Thermal and transport properties of neutron star matter.

Sanjay Reddy

- Introduction
- Specific heat of neutron stars
- Crust: Properties of solid and superfluid matter
- Core: Low energy modes and transport properties
- Summary

Relevant Papers: arXiv:1602.01831 arXiv:1409.7750 arXiv:1307.4455 arXiv:1210.5169 arXiv:1201.5602 arXiv:1102.5379 arXiv:1009.2303

INSTITUTE for NUCLEAR THEORY

Collaborators: Paulo Bedaque **Bridget Bertoni *** Vincenzo Cirigliano Andrew Cumming Nicolas Chamel Dany Page Chris Pethick **Alessandro Roggero Ermal Rrapaj *** Rishi Sharma

Phases of Dense Matter in Neutron Stars

(Dense) Material Characterization

• Fermions: Particle-hole excitations

$$
C_{F,i} = \pi^2 \ n_i \frac{E_{F,i} T}{p_{F,i}^2} \simeq 2 \times 10^{19} \ \left(\frac{E_{F,i}}{m_n}\right) \left(\frac{p_{F,i}}{400 \text{ MeV}}\right) \left(\frac{T}{10^8 \text{ K}}\right) \ \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Bosons: Collective excitations

$$
C_{B,i} = \frac{2\pi^2 c^3}{15 v_{B,i}^3} T^3 \simeq 1.5 \times 10^{10} \left(\frac{c}{v_{B,i}}\right)^3 \left(\frac{T}{10^8 \text{ K}}\right)^3 \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

• Fermions: Particle-hole excitations

$$
C_{F,i} = \pi^2 \ n_i \frac{E_{F,i} T}{p_{F,i}^2} \simeq 2 \times 10^{19} \ \left(\frac{E_{F,i}}{m_n}\right) \left(\frac{p_{F,i}}{400 \text{ MeV}}\right) \left(\frac{T}{10^8 \text{ K}}\right) \ \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Bosons: Collective excitations

$$
C_{B,i} = \frac{2\pi^2 c^3}{15 v_{B,i}^3} T^3 \simeq 1.5 \times 10^{10} \left(\frac{c}{v_{B,i}}\right)^3 \left(\frac{T}{10^8 \text{ K}}\right)^3 \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Pairing suppresses the nucleon contribution.

• Fermions: Particle-hole excitations

$$
C_{F,i} = \pi^2 \ n_i \frac{E_{F,i} T}{p_{F,i}^2} \simeq 2 \times 10^{19} \ \left(\frac{E_{F,i}}{m_n}\right) \left(\frac{p_{F,i}}{400 \text{ MeV}}\right) \left(\frac{T}{10^8 \text{ K}}\right) \ \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Bosons: Collective excitations

$$
C_{B,i} = \frac{2\pi^2 c^3}{15 v_{B,i}^3} T^3 \simeq 1.5 \times 10^{10} \left(\frac{c}{v_{B,i}}\right)^3 \left(\frac{T}{10^8 \text{ K}}\right)^3 \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Pairing suppresses the nucleon contribution.

• Fermions: Particle-hole excitations

$$
C_{F,i} = \pi^2 \ n_i \frac{E_{F,i} T}{p_{F,i}^2} \simeq 2 \times 10^{19} \ \left(\frac{E_{F,i}}{m_n}\right) \left(\frac{p_{F,i}}{400 \text{ MeV}}\right) \left(\frac{T}{10^8 \text{ K}}\right) \ \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Bosons: Collective excitations

$$
C_{B,i} = \frac{2\pi^2 c^3}{15 v_{B,i}^3} T^3 \simeq 1.5 \times 10^{10} \left(\frac{c}{v_{B,i}}\right)^3 \left(\frac{T}{10^8 \text{ K}}\right)^3 \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Pairing suppresses the nucleon contribution.

• Core contribution is large and electrons set the lower limit.

• Fermions: Particle-hole excitations

$$
C_{F,i} = \pi^2 \ n_i \frac{E_{F,i} T}{p_{F,i}^2} \simeq 2 \times 10^{19} \ \left(\frac{E_{F,i}}{m_n}\right) \left(\frac{p_{F,i}}{400 \text{ MeV}}\right) \left(\frac{T}{10^8 \text{ K}}\right) \ \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Bosons: Collective excitations

$$
C_{B,i} = \frac{2\pi^2 c^3}{15 v_{B,i}^3} T^3 \simeq 1.5 \times 10^{10} \left(\frac{c}{v_{B,i}}\right)^3 \left(\frac{T}{10^8 \text{ K}}\right)^3 \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Pairing suppresses the nucleon contribution.

• Core contribution is large and electrons set the lower limit.

• Fermions: Particle-hole excitations

$$
C_{F,i} = \pi^2 \ n_i \frac{E_{F,i} T}{p_{F,i}^2} \simeq 2 \times 10^{19} \ \left(\frac{E_{F,i}}{m_n}\right) \left(\frac{p_{F,i}}{400 \text{ MeV}}\right) \left(\frac{T}{10^8 \text{ K}}\right) \ \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Bosons: Collective excitations

$$
C_{B,i} = \frac{2\pi^2 c^3}{15 v_{B,i}^3} T^3 \simeq 1.5 \times 10^{10} \left(\frac{c}{v_{B,i}}\right)^3 \left(\frac{T}{10^8 \text{ K}}\right)^3 \frac{\text{ergs}}{\text{cm}^3 \text{ K}}
$$

•Pairing suppresses the nucleon contribution.

- Core contribution is large and electrons set the lower limit.
- Collective modes play a role in the crust.

Specific Heat & Phase Transitions

Canonial expectation:

$$
C_{NS} \approx 10^{38} \, \left(\frac{T}{10^8 \, {\rm K}} \right) \, \frac{\rm ergs}{\rm K}
$$

Nucleon superfluidity can suppress C_{NS} by a factor of about 10-20.

Additional un-gapped fermions (quarks, hyperons) can enhance it by a factor of a few.

Exception: A large CFL fraction in the core can reduce C_{NS} by a factor of 100.

the core can **Specific heat of a 1.4 M**® neutron star at $T=10^8$ K. Figure courtesy of Dany Page

Specific Heat & Phase Transitions

Canonial expectation:

$$
C_{NS} \approx 10^{38} \, \left(\frac{T}{10^8 \, {\rm K}} \right) \, \frac{\rm ergs}{\rm K}
$$

Nucleon superfluidity can suppress C_{NS} by a factor of about 10-20.

Additional un-gapped fermions (quarks, hyperons) can enhance it by a factor of a few.

Exception: A large CFL fraction in the core can reduce C_{NS} by a factor of 100.

the core can **Specific heat of a 1.4 M**® neutron star at $T=10^8$ K. Figure courtesy of Dany Page

Measuring the Heat Capacity

Cumming et al. (2016 in prep.)

Heat the star, allow it to relax, and observe the change in temperature:

$$
C_{NS} \, dT = dQ
$$

When
$$
C_{NS} = \alpha T
$$
:
\n
$$
\frac{\alpha}{2} (T_f^2 - T_i^2) = \Delta Q
$$
\nLower limit:
\n
$$
C_{NS}(T_f) > 2 \frac{\Delta Q}{T_f}
$$
\n
$$
\Delta Q = \dot{H} \times t_H - L_{\nu} \times (t_H + t_{obs})
$$
\nheating
\n*neutrino*
\n*function*
\nrotheating
\n
$$
T_f
$$
\nneutrino
\n*conling rate*
\n*function*
\

Transiently Accreting NSs

SXRTs: High accretion followed by periods of quiescence

Envelope

Nuclear reactions release: \sim 1-2 MeV / nucleon

Deep crustal heating.

Brown, Bildsten Rutledge (1998) Sato (1974), Haensel & Zdunik (1990)

Warms up old neutron stars

Transiently Accreting NSs

SXRTs: High accretion followed by periods of quiescence

Sato (1974), Haensel & Zdunik (1990)

Warms up old neutron stars

Image credit: NASA/CXC/Wijnands et al.

Observations of KS 1731-260 trons and protons, the heat capacity is expected to be *C* ⇠ ever, if the nucleons are superfluid the nucleons are superfluid the nucleons are superfluid their contribution to the nucleon to the nucleon the superfluid the nucleon to the nucleon to the nucleon to the nucleon to the n $\bigcap_{n=1}^{\infty}$ if the high density matter $\bigcup\bigcup\bigcup\bigcup\{f|V\}$ $\frac{1}{2}$ $\frac{1}{2}$ are 1.4 **M** and 1. reaches 2.4 model with a model with a model with a helium is a model with a helium is a model with a helium is
The dotted curve is a model with a helium is a hel

Accretion Phase: 12 yrs at dM/dt ≈10¹⁷ g/s Thermal Relaxation: $t \approx 8$ yrs Quiescent Surface Temperature (post relaxation): T_{eff}=63 eV AULTER HIGH DENSITY MATTER SITS **uutscent surface ferrit** temperature *T* ¹ *c* 2^{17} a $\sqrt{2}$ Accretion Phase: 12 yrs at dM/dt \approx 10¹⁷ g/s reaches 2.4 r erg s1. The dotted curve is a model with a helium is a mod envelope as in [9], and has a core temperature of *T* ¹ $\frac{1}{2}$ ature (post relaxation): T_{eff}=63 eV $\sum_{i=1}^n a_i = a_i$ in outburst for over 12 years at an accretion rate of *^M*˙ ⇡ Thorpol Delovation t \blacksquare incinial neidvalion, limit **it should be a core.** Oujescent Surface Temp has one of the lowest measured temperatures. Following the dl_1/d + $\frac{1}{2}$ dl_2/d $($ aluit (post itianation). Tempo tv

Wijnands et al. (2002) **cackett et al. (2002)**
Cackett et al. (2010) (e.g. [25]). For an increase of the relation between the relation between the relation between the relation be
The relation between the relation between the relationship in the relation between the relationship in the rel laxed neutron star is *Tc*,⁸ = 1.288 (*T*⁴ face gravity *g* = (*GM*/*R*2)(1 + *z*) as *g*¹⁴ = *g*/10¹⁴ cm s2, and

has one of the lowest measured temperatures. Following the Elleigy Deposition. Energy Deposition: strain the crust heating and envelope composition (§II). We

$$
E_{\rm dep} = \dot{H} \times t_H = 6 \times 10^{43} \text{ ergs} \left(\frac{Q_{\rm nuc}}{2 \text{ MeV}}\right) \left(\frac{\dot{M}}{10^{17} \text{g/s}}\right) \left(\frac{t_H}{10 \text{ yrs}}\right)
$$

compare the result in the result of the lower limit on the heat capacity to the heat capacity to the heat capacity of theoretical models (§III). We define insulating envelope sustains a temperature gradient near Inferred Core Temperature: the surface) II. THE LIMIT ON CORE HEAT CAPACITY FROM heat capacity from observations of \mathcal{L}

$$
T_c^{\infty} = 7.0 \times 10^7 \text{ K} \left(\frac{T_{\text{eff}}^{\infty}}{63.1 \text{ eV}}\right)^{1.82} \text{ (Fe envelope)}
$$

$$
T_c^{\infty} = 3.1 \times 10^7 \text{ K} \left(\frac{T_{\text{eff}}^{\infty}}{63.1 \text{ eV}}\right)^{1.65} \text{ (He envelope)}.
$$

Limits: Current & Future

FIG. 8. Bounds on the product *CV* ⇥ *Tc* from observations of KS 1731-260 as function of the neutrino luminosity. For small *L*⌫ the Cumming et al. (2016 in prep.)

Limits: Current & Future

FIG. 8. Bounds on the product *CV* ⇥ *Tc* from observations of KS 1731-260 as function of the neutrino luminosity. For small *L*⌫ the in prep.) Cumming et al. (2016 in prep.)

Limits: Current & Future

FIG. 8. Bounds on the product *CV* ⇥ *Tc* from observations of KS 1731-260 as function of the neutrino luminosity. For small *L*⌫ the in prep.) Cumming et al. (2016 in prep.)

ergs **8**
 $\frac{2}{r}$
 $\frac{1\%}{1\%}$ Limits: Current & Future $C_{NS}(T_f) > 2$ ΔQ *Tf*

FIG. 8. Bounds on the product *CV* ⇥ *Tc* from observations of KS 1731-260 as function of the neutrino luminosity. For small *L*⌫ the in prep.) Cumming et al. (2016 in prep.)

Crust Cooling

Watching NSs immediately after accretion ceases !

Crust

Envelope

Core Neutrino **Cooling**

The heated crust relaxes as heat is transported to the core.

Shternin & Yakovlev (2007) Cumming & Brown (2009)

Cackett, et al. (2006)

Accretion Induced Heating

Temperature profile depends on:

- •accretion rate and duration.
- •location of heat sources.
- •thermal conductivity
- •specific heat.
- •core temperature

Accretion Induced Heating

Temperature profile depends on: JH.

- •accretion rate and duration.
- •location of heat sources.
- •thermal conductivity
- · specific heat.
- •core temperature

Observations:

All known Quasi-persistent sources with post outburst cooling

•After a period of intense accretion the neutron star surface cools on a time scale of years.

•This relaxation was first discovered in 2001 and 6 sources have been studied to date.

•Expected rate of detecting new sources \sim 1/year.

Figure from Rudy Wijnands (2013)

Thermal Conductivity

Thermal Conductivity

Cooper Pairing

 $H = \sum$ $k, s = \uparrow, \downarrow$ $\left(\frac{k^2}{2}\right)$ $\frac{n}{2m} - \mu$) a_k^{\dagger} $\int_{k,s}^{\dagger}a_{k,s}+g$ $k,p,q,s{=}\uparrow,\downarrow$ $a_{k+q,s}^{\intercal}a_{p-q,s}^{\intercal}a_{k,s}a_{p,s}$ Attractive interactions destabilize the Fermi surface:

$$
\Delta = g \langle a_{k,\uparrow} a_{p,\downarrow} \rangle \quad \Delta^* = g \langle a_{k,\uparrow}^{\dagger} a_{p,\downarrow}^{\dagger} \rangle
$$

Cooper pairs leads to superfluidity

Energy gap for fermions:

$$
E(p) = \sqrt{(\frac{p^2}{2M} - \mu)^2 + \Delta^2}
$$

New collective mode: Superfluid Phonon

 $\omega(k) = v_s k$

Microscopic Structure of the Crust

Baym Pethick & Sutherland (1971) Negele & Vautherin (1973)

Microscopic Structure of the Crust

Baym Pethick & Sutherland (1971) Negele & Vautherin (1973)

Microscopic Structure of the Crust

Baym Pethick & Sutherland (1971) Negele & Vautherin (1973)

Electrons are (nearly) free

• Electrons are dense,degenerate and relativistic.

 $n_e = Z n_I$ $k_{\text{Fe}} \approx E_{\text{Fe}} \simeq 25 - 75 \text{ MeV} \gg m_e$

•Band gaps are small and restricted to small patches in the Fermi surface.

$$
\frac{V_{\rm e-i}}{E_{\rm Fe}} \simeq \alpha_{\rm em} Z^{2/3} \ll 1 \qquad \frac{\delta_{\rm e}}{E_{\rm F}}
$$

$$
\frac{\delta_{\rm e}}{E_{\rm Fe}}\simeq\frac{4\alpha_{\rm em}}{3\pi}\approx10^{-3}
$$

• Pairing energy is negligible.

\n
$$
T_c \simeq \omega_p^{\text{ion}} \exp\left(-\frac{v_{Fe}}{\alpha_{\text{em}}}\right) \approx 0
$$

Relevant Temperature Scales in the Crust

Relevant Temperature Scales in the Crust

Relevant Temperature Scales in the Crust

Separation of Scales

emperature Temperature

Low Energy Theory of Phonons

Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the fluctuation of the phase of the condensate.

 $\langle \psi_{\uparrow}(r, t) \psi \downarrow (r, t) \rangle = |\Delta(r, t)| \exp(2i\phi(r, t))$

Collective coordinates: Vector Field: $\xi_i(r,t)$ Scalar Field: $\phi(r,t)$

Low Energy Theory of Phonons

Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the fluctuation of the phase of the condensate.

 $\langle \psi_{\uparrow}(r, t) \psi \downarrow (r, t) \rangle = |\Delta(r, t)| \exp(2i\phi(r, t))$

Collective coordinates: Vector Field: $\xi_i(r,t)$ Scalar Field: $\phi(r,t)$

Symmetries & Derivative Expansion mmatrias ⁰ Darivativa Evnoncian The low of the low described in the field is described in the field of the field in the symmetries associated i condensate. Because of interactions, such as those between the neutrons and the protons in the neutron symmetries & Derivative Expansion $\bigcap_{x \in \mathcal{X}} \mathbb{R}^n$. Nonetheless in experiments $\bigcap_{x \in \mathcal{X}} \mathbb{R}^n$. Nonetheless in a low energy dynamics in all these cases the low energy dynamics in a low energy dynamics in a low energy dynamics in a low energ oyninielies a benvalive Expansion $\bigcap_{x \in \mathcal{X}} \mathbb{R}^n$. Nonetheless in experiments $\bigcap_{x \in \mathcal{X}} \mathbb{R}^n$ oyninetries & Denvauve Expansion $t_{\rm eff}$ associated with the superfluid mode σ is related to the space-time dependent phase of the space-

 $\mathcal{L}_{\mathcal{A}}$ are the effective lagrangian is given by, we can assume that $\mathcal{L}_{\mathcal{A}}$ The low energy theory symmetries of the $\phi(\mathbf{r},t)$ -The low energy theory
must respect
symmetries of the
underlying Hamiltonian with translation and number conservation require that the low energy that the low energy theory be invariant u
Low energy theory be invariant under the low energy theory be invariant under the low energy theory be in the l $t_{\rm eff}$ associated with the superfluid mode σ is related to the space-time dependent phase of the space-time depend star crust, one can not in general treatment is required. underlying Hamiltonian

with translation and number conservation require that the low energy theory be invariant under the low energy t
In the low energy theory be invariant under the low energy theory be invariant under the low energy the low en

with translation and number conservation require that the low energy theory be invariant under the transformation ^ξ^a=1..³(r, t) [→] ^ξ^a=1..³(r, t) + ^a^a=1..³ and ^φ(r, t) [→] ^φ(r, t) + ^θ where ^a^a=1..³ and ^θ are constant shifts. This naturally implies that the low energy lagrangian can contain only spatial and temporal gradients of these fields. Further, by requiring cubic symmetry for the crystalline state, the The low energy theory is described in terms of the fields φ and ξa. The symmetries associated with translation and number conservation require that the low energy theory be invariant under the transformation ^ξa=1..3(r, t) [→] ^ξa=1..3(r, t) + ^aa=1..³ and ^φ(r, t) [→] ^φ(r, t) + ^θ where ^aa=1..³ and ^θ are constant shifts. This naturally implies that the low energy lagrangian can contain only spatial and must respect star crust, one can not in general treat the two sectors separately and a unified treatment is required. the field associated with the superfluid mode φ(r, t) is related to the space-time dependent phase of the condensate. Because of interactions, such as those between the neutrons and the protons in the neutron condensate. Because of interactions, such as those between the neutrons and the protons in the neutron The low energy theory is described in terms of the fields φ and ξ^a. The symmetries associated It is the aim of this paper to provide such a framework. The low energy theory is described in terms of the fields φ and ξ^a. The symmetries associated with translation and number conservation require that the low energy theory be invariant under the transformation ^ξa=1..3(r, t) [→] ^ξa=1..3(r, t) + ^aa=1..³ and ^φ(r, t) [→] ^φ(r, t) + ^θ where ^aa=1..³ and ^θ are

symn . Lagrangian densit
etry: tr try:
∴ $\overline{1}$ Only derivative terms are allowed. Lagrangian density for the 2
2 ∞ phonon system with cubic symmetry: \mathbf{p} are only the effective lagrangian is given by , we have the effective lagrangian is given by, \mathbf{p} Uniy uchvalive tenno are allowed. Lagrangian ucholly for the low energy lagrangian can contain only spatial and priorion system with cubic symmetry for the cabic symmetry σ t_{c} gradients of these fields. Further, by requiring cubic symmetry for the crystalline state, the c

$$
\mathcal{L} = \frac{f_{\phi}^2}{2} (\partial_0 \phi)^2 - \frac{v_{\phi}^2 f_{\phi}^2}{2} (\partial_i \phi)^2 + \frac{\rho}{2} \partial_0 \xi^a \partial_0 \xi^a - \frac{1}{4} \mu (\xi^{ab} \xi^{ab}) - \frac{K}{2} (\partial_a \xi^a)(\partial_b \xi^b) - \frac{\alpha}{2} \sum_{a=1..3} (\partial_a \xi^a \partial_a \xi^a) + g_{\text{mix}} f_{\phi} \sqrt{\rho} \partial_0 \phi \partial_a \xi^a + \cdots,
$$

 ζab (ζ) ζb , ζ ζa) 2ζ as ζc such as ζ $\zeta^{\infty} = (\theta_a \zeta^{\in} + \theta_b \zeta^{\infty}) - \frac{2}{3} \theta_c \zeta^{\infty} \theta^{\infty}$ of the solid respectively. She some 2011 where $\xi^{ab} = (\partial_a \xi^b + \partial_b \xi^a) - \frac{2}{5} \partial_c \xi^c \delta^{ab}$ $\text{where} \qquad \xi^{ab} = (\partial_a \xi^b + \partial_b \xi^a) - \frac{2}{3} \partial_c \xi^c \delta^{ab}$ Cirigliano, Reddy, Sharma 2011 where h^{ab} order powers of the gradients of the gradients of the gradients of the ζ a ζ where $\xi^{ab} = (\partial_a \xi^b + \partial_b \xi^a) - \frac{2}{3} \partial_c \xi^c \delta^{ab}$ where $\xi^{ab} = (\partial_a \xi^b + \partial_b \xi^a) - \frac{2}{3} \partial_c \xi^c \delta^{ab}$ Cirigliano Reddy Sharma 2011 $\frac{2}{3}$ corder terms involved the gradients of th $\frac{2}{3} \partial_c \xi^c \delta^{ab}$ where

 \mathbf{C} of the velocities of the solid phase. Similarly, the velocity of the phonons in the phonons are relatively, the sheart mass density, the solid respectively. They determine the velocities of the phonons in the solid phase. Similarly, the phonons in the phonon are relation to the mass density of the mass of the compression of the compressibility of the solid respective
Solid respectively. Cirigliano, Reddy, Sharma 2011

Identifying the Low Energy Constants

- LECs must be related to thermodynamic properties.
- Each gradient produces a unique deformation of the ground state.
- The energy cost associated with these (small) deformations provide the LECs.

For a rigorous derivation of LECs in terms of thermodynamic derivatives see arXiv:1102.5379

Cirigliano, Reddy, Sharma 2011

The Coupled System at Leading Order
\n
$$
\mathcal{L}_{n+p} = \frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} v_s^2 (\partial_i \phi)^2 + \frac{1}{2} (\partial_t \xi_i)^2 - \frac{1}{2} (c_l^2 - g^2) (\partial_i \xi_i)^2
$$
\n
$$
+ g \partial_t \phi \partial_i \xi_i + \tilde{\gamma} \partial_i \phi \partial_t \xi_i
$$

Velocity:
$$
v_s^2 = \frac{n_f}{m\chi_n}
$$
 $c_l^2 = \frac{K + 4\mu_s/3}{m(n_p + n_b)}$

\nEntrainment: protons
\ndrag neutrons. **6**

\nBecause $n_b = \gamma n_n$ and $n_b = \gamma n_n$

\nFree neutrons: $n_f = n_n (1 - \gamma)$

Longitudinal lattice phonons and superfluid phonons are coupled:

$$
g = n_p \ E_{np} \ \sqrt{\frac{\chi_n}{m(n_p + n_b)}} \qquad \tilde{\gamma} = \frac{-n_b \ v_s}{\sqrt{(n_p + n_b)n_f}}
$$

The Coupled System at Leading Order
\n
$$
\mathcal{L}_{n+p} = \frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} v_s^2 (\partial_i \phi)^2 + \frac{1}{2} (\partial_t \xi_i)^2 - \frac{1}{2} (c_l^2 - g^2) (\partial_i \xi_i)^2
$$
\n
$$
+ g \partial_t \phi \partial_i \xi_i + \tilde{\gamma} \partial_i \phi \partial_t \xi_i
$$

Velocity:
$$
v_s^2 = \frac{n_f}{m\chi_n}
$$
 $c_l^2 = \frac{K + 4\mu_s/3}{m(n_p + n_b)}$

\nEntrainment: protons
\ndrag neutrons. **6**

\nEven outrons: $n_b = \gamma n_n$

\nFree neutrons: $n_f = n_n (1 - \gamma)$

Longitudinal lattice phonons and superfluid phonons are coupled:

$$
g = n_p \ E_{np} \ \sqrt{\frac{\chi_n}{m(n_p + n_b)}} \qquad \tilde{\gamma} = \frac{-n_b \ v_s}{\sqrt{(n_p + n_b)n_f}}
$$

Transverse lattice phonons:

$$
\mathcal{L}_t = \frac{1}{2} (\partial_t \xi_i)^2 - \frac{1}{2} c_t^2 (\partial_i \xi_j + \partial_j \xi_i)^2 \implies c_t^2 = \frac{\mu_s}{m(n_p + n_b)}
$$

Entrainment _{Chamel (2005)}

number of "bound" neutrons. $n_b \neq$

Bragg scattering off the lattice is important.

neutron single-particle energy

$$
n_f = \frac{m}{24\pi^3} \sum_{\alpha} \int_F |\nabla_{\mathbf{k}} \epsilon_{\alpha, \mathbf{k}}| dS^{\alpha}
$$

$$
n_b = n_n - n_f
$$

Complex interplay of nuclear and band structure effects. The nuclear surface and disorder are likely to play a role.

Longitudinal lattice phonons and superfluid phonons are strongly coupled by entrainment.

Carter, Chamel & Haensel (2006)

 $A=N+Z$

Mixed & Entrained Modes

transverse (right panel) considers a neutron star. Dotted collective exclusive exclusive collections of a new Shear mode velocity is reduced due to entrainment. Longitudinal modes are strongly mixed due to strong interactions.

Crustal Specific Heat ecific Heat (bespecific the specific heat per specific heat per specified the specified of \mathcal{C} and \mathbf{q} = **Heat** t_{total} $\bigcap_{n=1}^{\infty}$ of $\bigcup_{n=1}^{\infty}$ solids up to *T* ⇥ *Tp/*50 but fails when *T* ⇤ *Tp/*10 [18]. \mathcal{L} \sim *ific* \sim 1

Page & Reddy (2012) contribution to the heat capacity is negligible except perhaps in a sliver \mathbf{v}_i and $\mathbf{v}_$ *T* (2012)

Crustal Specific Heat

Transport: Thermal Conduction

$$
\kappa = \frac{1}{3} \ C_v \times v \times \lambda
$$

• Dissipative processes:

• Umklapp is important:

$$
\frac{k_{\rm Fe}}{q_{\rm D}} = \left(\frac{Z}{2}\right)^{1/3} > 1
$$

Electron Bragg scatters and emits a transverse phonon.

Flowers & Itoh (1976)

Superfluid Conduction reconfiguration, respectively [3, 4]. In current models the diusion of electrons is expected eutron stars due to heat released from neutron stars due to heat released from nuclear reactions and magnetic reconfiguration, respectively [3, 4]. In current models the diusion of electrons is expected

Its impossible to sustain a temperature gradient in bulk superfluid helium ! component of another with velocity σ temperature gradient in ∣IN SUPCHIU

$$
\vec{Q} = S^{(\mathrm{sPh})} T \vec{v}_n
$$

$$
S^{(\text{sPh})} = \frac{1}{3} C_v^{(\text{sPh})} = \frac{2\pi^2}{15 c_s^3} T^3
$$

Photographs: JF Allen and JMG Armitage (St Andrews University 1982).

 T^3 **T>T**_c **T<T**_c

 $\overline{\mathbf{S}}$ T adol: Co <mark>unte</mark>r T₃ (2) (2) The superfluid model (contained in the superfluid model in the superfluid model in the superfluid p (The superfluid phonon fiuld), and the superfluid phonon field), and contribute to the total the total theory (**THE SUBSET IS INCO THAT IS INCLUDED**
CONSERVAT **Phonon, and C(sPh)** \mathbf{v} is the phonon specific \mathbf{v} Two fluid model: Counter-flow transports heat. (The superfluid phonon fluid)

⌅3 is the velocity of the superfluid phonon, and *^C*(sPh) *^v* is the phonon specific heat. In the total the total theorem is the total theorem in the conductivity and particles that contribute to
The contribution all particles to the contribution of the contribution of the contribution of the contribution require the divided of the dividence with the same sign and aid in the same sign and aid in the same sign and a The velocity is limited only by fluid dynamics: (i) boundary shear viscosity or (ii) superfluid turbulence.

vwhy does this not occur ↵ Why does this not occur in neutron stars? Answer: Fluid motion is damped by electrons.

Aguilera, Cirigliano, Reddy & Sharma (2009)

Impurity Scattering in Coulomb Solids tering is *elastic* and the temperature independent scattering rative y society. v Scatteri *γ Qimp* ⟨*Z*⟩ ^Λ*imp ^κ ,* (11) *νimp κ* = *ν*⁰ *κ* ⟨*Z*⟩ ^Λ*imp ^κ ,* (11)

Uncorrelated impurities:

$$
\nu_{\kappa}^{imp} = \nu_{\kappa}^{0} \frac{Q_{imp}}{\langle Z \rangle} \Lambda_{\kappa}^{imp},
$$

urities:

$$
Q_{imp} = \langle Z^{2} \rangle - \langle Z \rangle^{2}
$$

$$
\Lambda_{\kappa}^{imp} = \left(\frac{\alpha_{em}}{\pi} + \frac{1}{2}\right) \ln\left(\frac{\alpha_{em} + \pi}{\alpha_{em}}\right) - 1
$$

111 $\breve{ }$ 250 350 atrib...t. PULATIC INC S . S . S . S P_{out} and Molecular Dynamics **Figure 19.5** PIMC - EUC (THEORY) ts tha \sim \sim 1.106 tion is not random. suggests that impurity

$$
Q_{\text{imp}} \to \tilde{Q}_{\text{imp}} = L(\Gamma) Q_{\text{imp}}
$$

EIGROUS IMPURITY SCATTERING DY A RUCION OF Z F. Impurity scattering by a factor of 2-4. α coattoring by a factor of 2α tering by a radio or L +. \blacksquare ions on equal limit, the continuum of the continuum of the classical limit, the dynamics of the dynamics of the dynamics of the continuum of the continuum of the dynamics of the continuum of the dynamics of the dyna lg by a factor of 2-4. \mathcal{L} Molecular Dynamics (MD), while the static structure t toning between minority species and correlationships and correlations and correlations and correlations and correlations \sim $\begin{array}{c} \text{letting by a factor of } 2-4. \end{array}$ Enhances impurity scattering by a factor of 2-4.

Roggero & Reddy (2016) calculating the Coulomb logarithm using *Simp*(*q*) from the rela- \mathbf{j} sical Monte Carlo (CMC) methods [12, 17, 17, 18]. In what is now the control of the con 016) structure factors using systems composed of different number

Unraveling Thermal Relaxation

•Late time signal is sensitive to inner crust thermal and transport properties.

Shternin & Yakovlev (2007) Brown & Cumming (2009)

•Variations in the pairing gap (changes the fraction of normal neutrons)may be discernible ! Page & Reddy (2012)

Unraveling Thermal Relaxation

•Late time signal is sensitive to inner crust thermal and transport properties.

Shternin & Yakovlev (2007) Brown & Cumming (2009)

•Variations in the pairing gap (changes the fraction of normal neutrons)may be discernible ! Page & Reddy (2012)

Time [days] A: Low T_c - large normal fraction R. Ligh T cmall normal fraction B: High T_c- small normal fraction sing panel are from the left parties of 8.5 kpc.

Transport Properties of the Core

Low energy excitations in the core tigy caulduul to in the Cule ature, density or microscopic interactions change $\mathbf{5}$ reasonable to assume that the ground state of $\mathsf{L}\mathsf{OW}$ (TABLE I. Ambient conditions, low energy constants and eigenmode velocities *v*¹ and *v*² in units of the velocity of light for the

Neutrons are superfluid (T<Tⁿc): Electrons + 4 Goldstone modes (3 neutron modes and 1 electron-proton mode). Bedaque, Rupak, Savage, (2003), Bedaque, Nicholson (2013), Bedaque and Reddy (2013). d 1 electron-proton mode). Bedaque, Rupak, Savage, form of the condensative condensative condensative condensative under the condensative conde i (2010), Deuayue and neur χ or σ . \overline{a} mode velocities *v*¹ and *v*² in units of the velocity of light for the *i* body of the nodice of the street procent process, Bodaque, Inapan, Savage,

(2003), Bedaque, Nicholson (2013), Bedaque and Reddy (2013). spontaneously the symmetry of the system under rotations,

Neutrons are normal (T>Tⁿ_c): Electrons, neutrons + 1 Goldstone boson (electron-proton mode). *I*J₂ *Flectrons* neutrons + 1 Goldstone \mathbf{S} spontaneously the system under the system under \mathbf{S} Neutrons are r the ground state in the beginning to respond the state $\frac{1}{2}$ tions of the condensate around the *x* and *y* axis. Angulons + the electron gas and is denoted by the scalar field ⇠. The ${\sf mal}$ (T>T" $_{\sf c}$): Electrons, neutrons + 1 Goldstone is well studied and the low energy Largrangian density α and the low energy Largrangian density α

$$
\mathcal{L}_{\text{phn}} = \frac{1}{2} (\partial_0 \phi)^2 - \frac{v_n^2}{2} (\partial_i \phi)^2 + \frac{1}{2} (\partial_0 \xi)^2 - \frac{v_p^2}{2} (\partial_i \xi)^2
$$

Superfluid Phonons:
$$
+ v_{\text{np}}^2 \partial_0 \phi \partial_0 \xi + \frac{1}{f_{\text{ep}}} \partial_0 \xi \psi_e^{\dagger} \psi_e + \cdots ,
$$

² (@)²

Angulons:

, (2)

were the studied in more detail in \mathbb{R}^n

$$
\mathcal{L}_{\text{ang}} = \sum_{i=1,2} \left[\frac{1}{2} (\partial_0 \beta_i)^2 - \frac{1}{2} v_{\perp}^i{}^2 ((\partial_x \beta_i)^2 + (\partial_y \beta_i)^2) + v_{\parallel}^2 (\partial_z \beta_i)^2 \right] + \frac{e g_n f_\beta}{2M \sqrt{-\nabla_{\perp}^2}} \left[\mathbf{B}_1 \partial_0 (\partial_y \beta_1 + \partial_x \beta_2) + \mathbf{B}_2 \partial_0 (\partial_x \beta_1 - \partial_y \beta_2) \right]
$$

Produced And Reddy (2013 They are given by $\overline{ }$ laque a s **Red** $\sqrt{20}$ Bedaque and Reddy (2013),

is given by

is well studied $\mathcal{I}(\mathcal{I})$

where we have also included the coupling to the electron

field *^e*. The coecients of the leading order terms in the

derivative expansion are related to simple thermodynamic

derivates and can be obtained from the equation of state.

Mixing and Damping of Goldstone Bosons

Modes decay rapidly due to the coupling to the large density of electron-hole states. Do not contribute to transport.

nucleon loops include and Reddy (2013)

Electron Scattering in the Core Such collisions are similar to electromagnetic scattering

Pethick and Heiselberg (1993), Shternin and Yakovlev (2006,2007)

$$
|M_{12}|^2 \propto \left| \frac{J_{1'1}^{(0)} J_{2'2}^{(0)}}{q^2 + \Pi_l} - \frac{J_{t1'1} \cdot J_{t2'2}}{q^2 - \omega^2 + \Pi_t} \right|^2
$$

$$
\Pi_t(\omega, \mathbf{q}) \simeq \alpha_{\mathsf{em}} k_{\mathsf{fp}}^2 \left(4\pi \frac{\Delta_{\mathsf{p}}}{\mathsf{q}} + 2\mathsf{i} \frac{\omega}{\mathsf{q}} \right)
$$

$$
\chi_p(\omega, q) = \mathcal{R}e \Pi_p^L(\omega, q) = \mathcal{R}e \int dt e^{i\omega t} \int d\mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} \langle n_p(\mathbf{r}, t) n_p(0, 0) \rangle
$$

Summary

- Accreting neutron stars provide a unique opportunity to study thermal and transport properties.
- Thermal relaxation in neutron stars is sensitive to the low temperature properties of the crust.
- Thermal and transport properties of the inner crust (super-solid) can be calculated in terms of a few lowenergy constants.
- Goldstone bosons in the crust and the core can decay into electron-hole states - this limits their contribution to transport.
- The induced interactions between electrons and neutrons can be relevant in the neutron star core.