Nuclear equation of state from chiral effective field theory

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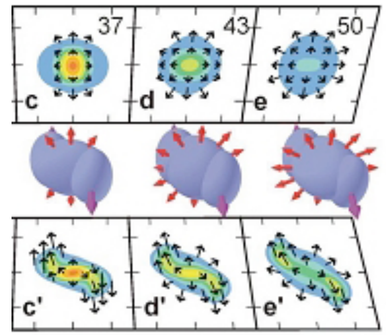
INT Program: The Phases of Dense Matter, July 12, 2016

NEUTRON-RICH MATTER IN THE STARS AND ON EARTH

Astronomical observations

Radio, optical,
X-ray, gamma ray,
gravitational wave,
neutrino astronomy

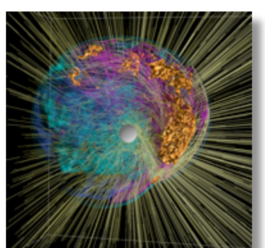
Nuclear experiments



Heavy ion collisions,
exotic isotope masses,
neutron skin thickness,
nuclear polarizabilities,

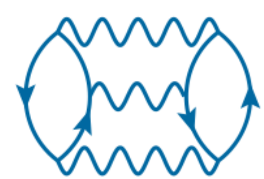
Astrophysical simulations

3-dimensional,
full general relativity,
neutrino transport,
magnetohydrodynamics



Nuclear Theory

Improved mean field
phenomenology &
chiral EFT: Equation of
state, neutrino response,
single-particle potentials



TOPICS OF FOCUS

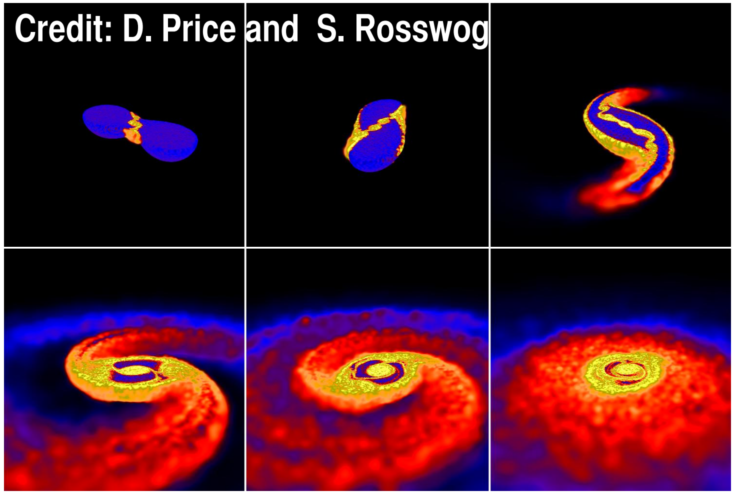
Nuclear equation of state

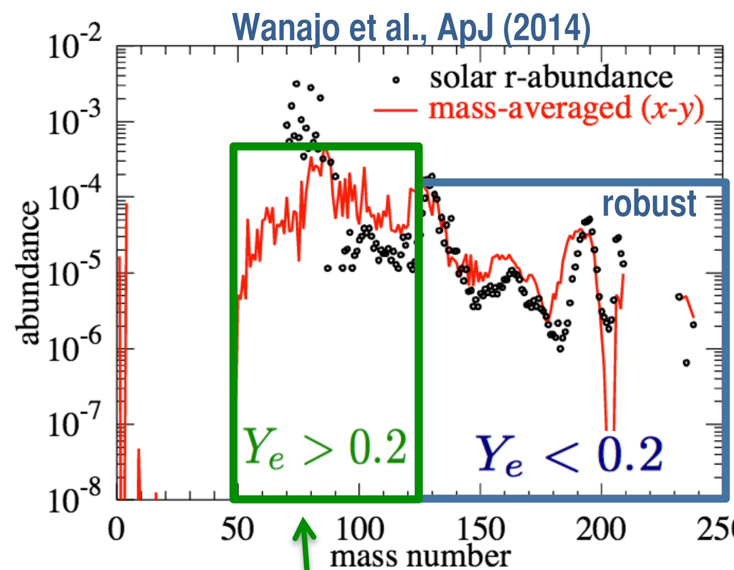
- **Shock wave energy** produced from stellar core collapse
- **Mass-Radius** relationship for cold neutron stars
- Shock heating in neutron star mergers and associated **nucleosynthesis**

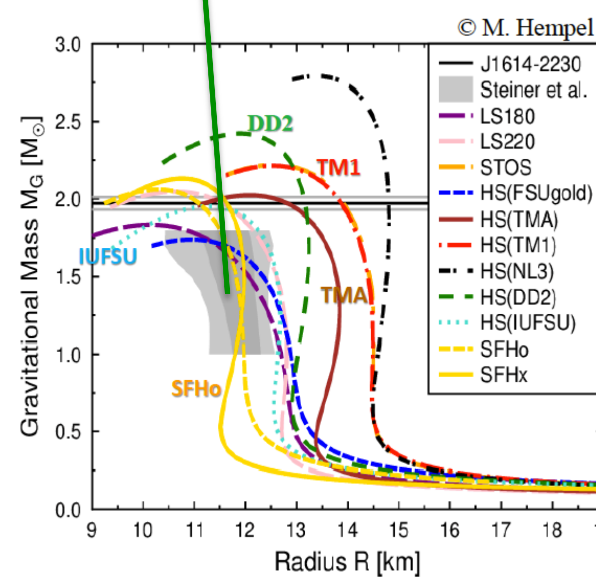
Nucleon single-particle potentials

- **Transport simulations** of medium-energy heavy-ion collisions
- **Optical potentials** for neutron-capture rates in r-process
- **Nucleosynthesis** outcome in supernova neutrino-driven wind

R-PROCESS NUCLEOSYNTHESIS IN NEUTRON STAR MERGERS







- Soft EoS (SFHo) required for favorable shock-heating in **full GR**
- Subsequent **neutrino processing** increases Y_e value for majority (60%) of ejecta

SCALES IN HOT/DENSE STELLAR MATTER

Large parameter space:

- Density

$$10^4 < \rho < 10^{15}\,\mathrm{g/cm^3}$$

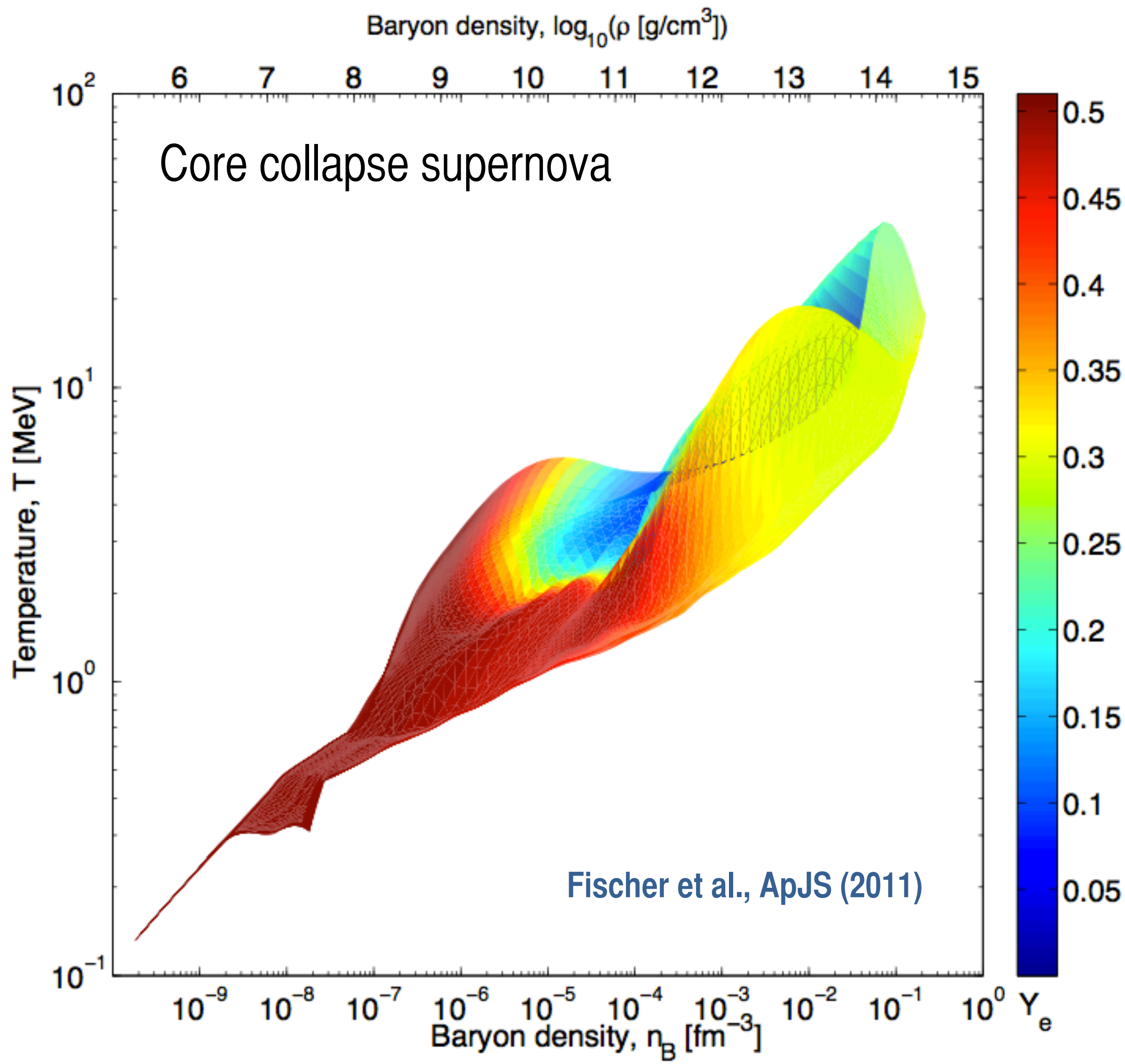
- Temperature

$$0 < T < 50\,\mathrm{MeV}$$

- Proton fraction

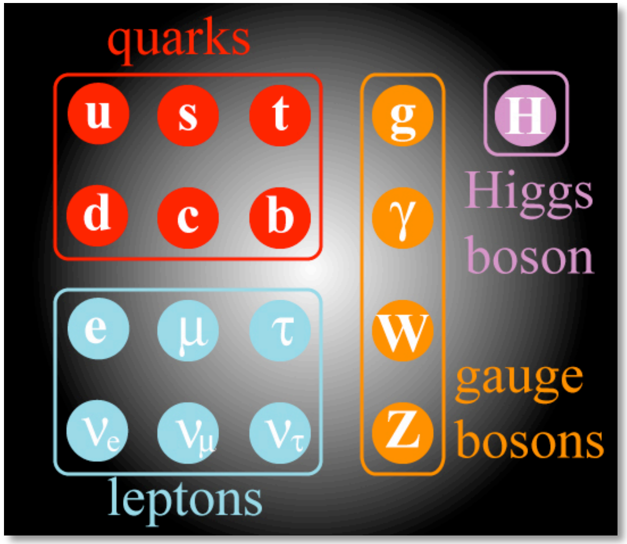
$$0 < Y_p < 0.6$$

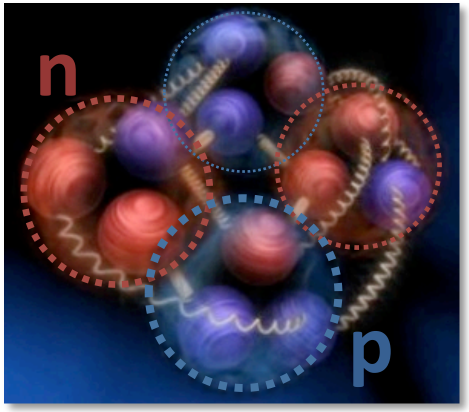
- **10 million configurations** in tabular form for EOS



CHIRAL EFFECTIVE FIELD THEORY DESCRIPTION

- 3D numerical simulations key to unraveling explosion mechanisms, …
- Parameter studies too computationally expensive: **incentive for improved nuclear modeling**
- **Consistent approach to nuclear microphysics:** multi-pion exchange processes, three-body forces, Pauli-blocking,…
- **Quantified uncertainty estimates** for the equation of state and neutrino response
  - Order-by-order convergence in chiral power counting
  - Order-by-order convergence in many-body perturbation theory
  - Scale dependence





Quark/gluon (high energy) dynamics

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}_L i\gamma_\mu D^\mu q_L$$
$$+\bar{q}_R i\gamma_\mu D^\mu q_R - \bar{q}\mathcal{M}q$$

- Approximate **chiral symmetry** (left- and right-handed quarks approximately decoupled)

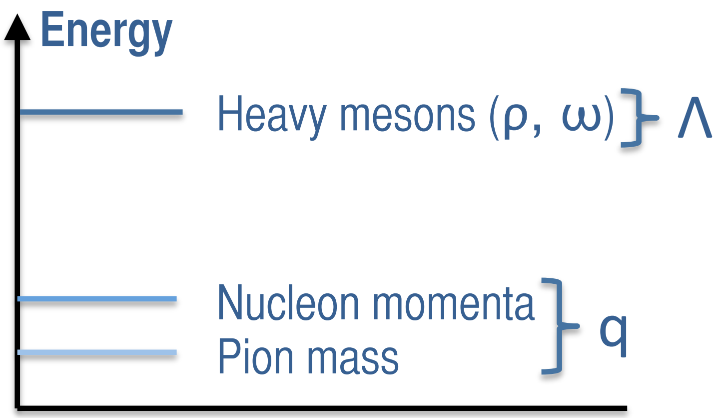
Nucleon/pion (low energy) dynamics

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(0)}_{NN} + \mathcal{L}^{(2)}_{NN} + \cdots$$

- Compatible with explicit and spontaneous **chiral symmetry breaking**

NUCLEAR FORCES IN CHIRAL EFFECTIVE FIELD THEORY

NATURAL SEPARATION OF SCALES



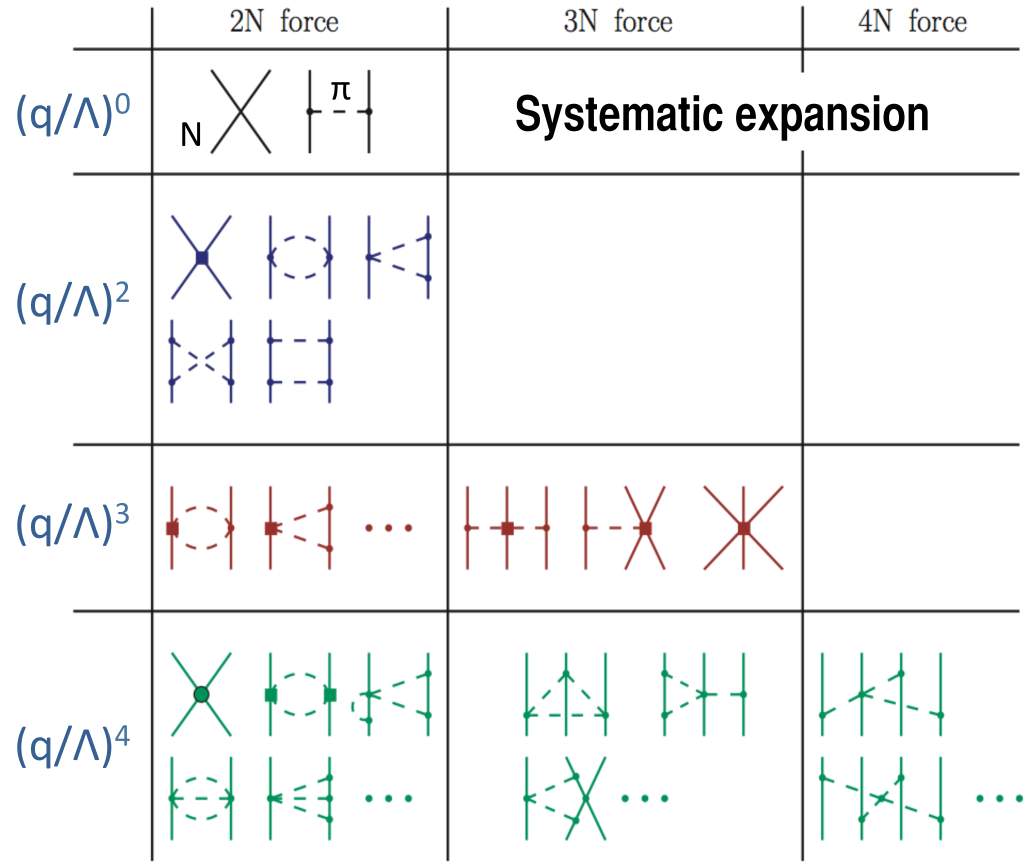
Pions weakly-coupled at low momenta

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{1}{2}\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{1}{2 f_\pi^2}(\partial_\mu \vec{\pi} \cdot \vec{\pi})^2$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}\left(i\gamma^\mu D_\mu - m - \frac{g_A}{2 f_\pi}\gamma^\mu \gamma_5 \vec{\tau} \cdot \partial_\mu \vec{\pi} \right) N$$

CHIRAL EFFECTIVE FIELD THEORY

Low-energy theory of nucleons and pions



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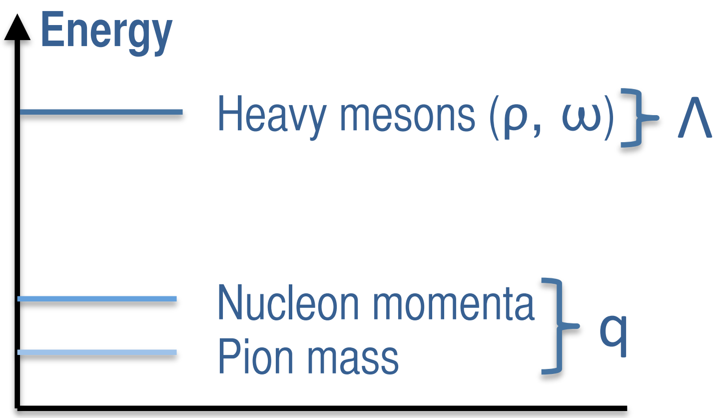

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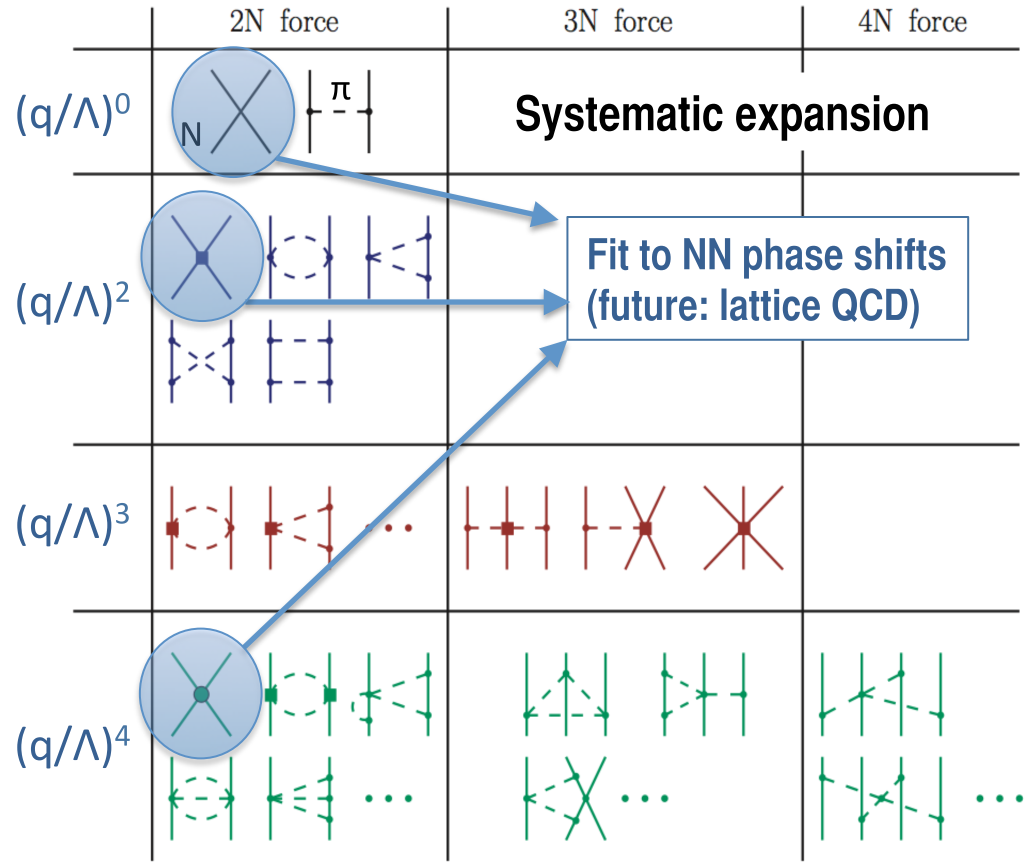

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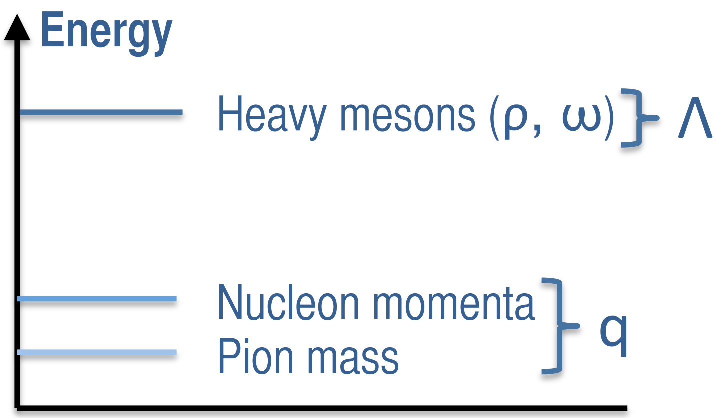

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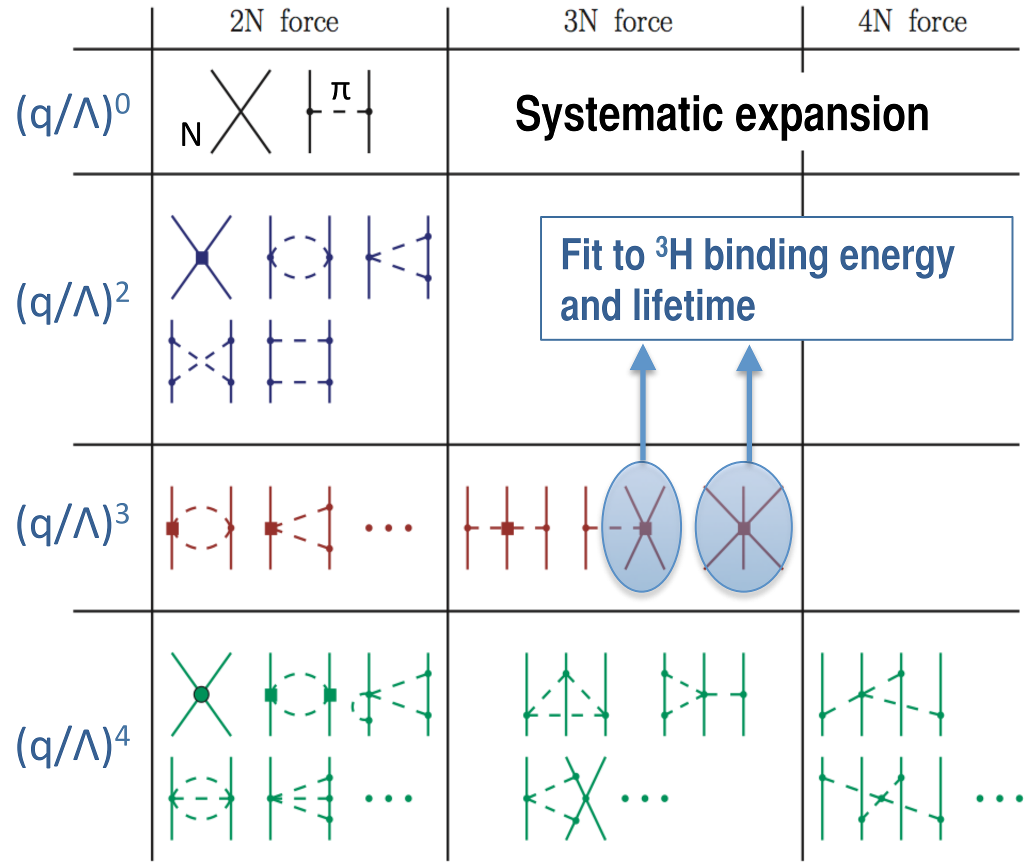


RESOLUTION SCALE DEPENDENCE

Regulating function

- $\langle \vec{p}\,'|V|\vec{p}\rangle$ $\exp[-(p/\Lambda)^{2n} - (p'/\Lambda)^{2n}]$

sets resolution scale

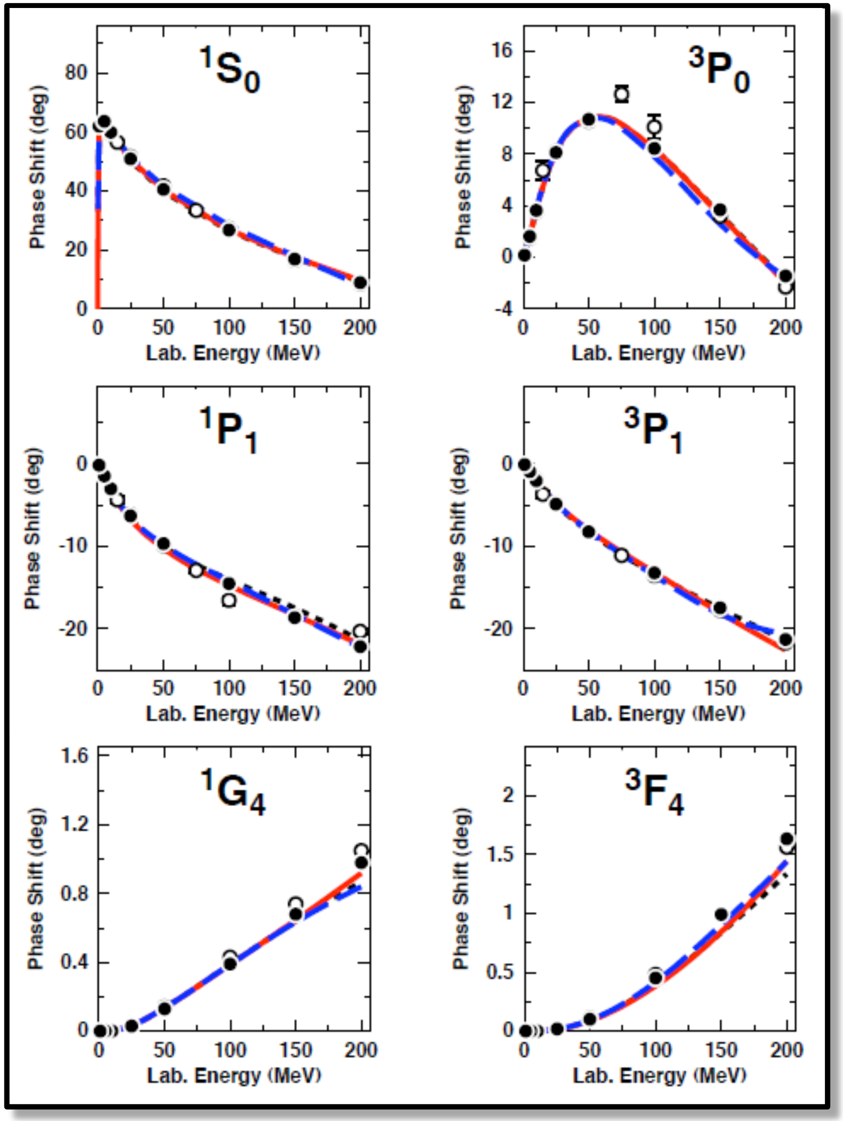
Variations in regulator

- Estimate of theoretical uncertainty

$\Lambda = 414\,\mathrm{MeV}\,(\Delta x \sim 1.50\,\mathrm{fm})$ (solid red)

$\Lambda = 450\,\mathrm{MeV}\,(\Delta x \sim 1.38\,\mathrm{fm})$ (dashed blue)

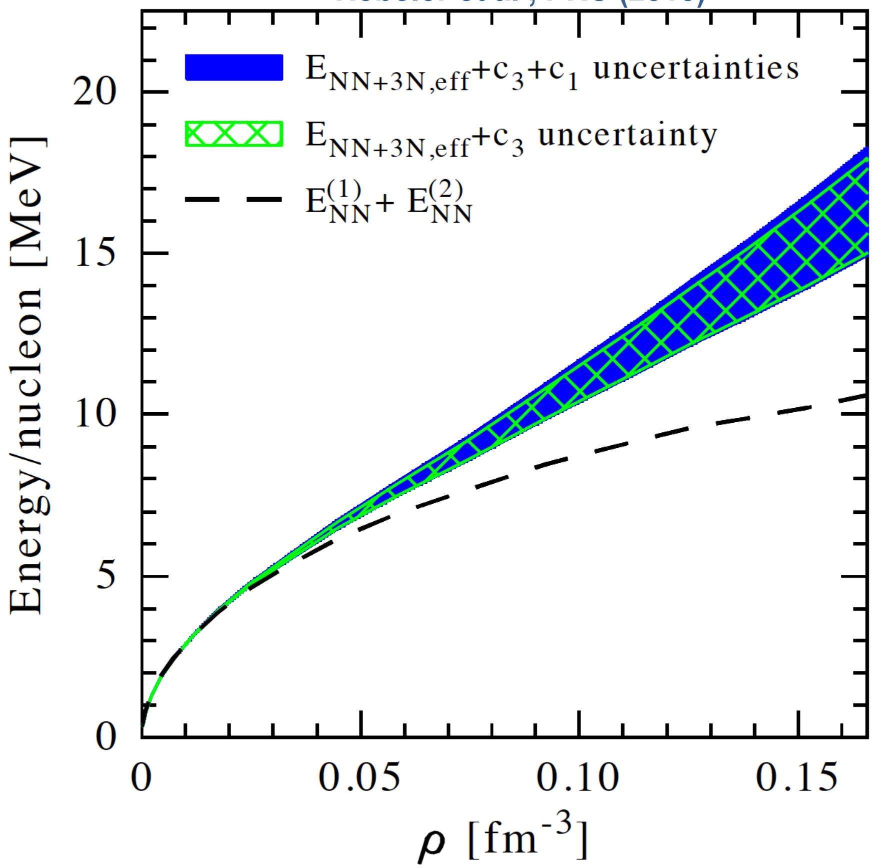
$\Lambda = 500\,\mathrm{MeV}\,(\Delta x \sim 1.25\,\mathrm{fm})$ (dotted black)



Coraggio, Holt, Itaco, Machleidt, Sammarruca, PRC (2013)

SCALE DEPENDENCE OF NEUTRON MATTER EOS

Hebeler et al., PRC (2010)



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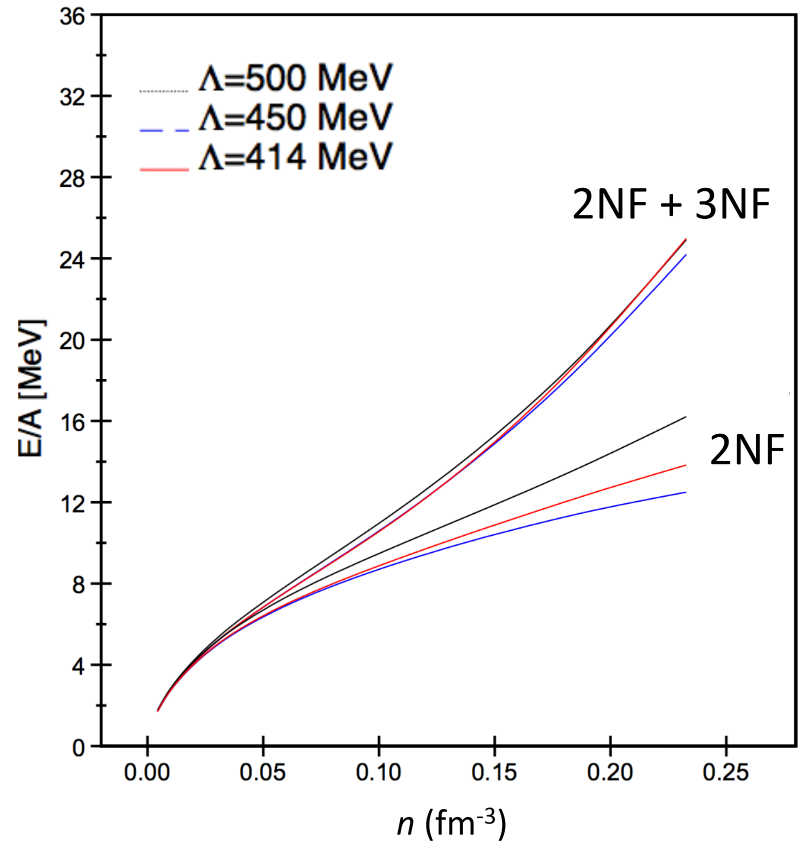
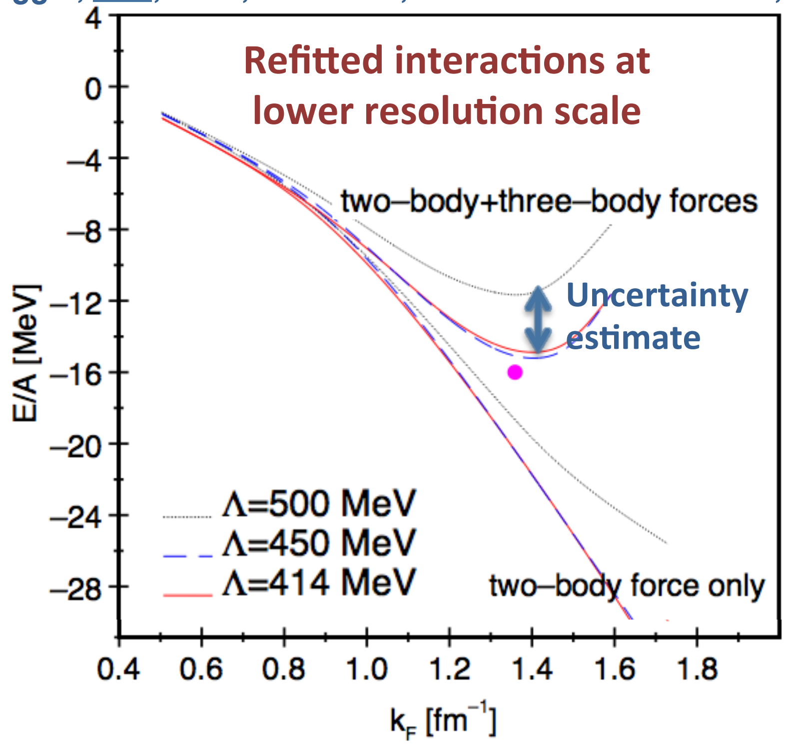


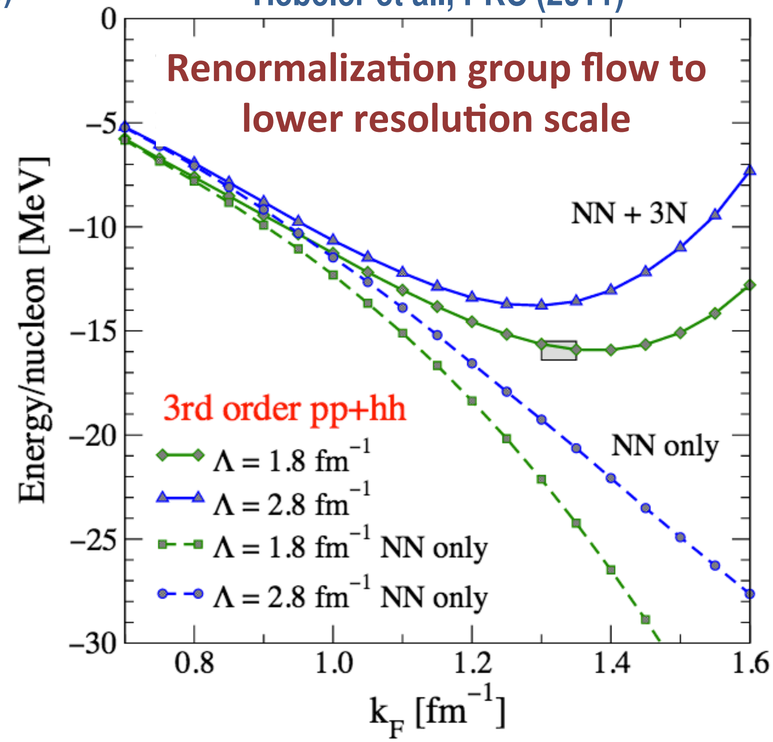
- Smaller scale dependence when consistent 2NF and 3NF employed

SATURATION OF SYMMETRIC NUCLEAR MATTER

Coraggio, Holt, Itaco, Machleidt, Marcucci & Sammarruca, PRC (2014)



Hebeler et al., PRC (2011)

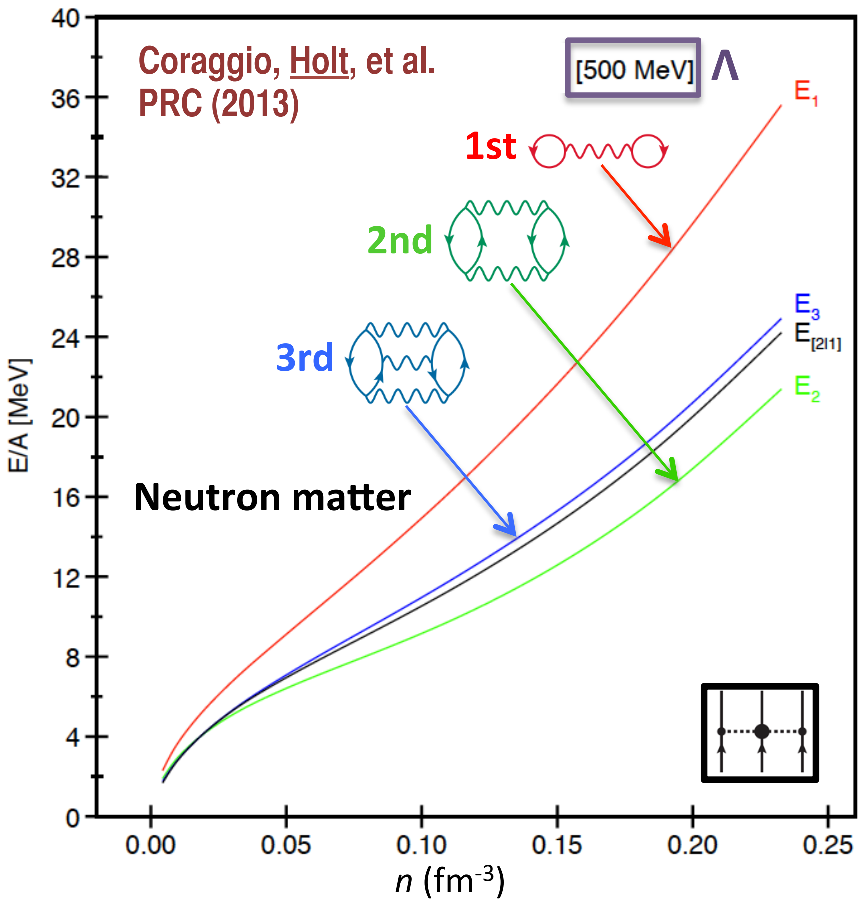


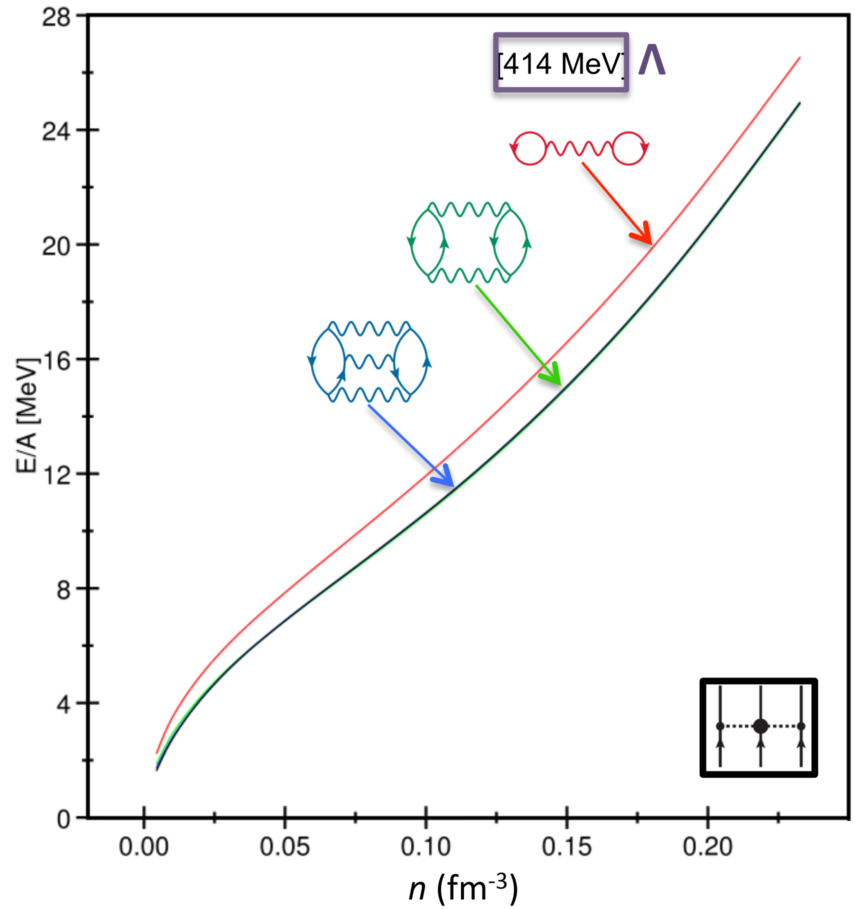
- Saturation energy: $E/A = -15.5 - 15.8$ MeV
- Saturation density: $\rho = 0.16 - 0.17\ \mathrm{fm}^{-3}$
- Asymmetry energy: $\beta = 31 - 33$ MeV
- Incompressibility: $\mathcal{K} = 220 - 240$ MeV

Good description of bulk nuclear matter properties

CHOICE OF Λ AND PERTURBATIVE NUCLEAR FORCES

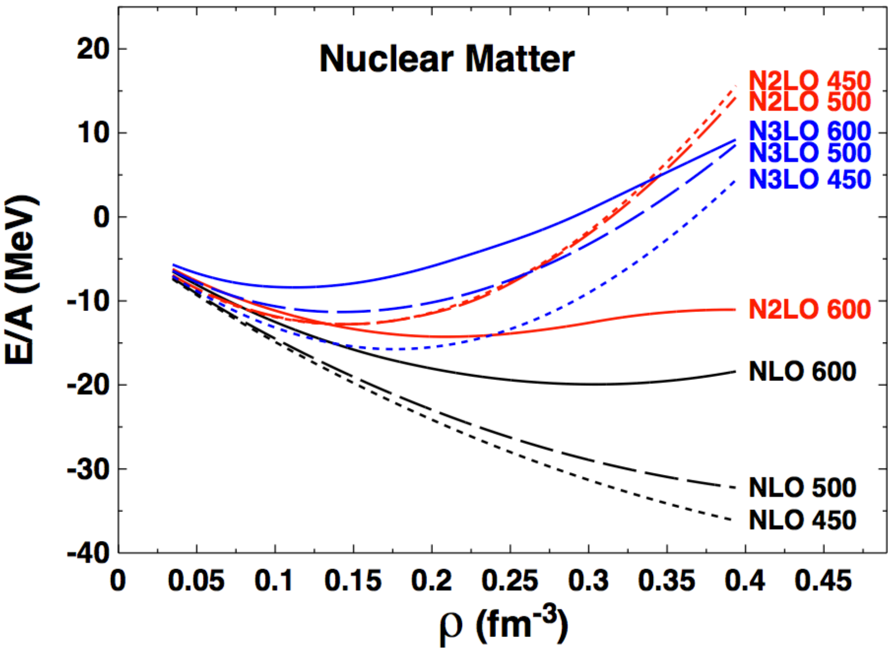
- Improved convergence in **many-body perturbation theory** with coarse-resolution chiral forces

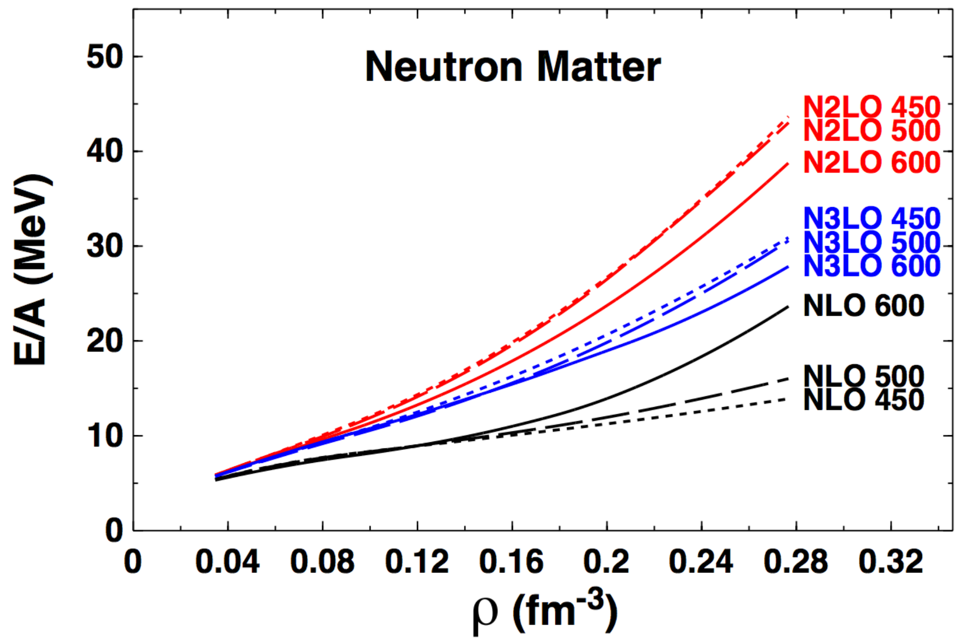




ORDER-BY-ORDER CONVERGENCE

Sammarruca, Coraggio, Holt, Itaco, Machleidt & Marcucci, PRC (2015)





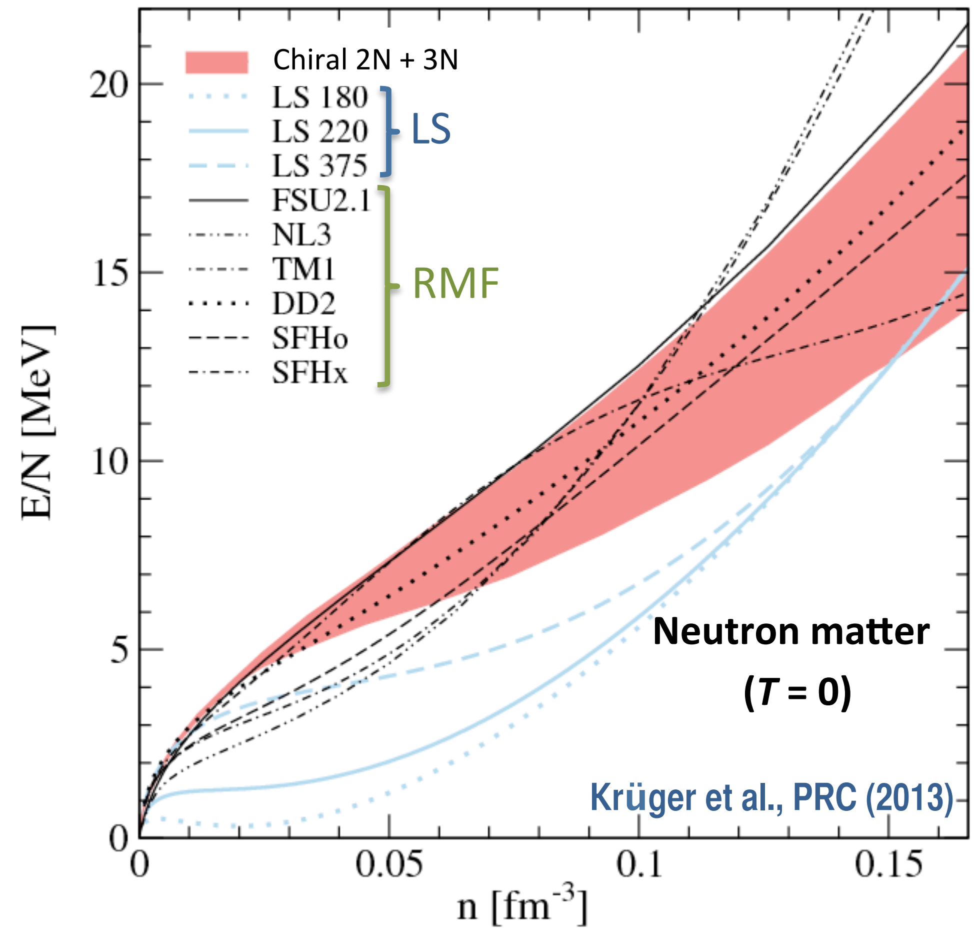
COMPARISON TO WIDELY USED EQUATIONS OF STATE

Lattimer & Swesty, 1991

- Skyrme + nonrelativistic liquid drop

Shen et al., 1998

- Relativistic mean field theory + Thomas-Fermi approximation
- Numerous other RMF equations of state

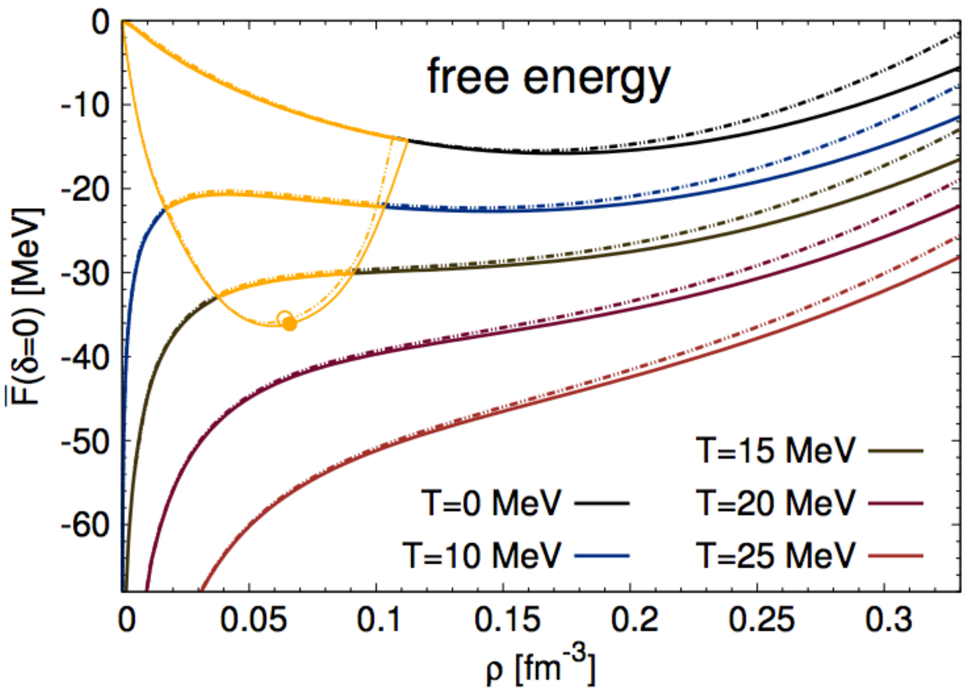


No supernova EoS is grounded in chiral EFT

FULL MICROSCOPIC TREATMENT

- Perturbation series of free-energy density in terms of grand canonical potential Ω

$$F(\mu_0,T) = F_0(\mu_0,T) + \lambda\Omega_1(\mu_0,T) + \lambda^2\left(\Omega_2(\mu_0,T) - \frac{1}{2}\frac{(\partial\Omega_1/\partial\mu_0)^2}{\partial^2\Omega_0/\partial\mu_0^2}\right) + \mathcal{O}(\lambda^3)$$



Wellenhofer, Holt, Kaiser & Weise, PRC (2014)

- All thermodynamic quantities derived from free energy, e.g., $P(\rho,T) = \rho^2 \frac{\partial \bar{F}(\rho,T)}{\partial \rho}$

LIQUID-GAS PHASE TRANSITION and THE CRITICAL POINT (CP)

Predicted critical endpoint

- Critical temperature:

$$T_c = 17.2 - 19.1\,\mathrm{MeV}$$

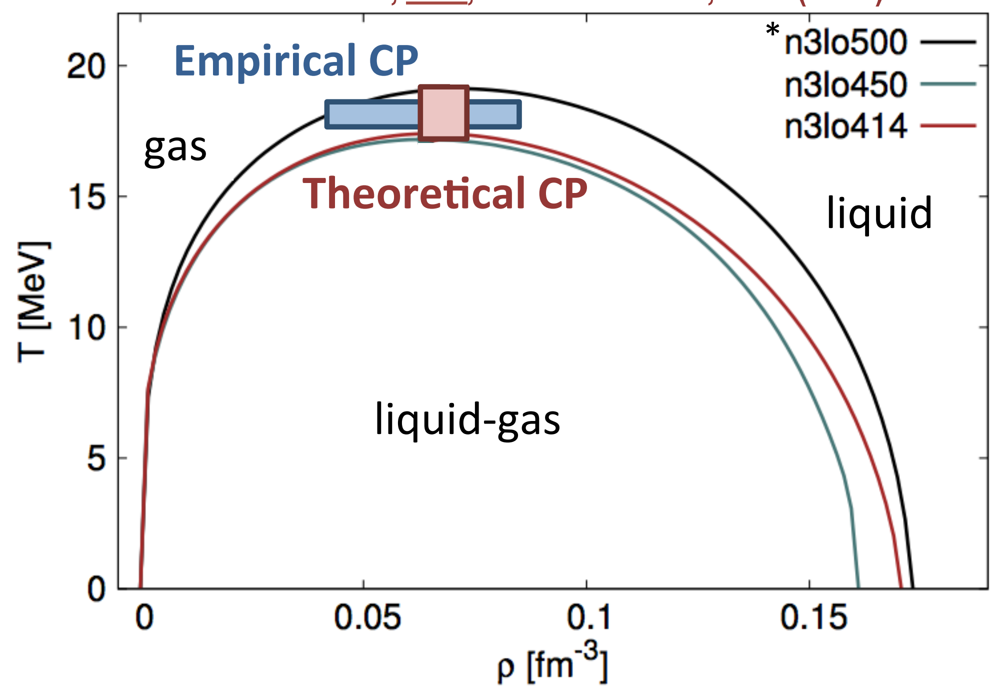
- Critical density:

$$\rho_c = 0.064 - 0.072\,\mathrm{fm}^{-3}$$

- Critical pressure:

$$P_c = 0.3 - 0.4\ \mathrm{MeV\ fm}^{-3}$$

Wellenhofer, Holt, Kaiser & Weise, PRC (2014)

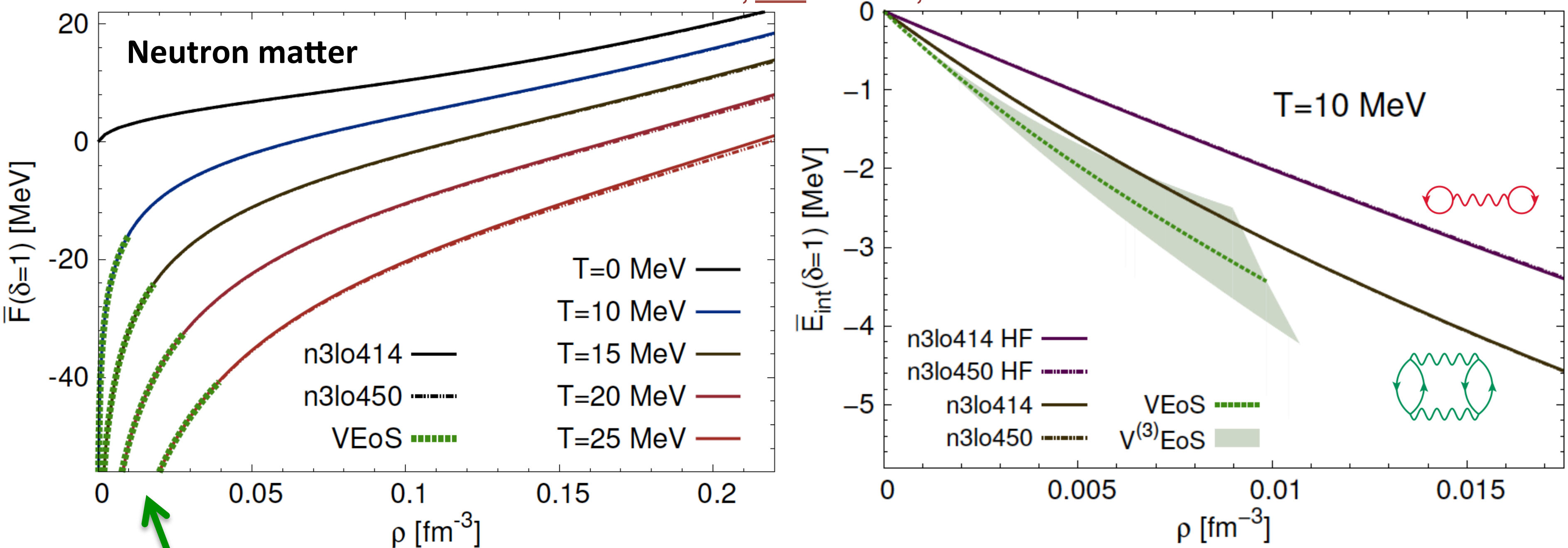


- Experiment (compound nucleus & multifragmentation) [J. B. Elliott et al., PRC (2013)]

$$T_c = 17.9 \pm 0.4\,\mathrm{MeV} \quad \rho_c = 0.06 \pm 0.02\ \mathrm{fm}^{-3} \quad P_c = 0.31 \pm 0.07\ \mathrm{MeV\ fm}^{-3}$$

FINITE TEMPERATURE NEUTRON MATTER EOS

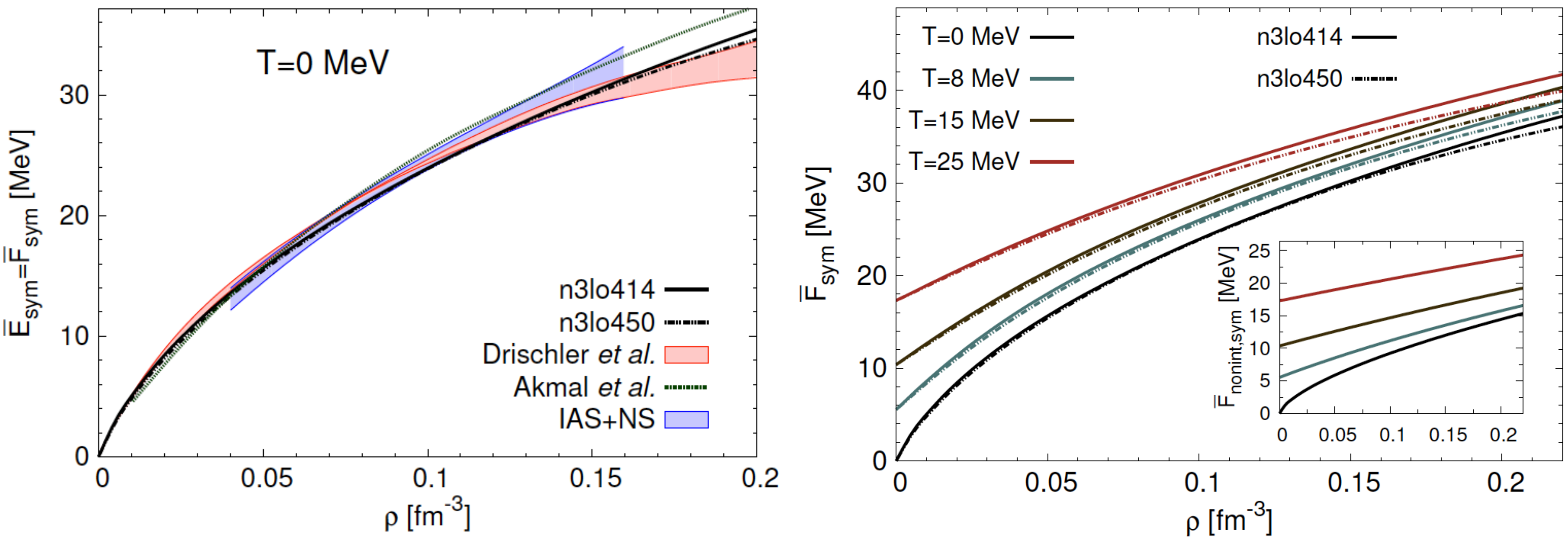
Wellenhofer, Holt & Kaiser, PRC 2015



Matches model-independent virial EoS at low densities and high temperatures

DENSITY-DEPENDENT SYMMETRY ENERGY

Wellenhofer, Holt & Kaiser, PRC 2015



- Slope of symmetry energy correlated with **neutron star radius** and **neutron skin thickness** in nuclei
- Density dependence of symmetry energy consistent with empirical constraints

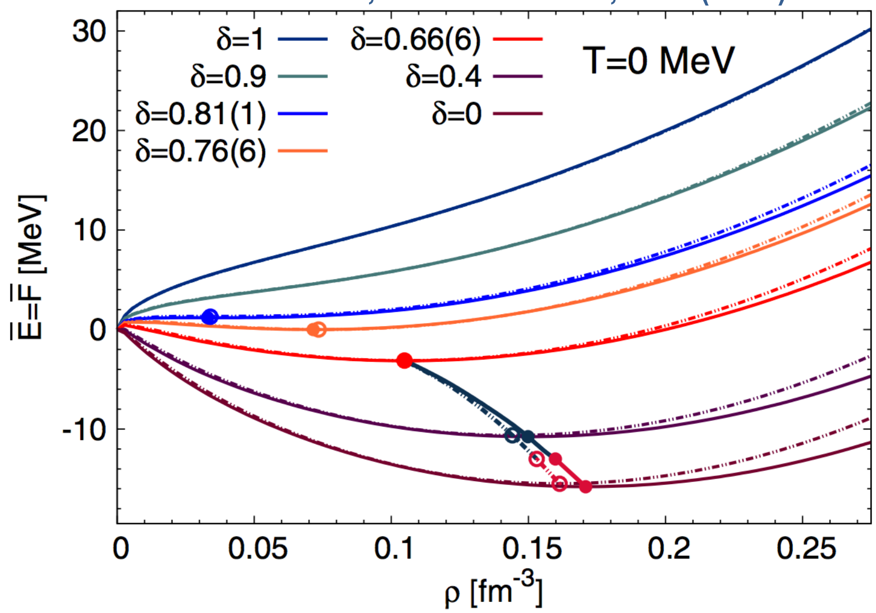
TERRESTRIAL EXPERIMENTS PROBE NEARLY SYMMETRIC MATTER

$$E(\rho, \delta) = A_0(\rho) + A_2(\rho)\delta^2 + \mathcal{O}(\delta^4)$$

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$A_0(\rho) = E_0 + \frac{1}{6}K\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \cdots$$

Wellenhofer, Holt and Kaiser, PRC (2015)



$$A_2(\rho) = J + \frac{1}{3}L\left(\frac{\rho - \rho_0}{\rho_0}\right) + \frac{1}{6}K_{sym}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \cdots$$

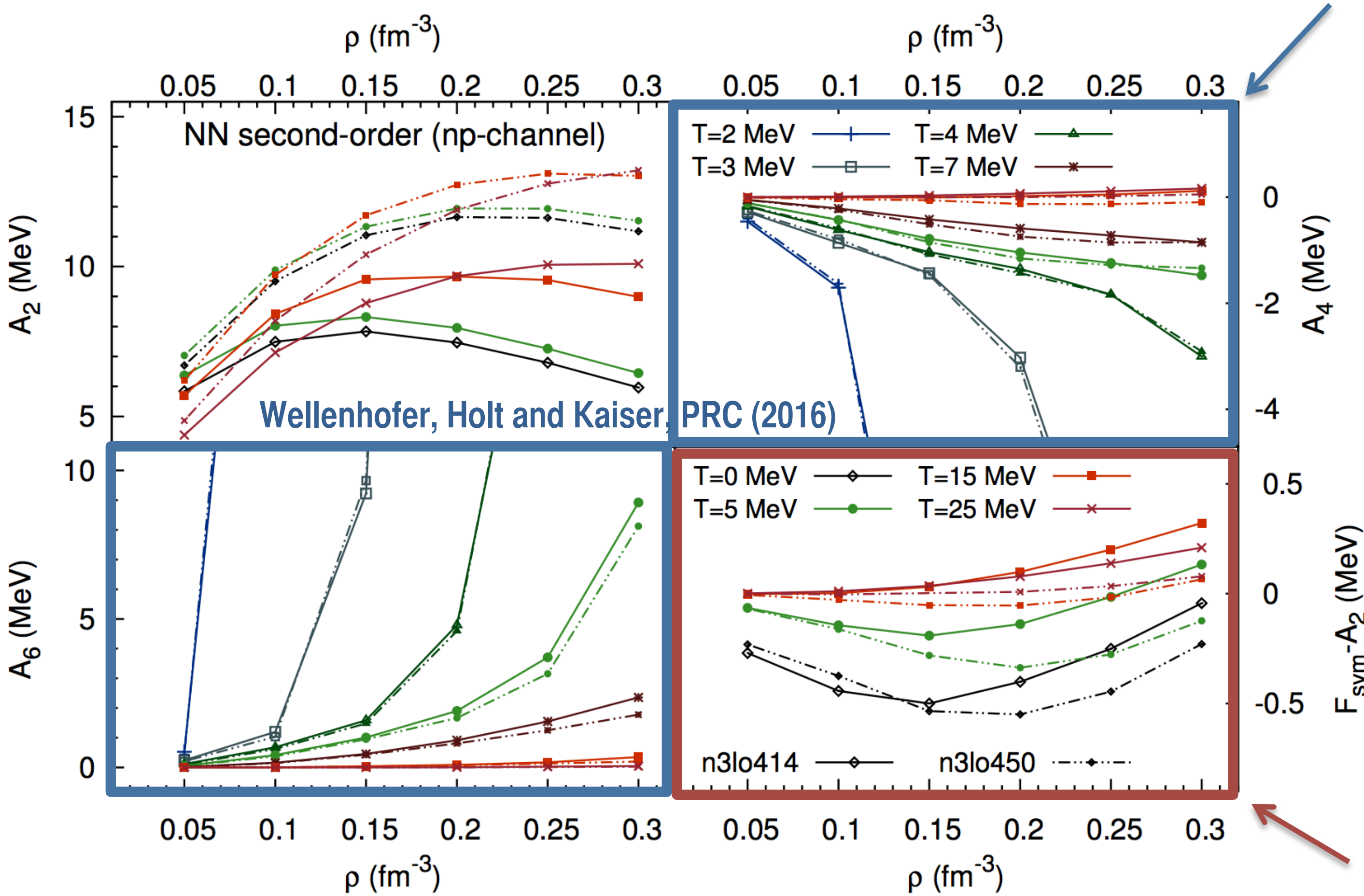
Role of higher-order δ^4 terms?

Crust-core transition density,...

NOVEL FEATURES AT SECOND-ORDER IN PERTURBATION THEORY

$$F(T,\rho,\delta) \simeq \sum_{n=0}^{N} A_{2n}(T,\rho)\,\delta^{2n}$$

Divergent expansion at low temperature



Sum of higher-order terms is finite

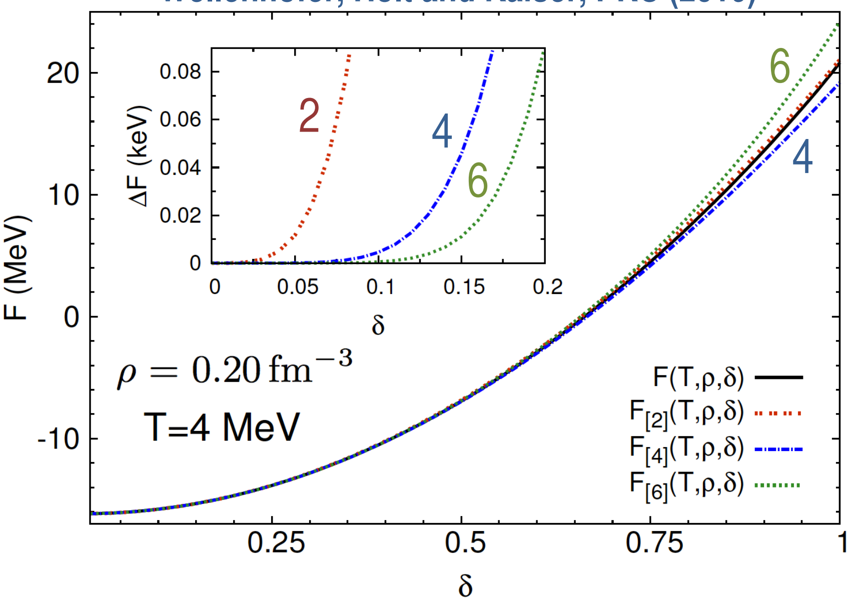
MODIFICATION OF ISOSPIN-ASYMMETRY EXPANSION

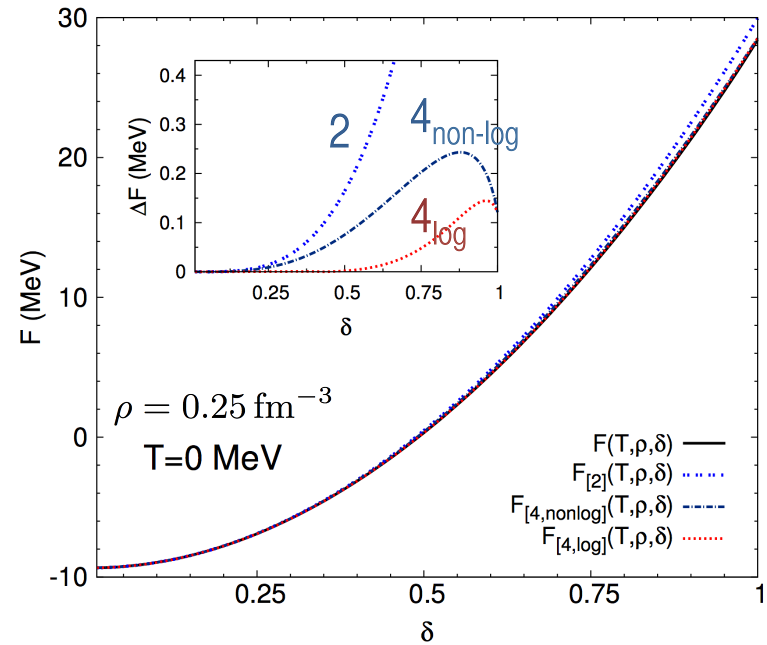
$$F(T=0,\rho,\delta) = A_0(T=0,\rho) + A_2(T=0,\rho)\,\delta^2 + \sum_{n=2}^{\infty} A_{2n,\mathrm{reg}}(\rho)\,\delta^{2n} + \sum_{n=2}^{\infty} A_{2n,\log}(\rho)\,\delta^{2n}\ln|\delta|$$

Kaiser, PRC 2015

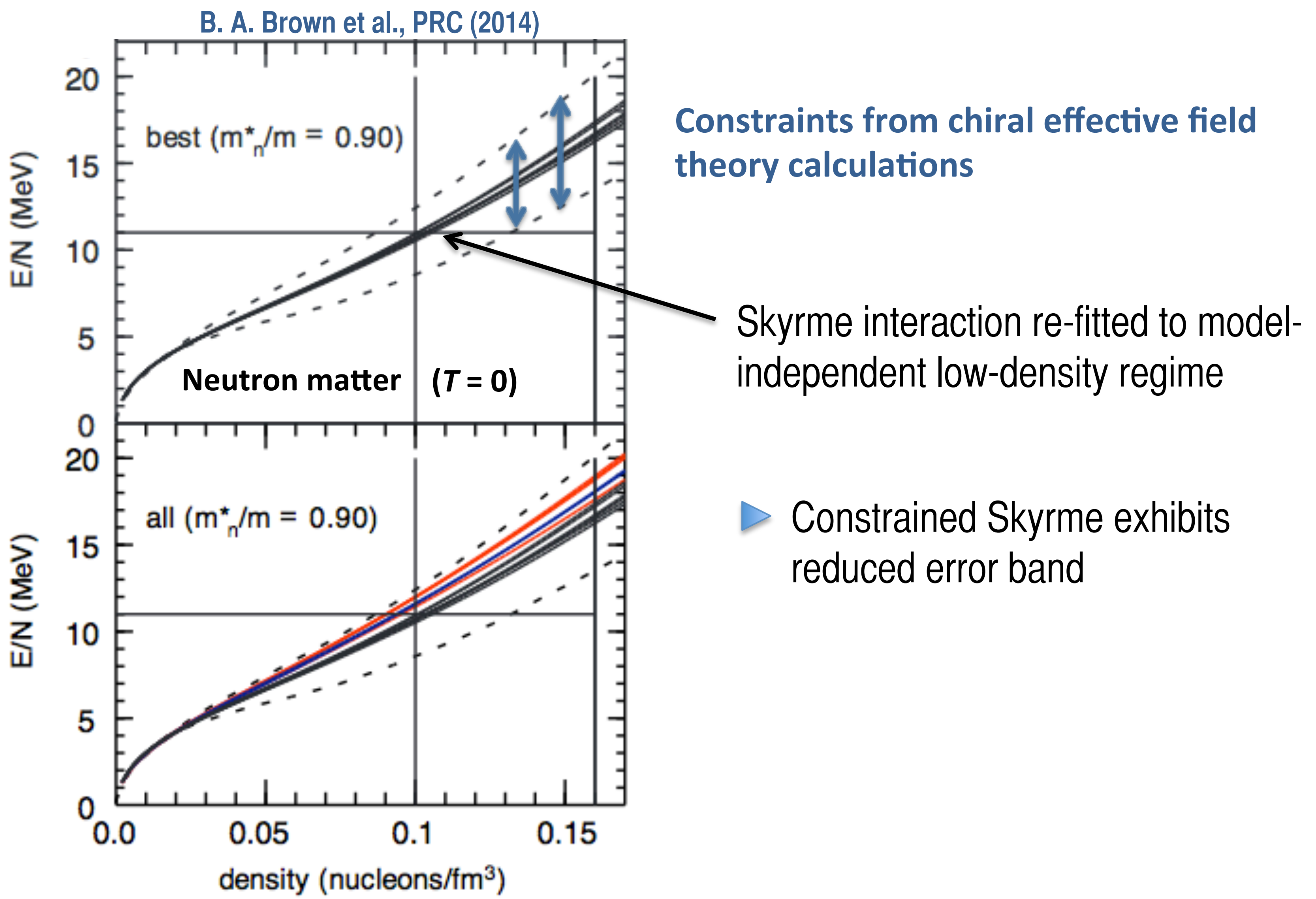
Logarithmic but finite

Wellenhofer, Holt and Kaiser, PRC (2016)





ALTERNATIVE: CONSTRAIN MEAN FIELD MODELS WITH CHIRAL EFT



- Constrained Skyrme exhibits reduced error band

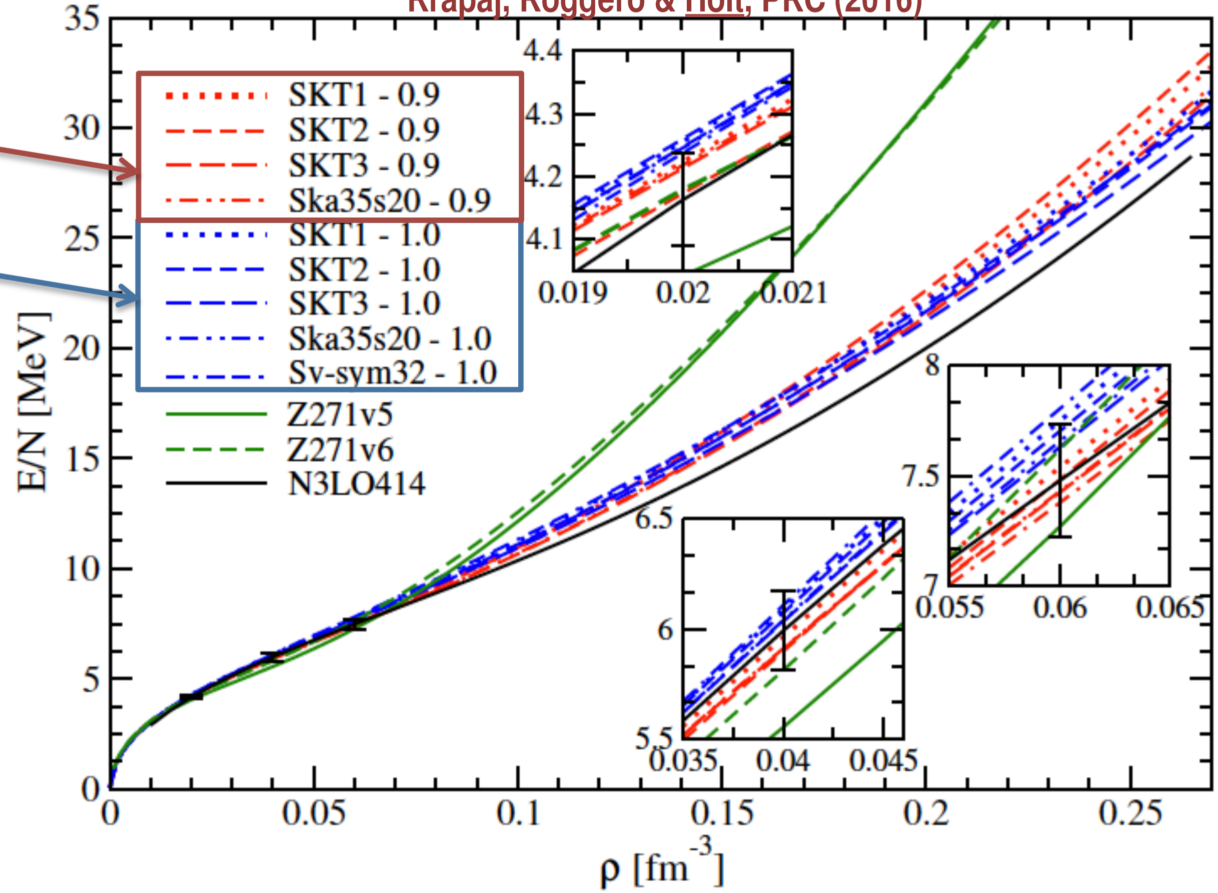
EXTRAPOLATE WITH MEAN FIELD MODELS

$$\frac{M^*}{M} = 0.9$$

$$\frac{M^*}{M} = 1.0$$

$$\left\{ \begin{array}{c} \text{Neutron matter} \\ \rho = 5/8\rho_0 \end{array} \right\}$$

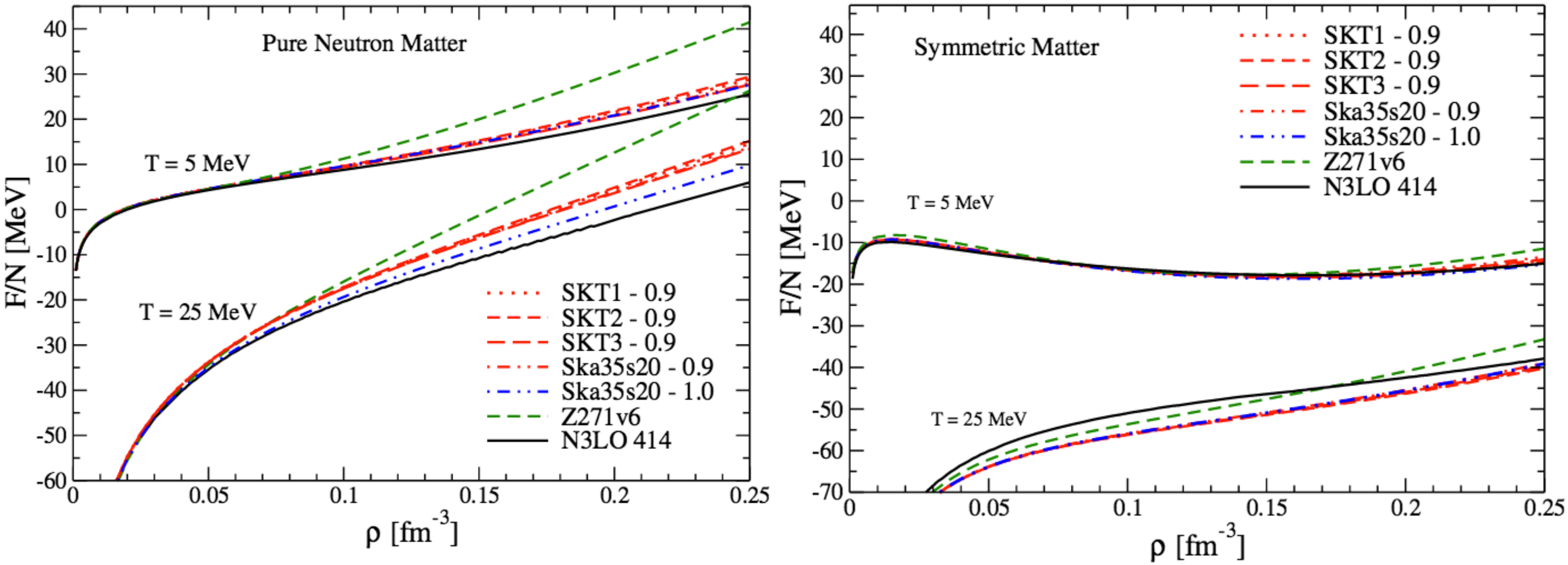
Rrapaj, Roggero & Holt, PRC (2016)



- Select Skyrme and RMF models consistent with low-density neutron matter EoS

CONSISTENT MEAN FIELD MODELS AT FINITE TEMPERATURE

Rrapaj, Roggero & Holt, PRC (2016)



- Larger discrepancies in free energy (coming from entropy contribution)

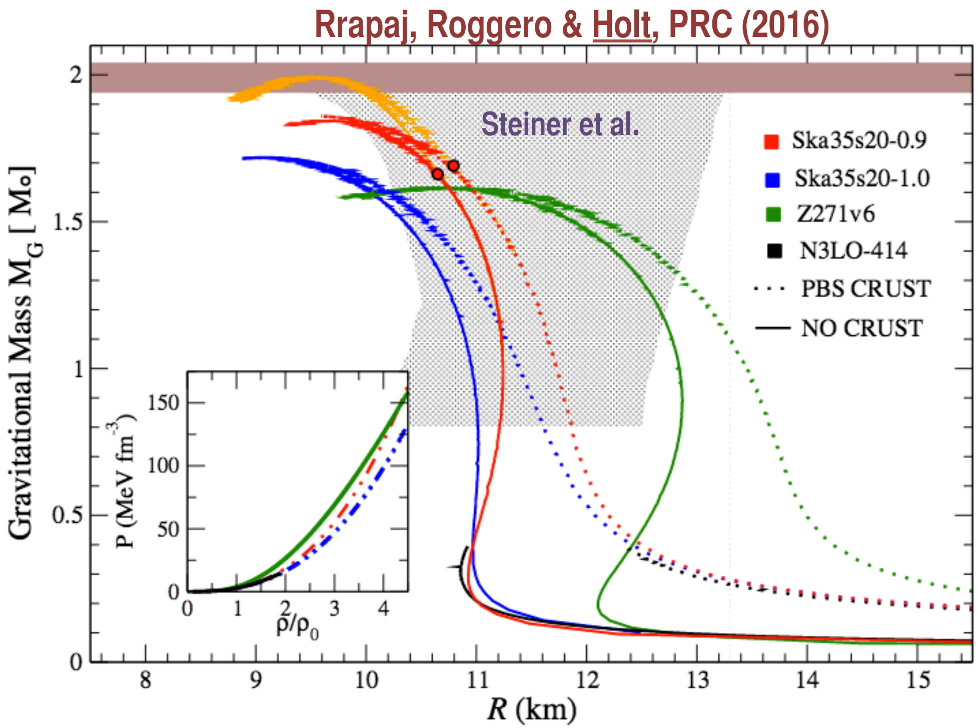
Density of states

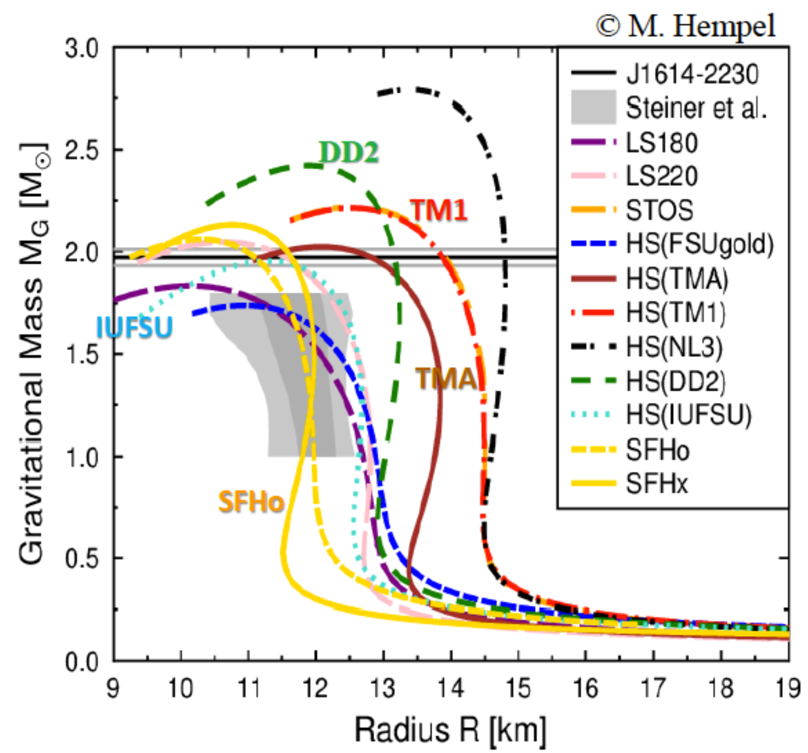
$$N(0) = \frac{1}{\pi^2}\sum_t M_t^* k_f^t$$

Entropy (low *T*)

$$\frac{S}{V} = \frac{T}{3}\sum_t M_t^* k_f^t$$

PREDICTED MASS VS. RADIUS FOR NEUTRON STARS

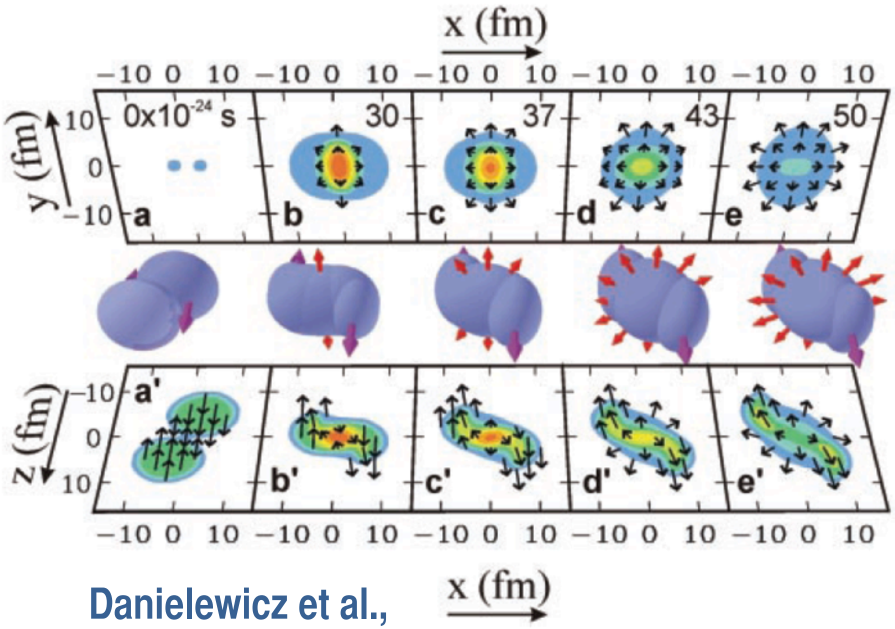




- Consistent Skyrme EoS generally soft (more powerful supernova shock waves and light r-process element production in NS mergers)
- Model-dependent extensions needed beyond $\rho > 4.5\rho_0$

HOW TO PROBE HOT/DENSE MATTER IN THE LAB?

- Intermediate-energy heavy ion collisions



Danielewicz et al., Science (2002)

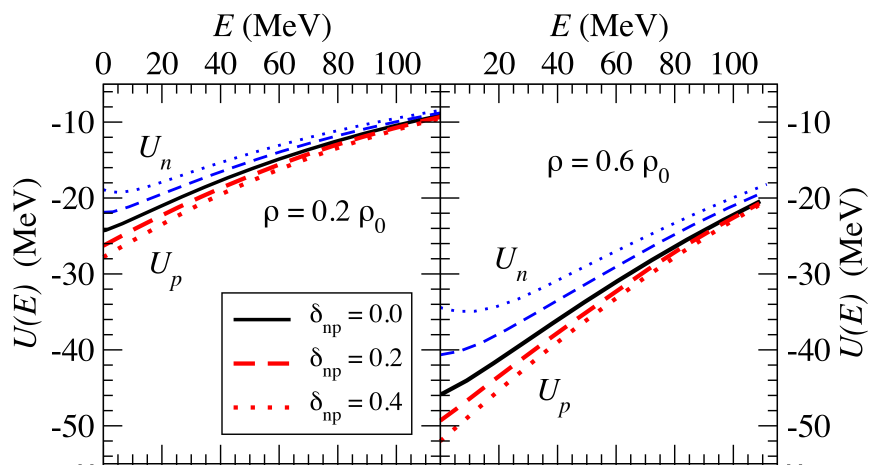
- Momentum-dependent nuclear mean field ε = *KE* + *U* from microscopic many-body theory

- Facilities: FRIB, SpiRIT, TAMU cyclotron, FAIR, SPIRAL2,...

- Observables: elliptic flow, transverse flow, fragment yields

- Model-dependent analysis with Boltzmann-like transport equation:

$$\frac{\partial f}{\partial t} + \nabla_p \varepsilon \cdot \nabla_r f - \nabla_r \varepsilon \cdot \nabla_p f = I$$



see [Holt et al., PRC (2013)]

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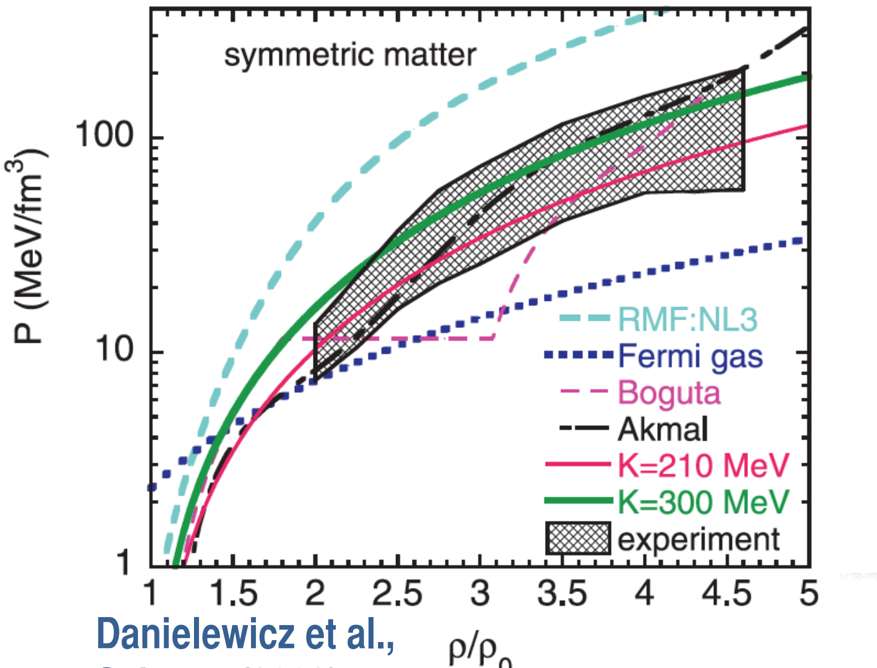

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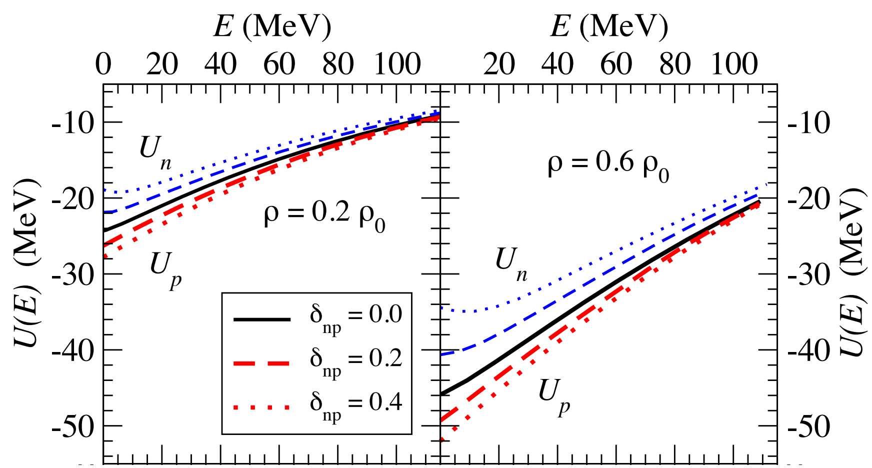


see [Holt et al., PRC (2013)]

GLOBAL OPTICAL POTENTIALS (PHENOMENOLOGICAL)

$$\mathcal{U}(r,E) = -\mathcal{V}_V(r,E) - i\mathcal{W}_V(r,E) - i\mathcal{W}_D(r,E) + \mathcal{V}_{SO}(r,E).\mathbf{l}.\sigma + i\mathcal{W}_{SO}(r,E).\mathbf{l}.\sigma + \mathcal{V}_C(r),$$

$$\mathcal{V}_V(r,E) = V_V(E) f(r, R_V, a_V),$$

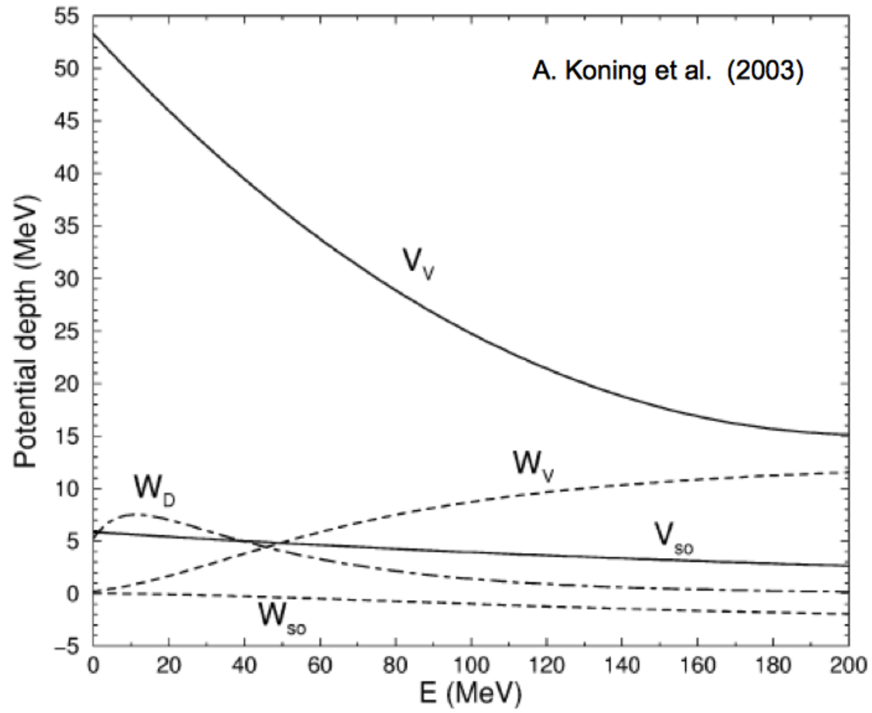
$$\mathcal{W}_V(r,E) = W_V(E) f(r, R_V, a_V),$$

$$\mathcal{W}_D(r,E) = -4a_D W_D(E) \frac{d}{dr} f(r, R_D, a_D),$$

$$\mathcal{V}_{SO}(r,E) = V_{SO}(E) \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr} f(r, R_{SO}, a_{SO}),$$

$$\mathcal{W}_{SO}(r,E) = W_{SO}(E) \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr} f(r, R_{SO}, a_{SO}).$$

$$f(r, R_i, a_i) = \left(1 + \exp[(r - R_i)/a_i]\right)^{-1}$$



$$V_V(E) = v_1\left[1 - v_2(E - E_f) + v_3(E - E_f)^2 - v_4(E - E_f)^3\right]$$

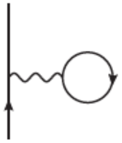
$$W_V(E) = w_1 \frac{(E - E_f)^2}{(E - E_f)^2 + (w_2)^2},$$

Energy dependence

MICROSCOPIC OPTICAL POTENTIALS (HOMOGENEOUS MATTER)

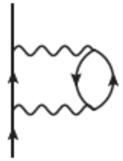
- Identified with the on-shell nucleon self-energy $\Sigma(\vec{r}_1, \vec{r}_2, \omega)$ **[Bell and Squires, PRL (2009)]**

- Hartree-Fock contribution (real, energy-independent):



$$\Sigma^{(1)}_{2N}(q; k_f) = \sum_1 \langle \vec{q}\vec{h}_1 s s_1 t t_1 | \bar{V}_{2N} | \vec{q}\vec{h}_1 s s_1 t t_1 \rangle n_1$$

- Second-order perturbative contibutions (complex, energy-dependent):

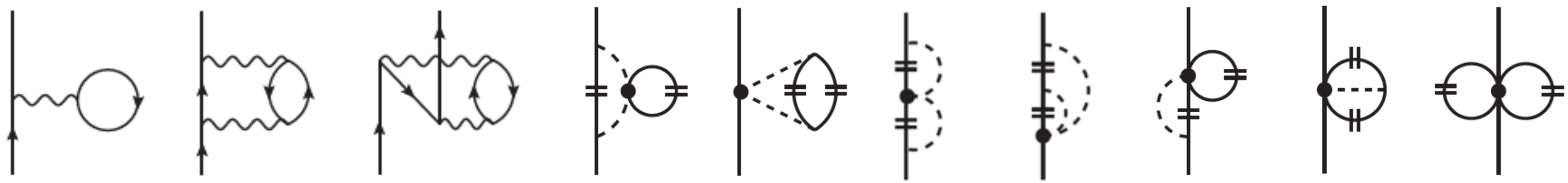


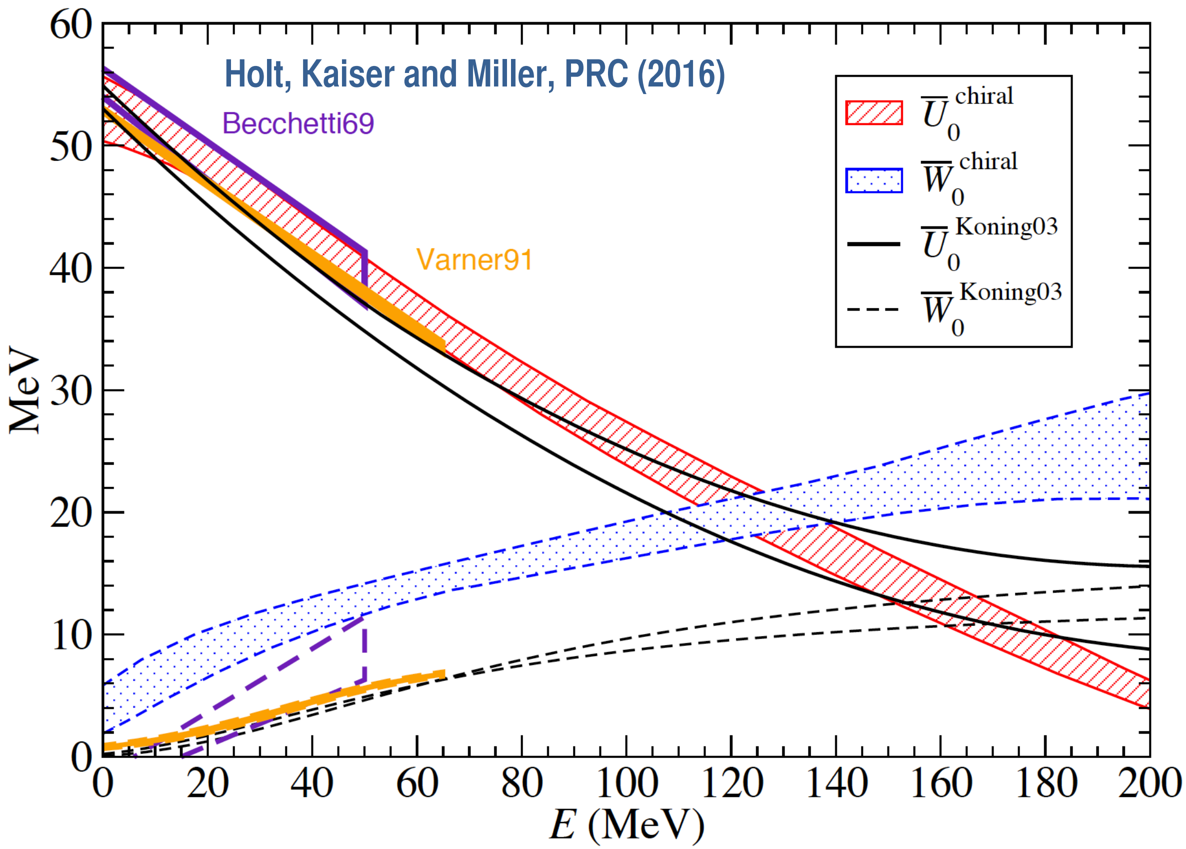
$$\Sigma^{(2a)}_{2N}(q, \omega; k_f) = \frac{1}{2}\sum_{123} \frac{|\langle \vec{p}_1 \vec{p}_3 s_1 s_3 t_1 t_3 | \bar{V} | \vec{q}\vec{h}_2 s s_2 t t_2 \rangle|^2}{\omega + \epsilon_2 - \epsilon_1 - \epsilon_3 + i\eta} \bar{n}_1 n_2 \bar{n}_3 (2\pi)^3 \delta(\vec{p}_1 + \vec{p}_3 - \vec{q} - \vec{h}_2)$$

Benchmarks:

- Depth and energy dependence of phenomenological volume parts (including isospin dependence)

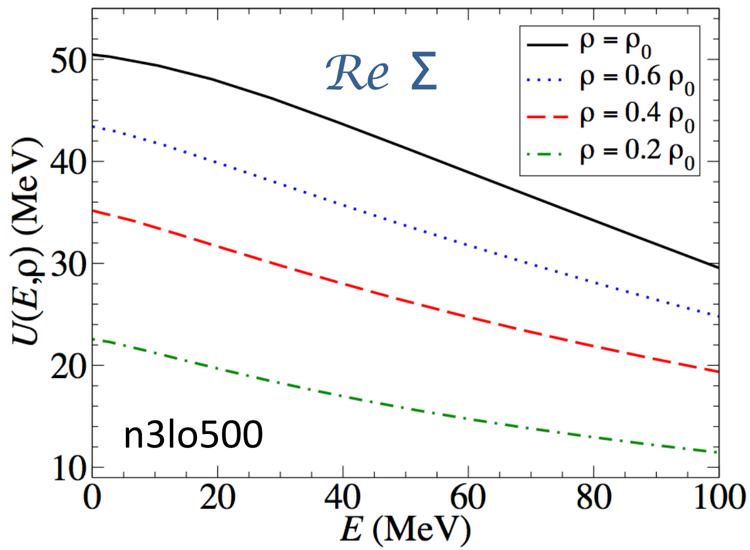
BENCHMARKS IN ISOSPIN-SYMMETRIC MATTER

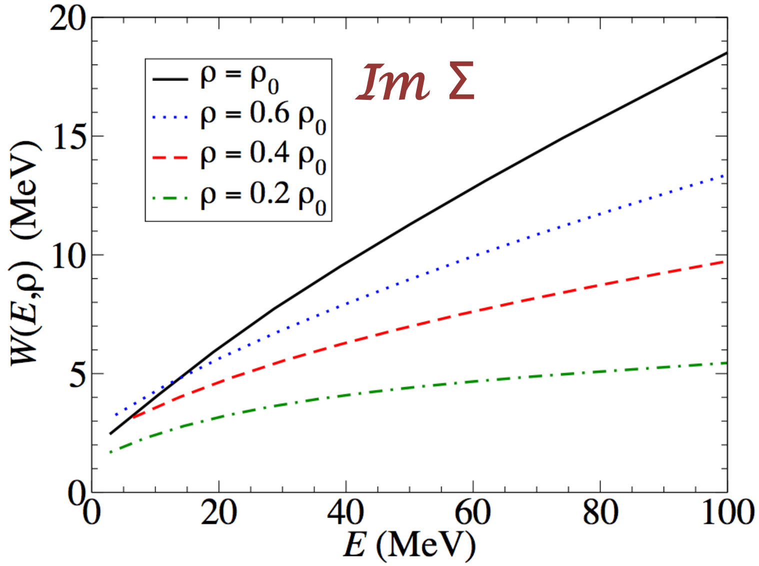




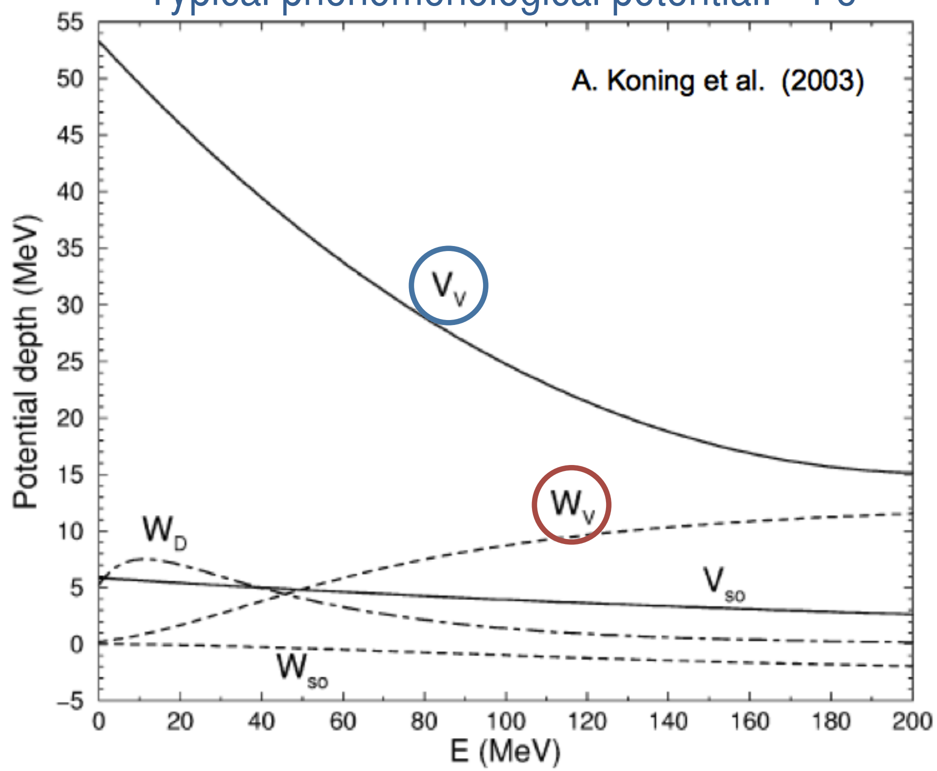
DENSITY DEPENDENCE OF REAL AND IMAGINARY PARTS

Holt, Kaiser, Miller & Weise, PRC (2013)





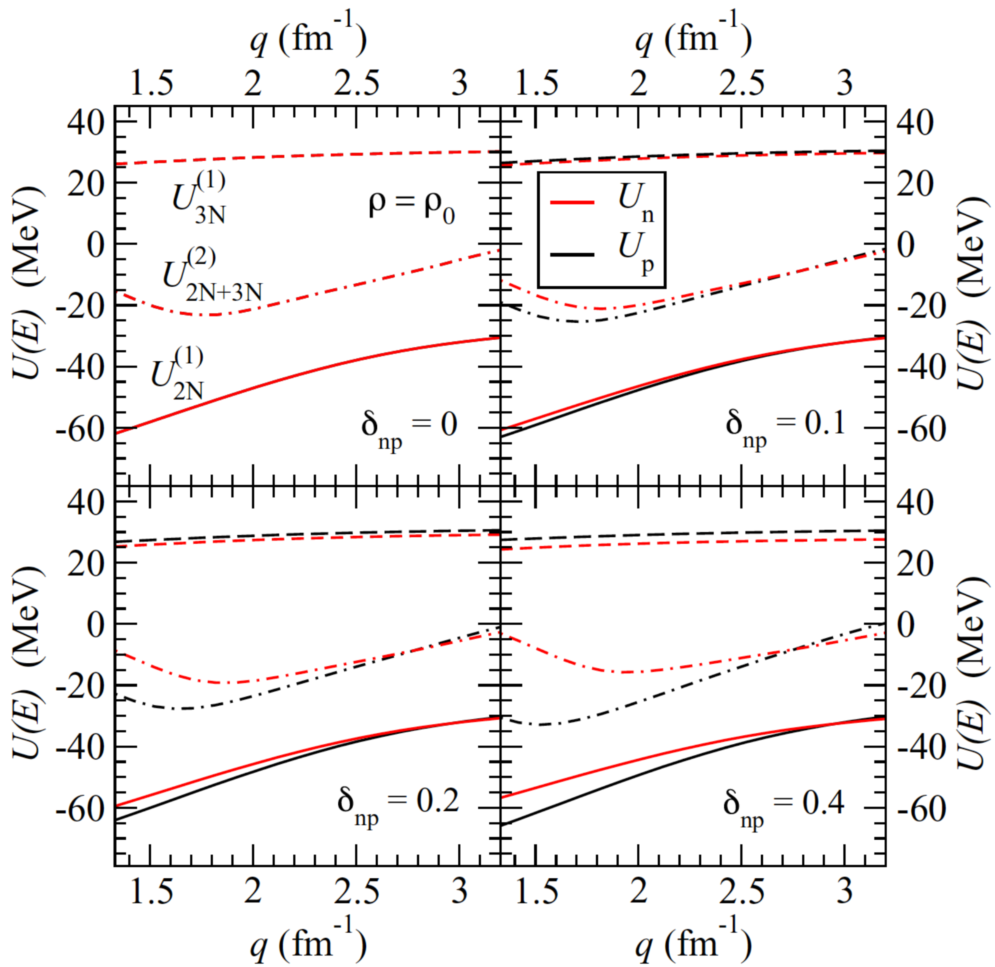
Typical phenomenological potential: ^{56}Fe



RELATIVE STRENGTH OF PERTURBATIVE CONTRIBUTIONS

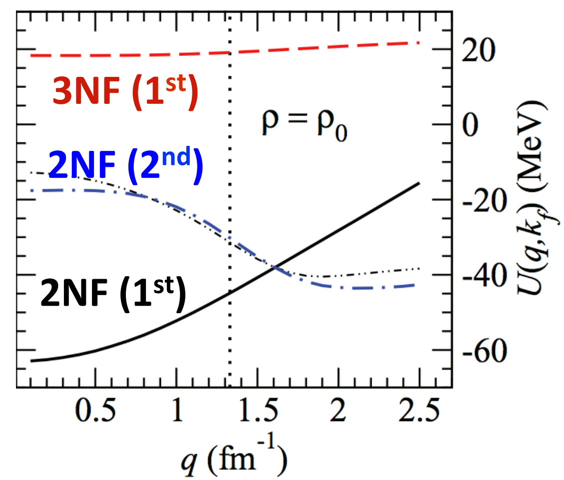
Holt, Kaiser and Miller, PRC (2016)

Λ = *450* MeV



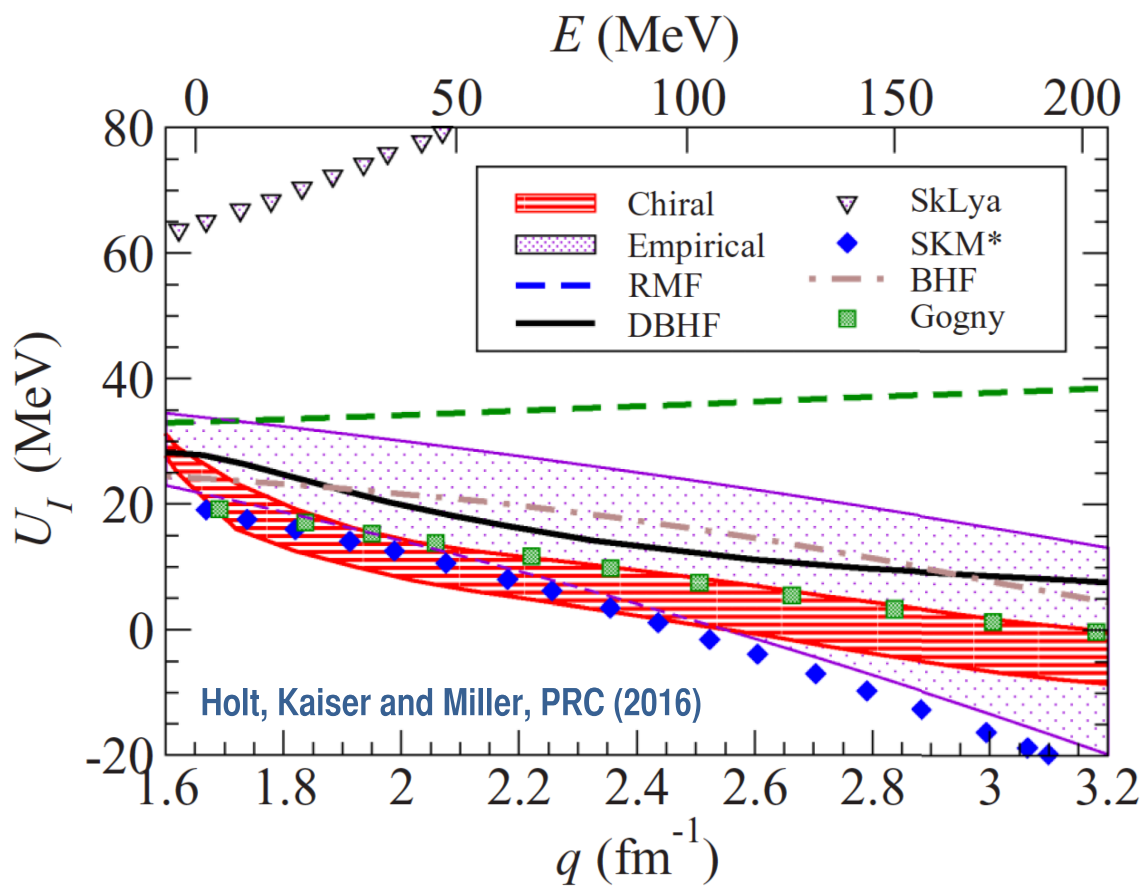
Holt, Kaiser, Miller
and Weise, PRC (2013)

Λ = *500* MeV



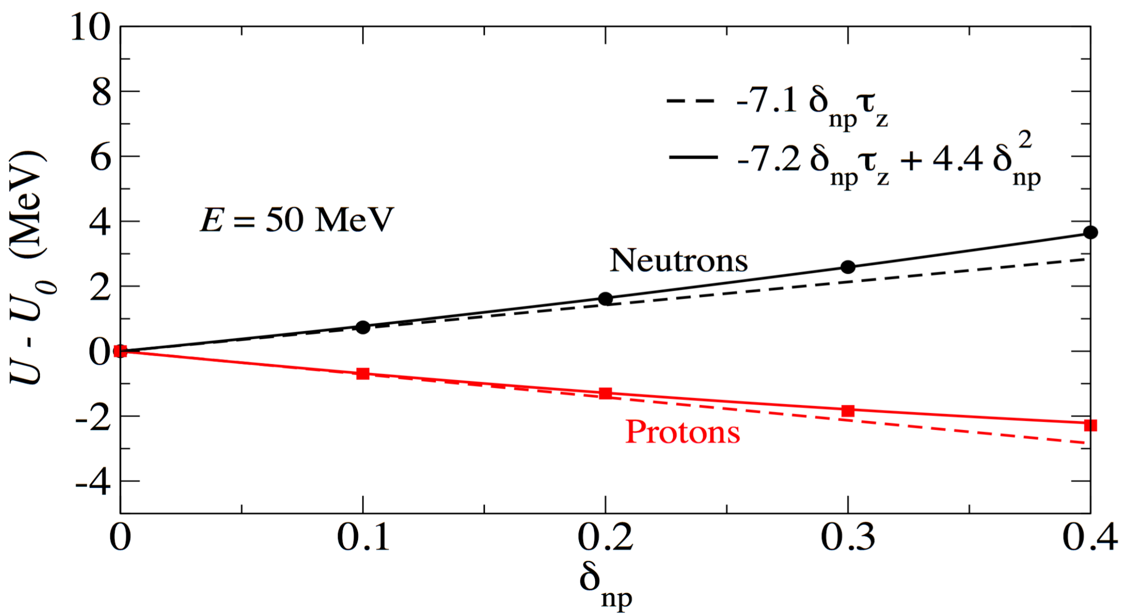
ISOVECTOR REAL OPTICAL POTENTIAL FROM CHIRAL EFT

$$U = U_0 - U_I \delta_{np} \tau_3$$

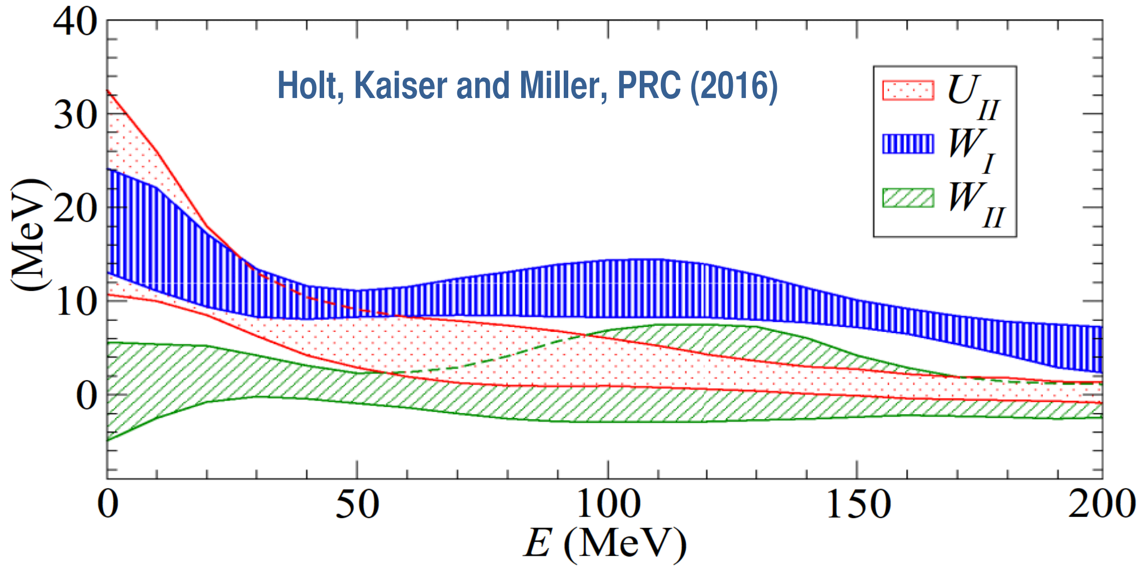


- Chiral EFT prediction consistent with broad empirical constraints

VALIDITY OF LANE APPROXIMATION



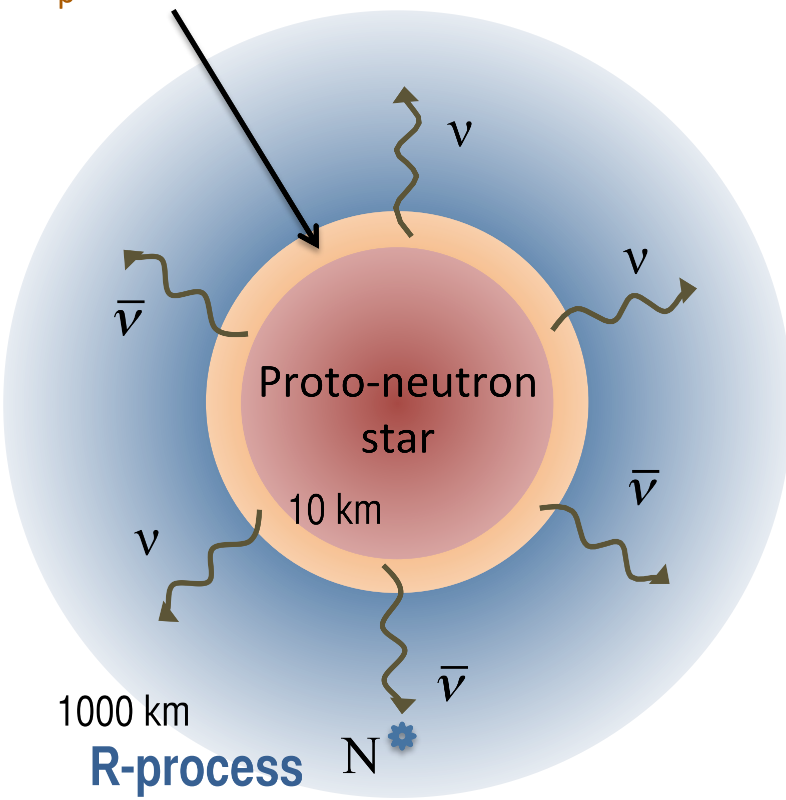
Real part has quadratic isoscalar contributions at low energies



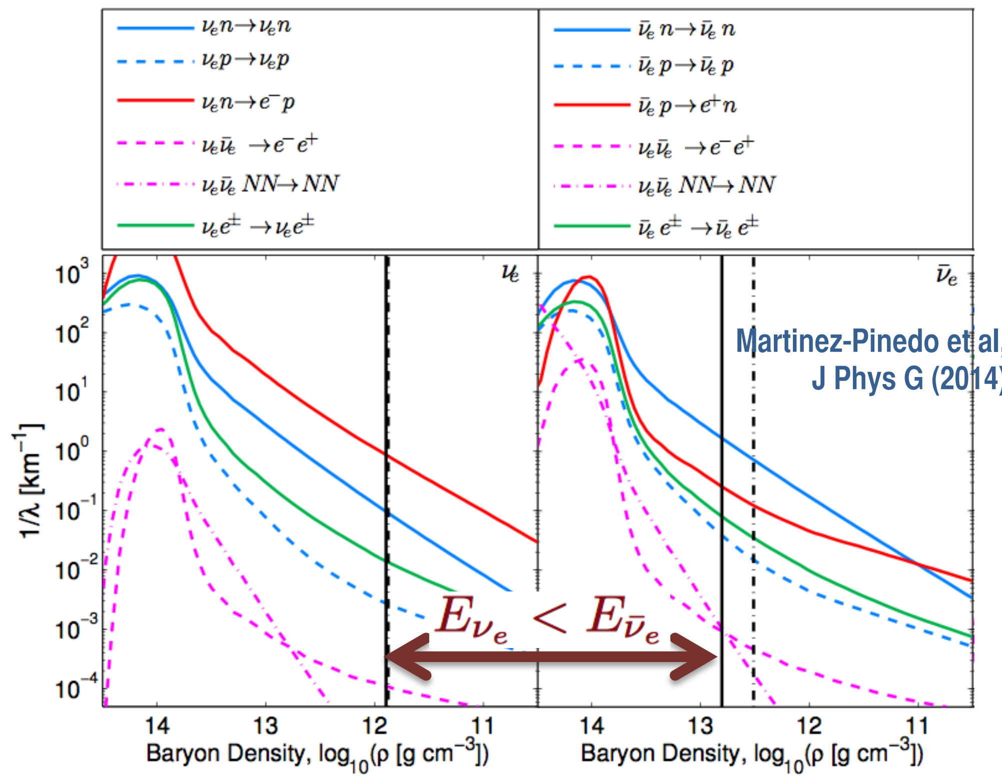
Imaginary part almost perfectly linear in isospin asymmetry

SUPERNOVA R-PROCESS AND THE NEUTRINOSPHERE

Neutrinosphere
T = 4 – 8 MeV,
$\rho = 10^{11} - 10^{13}$ g/cm^3,
$Y_p \sim 0.05 - 0.10$



Neutrino opacity	Anti-neutrino opacity
$\nu_e + n \longleftrightarrow e^- + p$	$\bar{\nu}_e + n \longrightarrow \bar{\nu}_e + n$



Governs energies of free-streaming neutrinos

NUCLEAR MEAN FIELDS AND CHARGED-CURRENT REACTIONS

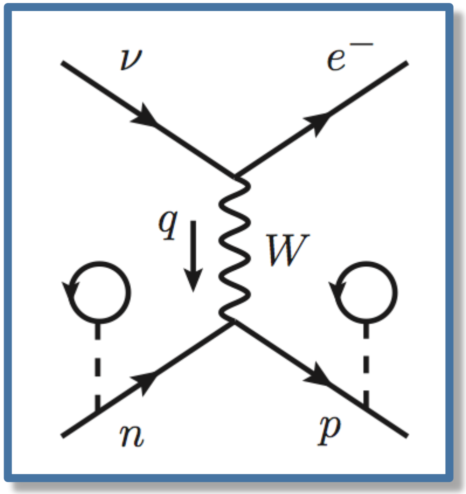
Neutrino-antineutrino spectral difference crucial for nucleosynthesis

$$\nu_e + n \longleftrightarrow e^- + p$$
$$\bar{\nu}_e + p \longleftrightarrow e^+ + n$$

Set proton fraction in region of r-process

$$N_p \lesssim 0.4$$
$$\langle E_{\bar{\nu}_e} \rangle - \langle E_{\nu_e} \rangle > 4(m_n - m_p)$$

Robust r-process



Nuclear mean fields enhance neutrino absorption

Skyrme & RMF calculations: Martinez-Pinedo et al, PRL (2012); Roberts et al, PRC (2012)

Resonant nucleon-nucleon interactions may enhance effect ($a_{nn} = -18\,\mathrm{fm}$)

MEAN FIELD EFFECTS ON NEUTRINO ABSORPTION CROSS SECTION

$$\frac{1}{V}\frac{d^2\sigma}{d\cos\theta\, dE_e} = \frac{G_F^2\cos^2\theta_C}{4\pi^2} |\vec{p}_e|\, E_e\,(1 - f_e(\xi_e))$$

Electron phase space

$$\times\left[(1+\cos\theta)S_\tau(q_0,q) + g_A^2(3-\cos\theta)S_{\sigma\tau}(q_0,q)\right]$$

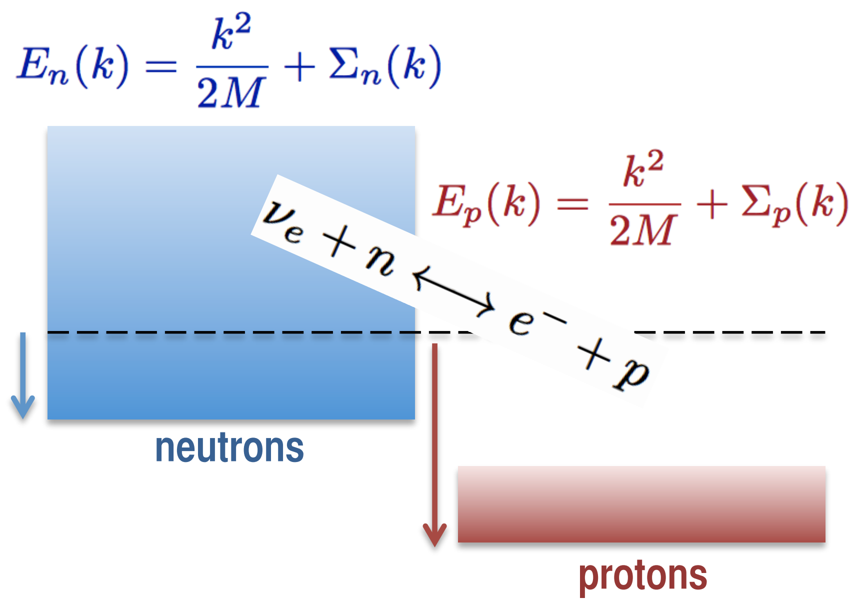
Nucleon response

- Nuclear interactions attractive at low momenta and

  $$|\langle np|V_{NN}|np\rangle| > |\langle nn|V_{NN}|nn\rangle|$$

- Mean field effects further **widen the energy gap** between protons and neutrons

- **Q-value** for neutrino absorption changes significantly



PHASE SPACE CONSIDERATIONS

Charged-current reactions ($\nu_e n \to e^- p$) with $E_\nu = 10\ \mathrm{MeV},\ p_n = 100\ \mathrm{MeV}$

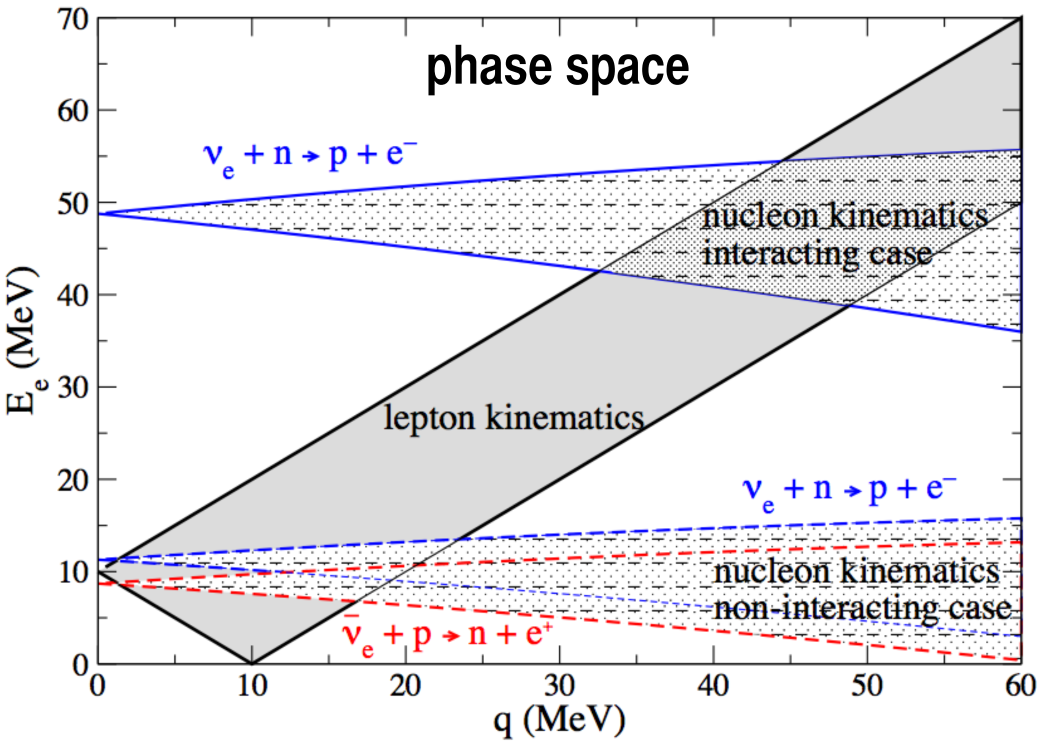
$$E_e = \sqrt{E_\nu^2 - 2E_\nu q\cos\theta + q^2 + m_e^2}$$

lepton

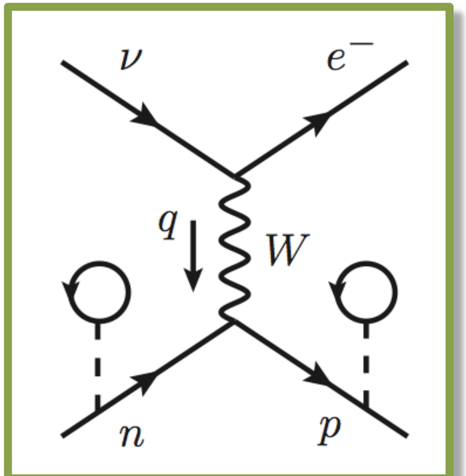
nucleon

$$E_e = E_\nu + (E_n - E_p) = E_\nu - \frac{1}{2\bar{M}}(q^2 + 2p_N q\cos\theta) + (M_n - M_p)$$

kinematic regions



Mean-field effects



MODELING RESONANT NN INTERACTIONS AT LOW DENSITIES

Virial expansion
Horowitz & Schwenk (2006)

- Equation of state and neutrino response for low-density, high-temperature matter

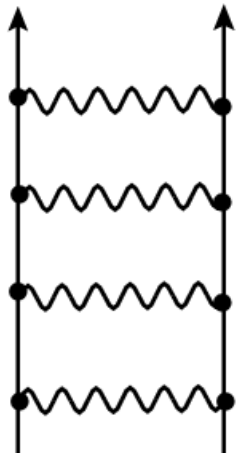
Many-body perturbation theory with chiral forces

- Leading Hartree-Fock contribution likely too weak
- Second-order perturbation theory may be sufficient (work in progress…)

Nuclear pseudo-potential:

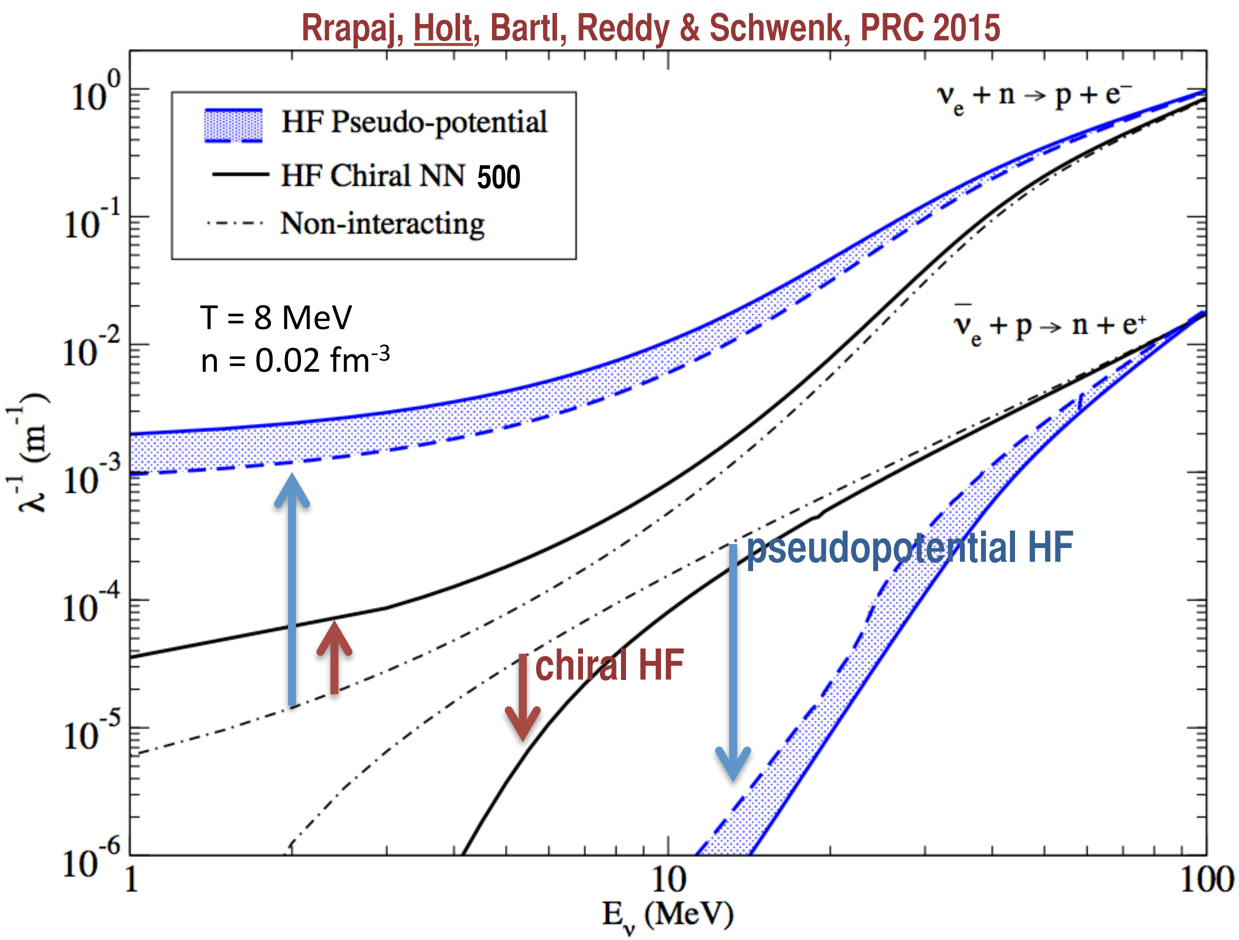
$$\langle p | V^{pseudo}_{llSJ} | p \rangle = -\frac{\delta_{lSJ}(p)}{p M_N}$$

Fumi (1955), Fukuda & Newton (1956)

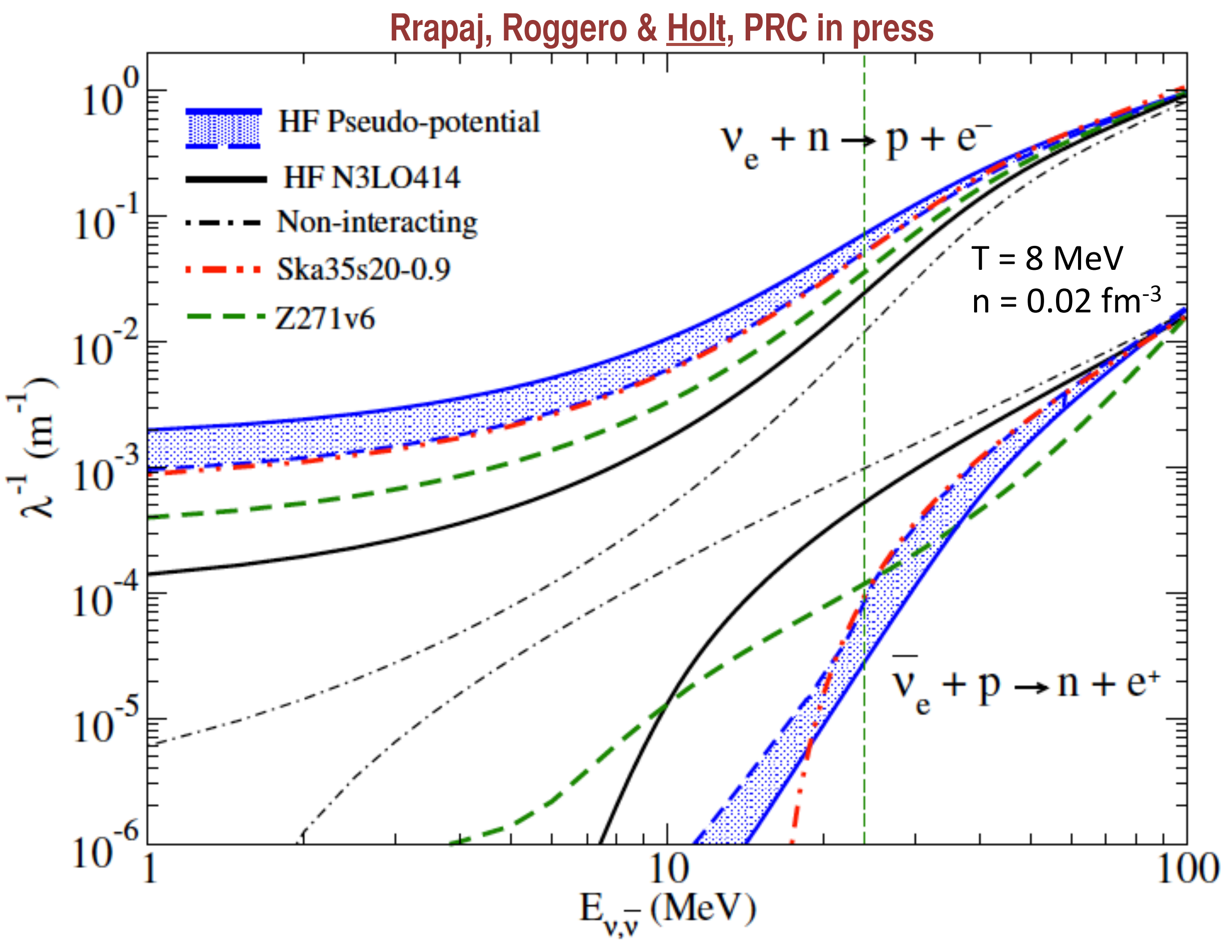
$\sim$ 

- Designed to reproduce **exact energy shift** when used at the mean field level (valid for low-density matter)

NEUTRINO MEAN FREE PATHS



COMPARISON WITH MEAN FIELD MODELS



SUMMARY & FUTURE WORK

Nuclear equation of state for astrophysical simulations

- Mean field extrapolations of chiral EFT to explore high-temperature, high-density regime
- Clustering and the low-density mixed phase
- Equation of state tables (for core-collapse supernovae) in progress

Single-particle energies

- Tabulate or parametrize in a form suitable for transport simulations of heavy-ion collisions
- Extend second-order calculations to finite temperature for neutrinosphere applications