Exploring different phases
Non-equilibrium properties of plasma
Transport coefficients in superfluid neutron star
Transport coefficients in Color super-conducting neutron star core
Summary

Exploring different phases of neutron star using Transport Coefficients

Sreemoyee Sarkar

In collaboration with: C. Manuel, R. Sharma, L. Tolos

TIFR and CBS, Mumbai

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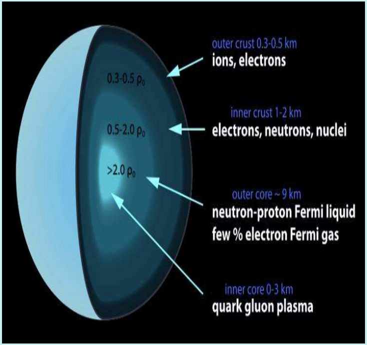
Outline

Outline

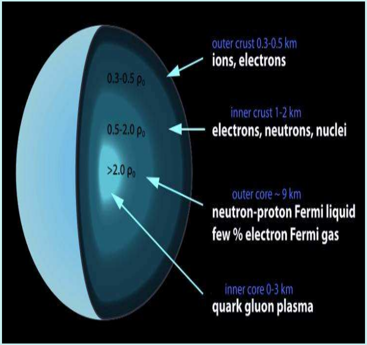
- Introducing Neutron stars: Different phases
- Transport coefficients in different phases
    - Thermal conductivity
    - Shear viscosity
- Summary

Profile



- Inhomogeneous matter profile consists of layers increases in density as we move deeper.
- The outer crust is made of lattice of ions embedded in a liquid of electrons.
- The inner region made up of protons, electrons and superfluid neutrons.
- In the inner core $\Rightarrow$ matter is expected to form degenerate matter.

Hypotheses about the inner core



- Deep inside difficult to observe core
- Phase transition to the quark matter composed of light deconfined u, d, s quarks.
- Composition of the inner core influences EOS, transport coefficients, cooling rate.
- Phase transitions in the core influences cooling rate and mechanical properties.

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Transport coefficients: Neutron star

Motivation

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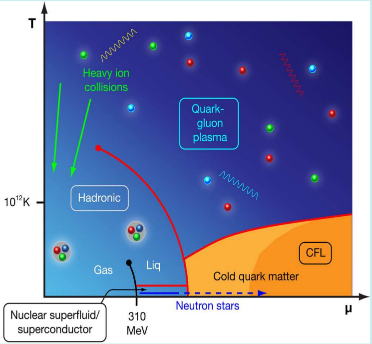
Transport coefficients of dense matter play a central role in the modeling of astrophysical phenomena in compact stars



Outline

- Thermal evolution ⇒ Thermal Conductivity
- Rotational dynamics ⇒ Shear and Bulk Viscosity

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Thermal evolution

κ is important for cooling of neutron stars

Cooling processes

Cooling of young neutron star

The cooling is realized via two channels $\Rightarrow$ by neutrino emission from the neutron star core and by transport of heat from the internal layers to the surface resulting in the thermal emission of photons.

- Powerful neutrino emission
- Thermal conduction

r-mode oscillation and shear viscosity

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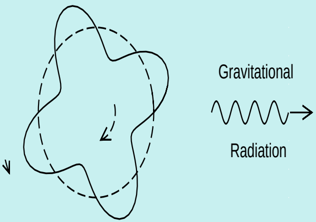
The r-modes are low-frequency toroidal modes and unstable due to the emission of gravitational waves. Damping mechanisms like viscous processes counteract the growth of an unstable r-mode.



- The non-radial pulsations of stars couple to gravitational radiation.
- GR produced by these oscillations carries away energy and angular momentum from the star.
- In non-rotating stars the effect of GR losses is dissipative, and the pulsations of the star are damped.
- Rotating stars $\Rightarrow$ angular momentum removed by GR lowers the angular momentum of such a mode, and therefore the amplitude of the mode grows.

Non-equilibrium properties of plasma

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- Non-equilibrium properties of plasma $\Rightarrow$ Transport coefficients

Plasma kinetic equation $\Rightarrow$ Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v_p}.\nabla_{\mathbf{r}} + \mathbf{F}.\nabla_{\mathbf{p}}\right) f_p = \mathcal{C}[f_p]$$

$$\mathcal{C}[f_p] \quad = \quad -\nu^{'} \int_{p^{'},k,k^{'}} [TF](2\pi)^4 \delta^4(p + k - p^{'} - k^{'})|\mathcal{M}|^2$$

- $\mathcal{C}[f_p] \rightarrow$ collision rate
- $\mathcal{M} \rightarrow$ Lorentz invariant transition rate
- Non-equilibrium state described by distribution functions small departure from equilibrium ($\delta f_p << f_p^0$) $f_p = f_p^0 + \delta f_p \Rightarrow \mathcal{C}[f_p] = \mathcal{C}[f_p^0] + \mathcal{C}[\delta f_p]$.

To extract transport coefficients

- Each transport coefficient under consideration will depend on the departure from equilibrium resulting from a particular form of the driving terms on the left-hand side of the linearized Boltzmann equation
- Thermal Conductivity $\Rightarrow$ response to a spatial variation to temperature.
- Diffusion $\Rightarrow$ response to a spatial variation in chemical potential.
- Shear Viscosity $\Rightarrow$ response to a spatial variation in fluid flow velocity.

Kinetic theory description

- **Stress-energy tensor**

  $$T_{ij} = \int \frac{d^3p}{(2\pi)^3} \frac{p_i p_j}{E_p} \delta f_p$$

  Thermal current

  $$J_T = \int \frac{d^3p}{(2\pi)^3} v_p(\epsilon_p - \mu)\delta f_p$$

Fluid description

- **Stress-energy tensor**

  $$T_{ij} = -\eta(\nabla_i u_j + \nabla_j u_i - \frac{2}{3}\delta_{ij}\nabla.\mathbf{u}) + \cdots$$

- **Thermal current**

  $$J_T = -\kappa \nabla T$$

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Thermal Conductivity in superfluid Neutron star

κ in Superfluid Neutron Star

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- Neutron star cores composed of neutrons, with an admixture of protons, electrons and muons.
- $\kappa \approx \kappa_b + \kappa_{e\mu}$
- $\kappa_{e\mu} = \kappa_e + \kappa_\mu$
- $\kappa_{e\mu}$ determined by electromagnetic interactions of electrons and muons with all charged particles.
- $\kappa_b \Rightarrow$ Conductivity of neutrons.

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- Migdals observation $\Rightarrow$ at low temperatures superfluidity of neutron matter may occur in the core of compact stars.
- Due to the onset of superfluidity$\Rightarrow$ a collective mode appears $\Rightarrow$ superfluid phonon.
- Transport coefficients are governed by superfluid phonons.
- Thermal Conductivity $\rightarrow$

$$\kappa = \frac{1}{3T^3} \int \frac{d^3p}{(2\pi)^3} f_p(1+f_p) g(p)\, v_p\, E_p\, p$$

- $$\delta f_p = -\frac{f_p^0(1+f_p^0)}{T^3} g(p) \mathbf{p} \cdot \boldsymbol{\nabla} T$$

- $g(p)$ is dimensionless variable $\Rightarrow$ $g(p) = \sum_s b_s B_s(p^2)$.
- $g(p)$ obtained solving Boltzmann equation

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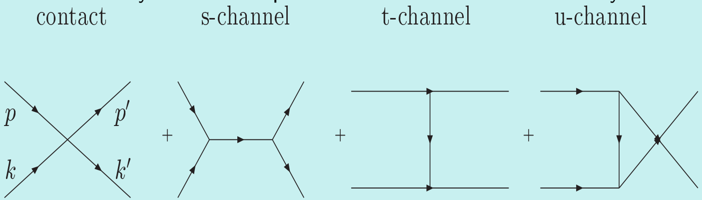
Superfluid phonon dispersion law: beyond leading order

Solution to the Boltzmann equation obeys the constraints of both energy and momentum conservation

$$\int \frac{d^3p}{(2\pi)^3} E_p\, \delta f_p = \int \frac{d^3p}{(2\pi)^3} p\, \delta f_p = \frac{\nabla T}{3\,T^3} \int \frac{d^3p}{(2\pi)^3}\, f_p(1+f_p) g(p)\, p^2 = 0$$

- Phonons $\Rightarrow$ exactly linear dispersion relation do not contribute to the thermal conductivity
- Beyond leading order $\Rightarrow$ $E_p = c_s p\left(1+\gamma p^2\right)$ where, $\gamma = -\frac{c_s^2}{15\Delta^2}$,
- Phonon dispersion law curves downward beyond linear order $\Rightarrow$ collisional processes of $1 \rightarrow 2$ kinematically forbidden
- Relevant binary collisions of phonons for the thermal conductivity are



Kinematically forbidden processes

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Sign of γ plays a crucial role in determining which processes are allowed

For $1 \to 2$ processes energy and momentum conservation impose

$$\begin{aligned} E_a &= E_b + E_c \\ \vec{p}_a &= \vec{p}_b + \vec{p}_c \end{aligned}$$

- Beyond leading order $\Rightarrow E_p = c_s p \left(1 + \gamma p^2\right)$
- First order in $\gamma \Rightarrow$ the NLO correction $\Rightarrow$

$$\theta_{bc} = \sqrt{6\gamma}\,(p_b + p_c)$$

- For the one to two processes to be kinematically allowed, it is necessary that $\gamma > 0$.

Variational solution to the Boltzmann equation

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- The polynomials are orthogonal with regard to the inner product

$$\kappa = \left(\frac{a_1(c_s,\Delta)^2}{T^2}\right) A_1(c_s,T)^2 M_{11}^{-1}$$

- $M_{11}^{-1} \Rightarrow$ (1,1) element of inverse of the truncated $N \times N$ matrix. The bound is saturated as $N \to \infty$.

$$\begin{aligned} M_{st} &= \int d\Gamma_{p,k,k',p'} \mathbf{Q}_s \cdot \mathbf{Q}_t \\ \mathbf{Q}_s &= B_s(p^2)\mathbf{p} + B_s(k^2)\mathbf{k} - B_s(k'^2)\mathbf{k}' - B_s(p'^2)\mathbf{p}' \end{aligned}$$

- Phonons with non-linear dispersion relation

$$a_1 = \frac{8c_s^4}{\sqrt{3}\times 15\Delta^2} \qquad A_1 = \frac{256\pi^6}{245c_s^9} T^9$$

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Phonon propagator

- Leading order phonon propagator

$$\mathcal{G}_{\rm ph}\left(p_i^0+p_j^0,\vec{p}_i+\vec{p}_j\right)=\frac{1}{(p_i^0+p_j^0)^2-E^2_{p_i+p_j}}$$

- Next to leading order phonon propagator

$$\mathcal{G}_{\rm ph}\left(p_i^0+p_j^0,\vec{p}_i+\vec{p}_j\right)\sim[2c_s^2p_ip_j(1-\cos\theta_{ij}-3\gamma(p_i+p_j)^2)]^{-1}$$

- In the collinear region $\Rightarrow \theta_{ij}\approx 0$ the propagator behaves as $\sim 1/p^4$.
- Region of large angle scattering the propagator behaves as $\sim 1/p^2$.

Temperature dependence of the thermal conductivity

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$$\text{Temperature dependence} \Rightarrow \kappa \propto \frac{T^{16}}{\Delta^4} M_{11}^{-1}$$
$$M_{11}^{-1} \propto T^{-10} |\mathcal{M}|^{-2}$$
$$|\mathcal{M}|^2 \propto T^{12} \times \frac{1}{G^2}$$

- For large angle collisions $\Rightarrow G^2 \propto T^{-4} \rightarrow |\mathcal{M}|^2 \propto T^8$,

$$\kappa \propto \frac{T^{16}}{\Delta^4} \frac{1}{T^{18}} \propto \frac{1}{T^2 \Delta^4} \qquad \text{for large angle collisions.}$$

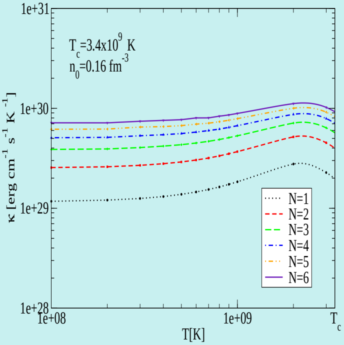
- In collinear region $\Rightarrow G^2 \propto \Delta^4 T^{-8} \rightarrow |\mathcal{M}|^2 \propto T^4 \Delta^4$,

$$\kappa \propto \frac{T^{16}}{\Delta^4} \frac{1}{T^{14} \Delta^4} \propto \frac{T^2}{\Delta^8} \qquad \text{for small angle collisions.}$$

- In combined large-small angle collisions $\Rightarrow G^2 \propto \Delta^2 T^{-6} \rightarrow |\mathcal{M}|^2 \propto T^6 \Delta^2$,

$$\kappa \propto \frac{T^{16}}{\Delta^4} \frac{1}{T^{16} \Delta^2} \propto \frac{1}{\Delta^6} \qquad \text{for combined large} - \text{small angle collisions}$$

Variational solution of κ



- Speed of sound at $T = 0$ and the different phonon selfcouplings $\Rightarrow$ the EoS for neutron matter in neutron stars.
- Nucleonic EoS $\Rightarrow$ APR98 Akmal, Pandharipande and Ravenhall Phys. Rev. C 58, 1804 (1998)
- The final value of the number $N \Rightarrow$ imposed the deviation with respect to the previous order should be $\lesssim 10\%$.
- For $T \lesssim 10^9 K$, below T_c, $\kappa \Rightarrow$ almost independent of T, with subleading corrections $\sim T$ and T^2.

S. Sarkar et. al Phys. Rev. C 90, 055803 (2014)
M. Braby et. al Phys. Rev. C 81, 045205 (2010)

Phonon contribution to κ

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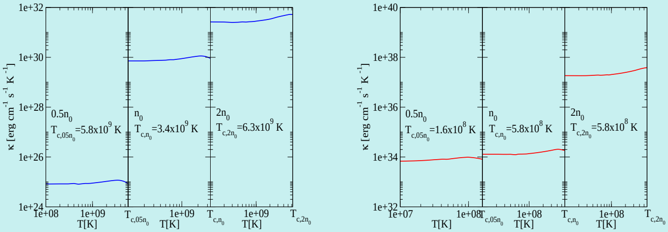

- For $N = 6 \Rightarrow$ a fit to our numerical results $\Rightarrow$
  $\kappa \sim (7.02 \times 10^{29} + 9.28 \times 10^{19}\ T + 9.08 \times 10^{10}\ T^2)\ \mathrm{erg\ cm^{-1}\ s^{-1}\ K^{-1}}$.
- For both the gaps $\Rightarrow$ the dominant processes to the phonon contribution to the thermal conductivity corresponds to the combined small and large angle collisions $\Rightarrow$ T independent behaviour of κ.
- The thermal conductivity grows with increasing density, with a non-linear dependence.

S. Sarkar et. al Phys. Rev. C 90, 055803 (2014)

Electromagnetic contributions to κ

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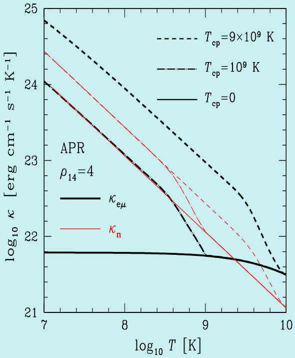

- In normal matter ($T_{cp} = 0$) κ_n dominates over $\kappa_{e\mu}$ at $T \leq 2 \times 10^9$ K.
- $T < T_{cp}$ proton superconductivity sets in $\Rightarrow$ $\kappa_{e\mu}$ starts to grow up much quicker than κ_n ($\kappa_{e\mu} \propto \Delta \propto T_{cp}$) and becomes comparable to or larger than κ_n.
- For a stronger superconductivity with $T_{cp} \gg 9 \times 10^9 K \Rightarrow \kappa_{e\mu}$ dominates over κ_n at any T.
- $10^{25} \lesssim \kappa_{ph} \lesssim 10^{32}$ erg cm^{-1} s^{-1} K^{-1} from 0.5 n_0 to 2 n_0 $\Rightarrow$ Thermal conductivity in the neutron star core is dominated by phonon-phonon collisions.

P. S. Shternin et. al Phys. Rev. D 75, 103004 (2007)

Thermal conductivity mean free path of the phonons

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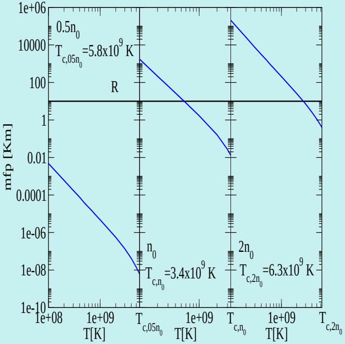
Thermal conductivity mean free path of the phonons $\Rightarrow l = \frac{\kappa}{\frac{1}{3} c_v c_s}$

heat capacity for phonons $\Rightarrow c_v = \frac{2\pi^2}{15 c_s^3} T^3$



- κ_{phn} is temperature independent, $c_v \propto T^3$. Temperature dependence of mfp $\Rightarrow l \propto 1/T^3$.
- Superfluid phonon mfp stays below the radius of the star
  - $n = 0.5 n_0$
  - $n = n_0,\ T \geq 6 \times 10^8 K$
  - $n = 2n_0,\ T \geq 3 \times 10^9 K$

S. Sarkar et. al Phys. Rev. C 90, 055803 (2014)

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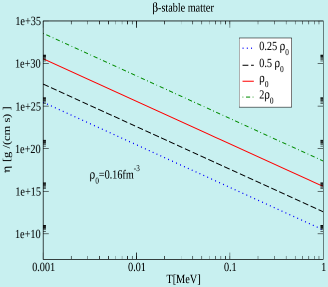
Shear Viscosity in Superfluid Neutron star

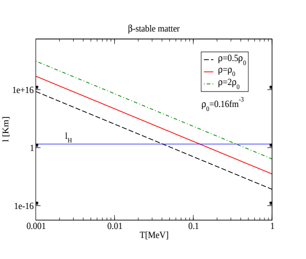
Shear Viscosity

- Shear Viscosity $\Rightarrow$

$$\eta = \frac{4c_s^2}{15T} \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{2E_p} f_{\rm eq}(1 + f_{\rm eq}) h(p)$$

- $\delta f = -h(p) p_{kl} V_{kl} \frac{f_{\rm eq}(1+f_{\rm eq})}{T} \Rightarrow p_{kl} = p_k p_l - 1/3 \delta_{kl} p^2$





- $\eta \propto 1/T^5$ for all four densities
- $T < 0.1$ MeV, the phonon mean free path is bigger than the size of the star.

C. Manuel et. al Phys. Rev. D 84, 123007 (2011)

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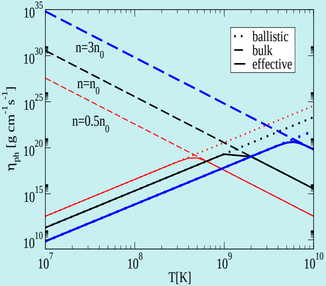

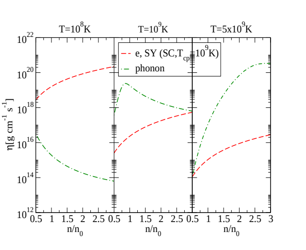

C. Manuel et. al Phys. Rev. D 88, 043001 (2013)

- $T < 0.1$ MeV $\rightarrow$ phonon mean free path is bigger than the size of the star.
- At low temperature phonons behave ballistically.
- Effective shear viscosity $\eta_{\rm eff} = \left(\eta_{\rm bulk}^{-1} + \eta_{\rm ball}^{-1}\right)^{-1}$
- $T < 10^8\,\mathrm{K}$ shear viscosity is described in terms of the ballistic contribution $\Rightarrow$ viscosity scales as $1/c_s^4$ with density
- T increases, the transition between the ballistic and hydrodynamic domains takes place.
- $T \sim 10^9\,\mathrm{K}$ and $n \lesssim n_0$, shear viscosity increases rapidly as function of the density following a hydrodynamical behavior. This is followed by a decrease with density as the ballistic description takes over

Color superconducting neutron star core

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- Quark matter in the core of a young neutron star (a few seconds old) below temperature $\sim 10 - 50$ MeV is in a color superconducting phase.
- At asymptotically high density the favored phase $\Rightarrow$ CFL phase.
- The nature of quark pairing at lower densities is uncertain.
- Two-flavor color-superconducting (2SC) phase $\rightarrow$ up and down quarks pair in a color antitriplet state, one of the colors remain unpaired
- Five quarks are left unpaired (the blue u and d quarks, and all three colors of the strange quark).
- The excitations that transport momentum and energy in 2SC superconductors are ungapped fermions
- Electron longer relaxation times than the ungapped quarks $\Rightarrow$ contribution dominates at high temperature.

M. Alford et. al Phys. Rev. C 90, 055205 (2014)

Crystalline Color superconducting phase

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Summary

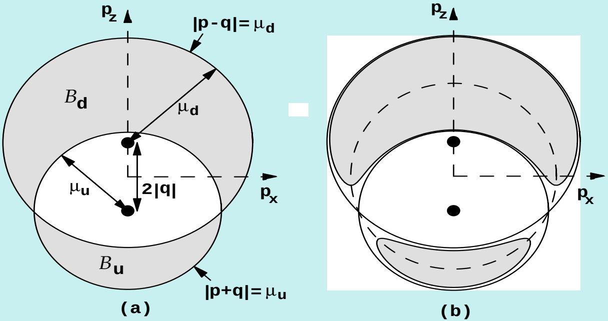
- At extremely high density and low temperature ⇒ BCS theory ⇒ different flavors have equal Fermi surfaces ⇒ condensate is dominated by pairs in the vicinity of Fermi surface.
- If the Fermi momenta are different ⇒ pairing with non-zero momentum is possible ⇒ crystalline color superconducting phase ⇒ *LOFF*.
- Mathematically,

$$p_F^{u,d} = \mu_{u,d}, \quad p_F^s = \sqrt{\mu_s^2 - m_s^2}, \quad p_F^e = \mu_e$$

$$\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$$

- The magnitude of the splitting between Fermi surfaces is $\delta p_F = m_s^2/4\mu$.
- Each Cooper pair in the condensate carries the same total momentum ⇒ $\langle\psi\psi\rangle \sim \Delta e^{2ibx}$

LOFF phase-space



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Shear Viscosity : LOFF phase

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- Pairing between $ur - dg$, $dg - ur$ quarks
- Isotropic system $\Rightarrow$ Shear stress tensor

$$\sigma^{\alpha\beta} = \eta\frac{1}{2}\left(\delta^{\alpha\gamma}\delta^{\beta\lambda} + \delta^{\alpha\lambda}\delta^{\beta\gamma} - 2/3\delta^{\alpha\beta}\delta^{\gamma\lambda}\right)V_{\gamma\lambda}$$

- Anisotropic system $\Rightarrow$ Shear stress tensor

$$\sigma^{\alpha\beta} = \eta\frac{3}{2}(b^{\alpha}b^{\beta} - \frac{1}{3}\delta^{\alpha\beta})(b^{\gamma}b^{\lambda} - \frac{1}{3}\delta^{\gamma\lambda})V_{\gamma\lambda}$$

- Relaxation time approximation $\Rightarrow$

$$\frac{\partial f}{\partial t} + F.\frac{\partial f}{\partial p} + v.\frac{\partial f}{\partial x} = \sum_n \frac{\delta f^{(n)}}{3\tau^{(n)}} .$$

$$\delta f^{(n)} \quad \propto \quad \tau^{(n)}\frac{\partial f_0}{\partial \epsilon}\Pi^{(n)\alpha\beta\gamma\lambda}(p_{\gamma}v_{\lambda} + p_{\lambda}v_{\gamma} - \frac{2}{3}\vec{p}\cdot\vec{v}).\Pi^{(n)}_{\alpha\beta\mu\nu}V^{\mu\nu}$$

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Anisotropic Shear Viscosity Coefficient

$$\eta^{(n)} \propto (\gamma^{(n)}\nu) \int \frac{d^3p}{(2\pi)^3} p_\alpha v_\beta \Pi^{(n)\alpha\beta\gamma\lambda}(p_\gamma v_\lambda + p_\lambda v_\gamma - \frac{2}{3}\vec{p}\cdot\vec{v})\tau^{(n)}$$

- Gapless fermions dominate $\Rightarrow E_1 = -\delta\mu + bv_F \cos\theta + \sqrt{\xi^2 + \Delta^2} \Rightarrow \delta\mu(\theta) > 0,\ \Delta > \delta\mu(\theta),\ E_1 < 0.$
- For gapless fermions

$$LHS_1 \quad \propto \quad \frac{\mu^4}{\gamma^{(n)}} \int_{\frac{\Delta-\delta\mu}{bv_F}}^{+1} d\cos\theta v_0(\theta) \left(\cos\theta^2 - \frac{1}{3}\right)^2$$

where,

$$v_0(\theta) = \left|\frac{\sqrt{(\delta\mu + bv_F\cos\theta)^2 - \Delta^2}}{\delta\mu + bv_F\cos\theta}\right|$$

continued ⋯

$$RHS \propto \frac{1}{T}\sum_j \mu_i^2 \mu_j^2 \int_{\xi_p,\xi_k,\cos\theta_p,\cos\theta_k,\phi_k,q_z,\omega} |\mathcal{M}|^2 \frac{1}{J_q}[f_1 f_2(1-f_3)(1-f_4)]$$
$$3\phi^1 \cdot \left[\left(\psi^{(0)1} - \psi^{(0)3}\right)\tau^{(0)1} + \left(\psi^{(0)2} - \psi^{(0)4}\right)\tau^{(0)2}\right]$$

- $\mu > \delta\mu, b, \Delta >> T$.
- LHS $\propto \mu^4 f_1(\delta\mu, bv_f, \Delta)$
- Shear viscosity $\eta \sim np\tau,\ n \sim \mu^3, p \sim \mu,\ \tau \sim \mu T^{-2} h(\delta\mu, bv_f, \Delta)$

$$\eta \propto \mu^5 T^{-2} g_1(\delta\mu, bv_f, \Delta)$$

- Unpaired quark matter $\eta \propto \mu^5 T^{-2}$
- LOFF phase shear viscosity ⇒ larger than unpaired quark matter.

Phonon contribution to η

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Summary

- Condensate spontaneously breaks rotational invariance
- Spontaneous breaking of the invariance $\Rightarrow$ single goldstone mode $\Rightarrow$ plane-wave structure of the crystalline condensate
- Quark-phonon scattering $\Rightarrow$ low temperature highly dense system $\Rightarrow$ $p_0 \sim T$, $p \sim \mu$, $m_D \sim g\mu \Rightarrow \eta \sim T^2\mu$.

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- Transport coefficients $\Rightarrow$ neutron star dynamics.
- The cooling of a neutron star depends on the rate of neutrino emission and the thermal conductivity.
- In rotating star modes of non-radial pulsations grow.
- Damping mechanisms like viscous processes counteract the growth of an unstable r-mode.
- Transport coefficients $\Rightarrow$ solving the Boltzmann equation.
- κ vanishes $\Rightarrow$ linear phonon dispersion law. We calculate first correction in dispersion relation which depends on the gap of neutron matter.
- Phonon dispersion law curves downward beyond linear order $\Rightarrow$ collisional processes of $1 \rightarrow 2$ kinematically forbidden.

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Exploring different phases

Non-equilibrium properties of plasma

Transport coefficients in superfluid neutron star

Transport coefficients in Color super-conducting neutron star core

Summary

- $\kappa \propto 1/\Delta^6$ the factor of proportionality depends on the density and EoS of the superfluid.
- Thermal conductivity in the neutron star core is dominated by phonon-phonon collisions when phonons are in a pure hydrodynamical regime.
- Different Fermi momenta pair with non-zero momentum $\Rightarrow$ crystalline color superconducting phase $\Rightarrow$ *LOFF*.
- Gapless fermions dominate shear viscosity in LOFF phase.
- Shear viscosity in LOFF more than the unpaired quark matter.

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THANK YOU

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- Phonon velocity $\Rightarrow$

$$v_{\rm ph} = \sqrt{\frac{\partial P}{\partial \tilde{\rho}}} \equiv c_s$$

  - $\tilde{\rho} \Rightarrow$ mass density, related to the particle density $(\rho) \Rightarrow \tilde{\rho} = m\rho$.

- Three phonon self-coupling constants $\Rightarrow$

$$g = \frac{1}{6\sqrt{m\rho}\ c_s}\left(1 - 2\frac{\rho}{c_s}\frac{\partial c_s}{\partial \rho}\right) , \qquad \eta_g = \frac{c_s}{6\sqrt{m\rho}\ g}$$

- Four phonon coupling constants

$$\lambda = \frac{1}{24\ m\rho\ c_s^2}\left(1 - 8\frac{\rho}{c_s}\frac{\partial c_s}{\partial \rho} + 10\frac{\rho^2}{c_s^2}\left(\frac{\partial c_s}{\partial \rho}\right)^2 - 2\frac{\rho^2}{c_s}\frac{\partial^2 c_s}{\partial \rho^2}\right) ,$$

$$\eta_{\lambda_2} = \frac{c_s^2}{8\ m\rho\ \lambda} , \qquad \eta_{\lambda,1} = 2\frac{\eta_{\lambda,2}}{\eta_g}$$

EoS for superfluid neutron star matter

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- Speed of sound at $T = 0$ and the different phonon selfcouplings $\Rightarrow$ the EoS for neutron matter in neutron stars.
- A common benchmark for nucleonic EoS is APR98
  Akmal, Pandharipande and Ravenhall Phys. Rev. C 58, 1804 (1998)
- Later parametrized $\Rightarrow$ H. Heiselberg, M. Hjorth-Jensen, Phys. Rep. 328, 237-327 (2000)

$$
\begin{aligned}
E/A &= \mathcal{E}_0 u \frac{u - 2 - \delta}{1 + \delta u} + S_0 u^{\gamma} (1 - 2x_p)^2 \\
u &= \rho/\rho_0 \qquad \mathcal{E}_0 = 15.8 \ \mathrm{MeV} \\
x_p &= \rho_p/\rho_0 \qquad S_0 = 32 \ \mathrm{MeV} \\
\delta &= 0.2 \qquad \gamma = 0.6 \\
\rho_0 &= 0.16 \ \mathrm{fm}^{-3}
\end{aligned}
$$

- For stable matter made up of neutrons, protons and electrons c_s at $T = 0$ is

$$
c_s(\rho, x_p) \approx \sqrt{\frac{1}{m} \frac{\partial P_N(\rho, x_p)}{\partial \rho_n}}
$$

Gap parameter

- $^1S_0$ and averaged $^3P_2$ neutron gaps
- Energy gap (Fermi surface) by the phenomenological formula

$$\Delta(k_F) = \Delta_0 \frac{(k_F - k_1)^2}{(k_F - k_1)^2 + k_2} \frac{(k_F - k_3)^2}{(k_F - k_3)^2 + k_4}$$

$$\underline{^1S_0(A) + ^3P_2(i),\ ^1S_0(c) + ^3P_2(k)}$$

model	Δ_0 (Mev)	k_1 (fm^{-1})	k_2 (fm^{-1})	k_3 (fm^{-1})	k_4 (fm^{-1})
A	9.3	0.02	0.6	1.55	0.32
c	22	0.3	0.09	1.05	4
i	10.2	1.09	3	3.45	2.5
k	0.425	1.1	0.5	2.7	0.5

Table : N. Andersson *et. al*, Nucl. Phys. A763, 212-229 (2005).

Model A is for the bare interaction and is relevant in a pure neutron (proton) medium

c is for the $^1S_0$ neutron pairing, i, k are for the $^3P_2$ neutron channel

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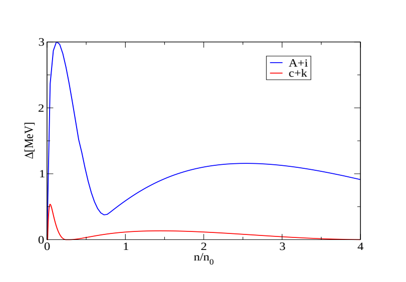

$^1S_0(A)+^3P_2(i)$, $^1S_0(c)+^3P_2(k)$

- $^1S_0(A) \Rightarrow$ maximum gap of about 3 MeV at $p_F \approx 0.85\mathrm{fm}^{-1}$
- $^3P_2(i)$ neutron angular averaged $\Rightarrow$ maximum value for the gap of approximately 1 MeV.
- $^1S_0(c) \Rightarrow$ corrections to the bare nucleon-nucleon potential.
- $^3P_2(k)$ parametrization assuming weak neutron superfluidity in the core with maximum value for the gap of the order of 0.1 MeV.

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Validity of result

- Close to T_c higher order corrections in the energy and momentum expansion should be taken into account in both the phonon dispersion law and self-interactions.
- Density of superfluid phonons becomes very dilute at very low $T \Rightarrow$ difficult to maintain a hydrodynamical description of their behavior.
- Phonons would behave in the low-T regime ballistically.
- Thermal conductivity due to phonons would be then dominated by the collisions of the phonons with the boundary $\Rightarrow \kappa = \frac{1}{3} c_v c_s R$

S. Sarkar et. al Phys. Rev. C 90, 055803 (2014)