Thermal and transport properties of neutron star matter.

Sanjay Reddy

- Introduction
- Specific heat of neutron stars
- Crust: Properties of solid and superfluid matter
- Core: Low energy modes and transport properties
- Summary

Relevant Papers:
arXiv:1602.01831
arXiv:1409.7750
arXiv:1307.4455
arXiv:1210.5169
arXiv:1201.5602
arXiv:1102.5379
arXiv:1009.2303

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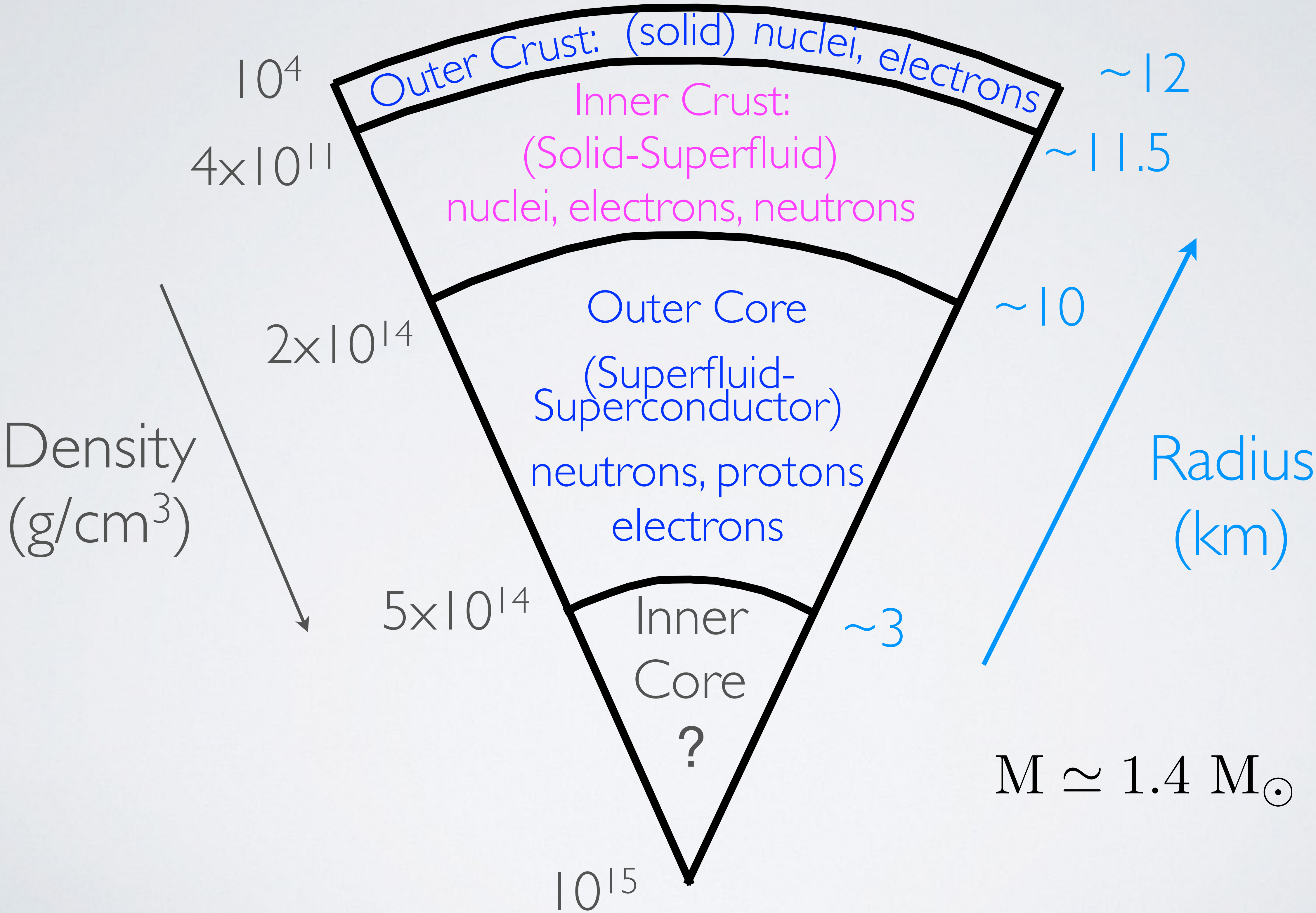
Phases of Dense Matter in Neutron Stars



(Dense) Material Characterization

Property	Observations
Equation of State	• Mass • Radius • Moment of Inertia • Tidal Polarizability
	• Seismology
Specific Heat Thermal Conductivity Neutrino Emissivity Neutrino Opacity	• Thermal Evolution • Supernova Neutrinos
Electrical Conductivity	• Magnetic Field Evolution
Shear Viscosity	• Spin Evolution (r-modes)

Specific Heat of Cold Dense Matter

- Fermions: Particle-hole excitations

$$C_{F,i} = \pi^2 \ n_i \frac{E_{F,i} \ T}{p_{F,i}^2} \simeq 2 \times 10^{19} \ \left(\frac{E_{F,i}}{m_n}\right) \left(\frac{p_{F,i}}{400 \ \mathrm{MeV}}\right) \left(\frac{T}{10^8 \ \mathrm{K}}\right) \ \frac{\mathrm{ergs}}{\mathrm{cm}^3 \ \mathrm{K}}$$

- Bosons: Collective excitations

$$C_{B,i} = \frac{2\pi^2 \ c^3}{15 \ v_{B,i}^3} \ T^3 \simeq 1.5 \times 10^{10} \ \left(\frac{c}{v_{B,i}}\right)^3 \left(\frac{T}{10^8 \ \mathrm{K}}\right)^3 \ \frac{\mathrm{ergs}}{\mathrm{cm}^3 \ \mathrm{K}}$$

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- Core contribution is large and electrons set the lower limit.

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- Pairing suppresses the nucleon contribution.

- Core contribution is large and electrons set the lower limit.

- Collective modes play a role in the crust.

Specific Heat & Phase Transitions

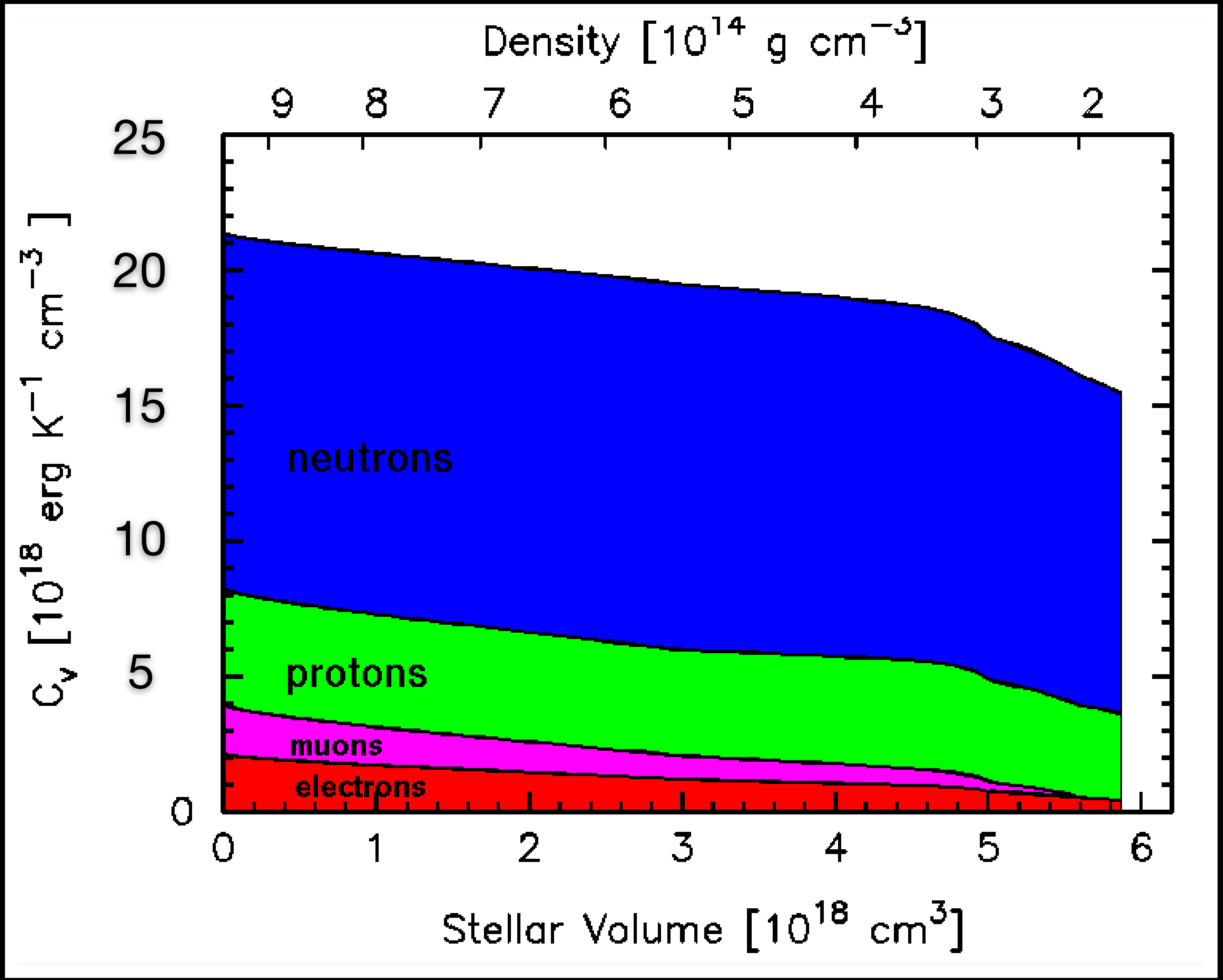
Canonial expectation:

$$C_{NS} \approx 10^{38} \left(\frac{T}{10^8\ \mathrm{K}}\right) \frac{\mathrm{ergs}}{\mathrm{K}}$$

Nucleon superfluidity can suppress C_{NS} by a factor of about 10-20.

Additional un-gapped fermions (quarks, hyperons) can enhance it by a factor of a few.

Exception: A large CFL fraction in the core can reduce C_{NS} by a factor of 100.



Specific heat of a 1.4 M⊙ neutron star at T=10^8 K. Figure courtesy of Dany Page

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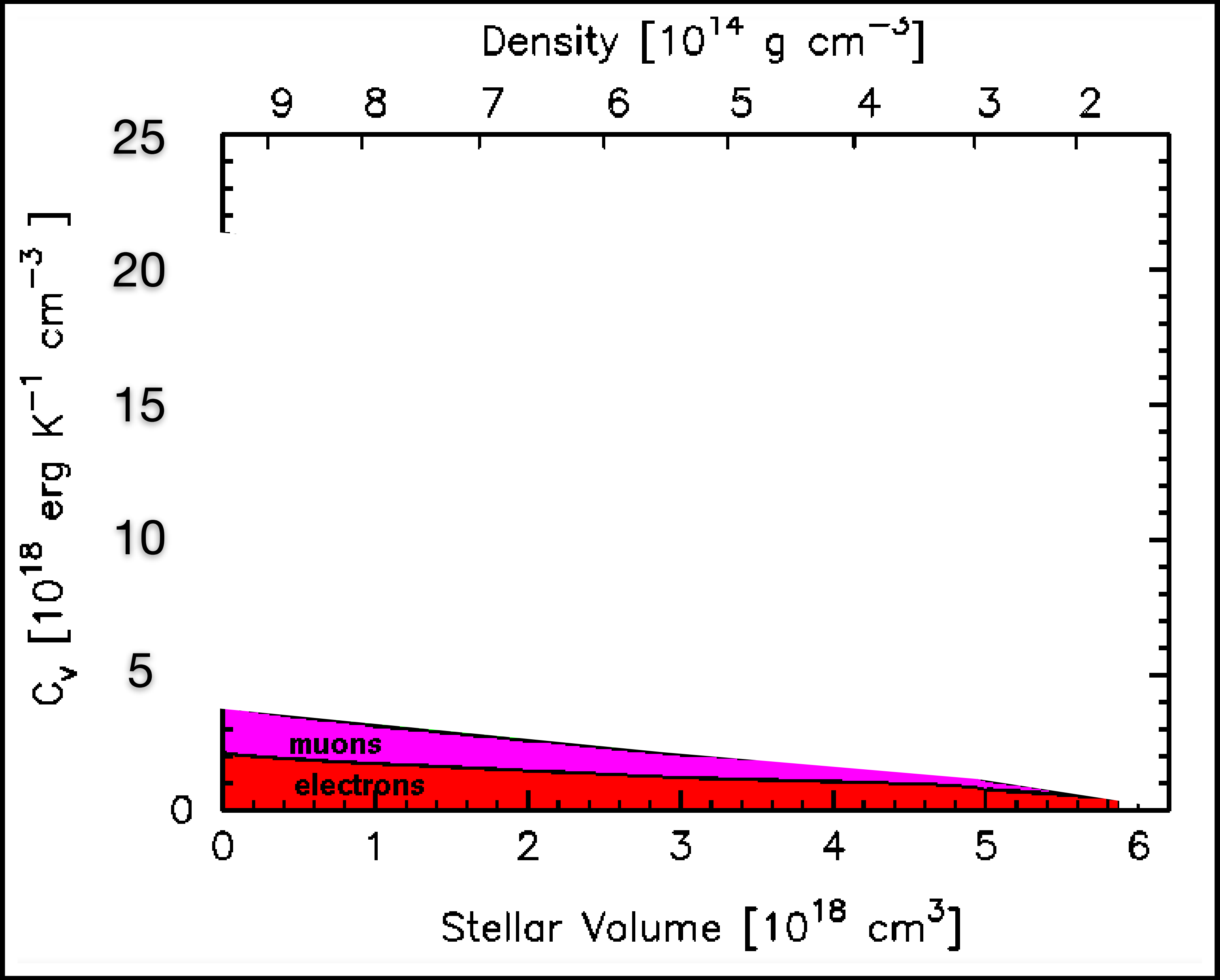


Specific heat of a 1.4 M⊙ neutron star at T=10^8 K. Figure courtesy of Dany Page

Measuring the Heat Capacity

Cumming et al. (2016 in prep.)

Heat the star, allow it to relax, and observe the change in temperature:

$$C_{NS}\ dT = dQ$$

When $C_{NS} = \alpha\, T$:

$$\frac{\alpha}{2}\,(T_f^2 - T_i^2) = \Delta Q$$

Lower limit:

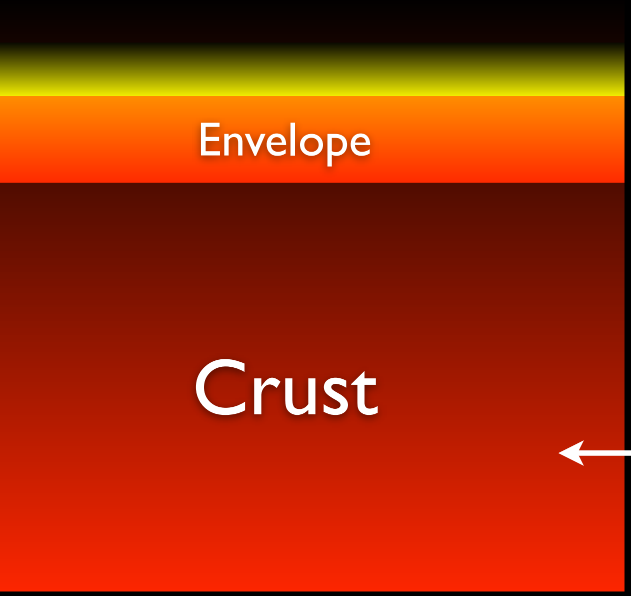
$$C_{NS}(T_f) > 2\frac{\Delta Q}{T_f}$$

$$\Delta Q = \dot{H} \times t_H - L_\nu \times (t_H + t_{obs})$$

- $\dot{H}$: heating rate
- t_H: duration of heating
- L_ν: neutrino cooling rate
- t_{obs}: time of observation (after heating ceases)

Transiently Accreting NSs

SXRTs: High accretion followed by periods of quiescence



Nuclear reactions release: ~ 1-2 MeV / nucleon

Deep crustal heating.

Brown, Bildsten Rutledge (1998)
Sato (1974), Haensel & Zdunik (1990)

Warms up old neutron stars

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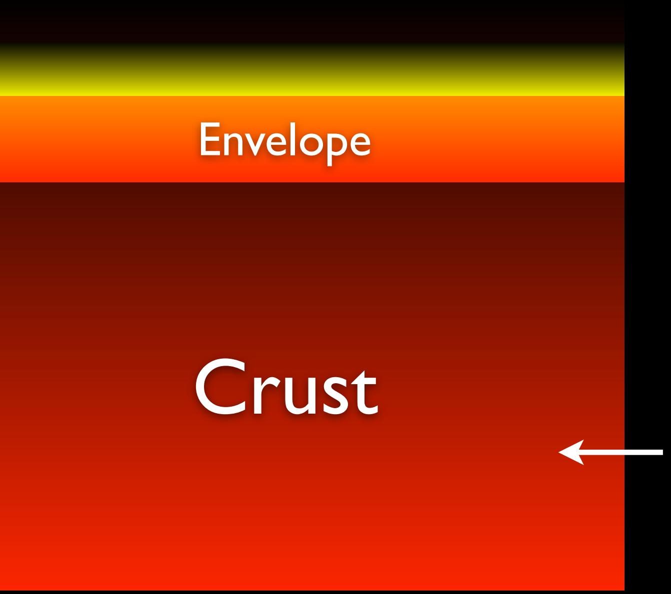

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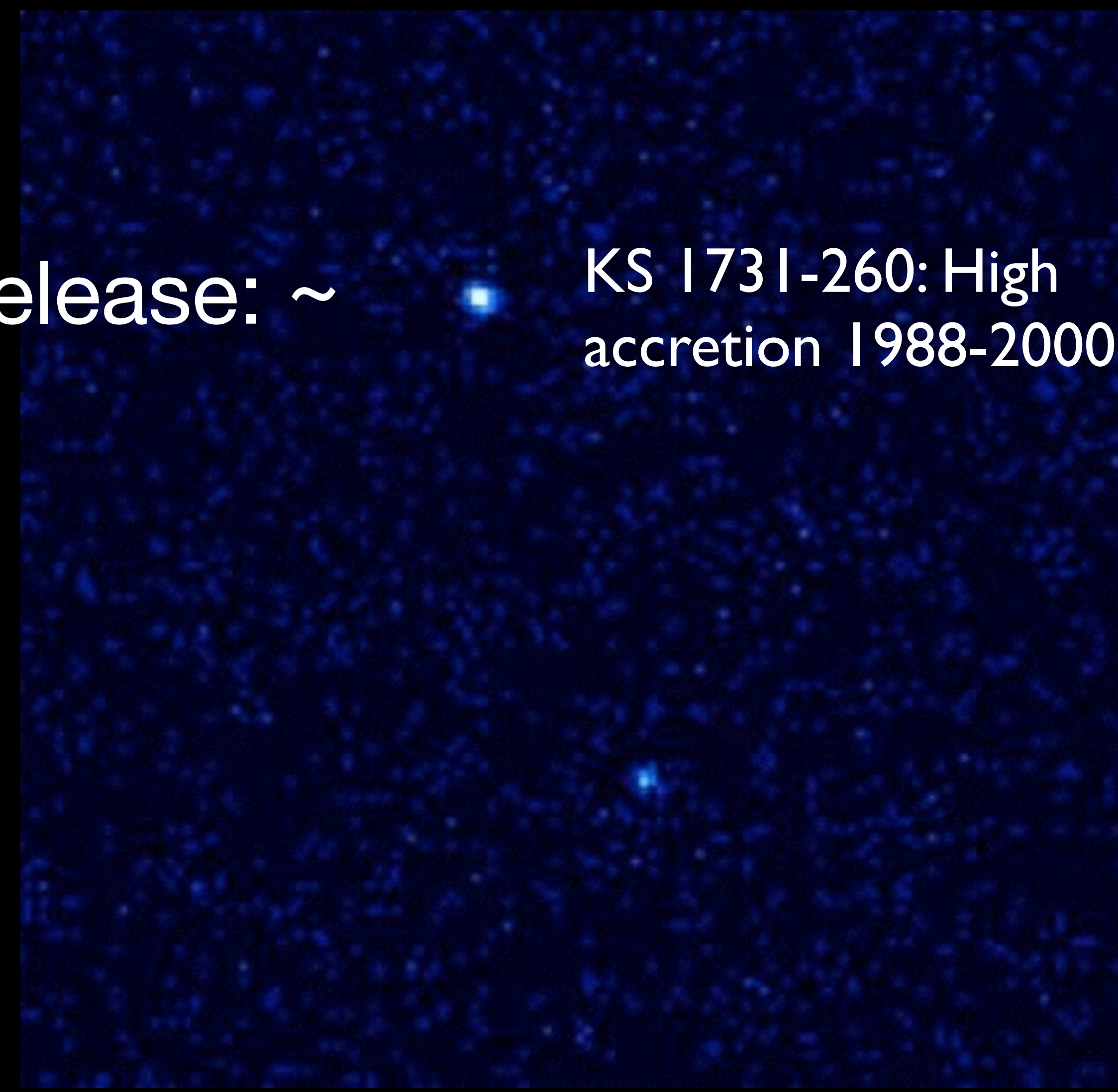

Image credit: NASA/CXC/Wijnands et al.

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Warms up old neutron stars

Observations of KS 1731-260

Accretion Phase: 12 yrs at dM/dt ≈10^{17} g/s
Thermal Relaxation: t ≈ 8 yrs
Quiescent Surface Temperature (post relaxation): T_{eff}=63 eV

Wijnands et al. (2002)
Cackett et al. (2010)

Energy Deposition:

$$E_{\rm dep} = \dot{H} \times t_H = 6 \times 10^{43}\ {\rm ergs} \left(\frac{Q_{\rm nuc}}{2\ {\rm MeV}}\right) \left(\frac{\dot{M}}{10^{17} {\rm g/s}}\right) \left(\frac{t_H}{10\ {\rm yrs}}\right)$$

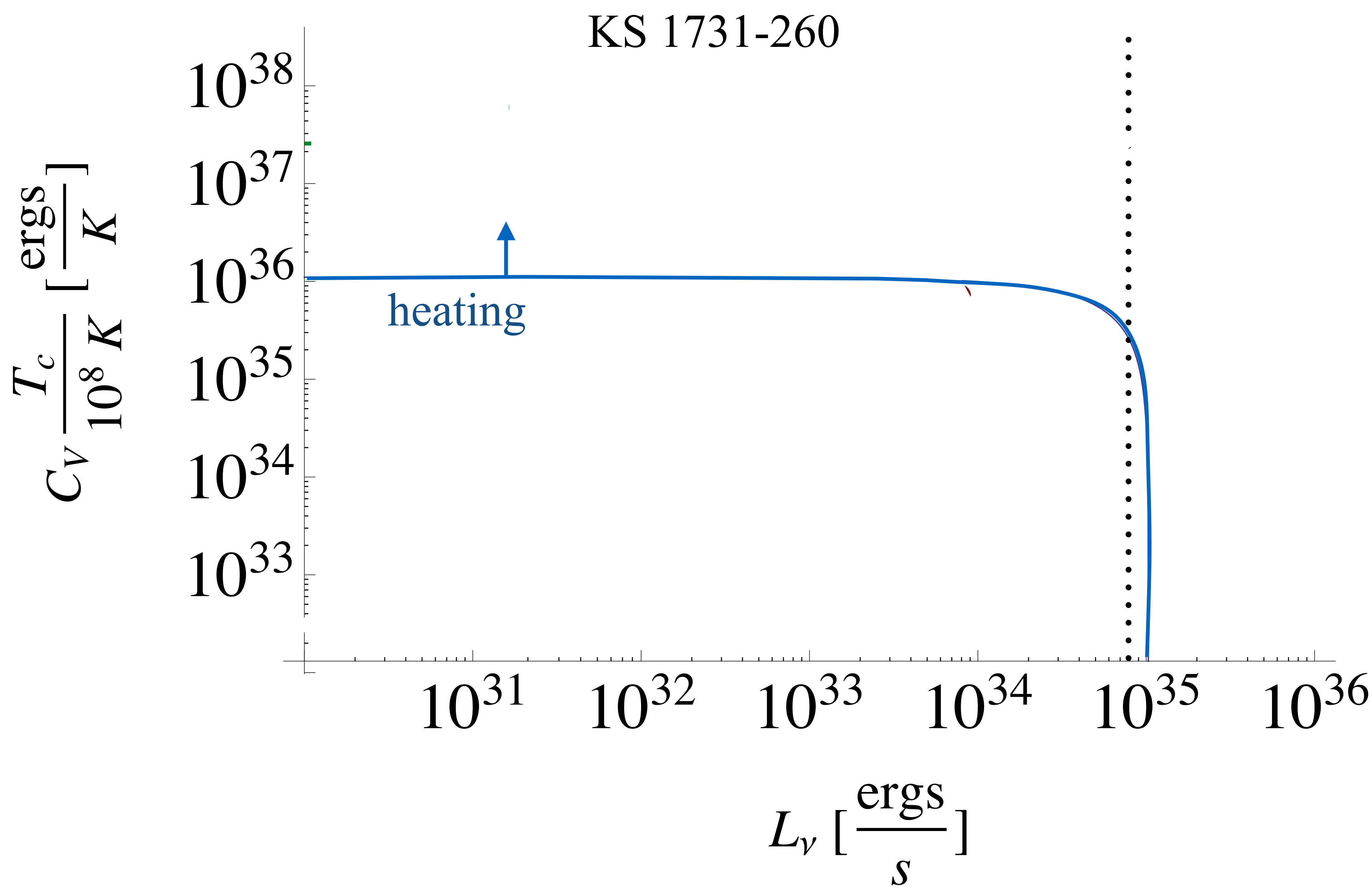
Inferred Core Temperature:
(insulating envelope sustains a temperature gradient near the surface)

$$T_c^\infty = 7.0 \times 10^7\ {\rm K} \left(\frac{T_{\rm eff}^\infty}{63.1\ {\rm eV}}\right)^{1.82} \quad \text{(Fe envelope)}$$

$$T_c^\infty = 3.1 \times 10^7\ {\rm K} \left(\frac{T_{\rm eff}^\infty}{63.1\ {\rm eV}}\right)^{1.65} \quad \text{(He envelope).}$$

Limits: Current & Future

$$C_{NS}(T_f) > 2\frac{\Delta Q}{T_f}$$



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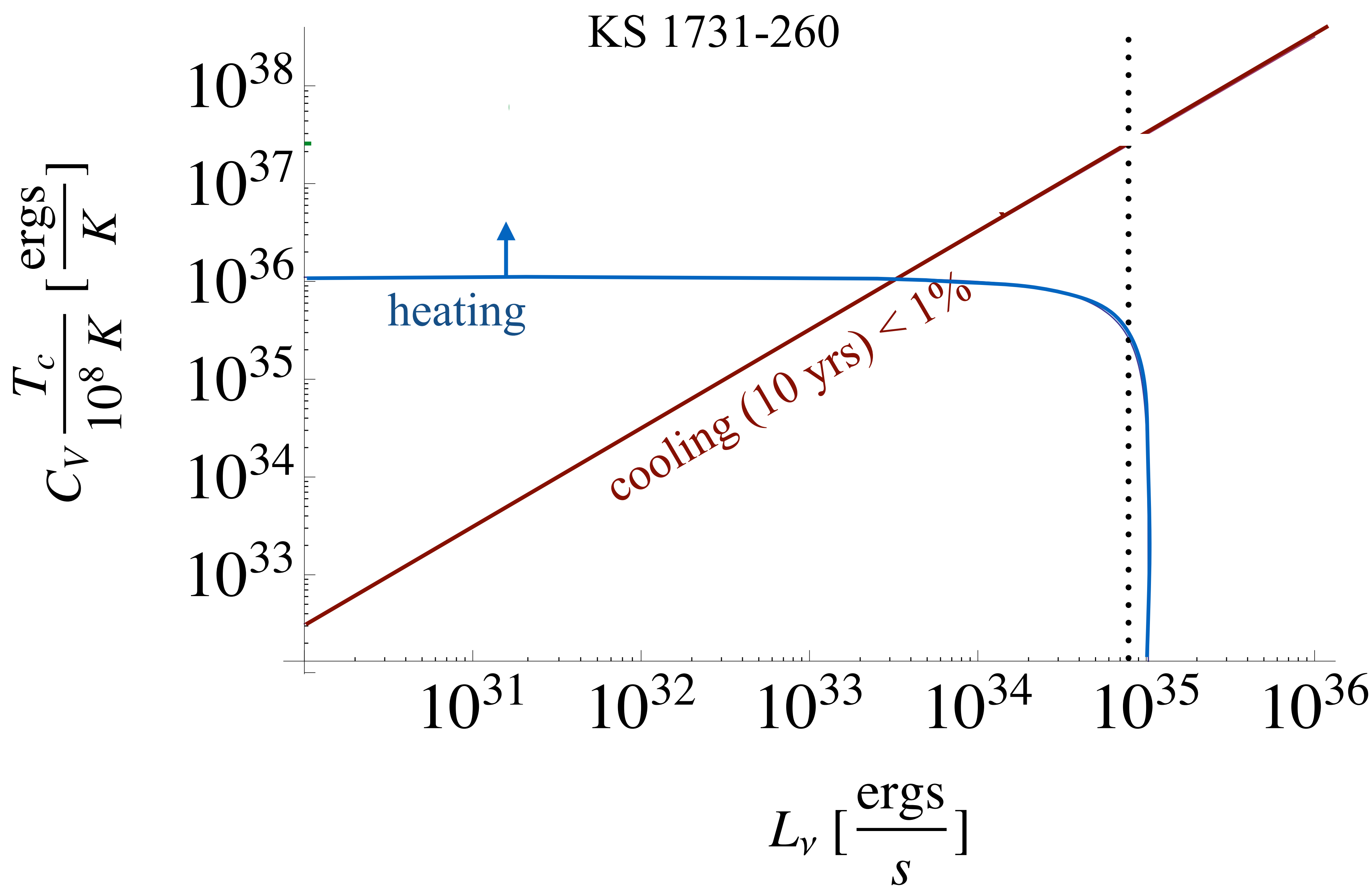

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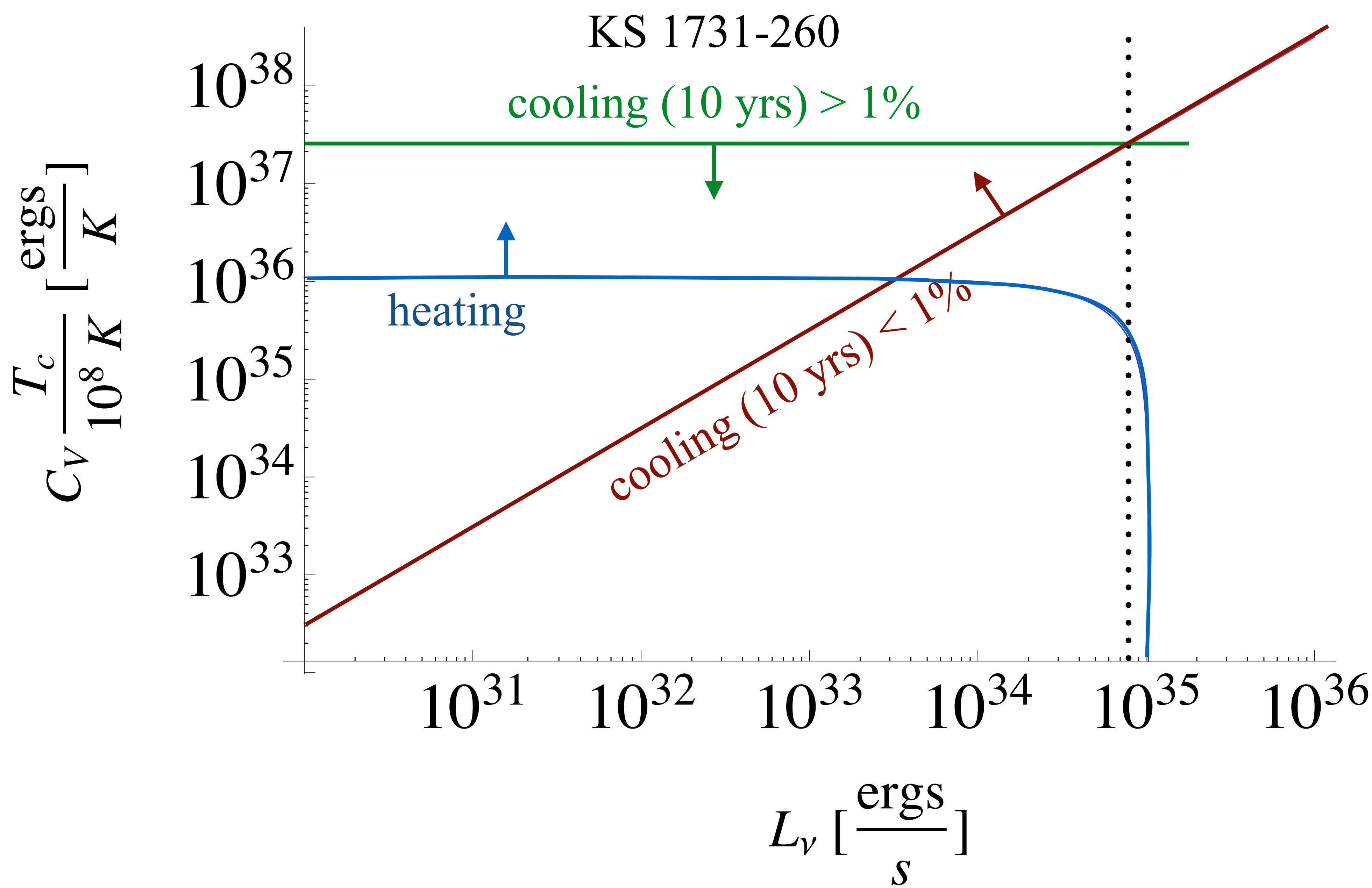

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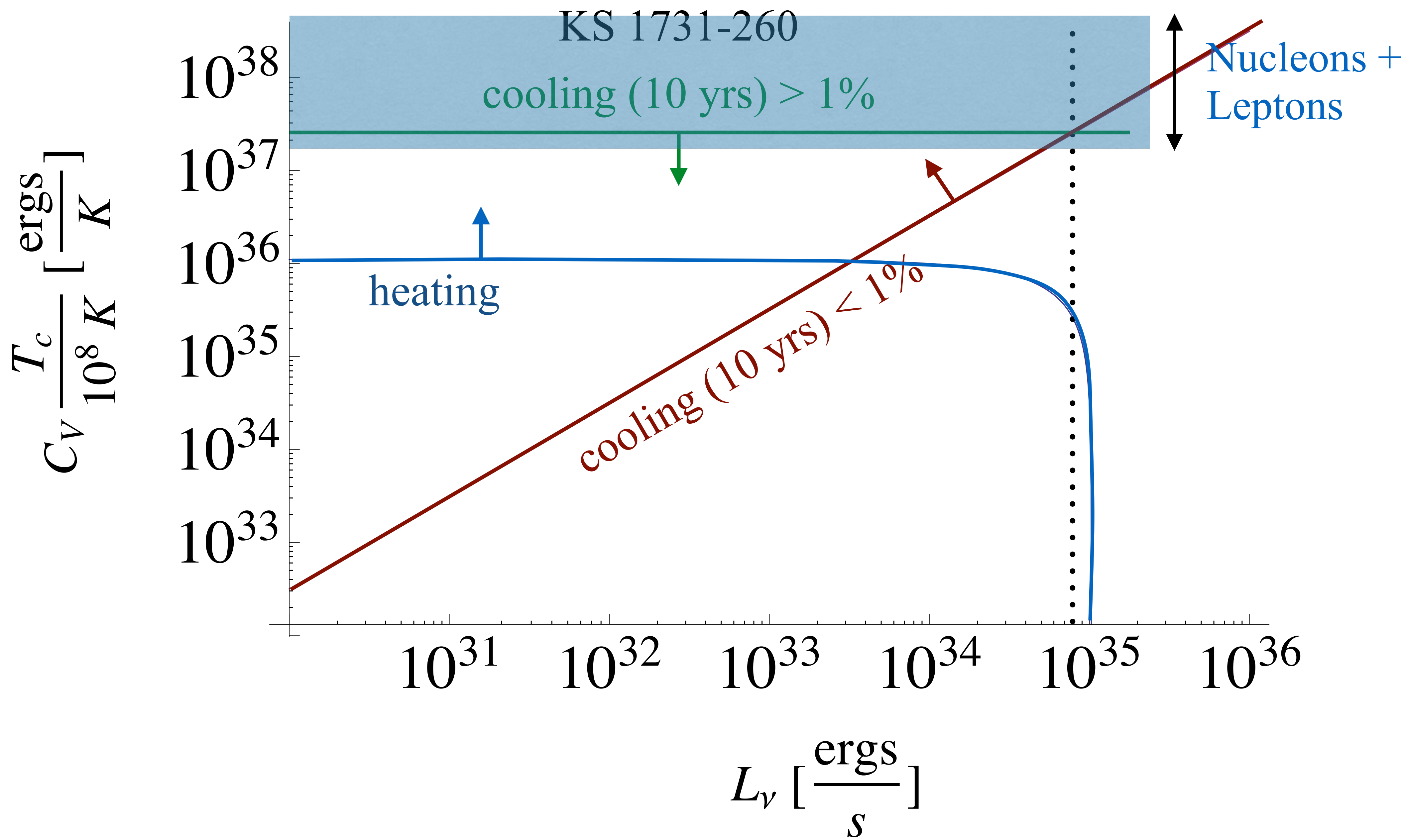


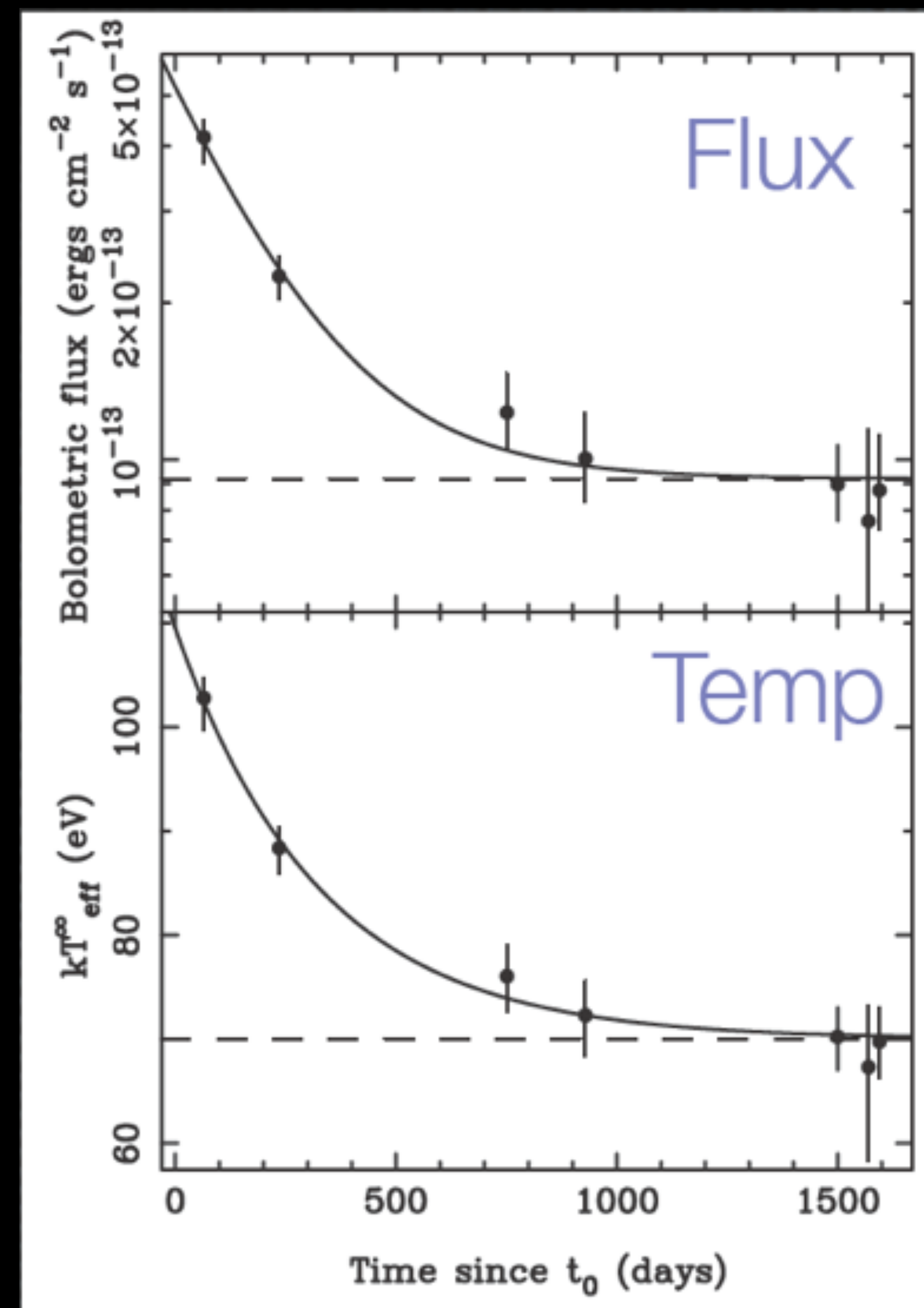
Cumming et al. (2016 in prep.)

Crust Cooling

Watching NSs immediately after accretion ceases !

The heated crust relaxes as heat is transported to the core.

Shternin & Yakovlev (2007)
Cumming & Brown (2009)

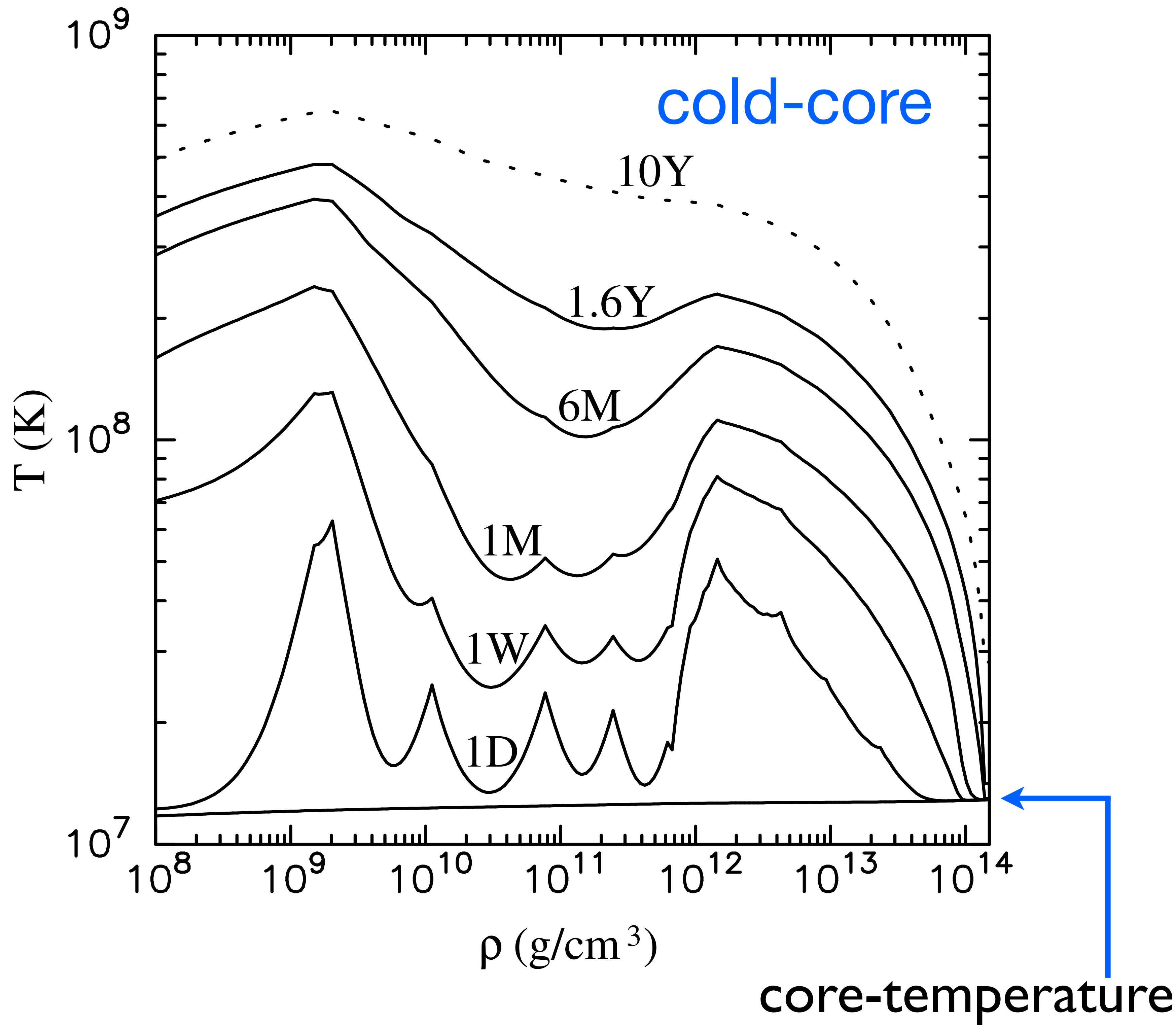


Cackett, et al. (2006)

Accretion Induced Heating

Temperature profile depends on:

- accretion rate and duration.
- location of heat sources.
- thermal conductivity
- specific heat.
- core temperature



Page & Reddy (2012,2013)

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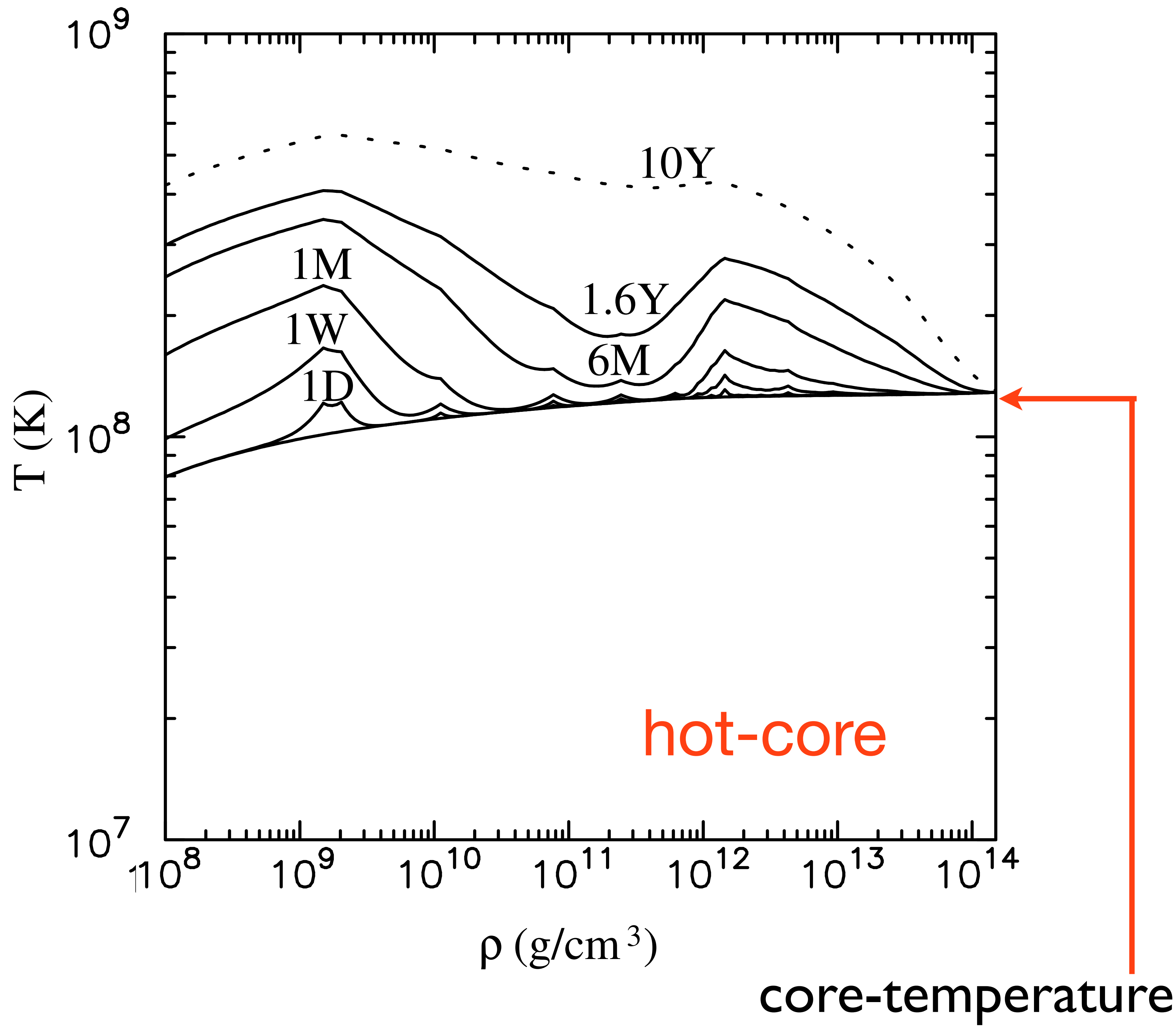


Page & Reddy (2012,2013)

Observations:

All known Quasi-persistent sources with post outburst cooling

- After a period of intense accretion the neutron star surface cools on a time scale of years.

- This relaxation was first discovered in 2001 and 6 sources have been studied to date.

- Expected rate of detecting new sources ~ 1/year.

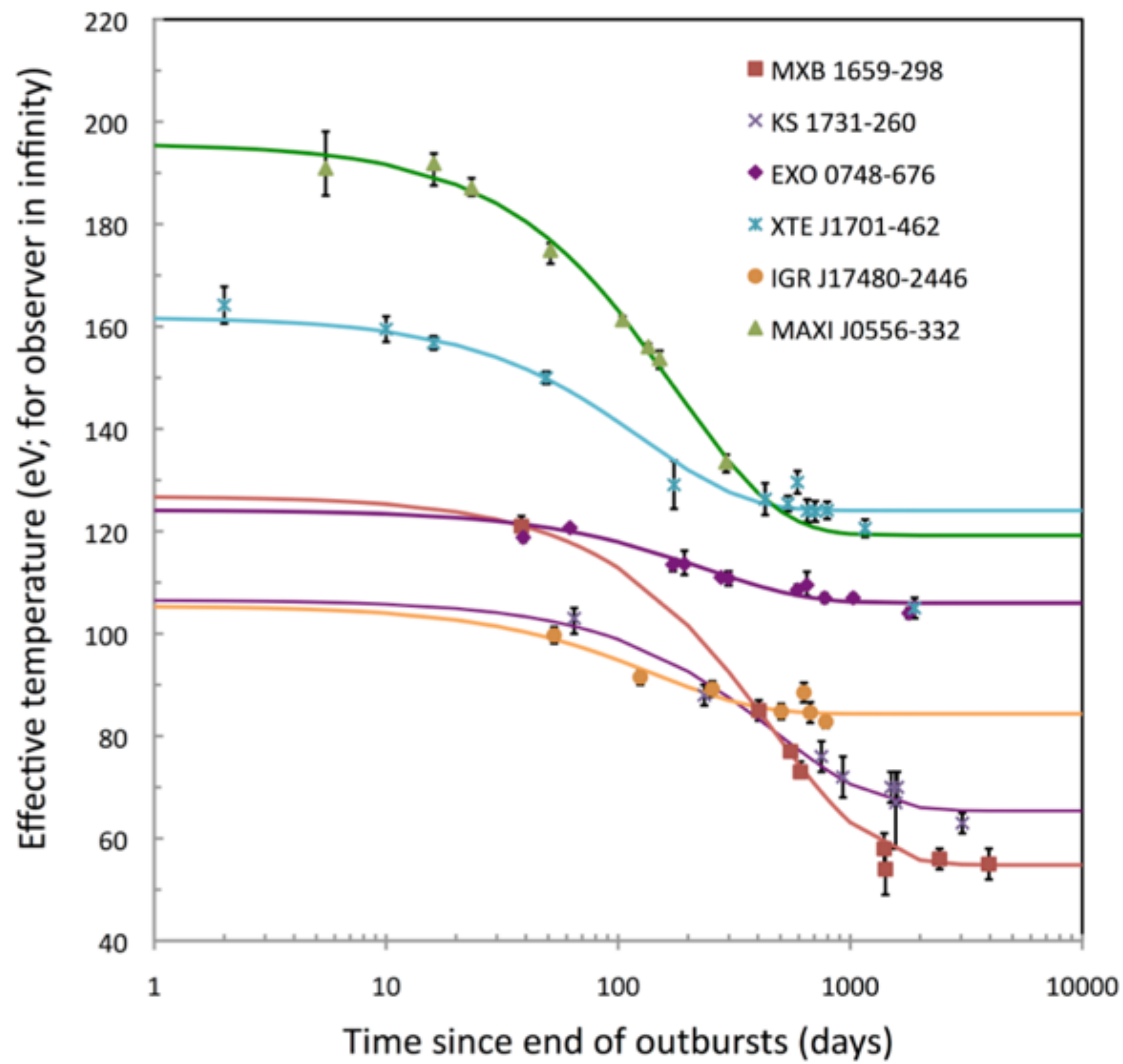


Figure from Rudy Wijnands (2013)

Connecting to Crust

$$\tau_{\mathrm{Cool}} \simeq \frac{C_V}{\kappa} \, (\Delta R)^2$$

Connecting to Crust

Crustal Specific Heat → C_V

$$\tau_{\rm Cool} \simeq \frac{C_V}{\kappa}\,(\Delta R)^2$$

Connecting to Crust

Crustal Specific Heat

$$\tau_{\mathrm{Cool}} \simeq \frac{C_V}{\kappa} \; (\Delta R)^2$$

Thermal Conductivity

Connecting to Crust

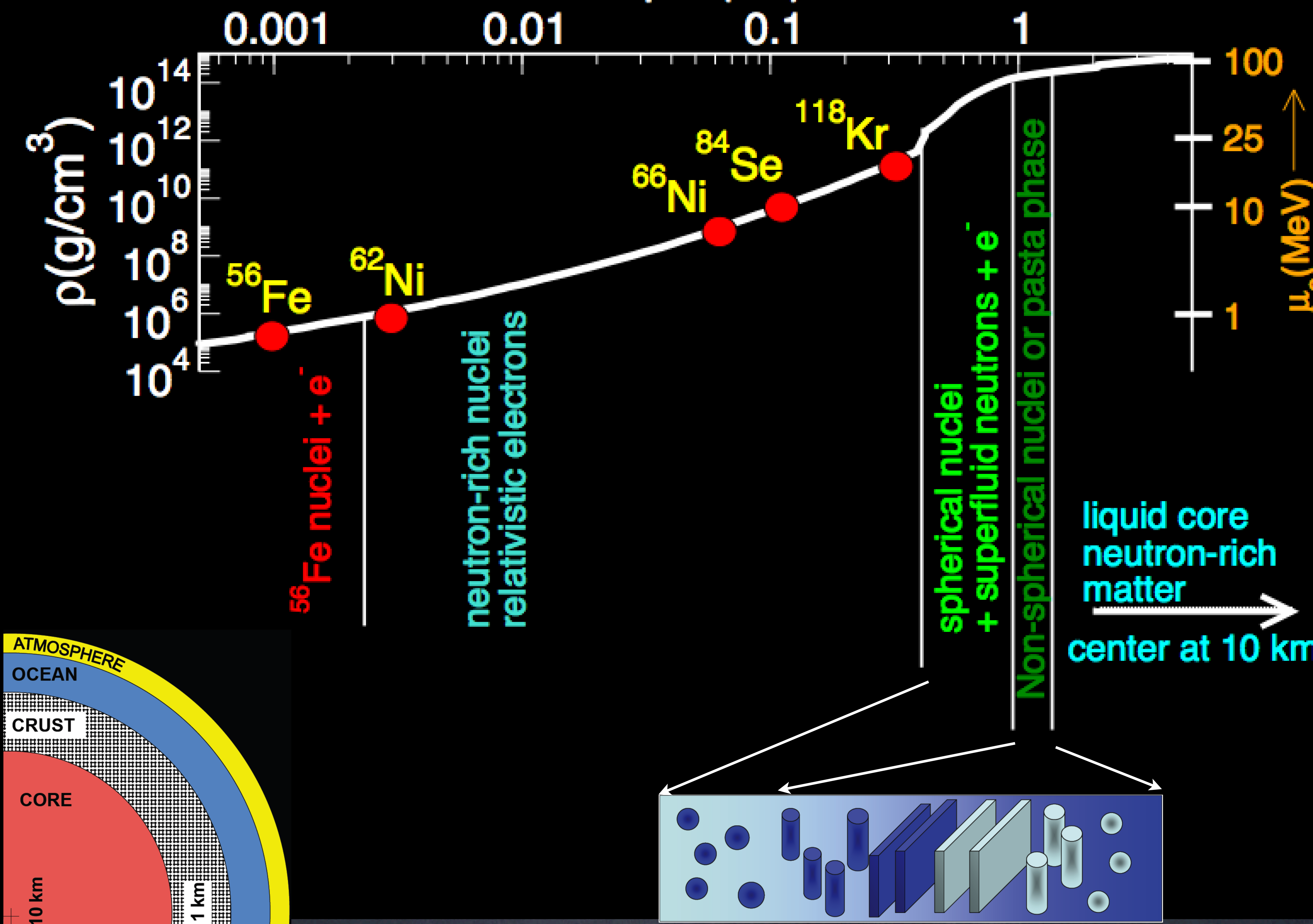
Crustal Specific Heat

Crust Thickness

$$\tau_{\mathrm{Cool}} \simeq \frac{C_V}{\kappa} (\Delta R)^2$$

Thermal Conductivity

Neutron Star Crust:



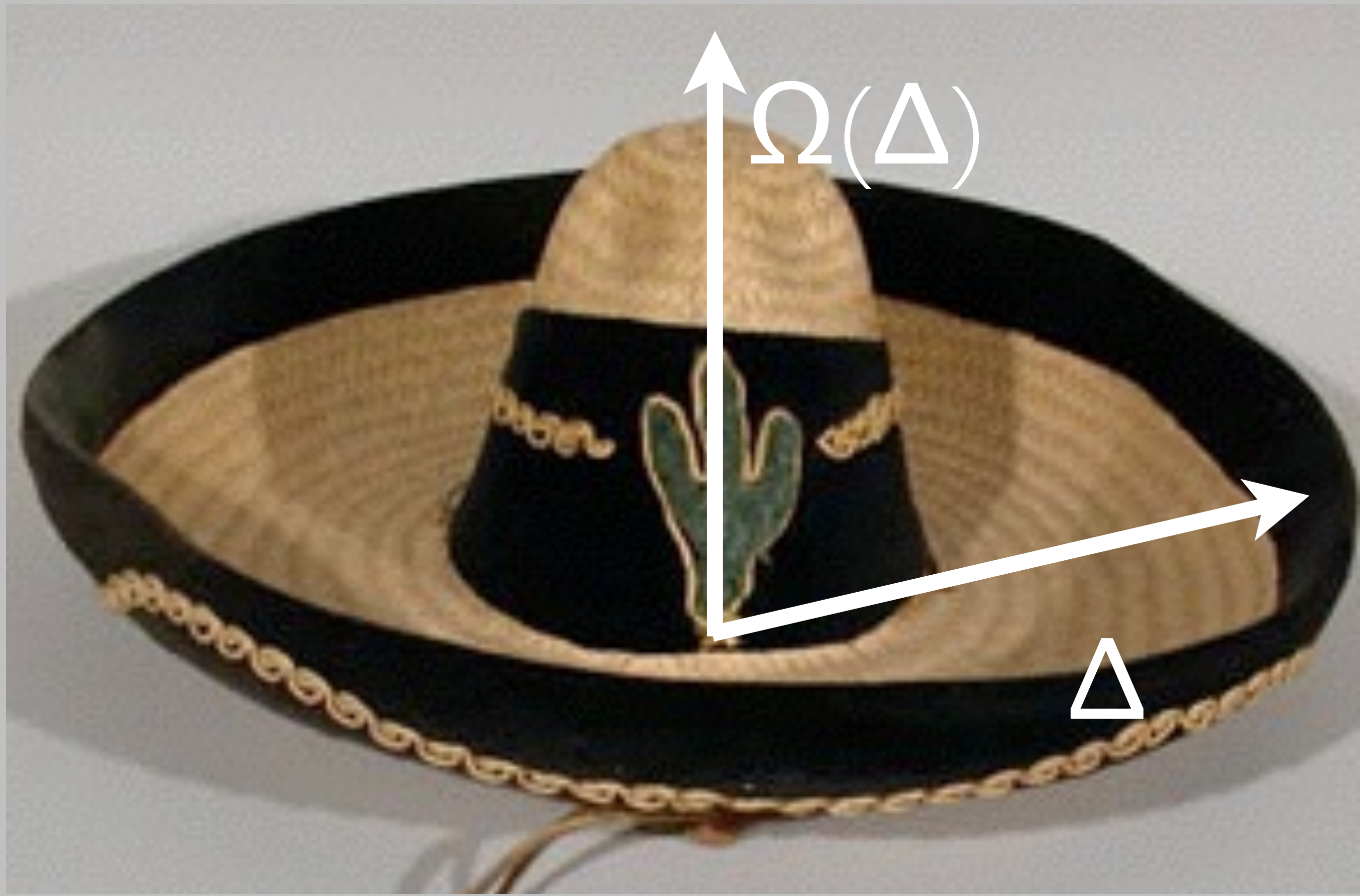
Cooper Pairing

Attractive interactions destabilize the Fermi surface:

$$H = \sum_{k,s=\uparrow,\downarrow} \left(\frac{k^2}{2m} - \mu\right) a^\dagger_{k,s} a_{k,s} + g \sum_{k,p,q,s=\uparrow,\downarrow} a^\dagger_{k+q,s} a^\dagger_{p-q,s} a_{k,s} a_{p,s}$$

$$\Delta = g\langle a_{k,\uparrow} a_{p,\downarrow}\rangle \quad \Delta^* = g\langle a^\dagger_{k,\uparrow} a^\dagger_{p,\downarrow}\rangle$$

Cooper pairs leads to superfluidity



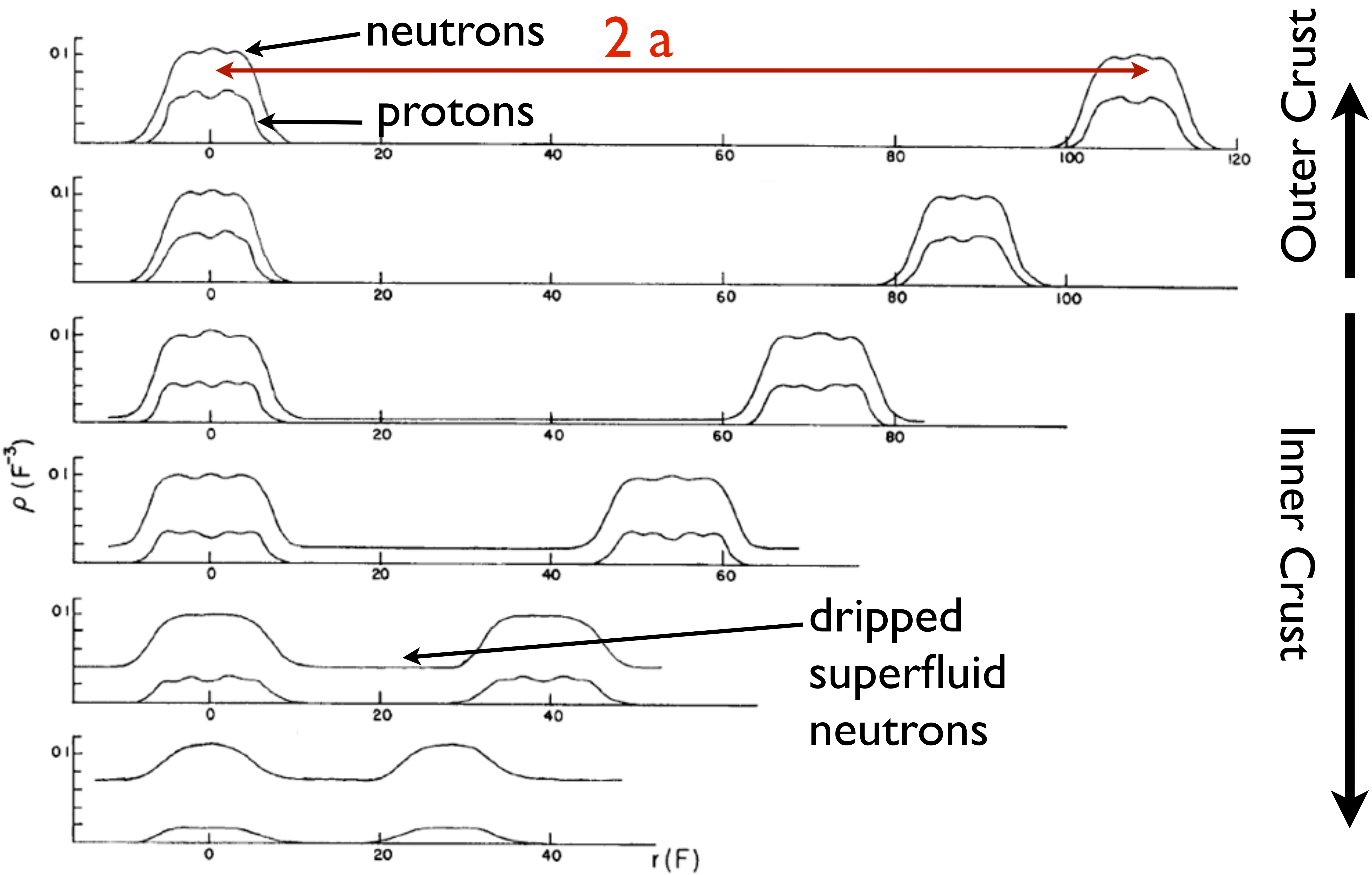
Energy gap for fermions:

$$E(p) = \sqrt{\left(\frac{p^2}{2M} - \mu\right)^2 + \Delta^2}$$

New collective mode:
Superfluid Phonon

$$\omega(k) = v_s\ k$$

Microscopic Structure of the Crust



Baym Pethick & Sutherland (1971)   Negele & Vautherin (1973)

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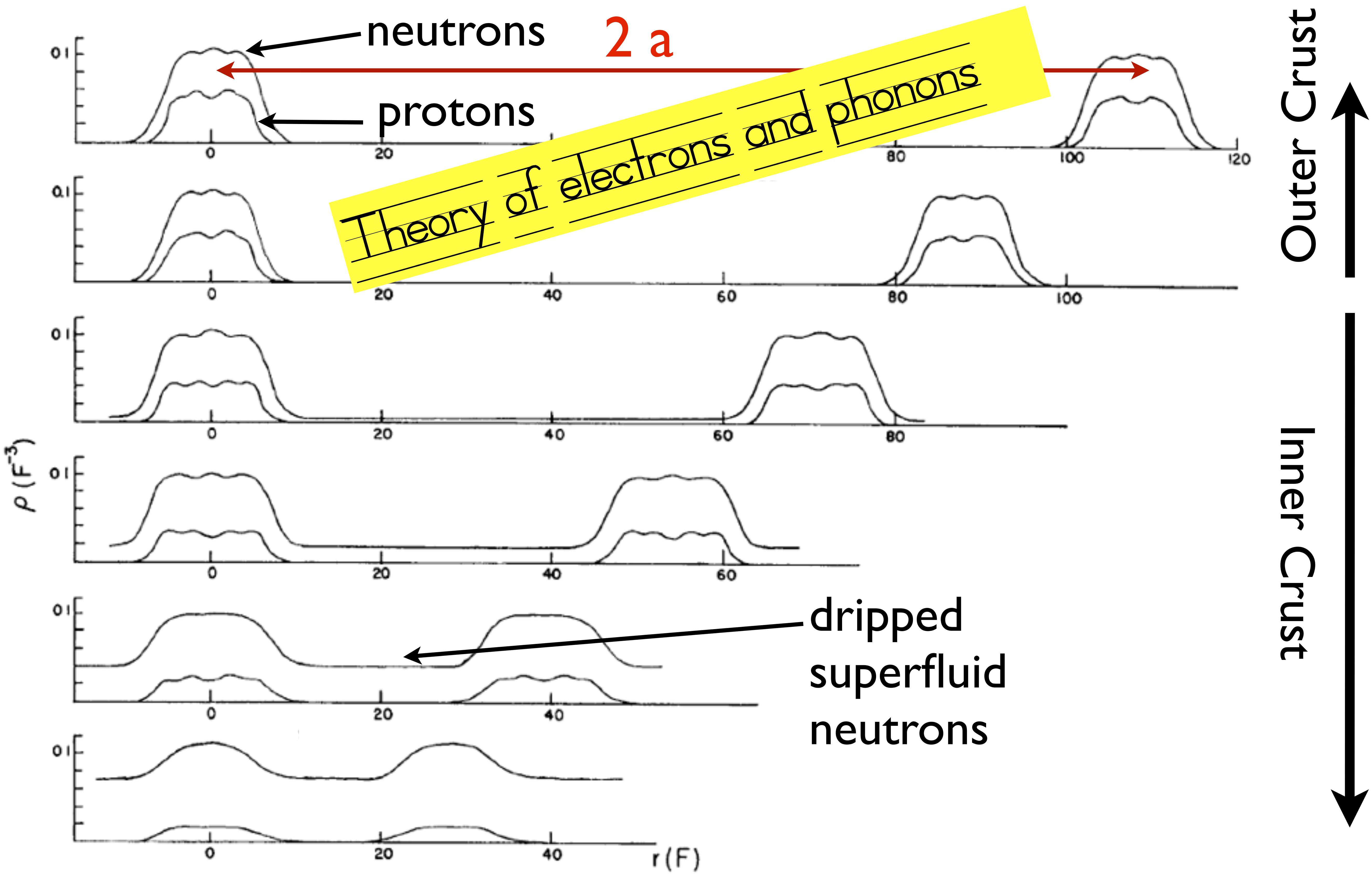

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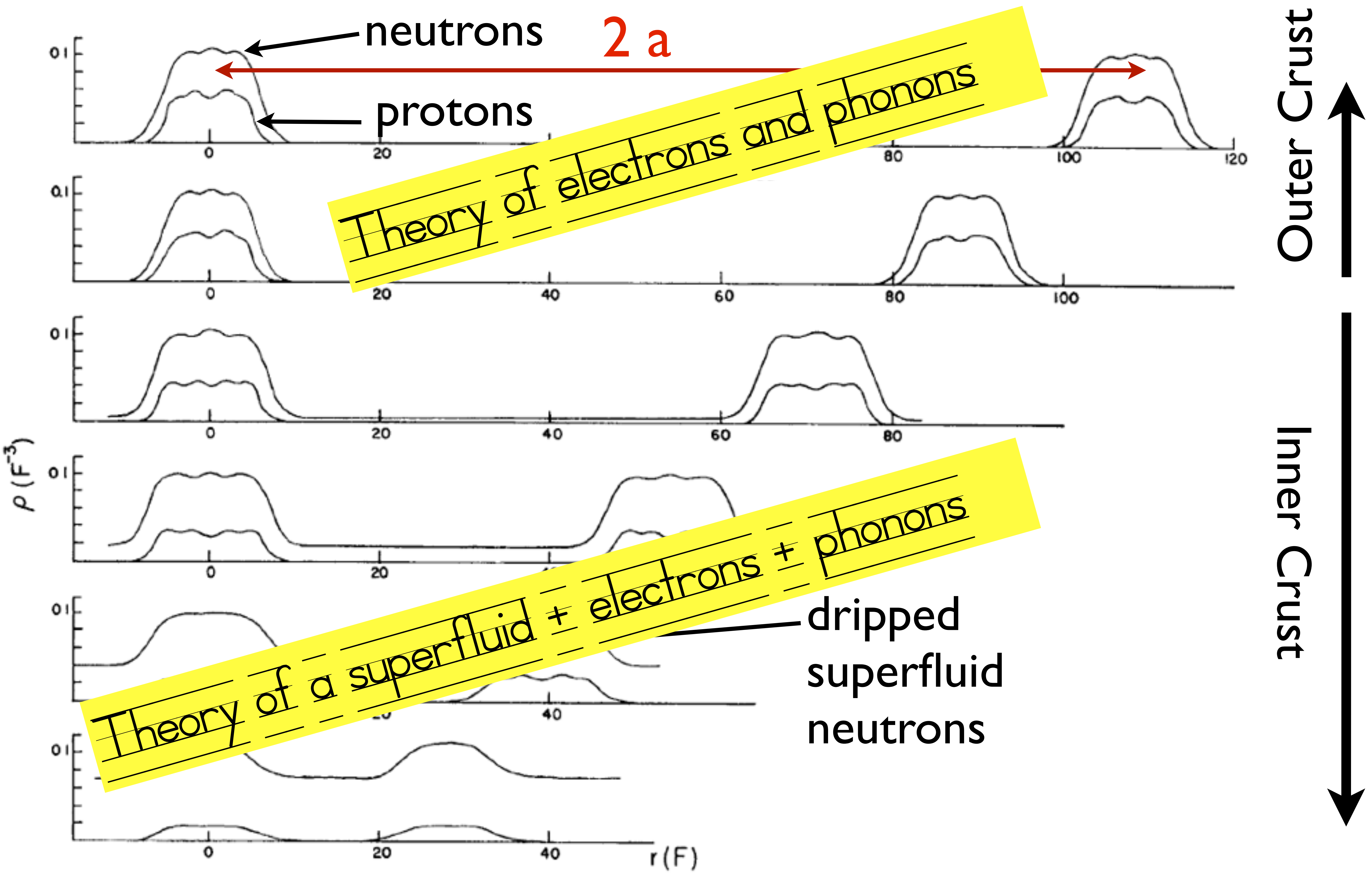

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Electrons are (nearly) free

- Electrons are dense,degenerate and relativistic.

$$n_e = Z\ n_I \qquad k_{\rm Fe} \approx E_{\rm Fe} \simeq 25 - 75\ {\rm MeV} \gg m_e$$

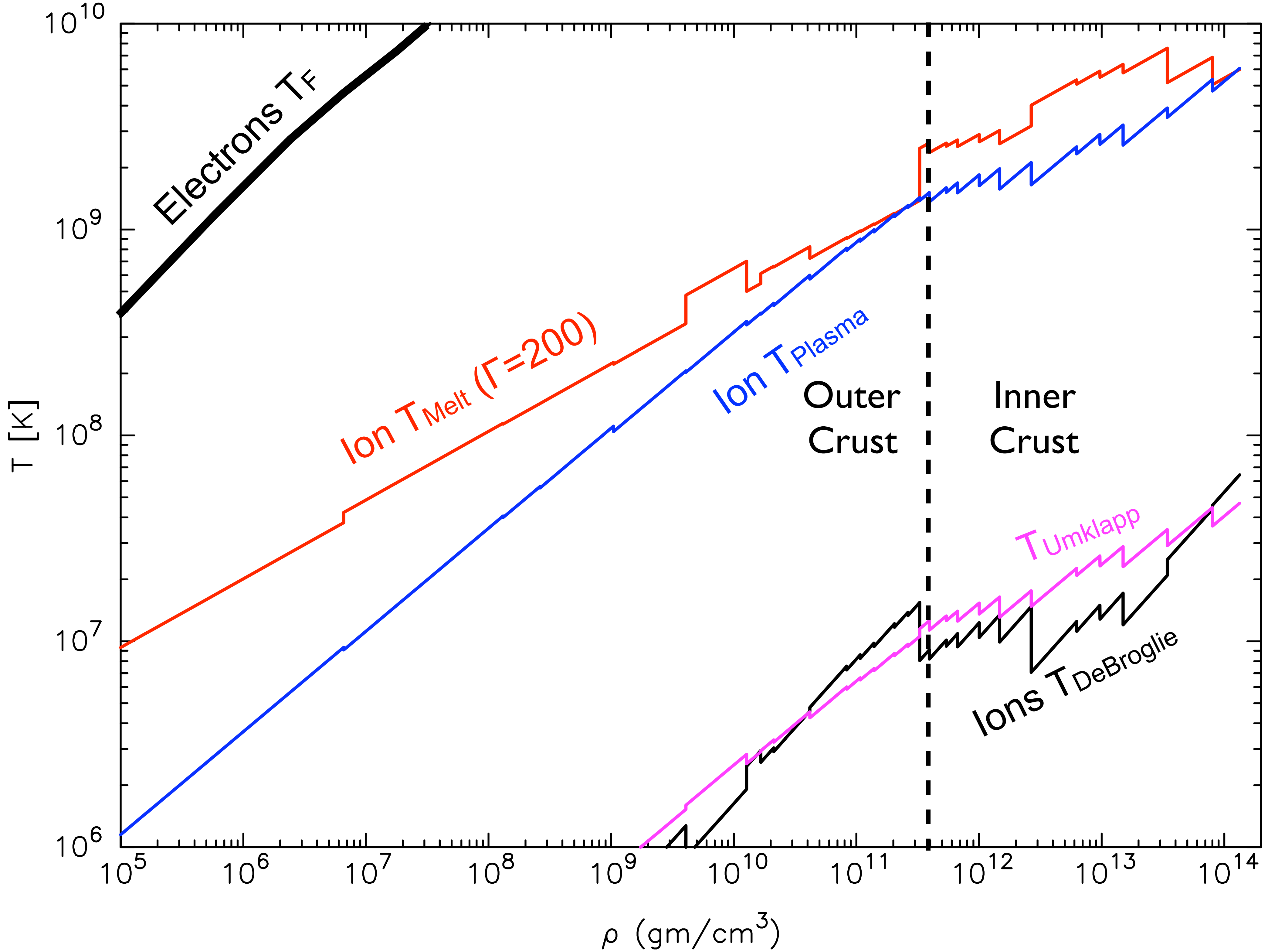
- Band gaps are small and restricted to small patches in the Fermi surface.

$$\frac{V_{\rm e-i}}{E_{\rm Fe}} \simeq \alpha_{\rm em}\ Z^{2/3} \ll 1 \qquad \frac{\delta_{\rm e}}{E_{\rm Fe}} \simeq \frac{4\alpha_{\rm em}}{3\pi} \approx 10^{-3}$$

- Pairing energy is negligible.

$$T_c \simeq \omega_p^{\rm ion}\ \exp\left(-\frac{v_{Fe}}{\alpha_{\rm em}}\right) \approx 0$$

Relevant Temperature Scales in the Crust



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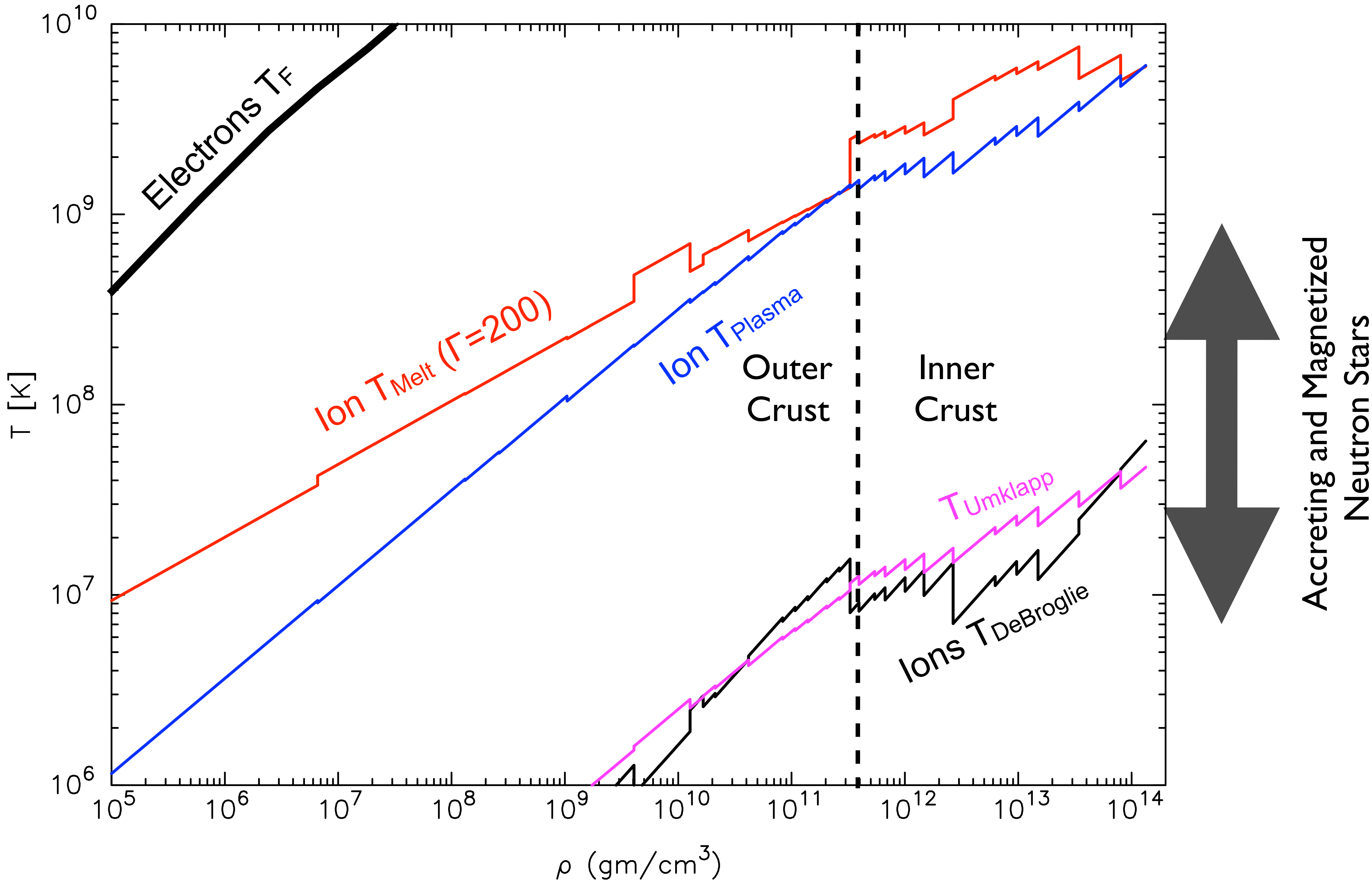

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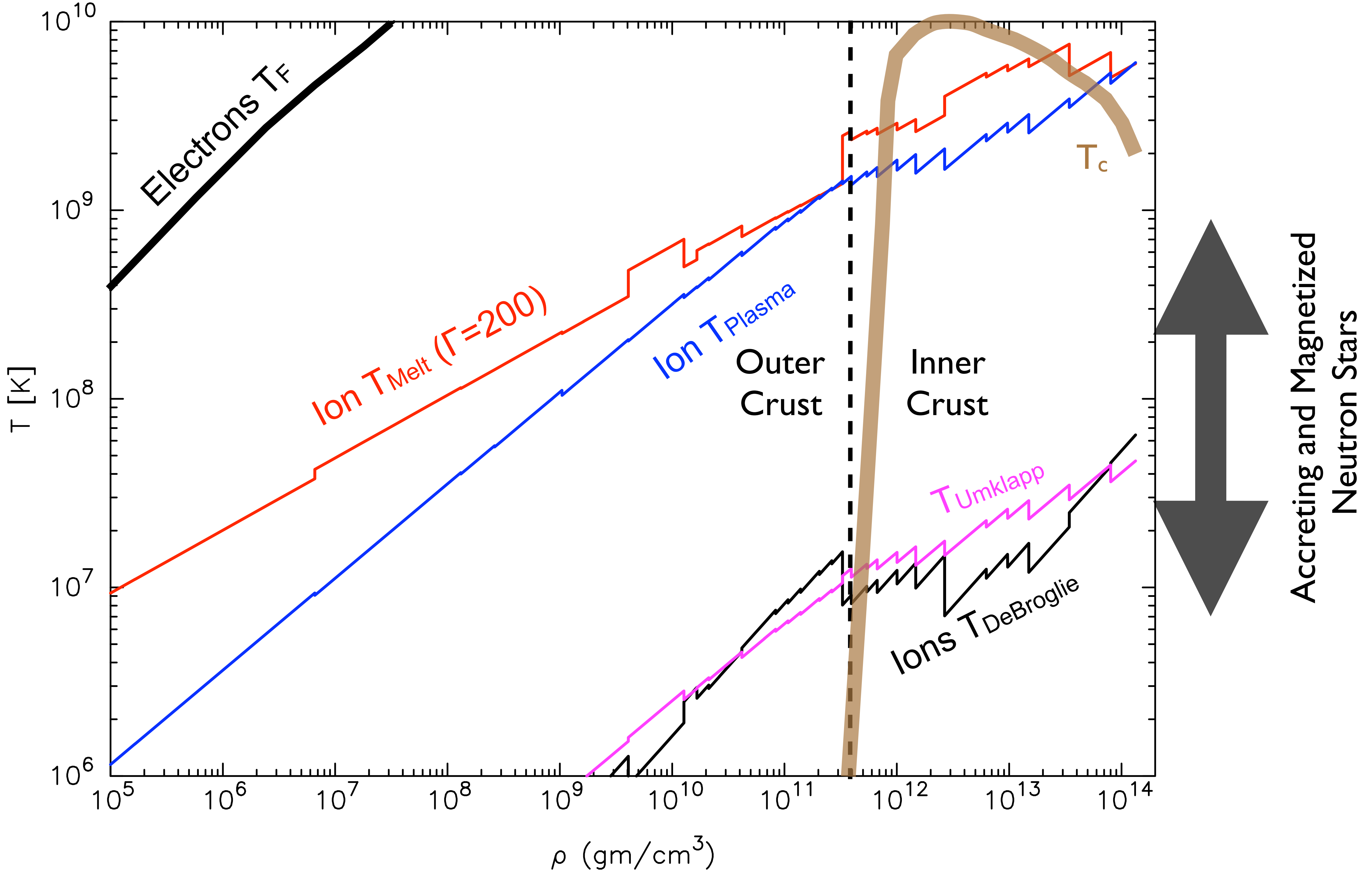
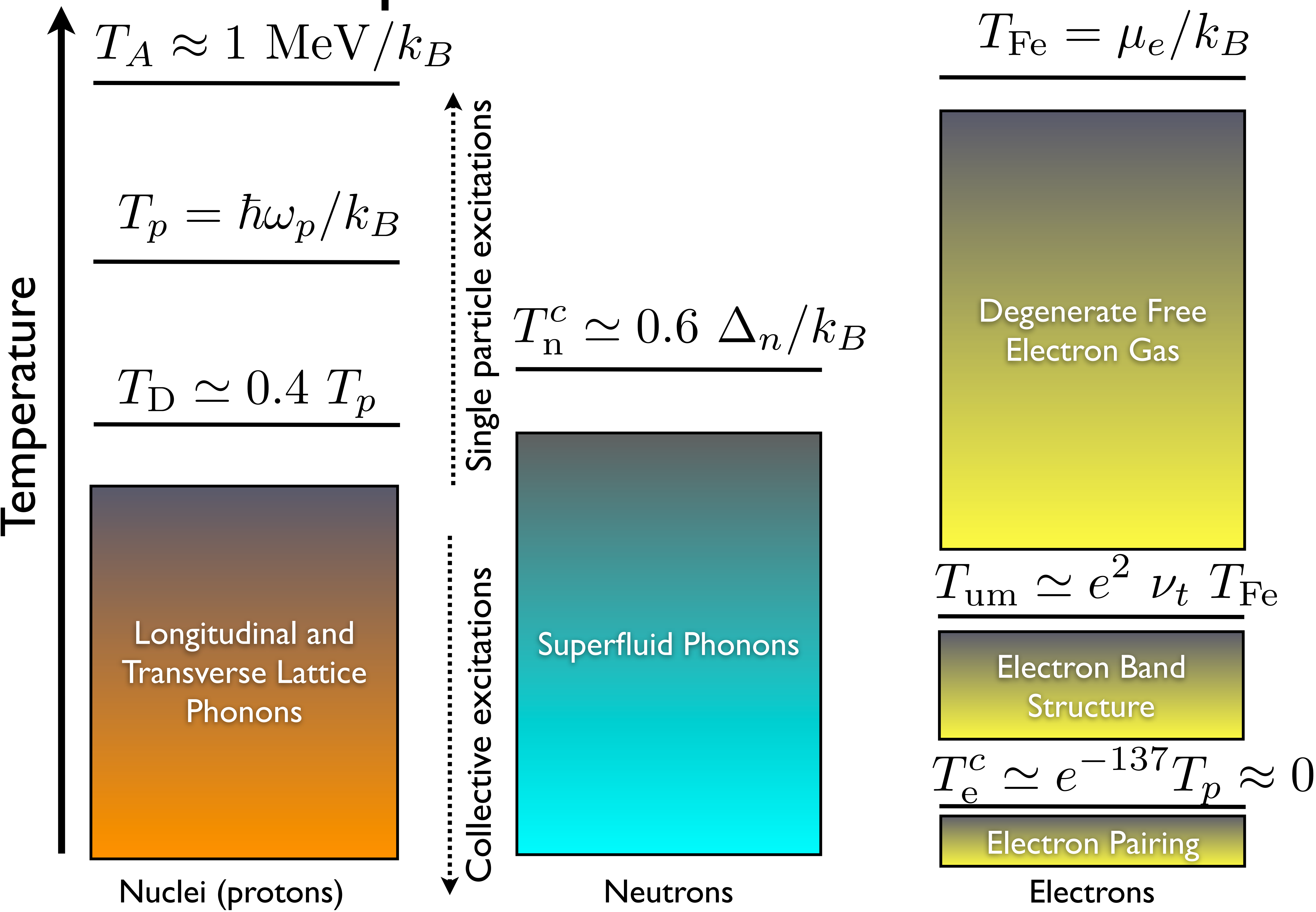


Separation of Scales



Low Energy Theory of Phonons

Proton (clusters) move collectively on lattice sites.
Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the fluctuation of the phase of the condensate.

$$\langle \psi_{\uparrow}(r,t)\psi\downarrow(r,t)\rangle = |\Delta(r,t)|\ \exp\left(2i\phi(r,t)\right)$$

Collective coordinates:

Vector Field: $\xi_i(r,t)$
Scalar Field: $\phi(r,t)$

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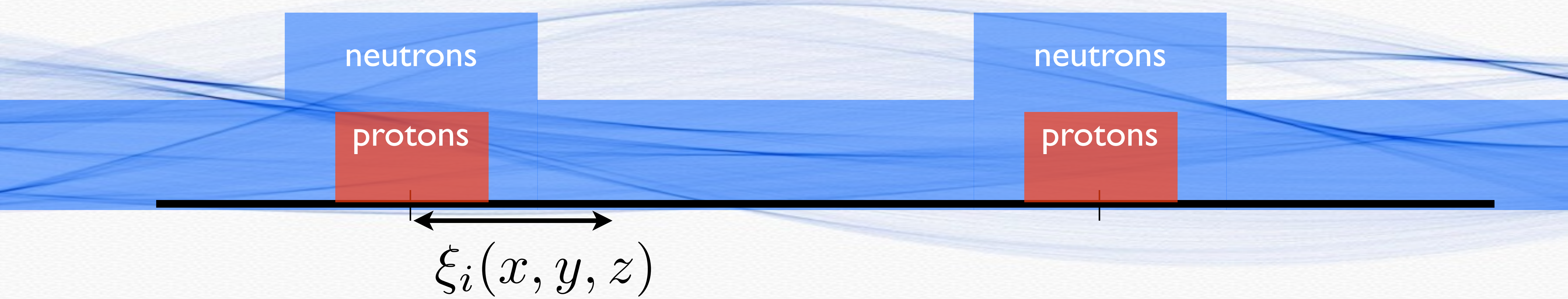

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Symmetries & Derivative Expansion

The low energy theory must respect symmetries of the underlying Hamiltonian

$$\begin{cases} \xi^{a=1..3}(\mathbf{r},t) \to \xi^{a=1..3}(\mathbf{r},t) + a^{a=1..3} \\ \phi(\mathbf{r},t) \to \phi(\mathbf{r},t) + \theta \end{cases}$$

Only derivative terms are allowed. Lagrangian density for the phonon system with cubic symmetry:

$$\mathcal{L} = \frac{f_\phi^2}{2}(\partial_0\phi)^2 - \frac{v_\phi^2 f_\phi^2}{2}(\partial_i\phi)^2 + \frac{\rho}{2}\partial_0\xi^a\partial_0\xi^a - \frac{1}{4}\mu(\xi^{ab}\xi^{ab}) - \frac{K}{2}(\partial_a\xi^a)(\partial_b\xi^b)$$
$$- \frac{\alpha}{2}\sum_{a=1..3}(\partial_a\xi^a\partial_a\xi^a) + g_{\rm mix} f_\phi\sqrt{\rho}\,\partial_0\phi\partial_a\xi^a + \cdots \quad ,$$

where $\xi^{ab} = (\partial_a\xi^b + \partial_b\xi^a) - \frac{2}{3}\partial_c\xi^c\delta^{ab}$

Cirigliano, Reddy, Sharma 2011

Identifying the Low Energy Constants

- LECs must be related to thermodynamic properties.
- Each gradient produces a unique deformation of the ground state.
- The energy cost associated with these (small) deformations provide the LECs.

For a rigorous derivation of LECs in terms of thermodynamic derivatives see arXiv:1102.5379

Cirigliano, Reddy, Sharma 2011

The Coupled System at Leading Order

$$\mathcal{L}_{n+p} = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2} v_s^2 \ (\partial_i \phi)^2 + \frac{1}{2}(\partial_t \xi_i)^2 - \frac{1}{2}(c_l^2 - g^2) \ (\partial_i \xi_i)^2$$
$$+ \ g \ \partial_t \phi \ \partial_i \xi_i + \tilde{\gamma} \ \partial_i \phi \ \partial_t \xi_i$$

Velocities : $v_s^2 = \dfrac{n_f}{m \chi_n}$ $\qquad c_l^2 = \dfrac{K + 4\mu_s/3}{m(n_p + n_b)}$

Entrainment: protons drag neutrons.

- Bound neutrons: $n_b = \gamma \ n_n$
- Free neutrons: $n_f = n_n \ (1 - \gamma)$

Longitudinal lattice phonons and superfluid phonons are coupled:

$$g = n_p \ E_{np} \ \sqrt{\frac{\chi_n}{m(n_p + n_b)}} \qquad \tilde{\gamma} = \frac{-n_b \ v_s}{\sqrt{(n_p + n_b) n_f}}$$

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$$+\ g\ \partial_t \phi\ \partial_i \xi_i + \tilde{\gamma}\ \partial_i \phi\ \partial_t \xi_i$$

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Transverse lattice phonons:

$$\mathcal{L}_t = \frac{1}{2}(\partial_t \xi_i)^2 - \frac{1}{2} c_t^2 (\partial_i \xi_j + \partial_j \xi_i)^2 \quad \Rightarrow \quad c_t^2 = \frac{\mu_s}{m(n_p + n_b)}$$

Phonon mixing and drag

$$\mathcal{L}_{\mathrm{sPh-lPh}} = g\ \partial_0\phi\ \partial_i\xi_i + \gamma\ \partial_i\phi\ \partial_0\xi_i$$

density-density interaction:

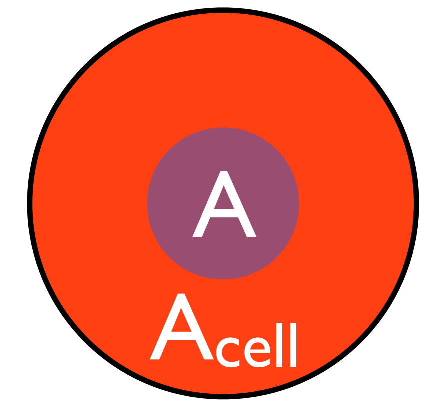
$$g = -\frac{n_p\ v_s}{\sqrt{n_f(n_p+n_b)}}\ \frac{\partial n_n}{\partial n_p}$$

velocity-velocity interaction:

$$\tilde{\gamma} = \frac{n_b\ v_s}{\sqrt{n_f(n_p+n_b)}}$$

Entrainment

Chamel (2005)
Carter, Chamel & Haensel (2006)



A=N+Z

$n_b \neq$ number of "bound" neutrons.

Bragg scattering off the lattice is important.

neutron single-particle energy

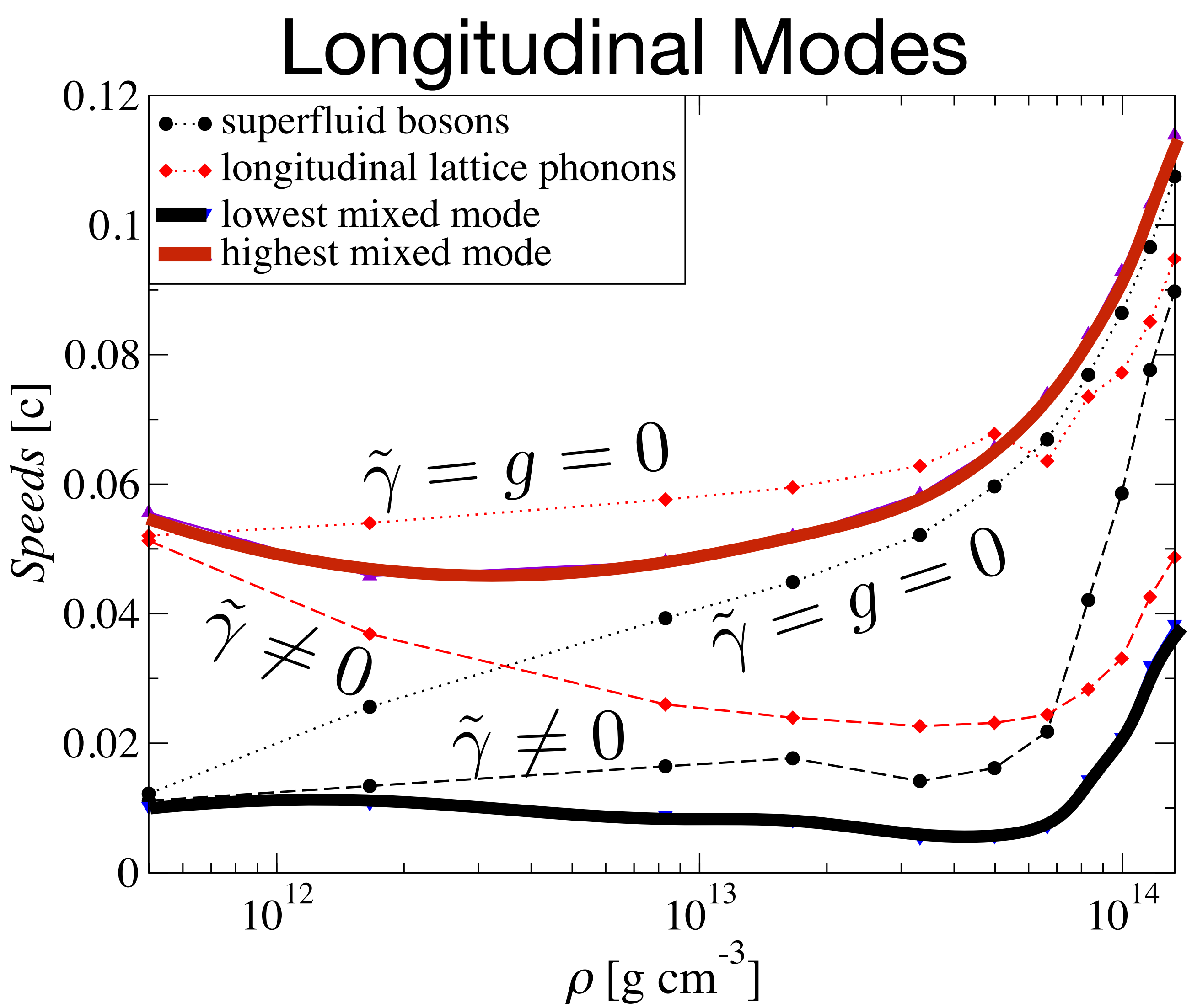
$$n_f = \frac{m}{24\pi^3} \sum_\alpha \int_F |\nabla_\mathbf{k} \epsilon_{\alpha,\mathbf{k}}| \, d\mathcal{S}^\alpha$$

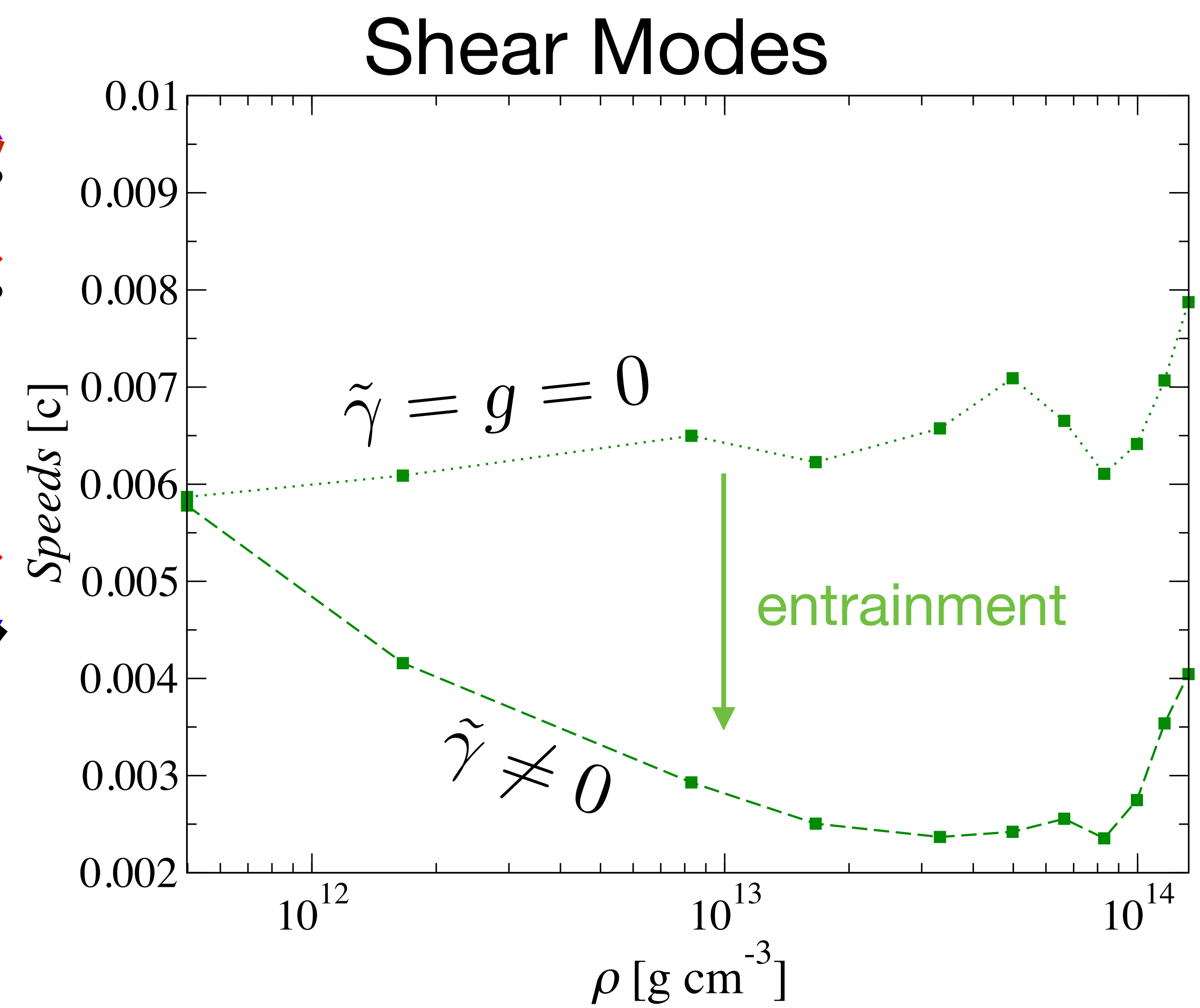
$$n_b = n_n - n_f$$

Complex interplay of nuclear and band structure effects.
The nuclear surface and disorder are likely to play a role.

Longitudinal lattice phonons and superfluid phonons are strongly coupled by entrainment.

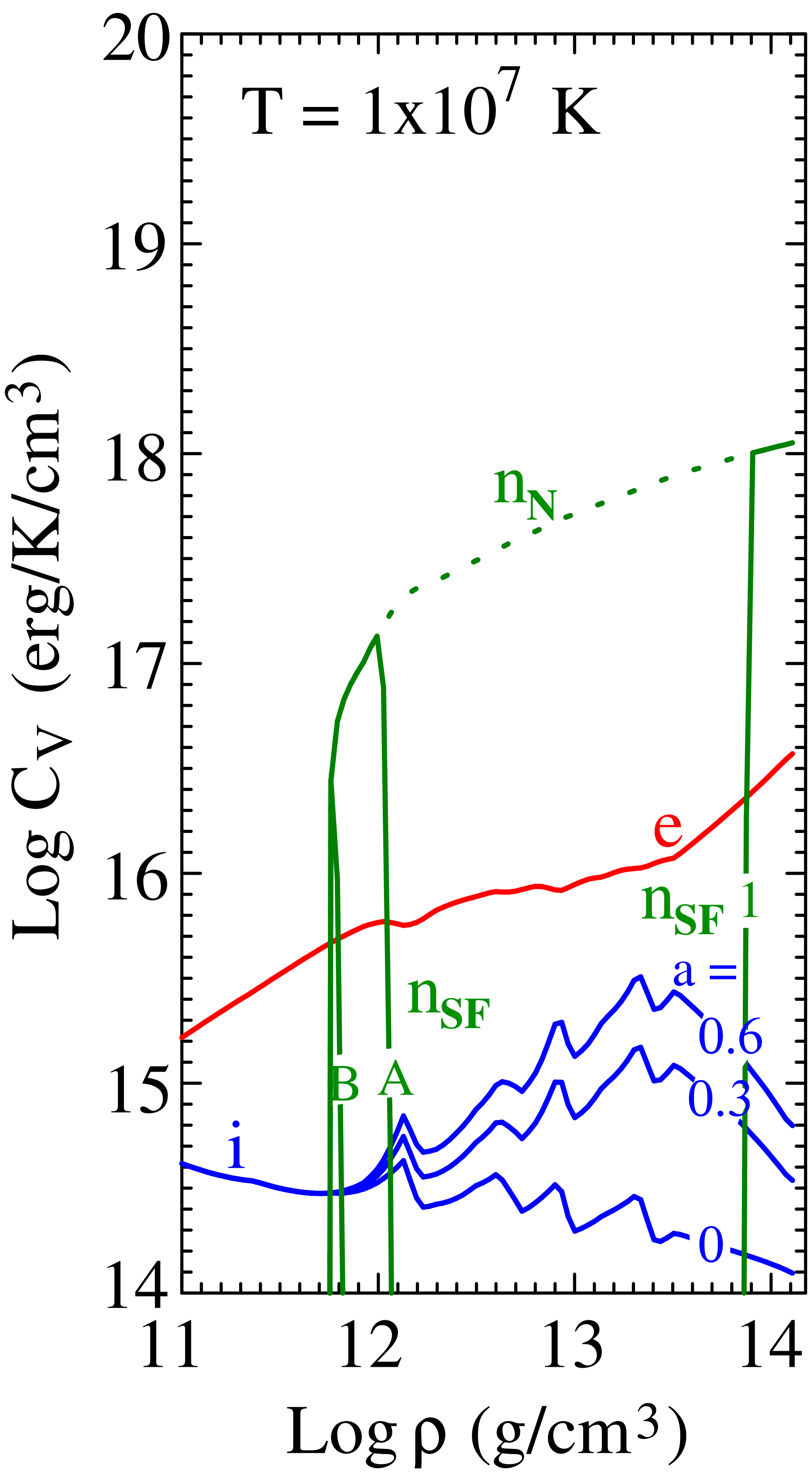
Mixed & Entrained Modes





Longitudinal modes are strongly mixed due to strong interactions.
Shear mode velocity is reduced due to entrainment.

Crustal Specific Heat



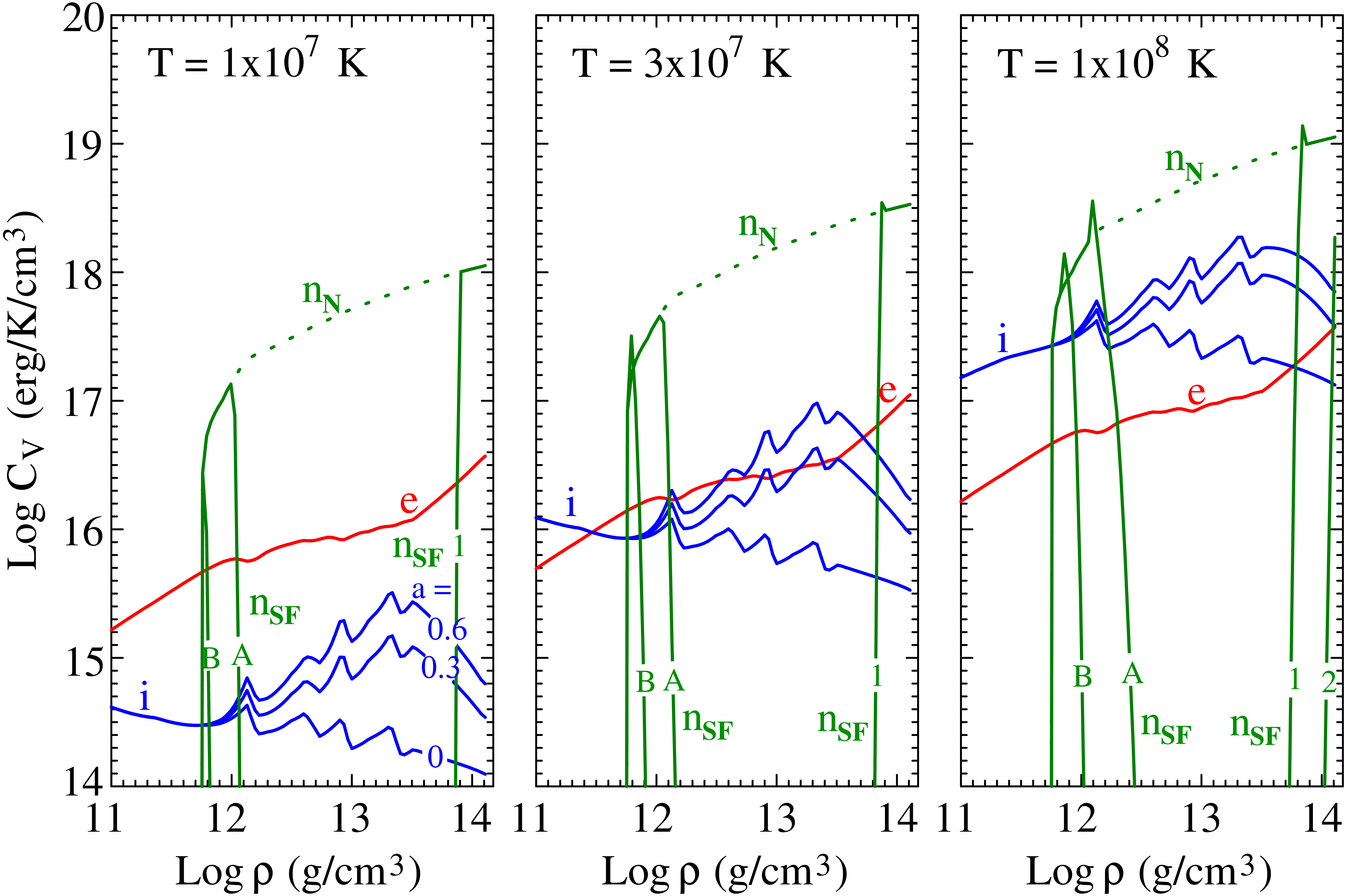
Electrons: $C_v^e = \frac{1}{3}\mu_e^2\, T$

Ions: $C_v^{\rm lph} = \frac{2\pi^2}{15}\left(\frac{T^3}{v_l^3} + \frac{2\,T^3}{v_t^3}\right)$

Neutrons:
$$\begin{cases} C_v^{\rm sph} = \frac{2\pi^2}{15}\frac{T^3}{v_\phi^3} & (T \ll T_c) \\ C_v^{\rm neutron} = \frac{1}{3}\, m_n\, k_{\rm Fn}\, T & (T > T_c) \end{cases}$$

Page & Reddy (2012)

Crustal Specific Heat

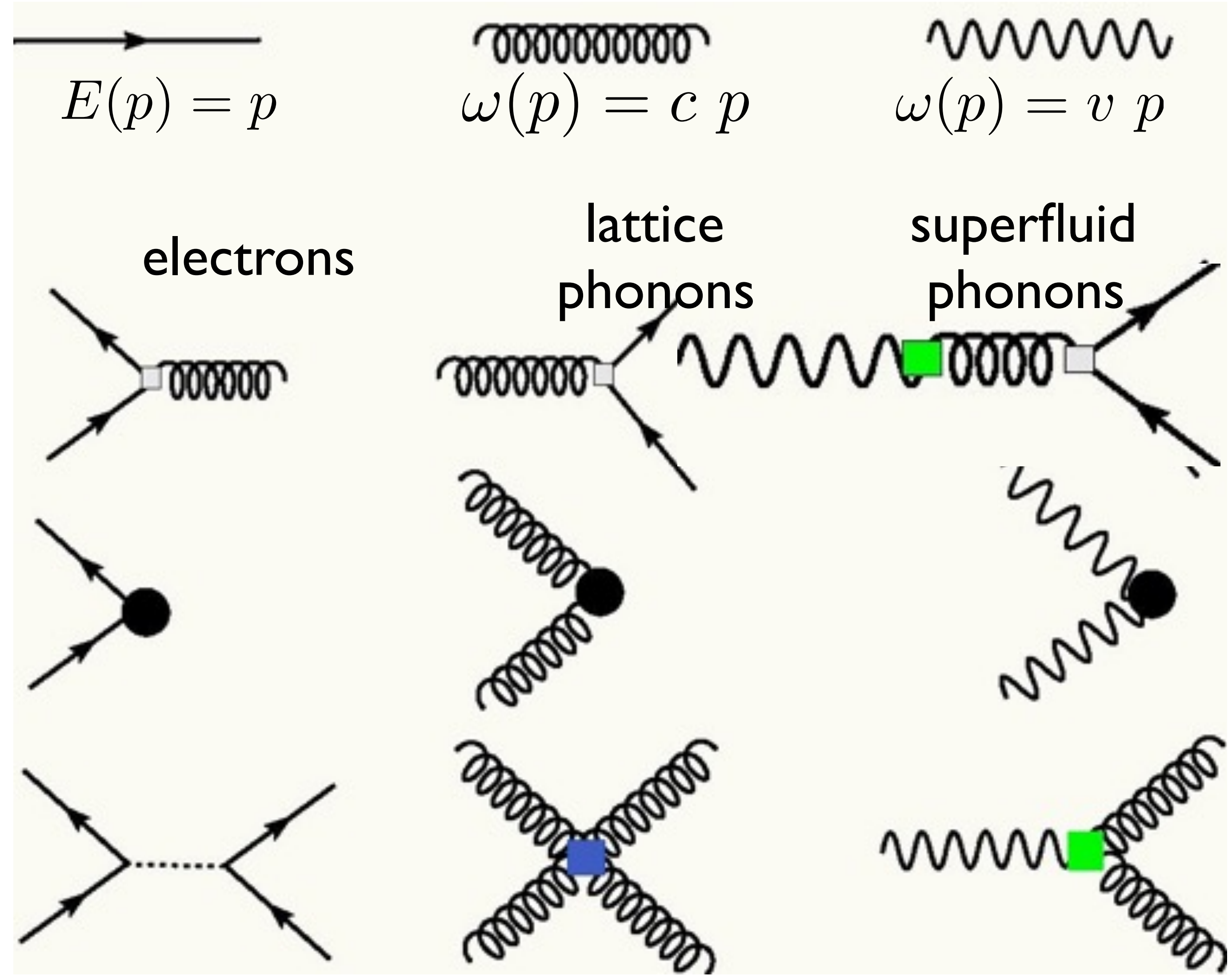


Page & Reddy (2012)

Transport: Thermal Conduction

$$\kappa = \frac{1}{3}\, C_v \times v \times \lambda$$

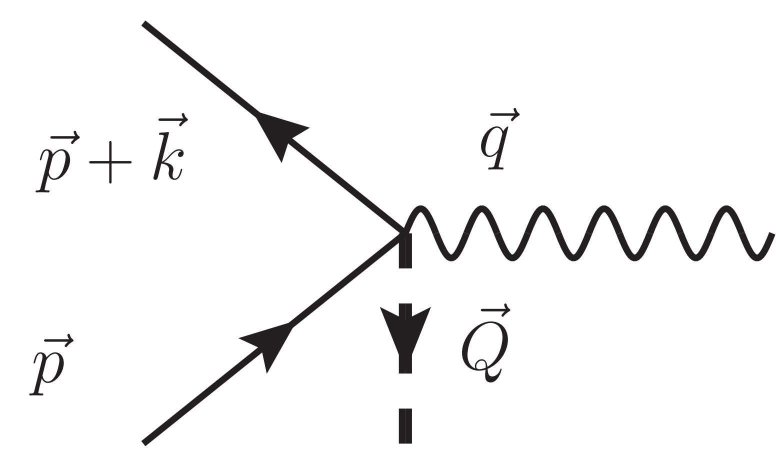
- Dissipative processes:



Cirigliano, Reddy & Sharma (2011)

- Umklapp is important:

$$\frac{k_{\rm Fe}}{q_{\rm D}} = \left(\frac{Z}{2}\right)^{1/3} > 1$$

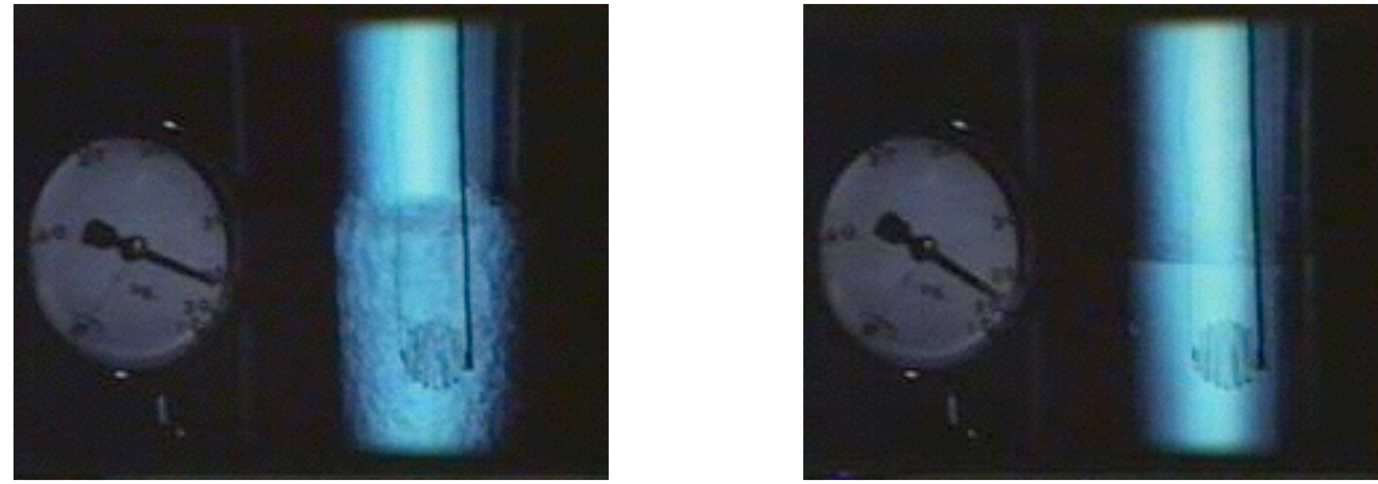


Electron Bragg scatters and emits a transverse phonon.

Flowers & Itoh (1976)

Superfluid Conduction

Photographs: JF Allen and JMG Armitage (St Andrews University 1982).



$T>T_c$ $\qquad$ $T<T_c$

Its impossible to sustain a temperature gradient in bulk superfluid helium !

$$\vec{Q} = S^{(\mathrm{sPh})} T \vec{v}_n$$

$$S^{(\mathrm{sPh})} = \frac{1}{3} C_v^{(\mathrm{sPh})} = \frac{2\pi^2}{15\ c_s^3} T^3$$

Two fluid model: Counter-flow transports heat.
(The superfluid phonon fluid)

The velocity is limited only by fluid dynamics: (i) boundary shear viscosity or (ii) superfluid turbulence.

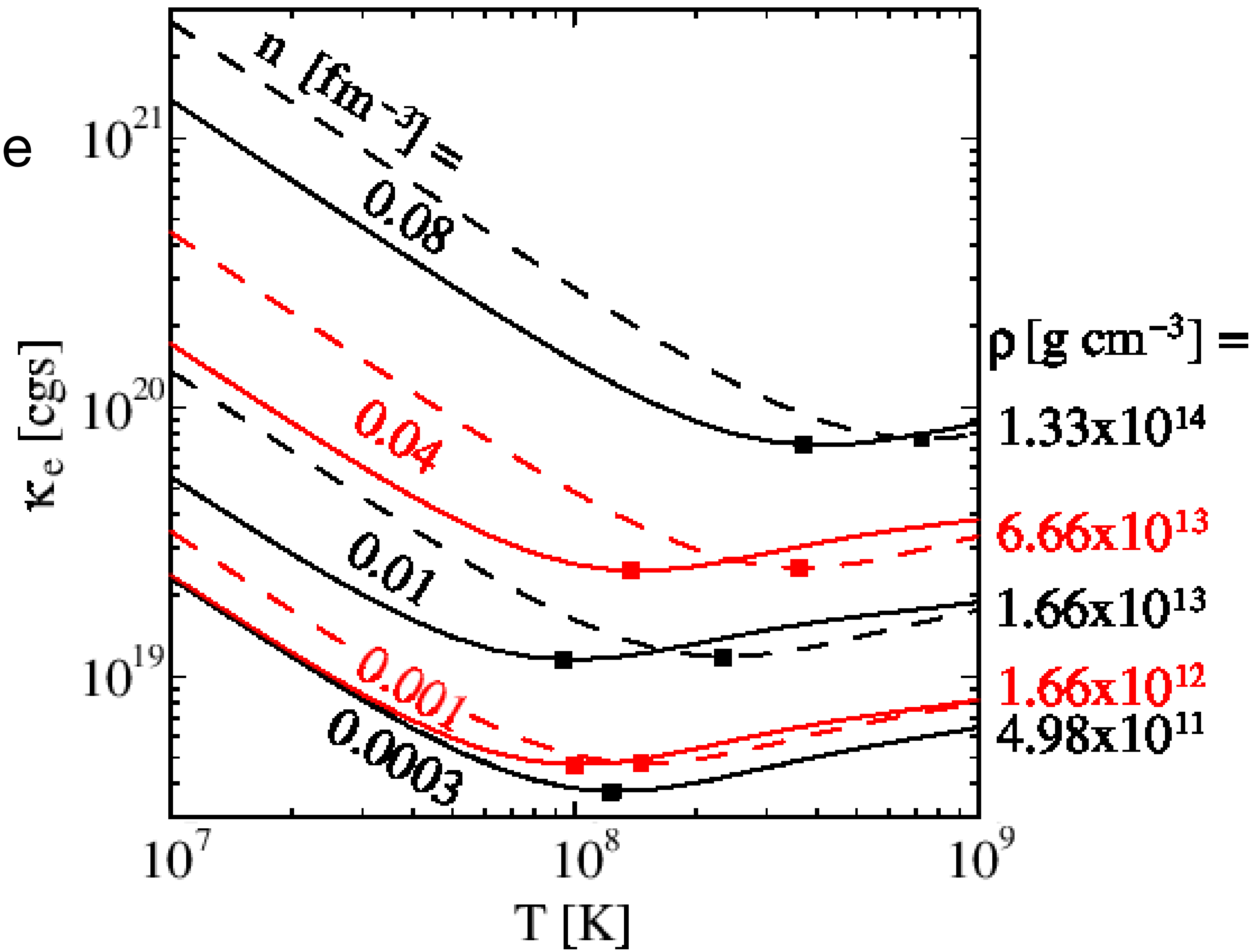
Why does this not occur in neutron stars ?
Answer: Fluid motion is damped by electrons.

Aguilera, Cirigliano, Reddy & Sharma (2009)

Electron Conduction

$$\kappa_e = \frac{1}{3} C_V v_{\mathrm{Fe}} \lambda_\kappa = \frac{\pi^2 T n_e}{3\epsilon_{\mathrm{Fe}}} \frac{1}{\nu_\kappa}$$

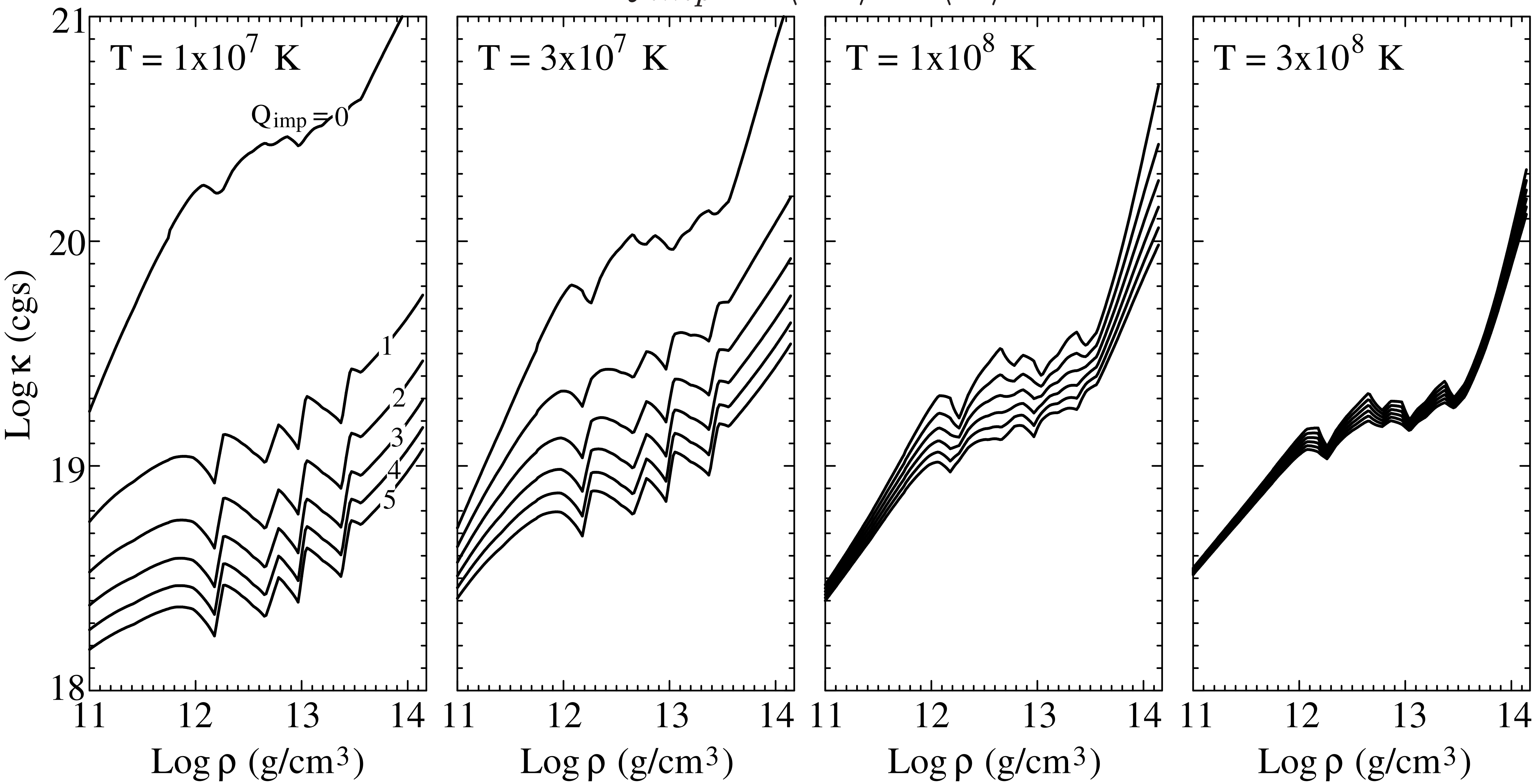
Changes to the phonon spectrum due to mixing and entrainment suppress the conductivity.



Electron Conduction

$$\kappa_e = \frac{1}{3} C_V v_{\mathrm{Fe}} \lambda_\kappa = \frac{\pi^2 T n_e}{3\epsilon_{\mathrm{Fe}}} \frac{1}{\nu_\kappa}$$

$$Q_{imp} = \langle Z^2 \rangle - \langle Z \rangle^2$$



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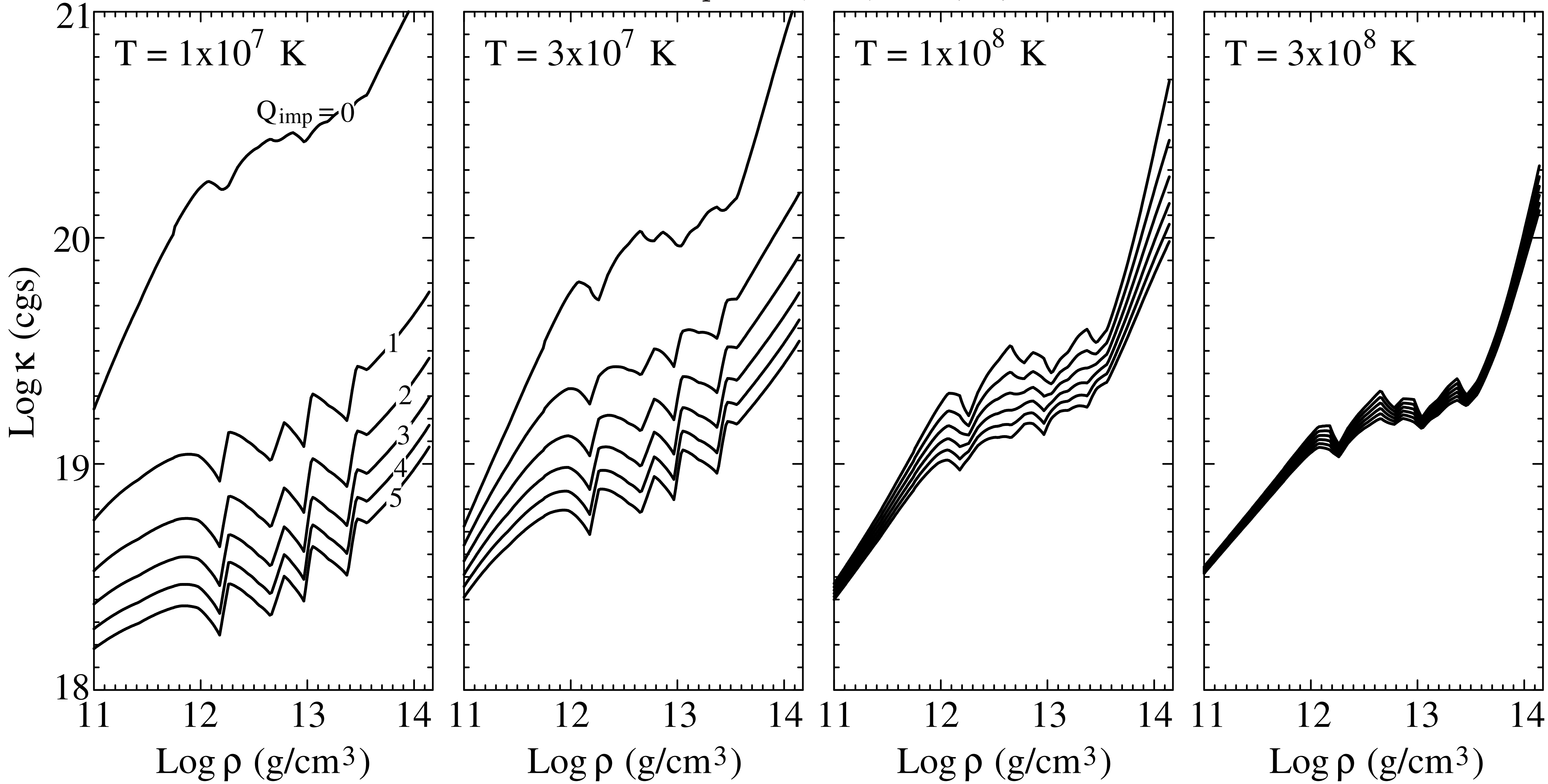


Impurity scattering is important at low temperature.

$$\nu_\kappa = \nu_\kappa^{ph} + \nu_\kappa^{imp}$$

Flowers & Itoh (1976)

Impurity Scattering in Coulomb Solids

Uncorrelated impurities:

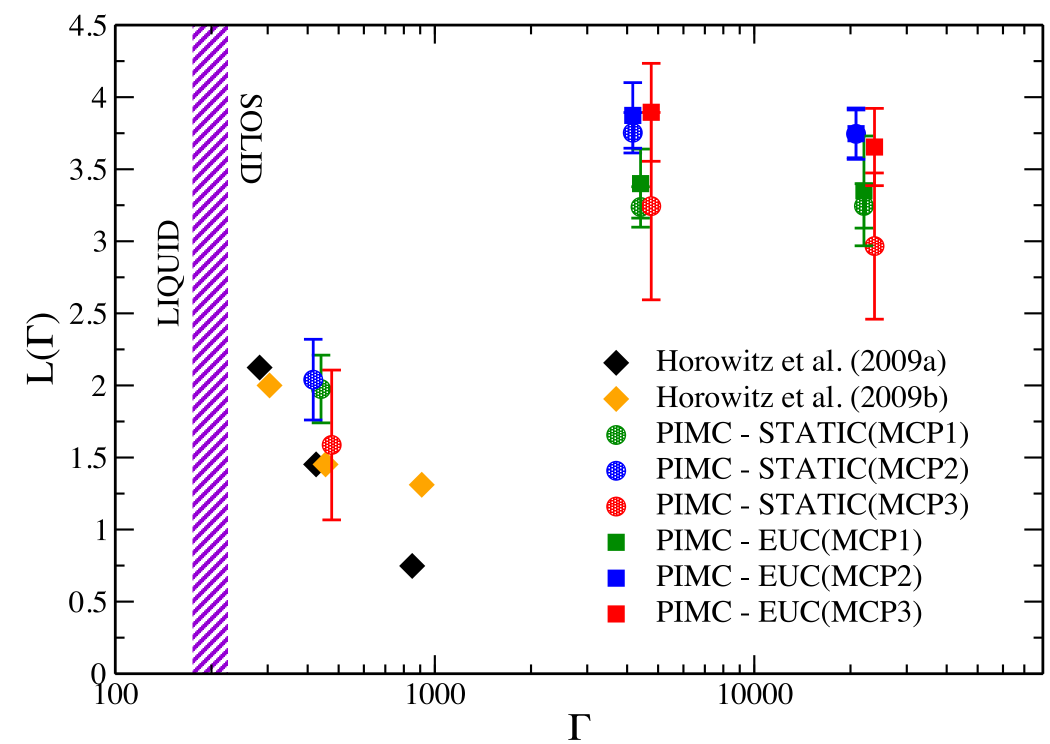
$$\nu_\kappa^{imp} = \nu_\kappa^0 \frac{Q_{imp}}{\langle Z \rangle} \Lambda_\kappa^{imp},$$

$$Q_{imp} = \langle Z^2 \rangle - \langle Z \rangle^2$$

$$\Lambda_\kappa^{imp} = \left(\frac{\alpha_{\rm em}}{\pi} + \frac{1}{2} \right) \ln \left(\frac{\alpha_{\rm em} + \pi}{\alpha_{\rm em}} \right) - 1$$

Path Integral Monte Carlo and Molecular Dynamics suggests that impurity distribution is not random.

$$Q_{\rm imp} \rightarrow \tilde{Q}_{\rm imp} = L(\Gamma)\ Q_{\rm imp}$$



Enhances impurity scattering by a factor of 2-4.

Roggero & Reddy (2016)

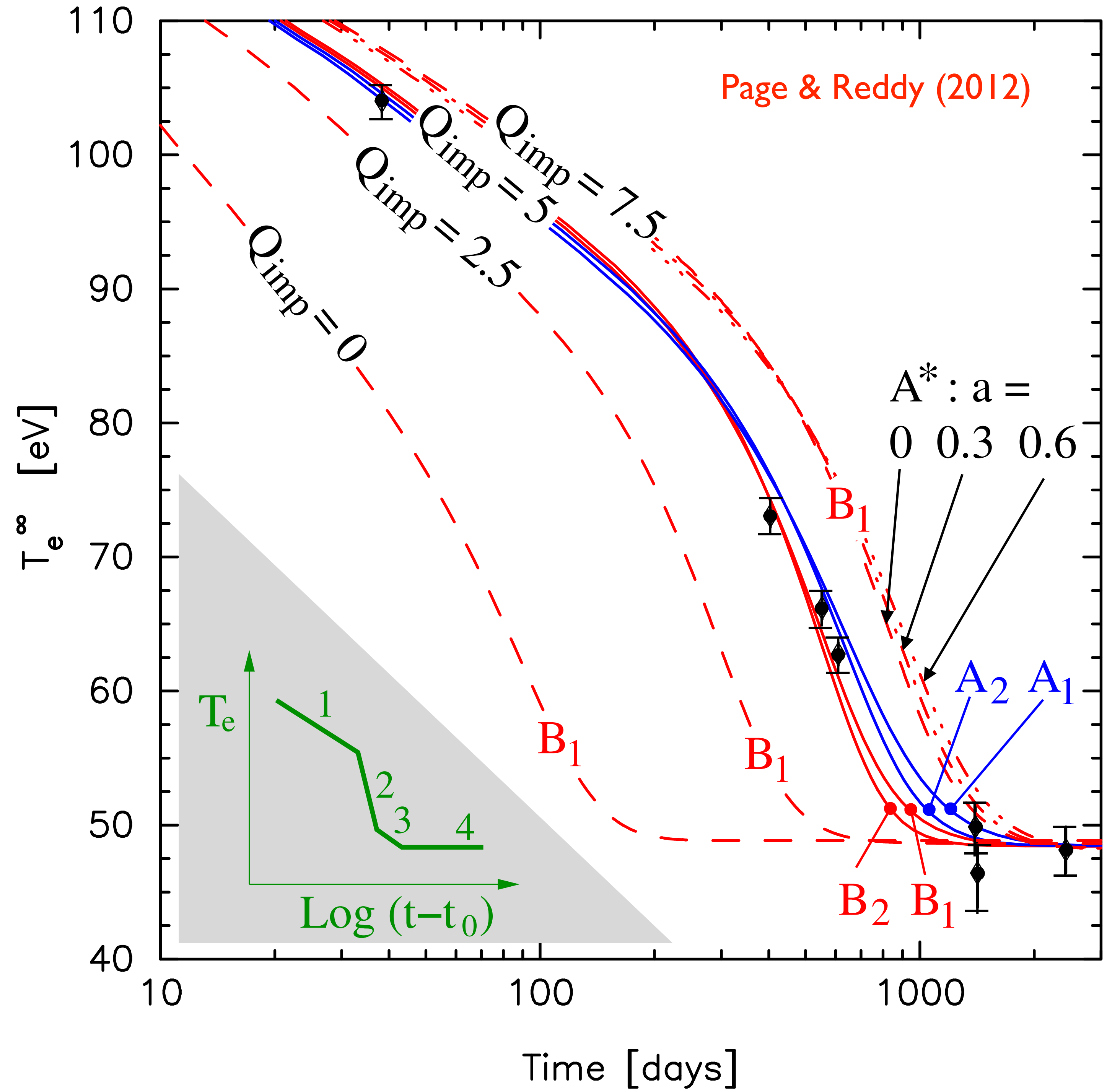
Unraveling Thermal Relaxation

- Late time signal is sensitive to inner crust thermal and transport properties.

  Shternin & Yakovlev (2007)
  Brown & Cumming (2009)

- Variations in the pairing gap (changes the fraction of normal neutrons)may be discernible !

  Page & Reddy (2012)



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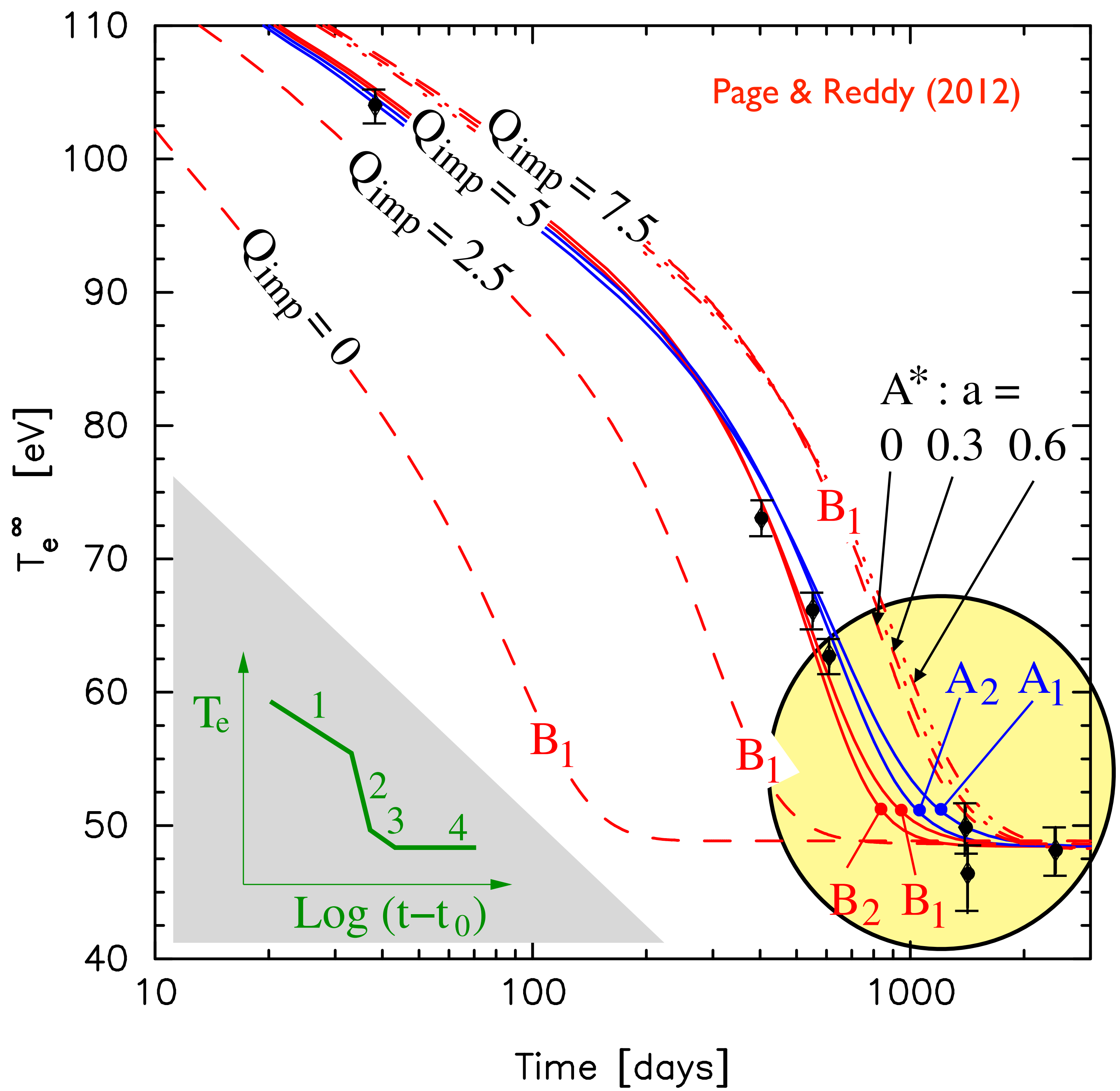


A: Low T_c - large normal fraction

B: High T_c- small normal fraction

Transport Properties of the Core

Low energy excitations in the core

Neutrons are superfluid ($T<T^n_c$): Electrons + 4 Goldstone modes (3 neutron modes and 1 electron-proton mode). Bedaque, Rupak, Savage, (2003), Bedaque, Nicholson (2013), Bedaque and Reddy (2013).

Neutrons are normal ($T>T^n_c$): Electrons, neutrons + 1 Goldstone boson (electron-proton mode).

Superfluid Phonons:

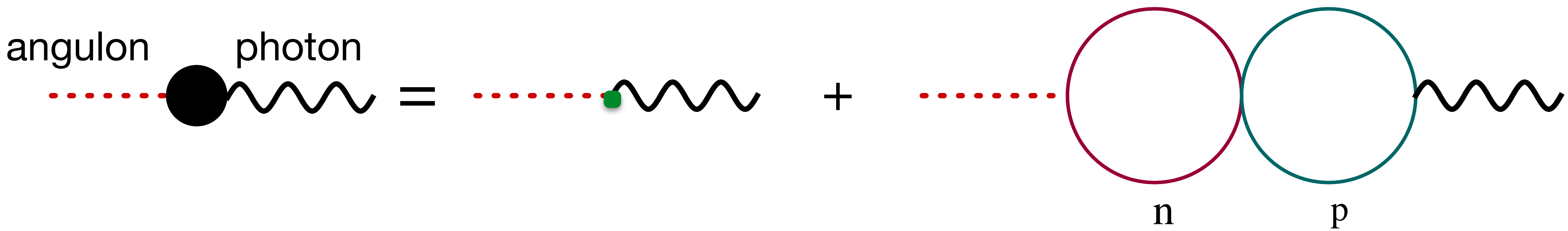
$$\mathcal{L}_{\rm phn} = \frac{1}{2}(\partial_0\phi)^2 - \frac{v_n^2}{2}(\partial_i\phi)^2 + \frac{1}{2}(\partial_0\xi)^2 - \frac{v_p^2}{2}(\partial_i\xi)^2 + v_{\rm np}^2\,\partial_0\phi\,\partial_0\xi + \frac{1}{f_{\rm ep}}\,\partial_0\xi\,\psi_{\rm e}^\dagger\,\psi_{\rm e} + \cdots\quad ,$$

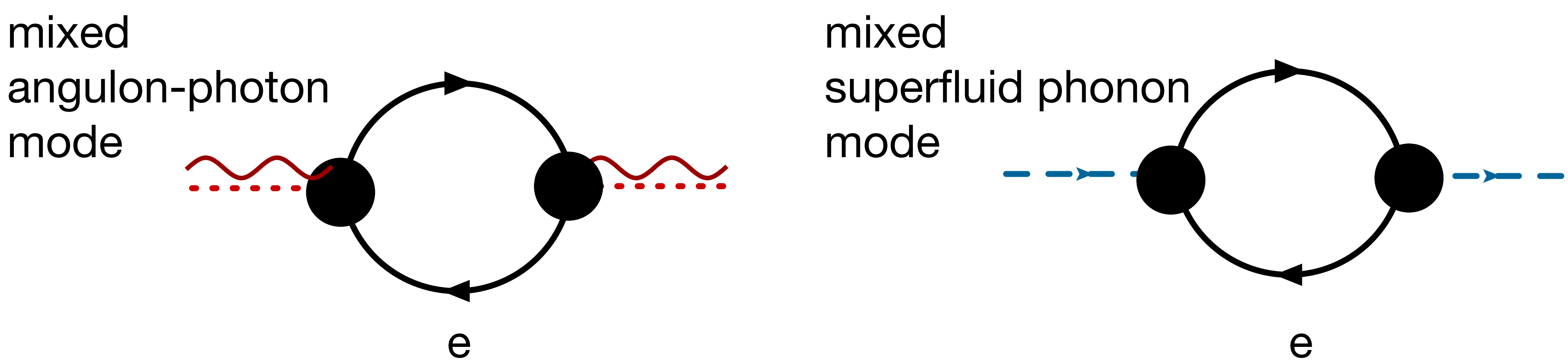
Angulons:

$$\mathcal{L}_{\rm ang} = \sum_{i=1,2}\left[\frac{1}{2}(\partial_0\beta_i)^2 - \frac{1}{2}{v_\perp^i}^2((\partial_x\beta_i)^2 + (\partial_y\beta_i)^2) + v_\parallel^2(\partial_z\beta_i)^2\right. + \frac{e g_n f_\beta}{2M\sqrt{-\nabla_\perp^2}}\left[\mathbf{B}_1\partial_0(\partial_y\beta_1 + \partial_x\beta_2) + \mathbf{B}_2\partial_0(\partial_x\beta_1 - \partial_y\beta_2)\right]$$

Bedaque and Reddy (2013),

Mixing and Damping of Goldstone Bosons

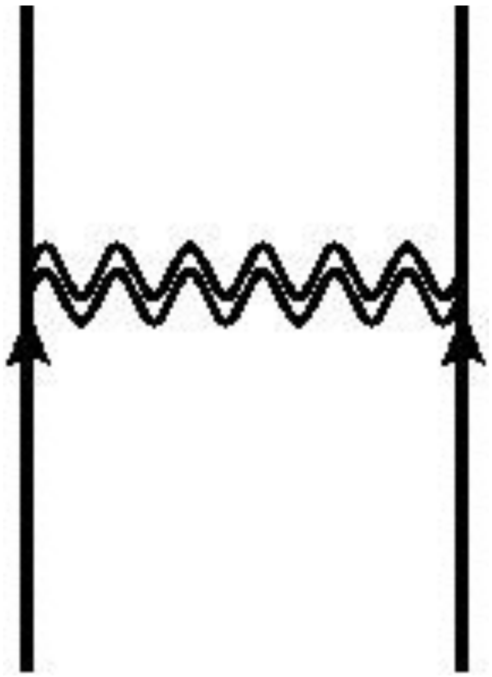




Modes decay rapidly due to the coupling to the large density of electron-hole states. Do not contribute to transport.

Bedaque and Reddy (2013)

Electron Scattering in the Core

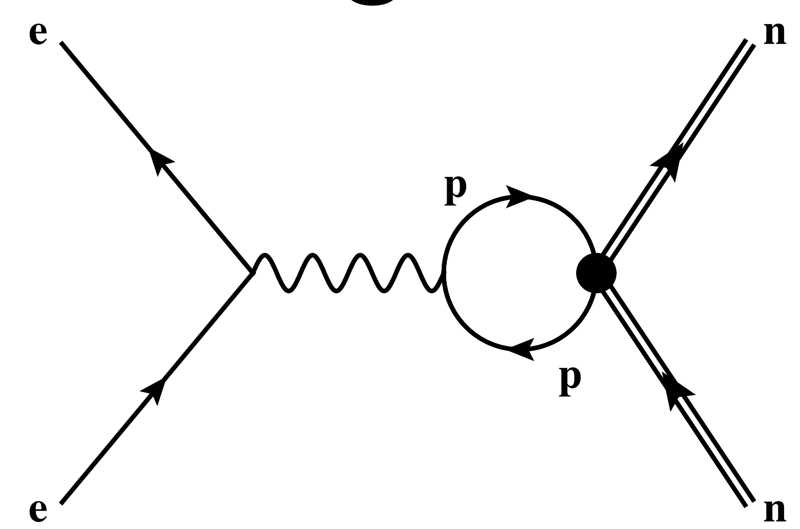


Pethick and Heiselberg (1993), Shternin and Yakovlev (2006,2007)

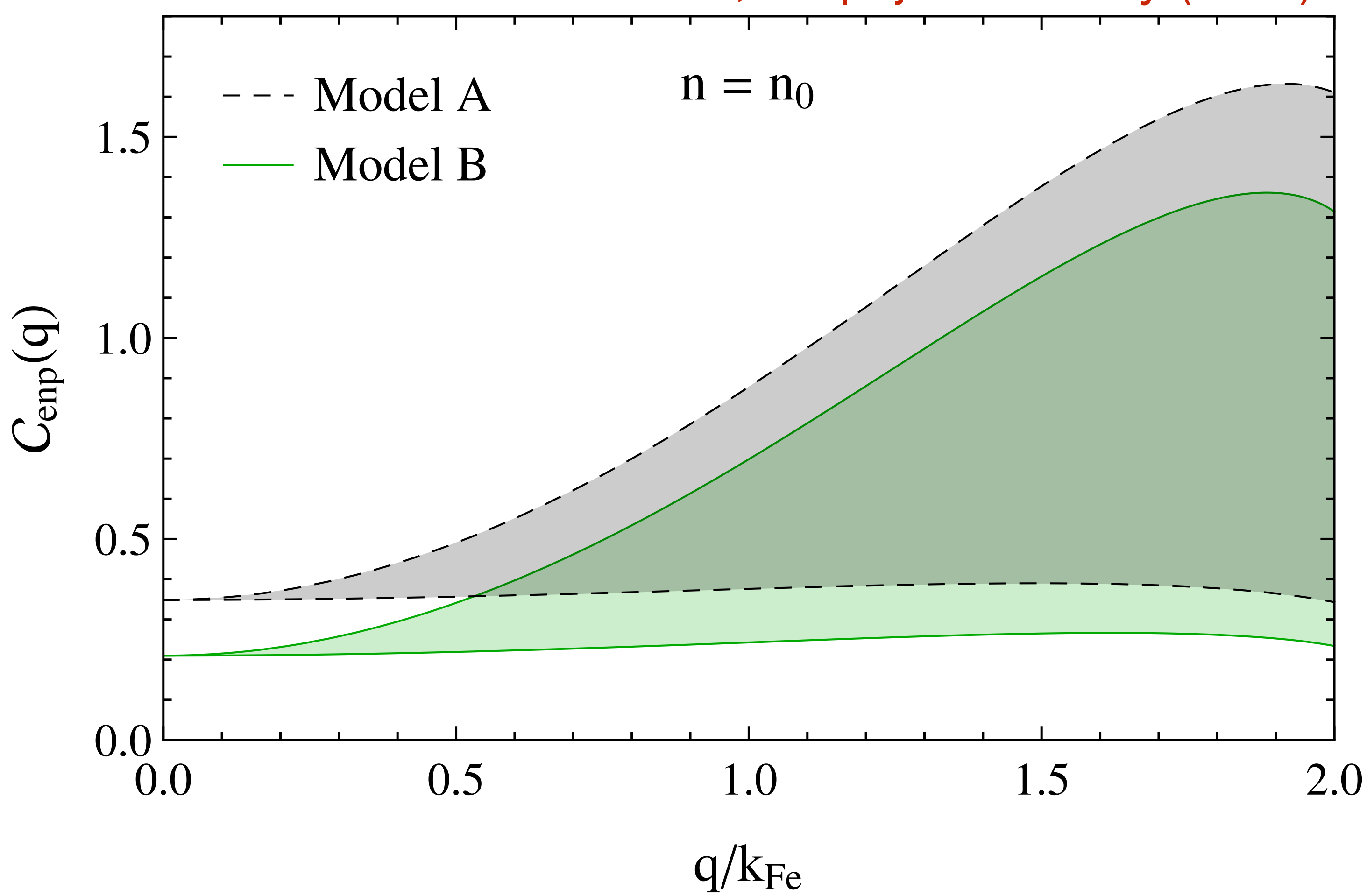
$$|M_{12}|^2 \propto \left| \frac{J^{(0)}_{1'1} J^{(0)}_{2'2}}{q^2 + \Pi_l} - \frac{\boldsymbol{J}_{t1'1} \cdot \boldsymbol{J}_{t2'2}}{q^2 - \omega^2 + \Pi_t} \right|^2$$

$$\Pi_t(\omega, q) \simeq \alpha_{em}\, k_{Fp}^2 \left(4\pi \frac{\Delta_p}{q} + 2i\, \frac{\omega}{q} \right)$$

Electron-Neutron Scattering



Bertoni, Rrapaj and Reddy (2014)



$$\mathcal{L}_{\gamma-\mathrm{n}} = -\sqrt{4\pi\alpha}\; V_{\mathrm{np}}\; \bar{n}\gamma_\mu n\; \Pi^{\mu\nu}_{\mathrm{p}} A_\nu$$

$$\mathcal{L}_{\mathrm{e-n}} = -\bar{e}\gamma_0 e\, \mathcal{U}_{\mathrm{enp}}(\omega, q)\; \bar{n}\gamma_0 n$$

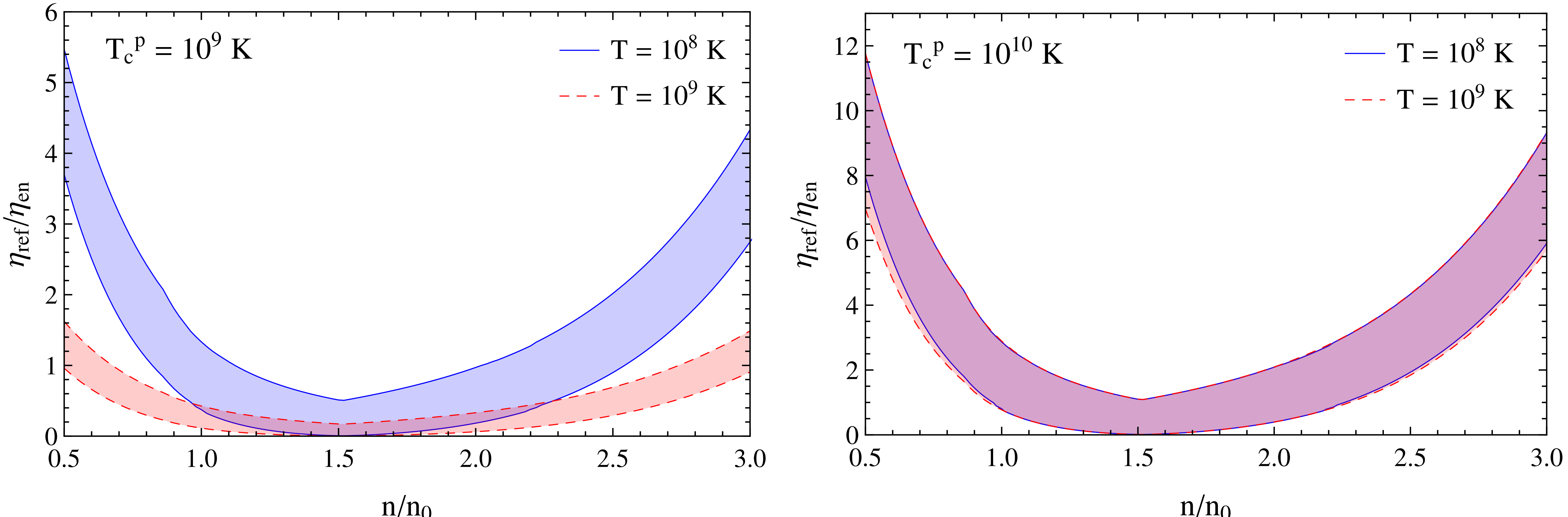
$$\mathcal{U}_{\mathrm{enp}}(\omega, q) = \frac{-4\pi\alpha\; \mathcal{C}_{\mathrm{enp}}(\omega, q)}{q^2 + q^2_{\mathrm{TF}}}$$

$$\mathcal{C}_{\mathrm{enp}}(\omega, q) = V_{\mathrm{np}}(q)\chi_p(\omega, q)$$

$$\chi_p(\omega, q) = \mathcal{R}e\; \Pi^L_p(\omega, q) = \mathcal{R}e\; \int dt\; e^{i\omega t} \int d\mathbf{r}\; e^{-i\mathbf{q}\cdot\mathbf{r}}\; \langle n_p(\mathbf{r}, t) n_p(0,0)\rangle$$

$$\eta_{\rm en} = \frac{k_{\rm Fe}^4}{15\pi^2} \langle \lambda_{en} \rangle_\eta$$

$$\eta_{\rm total} = \left(\frac{1}{\eta_{ee}} + \frac{1}{\eta_{ep}} + \frac{1}{\eta_{en}} \right)^{-1} \qquad \eta_{\rm ref} = \left(\frac{1}{\eta_{ee}} + \frac{1}{\eta_{ep}} \right)^{-1}$$



Bertoni, Rrapaj and Reddy (2014)

Summary

- Accreting neutron stars provide a unique opportunity to study thermal and transport properties.
- Thermal relaxation in neutron stars is sensitive to the low temperature properties of the crust.
- Thermal and transport properties of the inner crust (super-solid) can be calculated in terms of a few low-energy constants.
- Goldstone bosons in the crust and the core can decay into electron-hole states - this limits their contribution to transport.
- The induced interactions between electrons and neutrons can be relevant in the neutron star core.