



# Effect of Rapid Rotation on Neutron Star Radius Measurements

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# Outline

- Quasi-universality for rotating neutron stars
- Effect of Rapid Rotation on
  - Mass – Radius constraints
  - Luminosity Radius determination
  - Eddington Luminosity
  - Radius determinations with undetected hot spots

# Mass-Radius Constraints

- Pulse-profiles (cf Fred Lamb's talk for rotating ms pulsars ( $\sim 200$  Hz) and accreting ms pulsars (200 – 600 Hz))
- Some Eddington-Limited X-ray burst stars spin at  $\sim 600$  Hz
- Quiescent LMXB neutron stars don't have detected pulsations, but they \*might\* spin fast.

# Digression on Black Hole “No Hair”

- Black holes are very simple!
- Describe a rotating BH with 2 parameters:  $M$ ,  $a$
- Event Horizon radius given by:
$$R_{\text{EH}} = 2M + M[(1-(a/M)^2)^{1/2} - 1]$$
- Gravitational field outside of BH:
$$\Phi(r,\theta) = - M/r + \Phi_2/r^3 P_2(\cos\theta) + \dots$$
$$\Phi_2 = M a^2$$
- Properties of a BH are independent of the properties of the stuff that formed it.

# Neutron Stars Have “Hair”

- Given a mass and spin ( $M$  and  $\Omega$ ) different EOS predict different Radii ( $R$ )
- However, given  $M, R, \Omega$  dimensionless quantities:  $x = GM/Rc^2$   $y = \Omega^2 R^3/GM$
- Many secondary NS properties depend only on  $x, y$ .
- I Love Q (Moment of Inertia, Love number, Quadrupole moment) relationships
- “Neutron Star Universality” (Yagi & Yunes 2013)

# Example: Moment of Inertia

- Expect Moment of Inertia of the form:

$$I = \beta M R^2$$

Where  $\beta$  depends on how density varies inside the star. (Ravenhall & Pethick, 1994; Lattimer & Prakash, 2001)

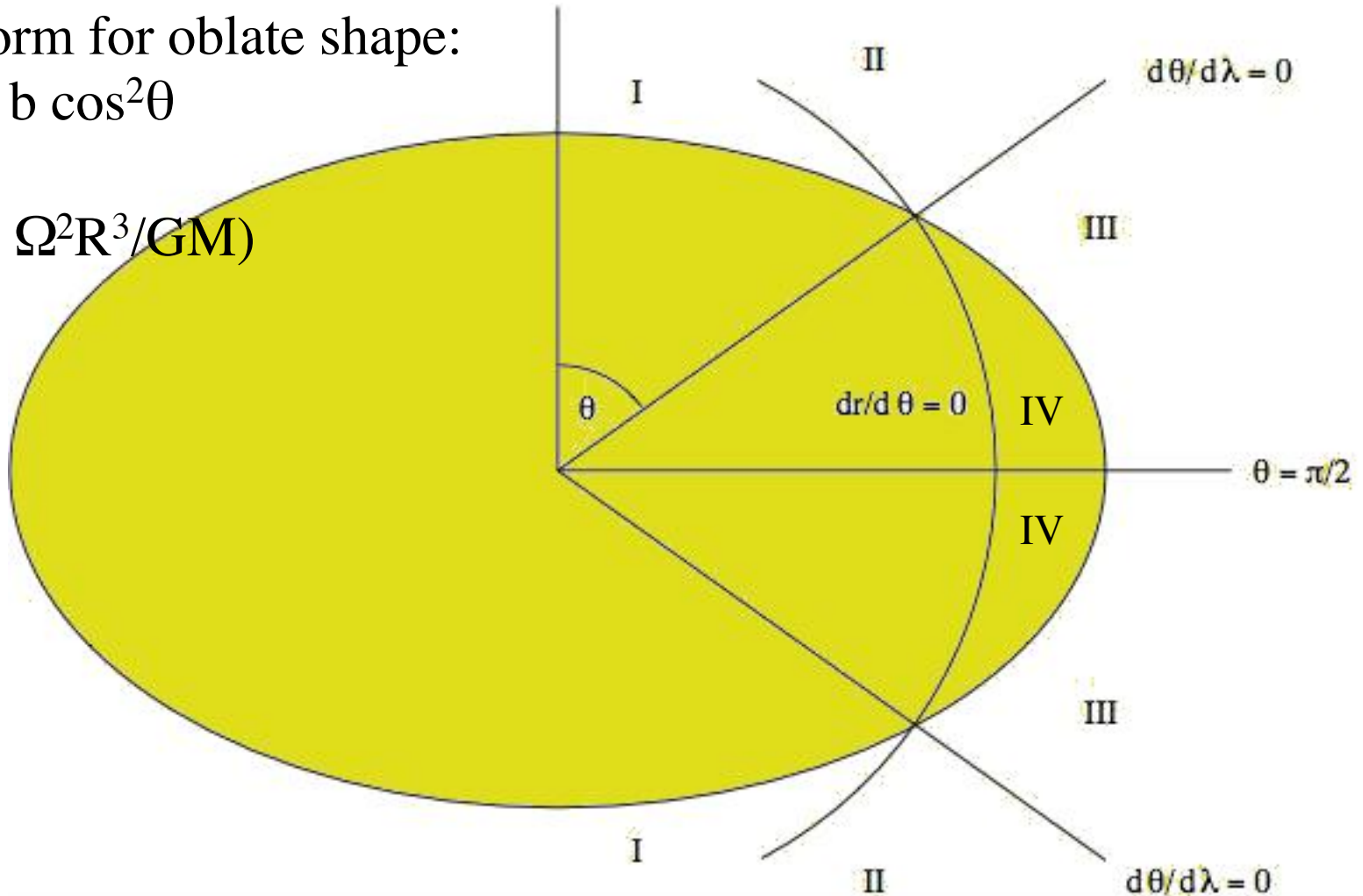
- For Neutron Stars  $\beta = \beta(M/R, \Omega^2 R^3/GM)$  with  $\beta$  a known function
- Similar functions for quadrupole moment, ellipticity, acceleration due to gravity, oscillation mode frequencies, etc...

# Stellar Oblateness

Universal form for oblate shape:

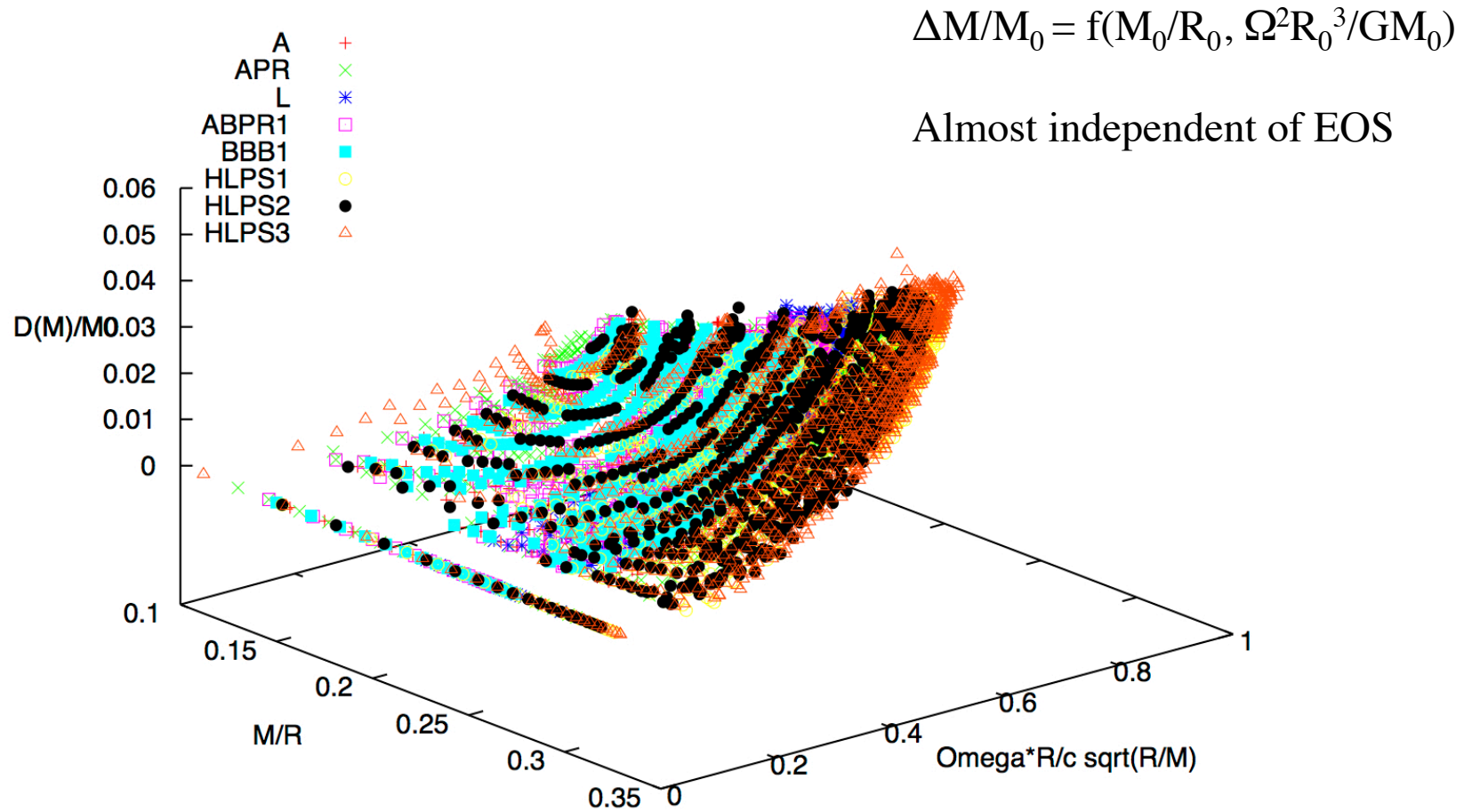
$$R(\theta) = R_e - b \cos^2\theta$$

$$b = b(M/R, \Omega^2 R^3/GM)$$



Morsink, Leahy, Cadeau & Braga 2007 ApJ

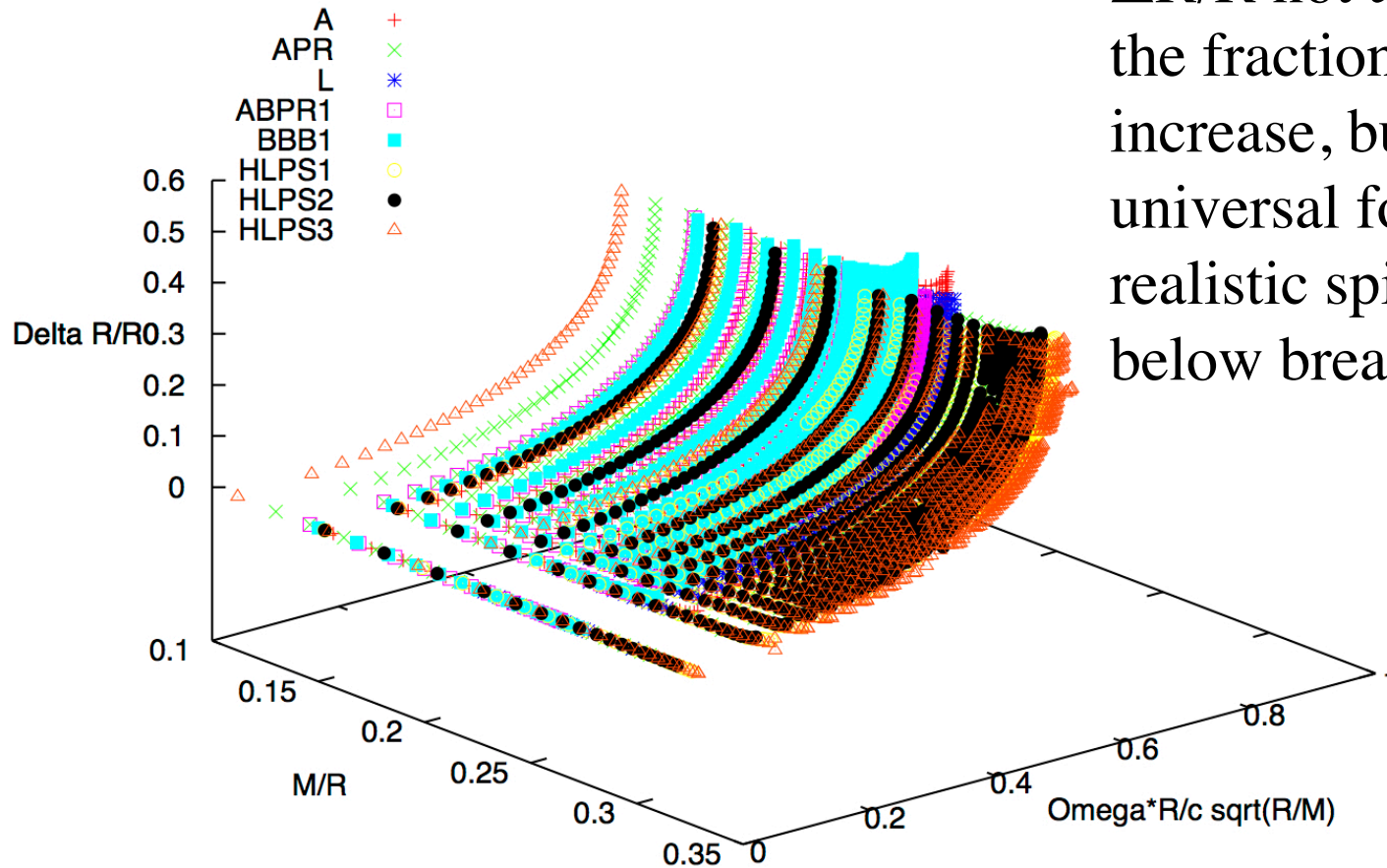
# Fractional Increase In Mass



$$\Delta M/M \sim -M/\Delta R + \Omega^2 R^2$$



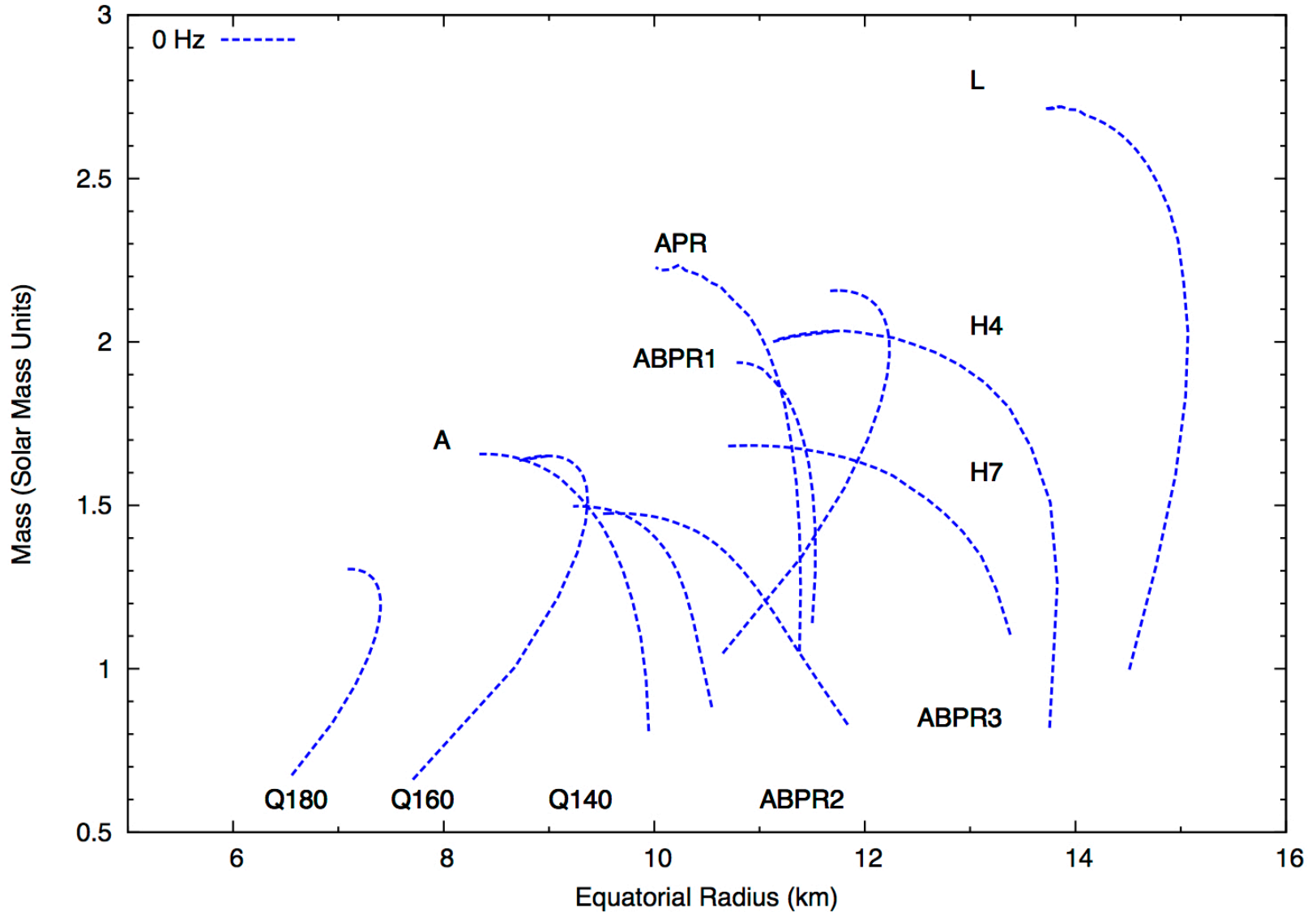
# Fractional Increase in Radius



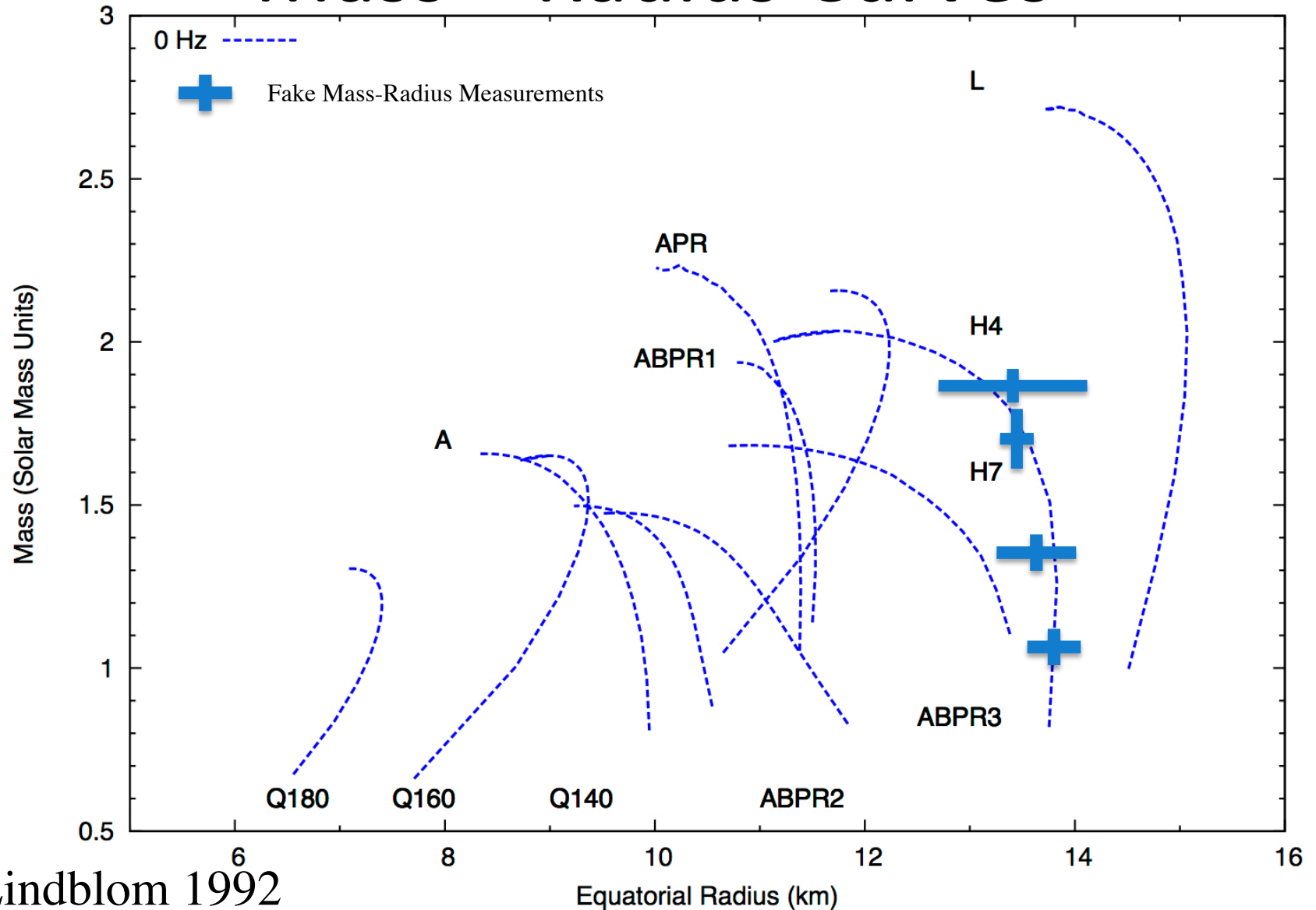
$\Delta R/R$  not as tight as the fractional mass increase, but almost universal form for realistic spins well below break-up

Morsink & Fedorowich, in prep...

# Mass – Radius Curves

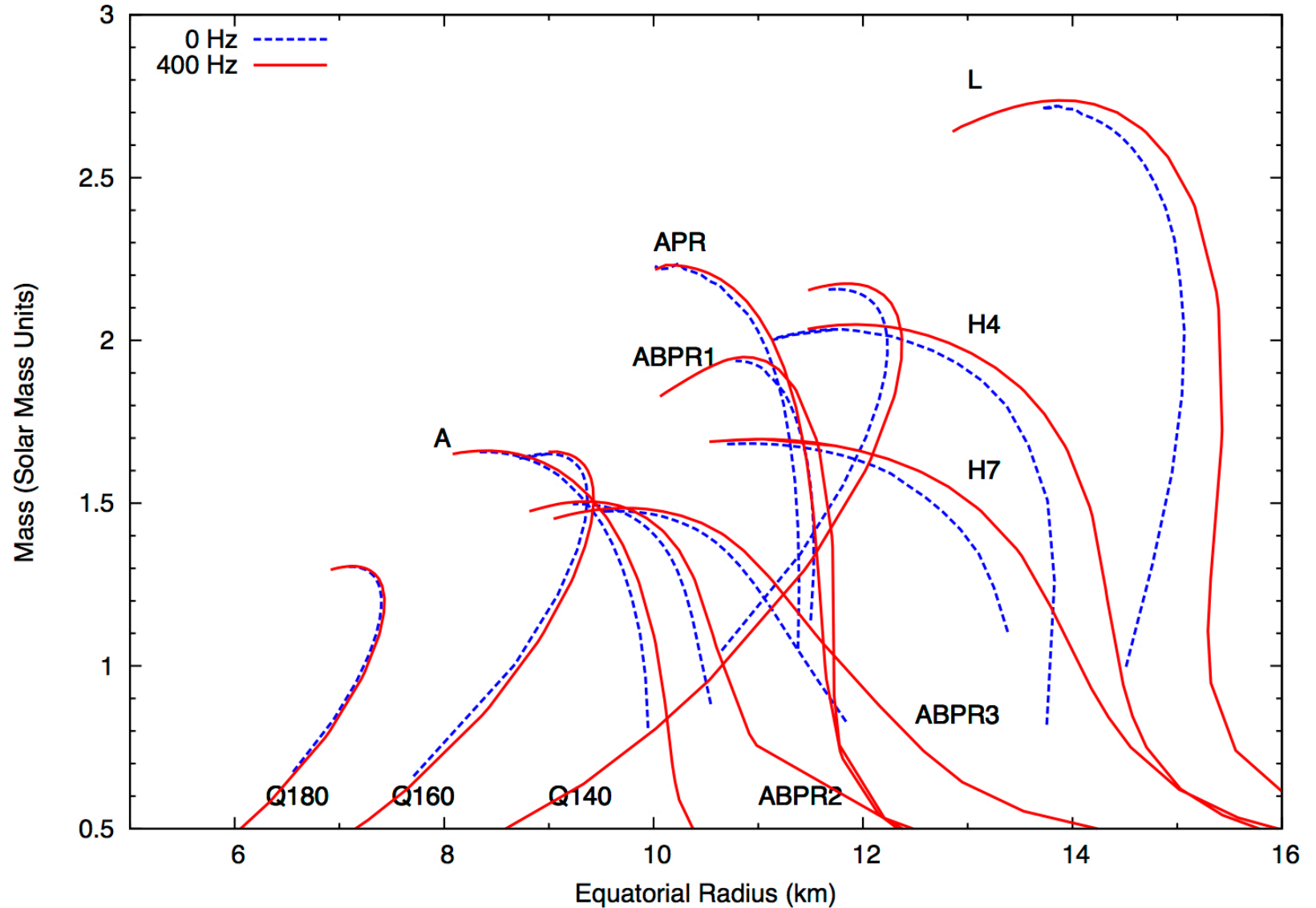


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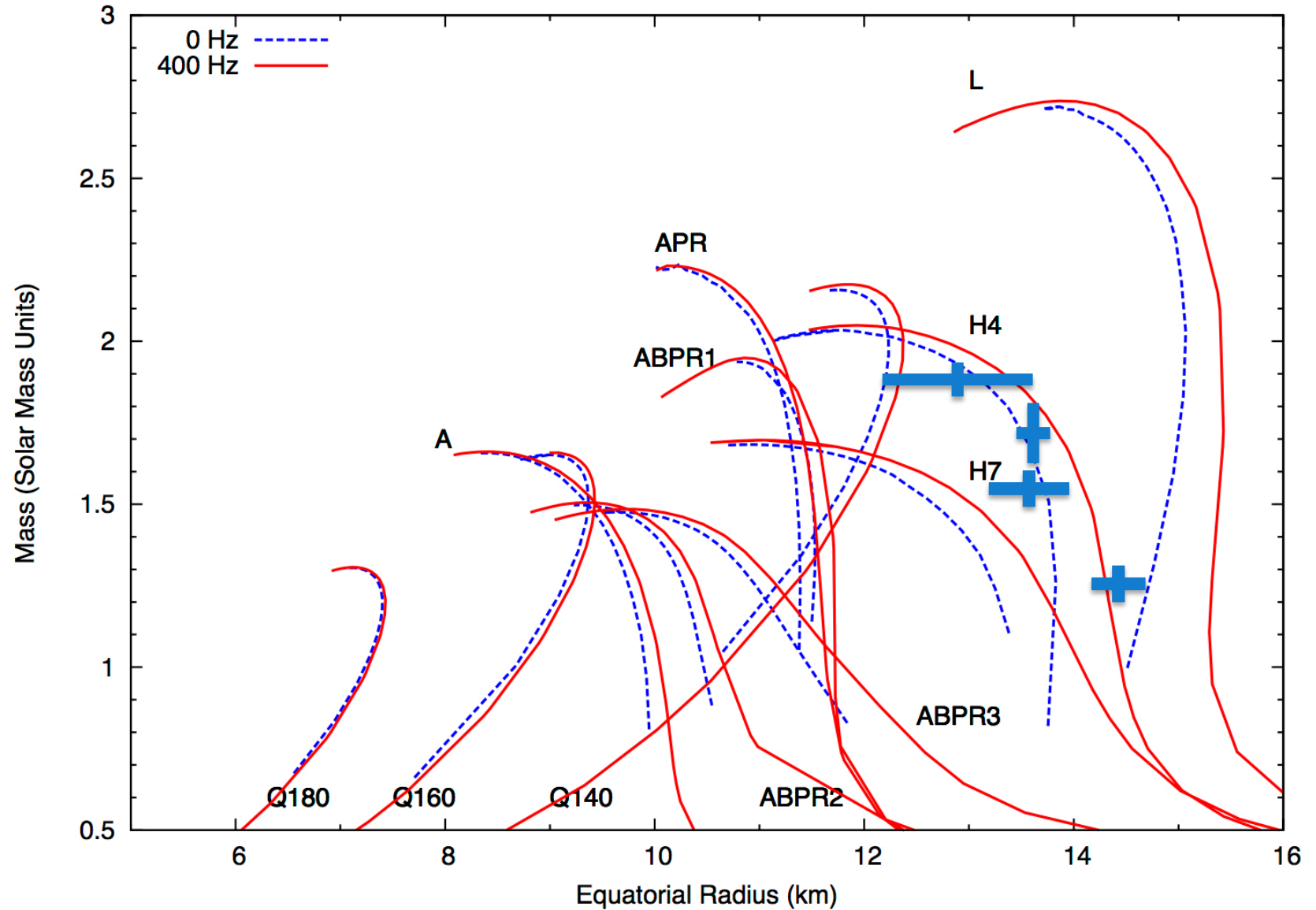


Lindblom 1992

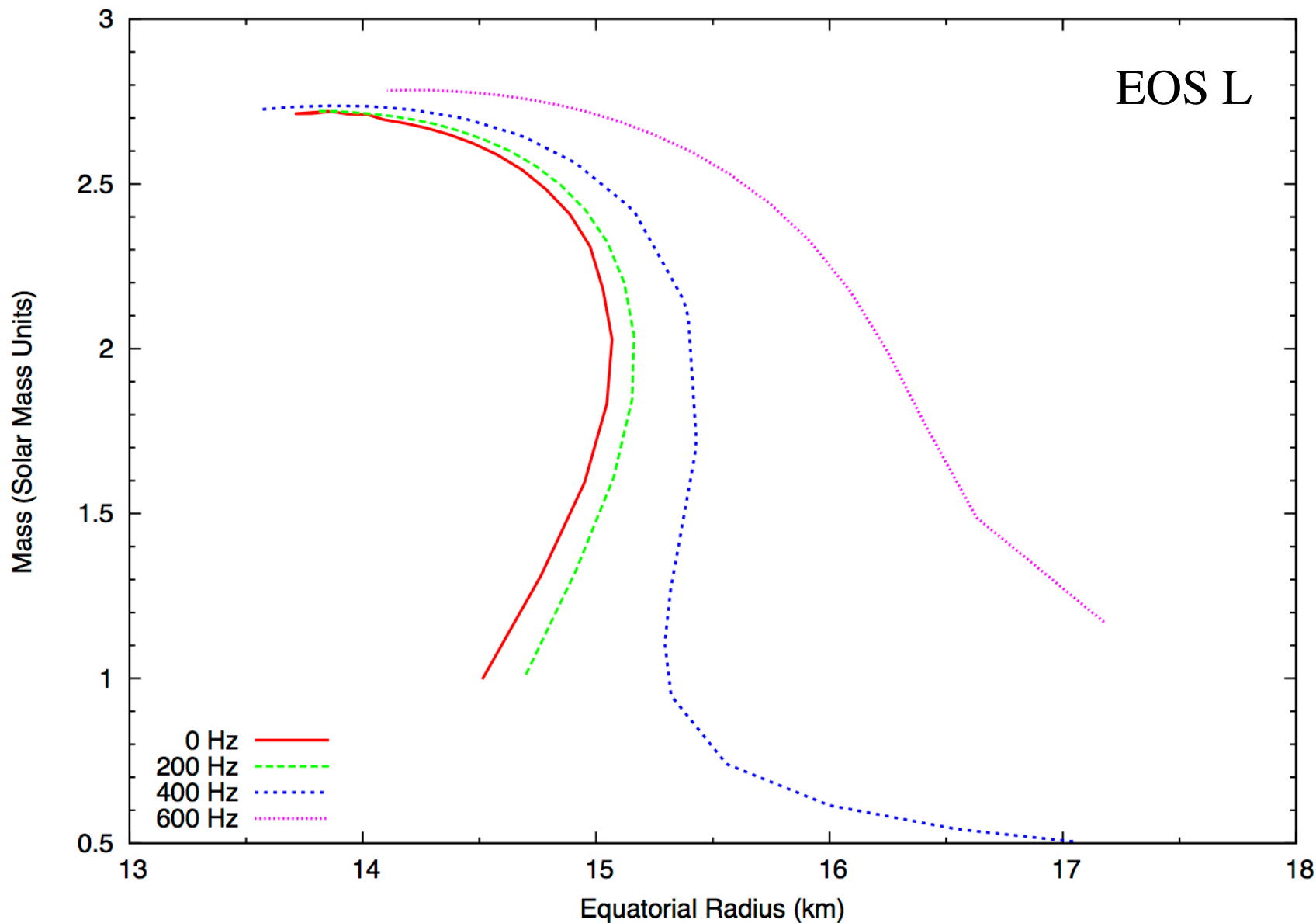
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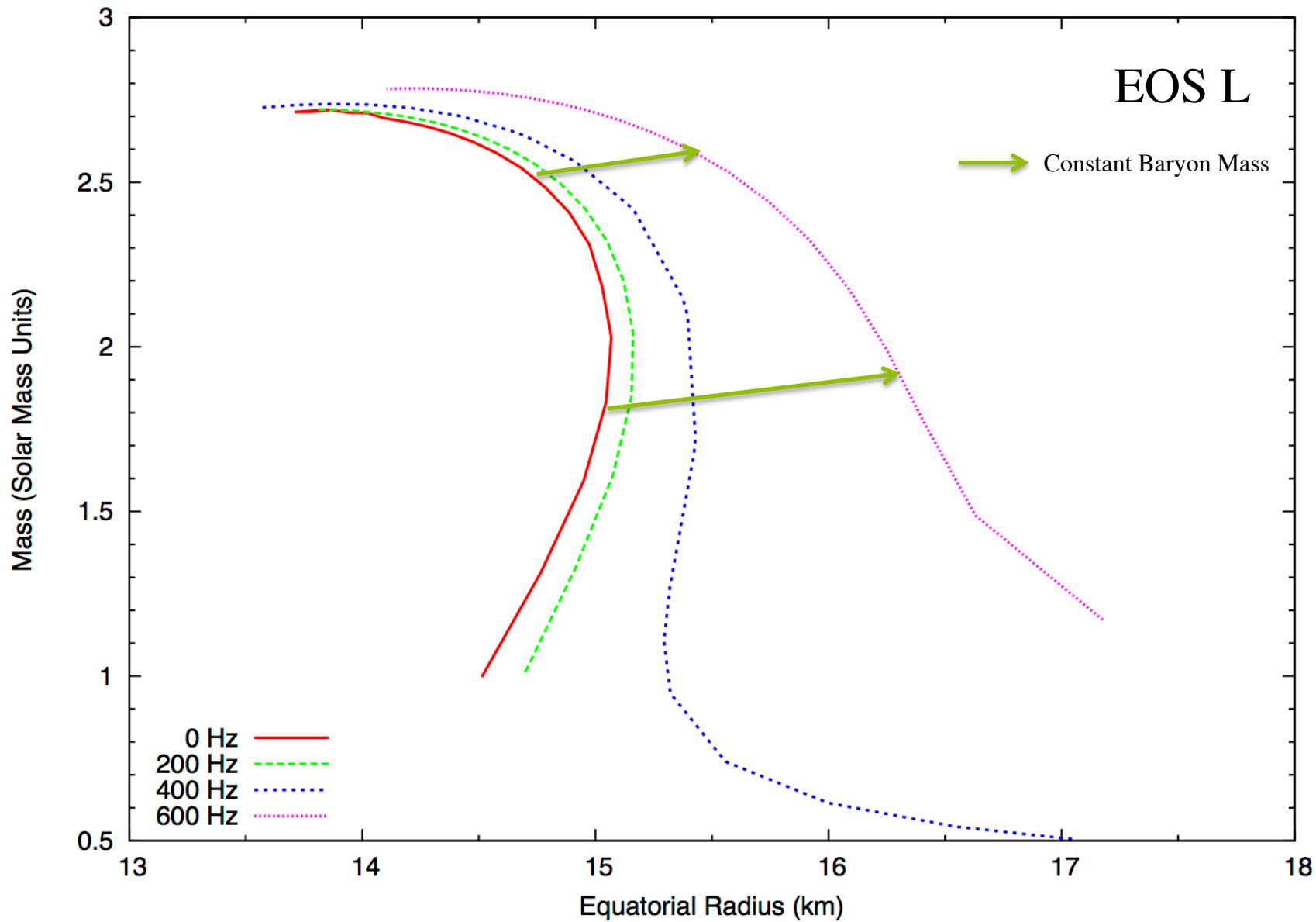
# Mass – Radius Curves



# Effect of Spin on a stiff EOS

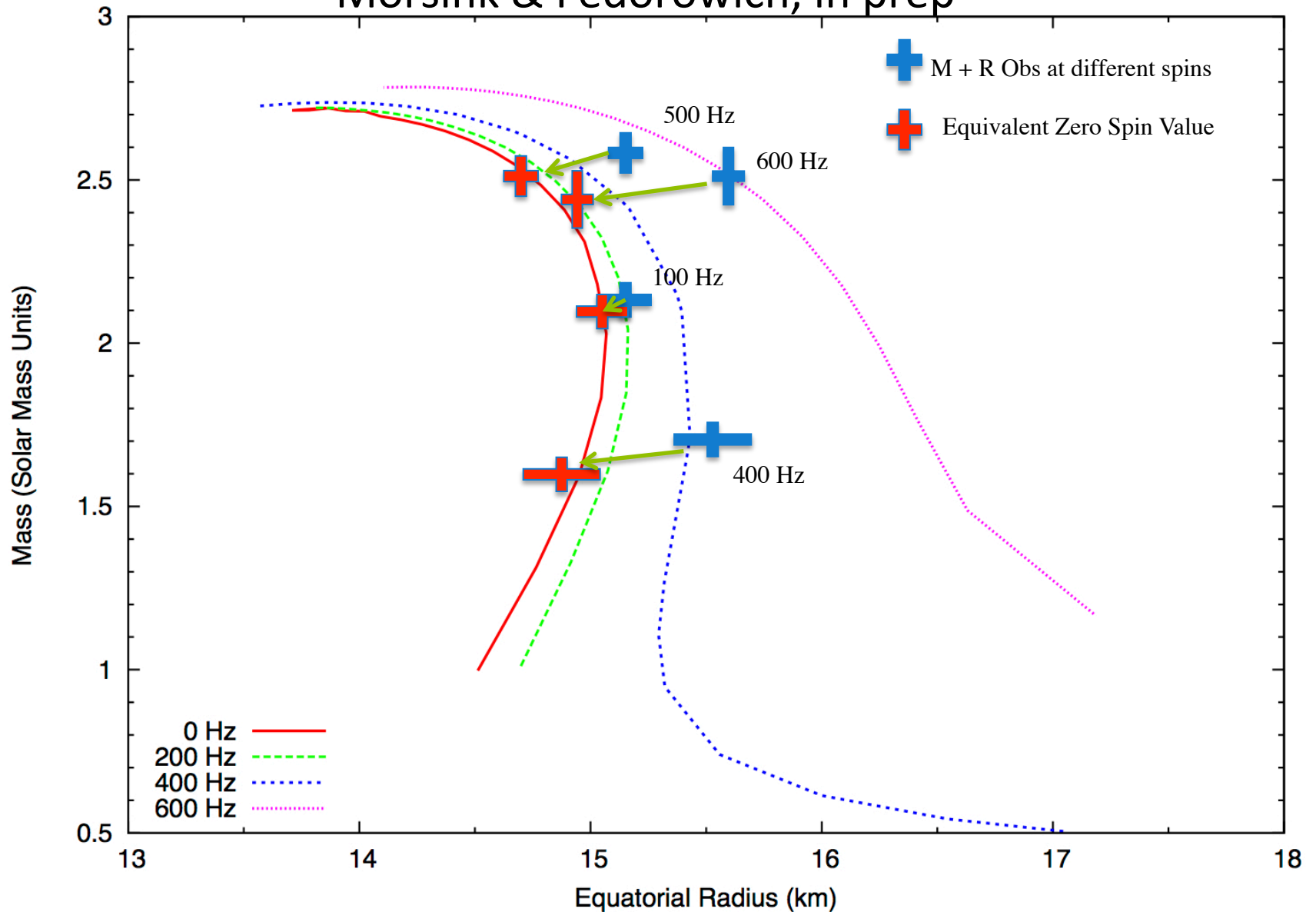


# Effect of Spin



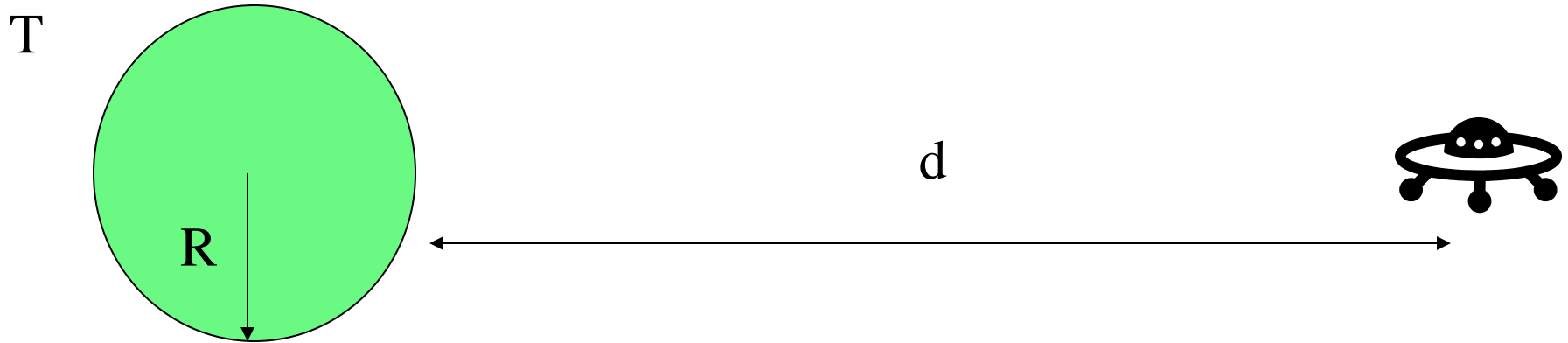
# Mapping Spin to Zero Spin Curve

Morsink & Fedorowich, in prep





# A Uniformly Emitting Blackbody Sphere



In Newtonian physics, if  $d$ , temperature ( $T$ ) and flux ( $F$ ) are measured then the “Luminosity Radius” is given by:

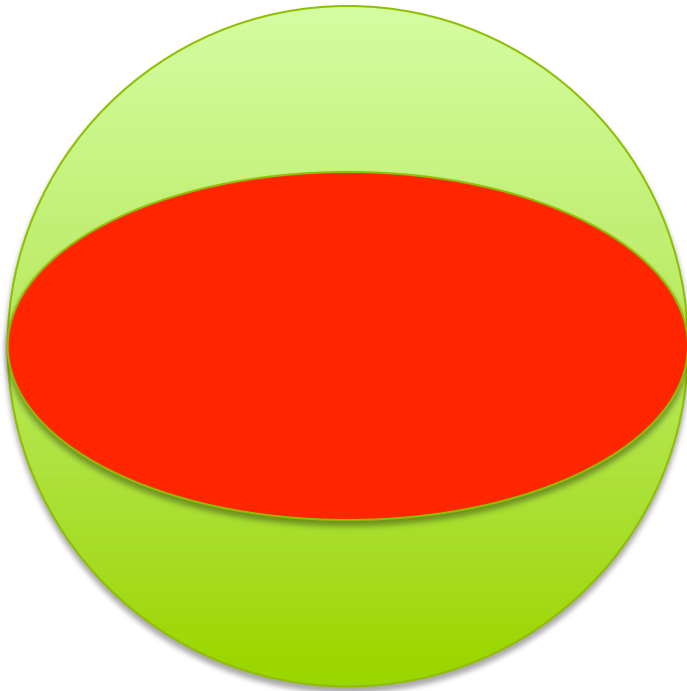
$$R_L = \frac{d}{T^2} \sqrt{\frac{F}{\sigma}}$$

In General Relativity, the gravitational redshift Effect must be corrected for, so the actual radius is:

$$R = R_L \sqrt{1 - \frac{2M}{R}}$$

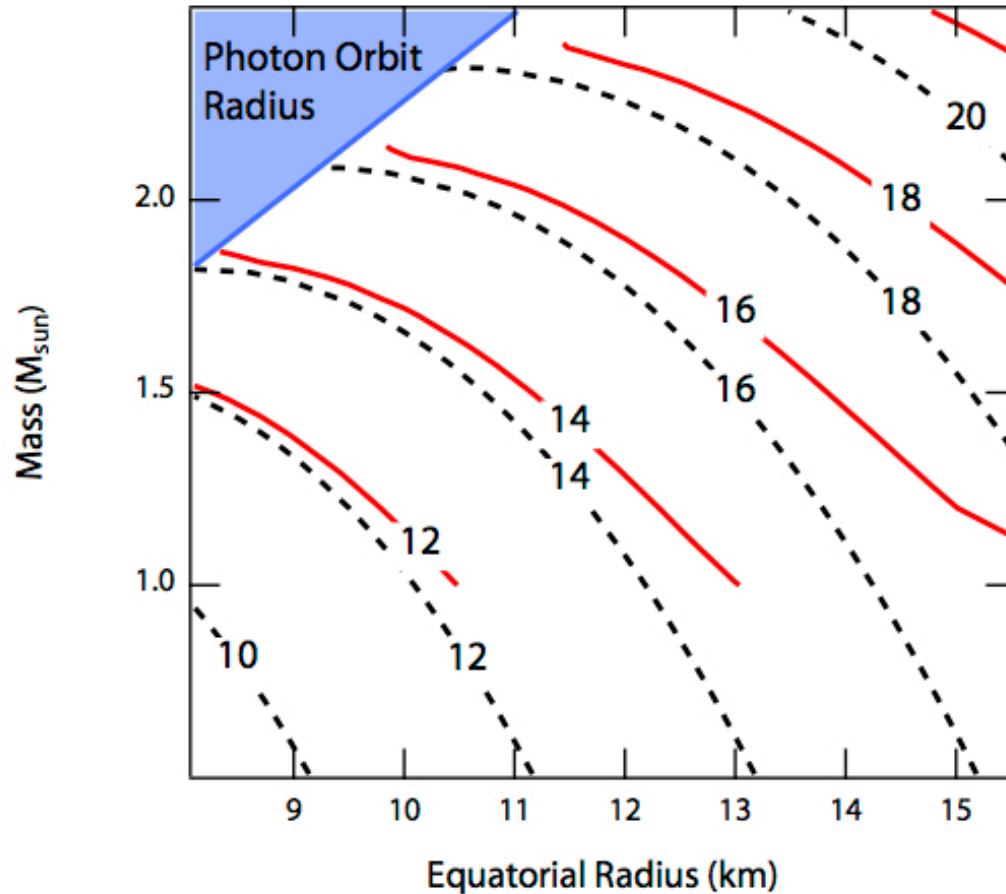
Unfortunately, real neutron stars aren't uniformly emitting blackbodies, so use more realistic spectra...

# Rotational Effect on Luminosity Radius



- An oblate star with the same equatorial radius as a spherical star has a smaller cross-sectional area  $A$
- Flux  $\sim A$  so assuming a sphere underestimates the equatorial radius of the star

# Luminosity Radius vs “Real” Radius



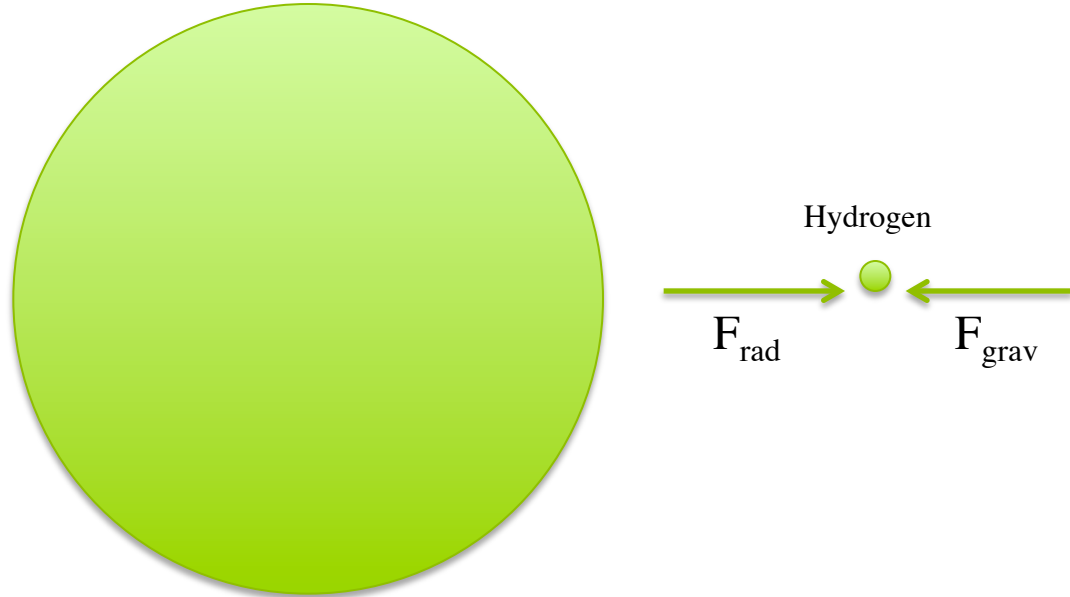
Baubock, Ozel, Psaltis,  
Morsink, ApJ 2015

(Calculation is for pure  
blackbody, also includes  
Doppler boosting effects)

Assuming a spherical star  
could lead to  
underestimating the radius  
by 3-5%

- Luminosity Radius for zero spin  $R_L = R(1-2M/R)^{-1/2}$
- Luminosity Radius for spinning star (600 Hz)

# Eddington Limit – Spherical Star Maximum Allowed Flux

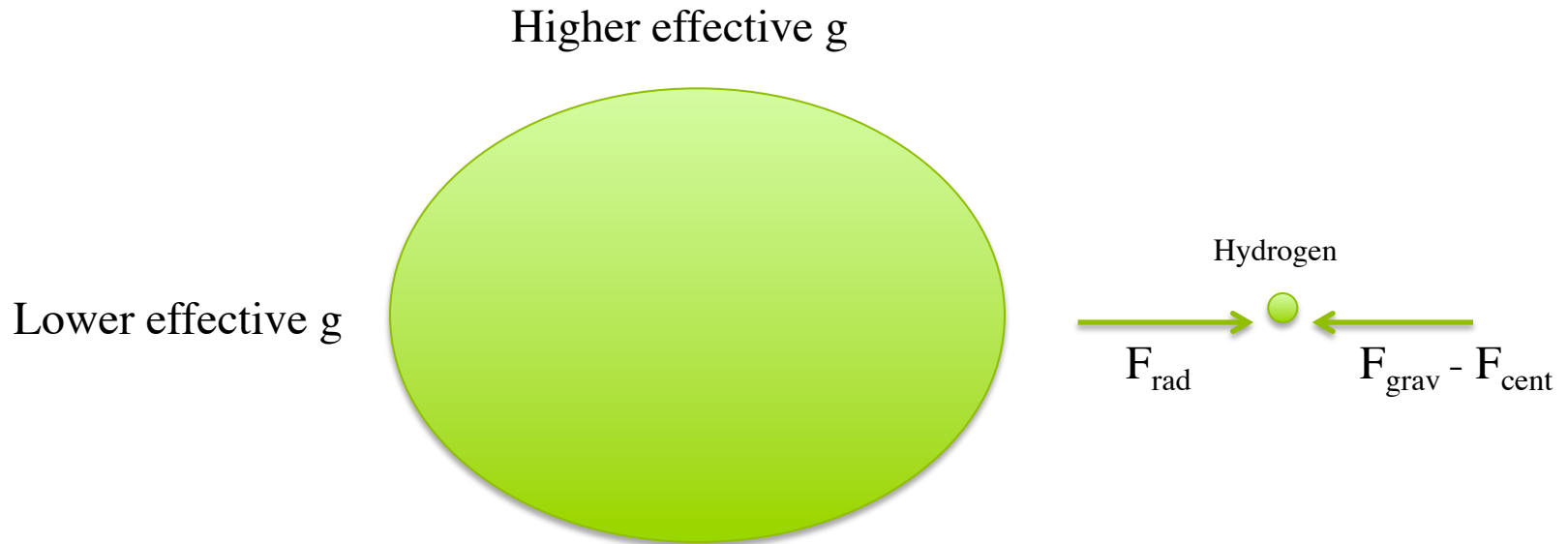


Newtonian Gravity:  $F_0 = 2GMm_p c/d^2(1+X)\sigma_T$

General Relativity:  $F_{\text{Edd}} = F_0 (1-2M/R)^{1/2}$

- (1) Acceleration due to gravity is stronger at the surface
- (2) Gravitational Redshift makes light appear dimmer

# Eddington Limit – Rotating Star

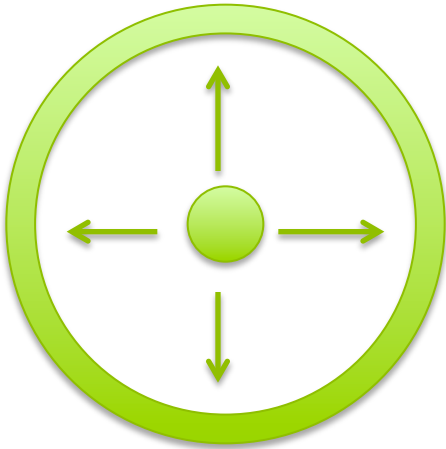


At Equator:  $F_{\text{Edd}} = F_0 (1 - 2M/R_e)^{1/2} (1 - a \Omega^2 R^3/GM) f_L$

- (1)  $a > 0$  centrifugal reduction in effective surface gravity
- (2)  $f_L < 1$  Luminosity radius reduction factor (due to rotation)

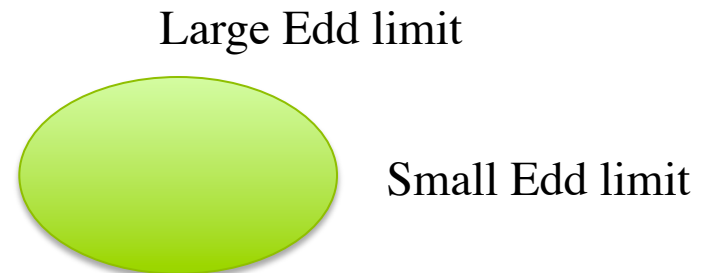
For 600 Hz, both corrections reduce Eddington by up to 15%  
(AlGendy & Morsink, ApJ 2014)

# Eddington-Limited X-ray Bursts



- During Type I X-ray burst radiation flux can exceed Eddington
- Atmosphere can be pushed off of star
- Away from star, Newtonian Eddington limit applies
- When atmosphere cools off it falls back onto star and “touches down”

- Rotating Relativistic Eddington limit only applies when atmosphere is in hydrostatic equilibrium with star
- Since Edd limit is largest at pole, touch-down takes place first at pole, and then moves down to equator.



# Effect of an undetected hot spot on luminosity radius

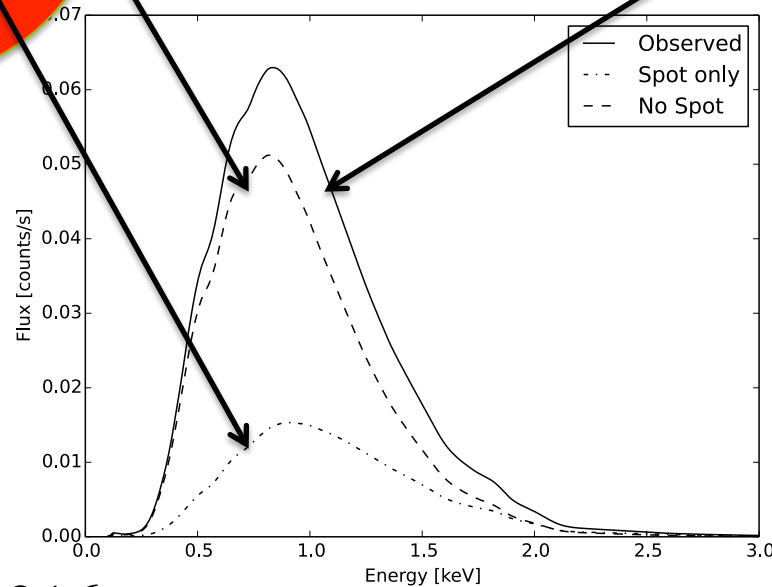
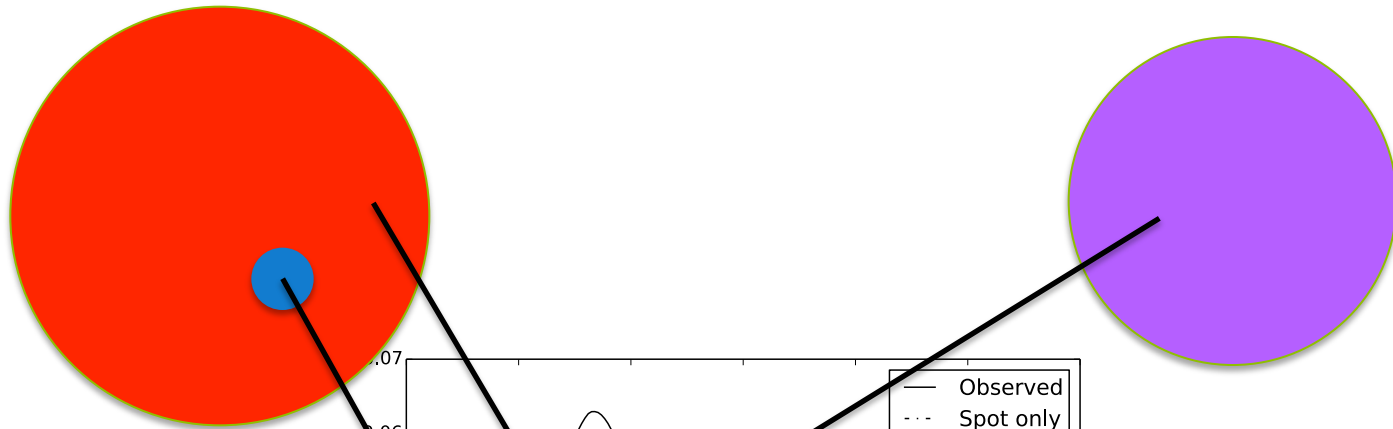
- Radius determined through luminosity radius method for many stars without pulsations – we normally assume that they are not rotating and they have a homogeneous temperature
- Example – Quiescent LMXBs
- BUT, what if they are rotating and have a hot spot that our telescope can't detect????

Elshamouty, Heinke, Morsink, Bogdanov, & Stevens, ApJ to appear in 2016

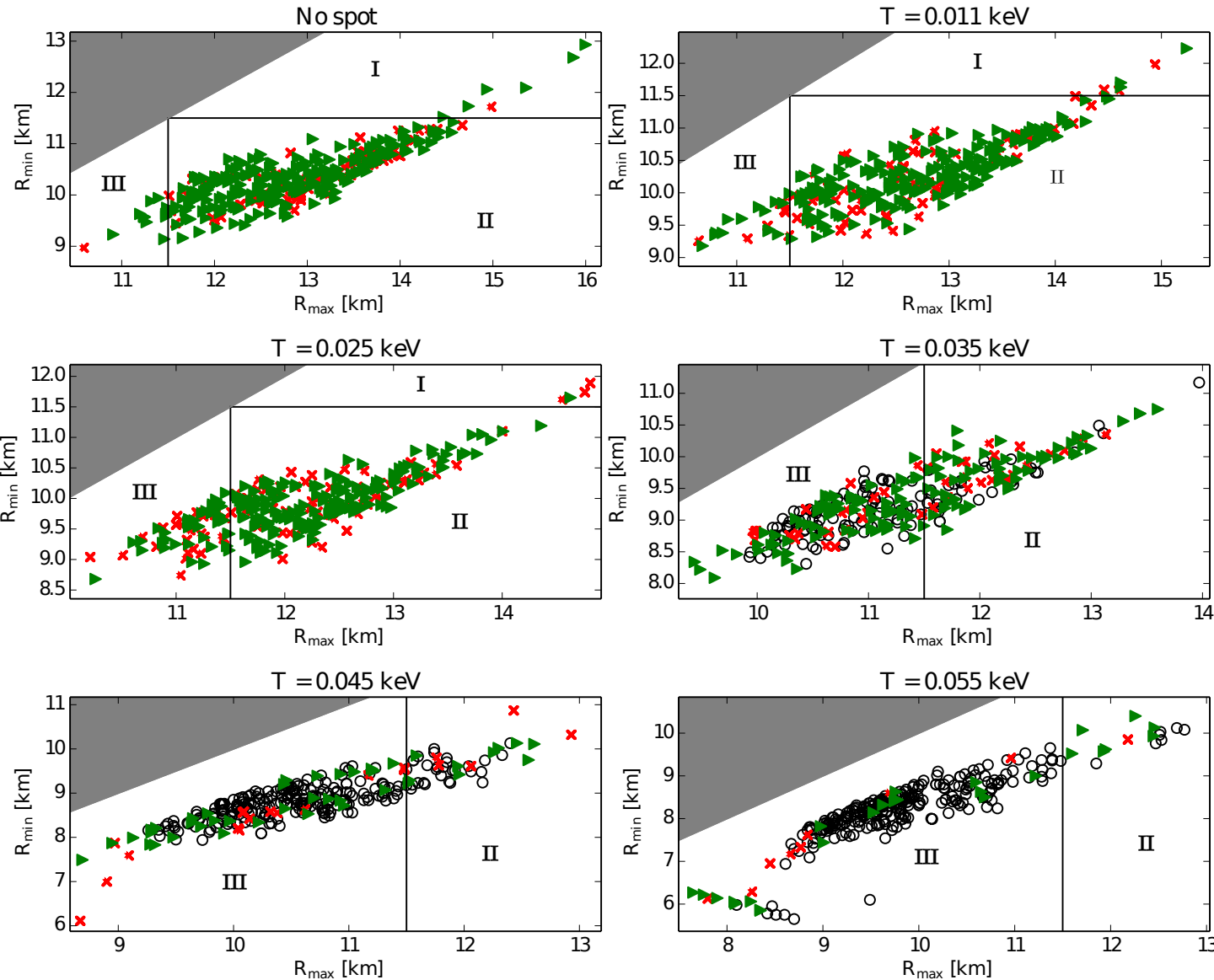
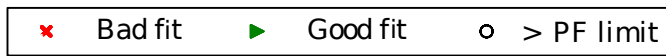
# What happens if we can't tell that the star has a hot spot?

Reality

What we infer: hotter T and smaller R







True  $R = 11.5$  km

I: Fit  $R > 11.5$  km

II: Fit allows true  $R$

III: Fit  $R < 11.5$  km

XSPEC fits assuming homogeneous surface – leads to bias towards smaller radius – Also true if star is not rotating

# Conclusions

- Spinning stars are more fun!
- If we know the spin of a star, we can factor in effect of rotation and find the equatorial radius of the star.
- If it is unknown if the star is spinning, then luminosity radius method tends to lead to underestimating the star's radius, if it is actually spinning.