Thermal Conductivity and Impurity Scattering in the Accreting Neutron Star's Crust

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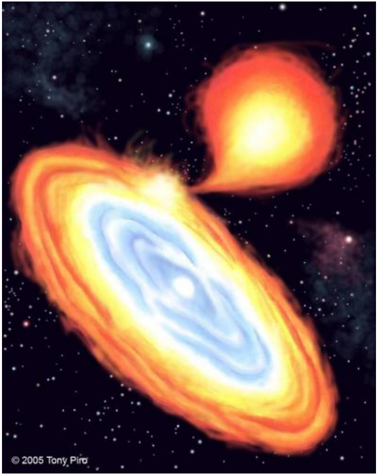
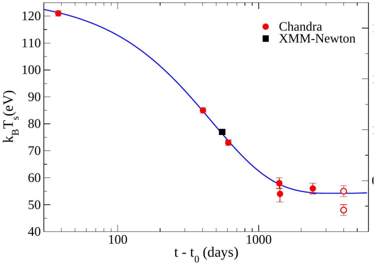
X-Ray Binaries and Neutron Star cooling



- unstable burning of accreted material produces X-Ray bursts
- thermal relaxation very sensitive to properties of the crust

$$\Delta t \approx \frac{C_V}{\kappa} (\Delta R)^2$$



Cackett et al. (2008) & (2013) [open symbols]

Thermal conductivity of electrons I

outer crust conditions: $\rho \approx 10^8 - 10^{11} \mathrm{g/cm^3}$ and $T \approx 10^7 - 10^9 K$

- electrons: relativistic, weakly coupled, degenerate
- nuclei: pressure ionized, large Z $(25 - 40)$

Strength of interactions: $\Gamma = \frac{Z^2 e^2}{aT}$

- $\Gamma \lessapprox 175$: liquid
- $\Gamma \gtrapprox 175$: crystalline solid

Electron thermal conductivity in the crust is limited by collisions

- electron–electron $[\nu_{ee}]$
- electron–ion $[\nu_{ei}]$

Electron-Ion scattering dominates rate:

- ions have much larger Z
- electrons are degenerate

$$\Longrightarrow \quad \nu_{ei} \gg \nu_{ee}$$

Thermal conductivity of electrons II

$$\frac{1}{\kappa} \propto \nu_{ei} \propto \int_0^{2p_F} dq q^3 |V(q)|^2 \int_{-\infty}^{\infty} d\omega S(q,\omega) w_k\left(\frac{\omega}{T}, q\right)$$

- scattering matrix element: $|V(q)|^2 = \frac{e^4}{\left(q^2+k_{TF}^2\right)^2}\left(1-\frac{q^2}{4p_F^2}\right)$

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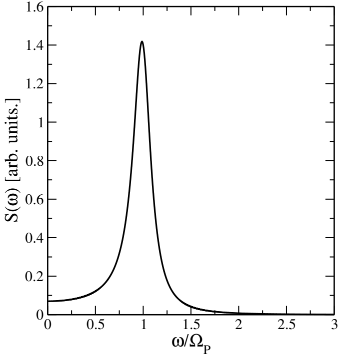

$S(q,\omega)$: dynamic structure factor

$$S(q,\omega) \approx \sum_n \left|\langle 0|\rho_q|n\rangle\right|^2 \delta(\omega - E_{n0})$$

- characteristic frequency: $\Omega_P^2 = \frac{4\pi\alpha Z^2}{M}\eta$

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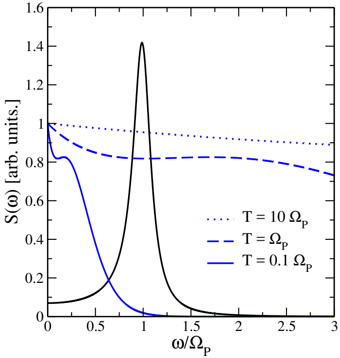

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$w_k\left(\frac{\omega}{T}, q\right)$ energy window: $w_k \xrightarrow{T \geq \Omega_P} 1$

Flowers & Itoh (1976), Baiko et al. (1998), Potekhin et al. (1999), Abbar et al. (2015)

Accreting Neutron Star Crust

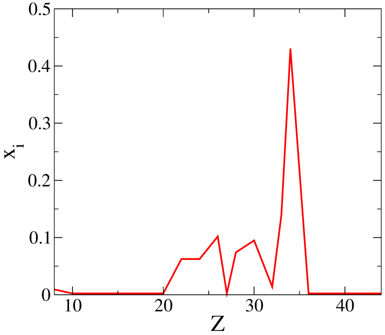
The composition of the crust at a given depth is a mixture of species!

impurity–parameter: $Q_{imp} = (\Delta Z)^2 = \langle Z^2 \rangle - \langle Z \rangle^2$

- rp-process ashes: $Q_{imp} \approx 100$ [Schatz et. al (2001), Gupta et al. (2007)]

Ashes purification

- electron capture [Gupta et al. (2008)]
- phase separation [Horowitz et al. (2007)]



from Horowitz et al. (2009)

- final composition expected in the solid: $Q_{imp} \approx 20 - 30$

Multi Component Plasmas (MCP): simple treatment

Itoh & Kohyama (1993)

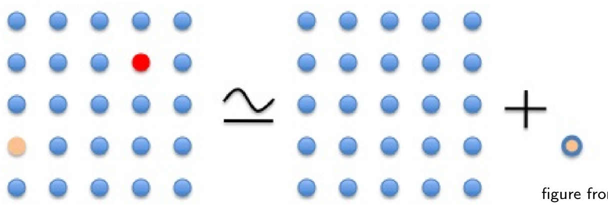


figure from Daligault & Gupta (2009)

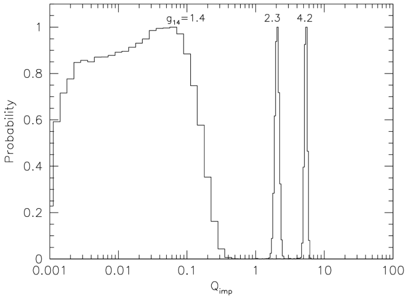
$$\nu_{ei}^{MCP} \simeq \nu_{ei}^{OCP} + \nu_{imp}$$
$$S(q,\omega)_{MCP} \simeq S(q,\omega)_{OCP} + S(q,\omega)_{imp}$$

- the ν_{ei}^{OCP} contribution can be safely computed
- the ν_{imp} can be estimated assuming:
  - elastic scattering from impurities: $S(q,\omega)_{imp} \approx S_{imp}(q)$
  - negligible correlations: $S_{imp}(q) \approx \langle (Z_k - \langle Z \rangle)^2 \rangle \equiv Q_{imp}$

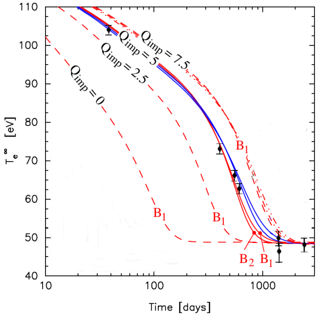
Hints from observations

Shternin et al. (2007),Turlione et al. (2015)

Brown & Cumming (2009)



Page&Reddy (2012)



Strong tension between Q_{imp} extracted from observations $(2 - 5)$ and microscopic predictions based on buried rp–process ashes $(20 - 30)$

- missing reactions?
- treatment of impurity scattering too simple?

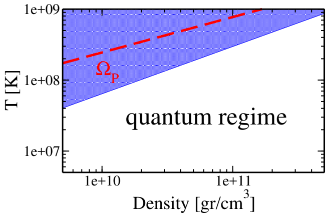
Multi Component Plasmas (MCP): full treatment

Ideally given a composition we would like to compute from first principles the full charge–charge response function of a finite system with N ions

$$S(q,\omega) \equiv FT\left[\langle \rho^\dagger(t,q)\rho(0,q)\rangle_\beta\right]$$

with

- charge density $\rho(t,\vec{q}) \equiv \frac{1}{\sqrt{N}}\sum_{i=1}^N Z_i e^{i\vec{q}\cdot\vec{r}_i(t)}$
- thermal expectation value $\langle \ldots \rangle_\beta$:
  - $T \geq T_c \rightarrow$ classical behaviour
  - $T < T_c \rightarrow$ quantum regime



- Impurity scattering important at low temperatures: $T \ll \Omega_P$
- Need quantum mechanical treatment

Path Integral Monte Carlo

$$\langle O \rangle_\beta \equiv \frac{Tr\left[\hat{O}e^{-\beta\hat{H}}\right]}{Tr\left[e^{-\beta\hat{H}}\right]} = \frac{1}{Z_\beta}\int d\mathbf{R}O(\mathbf{R})\langle\mathbf{R}|e^{-\beta\hat{H}}|\mathbf{R}\rangle$$

$$\approx \prod_{m=0}^{M}\int d\mathbf{R}d\mathbf{R}_1\dots d\mathbf{R}_M O(\mathbf{R})e^{-\frac{\beta}{M}V(\mathbf{R}_m)}e^{-M\frac{(\mathbf{R}_m-\mathbf{R}_{m+1})^2}{4\beta\lambda}} + O\left(\frac{\beta}{M}\right)$$

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- "simple" $3NM$–dimensional integral!! (usually $N \approx 10^3$ and $M \approx 10^2$) $\Longrightarrow$ use Monte Carlo integration [D. Ceperley, Rev.Mod.Phys (1995)]

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PROBLEM: difficult for time–dependent observables like $S(q,\omega)$

But we can acces the next best thing: the Laplace transform of $S(q,\omega)$

$$F(q,\tau) \equiv \int d\omega e^{-\tau\omega}S(q,\omega) \equiv \frac{1}{Z_\beta}Tr\left[\rho_{\mathbf{q}}e^{-\tau\hat{H}}\rho^\dagger_{\mathbf{q}}e^{-(\beta-\tau)\hat{H}}\right]$$

Euclidean correlation function

$$F(q,\tau) = \int d\omega e^{-\tau\omega} S(q,\omega) \approx \int d\omega e^{-\tau\omega} \sum_n |\langle 0|\rho_q|n\rangle|^2 \delta(\omega - E_{n0})$$

$$\approx \sum_n e^{-\tau E_{n0}} |\langle 0|\rho_q|n\rangle|^2 \xrightarrow{\tau \gg 1} |\langle 0|\rho_q|n_{min}\rangle|^2 e^{-\tau E_{min}}$$

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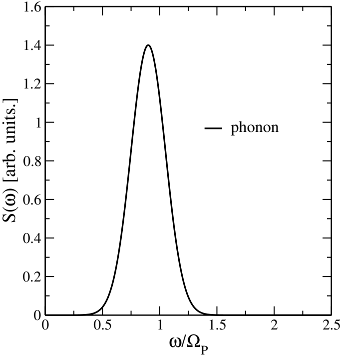
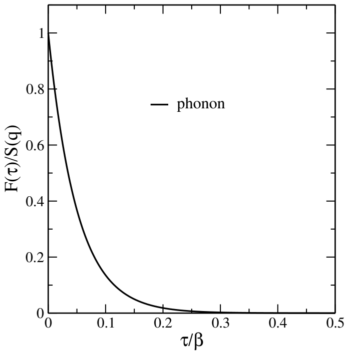


Laplace Transform →

← *Inverse Transform* / *Bayesian Methods*



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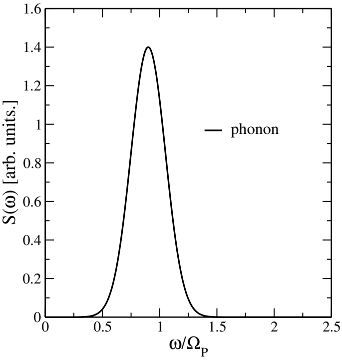
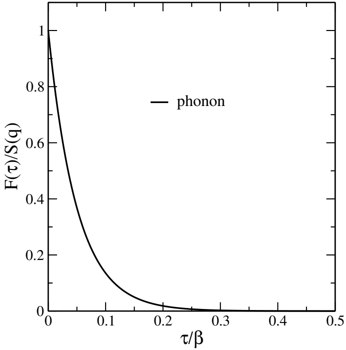


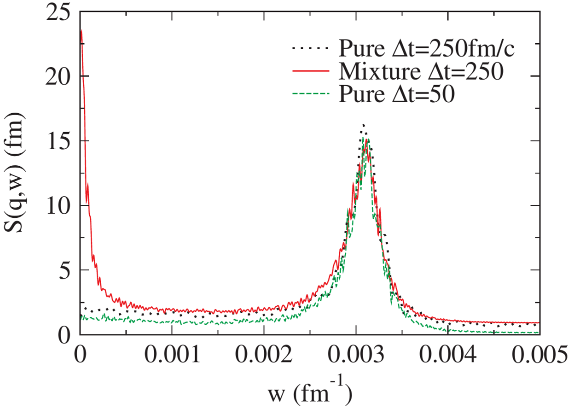
Laplace Transform ⟶

⟵ ~~*Inverse Transform*~~ / ~~*Bayesian Methods*~~



Previous results for Multi Component Plasma: MD

Caballero,Horowitz,Berry (2006)



Clean separation of scales: $S(q,\omega) \simeq S(q,\omega)_{OCP} + S_{imp}(q)$

At low temperature when $E_{min} \ll T \ll \Omega_P$: $F(q,\tau) \to e^{-\tau E_{min}} \approx$ const.

Euclidean correlation function with impurities

$$F(q,\tau) = \int d\omega e^{-\tau\omega} S(q,\omega) \approx \int d\omega e^{-\tau\omega} \sum_n |\langle 0|\rho_q|n\rangle|^2 \delta(\omega - E_{n0})$$

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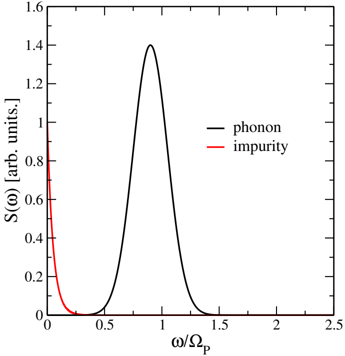
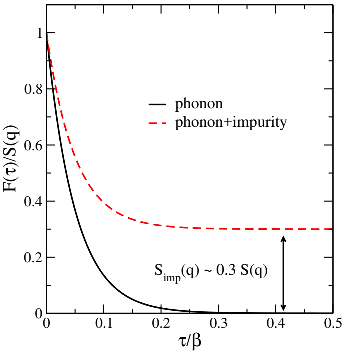
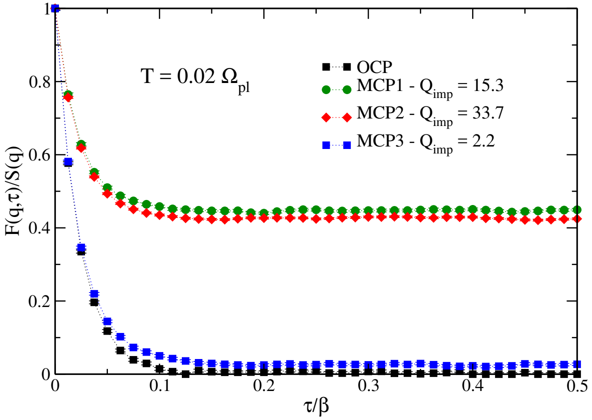




Results for Multi Component Plasmas

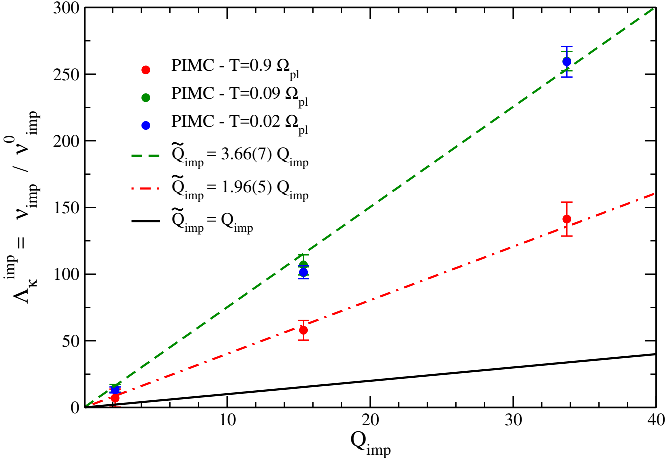
Roggero & Reddy, PRC (2016)



For low T a fit to the plateau provides a very good contraint on $S_{imp}(q)$.

Electron impurity scattering rate

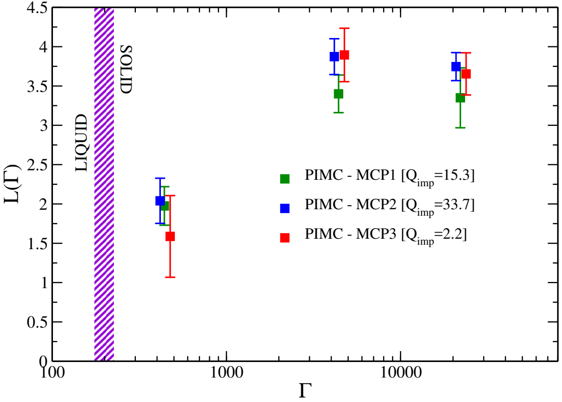
Roggero & Reddy, PRC (2016)



Collision frequency is still approximately linear!
Though with a different effective impurity parameter $\widetilde{Q}_{imp} \approx 2 - 4Q_{imp}$

Effective impurity parameter

$$\widetilde{Q}_{imp}(\Gamma) \equiv L(\Gamma) Q_{imp} \quad \text{with} \quad \Gamma \equiv \frac{\langle Z^2 \rangle e^2}{aT}$$



Summary

- Impurity scattering plays fundamental role in thermal relaxation of accreting neutron stars in X–ray binary systems
- Tension present between observations and theoretical predictions
- Need better theory for the density response in the crust

- PIMC helpful in determining the low–energy part of $S(q, \omega)$ $\implies$ reliable extraction of impurity contribution
- New results suggest stronger tension (Q_{imp} lower then expected)
  - need to perform calculations close to neutron–drip to make sure

Future work:

- compare with available MD results at higher T
- extend calculations into the inner–crust [$\rho \approx 10^{12} - 10^{13} \mathrm{g/cm^3}$]

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Thanks for your attention

Thermal density matrix

N–particles interacting trough a two–body Hamiltonian:

$$\hat{H} \equiv \hat{K} + \hat{V} = -\lambda \sum_{i}^{N} \nabla_i^2 + \sum_{i<j}^{N} V(\vec{r}_i - \vec{r}_j)$$

At finite–temperatures all properties of a many–body system are contained in the thermal density matrix: $\hat{\rho}(\beta) \equiv e^{-\beta \hat{H}}$.

- partition function: $Z_\beta \equiv Tr\left[e^{-\beta\hat{H}}\right]$
- expectation values:

$$\langle O \rangle_\beta = \frac{Tr\left[\hat{O} e^{-\beta\hat{H}}\right]}{Tr\left[e^{-\beta\hat{H}}\right]} = \frac{1}{Z_\beta} \iint d\mathbf{R} d\mathbf{R}' O(\mathbf{R},\mathbf{R}') \langle \mathbf{R}' | e^{-\beta\hat{H}} | \mathbf{R} \rangle \quad \mathbf{R} \in \mathbb{R}^{3N}$$

PROBLEM:

The matrix elements $\langle \mathbf{R}' | e^{-\beta\hat{H}} | \mathbf{R} \rangle$ are not known for non–trivial $\hat{H}$!

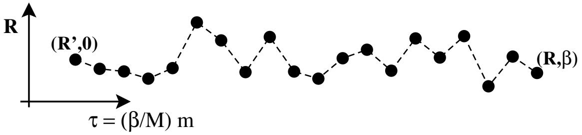
Path Integral representation

However we do know how to evaluate matrix elements of the form

$$\langle \mathbf{R}' | e^{-\beta \hat{K}} e^{-\beta \hat{V}} | \mathbf{R} \rangle \equiv \langle \mathbf{R}' | e^{-\beta \hat{K}} | \mathbf{R} \rangle e^{-\beta V(R)} \propto e^{-\frac{(\mathbf{R}'-\mathbf{R})^2}{4\beta\lambda}} e^{-\beta V(R)}$$

- Trotter–Suzuki expansion: $e^{-\beta\left(\hat{K}+\hat{V}\right)} = \lim_{M\to\infty} \left(e^{-\frac{\beta}{M}\hat{K}} e^{-\frac{\beta}{M}\hat{V}} \right)^M$

$$\langle \mathbf{R}' | e^{-\beta \hat{H}} | \mathbf{R} \rangle \approx \prod_{m=0}^{M} \int d\mathbf{R}_{1\dots M} e^{-\frac{\beta}{M} V(\mathbf{R}_m)} e^{-M \frac{(\mathbf{R}_m - \mathbf{R}_{m+1})^2}{4\beta\lambda}} \quad \text{with} \quad \begin{cases} \mathbf{R}_0 \equiv \mathbf{R}' \\ \mathbf{R}_{M+1} \equiv \mathbf{R} \end{cases}$$



matrix–element recovered after summing all paths

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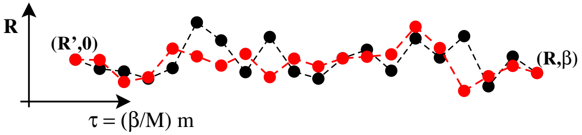

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