Proton-skins in momentum and neutron-skins in coordinate in heavy nuclei:
What we can learn from their correlations

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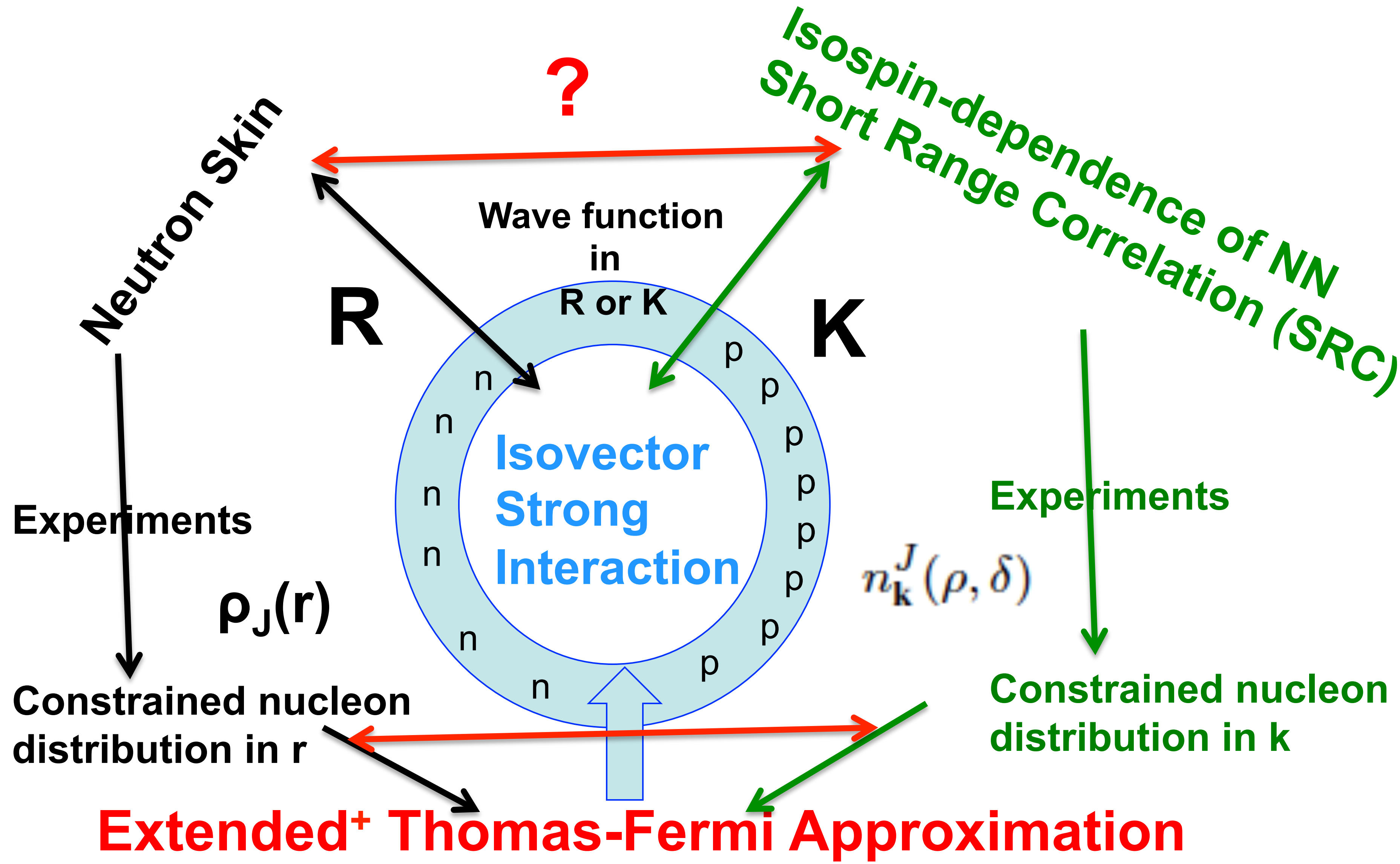
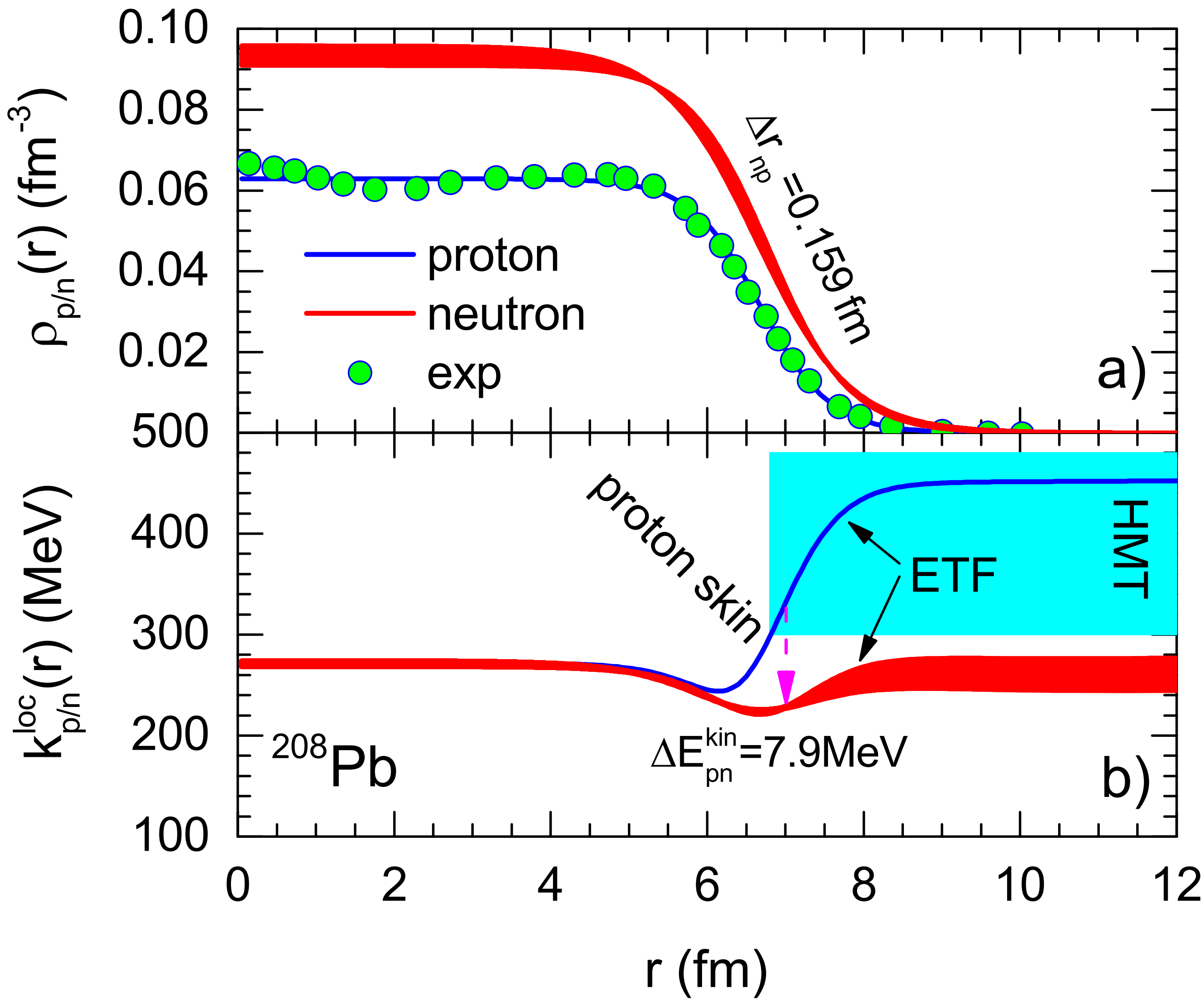
National Natural Science Foundation of China

Outline of my talk in Phase Space (R,K)

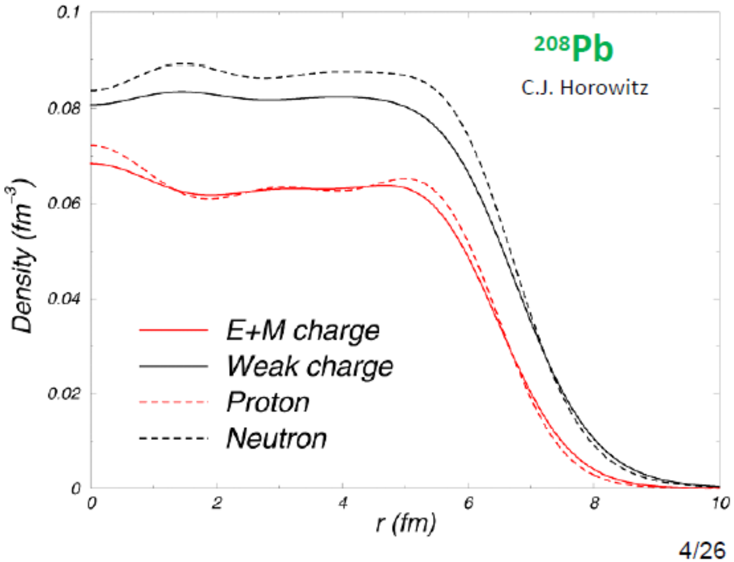


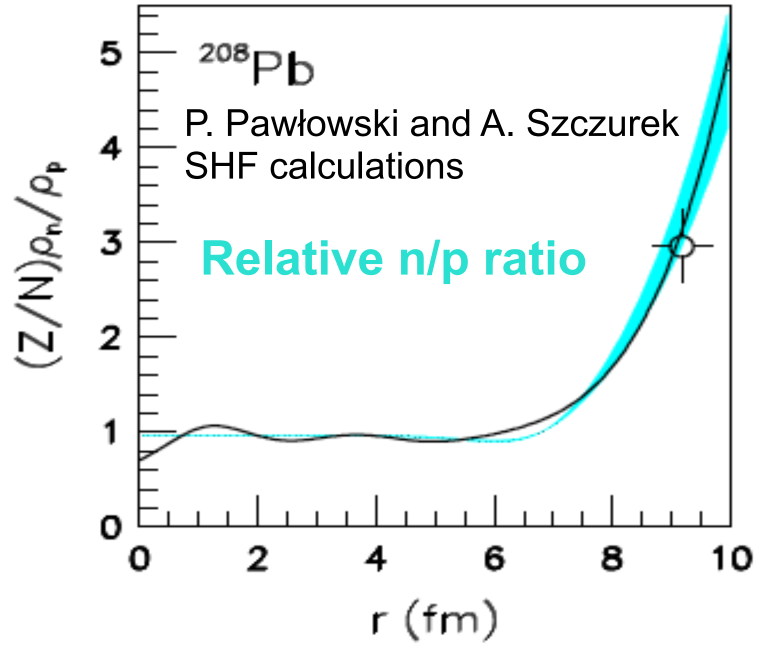
Protons are moving faster than neutrons in neutron-skins

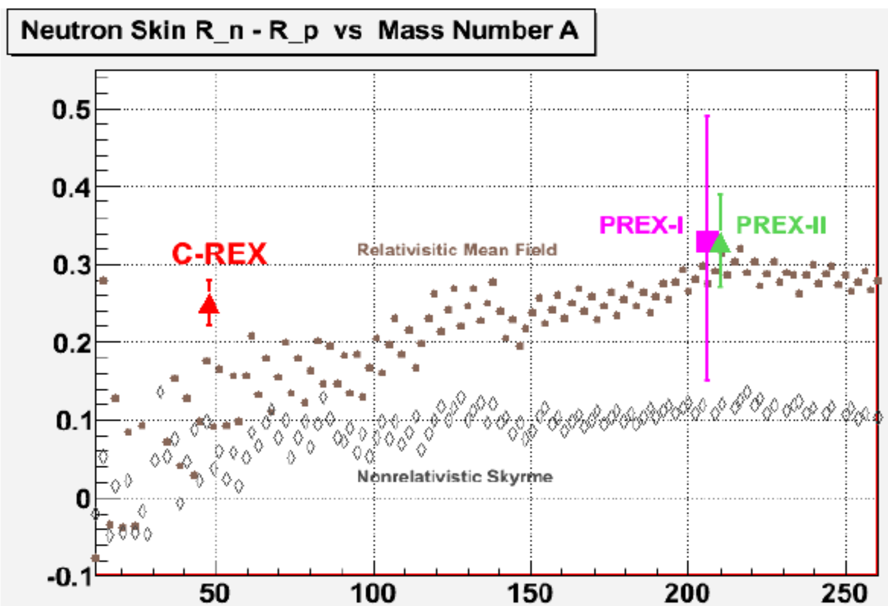


The average local momentum is defined via $k_J^{\mathrm{loc},2}(r)/2M = \varepsilon_J^{\mathrm{kin}}(r)/\rho_J(r)$

N-skins as a testing ground of the isovector part of strong interactions

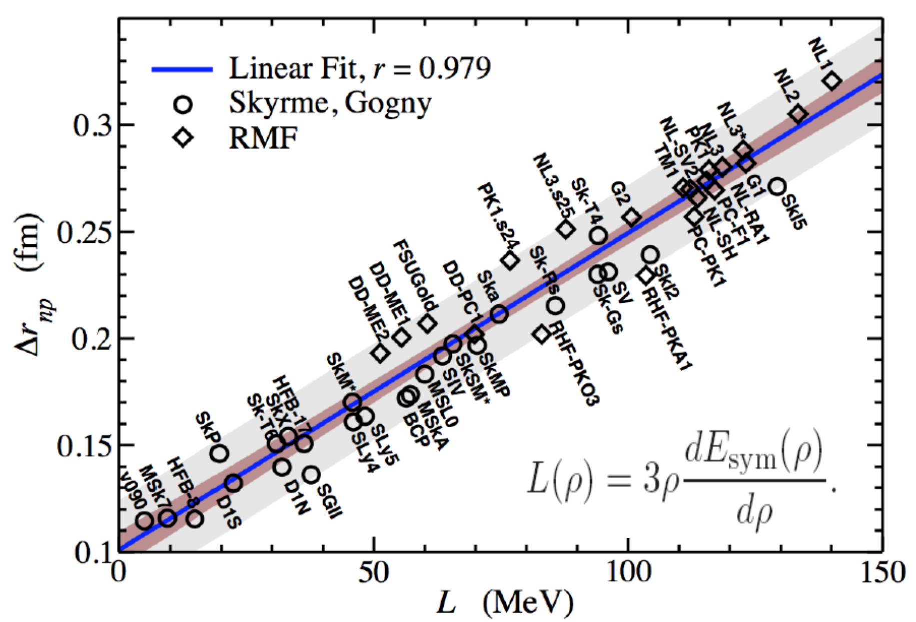






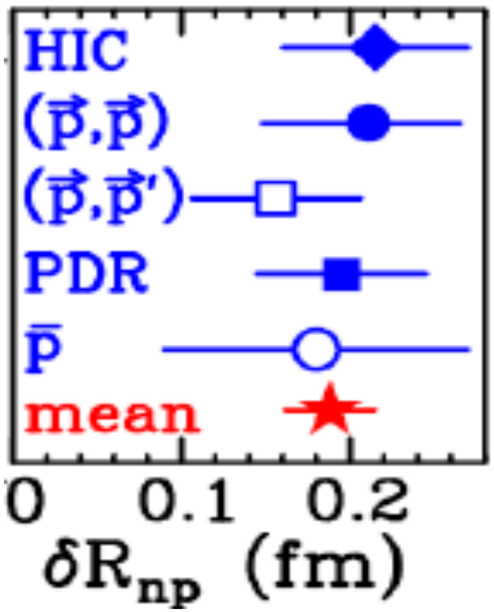
R. Michaels et al.

[nucl-th]1308.1008 X. Vinas, M. Centelles, X. Roca-Maza, M. Warda



Sizes of n-skin in ^{208}Pb extracted from various experiments

M.B. Tsang et al,
PRC86, 015803 (2012)

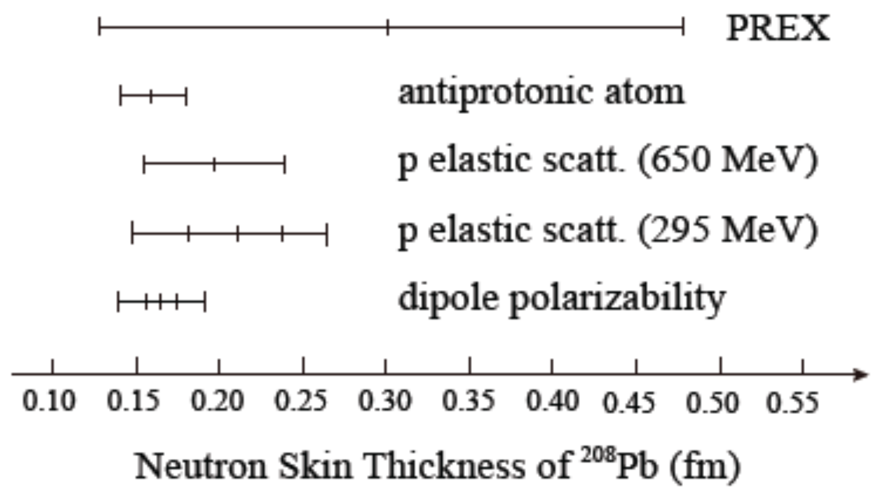


P. Danielewicz and J. Lee, Nucl. Phys. **A922**, 1 (2014).

Nucleus	Reference	Data Source	Δr_{np} [fm]
^{208}Pb	Starodubsky *et al.* [100]	elastic p scattering	0.197 ± 0.042
	Ray [95]	elastic $\vec{p}$ scattering	0.16 ± 0.05
	Clark *et al.* [96]	elastic p scattering	0.119 ± 0.045
	Zenihiro *et al.* [99]	elastic p scattering	0.211 ± 0.063
	Friedman [93]	elastic π^+ scattering	0.11 ± 0.06
	Friedman [93]	pionic atoms	0.15 ± 0.08
		combined results	$0.159 \pm 0.041^*$

A. Tamii[1], P. von Neumann-Cosel[2], and I. Poltoratska[2]

EPJA 50, 20 (2014)



Coherent pion photoproduction

C. M. Tarbert *et al.* (Crystal Ball at MAMI and A2 Collaboration)

$$\Delta r_{np} = 0.15 \pm 0.03(\text{stat.})^{+0.01}_{-0.03}(\text{sys.})\ \text{fm}.$$

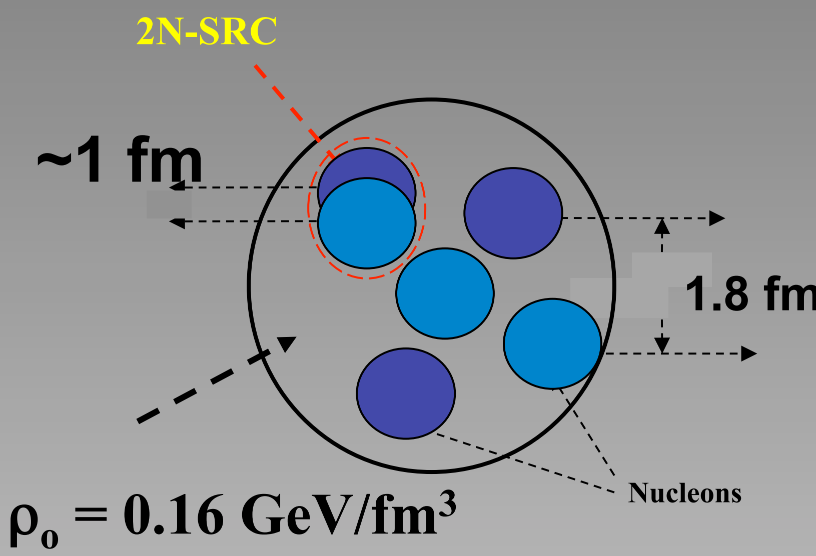
PRL **112**, 242502 (2014)

What are Short Range Correlations (SRC) in nuclei ?

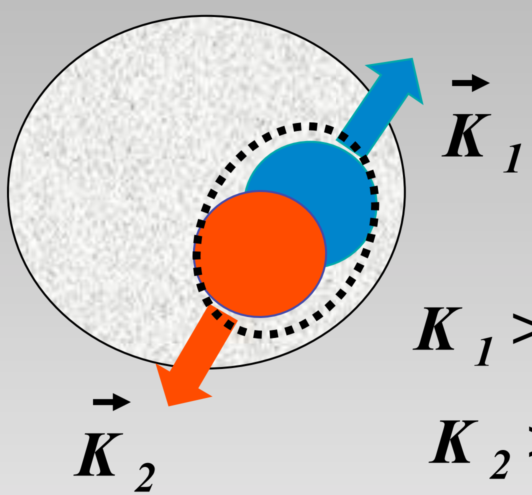
(Eli Piasetzky)

SRC $\sim R_N$ LRC $\sim R_A$

$k_F \sim 250$ MeV/c
High momentum tail:
300-600 MeV/c
$1.5\, K_F - 3\, K_F$



<u>In momentum space:</u>



$K_1 > K_F$,

$K_2 > K_F$

A pair with <u>large relative momentum</u> between the nucleons and <u>small CM momentum</u>.

Tensor force induced (1) high-momentum tail in nucleon momentum distribution and (2) isospin dependence of SRC

Theory of Nuclear matter
H.A. Bethe
Ann. Rev. Nucl. Part. Sci., 21, 93-244 (1971)

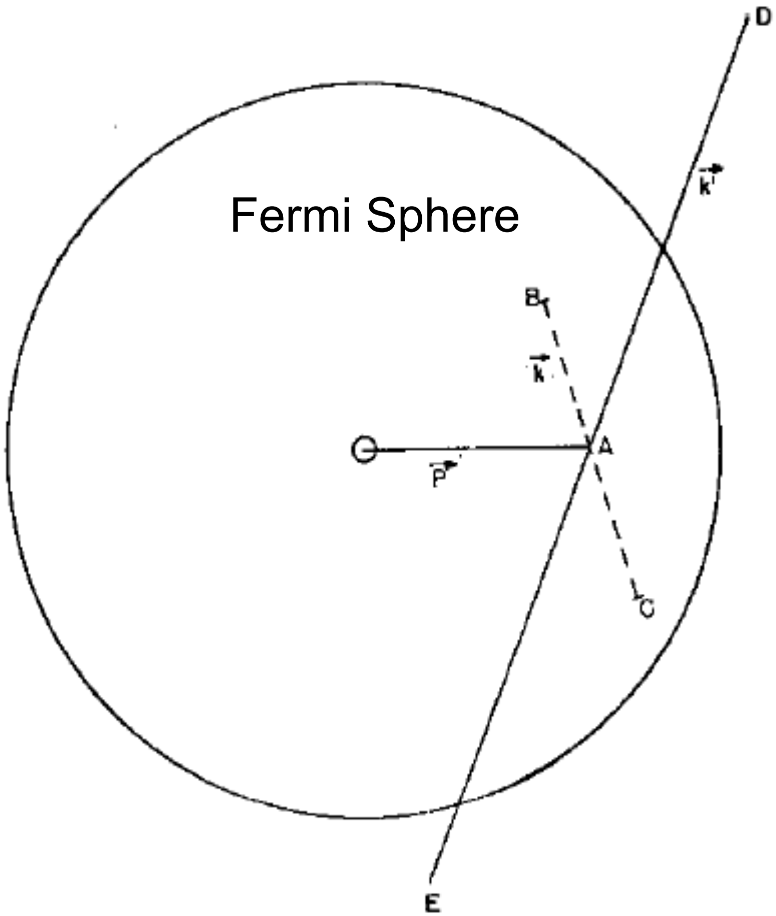
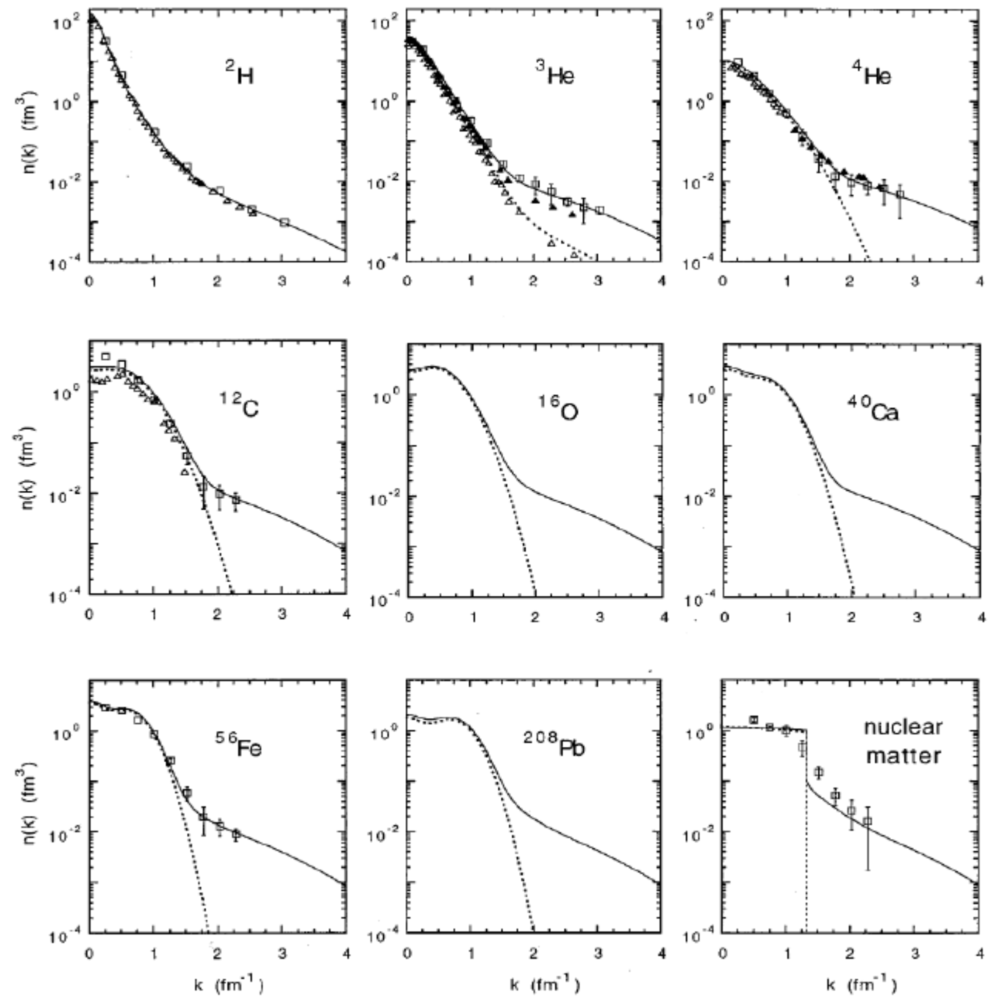


FIGURE 10. Two nucleons are initially in states B and C, having average momentum $\boldsymbol{P}$ and relative momentum $\boldsymbol{k}$. When they interact they are shifted to states D and E outside the Fermi sphere, with relative momentum $\boldsymbol{k}'$. If they are initially in a ^3S state and interact by tensor force, then they are in a $^3D_1$ state in DE.

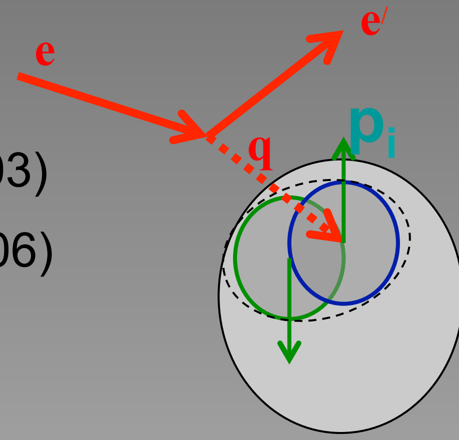


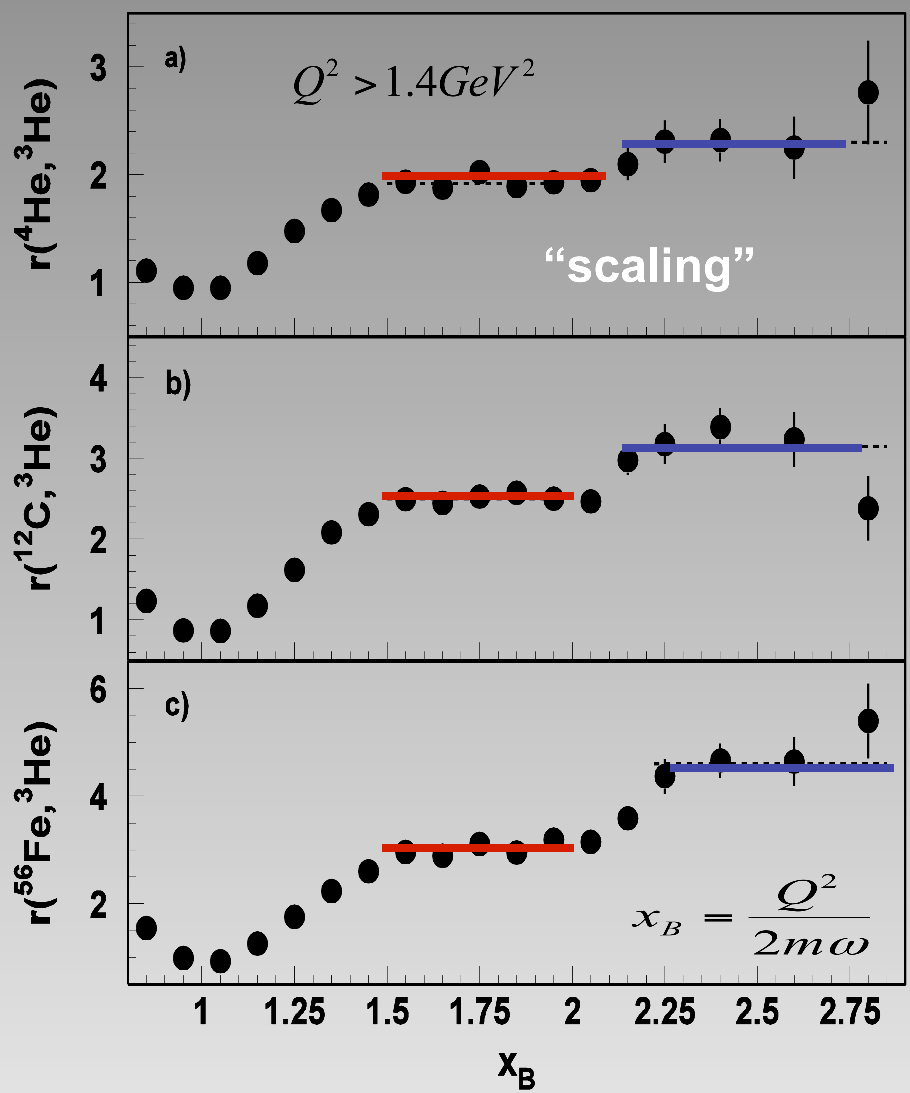
S. Fantoni and V. R. Pandharipande, Nucl. Phys. A **427**, 473 (1984).
C. Ciofi degli Atti and S. Simula, Phys. Rev. C **53**, 1689 (1996).

JLab. CLAS A(e,e') Result

K. Sh. Egiyan et al. PRC 68, 014313 (2003)

K. Sh. Egiyan et al. PRL. 96, 082501 (2006)





$a_{2N}(A/d)$	
^{3}He	2.08±0.01
^{4}He	3.47±0.02
Be	4.03±0.04
C	4.95±0.05
Cu	5.48±0.05
Au	5.43±0.06

$$P_{2N}(A) = a_{2N}(A/d) \cdot P_{2N}(D)$$

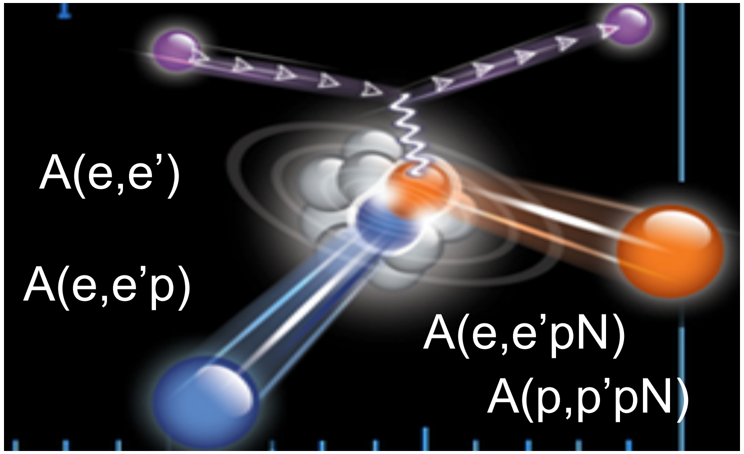
$P_{2N}(\infty) \approx 20\text{-}30\%$

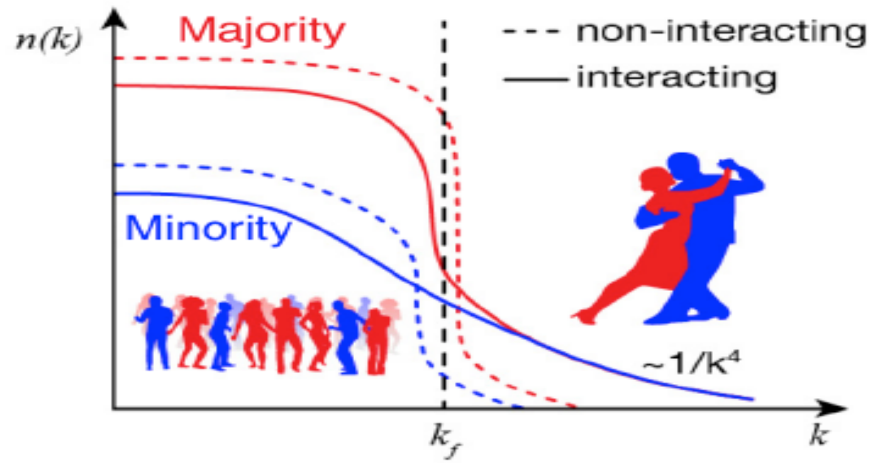
More r(A,d) data:

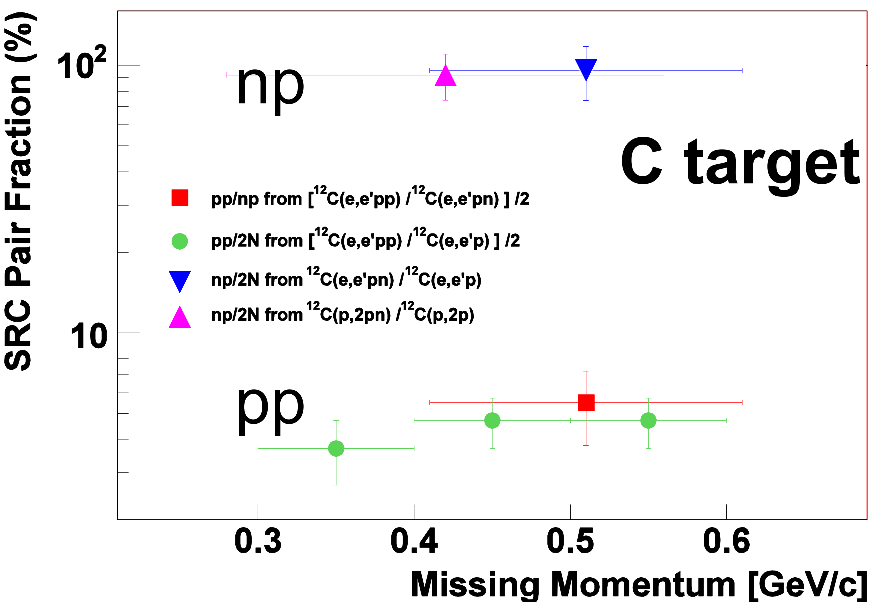
SLAC D. Day et al. PRL 59,427(1987)

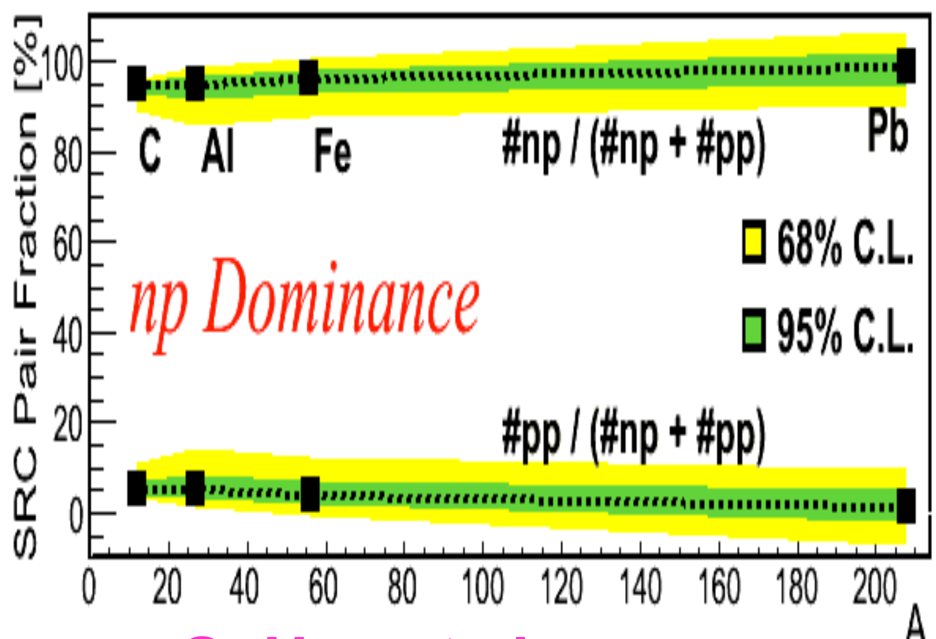
JLab. Hall C N. Fomin et al. PRL 108:092502, 2012.

Triple coincidence measurement of the isospin dependence of SRC









R. Subedi et al., Science 320, 1476 (2008)

O. Hen et al., Science 346, 614 (2014)

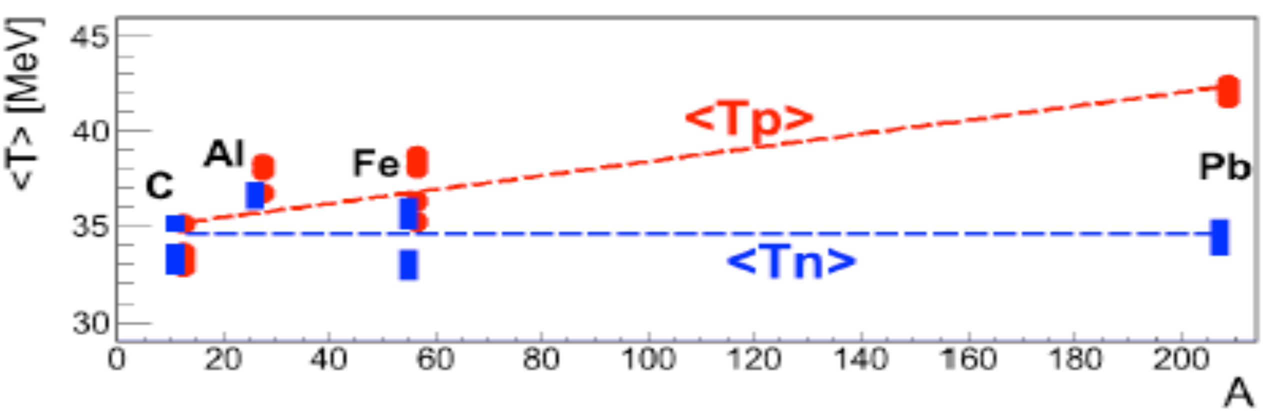
Average kinetic energies of neutrons and protons in nuclei

(1) Light nuclei: Predictions of the Variational Many-Body theory with AV18+UX interaction

Nucleus	Asymmetry (N-Z) / A	$\langle T_p \rangle$	$\langle T_n \rangle$	$\langle T_p \rangle / \langle T_n \rangle$
^{8}He*	0.50	30.13	18.60	1.62
^{6}He*	0.33	27.66	19.60	1.41
^{9}Li	0.33	31.39	24.91	1.26
^{3}He	-0.33	14.71	19.35	0.76
^{3}H	0.33	19.61	14.96	1.31
^{8}Li	0.25	28.95	23.98	1.21
^{10}Be	0.20	30.20	25.95	1.16
^{7}Li	0.14	26.88	24.54	1.09
^{9}Be	0.11	29.82	27.09	1.10
^{11}B	0.09	33.40	31.75	1.05

*Neutron Halo

(2) Heavy nuclei: Neutron-proton dominance model with parameters fixed by SRC data



(1) Where are these energetic protons located?

(2) How is the p-skin in momentum related to the n-skin in coordinate?

O. Hen et al. (Jlab CLAS), Science 346, 614 (2014)

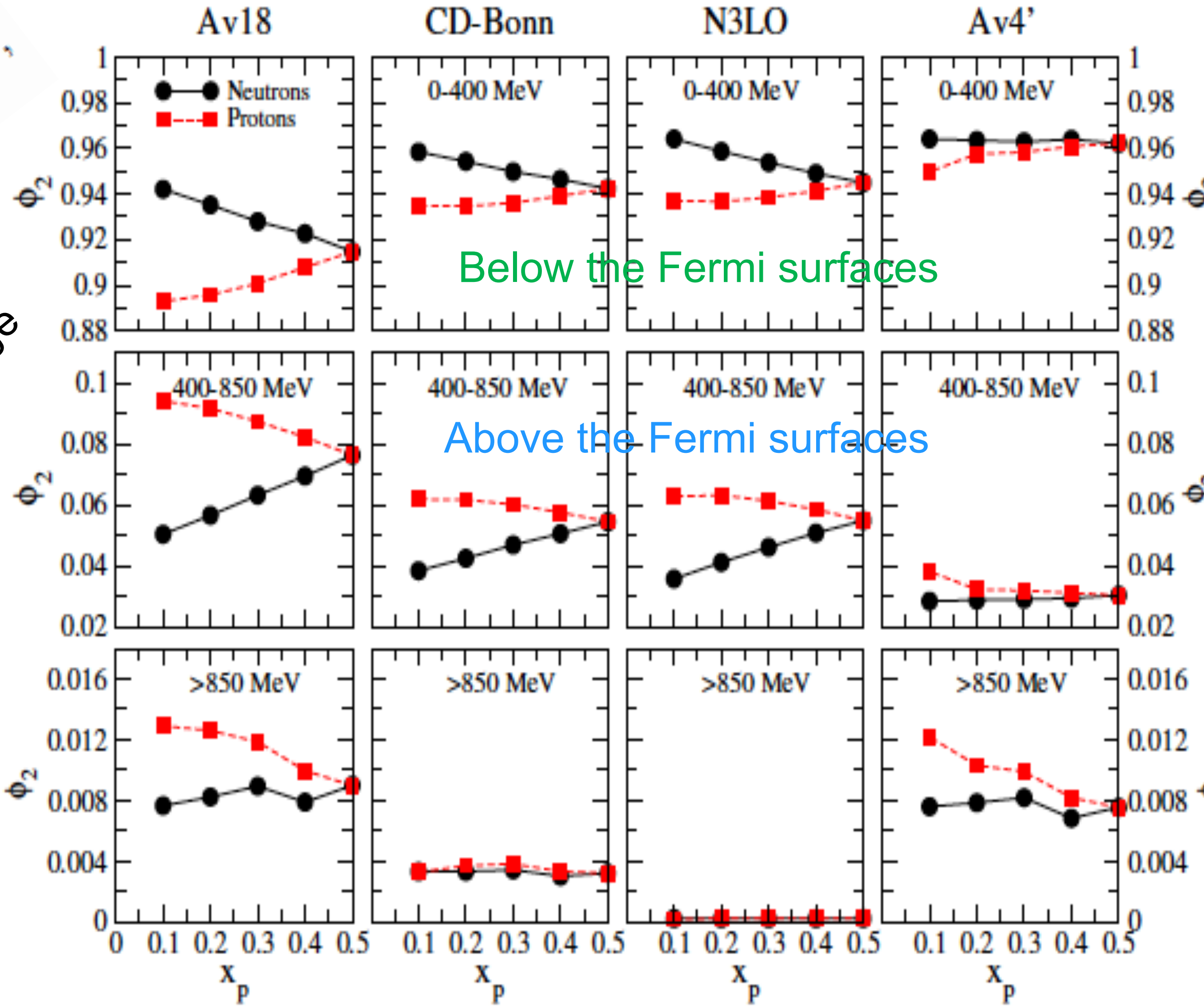
Isospin dependence of depletion (population) of Fermi sea (high momentum tail)

A. Rios, A. Polls and W. Dickhoff, PRC89, 044303 (2014),

A self-consistent Greens Function approach

$$\phi_2(k_i, k_f) = \frac{1}{\pi^2 \rho_\tau} \int_{k_i}^{k_f} dk\, k^2 n_\tau(k) \,,$$

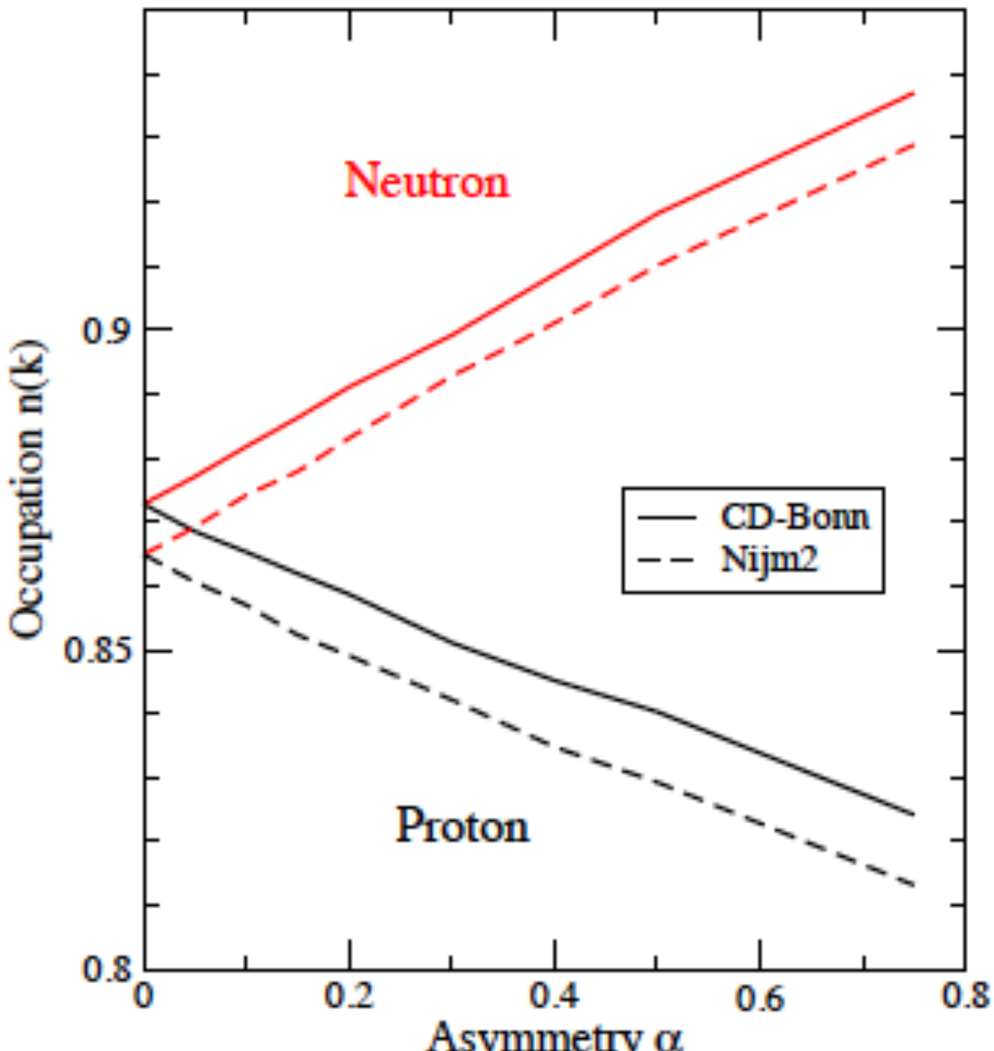
Relative population in the specified momentum range



Proton fraction

Isospin dependence of the average occupation of the Fermi sea at saturation density within BHF

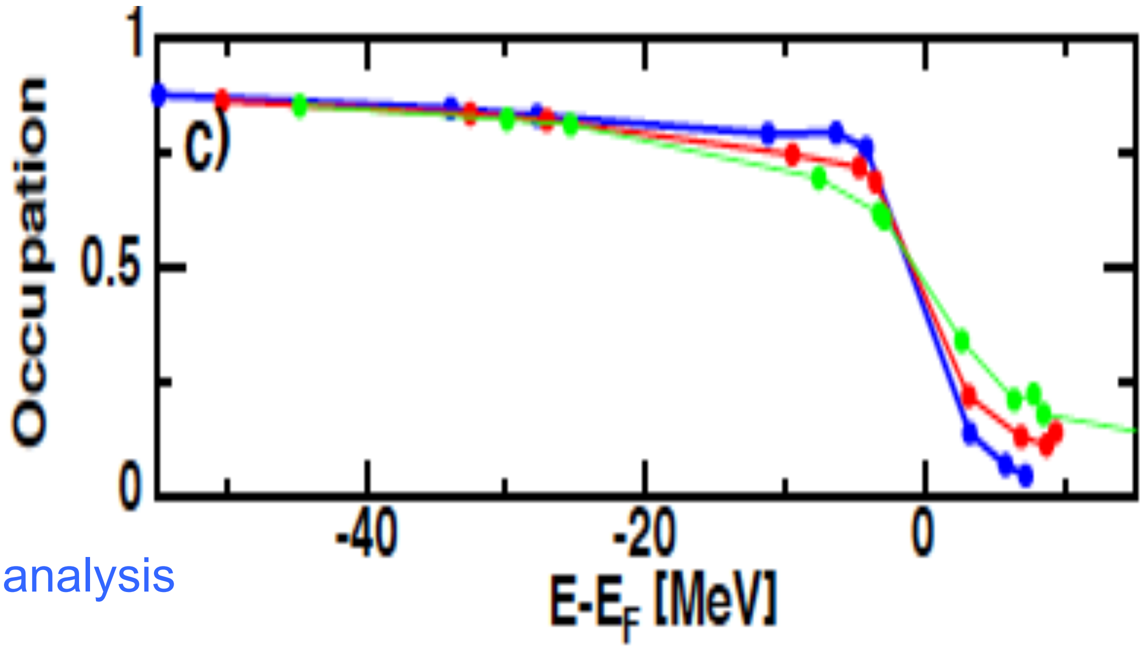
Kh.S.A. Hassaneen, H. Müther, Phys. Rev. C70 (2004) 054308



The Jlab finding is consistent with earlier findings from the spectroscopic factors of direction reactions and p+nucleus scattering

The minority component is more correlated near the Fermi surface!

Example I: proton occupation from p+^{40}Ca, p+^{48}Ca, and p+^{60}Ca (prediction)



Dispersive optical model analysis

PRL 97, 162503 (2006)

Asymmetry dependence of proton correlations.

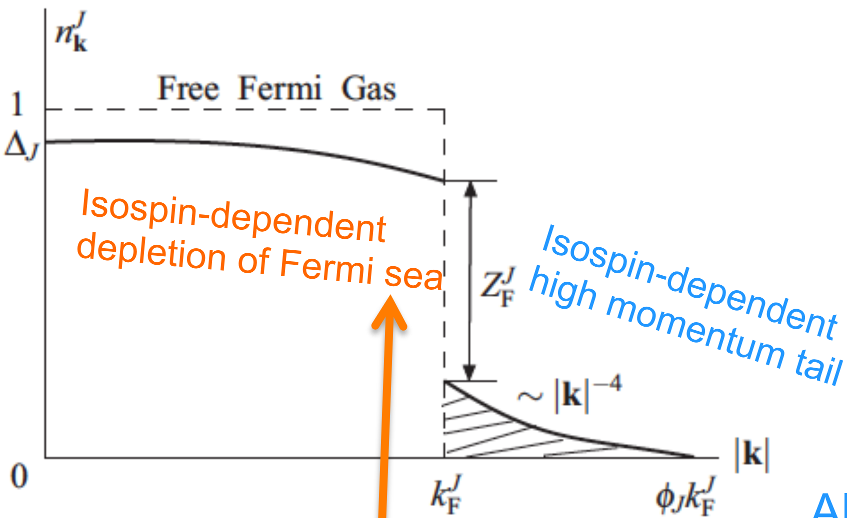
R. J. Charity[1], L. G. Sobotka[1,2], W. H. Dickhoff[2]

Phenomenological nucleon momentum distribution n(k) including SRC effects guided by microscopic theories and experimental findings

The n(k) is not directly measurable, but some of its features are observable

O. Hen, B.A. Li, W.J. Guo, L.B. Weinstein, and E. Piasetzky, PRC 91, 025803 (2015).

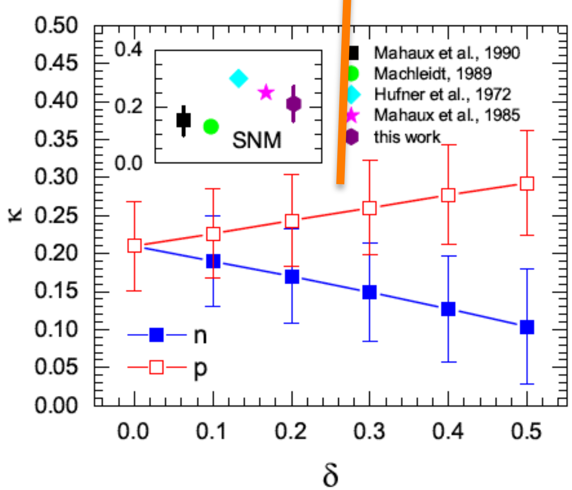
B.J. Cai and B.A. Li, PRC92, 011601(R) (2015); PRC93, 014619 (2016).



$$n_{\mathbf{k}}^J(\rho,\delta) = \begin{cases} \Delta_J + \beta_J I\left(|\mathbf{k}|/k_F^J\right), & 0 < |\mathbf{k}| < k_F^J, \\ C_J\left(k_F^J/|\mathbf{k}|\right)^4, & k_F^J < |\mathbf{k}| < \phi_J k_F^J. \end{cases}$$

$$\Phi_J = 1 + C_J(5\phi_J + 3/\phi_J - 8)$$

All parameters are fixed by
(1) Jlab data: HMT in SNM=25%, 1.5% in PNM,
(2) Contact C for SNM from deuteron wavefunction
(3) Contact C in PNM from microscopic theories



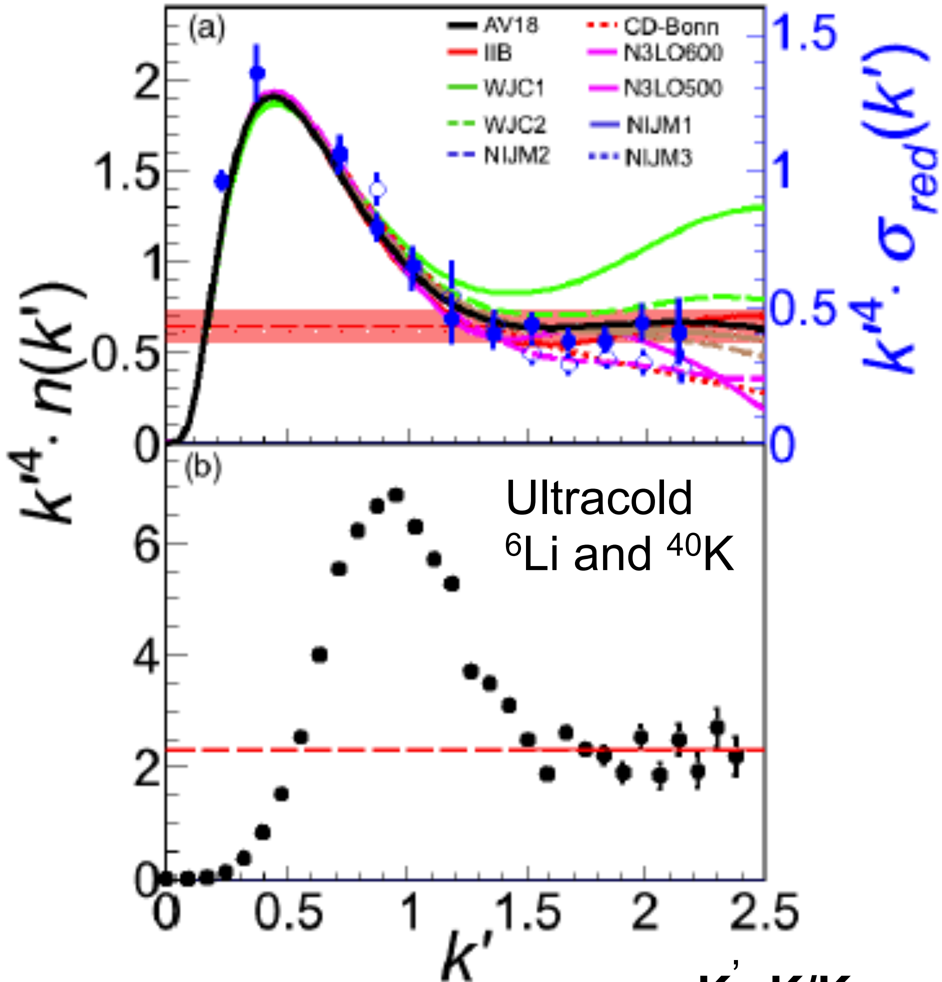
All parameters are assumed to have a linear dependence on isospin asymmetry as indicated by SCGF and BHF calculations

$$Y_J = Y_0(1 + Y_1^J \delta)$$

The high-momentum tail in deuteron scales as $1/K^4$

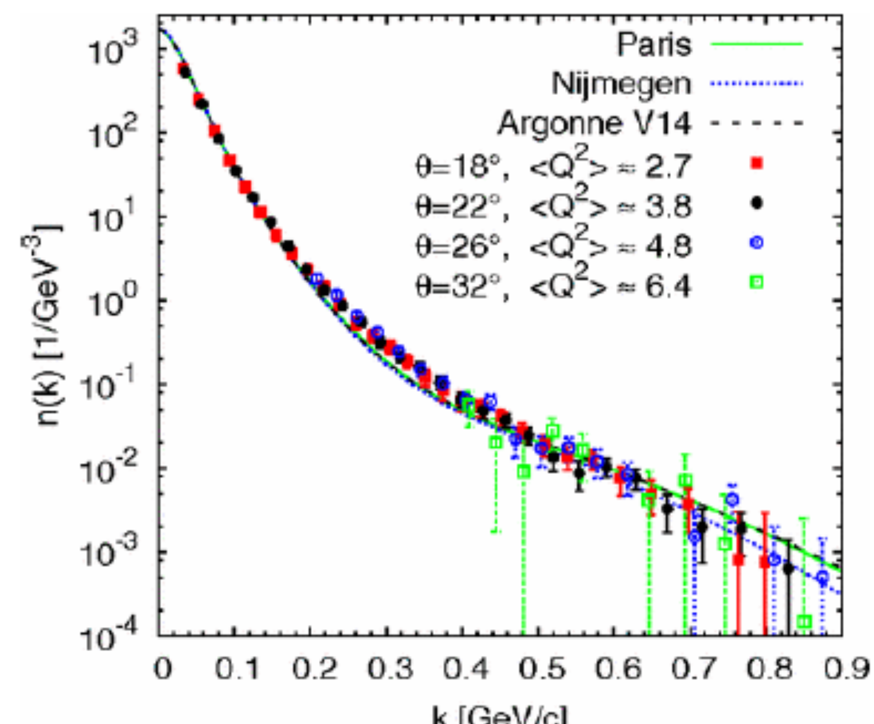
O. Hen, L. B. Weinstein, E. Piasetzky, G. A. Miller, M. M. Sargsian and Y. Sagi, PRC92, 045205 (2015).

VMB calculations



$K' = K/K_F$

Stewart et al. PRL **104**, 235301 (2010)
Kuhnle et al. PRL **105**, 070402 (2010)

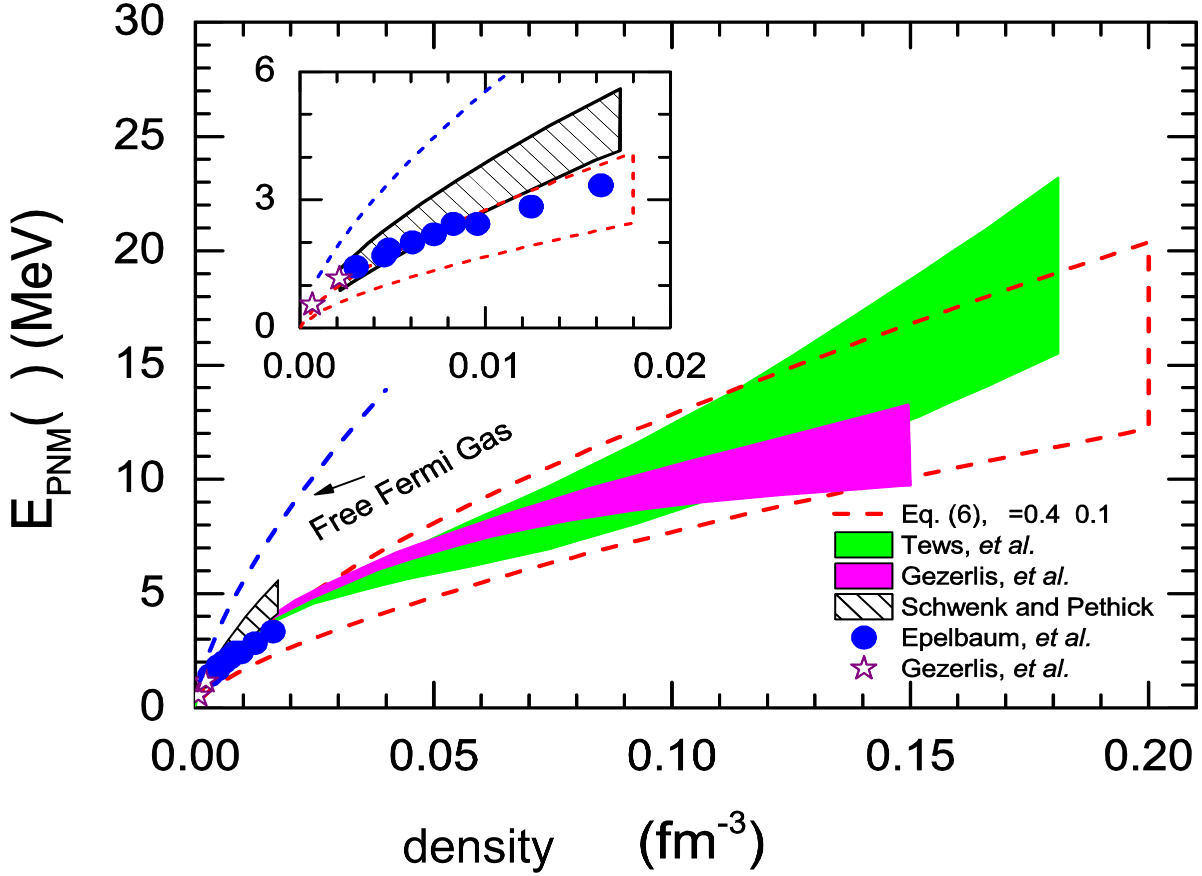


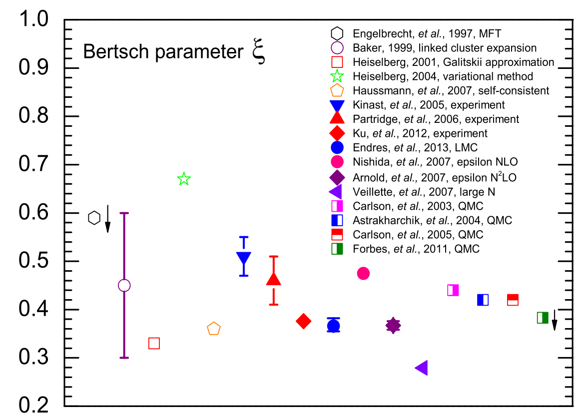
EOS and contact C of pure neutron matter

$$E_{\rm PNM}(\rho) \simeq \frac{3}{5}\frac{(k_{\rm F}^{\rm PNM})^2}{2M}\left[\xi - \frac{\zeta}{k_{\rm F}^{\rm PNM}a_{\rm nn}} - \frac{5\nu}{3(k_{\rm F}^{\rm PNM}a_{\rm nn})^2}\right]$$

$$C_{\rm n}^{\rm PNM} \approx 2\zeta/5\pi + 4\nu/(3\pi k_{\rm F}^{\rm PNM}a_{\rm nn}(^1{\rm S}_0)) \approx 0.12.$$

$$a_{\rm nn}(^1{\rm S}_0) = -18.8 \pm 0.3\ {\rm fm},$$





B.J. Cai and B.A. Li, PRC92, 011601(R) (2015).

The contact C of PNM is derived from its EOS using the **adiabatic sweep theorem**

n(k)=C/K⁴

$$\frac{\hbar^2\Omega C}{4\pi m} = \frac{{\rm d}E}{{\rm d}(-1/a)},$$

S. Tan, Annals of Physics 323 (2008) 2971-2986

Correlation between measurements in R and K spaces

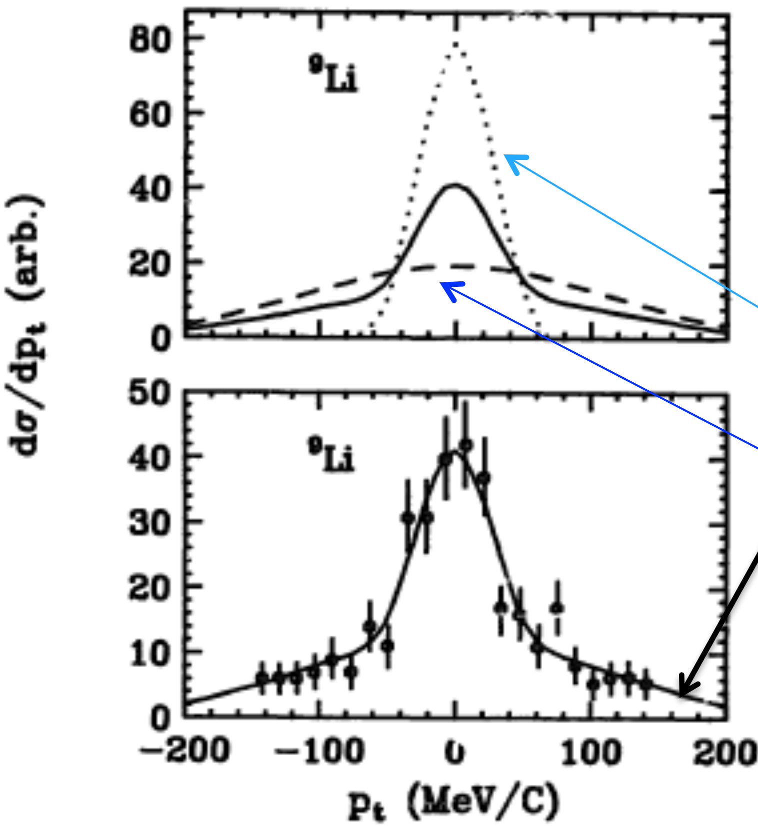
Same interaction→ different representations of the same wave functions in R or K space

Fundamental principles guiding physical intuitions:

(1) Liouville Theorem: $d\rho(r,k,t)/dt=0$ → $\langle r\rangle\bullet\langle k\rangle\approx$constant

(2) Uncertainty Principle → $\delta r \bullet \delta k \geq h$

Example: neutron momentum distribution in the halo nucleus ^{11}Li



Recoil momentum distribution of ^{9}Li in $^{11}Li+^{12}C \rightarrow ^{9}Li+2n+x$ reactions

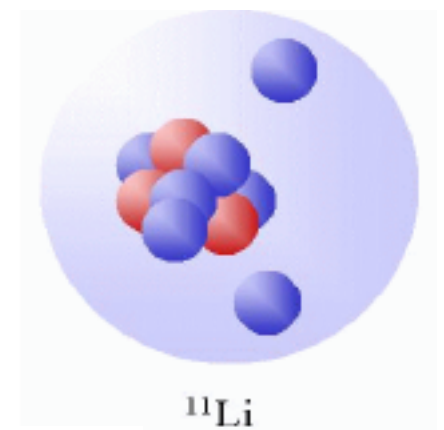
T. Kobayashi *et al.*, Phys. Rev. Lett. **60** (1988) 2599

2n randomly selected from

(1) A small Fermi sphere

(2) A large Fermi sphere

Mixed 2-Fermi spheres



Bao-An LI, Mahir S. HUSSEIN[1] and Wolfgang BAUER

Nuclear Physics A533 (1991) 749-760

The Extended Thomas-Fermi Approximation (ETF)

M. Brack et al, Phys. Rep. 123, 275 (1985)

Using the semi-classical ħ expansion of the Block-density matrix developed by Wigner (1932) and Kirkwood (1933)

The kinetic energy density in nuclei: $\tau_{ETF}[\rho] = \tau_{TF}[\rho] + \tau_2[\rho] + \tau_4[\rho]$

(1) The original Thomas-Fermi Approximation for nuclear matter: $\tau_{TF}[\rho] = \kappa\rho^{5/3}, \qquad \kappa = \frac{3}{5}(3\pi^2)^{2/3}$

(2) The Weizsacker term (1935): $\tau_2[\rho] = \frac{1}{36}\frac{(\nabla\rho)^2}{\rho} + \frac{1}{3}\Delta\rho\,.$

(sensitive to surface properties-a probe of n-skin!)

Laplacian term

(3) The ħ^4 term:

$$\tau_4[\rho] = \frac{1}{4320}(3\pi^2)^{-2/3}\rho^{1/3}\left[24\frac{\Delta^2\rho}{\rho} - 60\frac{\nabla\rho\cdot\nabla(\Delta\rho)}{\rho^2} - 28\left(\frac{\Delta\rho}{\rho}\right)^2 - 14\frac{\Delta(\nabla\rho)^2}{\rho^2} + \frac{280}{3}\frac{(\nabla\rho)^2\Delta\rho}{\rho^3} + \frac{184}{3}\frac{\nabla\rho\cdot\nabla(\nabla\rho)^2}{\rho^3} - 96\left(\frac{\nabla\rho}{\rho}\right)^4\right].$$

Extended^{+} Thomas-Fermi Approximation (ETF^{+}) considering the isospin-dependent SRC and effectively $\hbar^4$ and higher order terms

What to we add and modify?

Mimic effects of $\hbar^4$ and higher terms

H.Krivine and J. Treiner, PLB 88, 212 (1979)
X. Campi and S. Stringari, NPA 337, 313 (1980)
M. Barranco, M. Pi and X. Vinas, PLB124, 131 (1983)

$$\varepsilon_J^{\text{kin}}(r) = \frac{1}{2M}\left[\alpha_J^{\infty}\rho_J^{5/3} + \eta_J \frac{1}{36}\frac{(\nabla\rho_J)^2}{\rho_J} + \frac{1}{3}\Delta\rho_J\right]$$

$$\alpha_J^{\infty} = \frac{3}{5}(3\pi^2)^{2/3}\Phi_J,$$

Isospin-dependent SRC constrained by data
Φ_J=1 for sharp Fermi spheres, it is larger than 1 with SRC-induced high momentum tails

$$\Phi_{\rm p} \approx 2.0911 \pm 0.4982 \text{ and } \Phi_{\rm n} \approx 1.5978 \pm 0.3316 \text{ for } ^{208}\text{Pb}.$$

$$\varepsilon_J^{\text{kin}} = \frac{2}{(2\pi)^3}\int_0^{\phi_J k_F^J} \frac{\mathbf{k}^2}{2M} n_{\mathbf{k}}^J(\rho,\delta)\,\mathrm{d}\mathbf{k},$$

Connection with neutron-skin via the Extened Thomas-Fermi Approximation

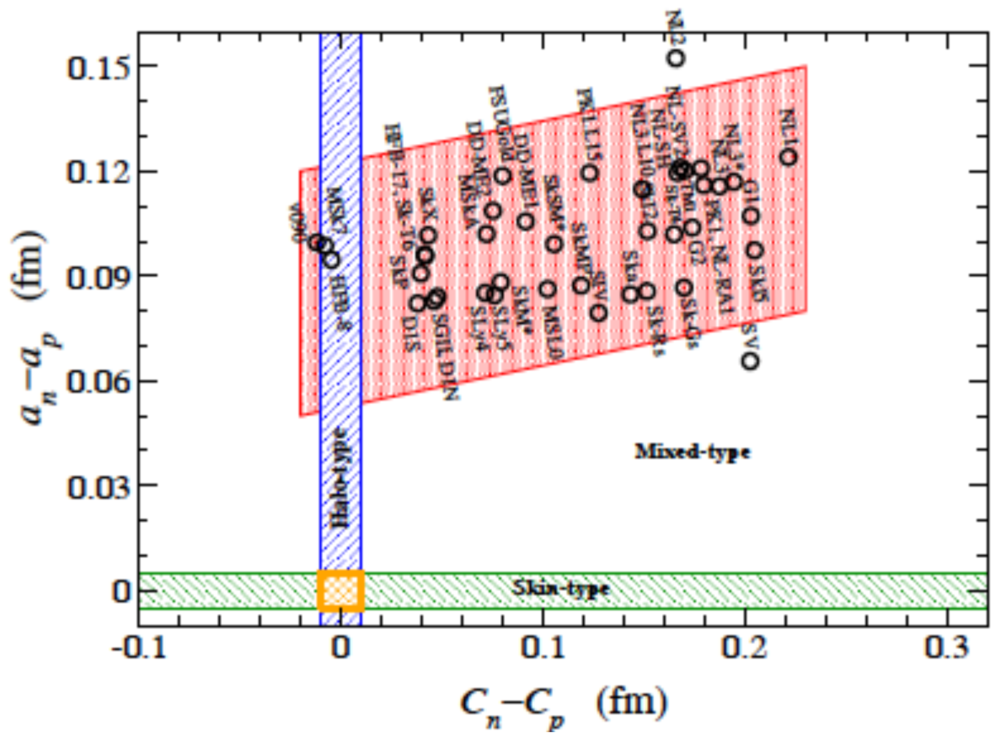
$$\varepsilon_J^{\mathrm{kin}}(r) = \frac{1}{2M}\left[\alpha_J^{\infty}\rho_J^{5/3} + \eta_J \frac{1}{36}\frac{(\nabla\rho_J)^2}{\rho_J} + \frac{1}{3}\Delta\rho_J\right]$$

$$\rho(r) = \frac{\rho_0}{1+\exp\left[(r-C)/a\right]}$$

$$\langle r^2\rangle = \frac{3}{5}c^2 + \frac{7}{5}(\pi a)^2$$

Both the half-radius C and surface diffuseness a contribute to the size of neutron-skin

$$a_p = 0.447 \text{ fm and } c_p = 6.680 \text{ fm.}$$



X. Viñas[1,a], M. Centelles[1], X. Roca-Maza[2], and M. Warda[3]
EPJA 50 (2014) 27

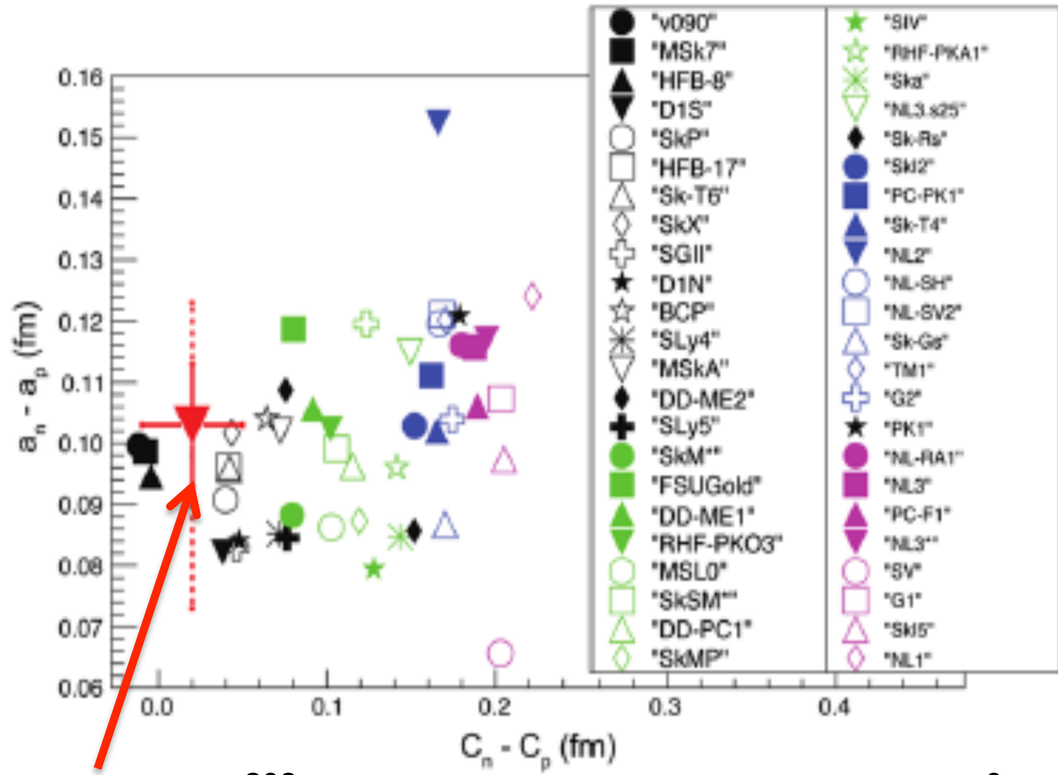
PRL 112, 242502 (2014) PHYSICAL REVIEW LETTERS week ending 20 JUNE 2014

Neutron Skin of ^{208}Pb from Coherent Pion Photoproduction

C. M. Tarbert,[1] D. P. Watts,[1,*] D. I. Glazier,[1] P. Aguar,[2] J. Ahrens,[2] J. R. M. Annand,[3] H. J. Arends,[2] R. Beck,[2,4]
et al.

(1) Need to go beyond inferring the n-skin from its correlation with an observable shown in models

(2) Better infer directly the neutron density profile from data analyses &understand the nature of n-skin



Experimental evidence of "Halo" domination in ^{208}Pb from photoproduction of π^0

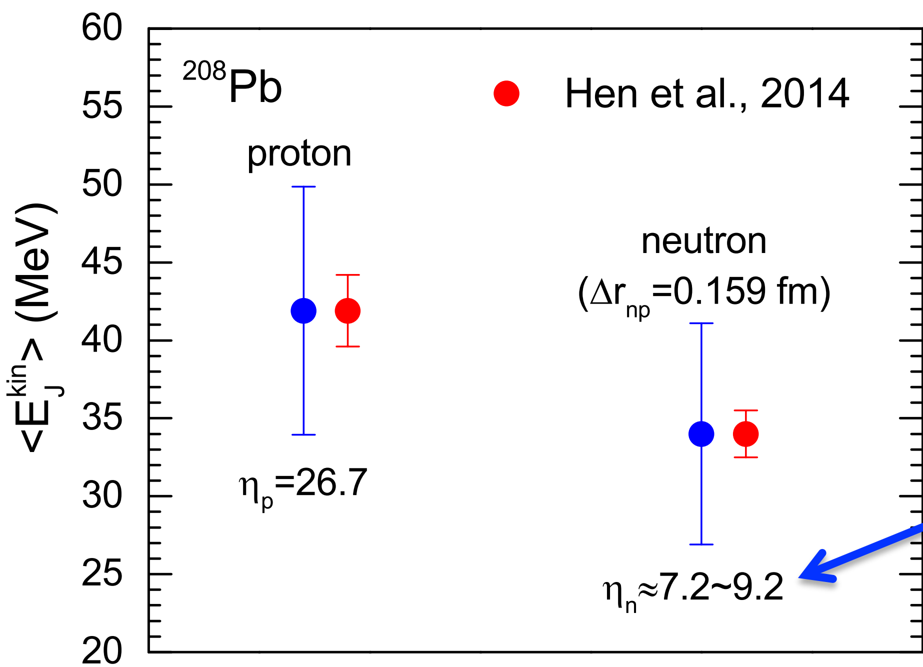
Constraining the ETF model parameters a_n, c_n and η_n for ^{208}Pb

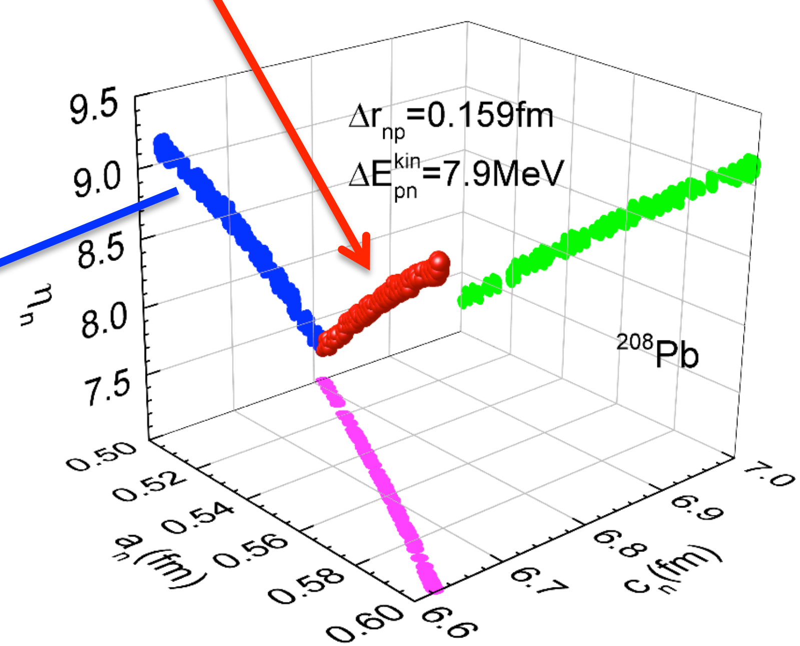
(1) For protons, η_p is the only parameter as the proton density profile is known

(2) For neutrons, given the size of neutron-skin, average kinetic energy, and the normalization condition, only a correlation among a_n, c_n and η_n is fixed

$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

$$\Delta E_{pn}^{kin} \equiv \langle E_p^{kin} \rangle - \langle E_n^{kin} \rangle$$

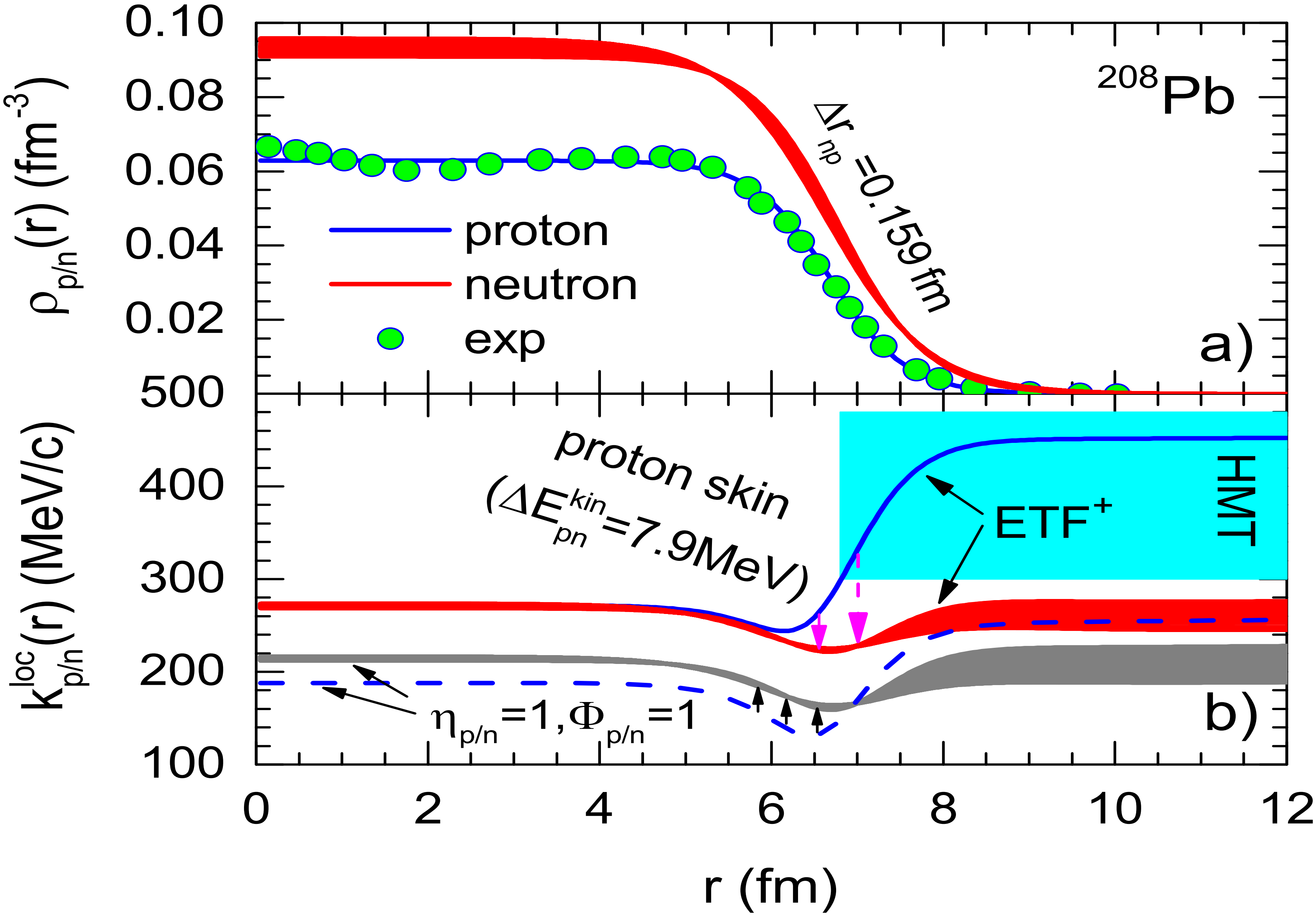




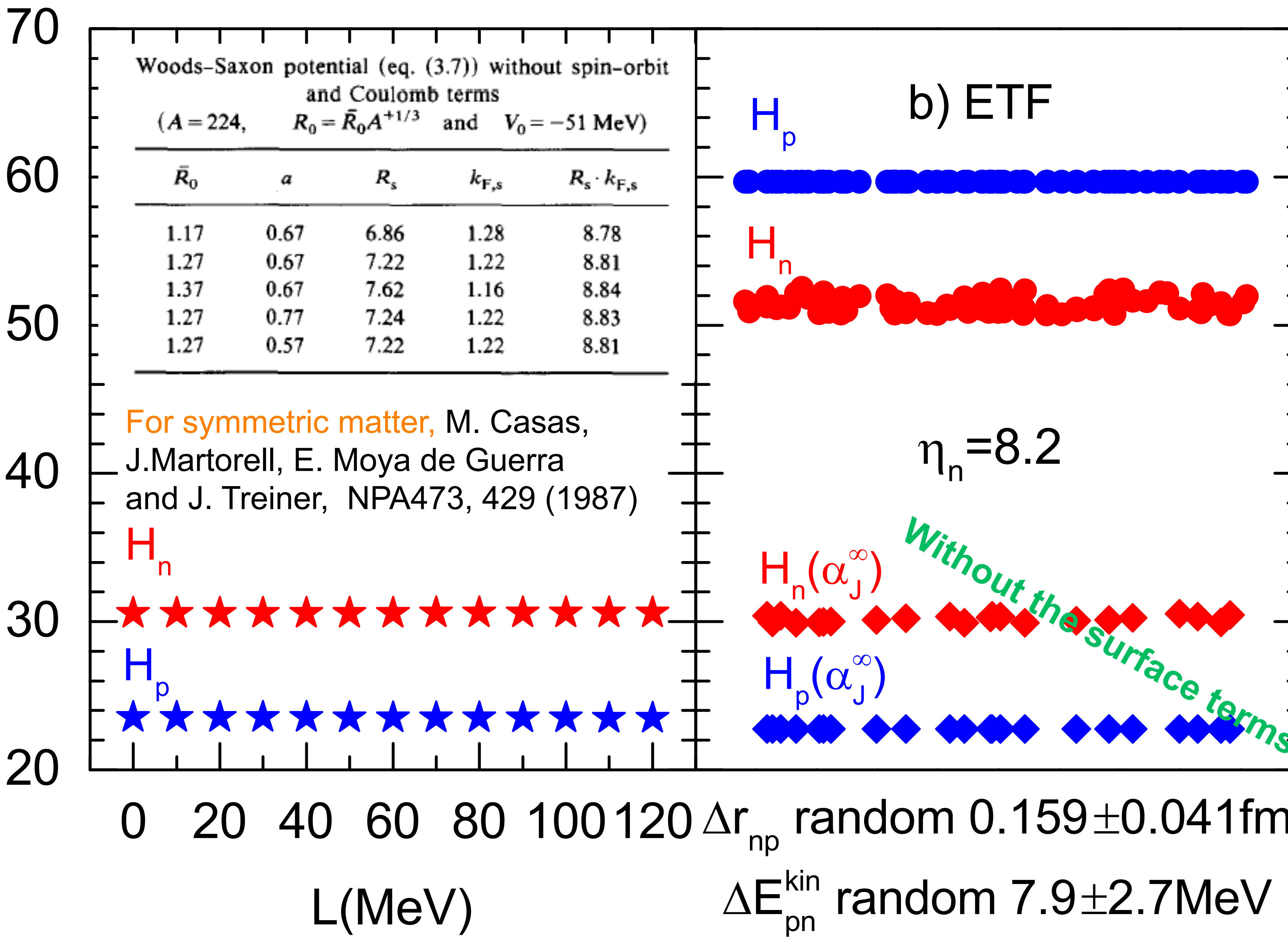
$$\langle E_J^{kin} \rangle = \int_0^\infty \varepsilon_J^{kin}(r) dr \Big/ \int_0^\infty \rho_J(r) dr \equiv \langle k_J^2 \rangle / 2M$$

Neutron-skin in coordinate and proton-skin in momentum

The average local momentum is defined via $k_J^{\mathrm{loc},2}(r)/2M = \varepsilon_J^{\mathrm{kin}}(r)/\rho_J(r)$



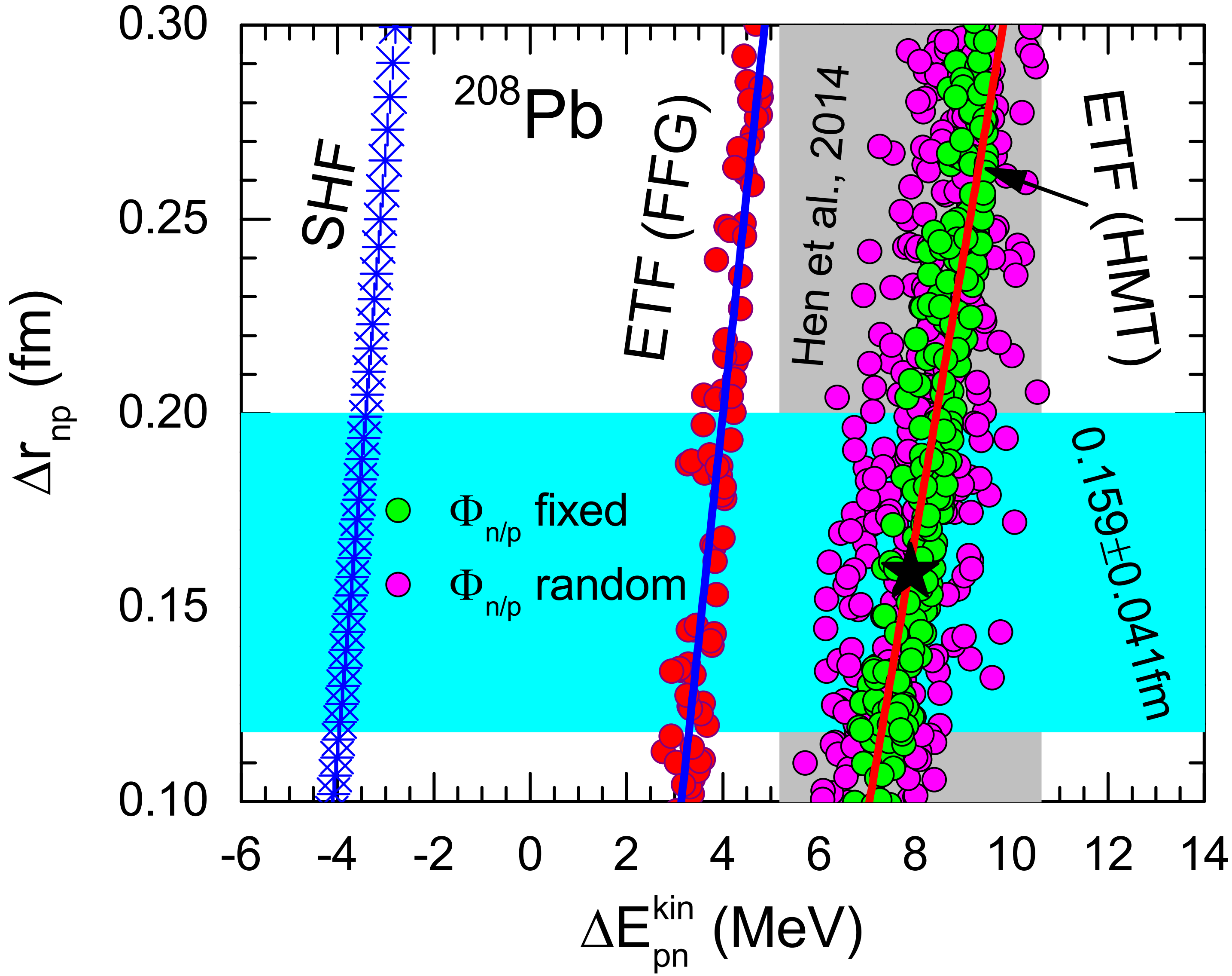
Constancy of $H_J = \langle r_J^2 \rangle \langle k_J^2 \rangle$ in a given model



Woods-Saxon potential (eq. (3.7)) without spin-orbit and Coulomb terms ($A = 224$, $R_0 = \bar{R}_0 A^{+1/3}$ and $V_0 = -51$ MeV)

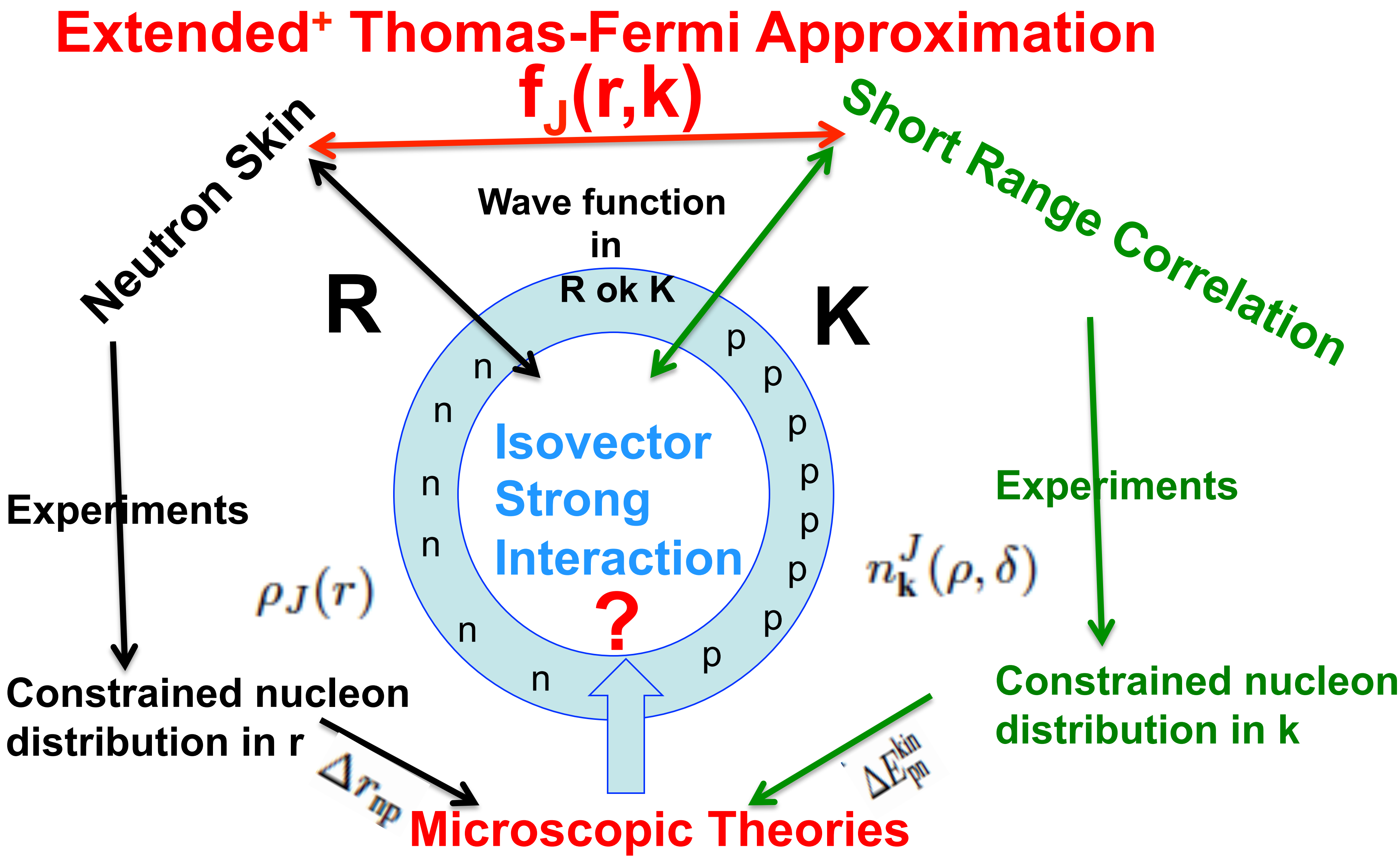
$\bar{R}_0$	a	R_s	$k_{F,s}$	$R_s \cdot k_{F,s}$
1.17	0.67	6.86	1.28	8.78
1.27	0.67	7.22	1.22	8.81
1.37	0.67	7.62	1.16	8.84
1.27	0.77	7.24	1.22	8.83
1.27	0.57	7.22	1.22	8.81

Two dimensional but correlated constraints on models using independent measurements of n-skin in R and p-skin in K



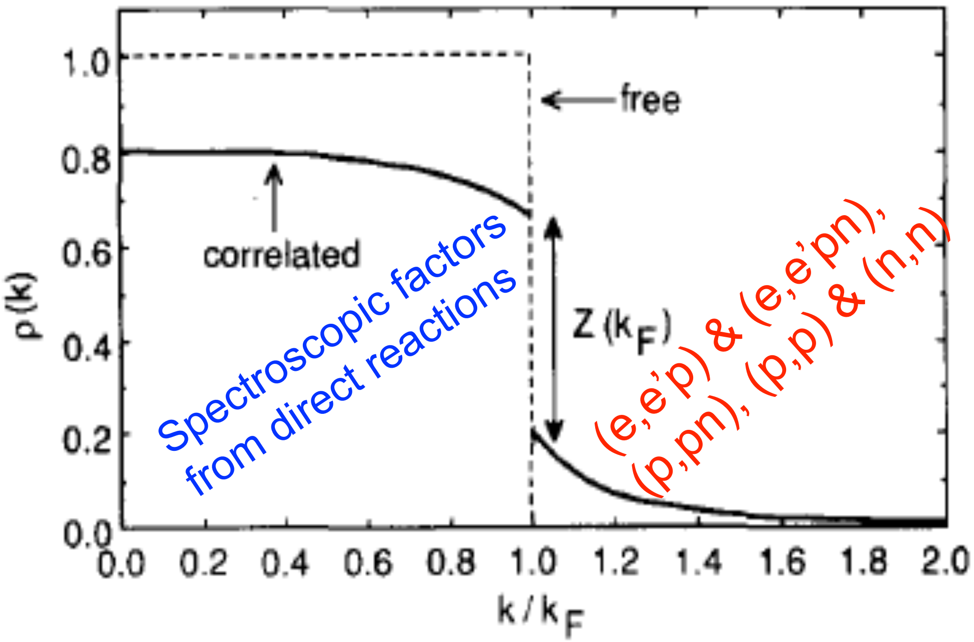
Summary

Protons are moving faster than neutrons in neutron-skins



The Migdal (1957)-Luttinger (1960) Theorem: $Z_F^J = n^J_{k_F^J-0} - n^J_{k_F^J+0} = M/M_E^{J,*}$

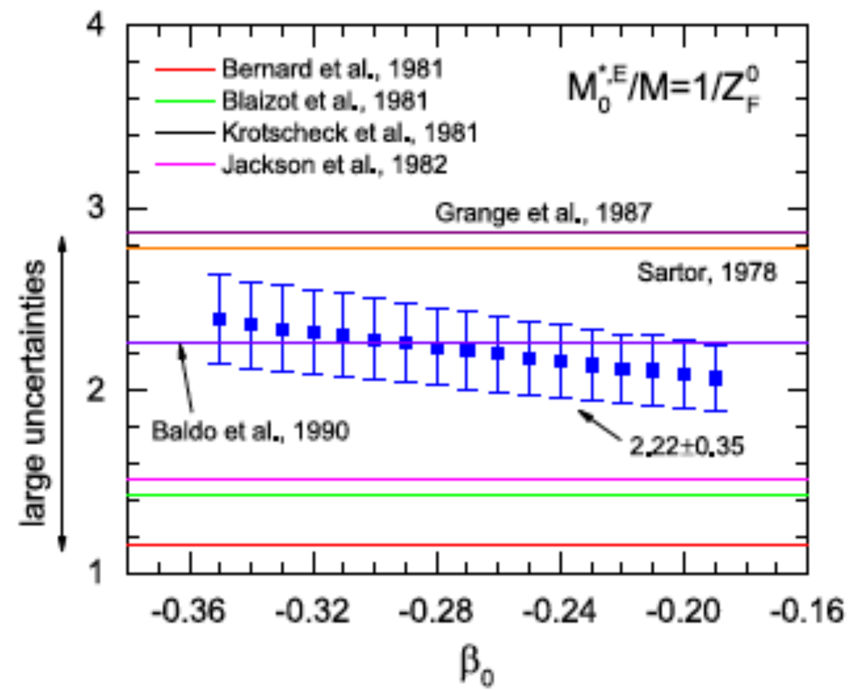
(occupation renormalization function)



C. MAHAUX and R. SARTOR

Physics Reports 211, 53 (1992).

The effective E-mass $\frac{M_J^{*,E}}{M} = 1 - \frac{\partial U_J}{\partial E}$



B.J. Cai and B.A. Li, PLB 757, 79 (2016)

Nuclear Neutron-Proton Contact and the Photoabsorption Cross Section

Ronen Weiss, Betzalel Bazak, and Nir Barnea*
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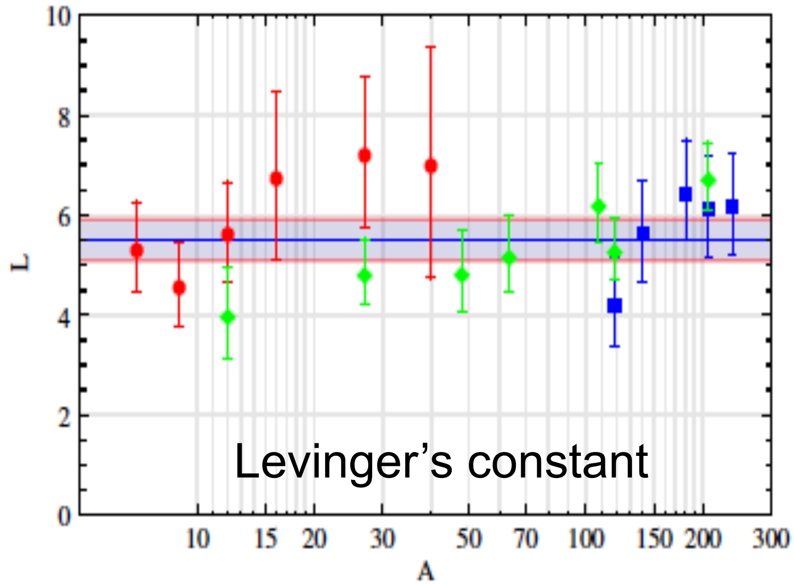
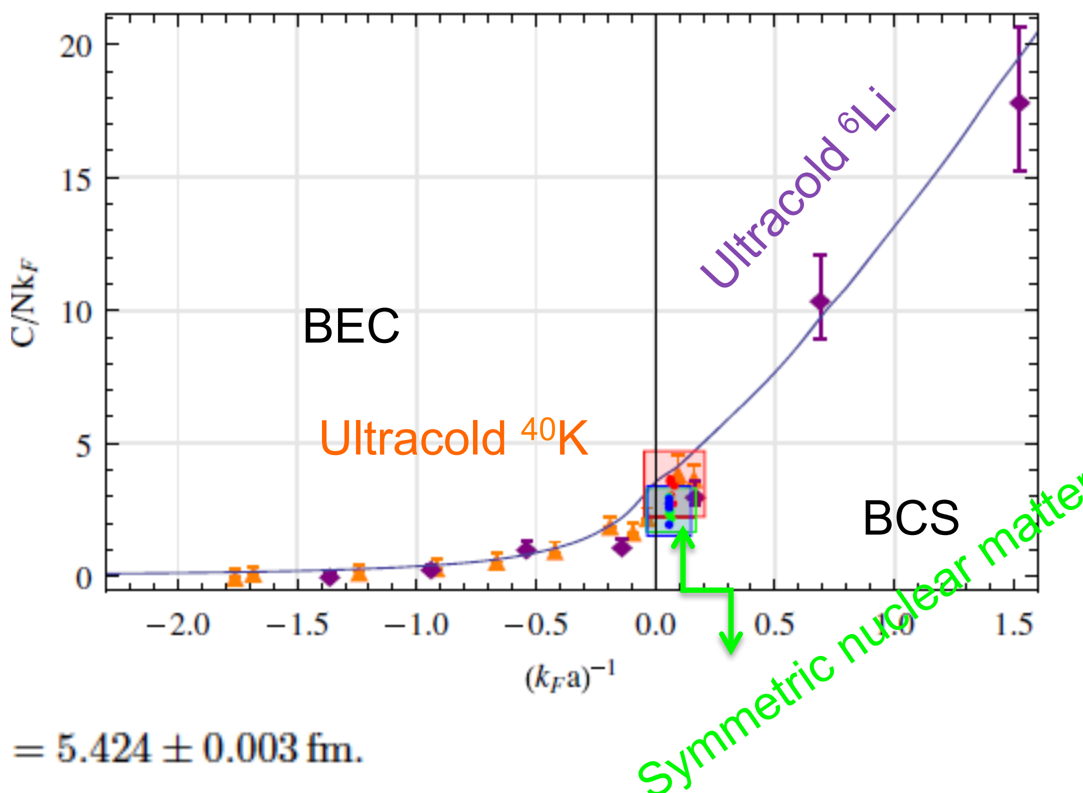




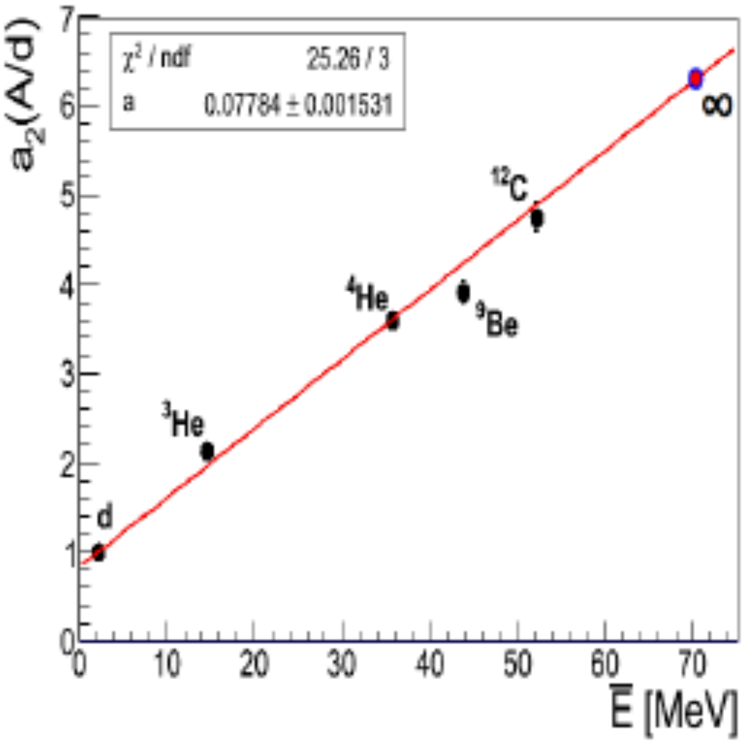
$$\sigma_A = L\frac{NZ}{A}\sigma_d,$$

$$a_{np}(^3S_1) = 5.424 \pm 0.003\ \text{fm}.$$

C=0.172±0.007

Relative probability of SRC in nucleus A with respect to that in deuteron

$a_2(A/d)$ extrapolated to infinite SNM

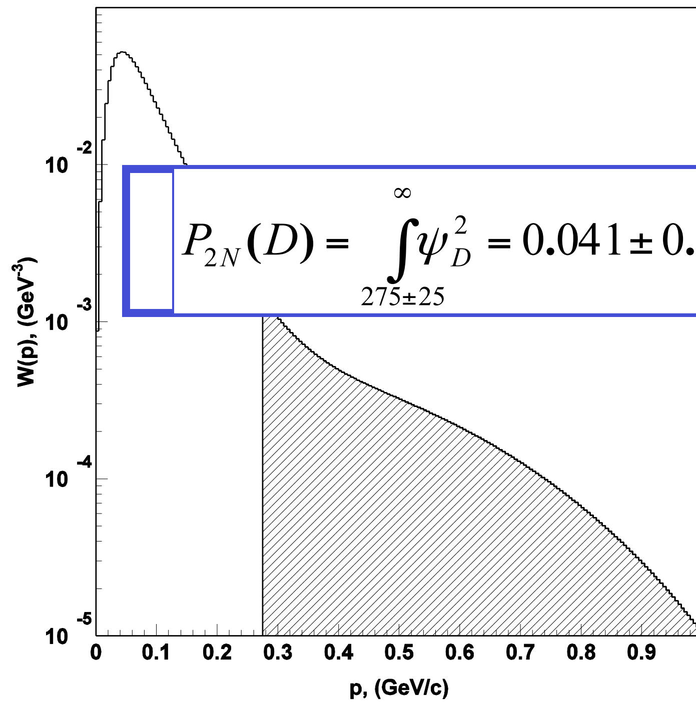


Nucleon removal energy

$$\bar{E} = \bar{T}\frac{A-2}{A-1} - \frac{E_0}{A}$$

Greens Function Monte-Carlo (GFMC)

E. Piasetzky, O. Hen, L. B. Weinstein
Proceedings of plenary talk at CIPANP 2012



$$P_{2N}(D) = \int_{275\pm25}^{\infty} \psi_D^2 = 0.041 \pm 0.008$$

$$P_{2N}(A) = a_{2N}(A/d) \cdot P_{2N}(D)$$

$P_{2N}(\infty)$≈20-30% in symmetric matter