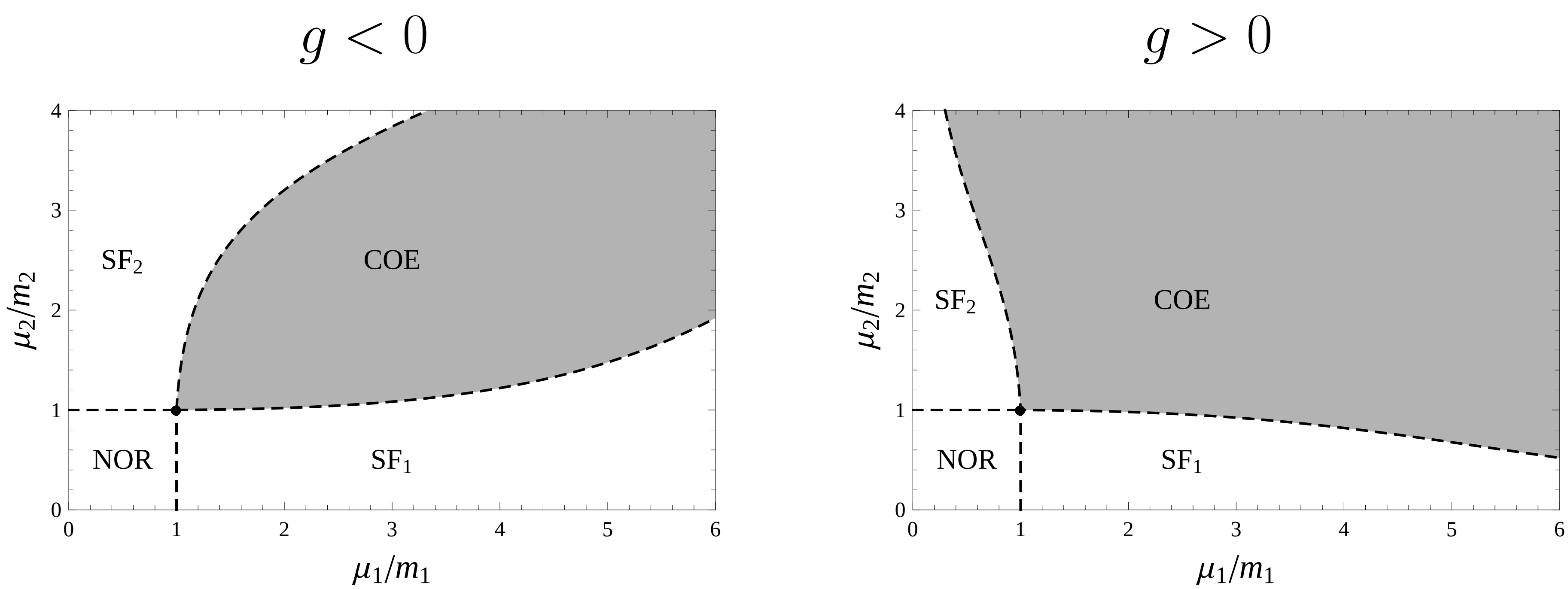
• Thermodynamics without superflow





- COE: both superfluids coexist, $U(1) \times U(1) \to 1$
- SF$_1$, SF$_2$: only one superfluid, $U(1) \times U(1) \to U(1)$ or $U(1)$
- NOR: normal phase, no superfluid, $U(1) \times U(1)$ intact

• Thermodynamics without superflow



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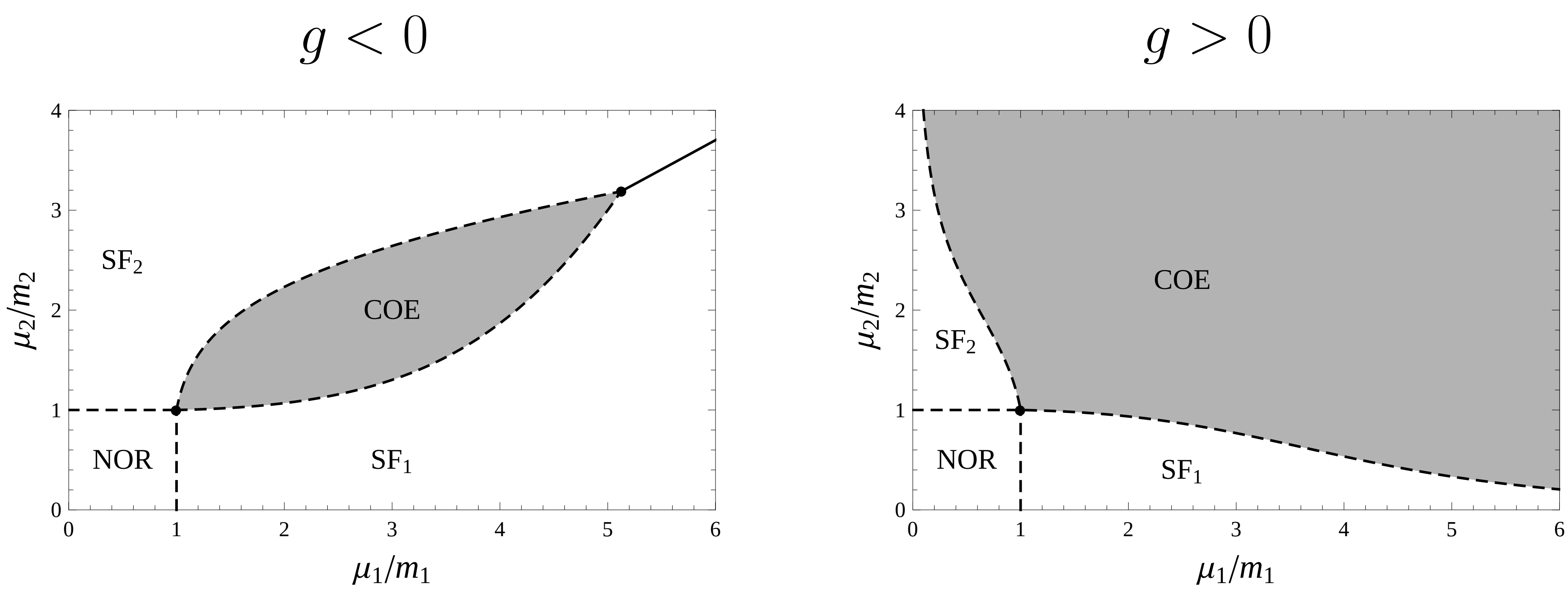

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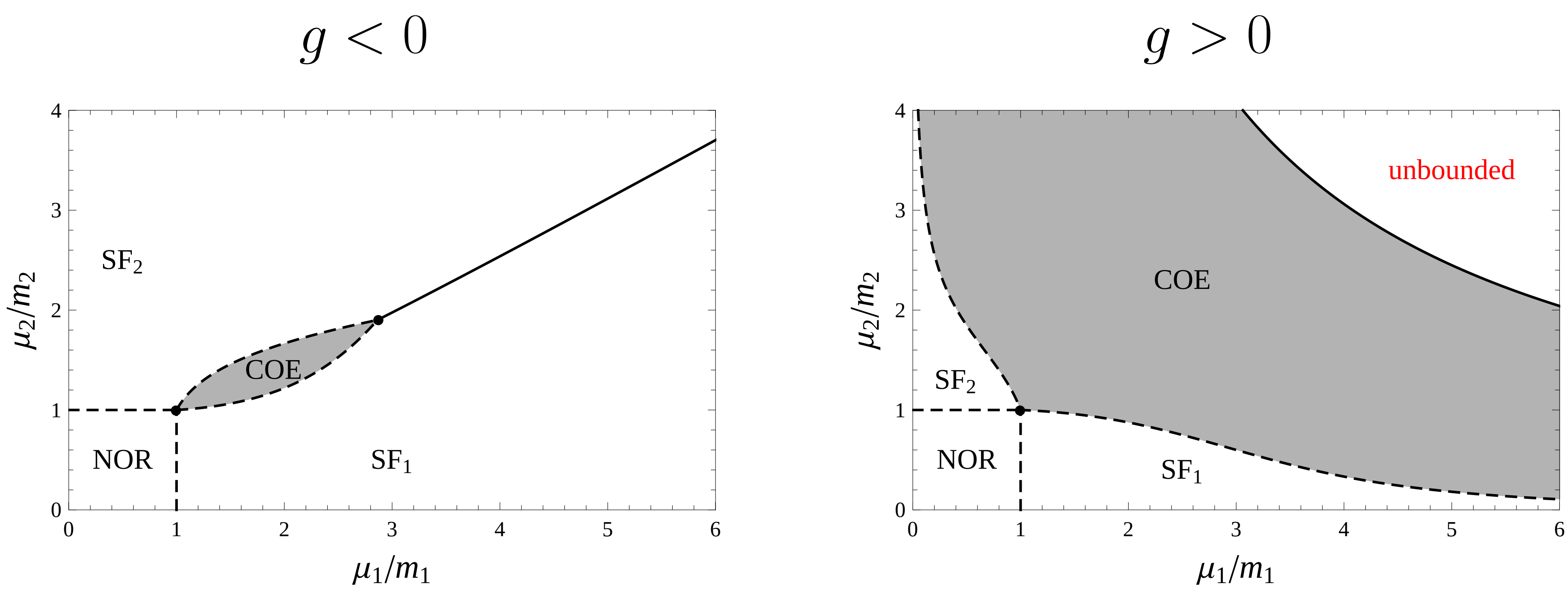

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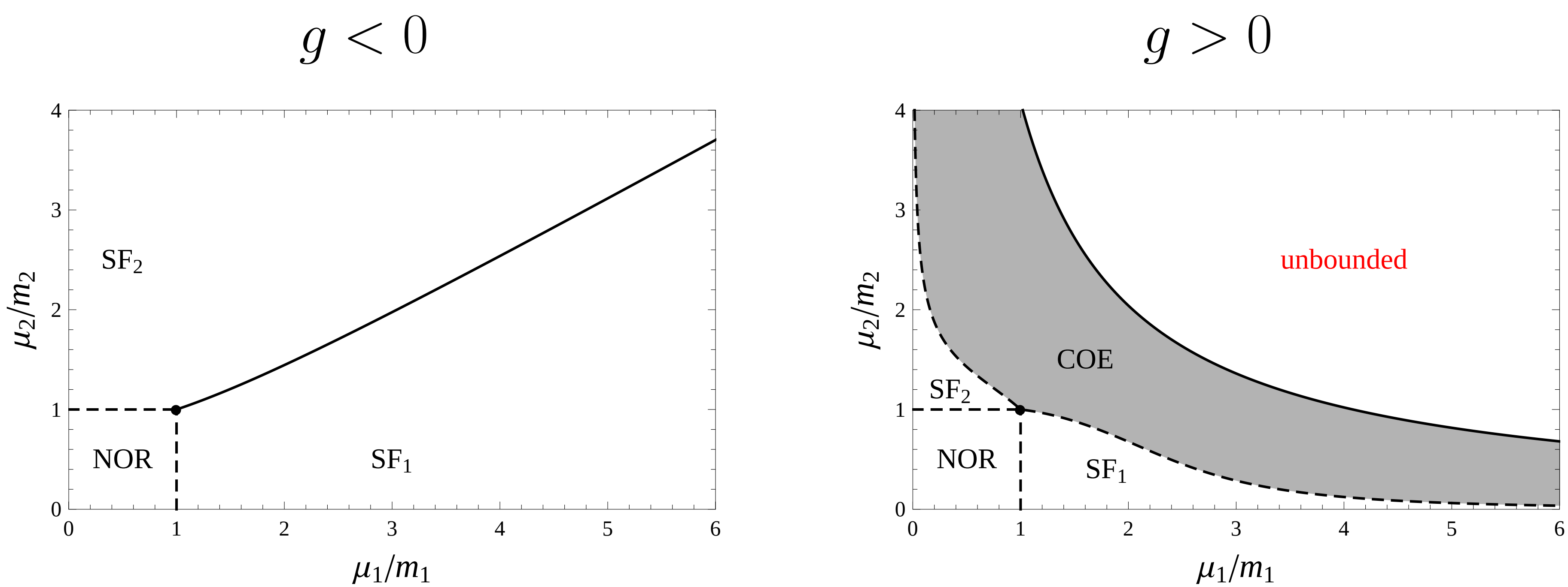

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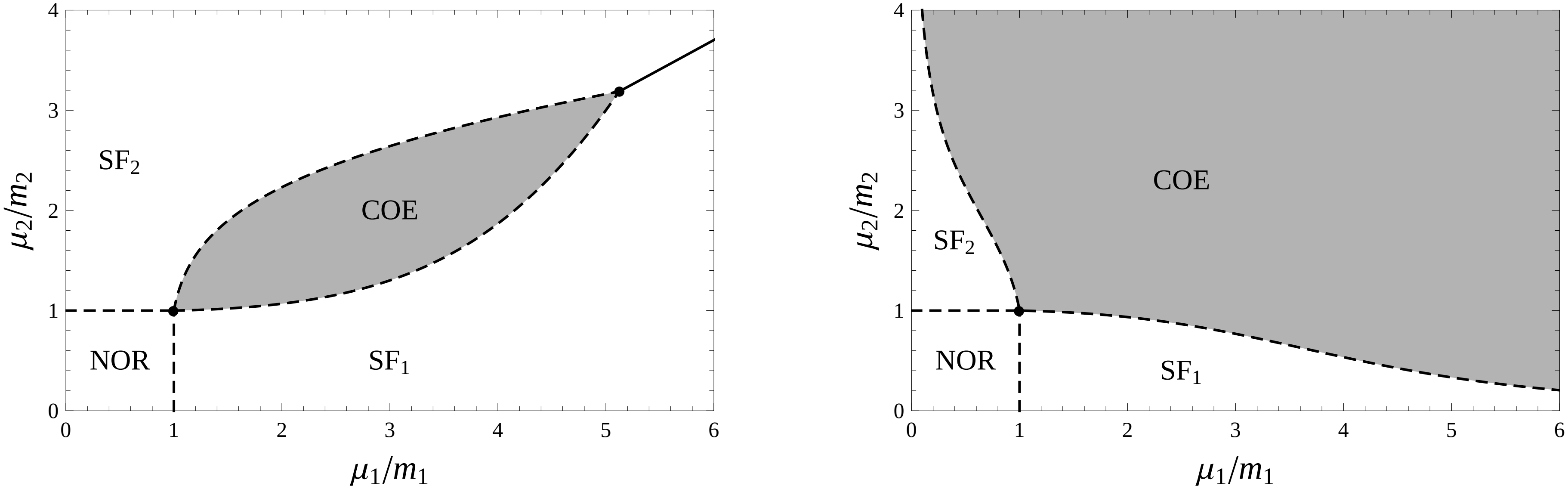
• Thermodynamics with (homogeneous) superflow

- equilibrium thermodynamics with $(\mu_1, \mu_2, \vec{v}_1, \vec{v}_2, T)$, all measured in "lab frame"
- $\vec{v}_2 = 0$: lab frame = rest frame of superfluid 2



$$v_1 = 0$$

• Thermodynamics with (homogeneous) superflow

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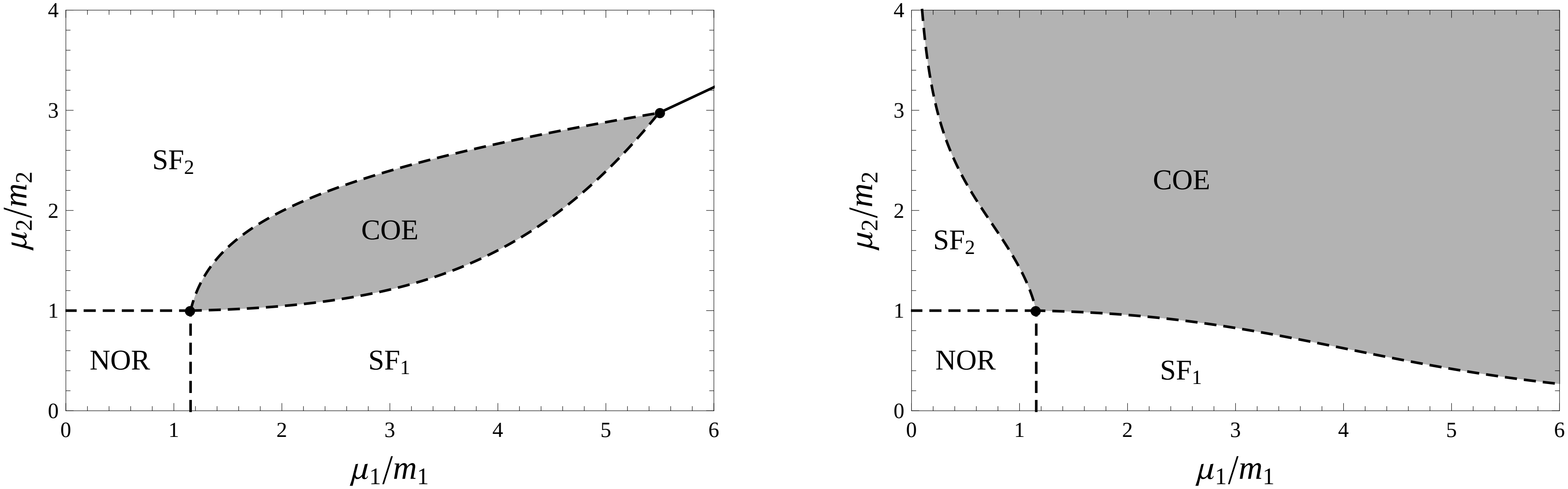
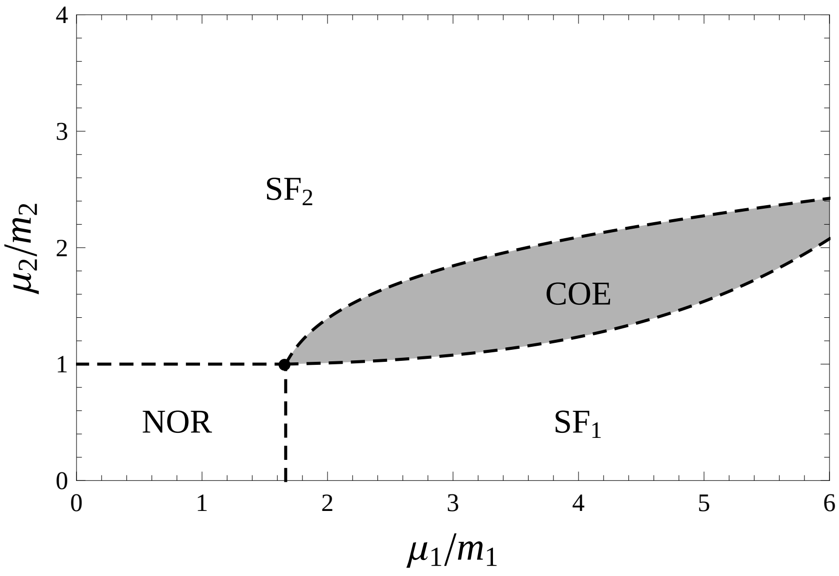
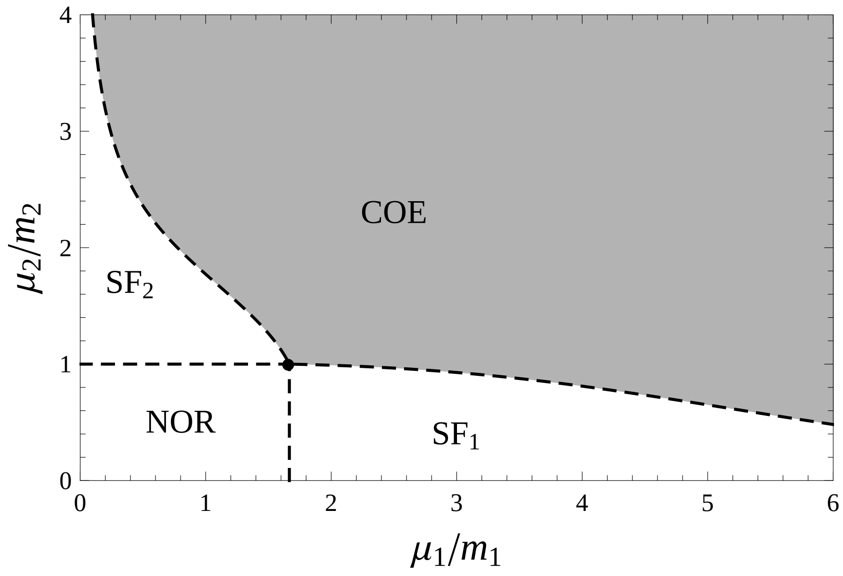


$$v_1 = 0.5$$

• Thermodynamics with (homogeneous) superflow

- equilibrium thermodynamics with $(\mu_1, \mu_2, \vec{v}_1, \vec{v}_2, T)$, all measured in "lab frame"
- $\vec{v}_2 = 0$: lab frame = rest frame of superfluid 2

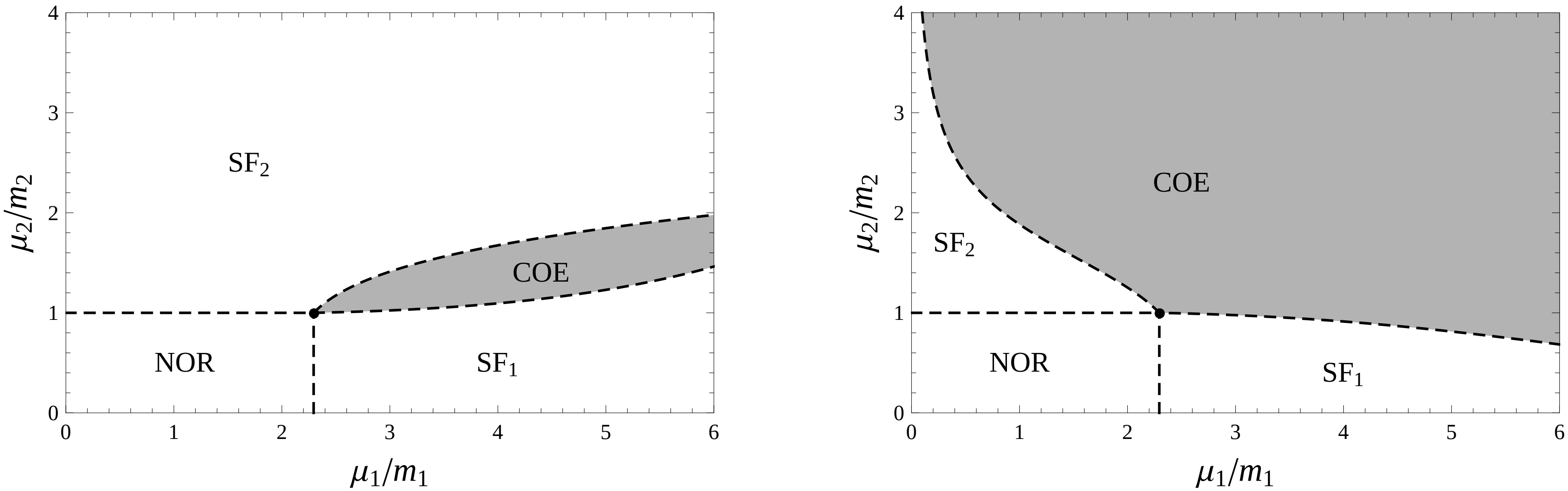




$$v_1 = 0.8$$

• Thermodynamics with (homogeneous) superflow

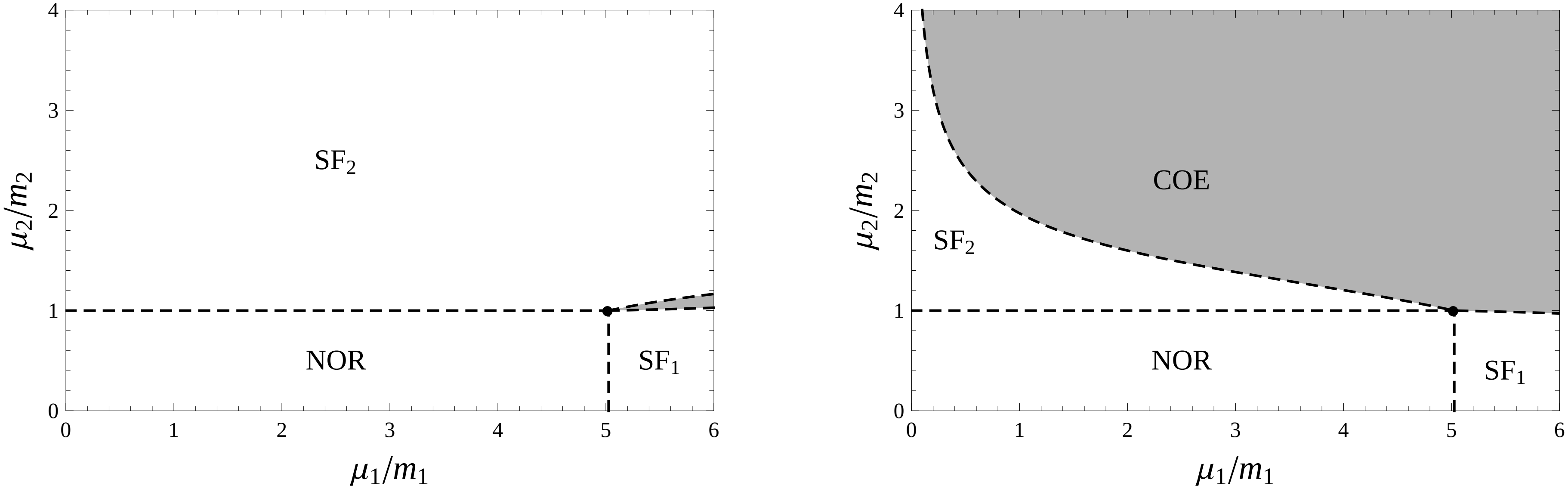
- equilibrium thermodynamics with $(\mu_1, \mu_2, \vec{v}_1, \vec{v}_2, T)$, all measured in "lab frame"
- $\vec{v}_2 = 0$: lab frame = rest frame of superfluid 2



$$v_1 = 0.9$$

• Thermodynamics with (homogeneous) superflow

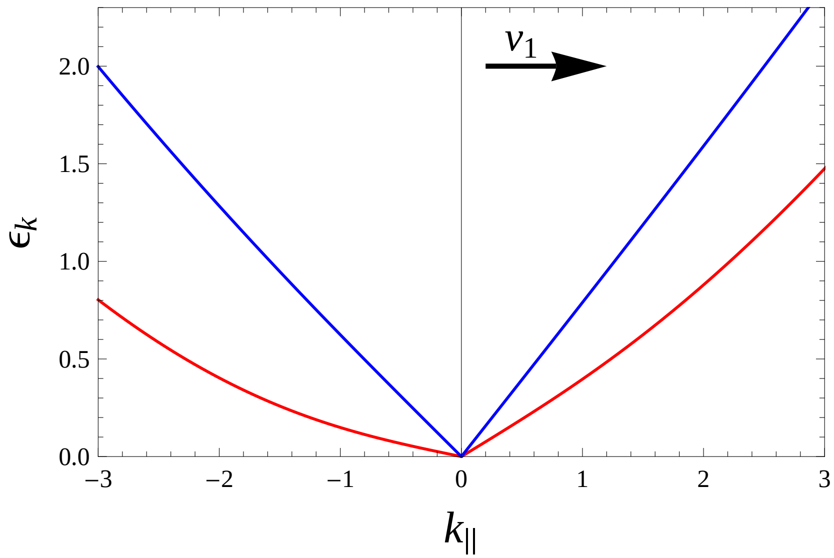
- equilibrium thermodynamics with $(\mu_1, \mu_2, \vec{v}_1, \vec{v}_2, T)$, all measured in "lab frame"
- $\vec{v}_2 = 0$: lab frame = rest frame of superfluid 2



$$v_1 = 0.98$$

• Excitations and sound modes

- excitations = poles of (tree-level) propagator



- 2 Goldstone modes

$$\epsilon_{i,k} = c_i(\theta)k + d_i(\theta)k^3 + \ldots$$

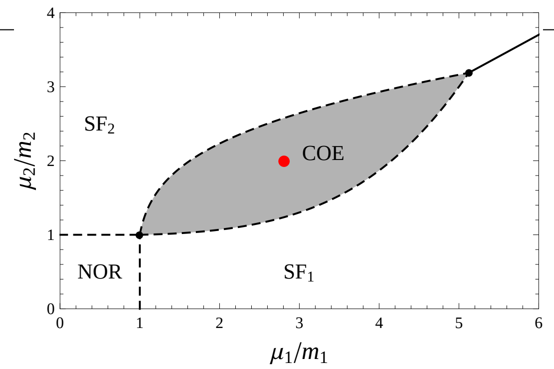
(+ 2 massive modes)

- alternatively: wave equations from (linearized) hydro

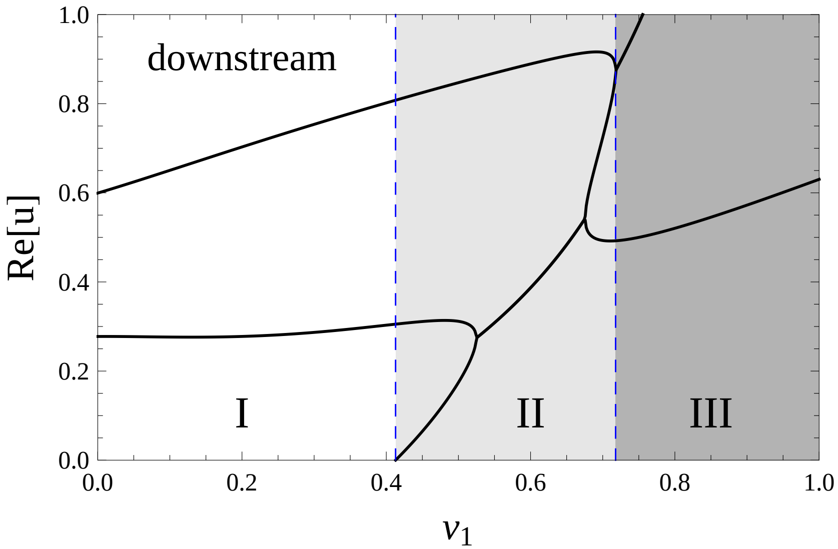
$$\partial_\mu j_1^\mu = 0\,, \qquad \partial_\mu j_2^\mu = 0\,, \qquad \partial_\mu T^{\mu\nu} = 0$$

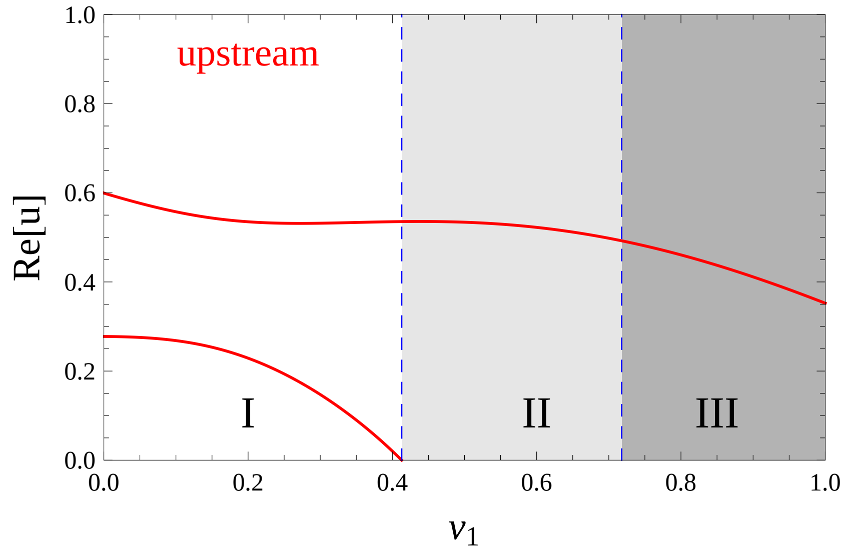
- 2 "first sounds" with sound velocities $c_i(\theta)$

($T > 0$: speeds of first and second sound in general different from Goldstone mode!)



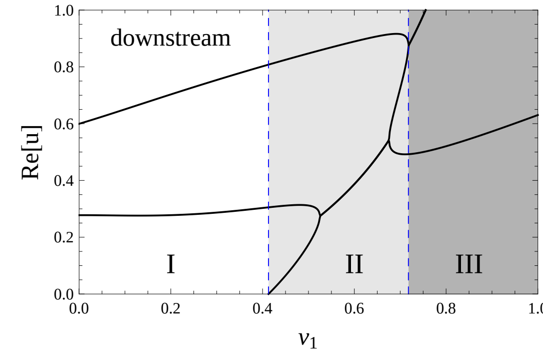
• Instabilities with superflow

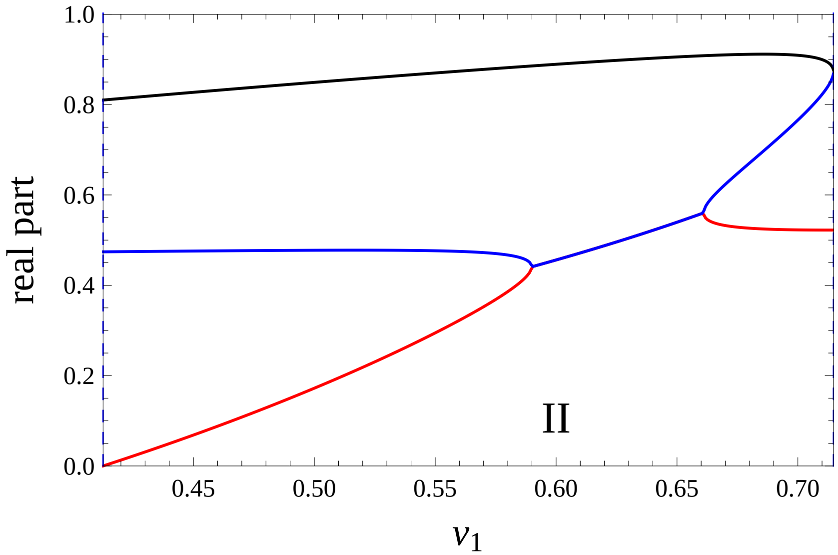


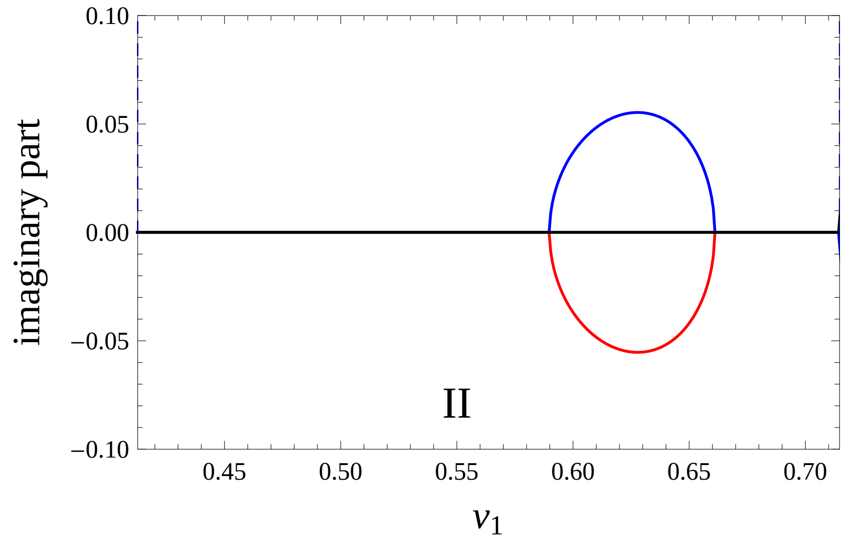


- region I: stable
- region III: SF$_2$ preferred

- **Two-stream instability (or "counterflow instability")**





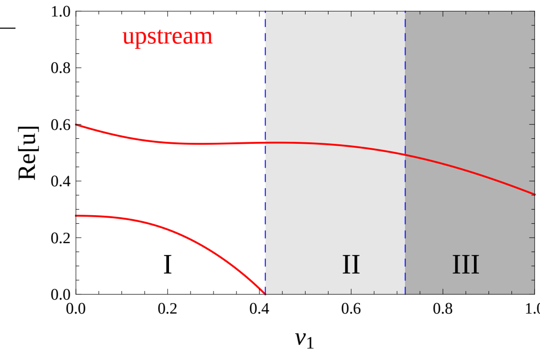


- complex sound speeds $\longrightarrow$ one mode damped, one mode explodes

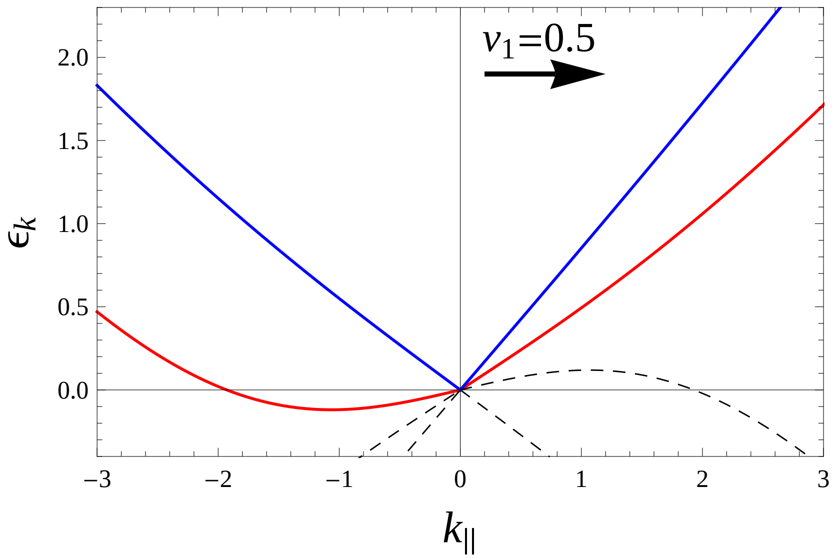
plasma physics: O. Buneman, Phys.Rev. 115, 503 (1959); D.T. Farley, PRL 10, 279 (1963)

general two-fluid system: L. Samuelsson *et al.* Gen. Rel. Grav. 42, 413 (2010)

atomic gases: M. Abad, A. Recati, S. Stringari, F. Chevy, EPJD 69, 126 (2015)



• Landau's critical velocity



- negative energies in Goldstone dispersion $\epsilon_k(\vec{v}) < 0$
- Landau's original argument

$$\epsilon_k - \vec{k} \cdot \vec{v} < 0$$

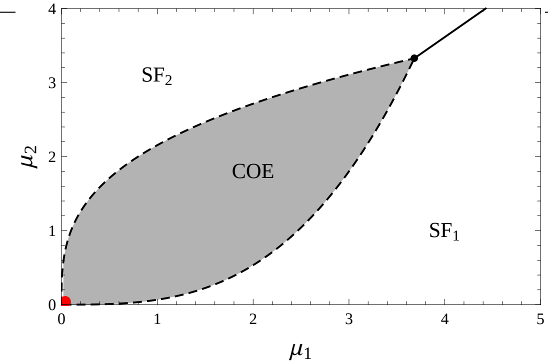
(for a single fluid)

• Two qualitatively different instabilities

“energetic instability” (Landau)
vs.
“dynamical instability” (two-stream)

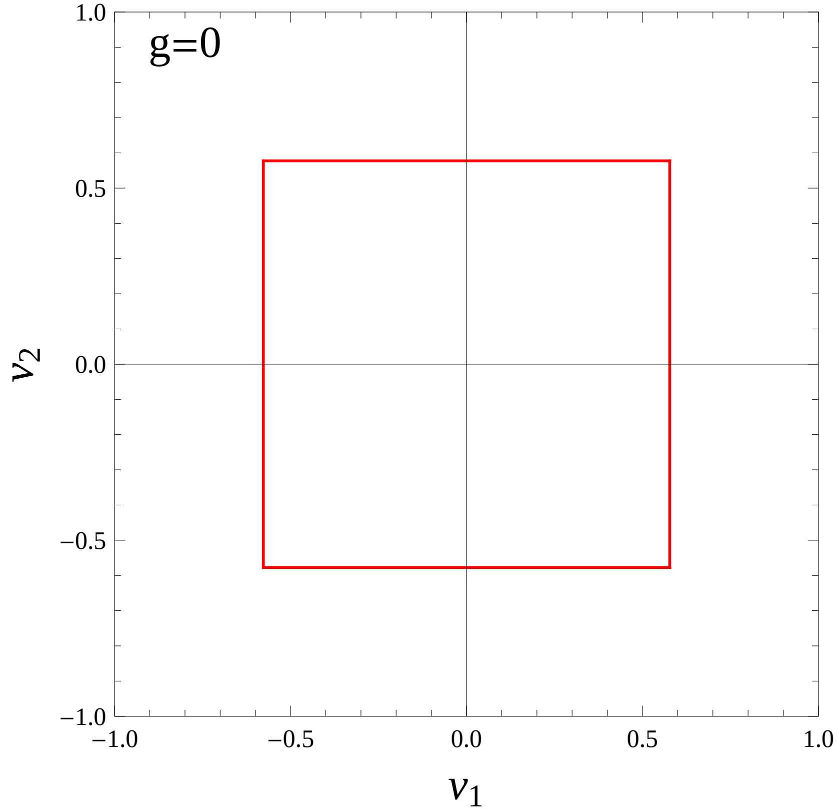
- any meaning of two-stream instability if it occurs ”after” (= at larger critical velocity than) $\epsilon_k(\vec{v}) < 0$?
  new (inhomogeneous) ground state?

- does it always occur after $\epsilon_k(\vec{v}) < 0$?



- **Analysis of the onset of instabilities**

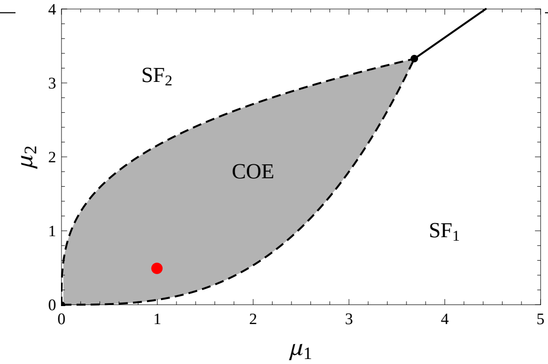
  - $\vec{v}_1$, $\vec{v}_2$ (anti-)aligned
  - $m_1 = m_2 = 0$ for simplicity



- Landau's critical velocity

$$v_1, v_2 = \pm\frac{1}{\sqrt{3}}$$

- no two-stream instability



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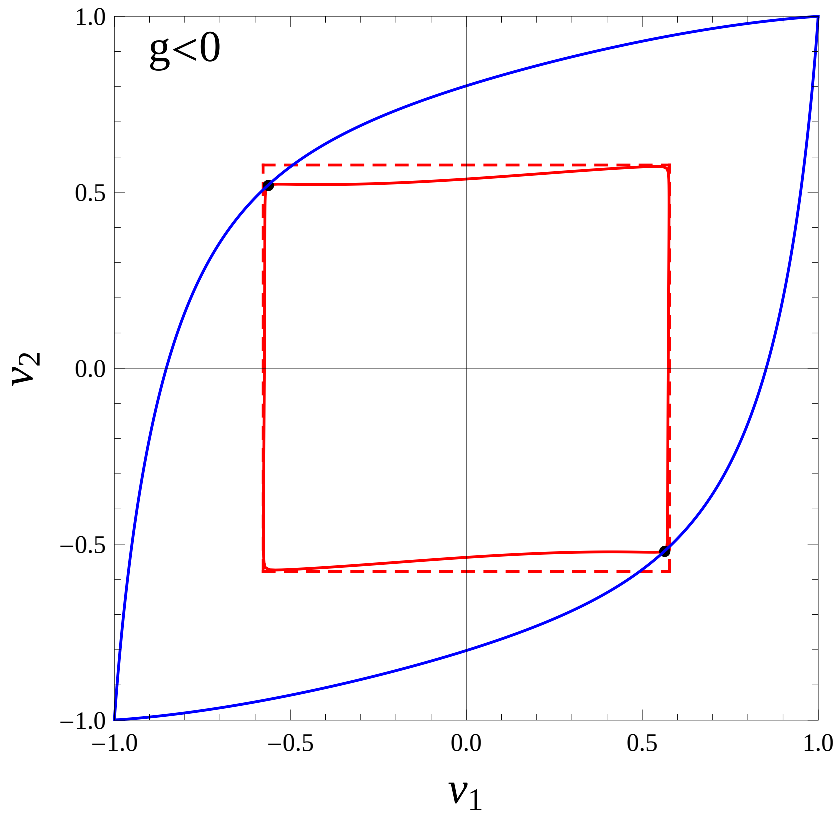
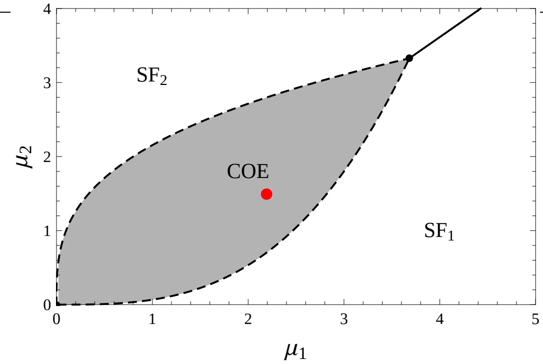


- Landau's critical velocity reduced
- two-stream instability $\forall v_1 \neq v_2$
- two-stream always "after" Landau
- $v_2 = 0$:

$$v_{\mathrm{two-stream}} = \frac{\sqrt{3}}{2} + \mathcal{O}(g)$$



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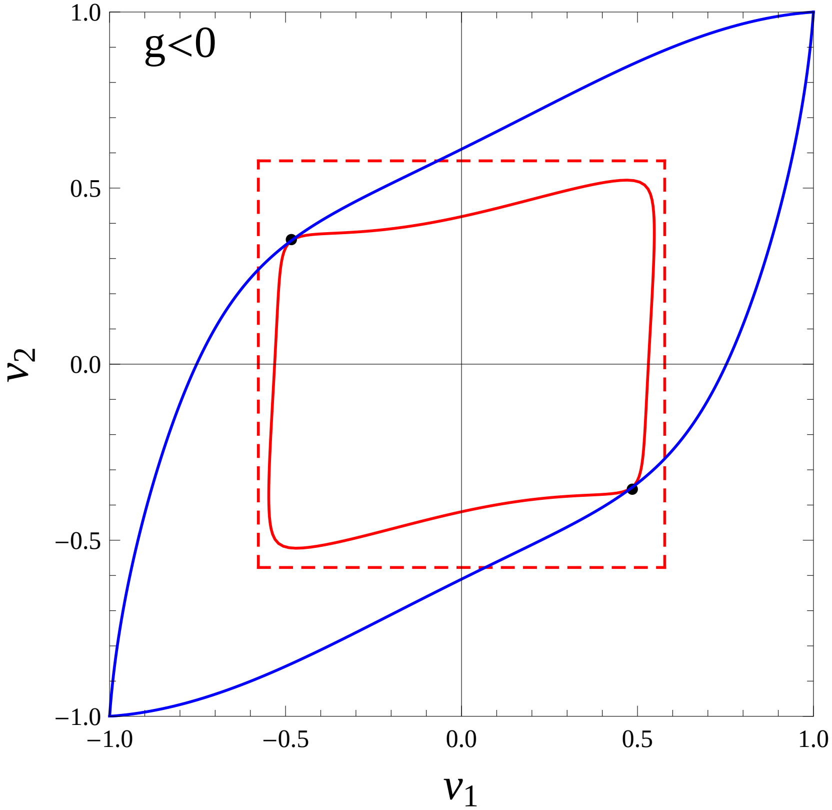

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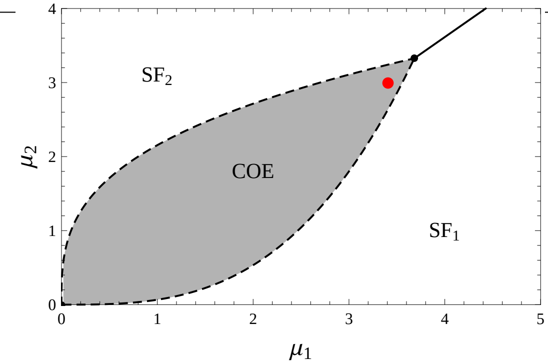

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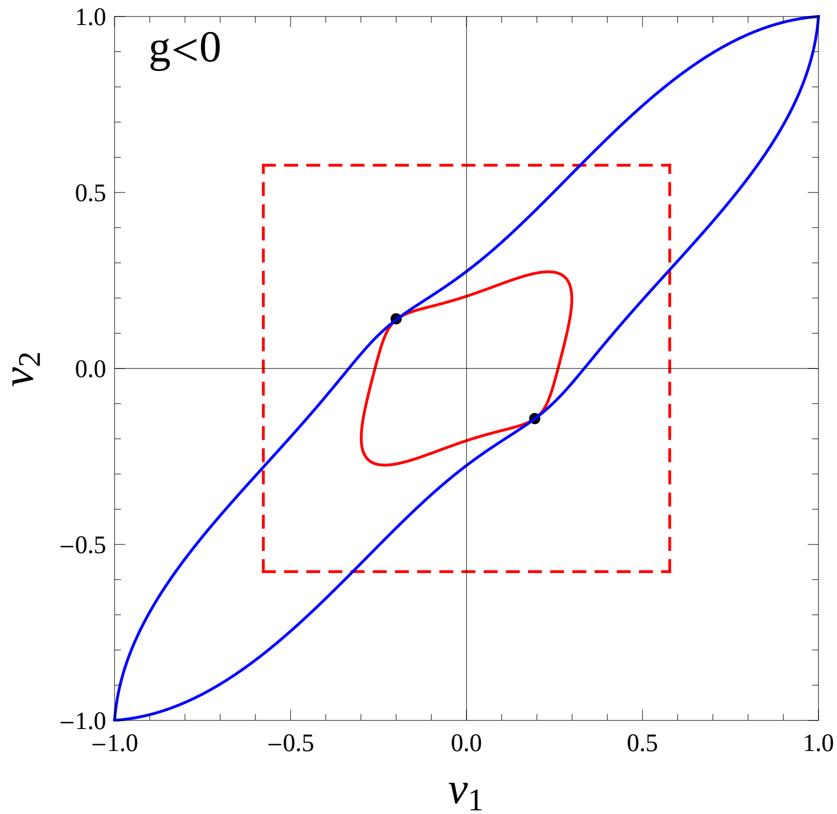

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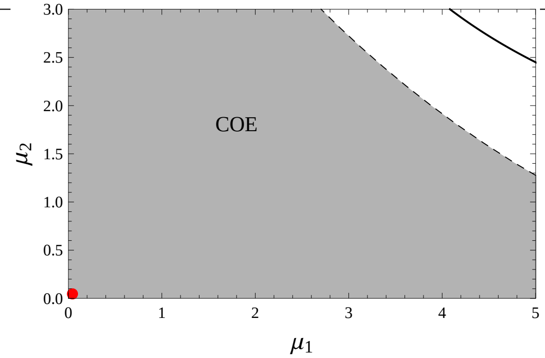

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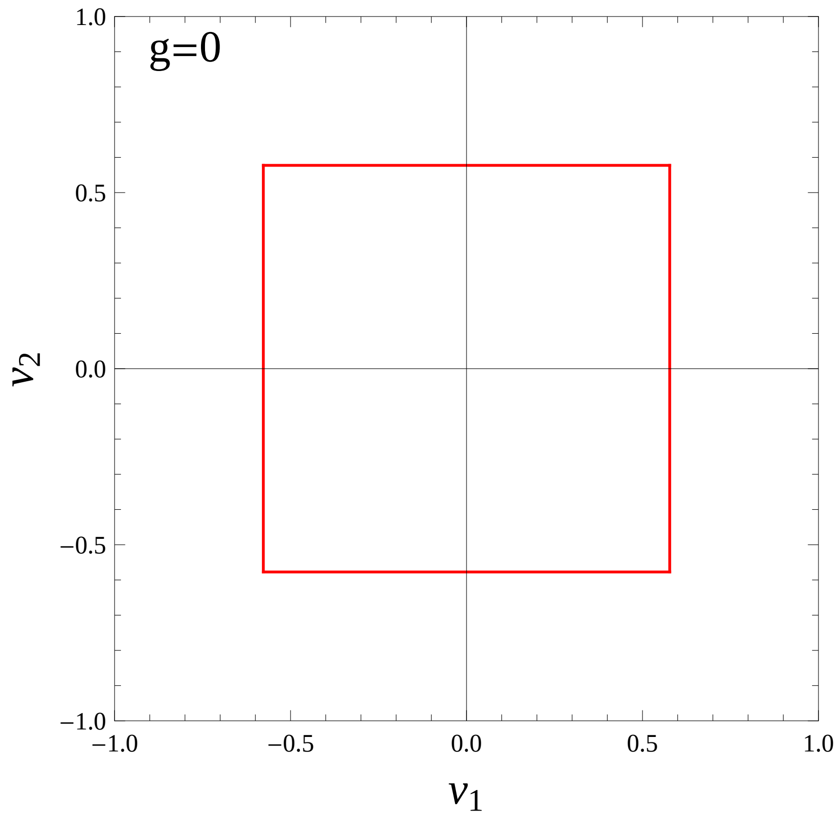

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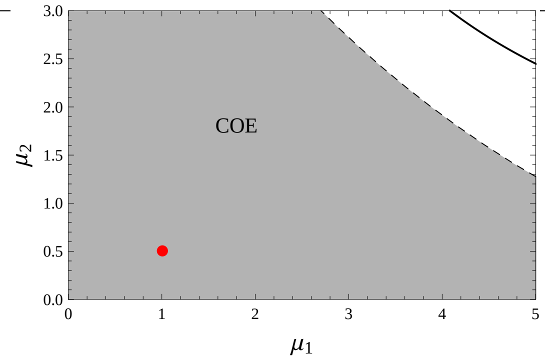

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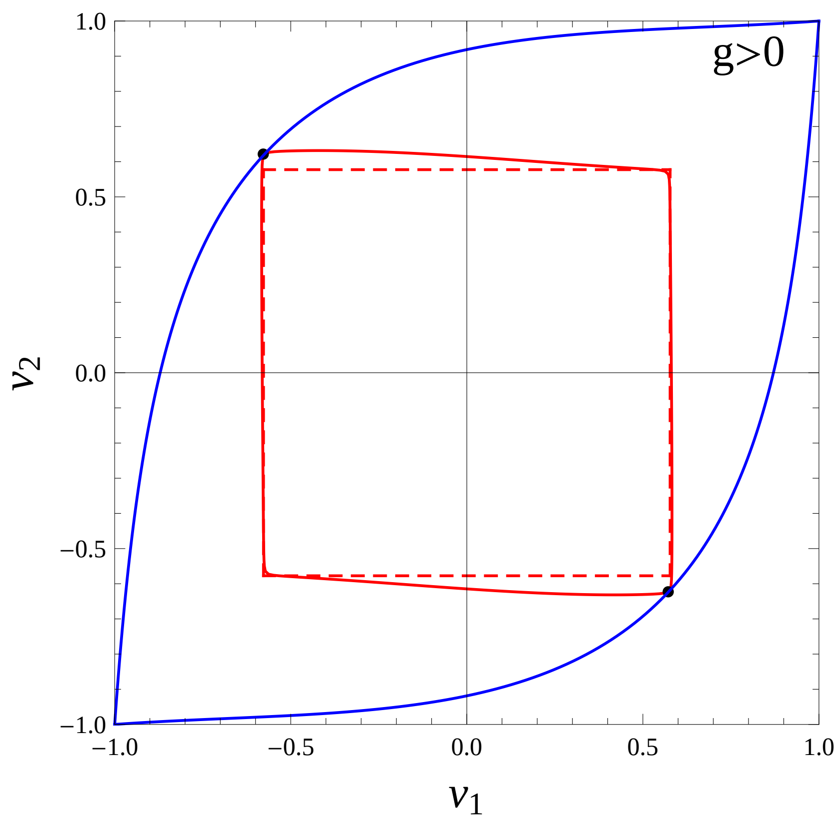
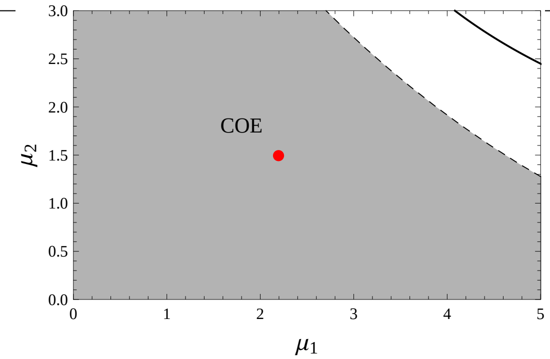


- Landau's critical velocity enhanced for anti-aligned flow
- two-stream always "after" Landau



- **Analysis of the onset of instabilities**

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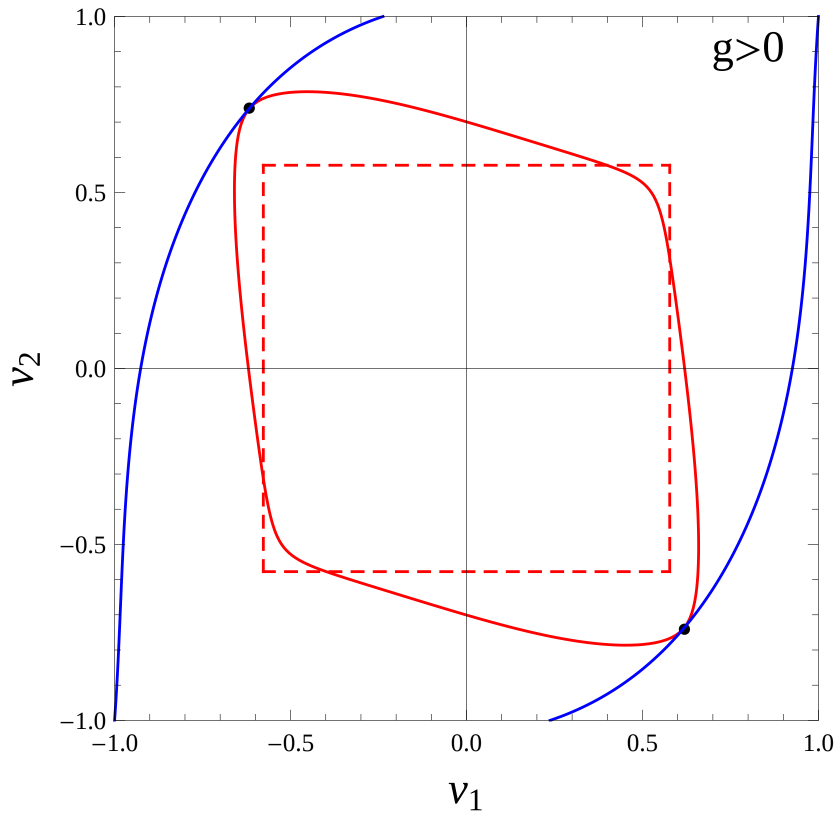
  - $m_1 = m_2 = 0$ for simplicity



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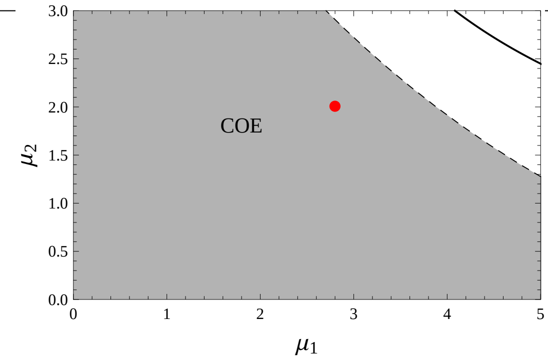

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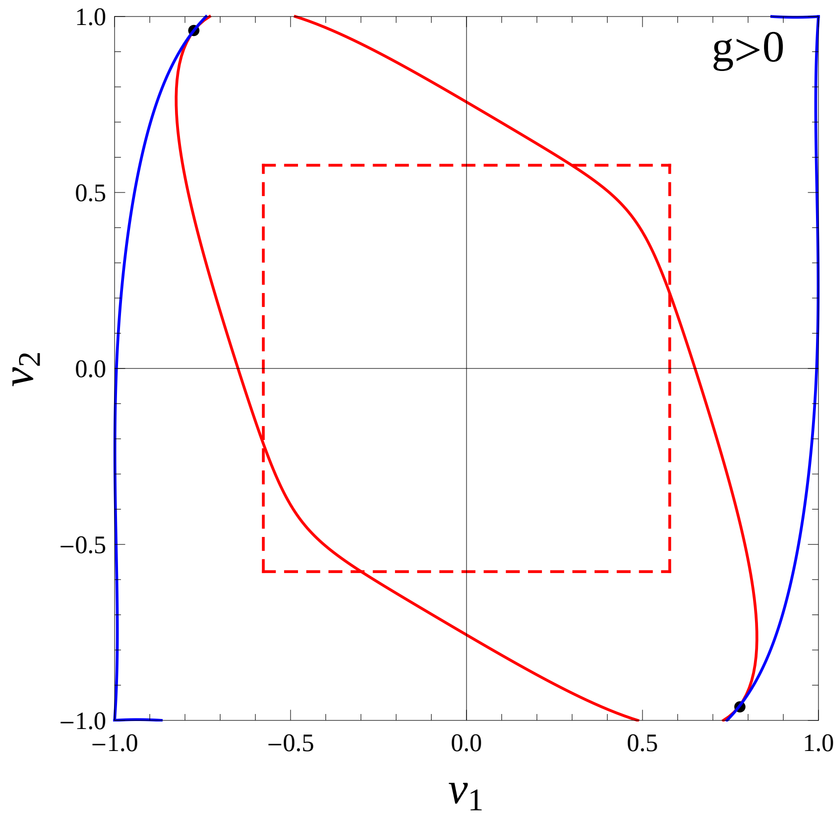

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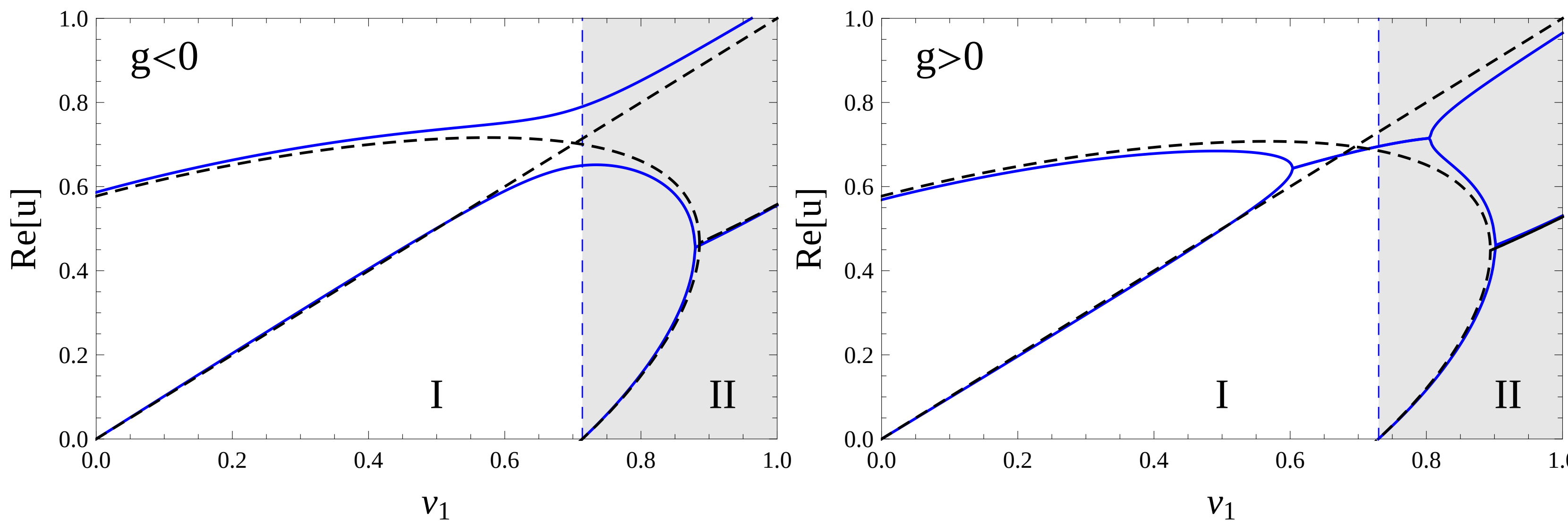
• Normal fluids

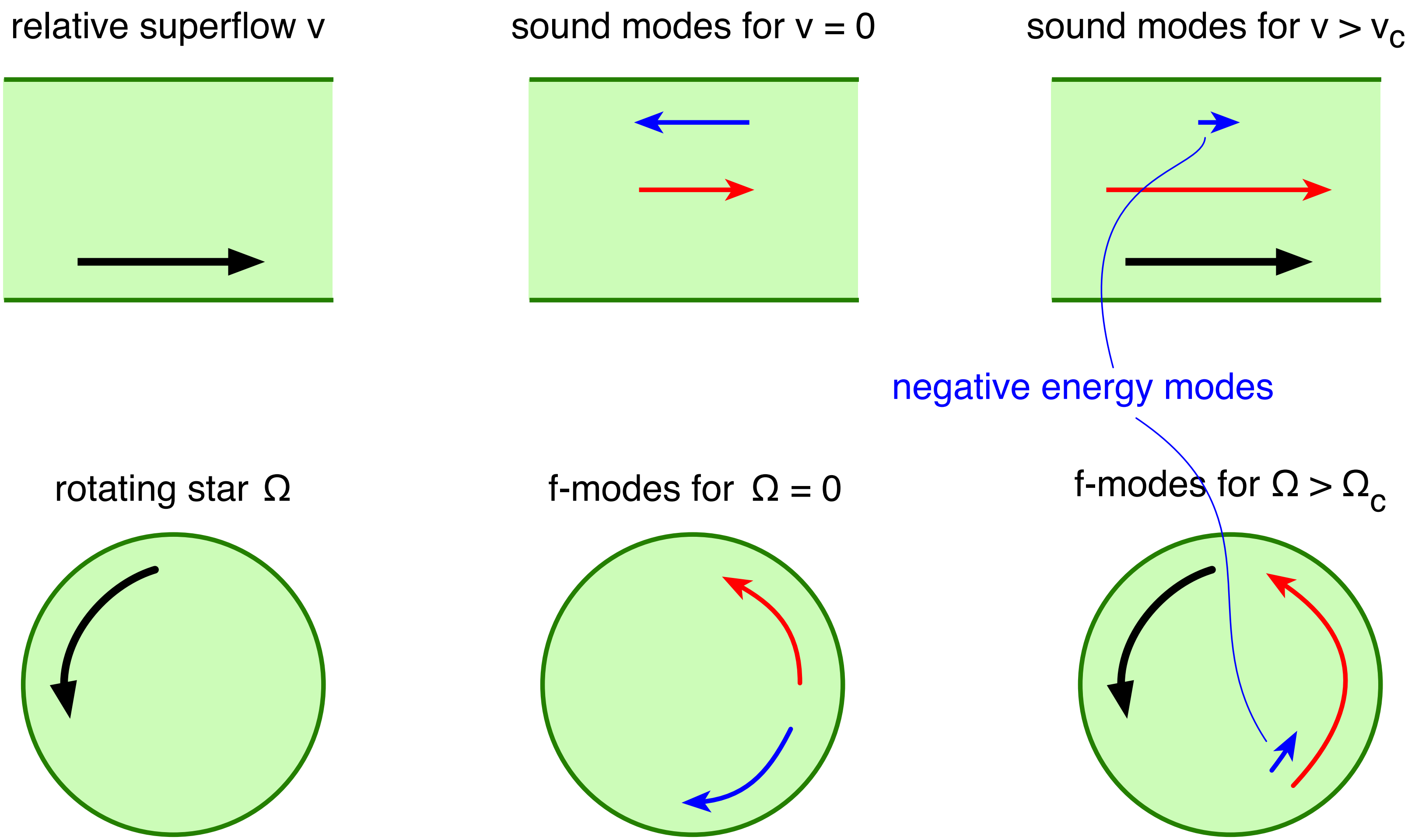
- superfluid: only longitudinal modes allowed, $\omega\delta(\mu\vec{v}) = \vec{k}\delta\mu$
  (because $\vec{v}$ and μ are not independent, both related to ψ)
- normal fluid: no such restriction

- downstream modes without (black dashed) and with (blue solid) entrainment coupling, including the transverse mode $u = v_1 \cos\theta$



Two-stream instability "before" energetic instability for $g > 0$!

• Energetic instability: analogy to star pulsations



(slight difference to r-modes, which only exist in rotating star, and have $\Omega_c = 0$)

• General picture

(I) fluid (star, superfluid, ...) with
propagating modes (sound modes, f-modes, ...)

$+$

(II) second rest frame
(non-rotating frame, second fluid, walls of a capillary, ...)

relative (angular) velocity between (I) and (II) sufficiently fast to flip direction of propagating mode
$\rightarrow$ **energetic ("secular") instability**

negative energy mode can become **exponentially growing mode** if (angular) momentum is exchanged (gravitational waves, dissipation, interaction with the walls of the capillary...)

• How dissipation can induce unstable modes (page 1/3)

A. Schmitt, work in progress

- go back to single superfluid, but $T > 0$, from

$$\mathcal{L} = \partial_\mu \varphi \partial^\mu \varphi^* - m^2 |\varphi|^2 - \lambda |\varphi|^4$$

- add dissipative terms to stress-energy tensor

$$T^{\mu\nu} = (\epsilon_n + P_n) u^\mu u^\nu - g^{\mu\nu} P_n + (\epsilon_s + P_s) v^\mu v^\nu - g^{\mu\nu} P_s + \delta T^{\mu\nu}(\eta, \zeta_1, \zeta_2, \zeta_3, \zeta_4, \kappa)$$

$$j^\mu = n_n u^\mu + n_s v^\mu$$

(in "Eckart frame")

  – set $\zeta_1 = \zeta_3 = \zeta_4 = 0$ for simplicity, keep shear viscosity η, bulk viscosity ζ_2, and heat conductivity κ as free parameters

- compute sound modes from

$$\partial_\mu T^{\mu\nu} = \partial_\mu j^\mu = 0 \, , \qquad u^\mu \partial_\mu \psi = \mu$$

• How dissipation can induce unstable modes (page 2/3)

- damped sound mode in normal fluid, $T = 0$

$$\omega \simeq ck + ik^2\Gamma + \mathcal{O}(k^3)\,, \qquad c^2 = \frac{n}{\mu}\left(\frac{\partial n}{\partial \mu}\right)^{-1}\,, \qquad \Gamma = \frac{4\eta + 3\zeta}{6\mu n}$$

- superfluid, vanishing superflow, low-T expansion $(x \equiv \frac{m}{\mu} < 1)$

conformal limit $x = 0$: C. P. Herzog, N. Lisker, P. Surowka, A. Yarom, JHEP 1108, 052 (2011)

– first sound

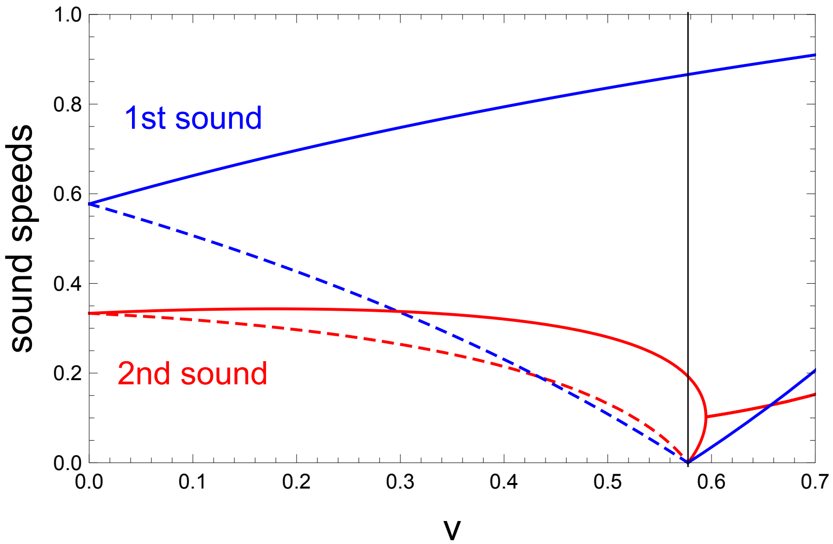
$$c_1 = \frac{1}{\sqrt{3}}\sqrt{\frac{1-x^2}{3-x^2}} + \mathcal{O}(m^2T^4)\,, \quad \Gamma_1 = \frac{\lambda(4\eta + 3\zeta_2)}{4\mu^4}\frac{54 - 27x^2 - 2x^6}{(1-x^2)(3-x^2)^4} + \mathcal{O}(T)$$

– second sound

$$c_2 = \frac{c_1}{\sqrt{3}} + \mathcal{O}(T^4)\,, \quad \Gamma_2 = \frac{15(4\eta + 3\zeta_2)c_1^5}{4\pi^2T^4} + \frac{15\kappa c_1^3}{\pi^2T^3}\frac{2 - x^2}{(3-x^2)^2} + \mathcal{O}(T^{-2})$$

• How dissipation can induce unstable modes (page 3/3)

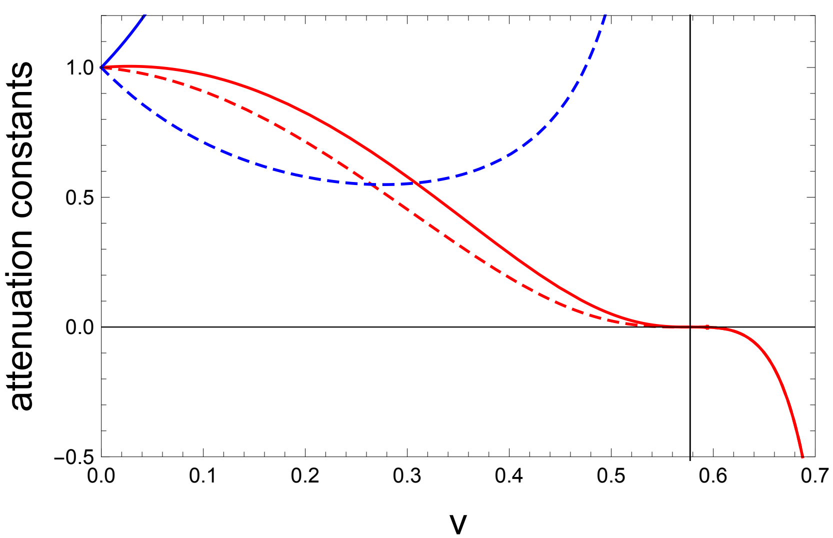
- nonvanishing superflow v, in rest frame of normal fluid



- $m = 0$ for simplicity
- sound mode gets "flipped" at

$$v_c = \frac{1}{\sqrt{3}}$$

(independent of dissipation)



- viscous effects induce negative Γ $\rightarrow$ exponentially growing mode
- close to v_c:

$$\Gamma_2 \simeq -\frac{135(4\eta + 3\zeta_2)}{2\pi^2 T^4}(v - v_c)^3$$

• Summary

- (relativistic) two-component superfluids exist in compact stars and can be created in the laboratory
- they show hydrodynamic instabilities in the presence of a sufficiently large relative flow
- in $U(1) \times U(1)$ model at $T = 0$: energetic (Landau) instability and dynamical (two-stream) instability with

$$v_{\rm Landau} < v_{\rm two-stream}$$

(exception: two normal fluids with entrainment)

- energetic instability can become dynamical through dissipation, as demonstrated for a single superfluid at $T > 0$

• Outlook

- add electric charge to (one of) the fields
  (→ neutron/proton system)
  - instabilities in the presence of electromagnetism
  - use model for Meissner and flux tube phases in coupled system
    M. G. Alford and G. Good, PRB 78, 024510 (2008)
    A. Haber, A. Schmitt, work in progress
- inhomogeneous condensates as resolution for energetic instability?
  L. A. Melnikovsky, JPhCS 150, 032057 (2009)
  I. S. Landea, 1410.7865
- time evolution of two-stream instability
  I. Hawke, G. L. Comer and N. Andersson, Class. Quant. Grav. 30, 145007 (2013)
- relevance of instabilities for pulsar glitches
  N. Andersson, G. L. Comer, R. Prix, MNRAS 354, 101 (2004)
  B. Haskell and A. Melatos, Int. J. Mod. Phys. D 24, 1530008 (2015)