Phases of Dense Matter

INT, Seattle, July 2016

Nuclear matter from an effective lattice theory of heavy QCD

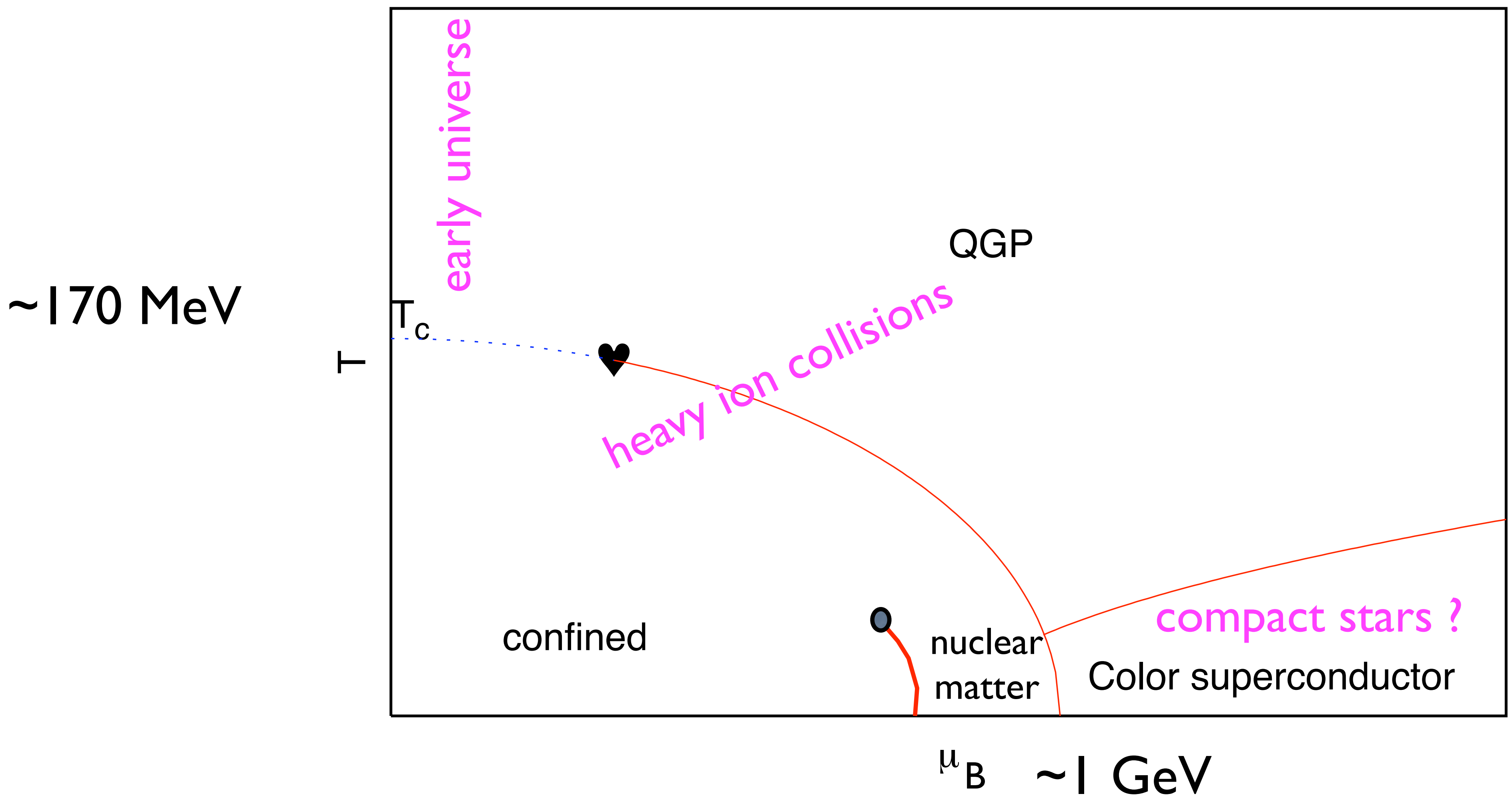
HIC for FAIR
Helmholtz International Center

Owe Philipsen

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- A 3d effective theory for heavy QCD: Strong coupling + hopping expansions
- The (heavy) nuclear equation of state in the continuum
- A 4d effective theory for chiral QCD at strong coupling

QCD phase diagram: theorist's science fiction

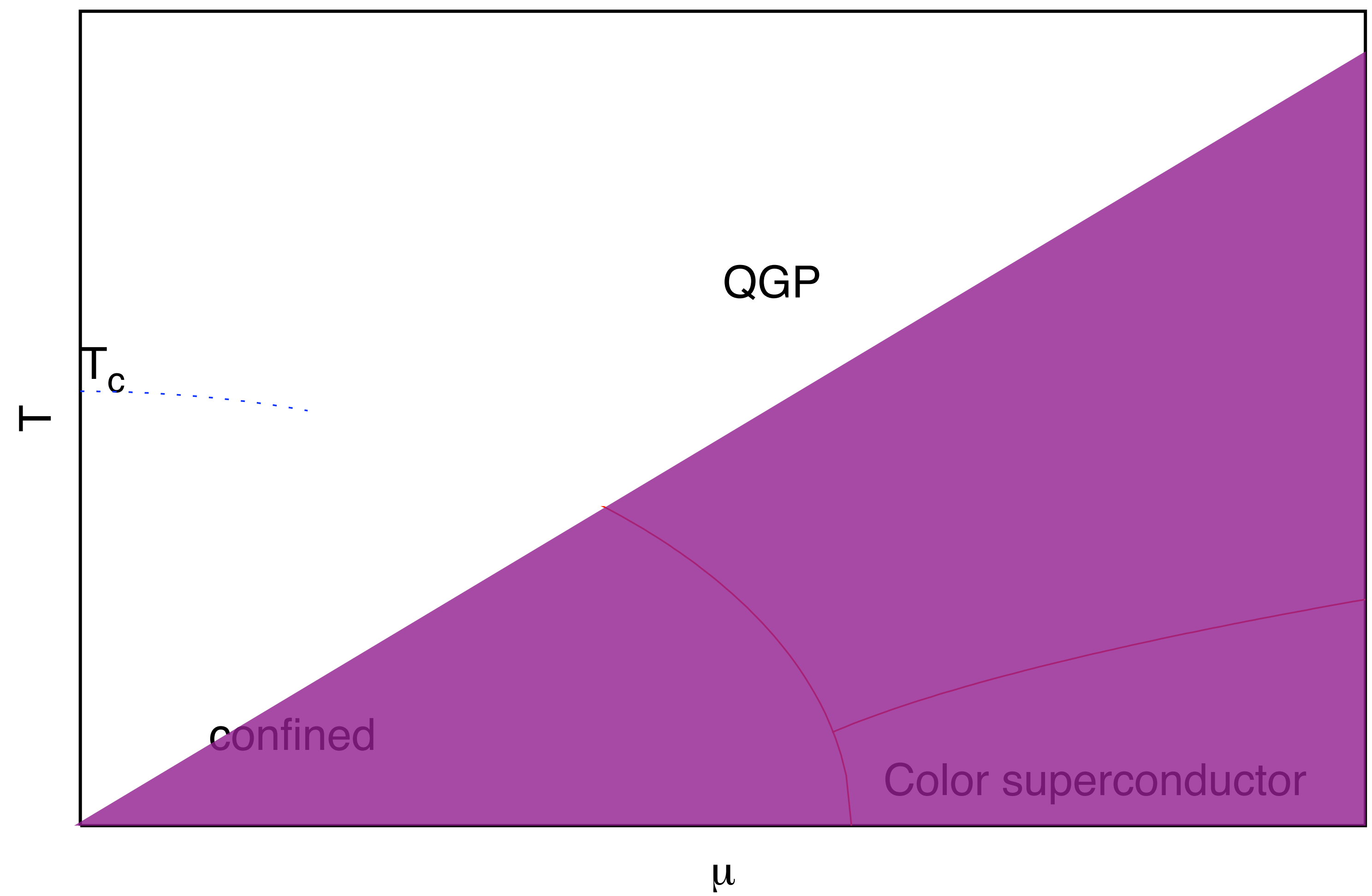


Until 2001: no finite density lattice calculations, sign problem!

Expectation based on simplifying models (NJL, linear sigma model, random matrix models, ...)

Check this from first principles QCD!

The lattice-calculable region of the phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region, some signals beyond
- Complex Langevin: lots of progress, but not in all parameter space, no "guarantees"

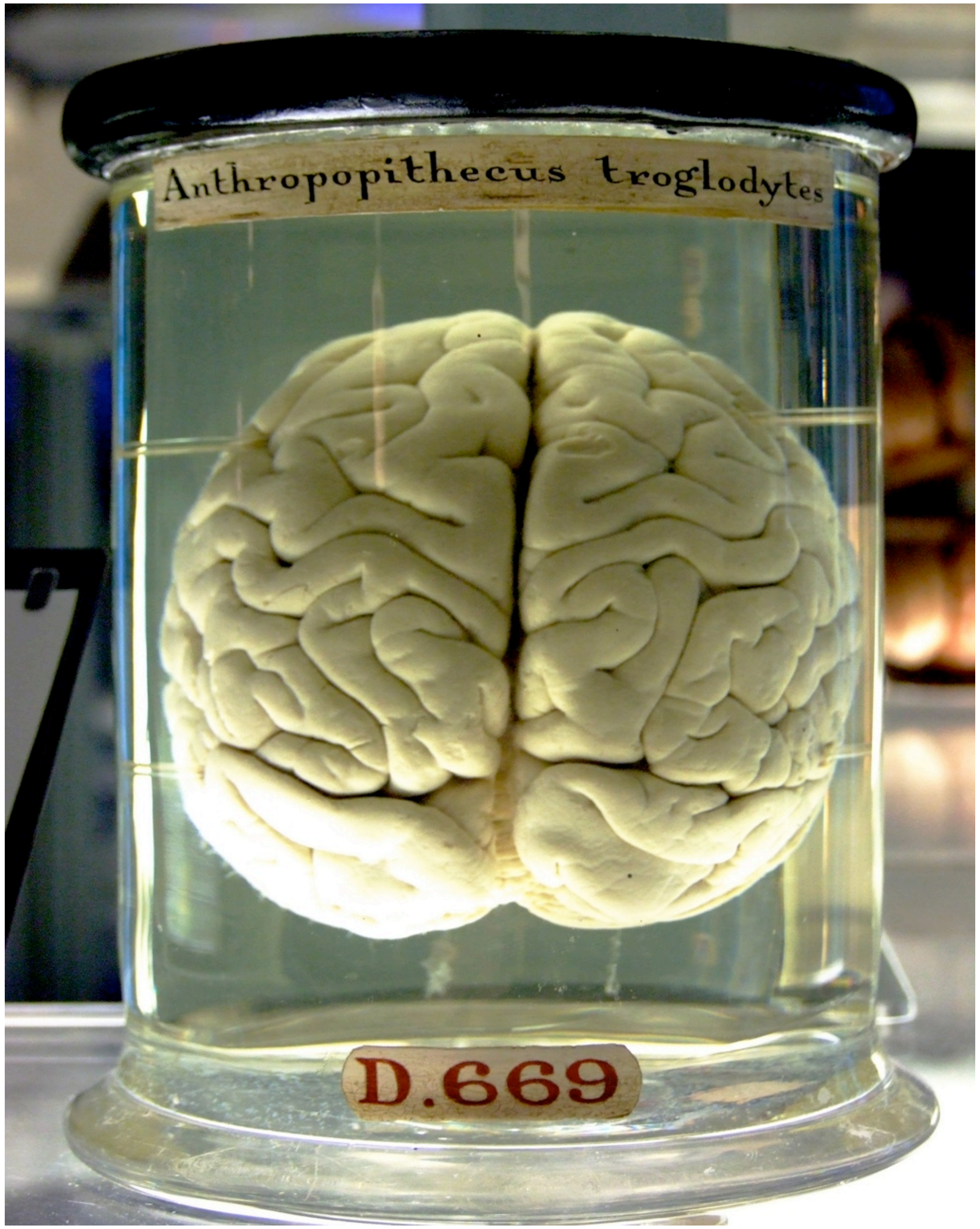
New computational avenues in LQCD:

"(Wall)Time is Money (CPU hrs)"

CPU ⟶ GPU

Here, very old-fashioned approach: BPU!

Biological Processing Unit!



Large densities? Effective theories!

Effective lattice theory for heavy and dense QCD

with M.Fromm, J.Langelage, S.Lottini, M.Neuman, J.Glesaaen

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \; \det Q \; e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \; e^{-S_{eff}[L]}$$

  Spatial integration after analytic strong coupling and hopping expansion

- Truncation valid for heavy quarks on reasonably fine lattices, a~0.1 fm

- Step II.: Mild sign problem, complex Langevin, Monte Carlo

  Check in SU(2): Scior, von Smekal 15

- New Step II.: Analytic solution by cluster expansion!

Starting point: Wilson's lattice Yang-Mills action

Partition function; link variables as degrees of freedom

$$Z = \int \prod_{x,\mu} dU(x;\mu) \exp\left(-S_g[U]\right) \equiv \int DU \exp\left(-S_g[U]\right)$$

Wilson's gauge action

$$S_g[U] = \sum_x \sum_{1\le\mu<\nu\le4} \beta\left(1 - \frac{1}{3}\mathrm{ReTr}U_p\right) \equiv \sum_p S_p \qquad \beta = \frac{2N}{g^2}$$

Plaquette:

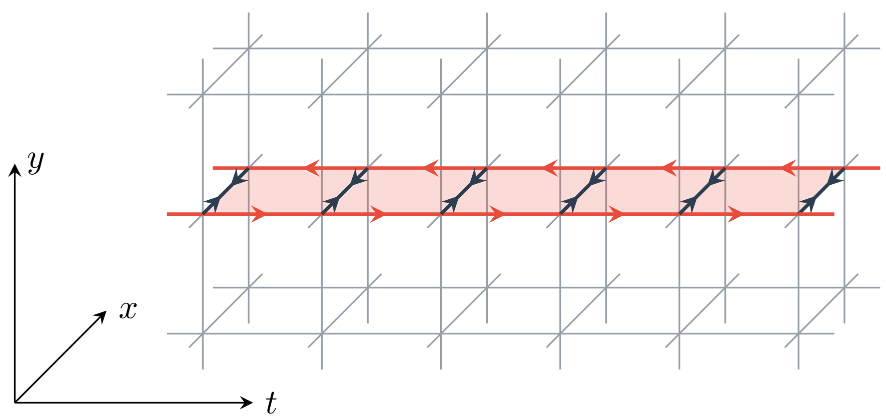
$$\square \rightarrow 1 + ia^2 gF_{\mu\nu} - \frac{a^4g^2}{2}F_{\mu\nu}F^{\mu\nu} + O(a^6) + \ldots$$

$$U_\mu(x) = \mathrm{e}^{-iagA_\mu(x)}$$

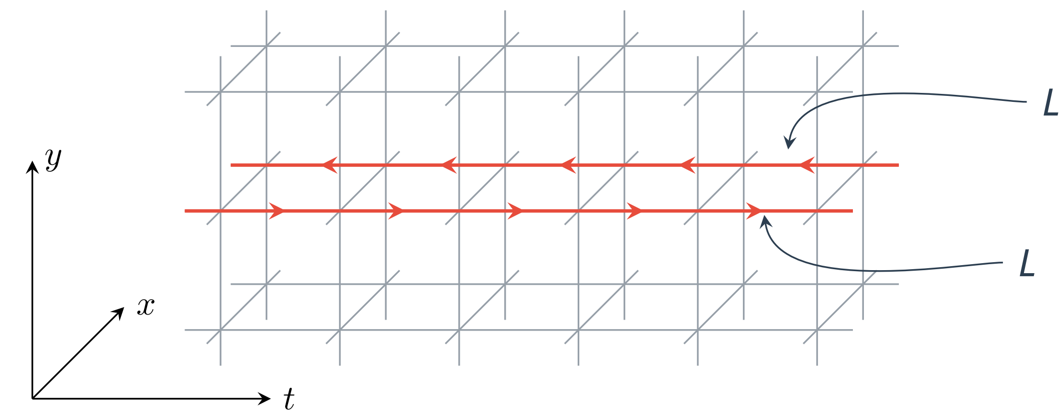
$$T = \frac{1}{aN_t}$$ continuum limit $a \rightarrow 0, N_t \rightarrow \infty$

Small $\beta(a) \Rightarrow$ small T

LO 3d effective theory for lattice YM



Integrate over all spatial gauge links



What remains is an interaction between Polyakov Loops

$$-S_1 = u^{N_\tau} \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \, \mathrm{tr}\, W_j$$

Polonyi, Szachlanyi 82

Character expansion: $u = \frac{\beta}{18} + O(\beta^2) < 1$

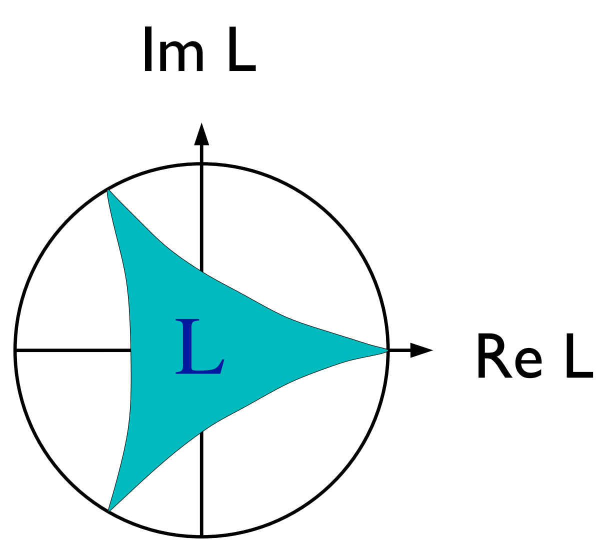
- larger distances between loops, higher powers of loops
- higher representations of loops
- decorations of LO graphs by additional plaquettes

Effective one-coupling theory for SU(3) YM

Langelage, Lottini, O.P. 10

(L= Tr W)

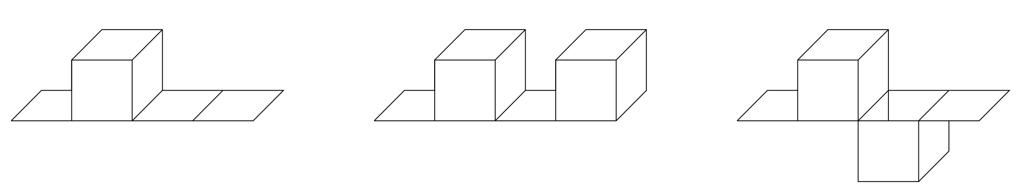
$$Z = \int [dL] \exp\left[-S_1 + V_{SU(3)}\right]$$
$$= \int [dL] \prod_{\langle ij \rangle} \left[1 + 2\lambda_1 \mathrm{Re}\left(L_i L_j^*\right)\right] *$$
$$* \prod_i \sqrt{27 - 18|L_i|^2 + 8\mathrm{Re}L_i^3 - |L_i|^4}$$



Resummations:

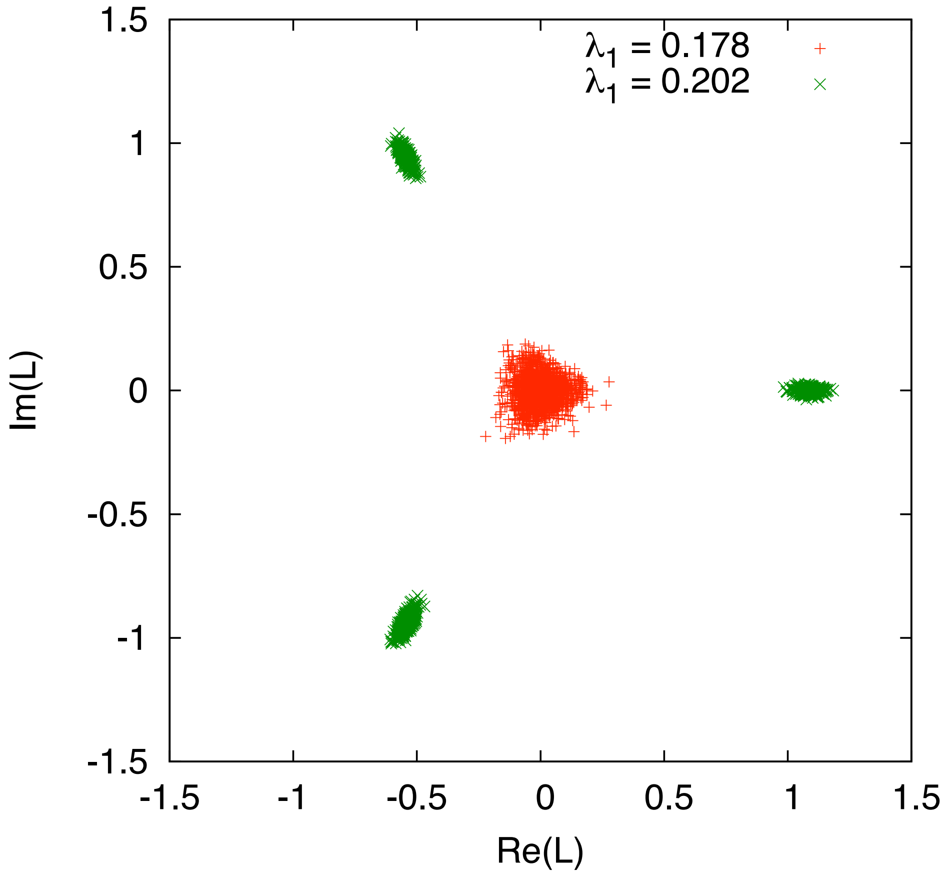
$$\sum_{<ij>} \left(\lambda_1 L_i L_j - \frac{\lambda_1^2}{2} L_i^2 L_j^2 + \frac{\lambda_1^3}{3} L_i^3 L_j^3 - \ldots\right) = \sum_{<ij>} \ln\left(1 + \lambda_1 L_i L_j\right)$$

$$\lambda(u, N_\tau \geq 5) = u^{N_\tau} \exp\left[N_\tau \left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2} u^8 + \frac{1851}{10} u^9 + \frac{1055797}{5120} u^{10}\right)\right]$$

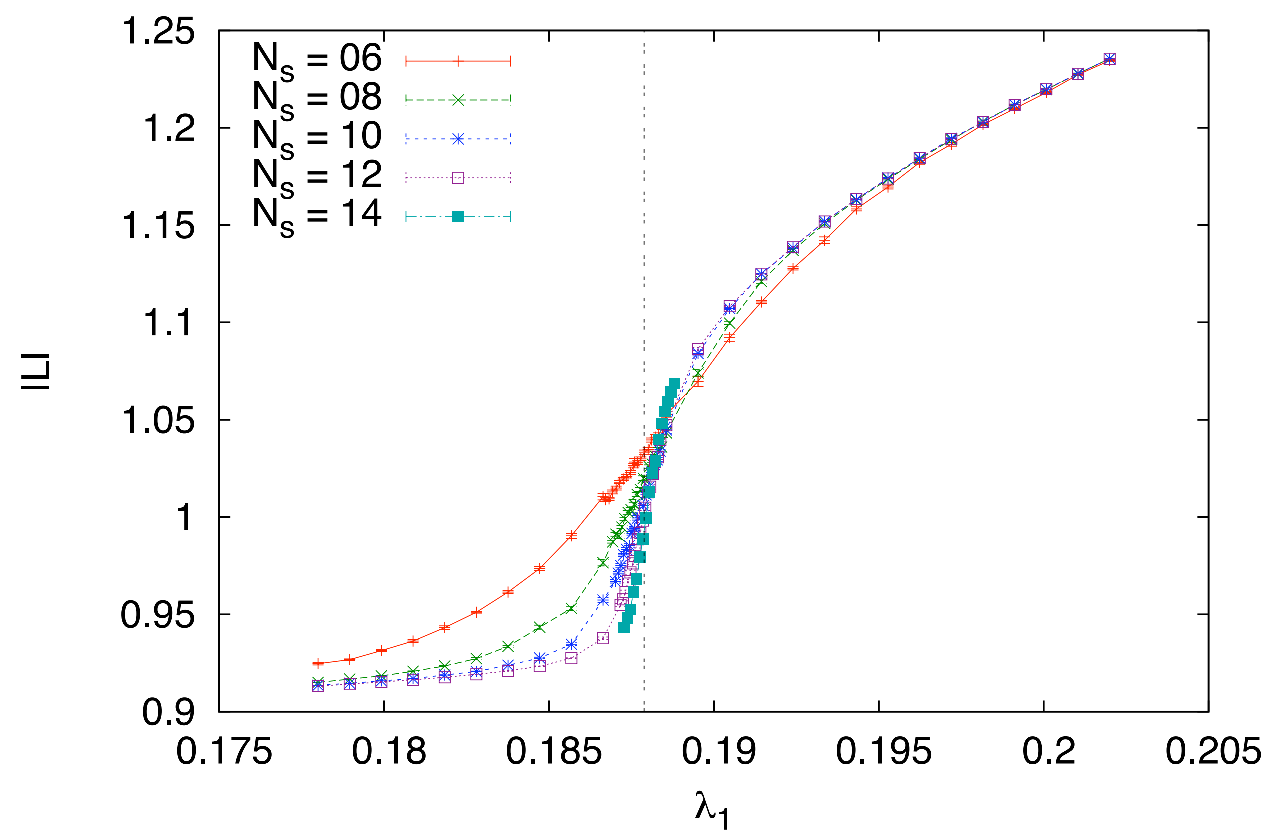


...

Numerical results for SU(3), one coupling



Order-disorder transition
=Z(3) breaking

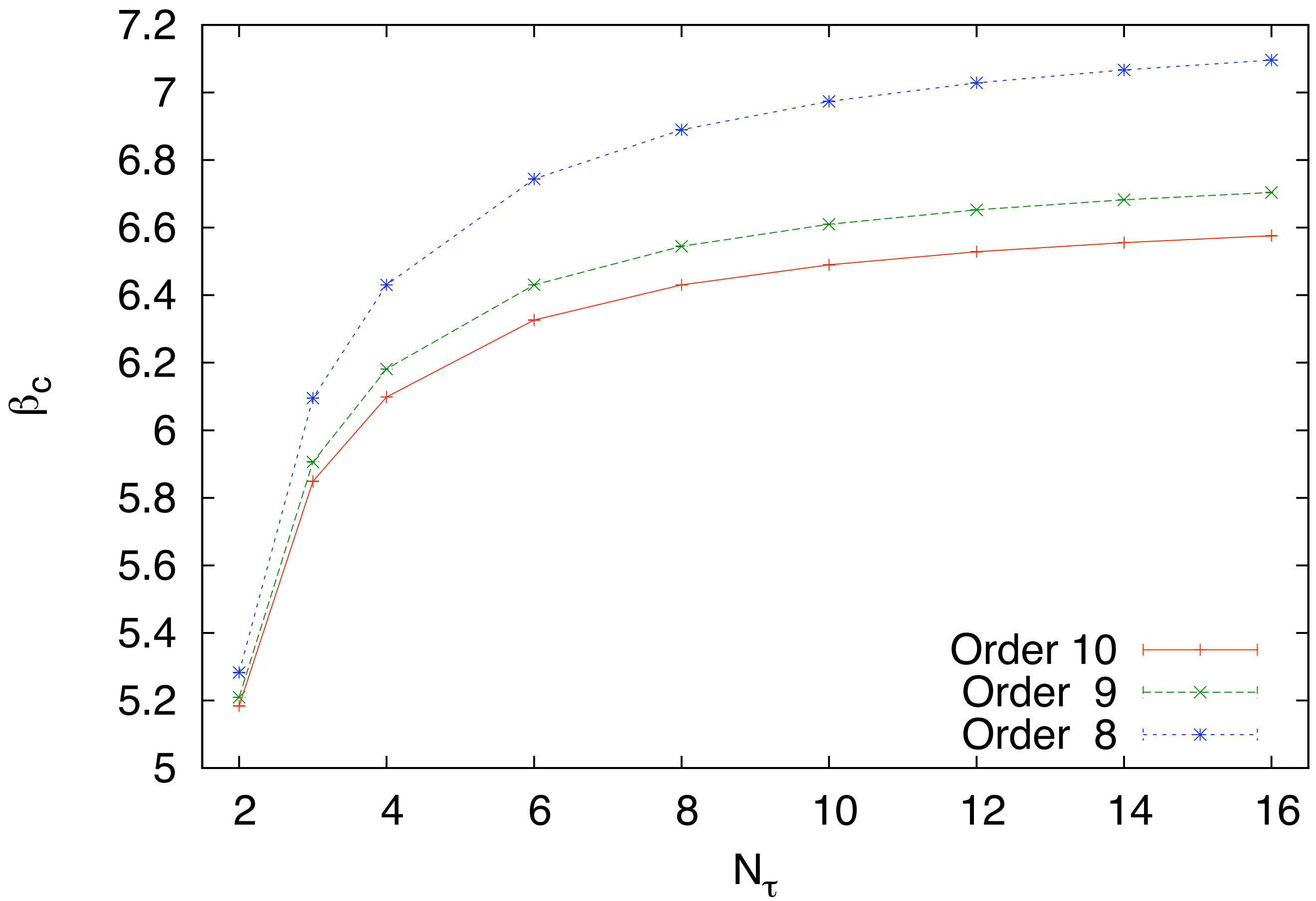


Mapping back to 4d finite T Yang-Mills

Inverting

$$\lambda_1(N_\tau, \beta) \rightarrow \beta_c(\lambda_{1,c}, N_\tau)$$

...points at reasonable convergence



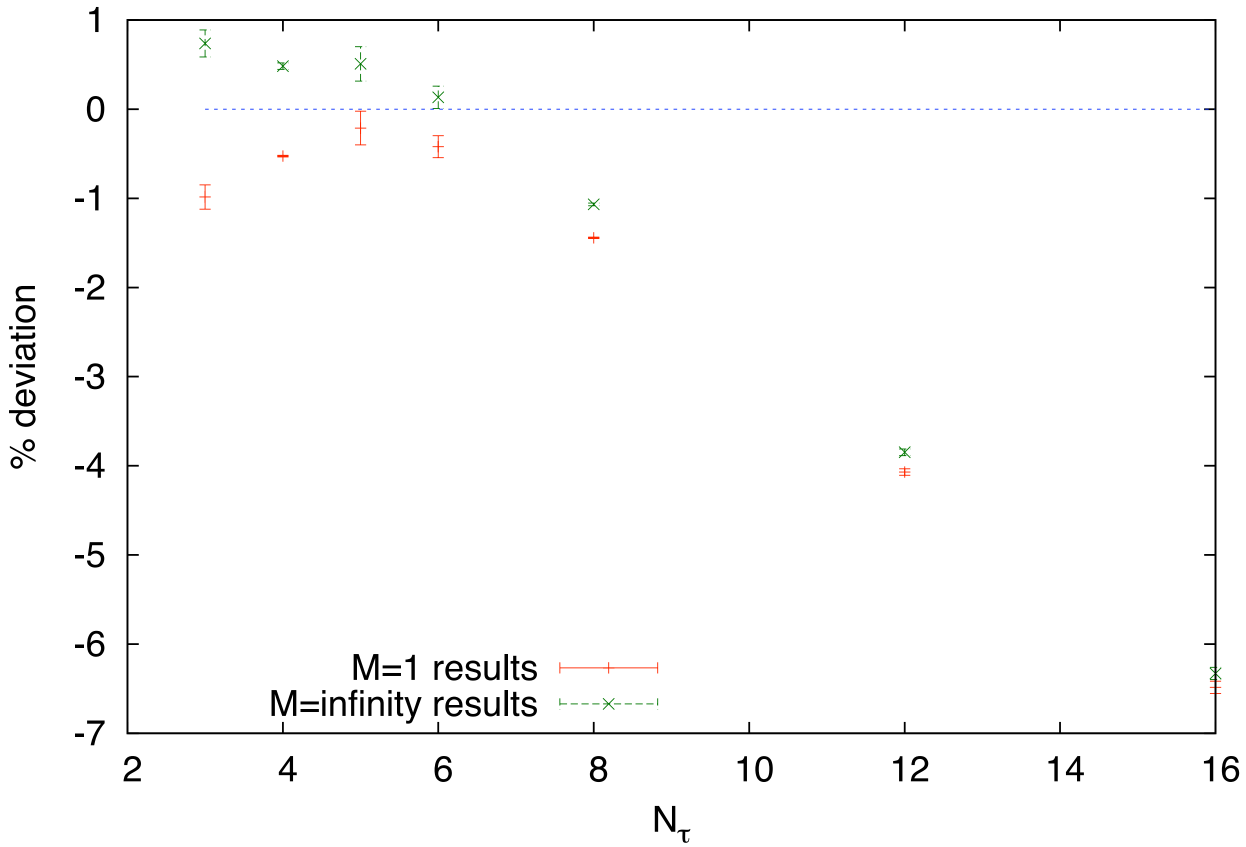
SU(3)

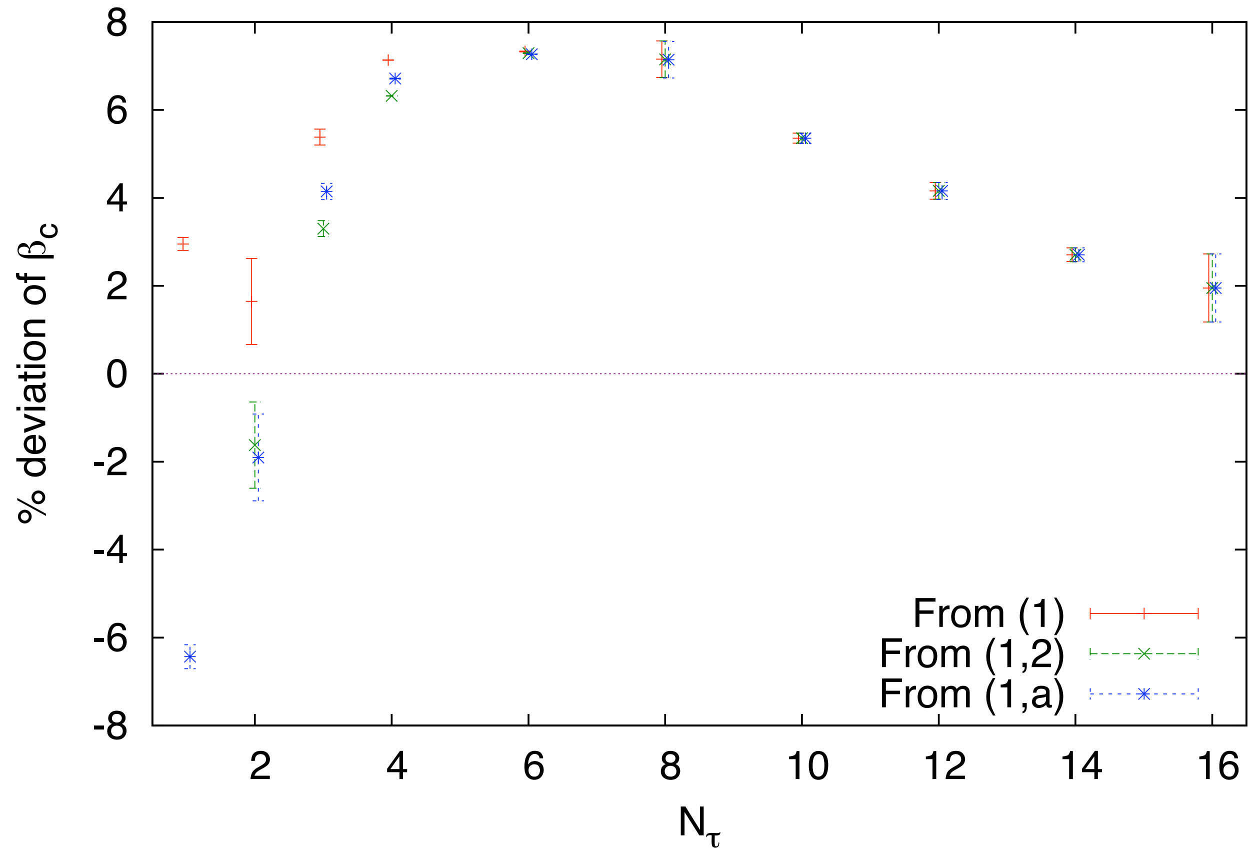
Comparison with 4d Monte Carlo

Relative accuracy for β_c compared to the full theory

SU(2)

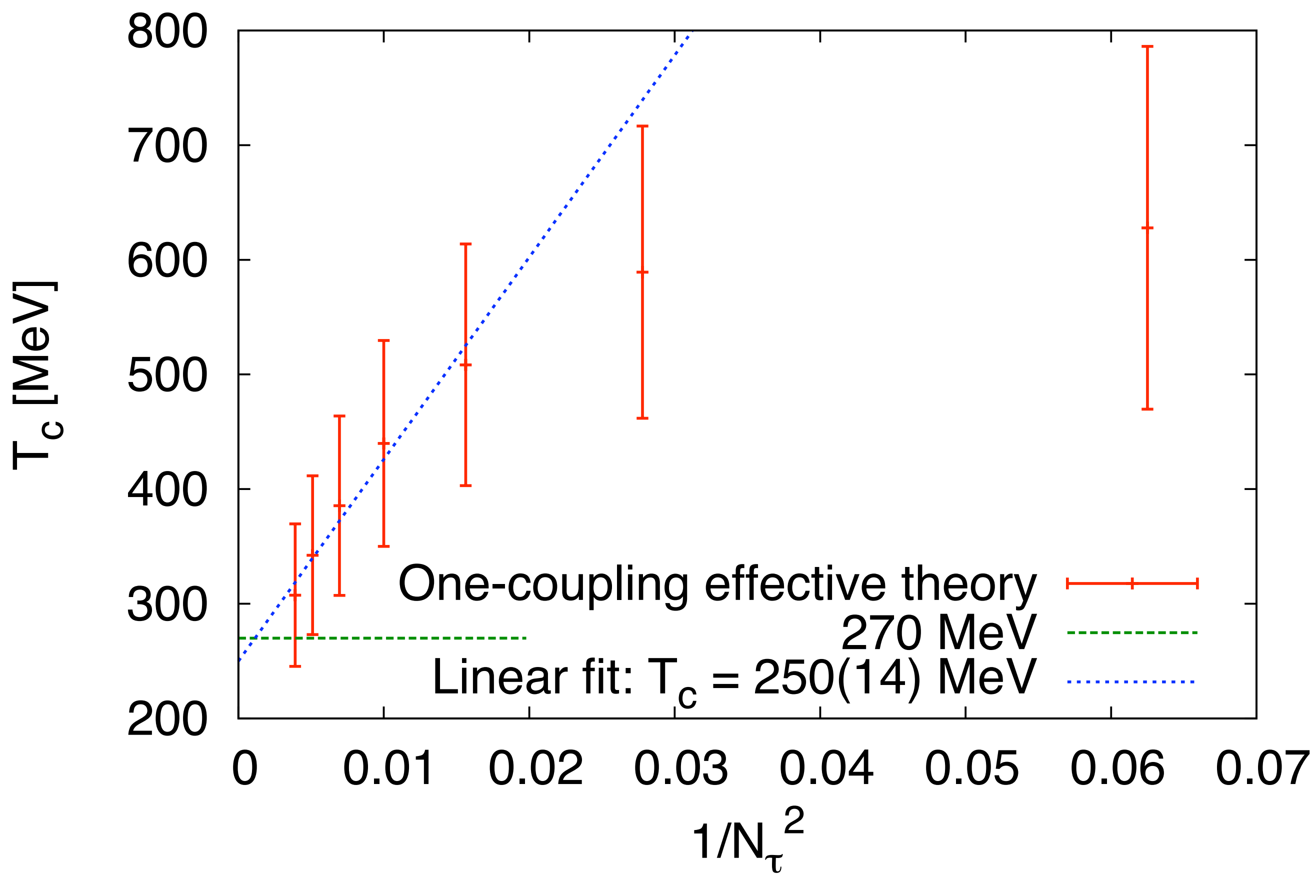
SU(3)





Note: influence of additional couplings checked explicitly!

Continuum limit feasible!



-error bars: difference between last two orders in strong coupling exp.

-using non-perturbative beta-function (4d T=0 lattice)

-all data points from one single 3d MC simulation!

Including dynamical Wilson fermions

Integrate the Grassmann variables $\psi, \overline{\psi}$:

$$S = S_{\rm gauge} - N_f \mathrm{Tr} \log(\mathbb{1} - \kappa H)$$

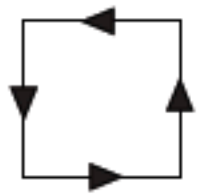
Expand in the *hopping parameter* $\kappa = 1/(2aM + 8)$

$$\begin{aligned} Z_{\rm eff}(\lambda_1, h_1, \overline{h}_1; N_\tau) \quad &= \quad \int [\mathrm{d}L] \Big(\prod_{<ij>} [1 + 2\lambda_1 \mathrm{Re} L_i L_j^*] \Big) \\ &\quad \Big(\prod_x \underbrace{\det[(1 + h_1 W_x)(1 + \overline{h}_1 W_x^\dagger)]^{2N_f}}_{\equiv Q(L_x, L_x^*)^{N_f}} \Big) \end{aligned}$$

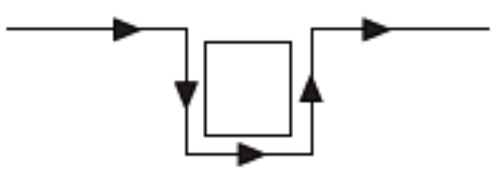
Corrections: exact expand in spatial hops

$$\begin{aligned} \det[Q] &\equiv \det[Q_{\rm stat}] \det[Q_{\rm kin}] \ , \\ \det[Q_{\rm kin}] &= \det[1 - (1 - T)^{-1}(S^+ + S^-)] \\ &\equiv \det[1 - P - M] = \exp\left[\mathrm{Tr} \ \ln(1 - P - M)\right] \end{aligned}$$

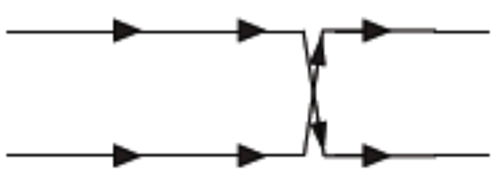
Fromm, Langelage, Lottini, Neuman, Glesaaen, O.P. 12-15

spatial:  reabsorbed in gauge part: $\begin{cases} \beta \to \beta + \mathcal{O}(\kappa^4) \\ u(\beta) \to u(\beta,\kappa) \end{cases}$

 LO Polyakov “magnetic” term $\sim \begin{cases} \underbrace{(2\kappa e^{+a\mu})^{N_\tau}}_{h_1} L \\ \underbrace{(2\kappa e^{-a\mu})^{N_\tau}}_{\bar{h}_1} L^* \end{cases}$

 higher corrections to the above:

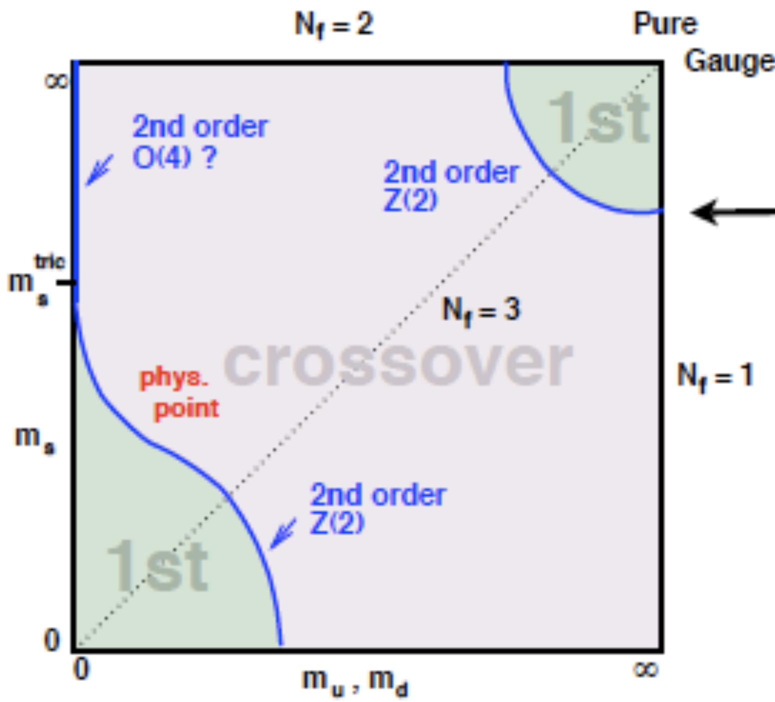
$$h_1 = (2\kappa e^{a\mu})^{N_\tau} \left[1 + \mathcal{O}(k^2) f(u) + \dots\right]$$

 other (suppressed) terms, such as $h_2(L_x L_{x+\hat{i}})$,

$$h_2 \sim (2\kappa e^{a\mu})^{2N_\tau} \kappa^2 N_\tau \,.$$

The deconfinement transition for heavy quarks

NLO: $\sim \kappa^2$



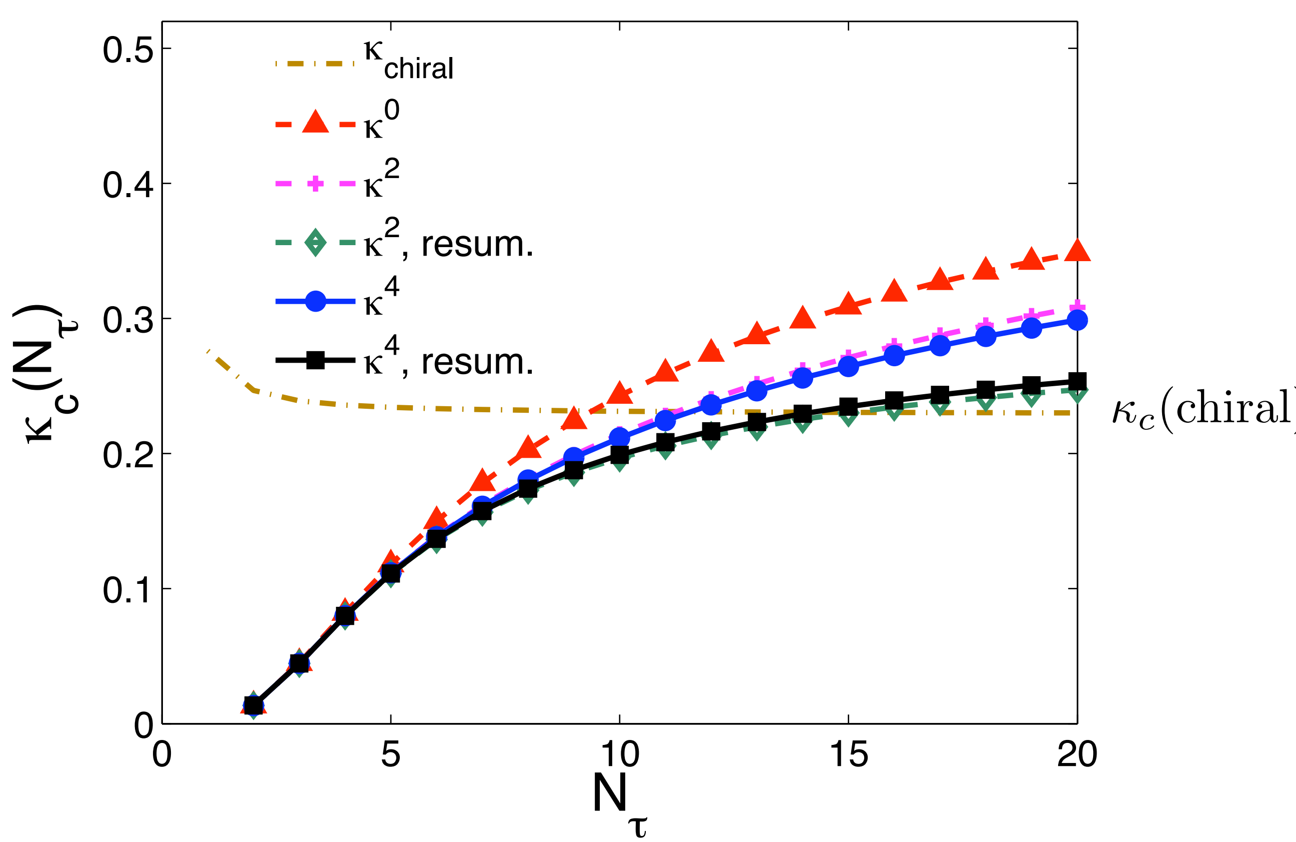
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
	eff. theory		4d MC, WHOT	4d MC, de Forcrand et al
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

Accuracy ~5%, predictions for Nt=6,8,... available!

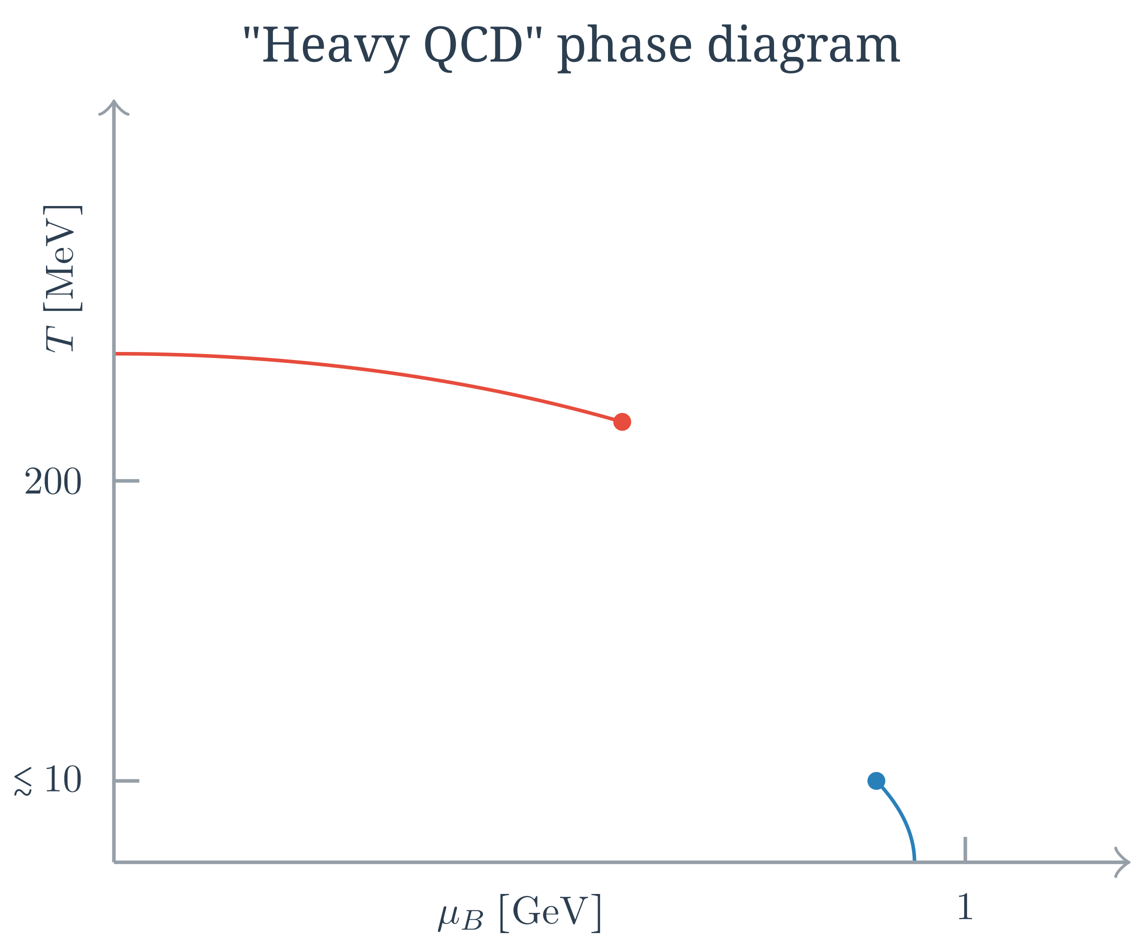
Fromm, Langelage, Lottini, O.P. 11

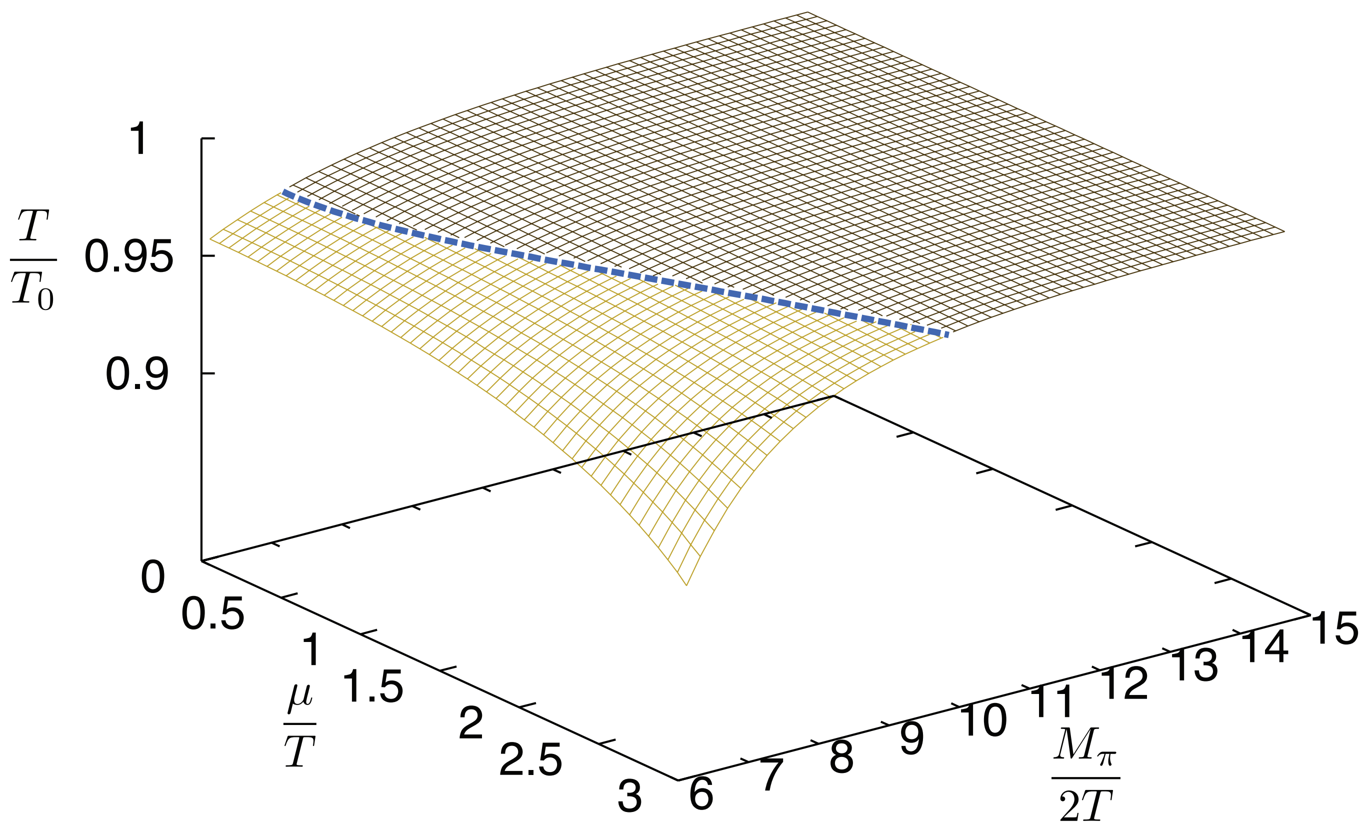
Continuum:

Friman, Lo, Redlich 14
Fischer, Lücker, Pawlowski 15



The fully calculated deconfinement transition





Fromm, Langelage, Lottini, O.P. 11

Cold and dense QCD: static strong coupling limit

Fromm, Langelage, Lottini, Neuman, O.P. , PRL 13

For T=0 (at finite density) anti-fermions decouple $N_f=1, h_1=C, h_2=0$

$$C_f \equiv (2\kappa_f e^{a\mu_f})^{N_\tau} = e^{(\mu_f - m_f)/T}, \; \bar{C}_f(\mu_f) = C_f(-\mu_f)$$

$$Z(\beta=0) \xrightarrow{T\to 0} \left[\prod_f \int \mathrm{d}W \left(1 + C_f L + C_f^2 L^* + C_f^3\right)^2 \right]^{N_s^3}$$

$$= \left[1 + 4C^{N_c} + C^{2N_c}\right]^{N_s^3}$$

Free gas of baryons!

Quarkyonic?

$$n = \frac{T}{V}\frac{\partial}{\partial\mu}\ln Z = \frac{1}{a^3}\frac{4N_c C^{N_c} + 2N_c C^{2N_c}}{1 + 4C^{N_c} + C^{2N_c}}$$

$$\lim_{\mu\to\infty}(a^3 n) = 2N_c$$

Silver blaze property + saturation!

$$\lim_{T\to 0} a^3 n = \begin{cases} 0, & \mu < m \\ 2N_c, & \mu > m \end{cases}$$

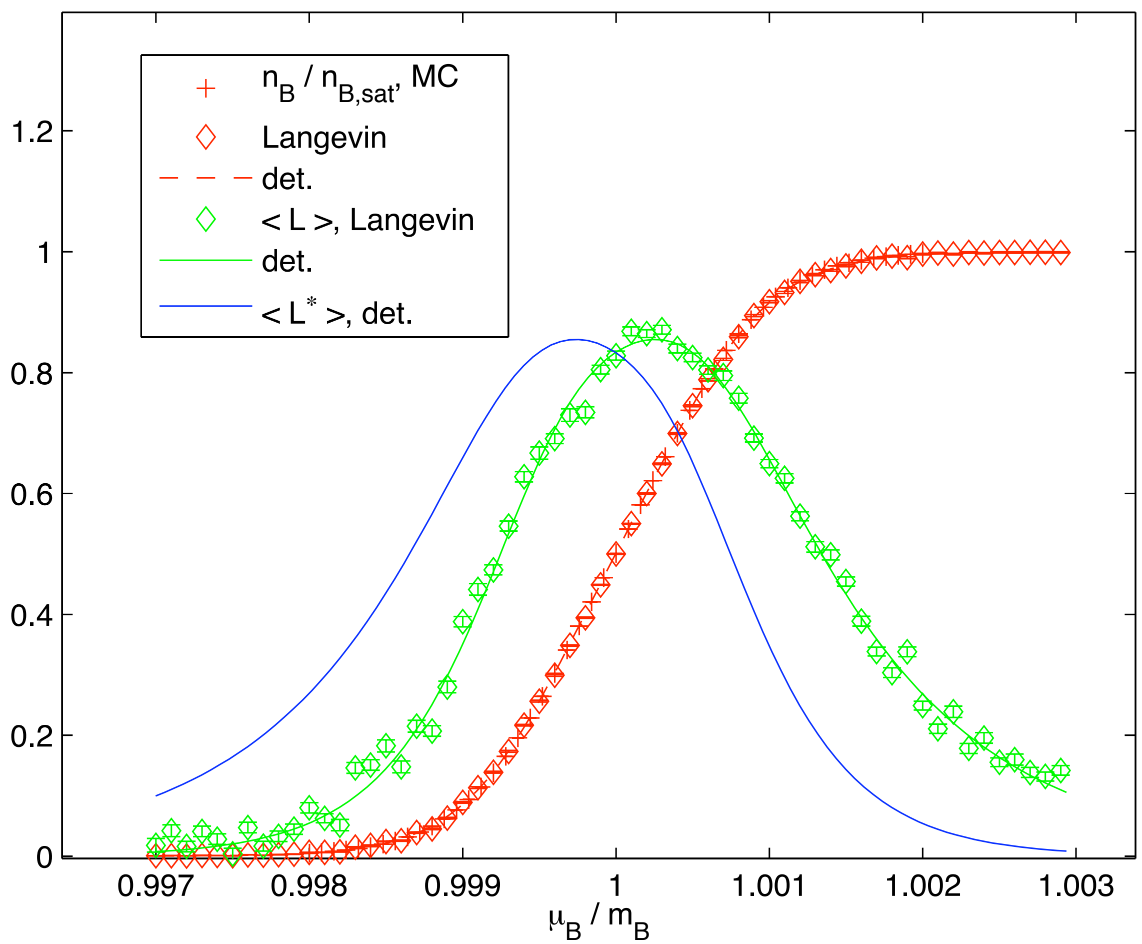
$$N_f = 2$$

$$\begin{aligned} z_0 = & \ (1 + 4h_d^3 + h_d^6) + (6h_d^2 + 4h_d^5)h_u + (6h_d + 10h_d^4)h_u^2 + (4 + 20h_d^3 + 4h_d^6)h_u^3 \\ & +(10h_d^2 + 6h_d^5)h_u^4 + (4h_d + 6h_d^4)h_u^5 + (1 + 4h_d^3 + h_d^6)h_u^6 \ . \end{aligned} \tag{3.11}$$

Free gas of baryons: complete spin flavor structure of vacuum states!

Cold and dense, interacting: onset to nuclear matter

Fromm, Langelage, Lottini, Neuman, O.P. , PRL 13



$$m_\pi = 20 \text{ GeV}, T = 10 \text{ MeV}, a = 0.17 \text{ fm}$$

$$\beta = 5.7, \kappa = 0.0000887, N_\tau = 116$$

$$\lambda_1(\beta, \kappa, N_\tau) \sim 10^{-26}$$

- Silver blaze property

  no dependence on chem. pot. until onset

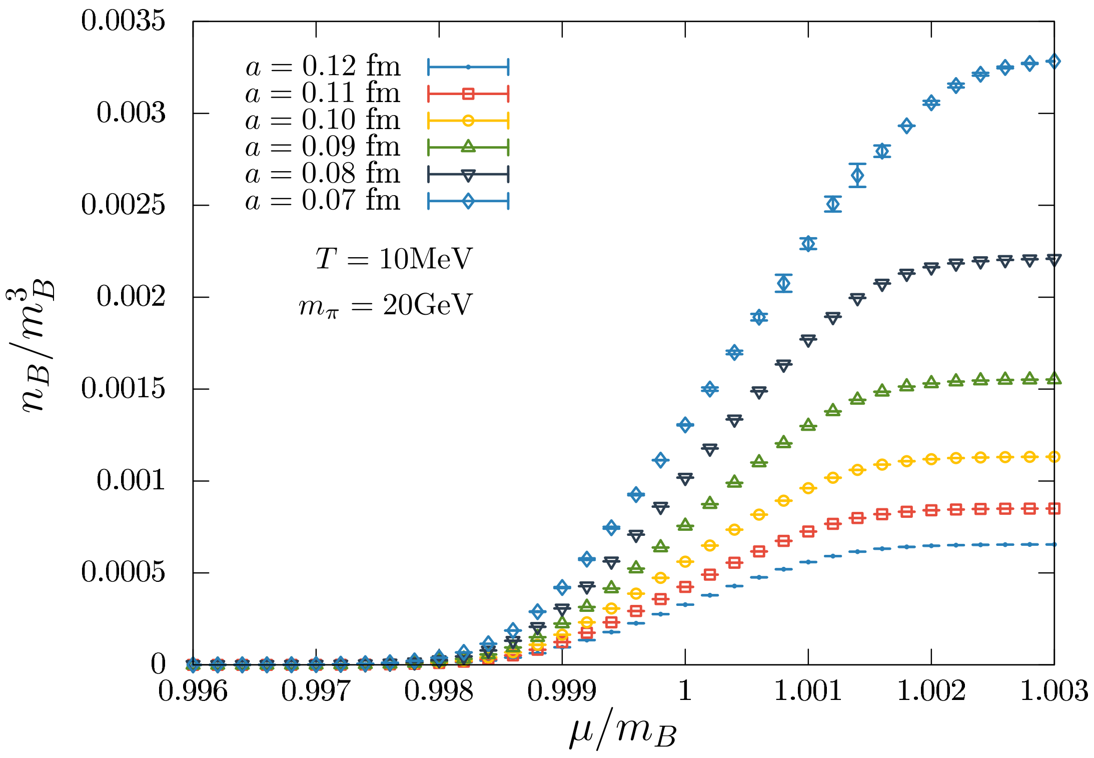
- Lattice saturation

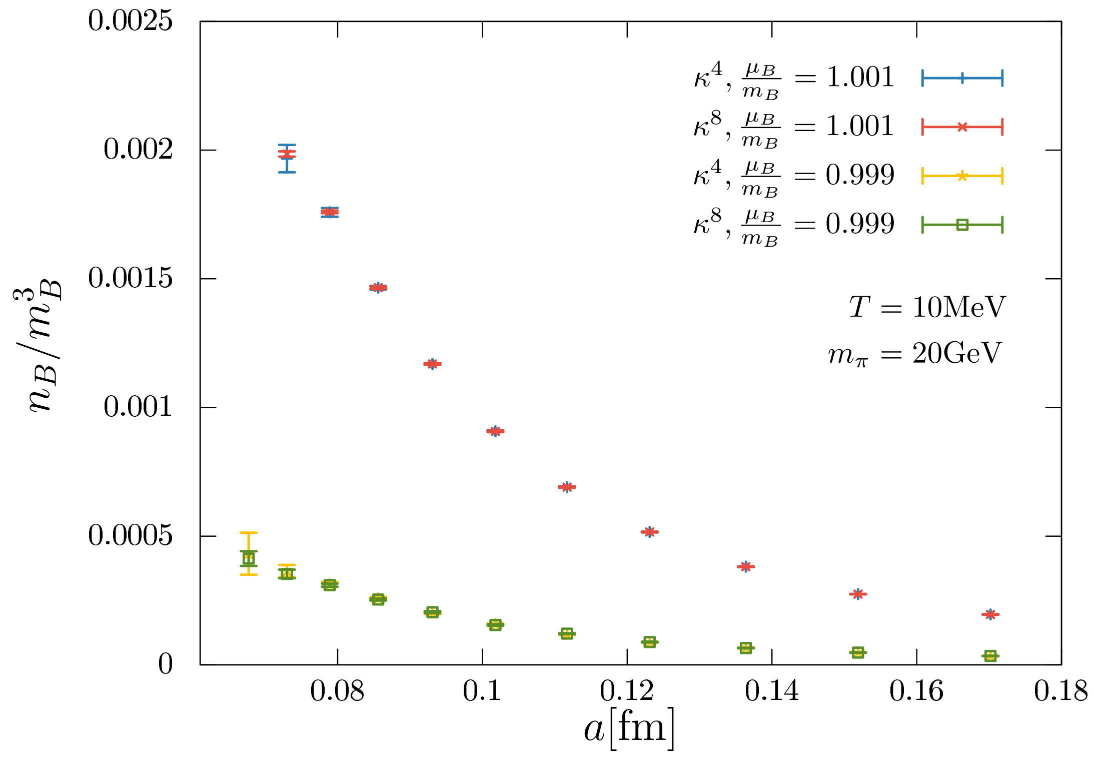
  Pauli principle, strongly limits density!

- Screening of Polyakov loop

  But no deconfinement!

Continuum approach



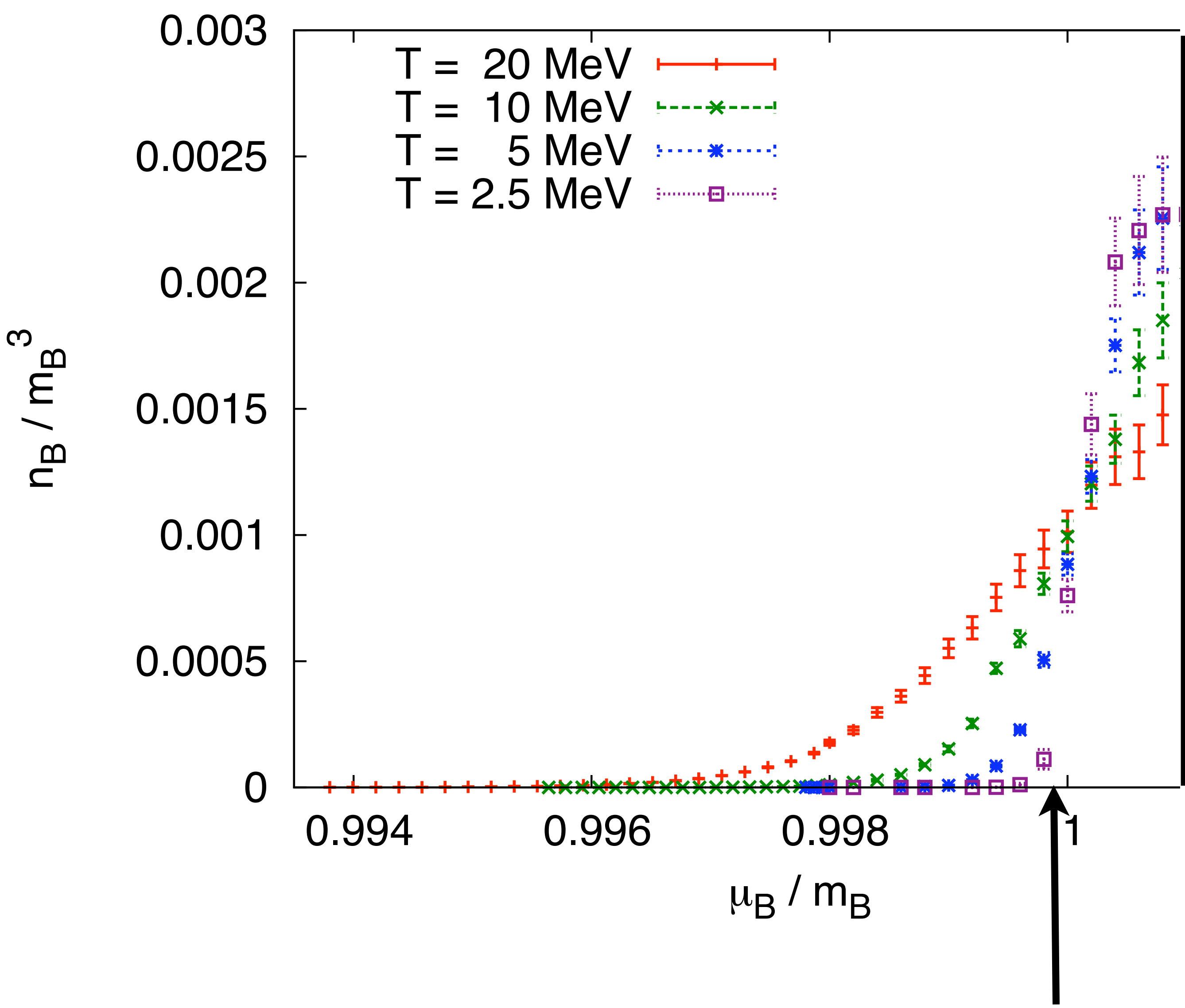


- Continuum approach ~a as expected for Wilson fermions
- Cut-off effects grow rapidly beyond onset transition
- Finer lattice necessary for larger density to avoid saturation

Cold and dense, interacting: onset to nuclear matter

continuum extrapolated

$m_\pi = 20\ \mathrm{GeV}$



$\frac{\mu}{T} \sim 4000$

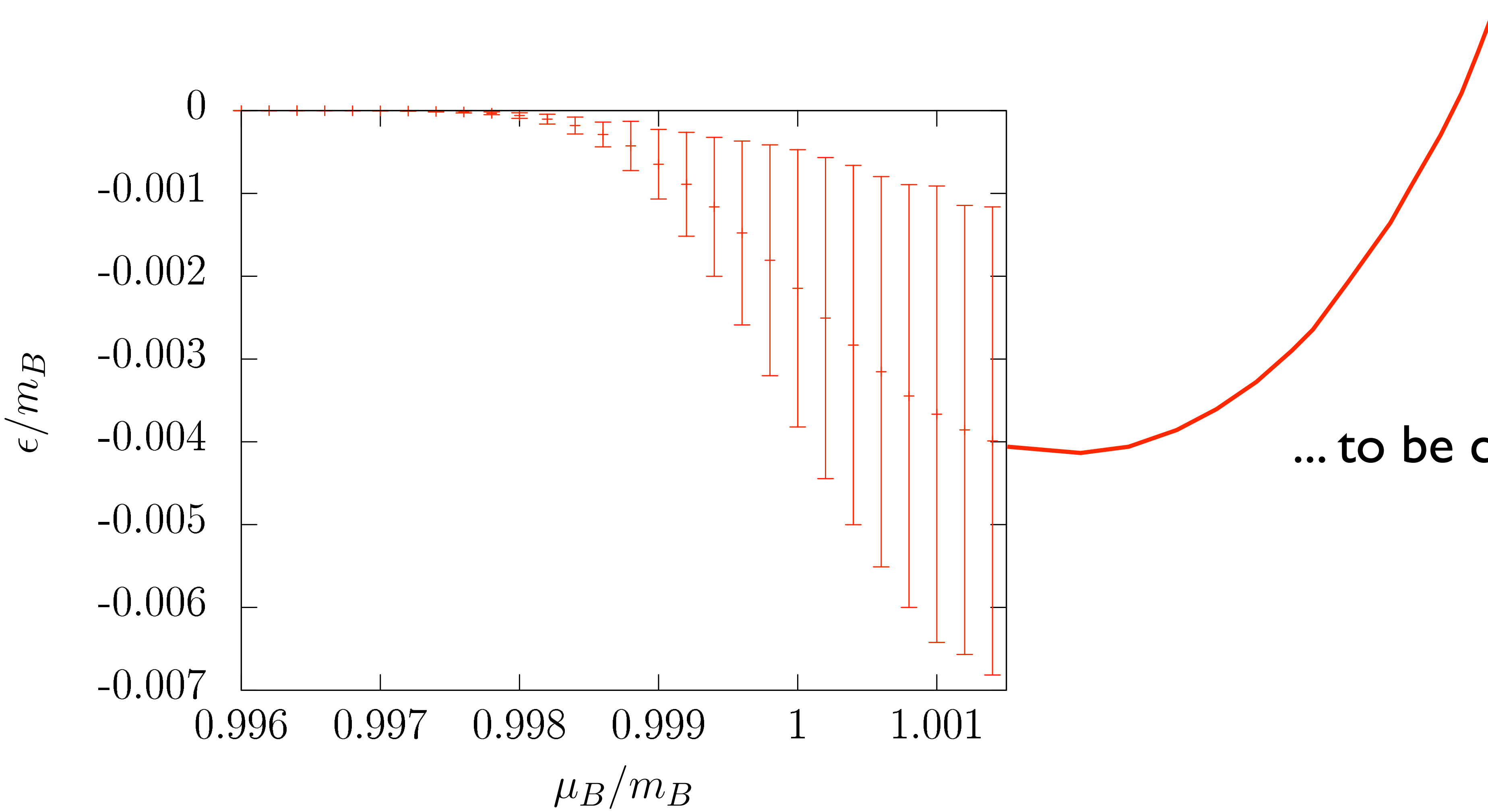
Effect of binding between baryons: $\mu_c < m_B$

Binding energy per nucleon: $\epsilon = \frac{\mu_c - m_B}{m_B} \sim 10^{-3}$

Transition is smooth crossover: $T > T_c \sim \epsilon\, m_B$

Binding energy per nucleon

$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$

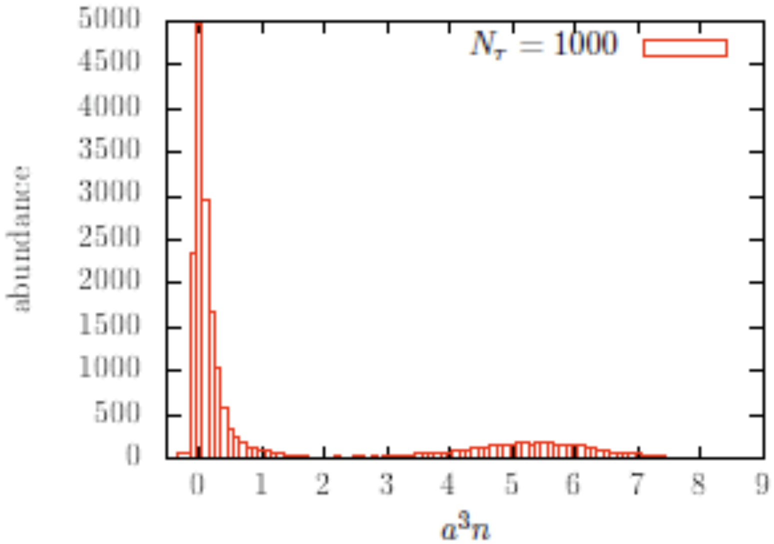


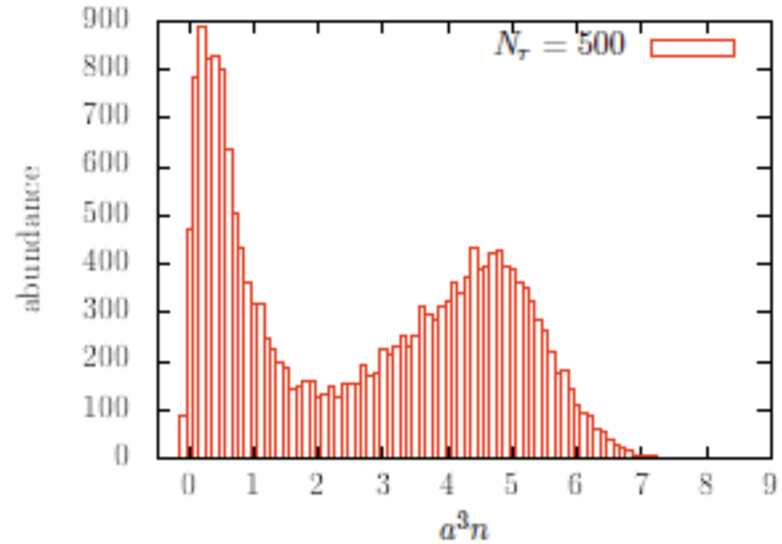
... to be continued...

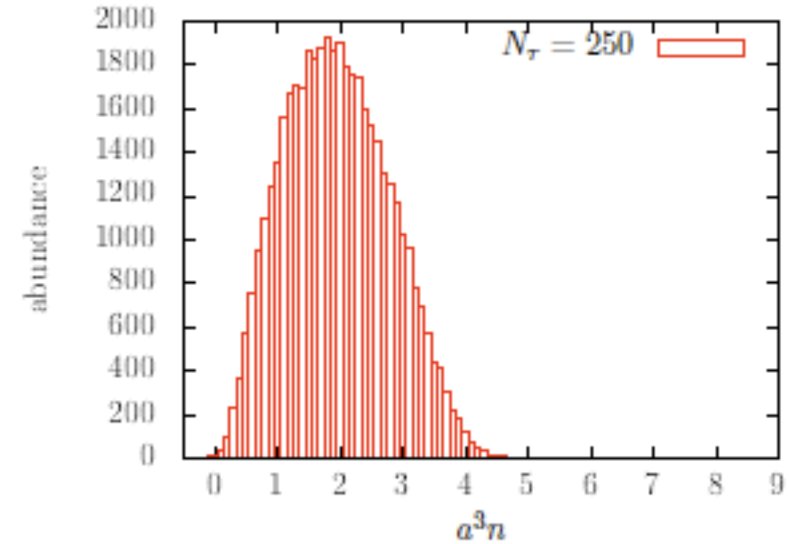
Minimum: access to nucl. binding energy, nucl. saturation density!

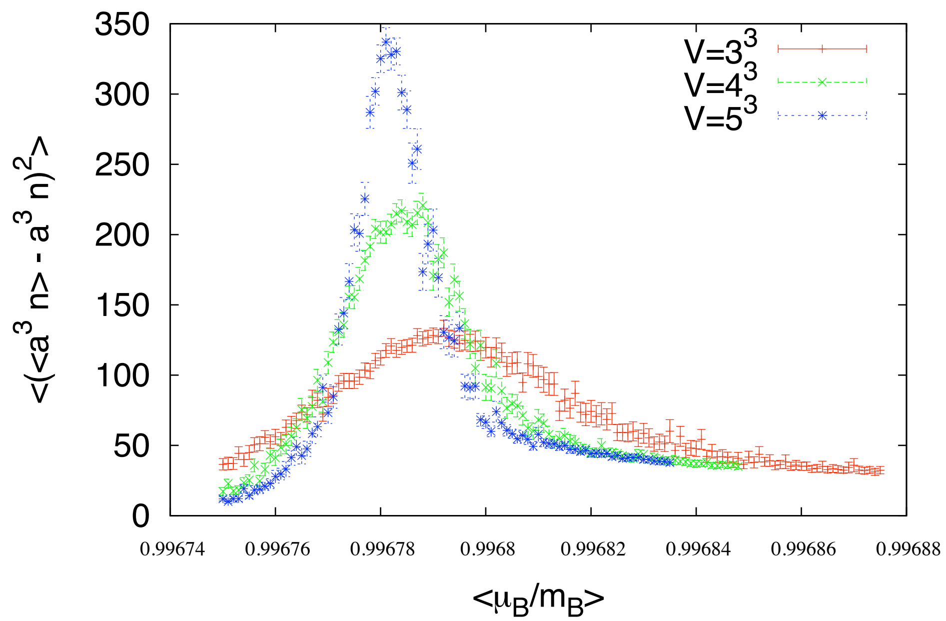
$\epsilon \sim 10^{-3}$ consistent with the location of the onset transition

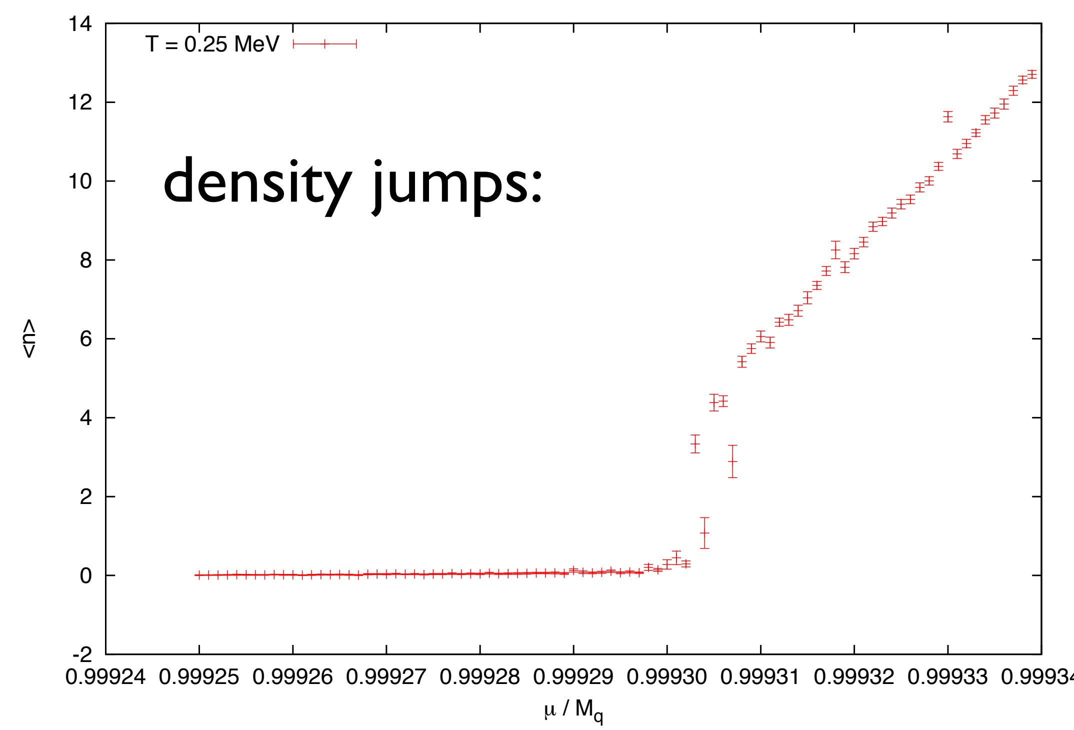
Light quarks: first order transition + endpoint









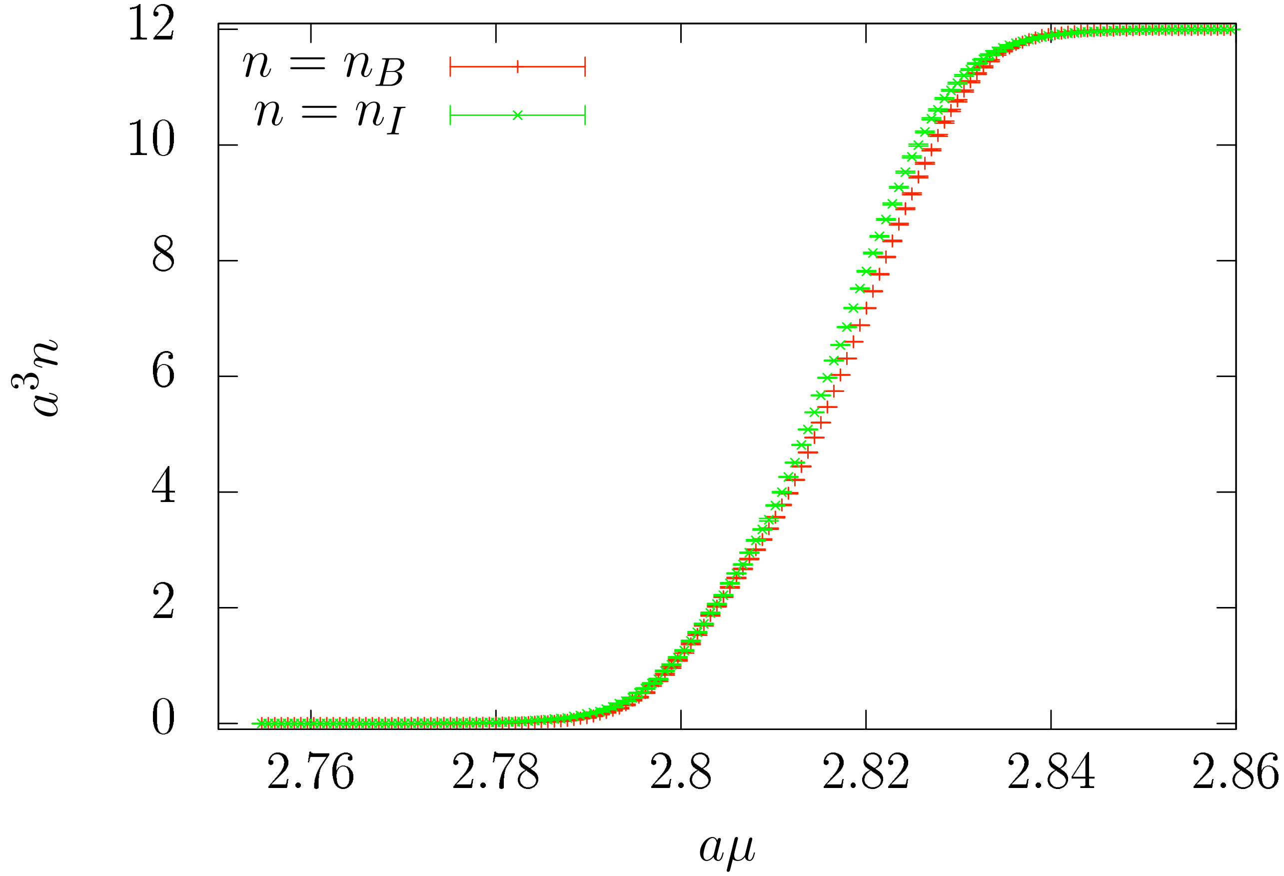


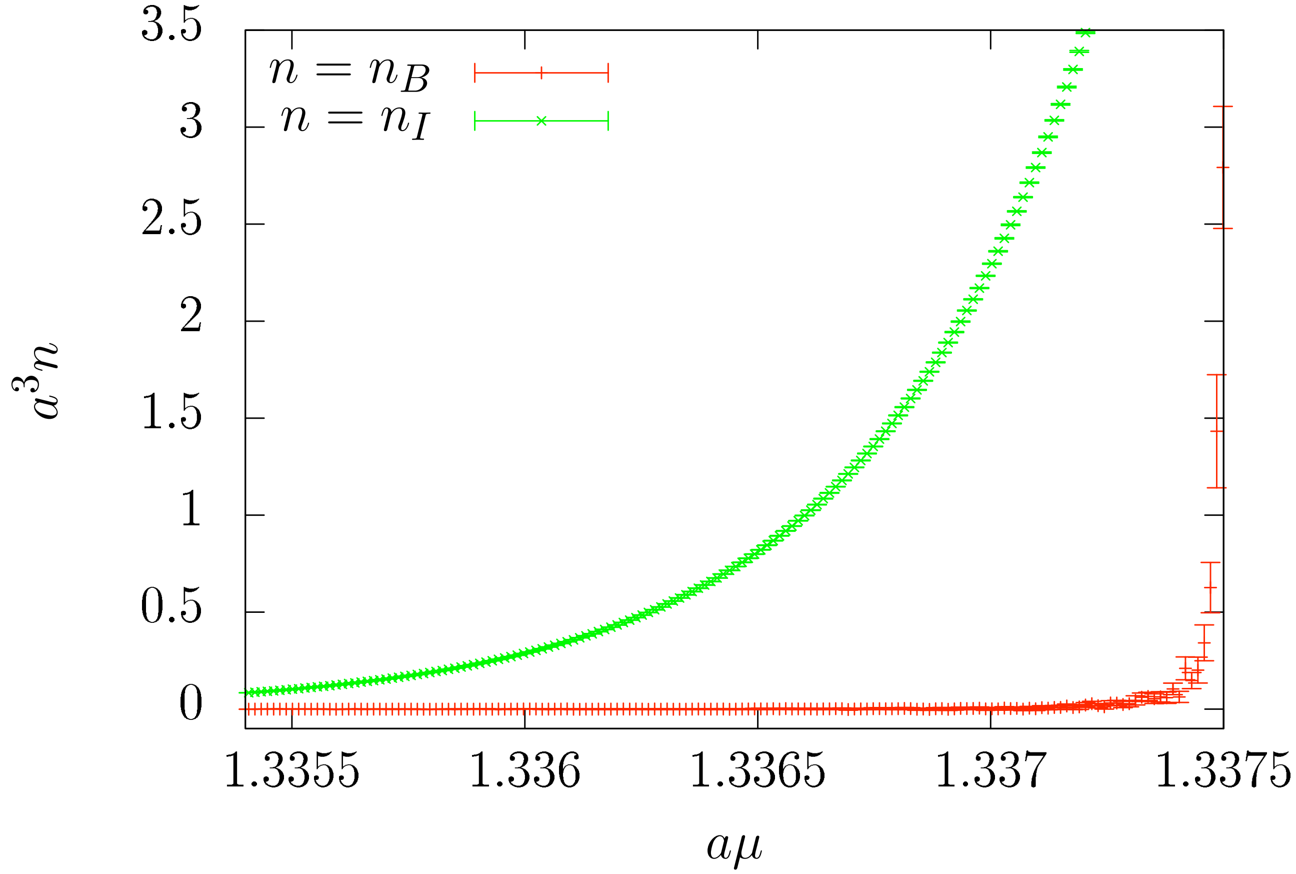
For sufficiently light quarks: $\kappa \sim 0.1$

- Coexistence of vacuum and finite density phase: 1st order
- If the temperature $T = \frac{1}{aN_\tau}$ or the quark mass is raised this changes to a crossover

nuclear liquid gas transition!!!

Finite isospin vs baryon chemical potential





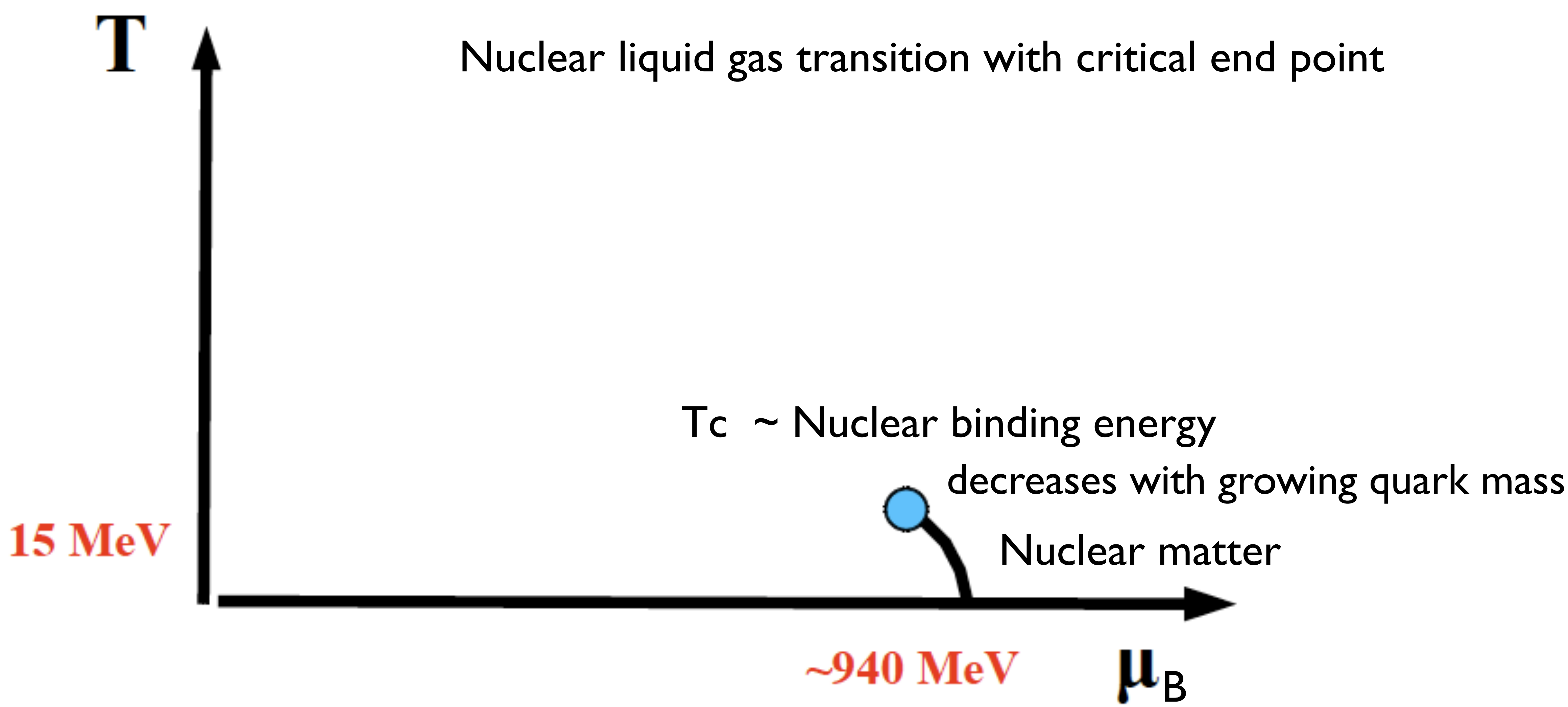
nearly static quarks

$$\frac{m_\pi}{2} \approx \frac{m_B}{3}$$

lighter quarks

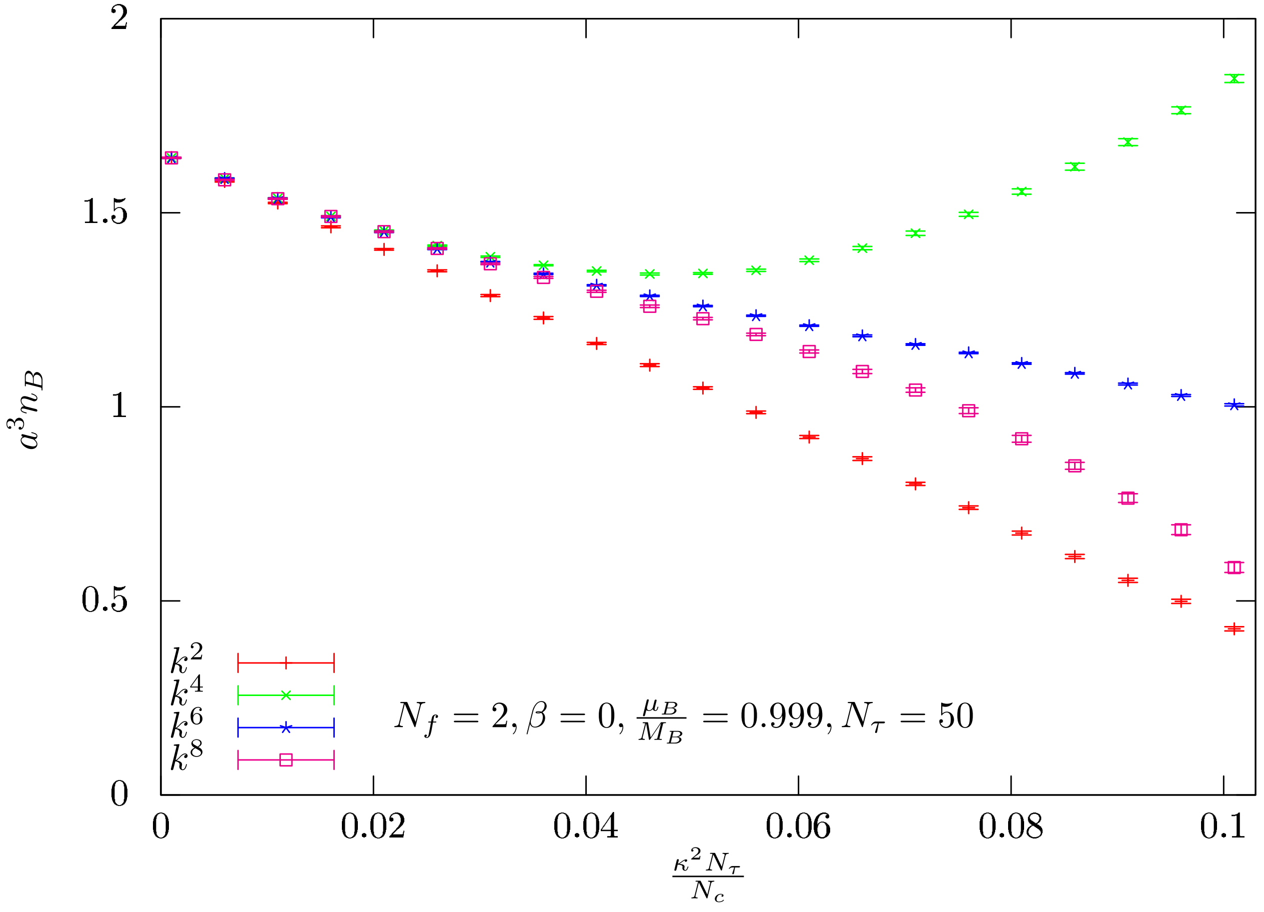
$$\frac{m_\pi}{2} < \frac{m_B}{3}$$

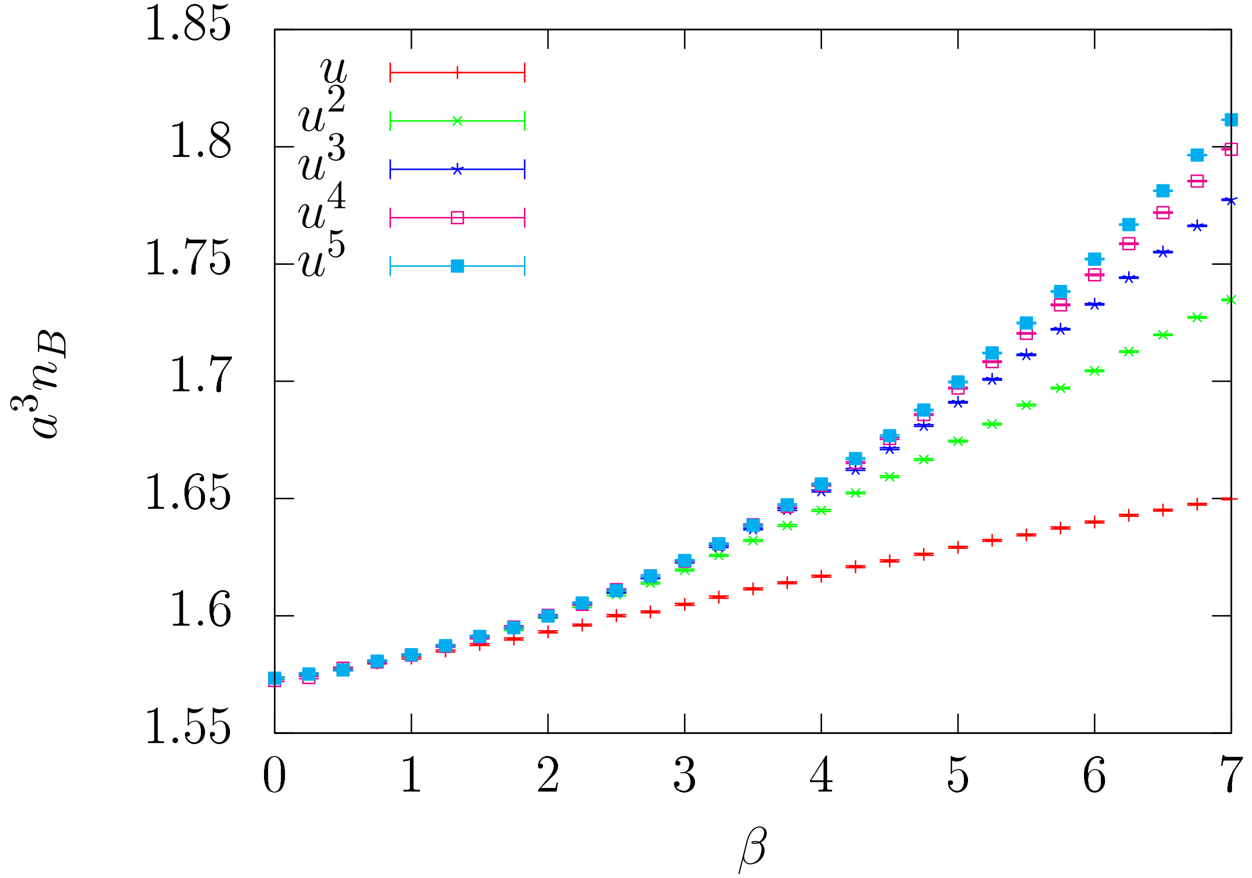
onset at smaller chemical potential



$$\epsilon = -\frac{4}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 \kappa^2 = -\frac{1}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 e^{-am_M} \quad + \quad \ldots$$

Convergence of the effective theory





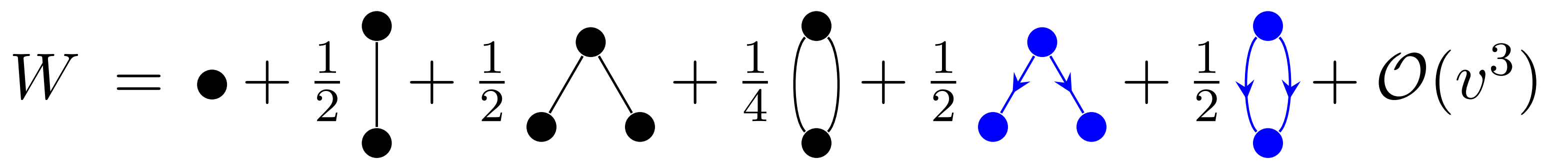
hopping expansion

strong coupling expansion κ^8

Linked cluster expansion of effective theory

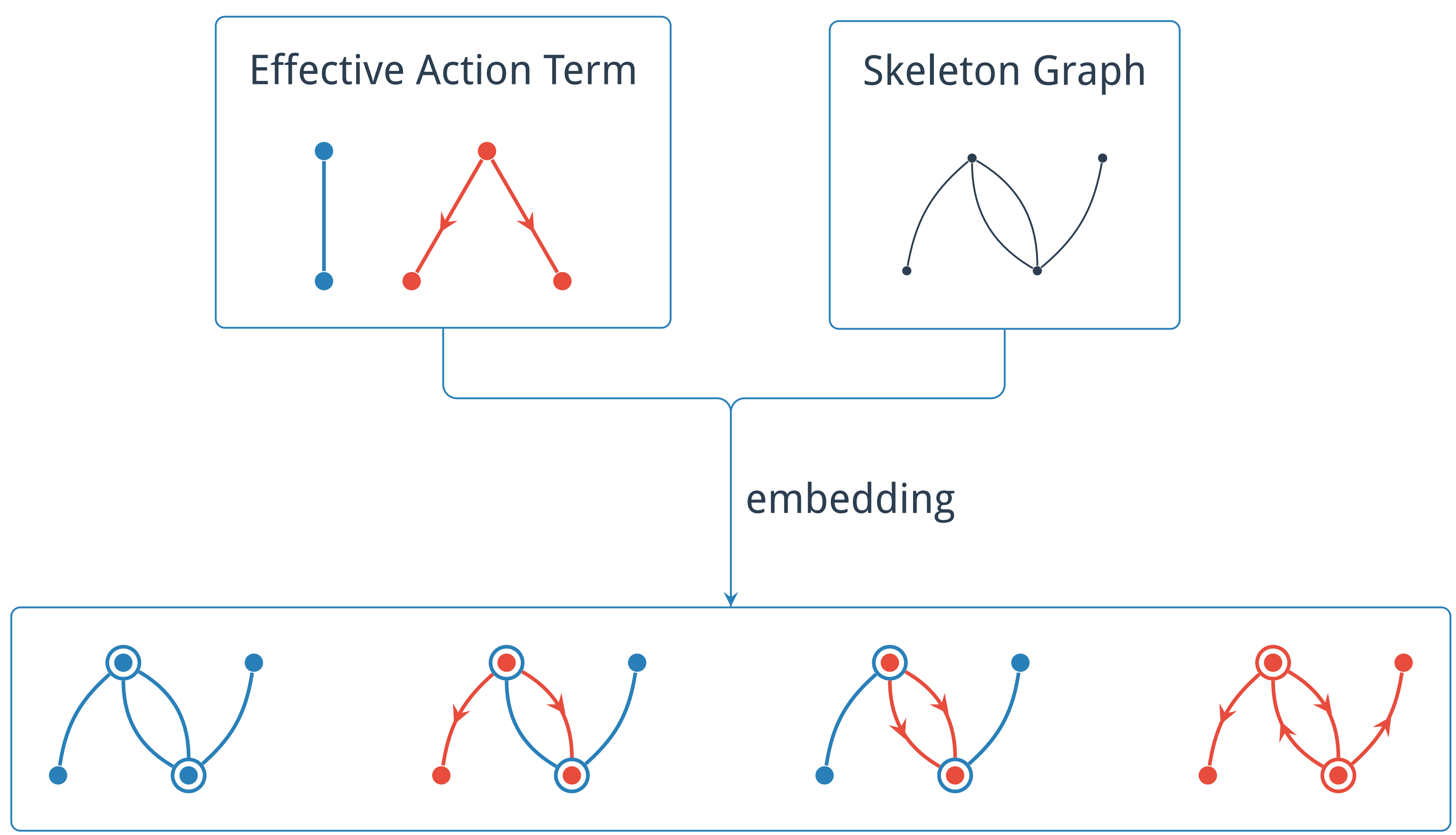
Glesaaen, Neuman, O.P. 15

$$\mathcal{Z} = \int \mathcal{D}\phi \, e^{-S_0[\phi] + \frac{1}{2}\sum v_{ij}(x,y)\phi_i(x)\phi_j(y) + \frac{1}{3!}\sum u_{ijk}(x,y,z)\phi_i(x)\phi_j(y)\phi_k(z) + \dots}$$

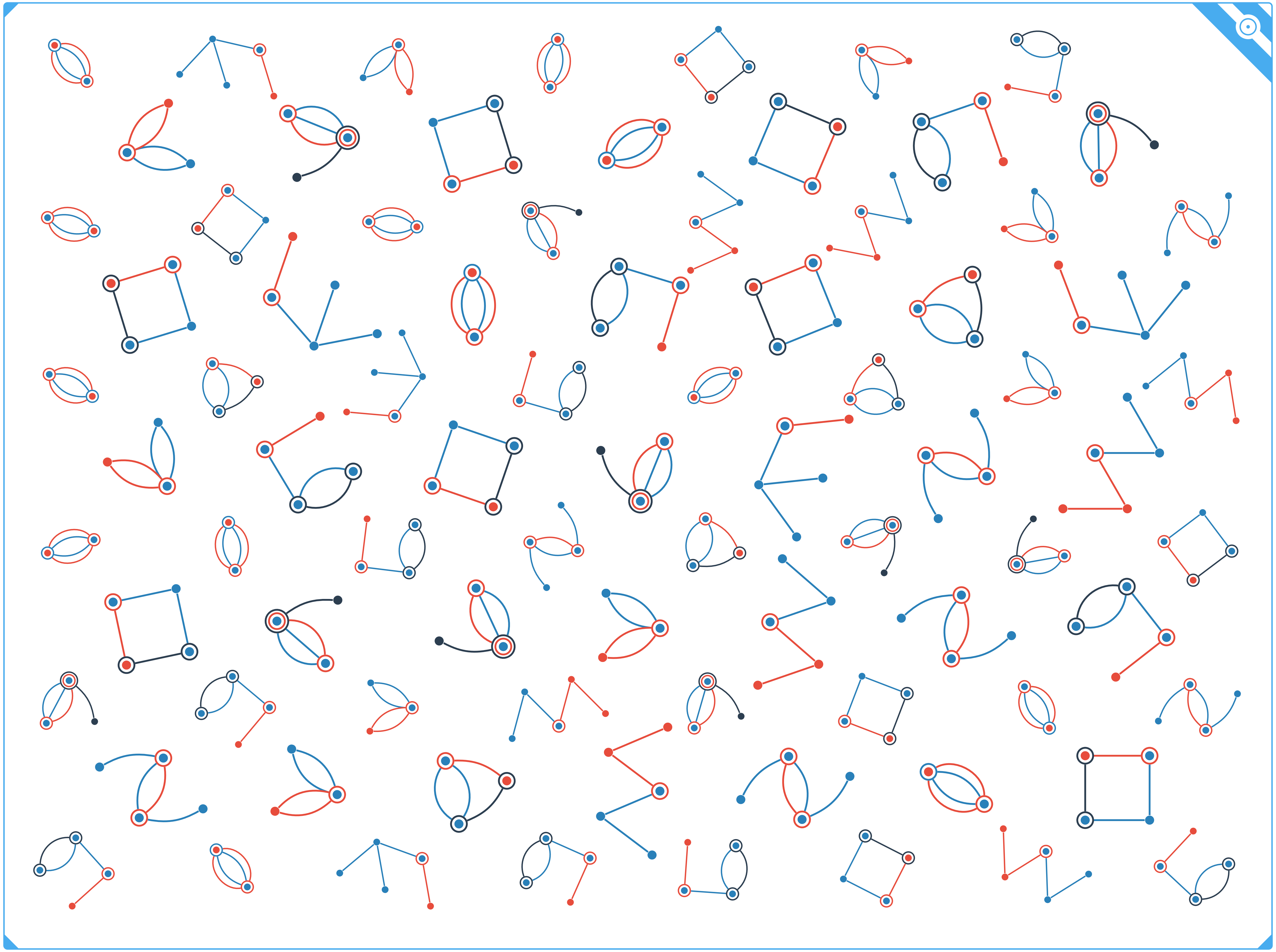


Required generalization: n-point interactions

Mapping of the effective theory by embedding:

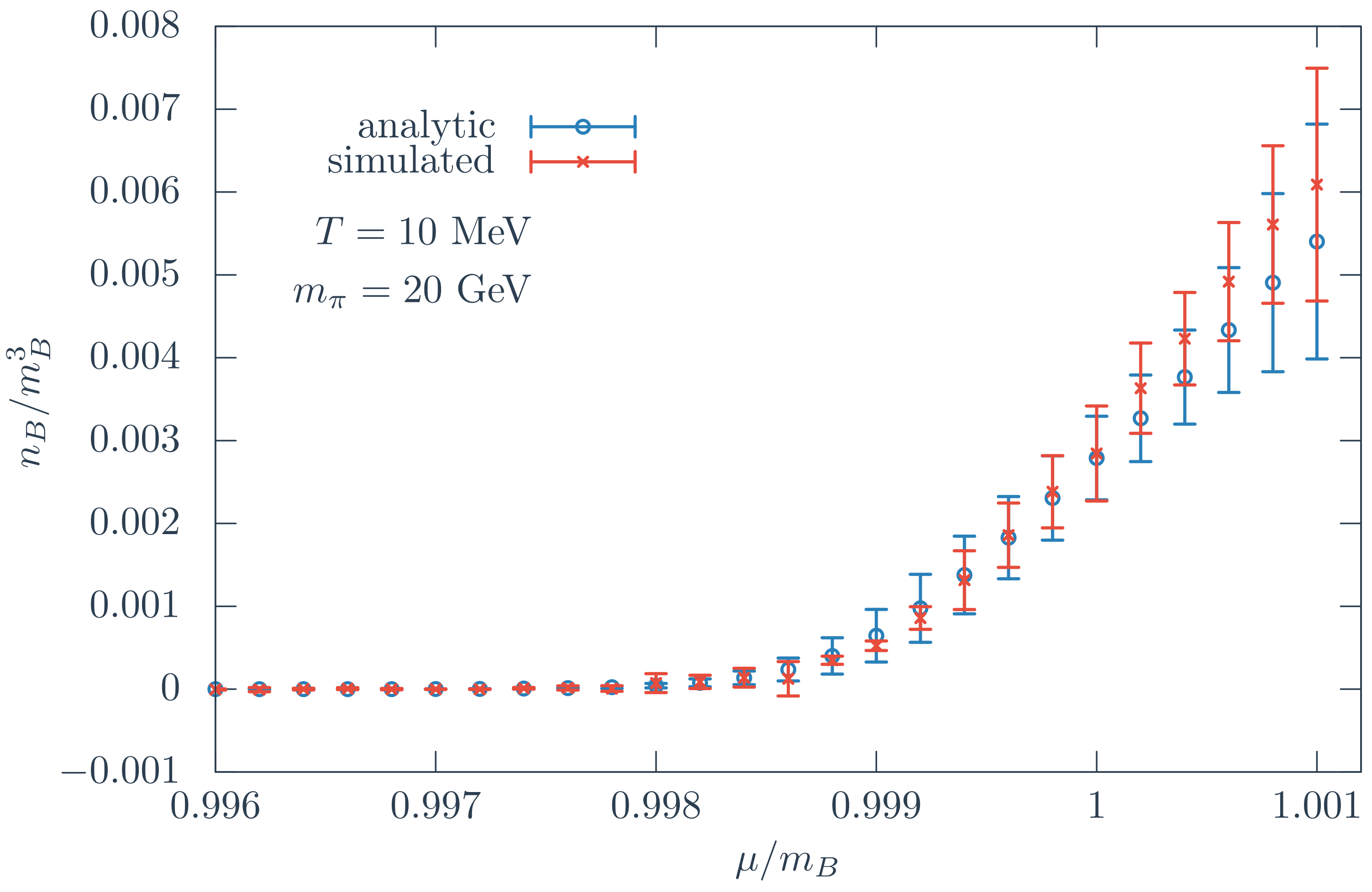


Fun with diagrams….

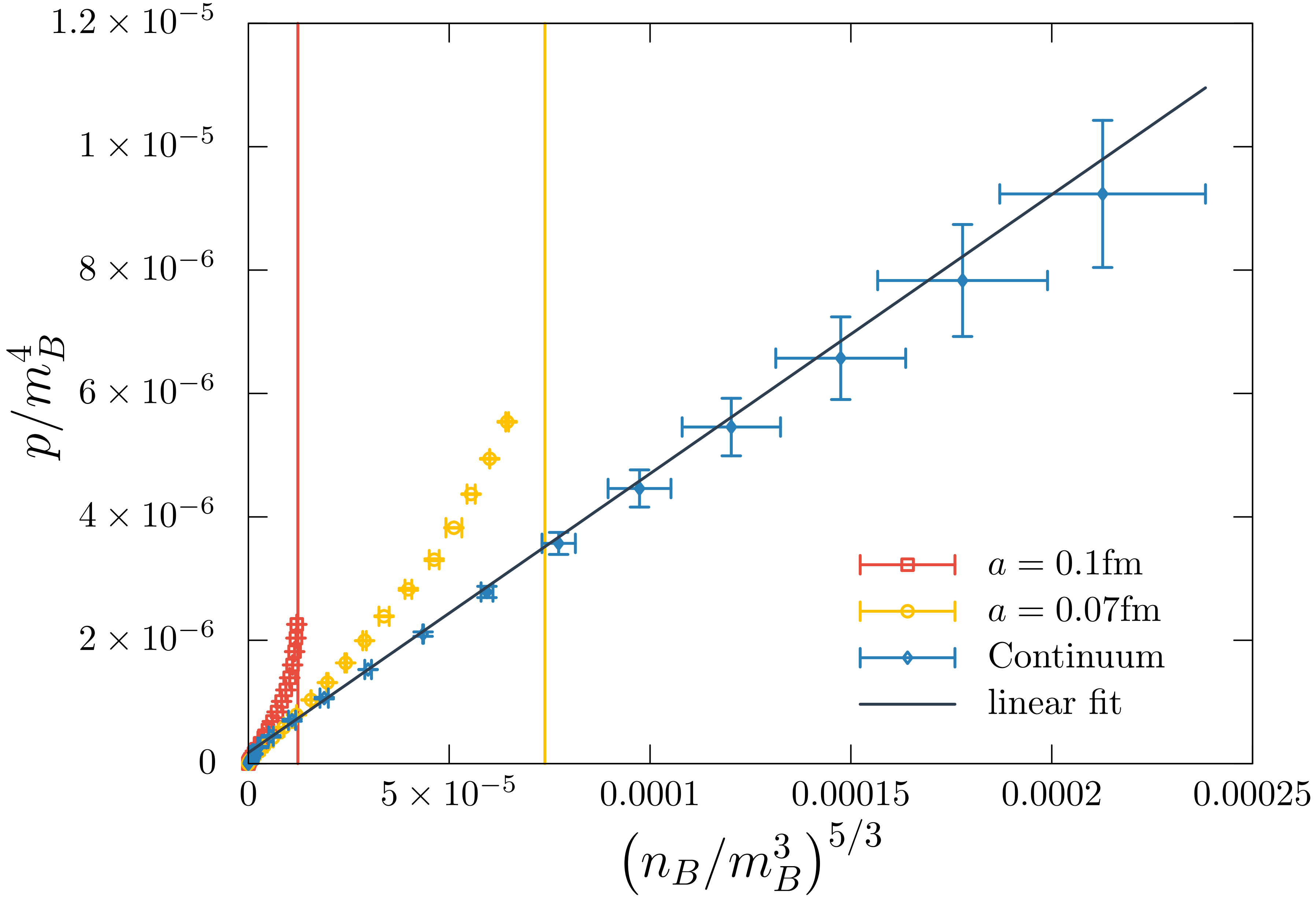


Compare continuum extrapolated results

through $u^5 \kappa^8$



Equation of state of heavy nuclear matter, continuum

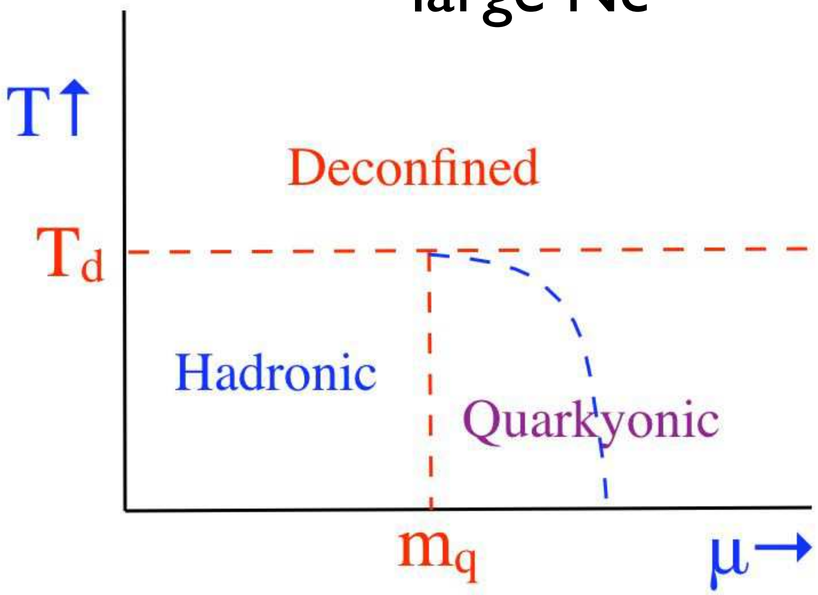


- EoS fitted by polytrope, non-relativistic fermions!
- Can we understand the pre-factor? Interactions, mass-dependence…

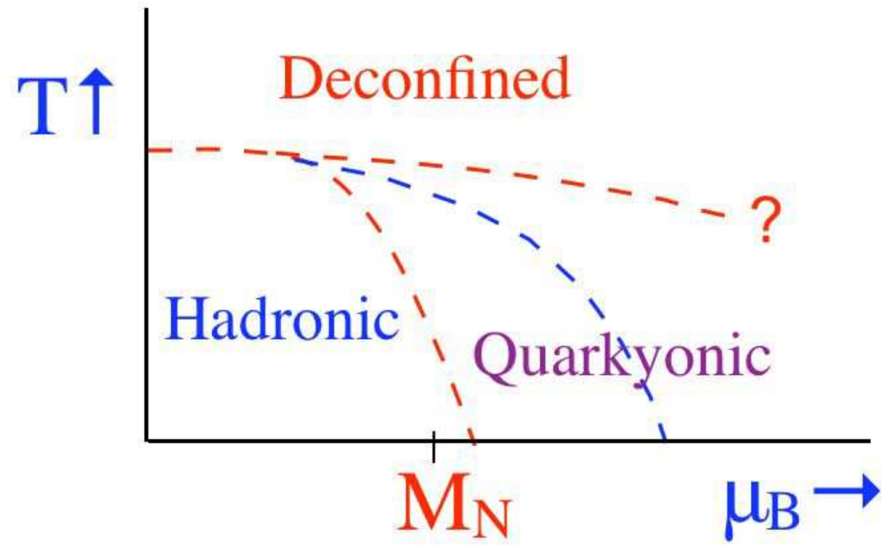
Quarkyonic vs baryonic phase

McLerran, Pisarski 07:

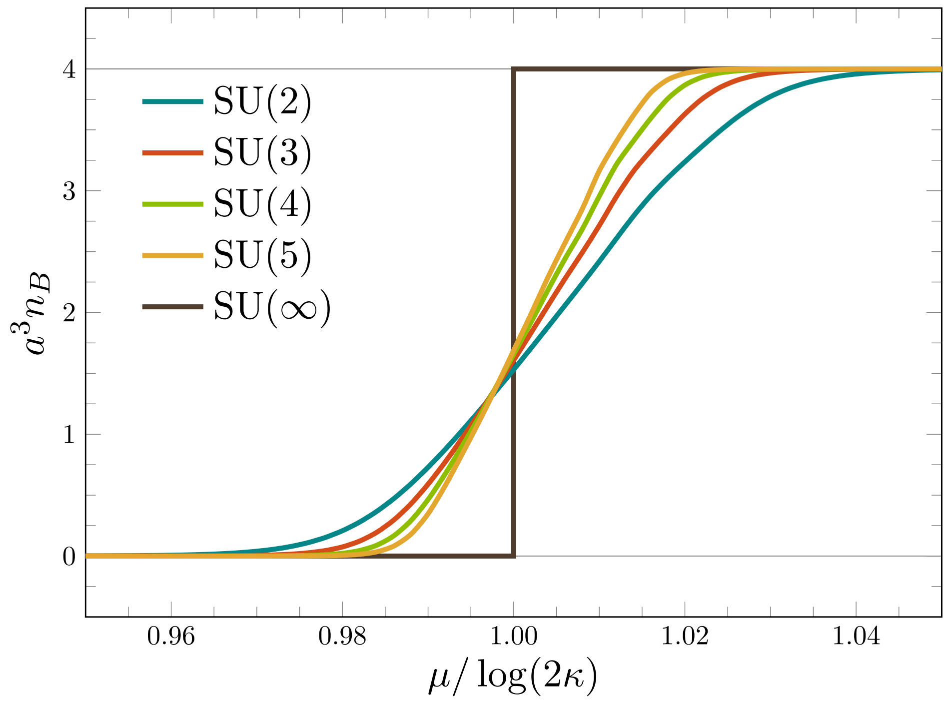
large Nc



QCD



Glesaaen, O.P. in progress:



The effective lattice theory approach II

- Two-step treatment:

  de Forcrand, Langelage, O.P., Unger
  Phys.Rev.Lett. 113 (2014) 152002

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: integrate over gauge links in strong coupling expansion, leave fermions

  Wolff; Karsch Mütter

$$Z_{\rm QCD} = \int d\psi d\bar\psi dU\, e^{S_F+S_G} = \int d\psi d\bar\psi Z_F \left\langle e^{S_G}\right\rangle_{Z_F}$$

$$\left\langle e^{S_G}\right\rangle_{Z_F} \simeq 1+\langle S_G\rangle_{Z_F} = 1+\frac{\beta}{2N_c}\sum_P \left\langle {\rm tr}[U_P+U_P^\dagger]\right\rangle_{Z_F} \qquad Z_F(\psi,\bar\psi)=\int dU\, e^{S_F}$$

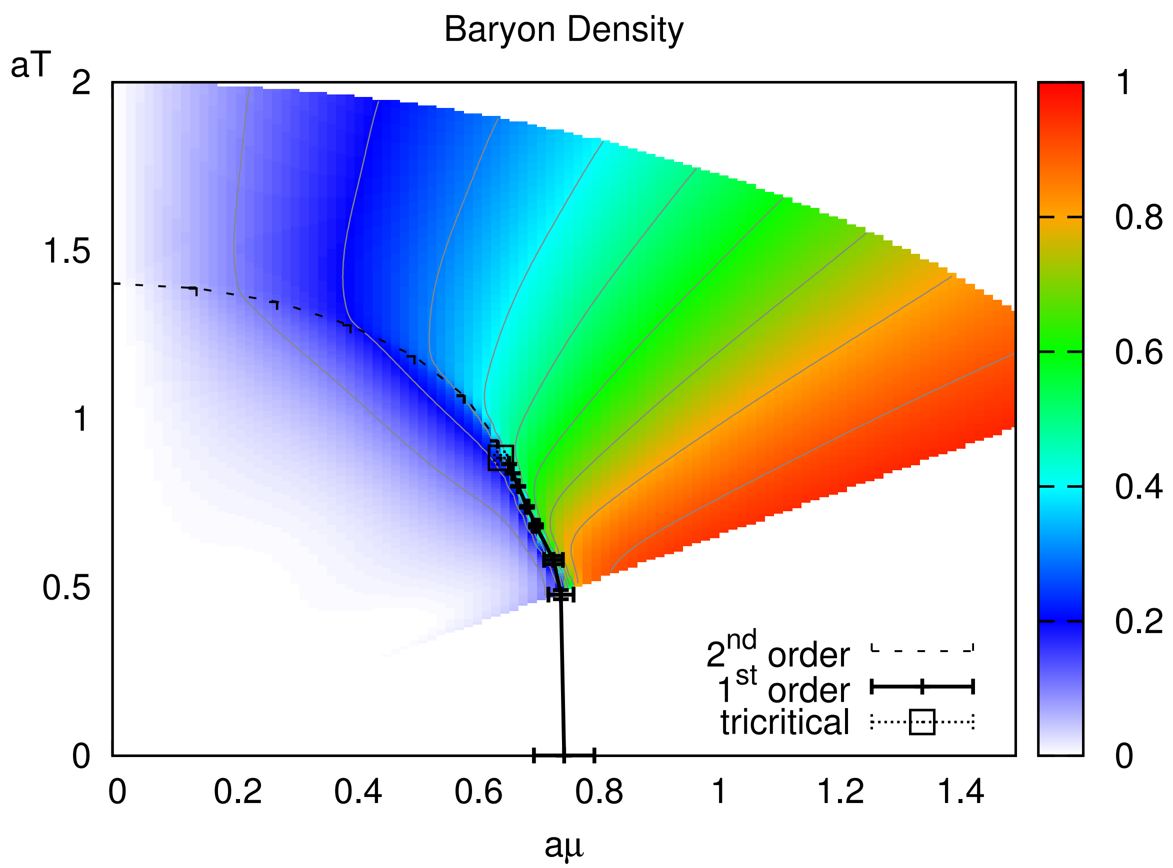
- Result: 4d "polymer" model of QCD (hadronic degrees of freedom!)
  Valid for all quark masses (also m=0!), at strong coupling (very coarse lattices)

- Step II: sign problem milder: Monte Carlo with worm algorithm

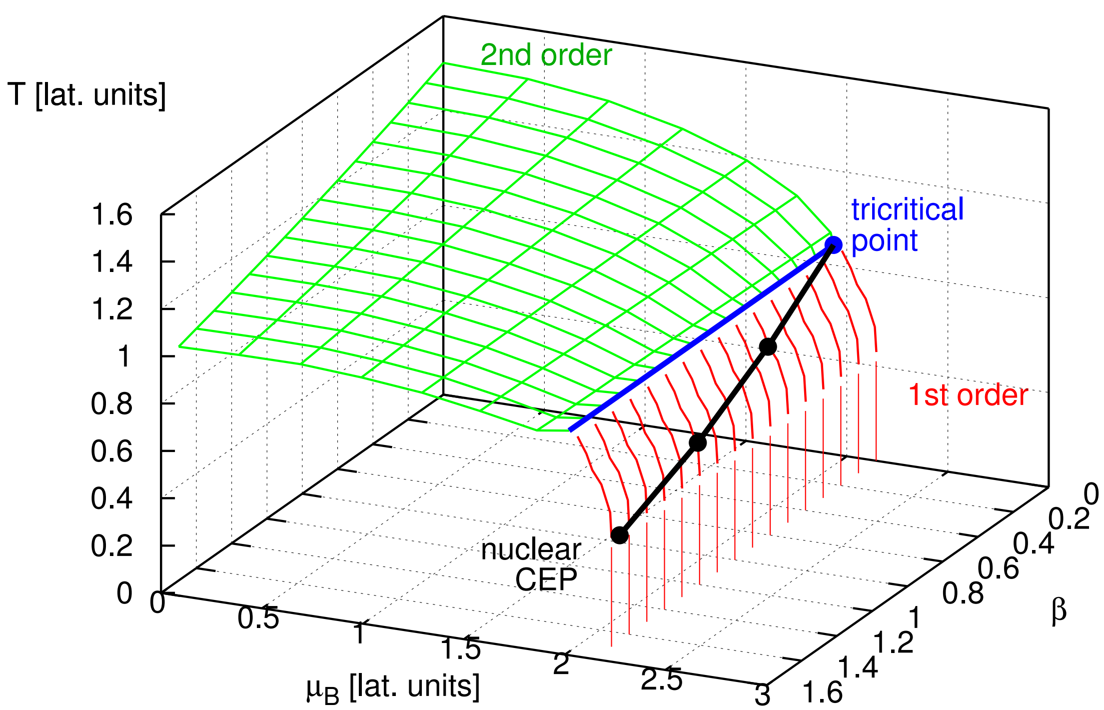
- Numerical simulations without fermion matrix inversion, very cheap!

Chiral QCD at strong coupling

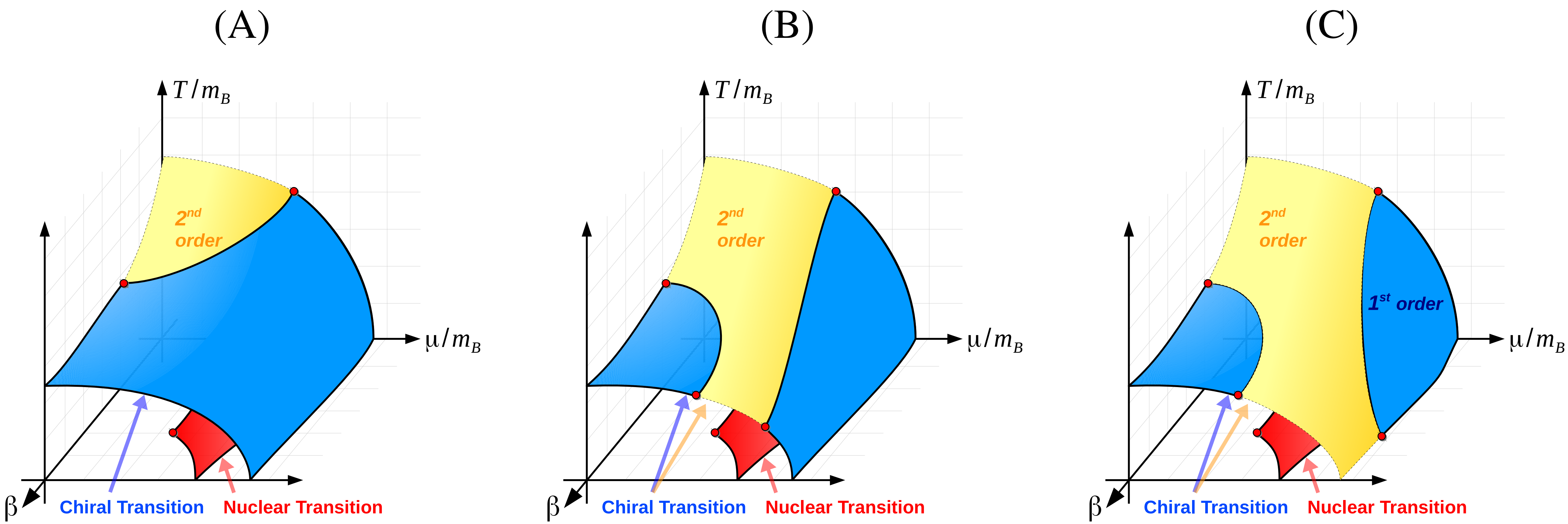
Strong coupling limit:



LO gauge corrections:



Possibilities for continuum Nf=4 phase diagram:



Nf=4 is known to have first order transition at zero density

Conclusions

- Nuclear matter directly from QCD in “one-parameter distortions”:
- Heavy dense QCD near continuum with fully analytic methods
- Chiral dense QCD on coarse lattices
- Larger than nuclear densities out of reach because of lattice saturation

Backup slides

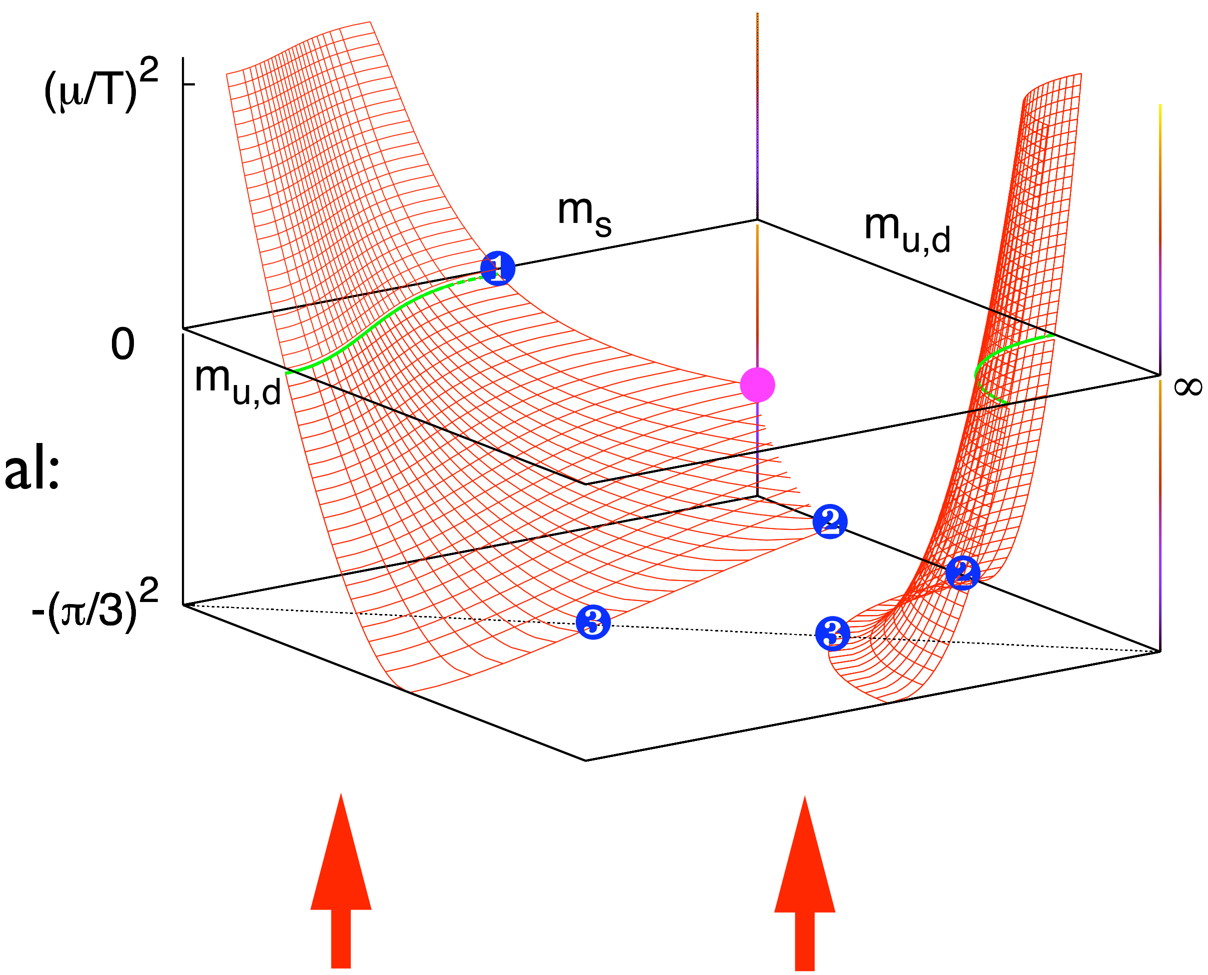
Chiral and deconfinement critical surfaces

Qualitative structure done on coarse Nt=4 lattices:

Real chemical potential:
sign problem

Imaginary chemical potential:
no sign problem

Structure related to
Z(3) symmetry



staggered,Nf=3:
de Forcrand, O.P. 10

staggered,Nf=2:
D'Elia, Sanfillippo 11

Wilson,Nf=2:
Pinke, O.P. 14

shape, sign of curvatures determined by tricritical scaling!

Strong coupling expansion (pure gauge)

Wilson action: $$S_g[U] = \sum_x \sum_{1\le\mu<\nu\le 4} \beta\left(1-\frac{1}{3}\mathrm{Re}\mathrm{Tr}U_p\right) \equiv \sum_p S_p$$ Plaquette action

Character of rep. r: $\chi_r(U) = \mathrm{Tr}D_r(U)$

group element representation matrix of group element

Character expansion: $$\exp -S_p = c_0(\beta)[1+\sum_{r\neq 0} d_r c_r(\beta)\chi_r(U_p)]\ ,$$ convergent inside radius of c.

dimension of rep. matrix

Expansion coefficients: combinations of modified Bessel fcns. for SU(N)

$c_f \equiv u = \frac{\beta}{18} + O(\beta^2)$<1,all others can be expressed by fundamental one

Wilson 74: static potential, string tension Münster, Seo 80-82: glueball masses,
Polonyi, Szachlanyi 82: strong coupling limit of free energy, effective action, Green 83: finite T string
Langelage, Münster, O.P. 08: strong coupling series for finite T

Subleading couplings

Subleading contributions for next-to-nearest neighbours:

$$\lambda_2 S_2 \quad \propto \quad u^{2N_\tau+2} \sum_{[kl]}' 2\mathrm{Re}(L_k L_l^*) \quad \text{distance} \quad = \sqrt{2}$$

$$\lambda_3 S_3 \quad \propto \quad u^{2N_\tau+6} \sum_{\{mn\}}'' 2\mathrm{Re}(L_m L_n^*) \quad \text{distance} \quad = 2$$

as well as terms from loops in the *adjoint* representation:

$$\lambda_a S_a \propto u^{2N_\tau} \sum_{<ij>} \mathrm{Tr}^{(a)} W_i \mathrm{Tr}^{(a)} W_j \quad ; \quad \mathrm{Tr}^{(a)} W = |L|^2 - 1$$

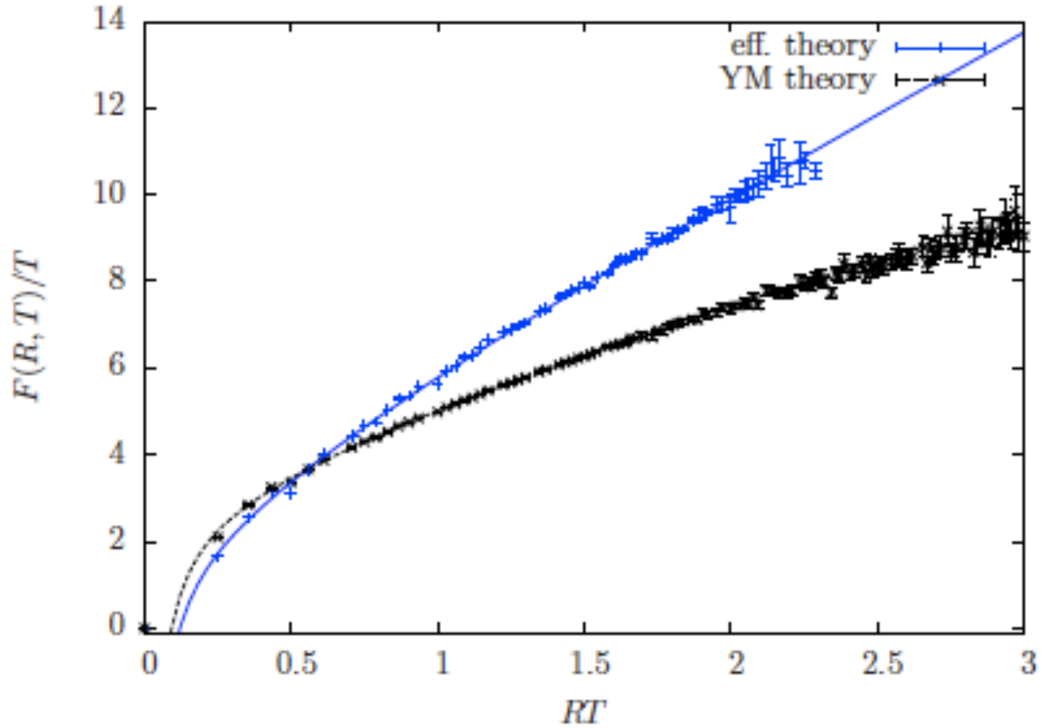
Simplification for T=0, only leading powers in Nt, fermion contribution:

$$
\begin{aligned}
-S_{\mathrm{eff}} = & -\log \sum_{\vec{x}} (1 + h_1 \mathrm{Tr} W_{\vec{x}} + h_1^2 \mathrm{Tr} W_{\vec{x}}^{\dagger} + h_1^3)^2 - 2h_2 \sum_{\vec{x},i} \mathrm{Tr} \frac{h_1 W_{\vec{x}}}{1 + h_1 W_{\vec{x}}} \mathrm{Tr} \frac{h_1 W_{\vec{x}+i}}{1 + h_1 W_{\vec{x}+i}} \\
& + 2 \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i} \mathrm{Tr} \frac{h_1 W_{\vec{x}}}{(1 + h_1 W_{\vec{x}})^2} \mathrm{Tr} \frac{h_1 W_{\vec{x}+i}}{(1 + h_1 W_{\vec{x}+i})^2} \\
& + \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr} \frac{h_1 W_{\vec{x}}}{(1 + h_1 W_{\vec{x}})^2} \mathrm{Tr} \frac{h_1 W_{\vec{x}-i}}{1 + h_1 W_{\vec{x}-i}} \mathrm{Tr} \frac{h_1 W_{\vec{x}-j}}{1 + h_1 W_{\vec{x}-j}} \\
& + 2 \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr} \frac{h_1 W_{\vec{x}}}{(1 + h_1 W_{\vec{x}})^2} \mathrm{Tr} \frac{h_1 W_{\vec{x}-i}}{1 + h_1 W_{\vec{x}-i}} \mathrm{Tr} \frac{h_1 W_{\vec{x}+j}}{1 + h_1 W_{\vec{x}+j}} \\
& + \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr} \frac{h_1 W_{\vec{x}}}{(1 + h_1 W_{\vec{x}})^2} \mathrm{Tr} \frac{h_1 W_{\vec{x}+i}}{1 + h_1 W_{\vec{x}+i}} \mathrm{Tr} \frac{h_1 W_{\vec{x}+j}}{1 + h_1 W_{\vec{x}+j}} \; .
\end{aligned}
$$

Current state of the art for fermionic sector: $u^5 \kappa^8$

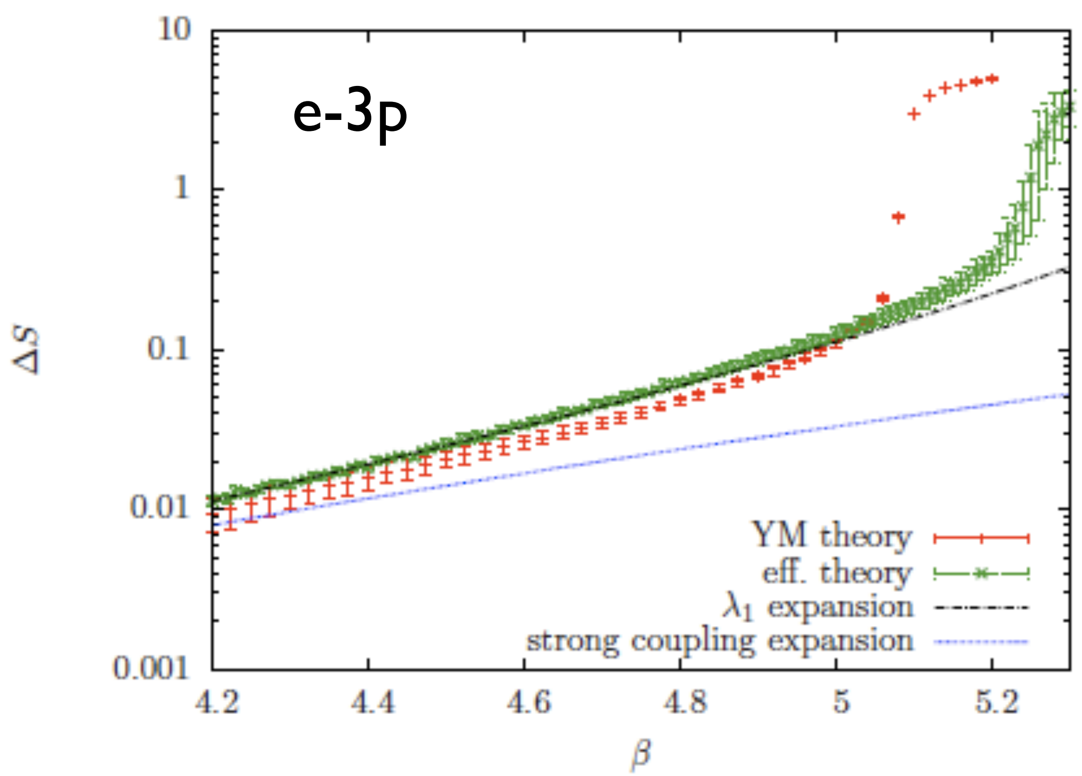
One coupling: What does and does not work?

G.Bergner, J.Langelage, O.P. 14,15



Correlation functions and spectrum:
NO

couplings over large distances needed



Thermodynamics and critical coupling:
YES

partition function needed, local!

The equation of state for nuclear matter, Nf=2