

Deconfinement at high temperatures and moderately high baryon densities

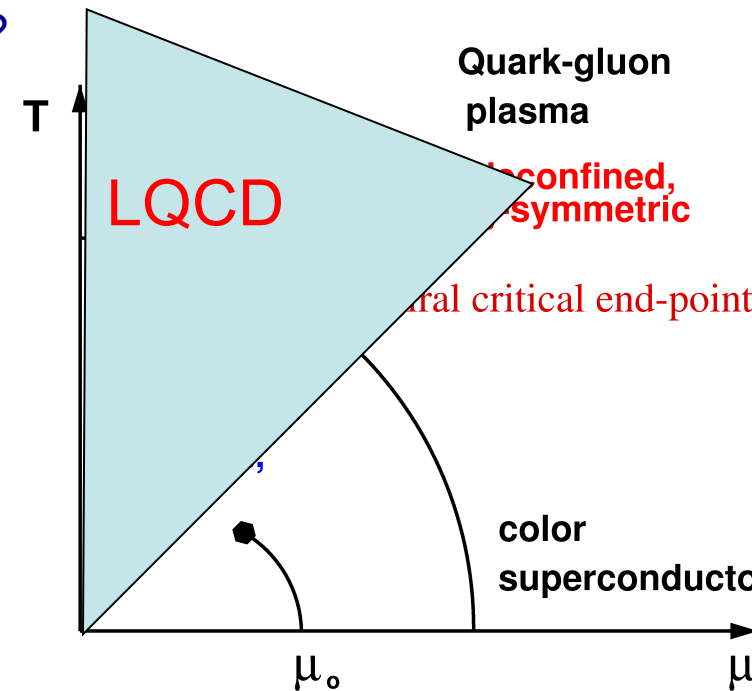
Péter Petreczky



What is the limiting temperature on hadronic matter ?
What is the nature of the deconfined matter ?

In this talk:

- Chiral transition in QCD at $T > 0$
- Color screening deconfinement
- Equation of state
- Fluctuations of conserved charges
- How does the transition and EoS changes with baryon density



The temperature dependence of chiral condensate

Renormalized chiral condensate introduced by Budapest-Wuppertal collaboration

$$\langle \bar{\psi}\psi \rangle_q \Rightarrow \Delta_q^R(T) = m_s r_1^4 \left(\langle \bar{\psi}\psi \rangle_{q,T} - \langle \bar{\psi}\psi \rangle_{q,T=0} \right) + d, \quad q = l, s$$

With choice : $d = \langle \bar{\psi}\psi \rangle_{m_q=0}^{T=0}$

Bazavov et al (HotQCD), Phys. Rev. D85 (2012) 054503;

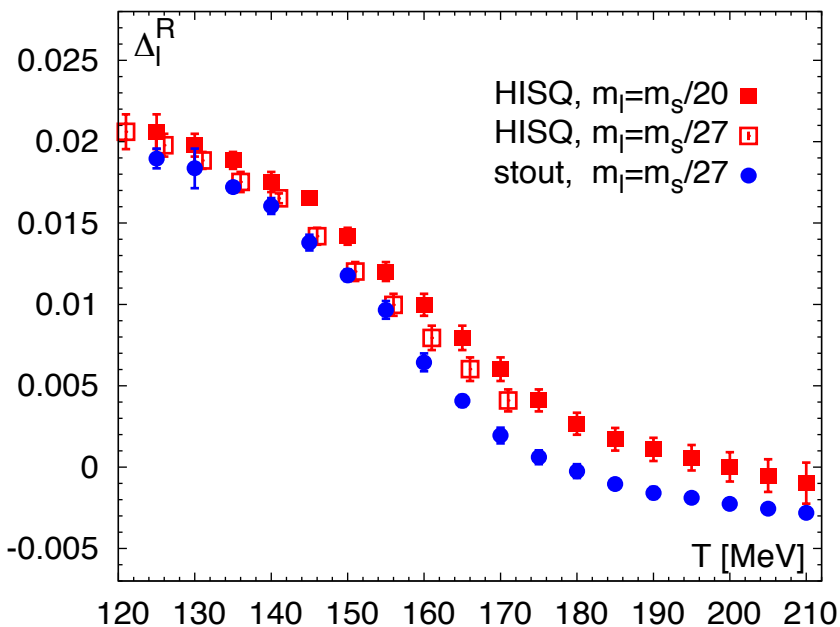
Bazavov, PP, PRD 87(2013)094505,

Borsányi et al, JHEP 1009 (2010) 073

Calculations with chiral
(Domain Wall) fermions:

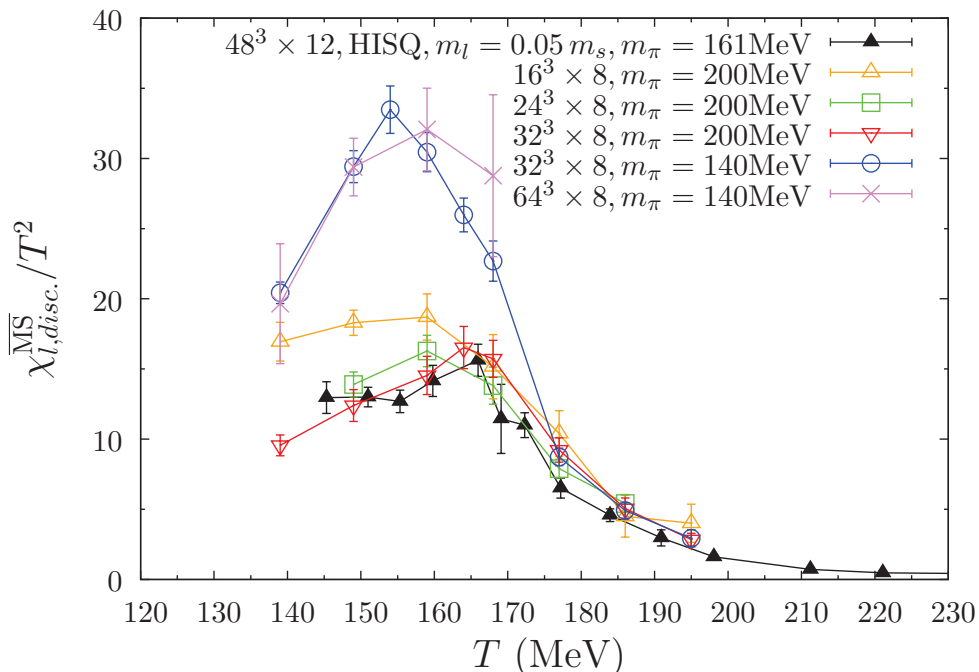
$$\chi_{disc} = \frac{V}{T} \left(\langle (\bar{\psi}\psi)^2 \rangle - \langle \bar{\psi}\psi \rangle^2 \right)$$

Bhattacharya et al (HotQCD), PRL 113 (2014)082001



- O(4) scaling analysis and continuum limit:

$$T_c = (154 \pm 8 \pm 1(\text{scale}))\text{MeV}$$



Transition is a crossover (no volume dep.)

$$T_c = (155 \pm 8 \pm 1)\text{MeV}$$

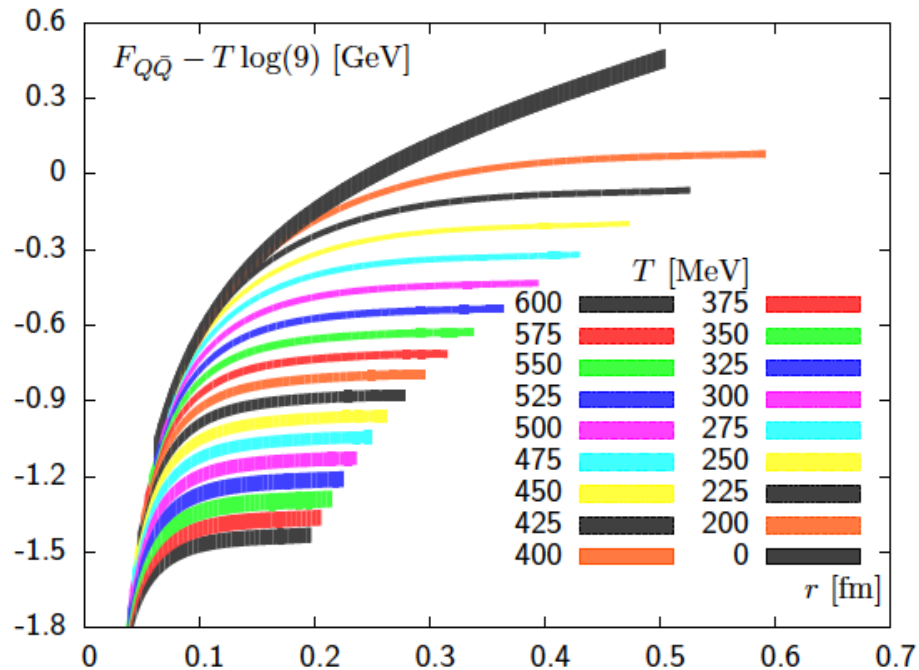
Deconfinement and color screening

Onset of color screening is described by Polyakov loop (order parameter in SU(N) gauge theory)

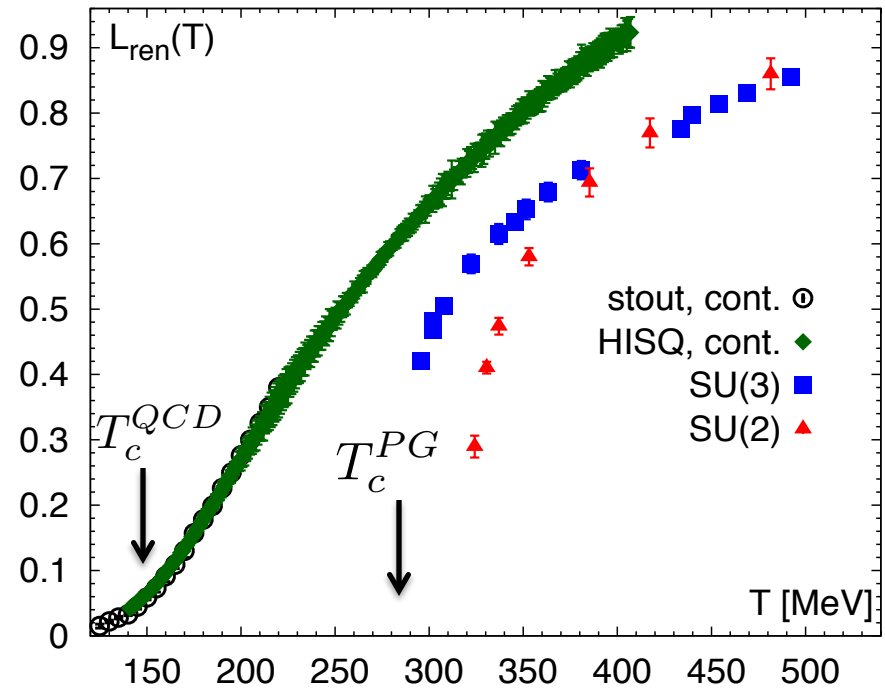
$$L = \text{tr} \mathcal{P} e^{ig \int_0^{1/T} d\tau A_0(\tau, \vec{x})} \quad \exp(-F_{Q\bar{Q}}(r, T)/T) = \frac{1}{9} \langle \text{tr} L(r) \text{tr} L^\dagger(0) \rangle$$

$$F_{Q\bar{Q}}(r \rightarrow \infty, T) = 2F_Q(T) \quad \Rightarrow \quad L_{\text{ren}} = \exp(-F_Q(T)/T)$$

2+1 flavor QCD, continuum extrapolated (work in progress with Bazavov, Weber ...)



free energy of static quark anti-quark pair shows Debye screening at high temperatures



SU(N) gauge theory \neq QCD !

Similar results with stout action [Borsányi et al, JHEP04\(2015\) 138](#)

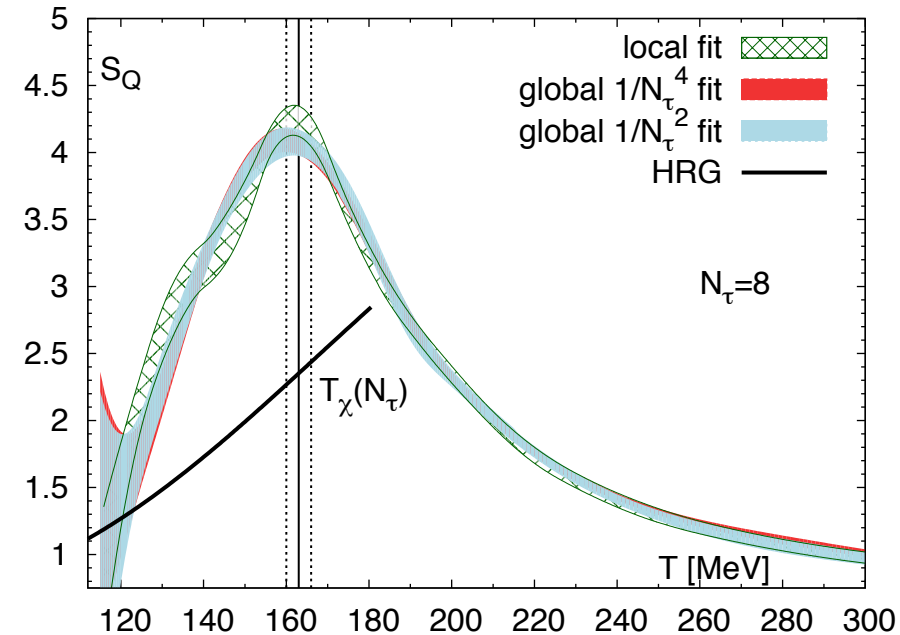
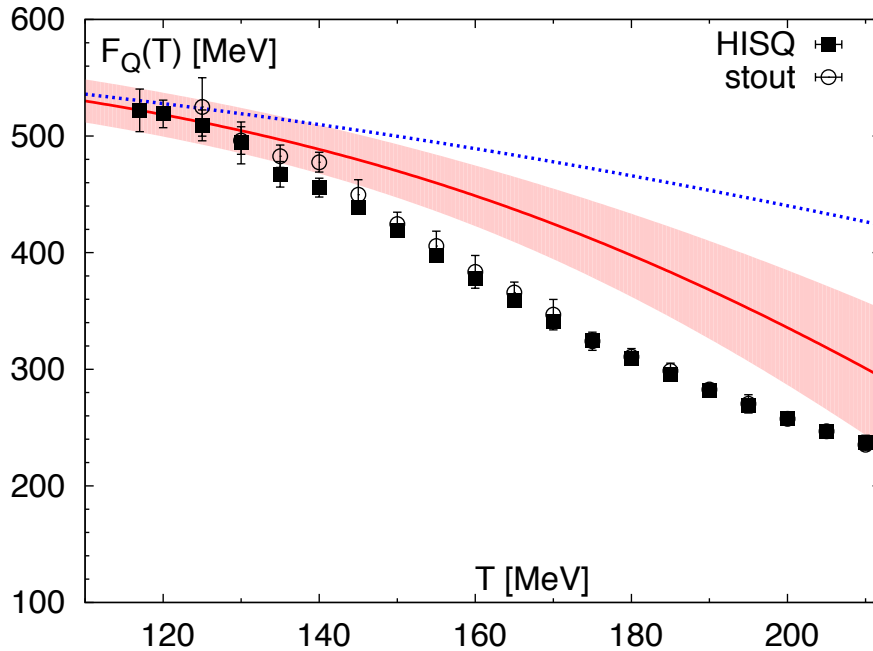
The entropy of static quark

Bazavov, PP, PRD 87 (2013) 094505

Bazavov et al, PRD 93 (2016) 114502

Hadron gas (of static-light) vs. LQCD

$$S_Q = -\frac{\partial F_Q}{\partial T}$$



At low T the entropy S_Q increases reflecting the increase of states the heavy quark can be coupled to; at high temperature the static quark only “sees” the medium within a Debye radius, as T increases the Debye radius decreases and S_Q also decreases

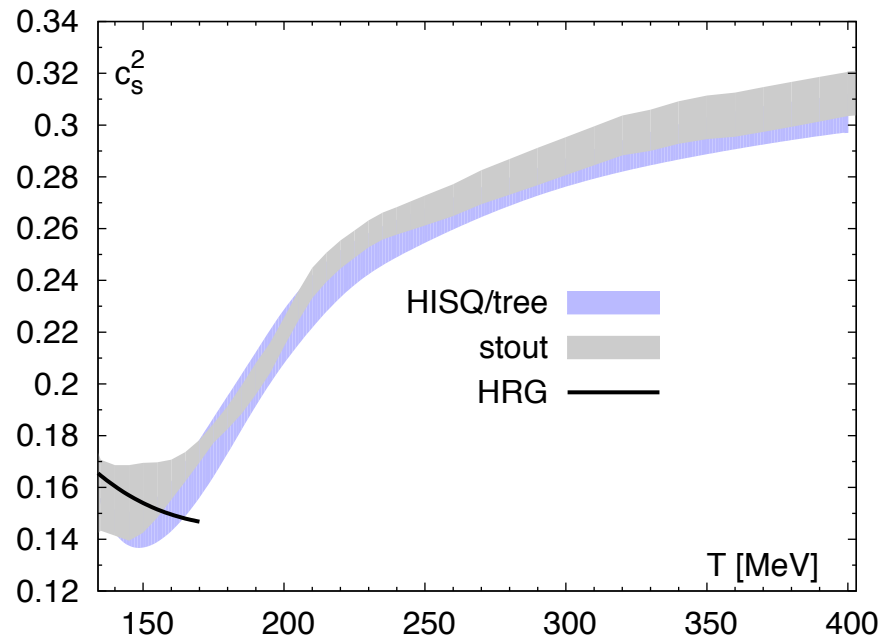
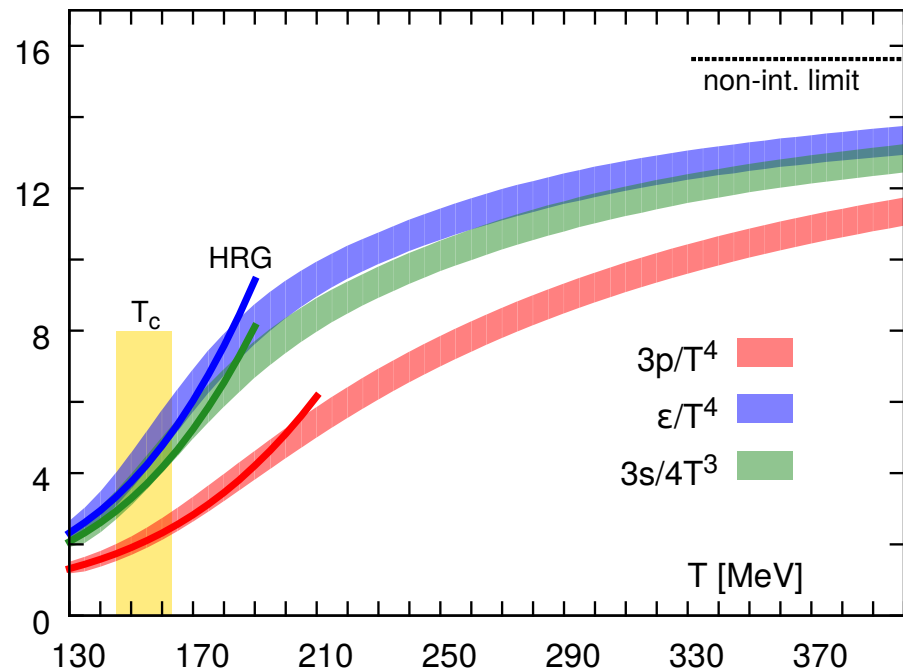
The onset of screening corresponds to peak in S_Q and its position coincides with T_c

Equation of state in the continuum limit

Equation of state has been calculated in the continuum limit up to $T=400$ MeV using two different discretizations and the results agree well

Bazavov et al (HotQCD), PRD 90 (2014) 094503

Borsányi et al, PLB 730 (2014) 99



Hadron resonance gas (HRG):
Interacting gas of hadrons = non-interacting
gas of hadrons and hadron resonances

← virial expansion

Prakash, Venugopalan, NPA546 (1992) 718

HRG agrees with the lattice for $T < 155$ MeV

$$T_c = (154 \pm 9) \text{ MeV}$$



$$\epsilon_c \simeq 300 \text{ MeV}/\text{fm}^3$$

$$\epsilon_{low} \simeq 180 \text{ MeV}/\text{fm}^3 \leftrightarrow \epsilon_{nucl} \simeq 150 \text{ MeV}/\text{fm}^3$$

$$\epsilon_{high} \simeq 500 \text{ MeV}/\text{fm}^3 \leftrightarrow \epsilon_{proton} \simeq 450 \text{ MeV}/\text{fm}^3$$

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T, \mu_B, \mu_Q, \mu_S, \mu_C)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{BQSC} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_B}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k \left(\frac{\mu_C}{T}\right)^l \quad \text{hadronic}$$

$$\frac{p(T, \mu_u, \mu_d, \mu_s, \mu_c)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{udsc} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k \left(\frac{\mu_c}{T}\right)^l \quad \text{quark}$$

$$\chi_{ijkl}^{abcd} = T^{i+j+k+l} \frac{\partial^i}{\partial \mu_b^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{\partial^l}{\partial \mu_d^l} \ln Z(T, V, \mu_a, \mu_b, \mu_c, \mu_d) \Big|_{\mu_a=\mu_b=\mu_c=\mu_d=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3} (\langle X^2 \rangle - \langle X \rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3} (\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$



information about carriers of the conserved charges (hadrons or quarks)



probes of deconfinement

Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3} (\langle B^2 \rangle - \langle B \rangle^2)$$

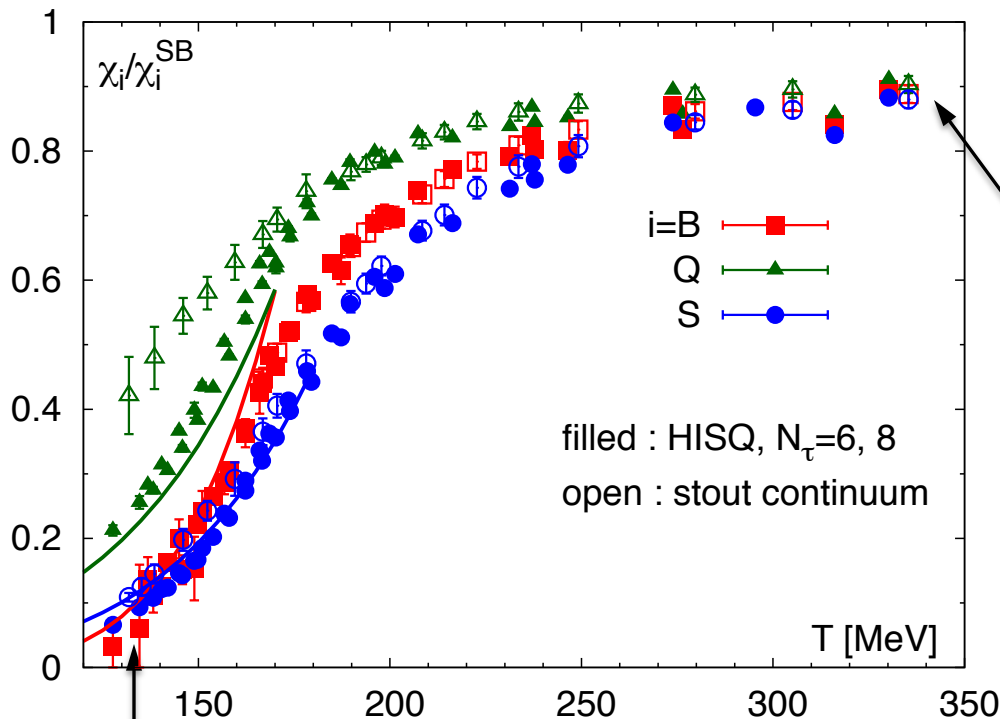
baryon number

$$\chi_Q = \frac{1}{VT^3} (\langle Q^2 \rangle - \langle Q \rangle^2)$$

electric charge

$$\chi_S = \frac{1}{VT^3} (\langle S^2 \rangle - \langle S \rangle^2)$$

strangeness



Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \quad \chi_Q^{SB} = \frac{2}{3}$$

$$\chi_S^{SB} = 1$$

conserved charges carried by light quarks

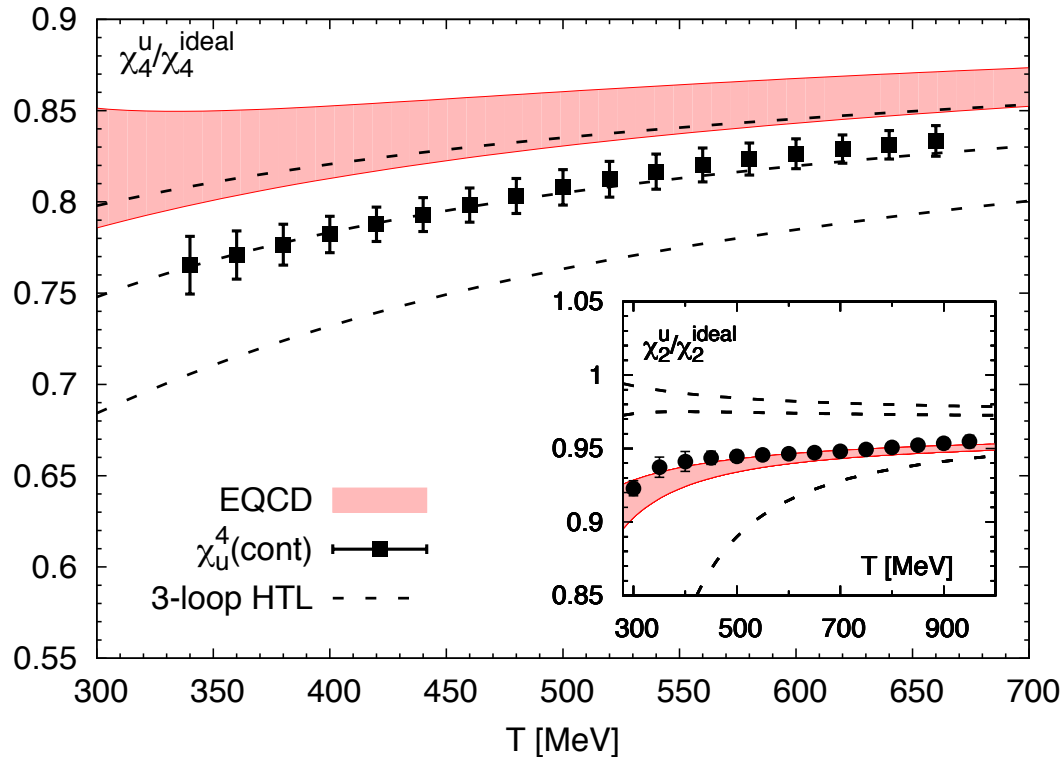
Bazavov et al (HotQCD) PRD86 (2012) 034509
 Borsányi et al, JHEP 1201 (2012) 138

conserved charges are carried by massive hadrons

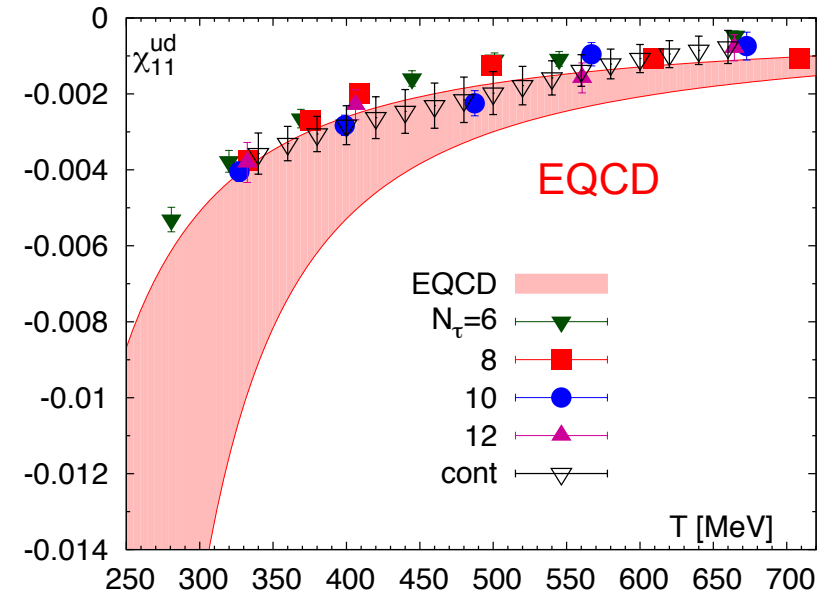
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

quark number fluctuations



quark number correlations



- Lattice results converge as the continuum limit is approached
- Good agreement between lattice and the weak coupling approach for 2nd and 4th order quark number fluctuations as well as for correlations

Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

$$P_S = \frac{p(T) - p_{S=0}(T)}{T^4} = M(T) \cosh\left(\frac{\mu_S}{T}\right) +$$

$$B_{S=1}(T) \cosh\left(\frac{\mu_B - \mu_S}{T}\right) + B_{S=2}(T) \cosh\left(\frac{\mu_B - 2\mu_S}{T}\right) + B_{S=3}(T) \cosh\left(\frac{\mu_B - 3\mu_S}{T}\right)$$



$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

$$v_2 = \frac{1}{3} (\chi_4^S - \chi_2^S) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

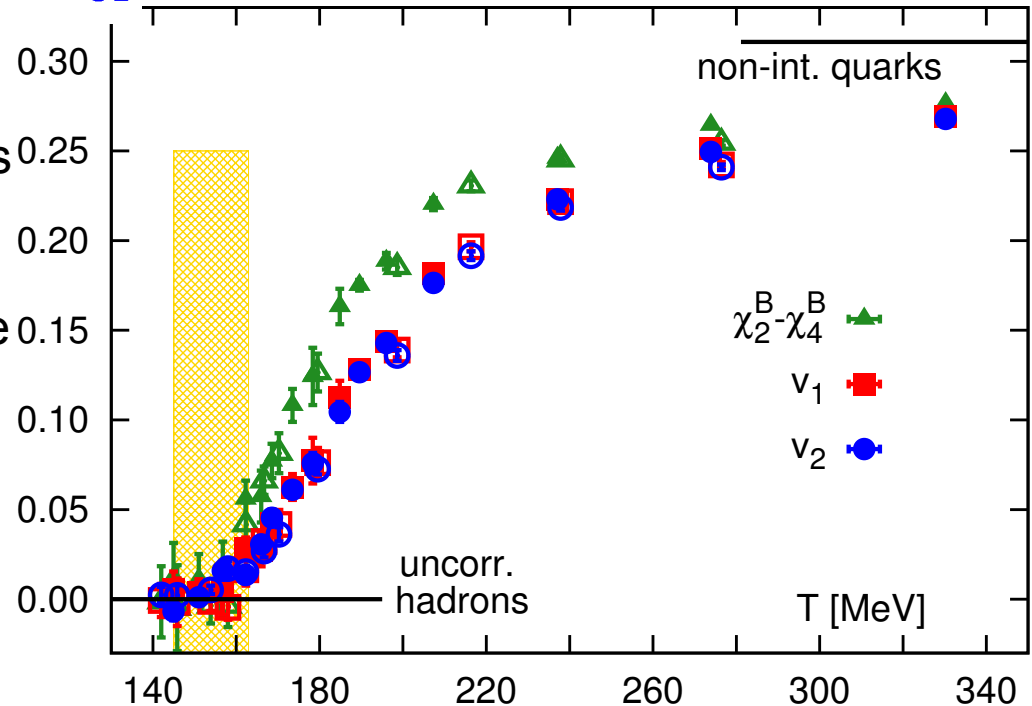
should vanish !

- v_1 and v_2 do vanish within errors at low T

- v_1 and v_2 rapidly increase above the transition region, eventually reaching non-interacting quark gas values

Strange hadrons are heavy \rightarrow treat them as Boltzmann gas

Bazavov et al, PRL 111 (2013) 082301



Deconfinement of strangeness (cont'd)

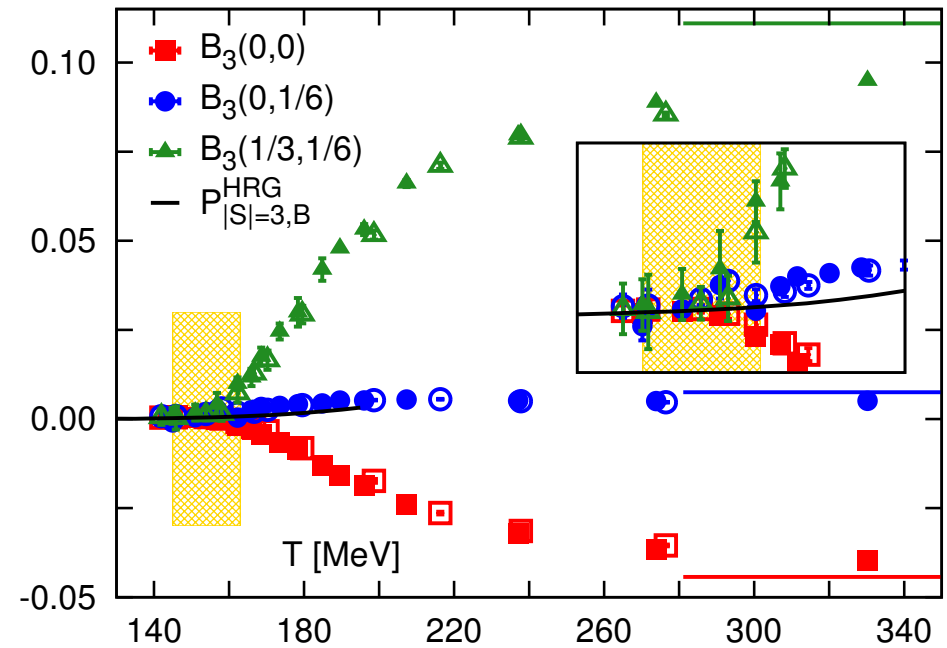
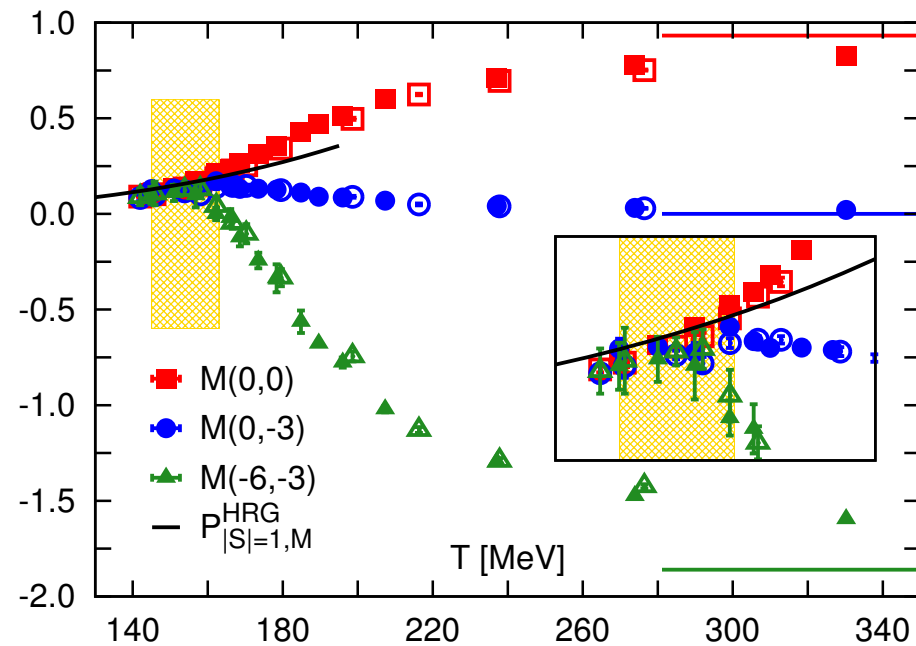
Using the six Taylor expansion coefficients related to strangeness

$$\chi_2^S, \chi_4^S, \chi_{13}^{BS}, \chi_{22}^{BS}, \chi_{31}^{BS}$$

it is possible to construct combinations that give

$$M(T), B_{S=1}(T), B_{S=2}(T), B_{S=3}(T)$$

up to terms $c_1 v_1 + c_2 v_2$



Hadron resonance gas descriptions breaks down for all strangeness sectors above T_c

Bazavov et al, PRL 111 (2013) 082301

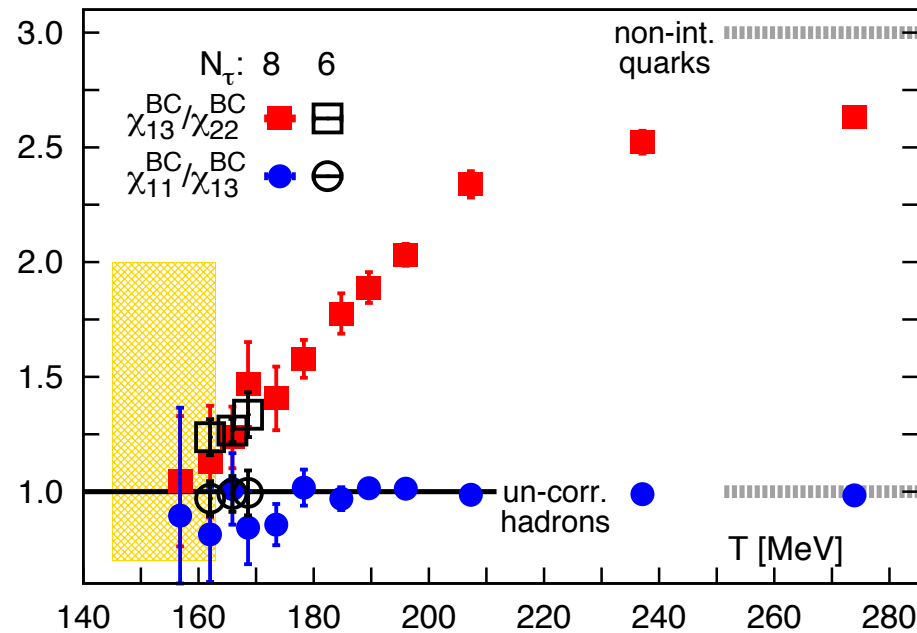
Deconfinement of charm

$$\chi_{nml}^{XYC} = T^{m+n+l} \frac{\partial^{n+m+l} p(T, \mu_X, \mu_Y, \mu_C) / T^4}{\partial \mu_X^n \partial \mu_Y^m \partial \mu_C^l}$$

Bazavov et al, PLB 737 (2014) 210

$m_c \gg T \Rightarrow$ only $|C|=1$ sector contributes

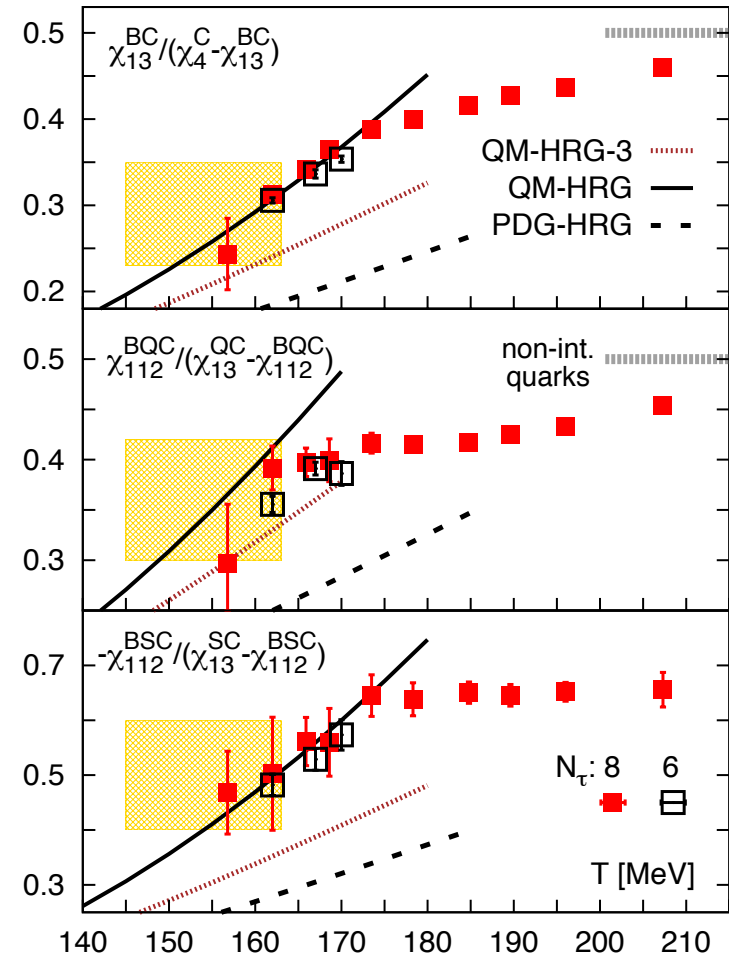
In the hadronic phase all BC -correlations are the same !



Hadronic description breaks down just above T_c
 \Rightarrow open charm deconfines above T_c

The charm baryon spectrum is not well known (only few states in PDG), HRG works only if the “missing” states are included

Charm baryon to meson pressure



Quasi-particle model for charm degrees of freedom

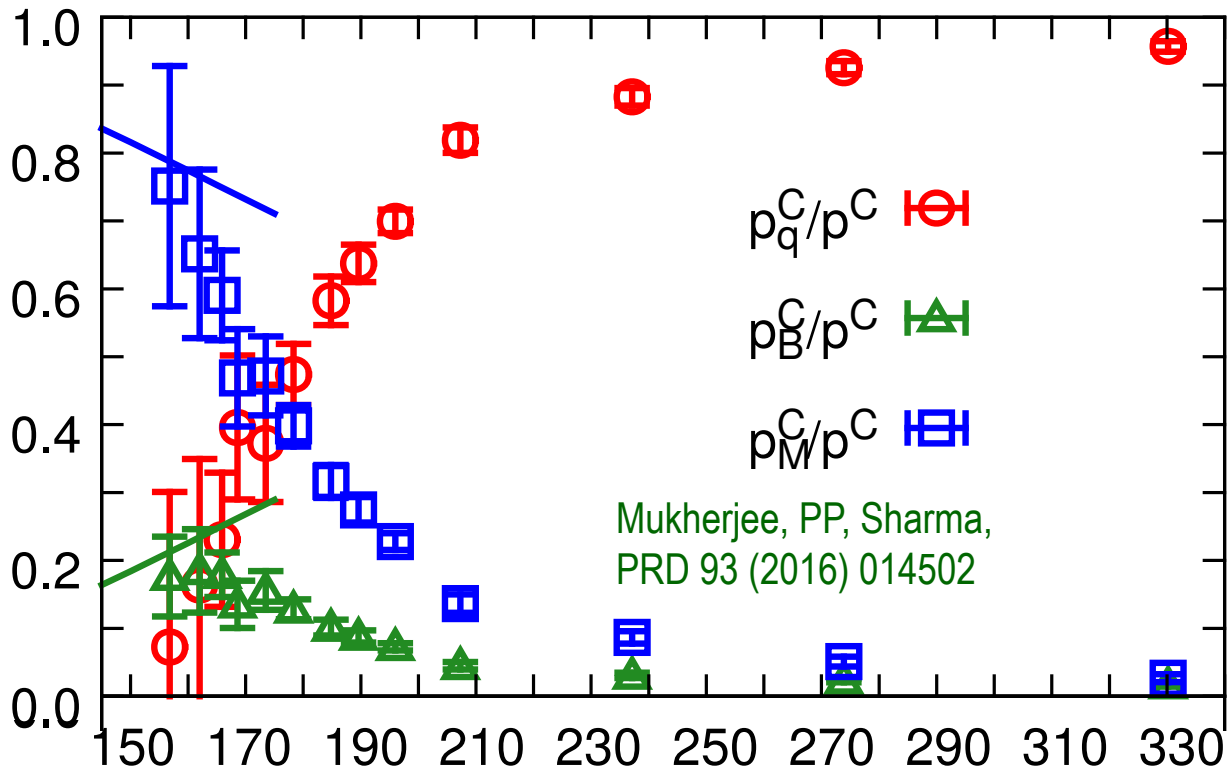
Charm dof are good quasi-particles at all T because $M_c \gg T$ and Boltzmann approximation holds

$$p^C(T, \mu_B, \mu_c) = p_q^C(T) \cosh(\hat{\mu}_C + \hat{\mu}_B/3) + p_B^C(T) \cosh(\hat{\mu}_C + \hat{\mu}_B) + p_M^C(T) \cosh(\hat{\mu}_C)$$

$$\chi_2^C, \chi_{13}^{BC}, \chi_{22}^{BC} \Rightarrow p_q^C(T), p_M^C(T), p_B^C(T)$$

$$\hat{\mu}_X = \mu_X/T$$

Partial meson and baryon pressures described by HRG at T_c and dominate the charm pressure then drop gradually, charm quark only dominant dof at $T > 200$ MeV or $\epsilon > 6$ GeV/fm³



Partial pressures drop because hadronic excitations become broad at high temperatures (bound state peaks merge with the continuum)

See
 Jakovác, PRD88 (2013), 065012
 Biró, Jakovác, PRD(2014)065012

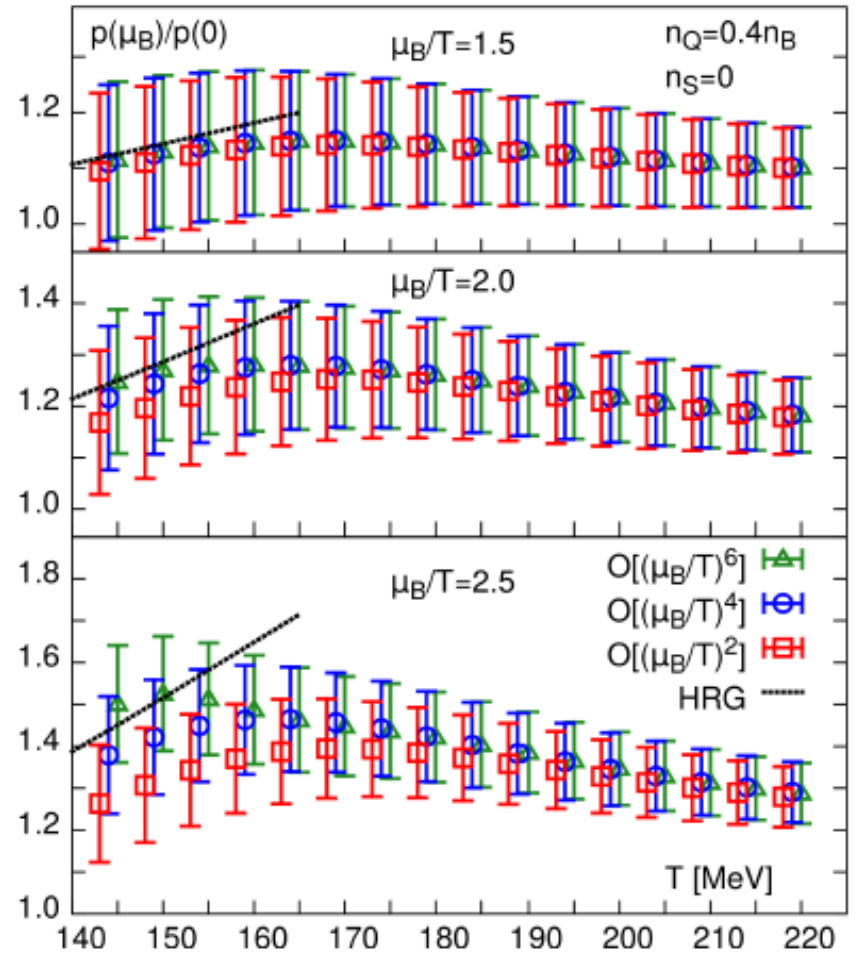
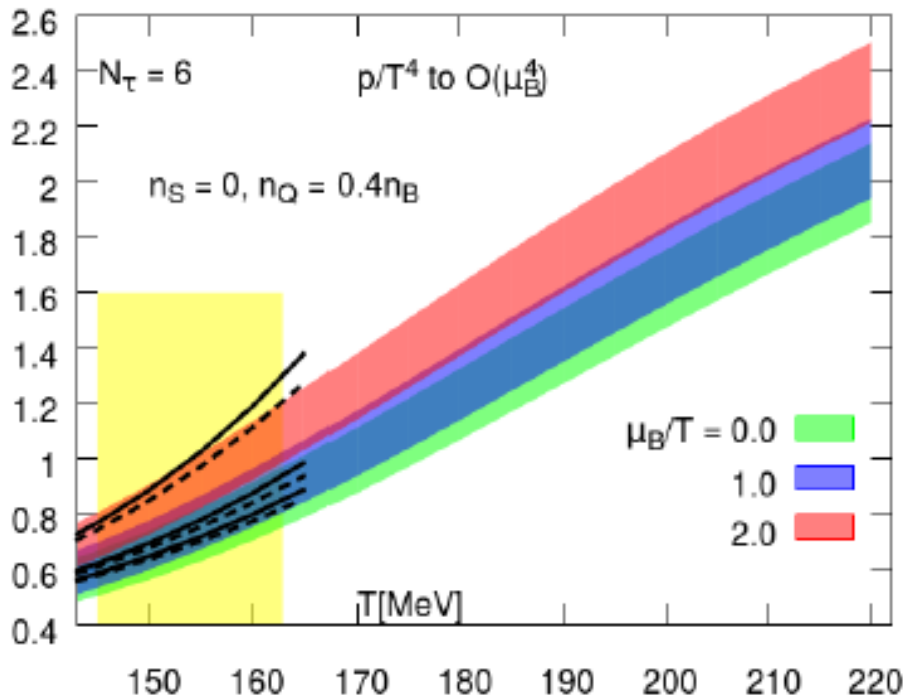
Vice versa for quarks

Equation of state at non-zero baryon density

At the highest energy heavy ion collisions create a system with almost zero net baryon density. by decreasing the the center of mass energy of the collisions $E_{CM} = \sqrt{s}$ the net baryon number can be increased and the EoS calculated in LQCD using Taylor expansion

Taylor expansion up to 4th order for net zero strangeness $n_S = 0$ and

$$r = n_Q/n_B = Z/A = 0.4$$



Moderate effects due to non-zero baryon density up to $\mu_B/T = 2 \leftrightarrow \sqrt{s} \sim 20\text{GeV}$

and Taylor expansion works well

Freeze-out and transition at non-zero baryon density

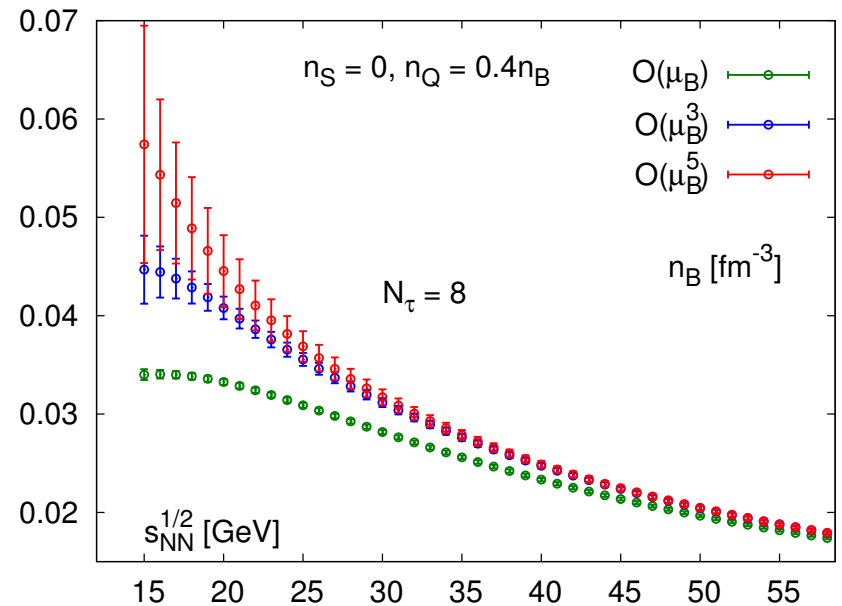
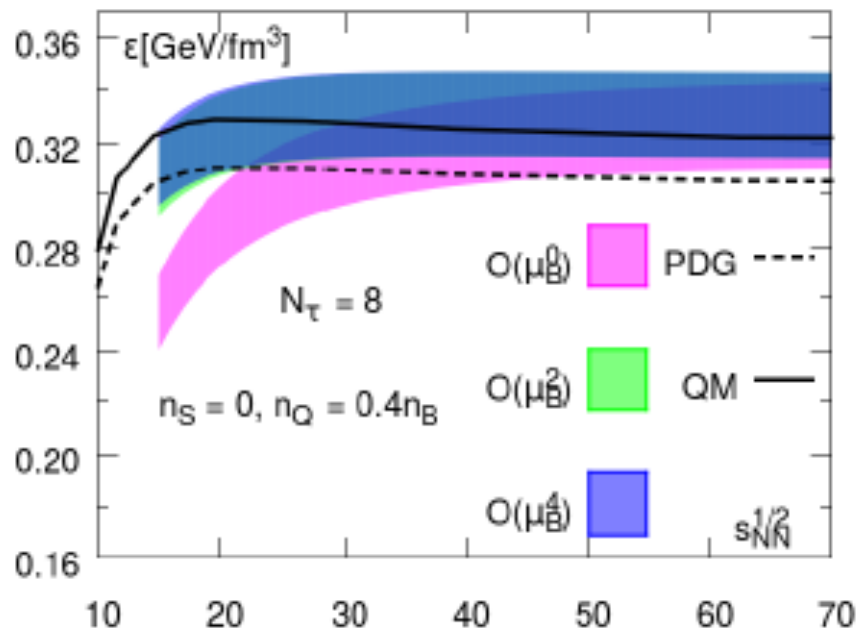
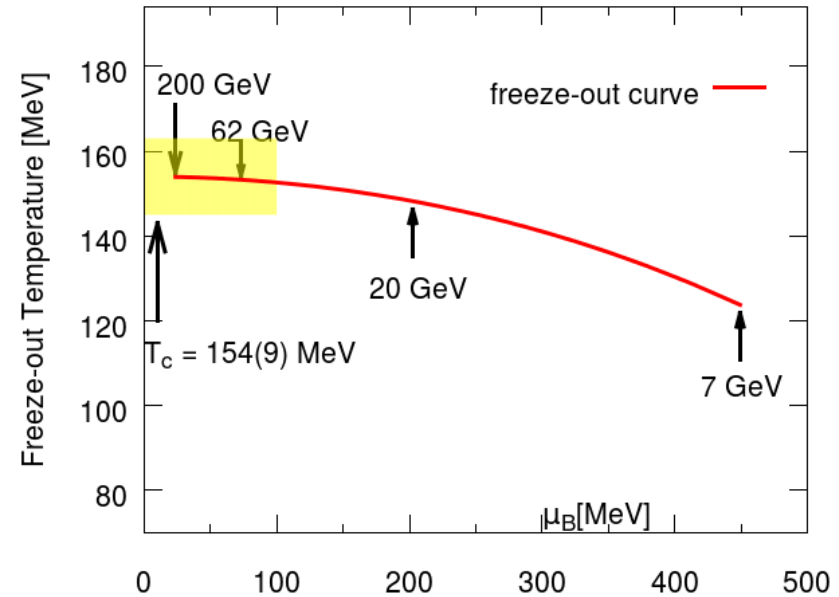
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c} \right)^2 + \dots$$

$\kappa = 0.020(4)$ Cea et al, PRD 93 (2016) 014507

$\kappa = 0.0135(20)$ Bonati et al, PRD 92 (2015) 054503

$\kappa = 0.0149(21)$ Bellweid et al, PLB 751 (2015) 559

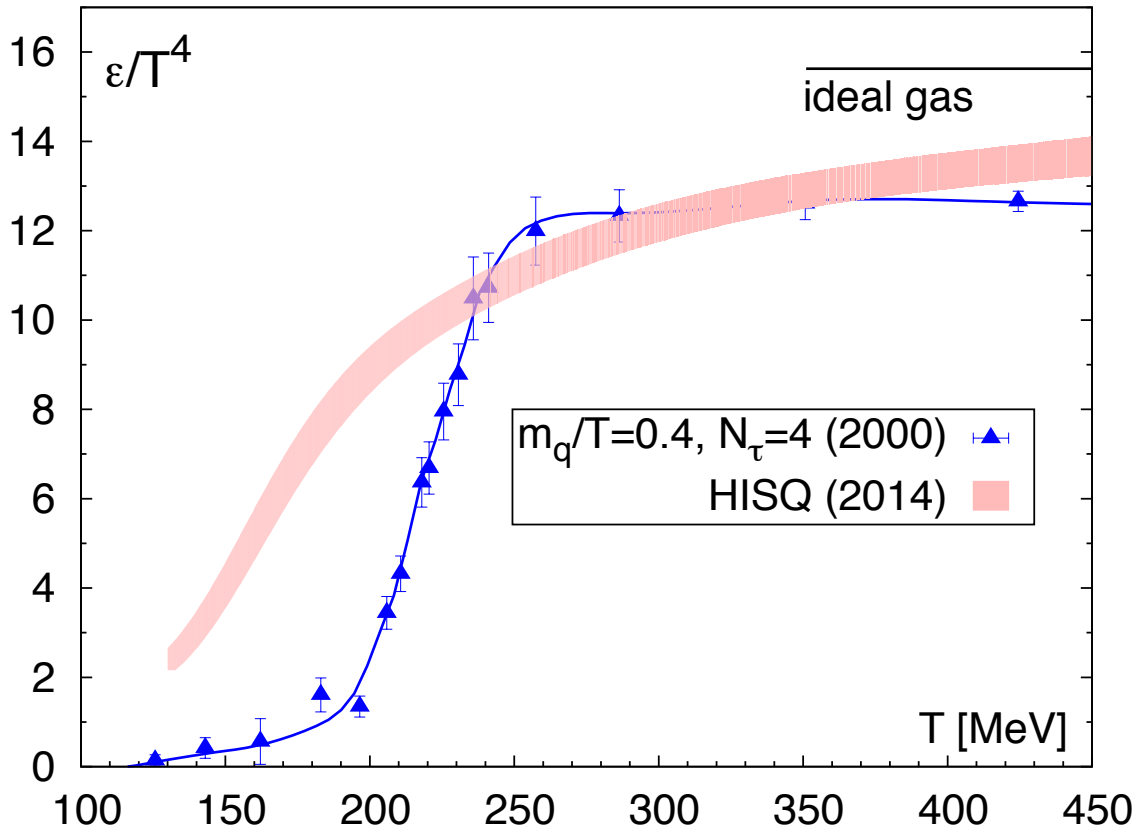
$\kappa = 0.0066(7)$ Kaczmarek et al, PRD 83 (2011) 014504



Summary

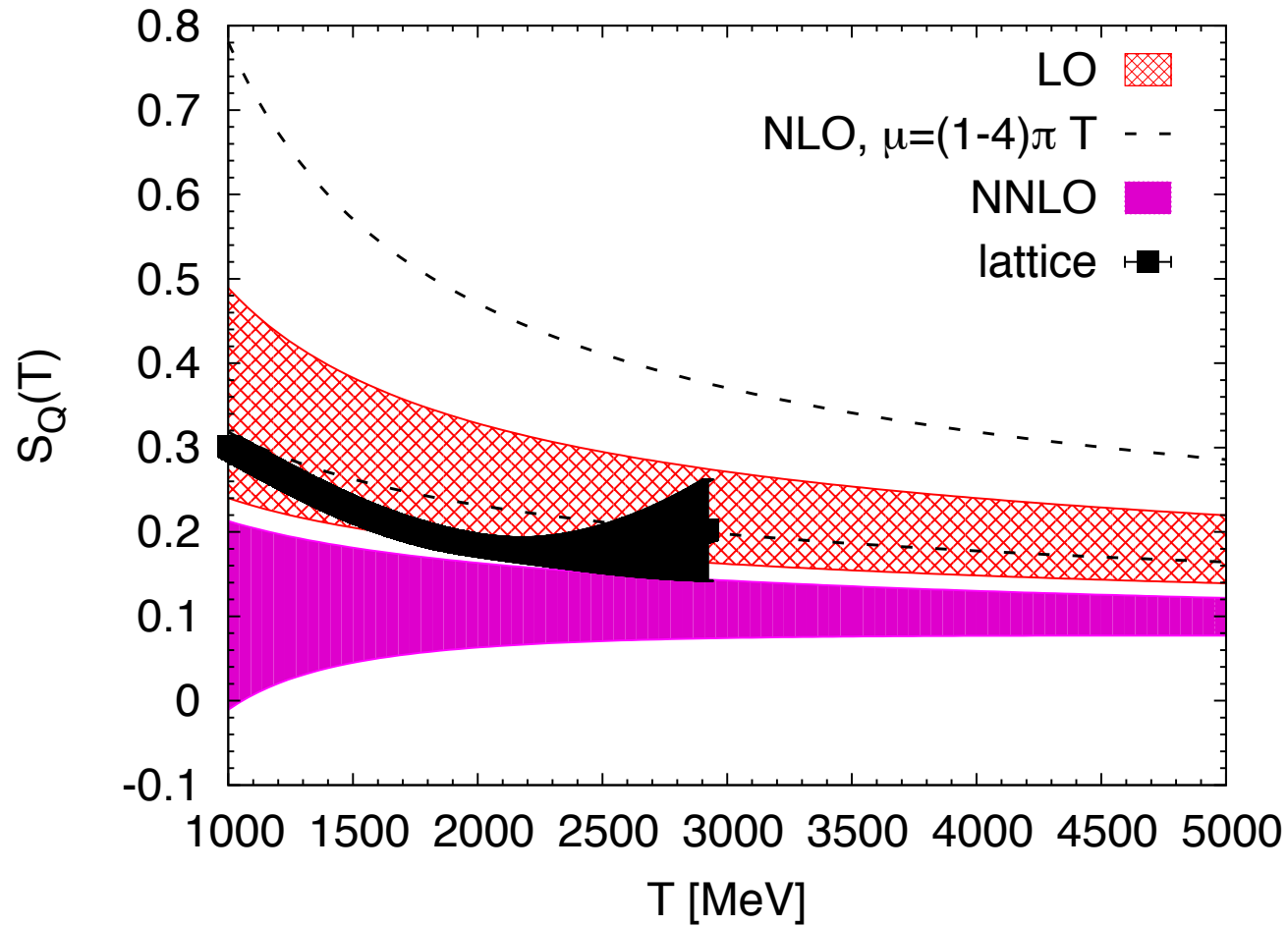
- The deconfinement transition temperature coincides with the chiral transition temperature $T_c = 154(9) \text{ MeV}$
- Equation of state are known in the continuum limit up to $T=400 \text{ MeV}$ at zero baryon density and the energy density at the transition is small $\approx 300 \text{ MeV/fm}^3$
- Hadron resonance gas (HRG) can describe various thermodynamic quantities at low temperatures
- Deconfinement transition can be studied in terms of fluctuations and correlations of conserved charges; deconfinement means an abrupt breakdown of hadronic description that occurs at T_c and appearance of quark like excitations
- Hadronic excitations containing charm quark can exist above T_c
- For $T > (300-400) \text{ MeV}$ weak coupling expansion may work for certain quantities (e.g. quark number susceptibilities)
- The region of moderately large baryon densities relevant for RHIC can be accessed in LQCD using Taylor expansion. The effects of the baryon densities are not large and consistent with HRG (no hint for critical point ?); Much more work is needed to obtain reliable results, e.g. $T_c(\mu_B)$, energy density at T_c

How Equation of state changed since 2002 ?



- Much smoother transition to QGP
- The energy density keeps increasing up to 450 MeV instead of flattening

Entropy of static quark at high temperature



M. Berwein, N. Brambilla, P. Petreczky and A. Vairo, arXiv:1512.08443 [hep-ph]

Does the quasi-particle model makes sense ?

4 non-trivial constraints on the model provided by : χ_{31}^{BC} , χ_{31}^{SC} , χ_{121}^{BSC} , χ_{211}^{BSC}

$$c_1 \equiv \chi_{13}^{BC} - 4\chi_{22}^{BC} + 3\chi_{31}^{BC} = 0,$$

$$c_2 \equiv 2\chi_{121}^{BSC} + 4\chi_{112}^{BSC} + \chi_{22}^{SC} + 2\chi_{13}^{SC} - \chi_{31}^{SC} = 0$$

$$c_3 \equiv 6\chi_{112}^{BSC} + 6\chi_{121}^{BSC} + \chi_{13}^{SC} - \chi_{31}^{SC},$$

$$c_4 \equiv \chi_{211}^{BSC} - \chi_{112}^{BSC} . \quad \longleftarrow \text{Diquark pressure is zero !}$$

