QED, Neutron Stars, X-ray Polarimetry

Jeremy Heyl

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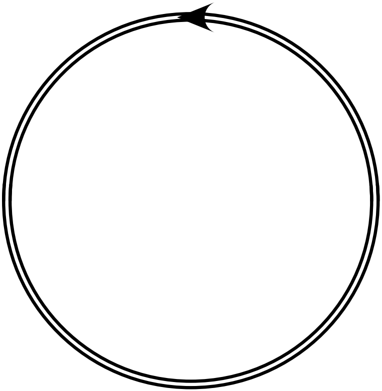
Ilaria Caiazzo; Yoram Lithwick, Don Lloyd, Dan Mazur, Nir Shaviv; Roberto Turolla, Roberto Taverna; Wynn Ho, Dong Lai, Rosalba Perna and others.

Effective Action



For a magnetic field the effective action is the free energy of the system (actually minus the free energy).

$$\Gamma[A^0_\mu] = \int \mathrm{d}x^4 \left(-\frac{1}{4} F^0_{\mu\nu} F^{0,\mu\nu} \right) - i\hbar \mathrm{Tr} \ln \left[\frac{\not{\Pi} - m}{\not{p} - m} \right]$$

The QED Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{\hbar}{8\pi^2} B_k^2 \int_0^\infty \frac{d\zeta}{\zeta} e^{-i\zeta} \left[\frac{ab}{B_k^2} \coth\left(\zeta \frac{a}{B_k}\right) \cot\left(\zeta \frac{b}{B_k}\right) - \mathrm{CT} \right]$$

where

$$(b - ia)^2 = (\mathbf{B} - i\mathbf{E})^2 = |\mathbf{B}|^2 - |\mathbf{E}|^2 - 2i\mathbf{E}\cdot\mathbf{B}$$

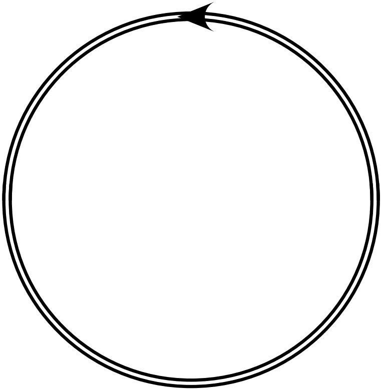
$$\left[2(b-ia)^2\right] = F^{\mu\nu}F_{\mu\nu} + i\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} \equiv I + iJ$$

and

$$\mathrm{CT} = \frac{1}{\zeta^2} + \frac{1}{3}\frac{a^2 - b^2}{B_k^2}(a^2 - b^2)$$

Heisenberg-Euler, Weisskopf, Schwinger

What do the poles mean?



- The poles along the real axis lie at

$$\zeta \frac{B}{B_k} = \zeta \frac{\omega_B}{\frac{mc^2}{\hbar}} = \tau \omega_B = n\pi$$

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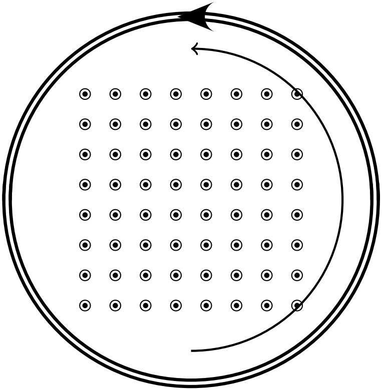

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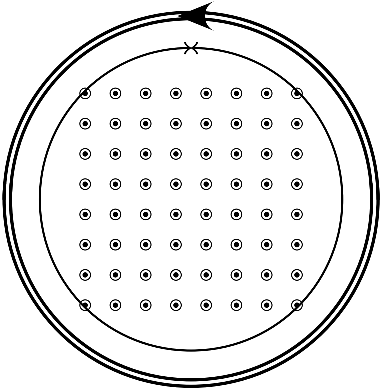

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- Take $n = 1$. After a proper time of $\tau = n\pi/\omega_B$, an electron performs half a revolution.
- In the same proper time, a positron does the same.

Field and Photons

To understand the interaction of light with the magnetized vacuum, we imagine expanding the action for a uniform field plus a small photon field,

$$\mathbf{E} = \mathbf{E}_0 + \delta\mathbf{E}, \mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}, F^{\mu\nu} = (F_0)^{\mu\nu} + f^{\mu\nu}.$$

We have two possibilities.

1. $k\lambda\!\!\!^{-}_e \ll 1$: we pretend that the photon field is also uniform and expand the effective Lagrangian density.

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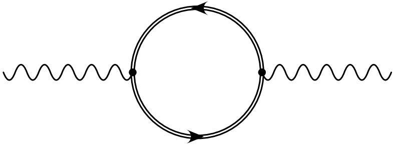
We have two possibilities.

1. $k\lambda\!\!\!^{-}_e \ll 1$: we pretend that the photon field is also uniform and expand the effective Lagrangian density.
2. $k\lambda\!\!\!^{-}_e \gtrsim 1$: we have to expand the action itself.

How It Works



$$S = S_0 + \frac{1}{2} f^{\mu\nu} f^{\alpha\beta} \frac{\delta^2 S}{\delta f^{\mu\nu} \delta f^{\alpha\beta}}$$

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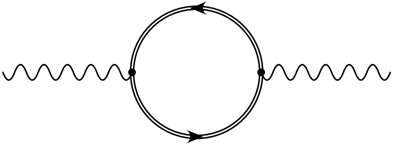

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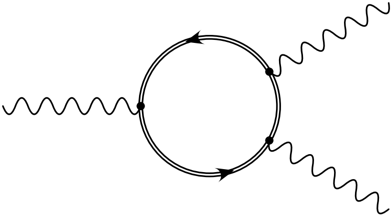


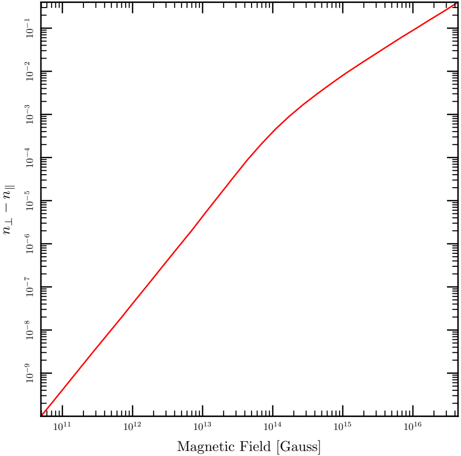
$$S = S_0 + \frac{1}{2} f^{\mu\nu} f^{\alpha\beta} \frac{\delta^2 S}{\delta f^{\mu\nu} \delta f^{\alpha\beta}} + \frac{1}{6} f^{\mu\nu} f^{\alpha\beta} f^{\sigma\tau} \frac{\delta^3 S}{\delta f^{\mu\nu} \delta f^{\alpha\beta} \delta f^{\sigma\tau}}$$

Index of Refraction

$$\Delta n = 4 \times 10^{-24} \mathrm{T}^{-2} B^2$$

What could be a signature of this birefringence?

- A time delay: $\Delta t \sim 10^{-3} R/c \sim 10\mathrm{ns}$?



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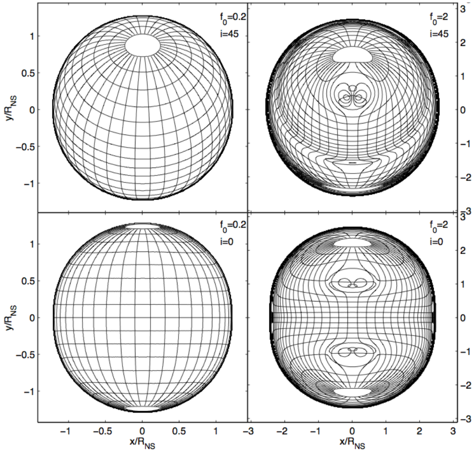
Shaviv et al. 99



3×10^{16}G

3×10^{17}G

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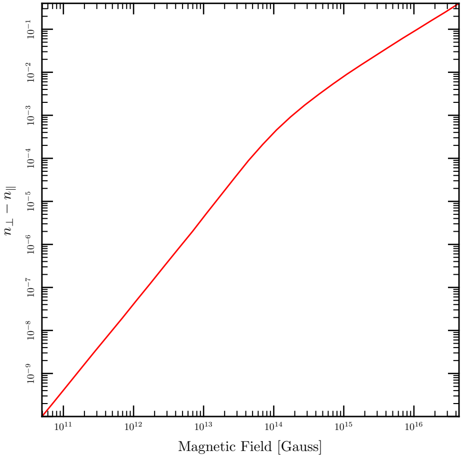

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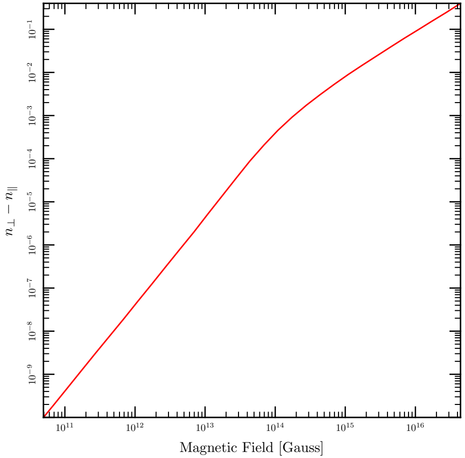
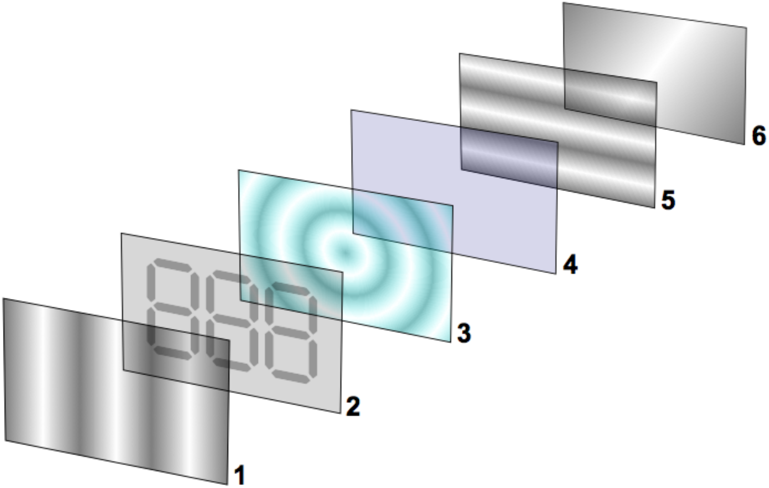
- These all seemed a bit too subtle.
- We were literally staring at the answer.



Liquid Crystal Displays



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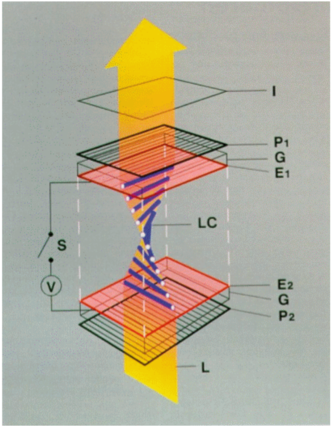

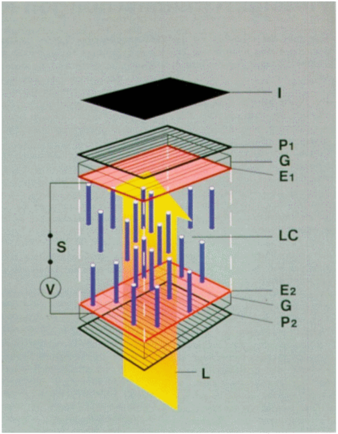

Wikipedia

Propagation through a twisting magnetic field

Kubo and Nagata (1983) present a concise way to characterize the evolution of the polarization of light through a medium; they simply write an equation to track the four Stokes parameters of the polarization light.

$$\frac{\partial \mathbf{s}}{\partial l} = \hat{\boldsymbol{\Omega}} \times \mathbf{s}$$

where $\left|\hat{\boldsymbol{\Omega}}\right| = \Delta k$. The vector $\mathbf{s} = (S_1, S_2, S_3)/S_0$ or $(Q, U, V)/I$.

Stokes Parameters and the Poincaré Sphere

An important analytic solution. What if $\frac{\partial \hat{\mathbf{\Omega}}}{\partial l} = \hat{\mathbf{\Upsilon}} \times \hat{\mathbf{\Omega}}$?

1. Move into frame that corotates with $\hat{\mathbf{\Omega}}$.
2. In this frame we have

$$\frac{\partial \mathbf{s}}{\partial l} = \left(\hat{\mathbf{\Omega}} - \hat{\mathbf{\Upsilon}} \right) \times \mathbf{s} = \hat{\mathbf{\Omega}}_{\mathrm{Eff}} \times \mathbf{s}$$

3. $\mathbf{s}$ orbits $\hat{\mathbf{\Omega}}_{\mathrm{Eff}}$ if

$$\left| \hat{\mathbf{\Omega}} \left(\frac{1}{|\hat{\mathbf{\Omega}}|} \frac{\partial |\hat{\mathbf{\Omega}}|}{\partial l} \right)^{-1} \right| \gtrsim 0.5$$

Polarization-Limiting Radius

The radius at which the polarization stops following $\hat{\mathbf{\Omega}}$ is called the polarization-limiting radius. Beyond here the modes are coupled.

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$$\left|\hat{\mathbf{\Omega}}\right| = \frac{\alpha}{4\pi}\frac{2}{15}\left(\frac{B}{B_{\rm QED}}\right)^2\frac{c}{\omega}\sin^2\theta; \left|\hat{\mathbf{\Omega}}\left|\frac{1}{|\hat{\mathbf{\Omega}}|}\frac{\partial|\hat{\mathbf{\Omega}}|}{\partial l}\right|^{-1}\right| = \Delta n\frac{c}{\omega}\frac{r}{6}$$

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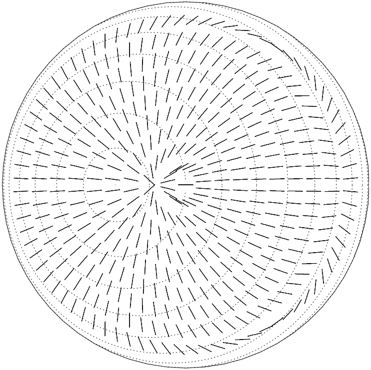
Therefore, the polarization-limiting radius is

$$\begin{aligned} r \lesssim r_{\rm pl} &\equiv \left(\frac{\alpha}{45}\frac{\nu}{c}\right)^{1/5}\left(\frac{\mu}{B_{\rm QED}}\sin\beta\right)^{2/5} \\ &\approx 1.2\times 10^7 \left(\frac{\mu}{10^{30}\ {\rm Gcm}^3}\right)^{2/5}\left(\frac{\nu}{10^{17}\ {\rm Hz}}\right)^{1/5}(\sin\beta)^{2/5}\,{\rm cm}, \end{aligned}$$

Why does this matter?



$r_{\rm pl}/R = 0$
Heyl, Shaviv, Lloyd 03

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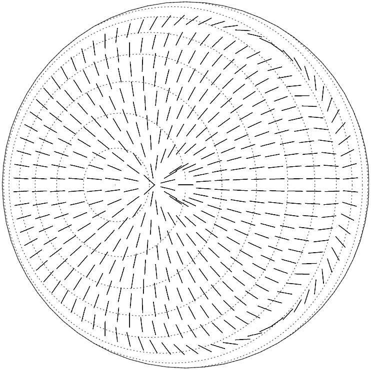

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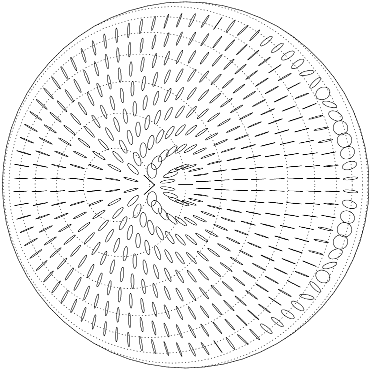

$r_{\rm pl}/R = 1.9$ (AM Her, AMSP)

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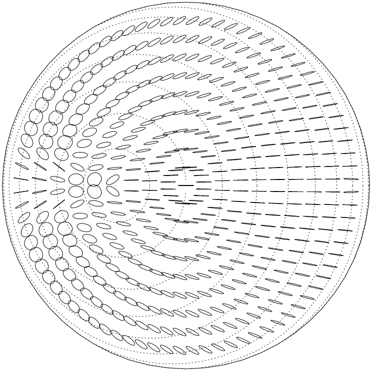

$r_{\rm pl}/R = 12$ (XRP)

Heyl, Shaviv, Lloyd 03

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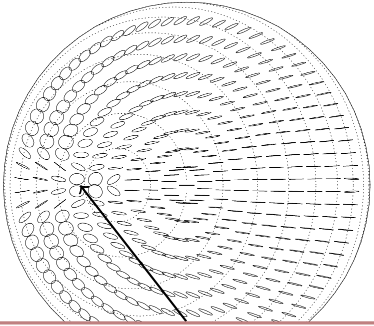

Quasi-Tangential Region *Wang, Lai 09*

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Heyl, Shaviv, Lloyd 03

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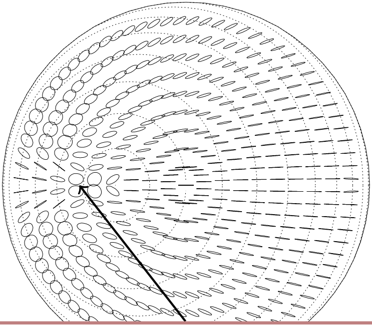

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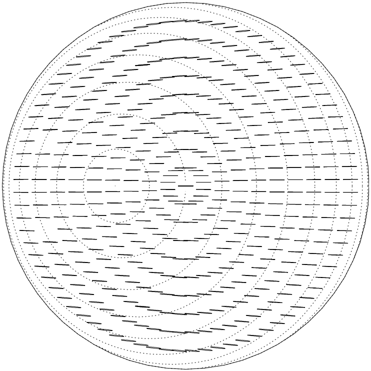

$r_{\rm pl}/R = 76$ (Magnetar)

Places to Look

	Radius	Magnetic Field	μ_{30}	$r_{\rm pl}$ at 4 keV
Magnetar	10^6	10^{15}	10^{33}	3.0×10^8
XRP	10^6	10^{12}	10^{30}	1.9×10^7
ms XRP	10^6	10^9	10^{27}	1.2×10^6
AM Her	10^9	10^8	10^{35}	1.9×10^9

This is not subtle.

Let's recap.

- Neutron star atmospheres emit polarized light.

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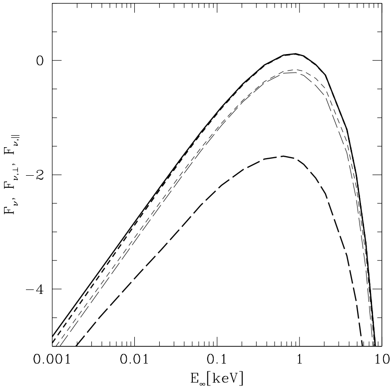
Let's recap.

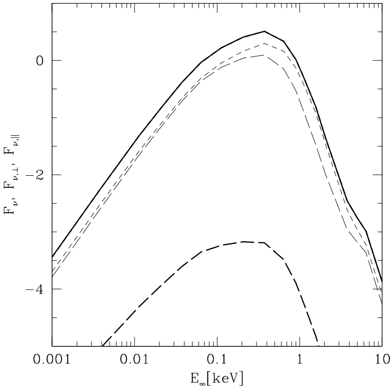
- Neutron star atmospheres emit polarized light.
- The emission varies across the surface.
- The rotating magnetic field twists the polarization.

Realistic Hydrogen Atmosphere



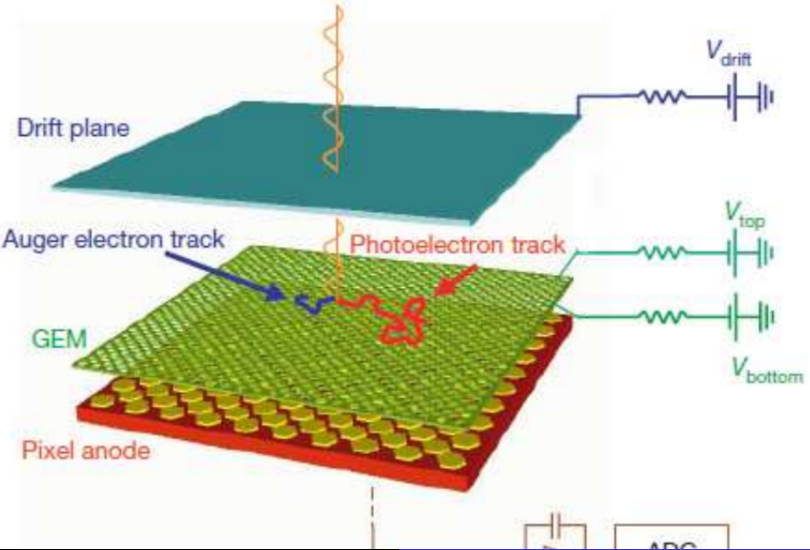
10^{12}G; 60-degree inclination

Heyl, Shaviv, Lloyd 05

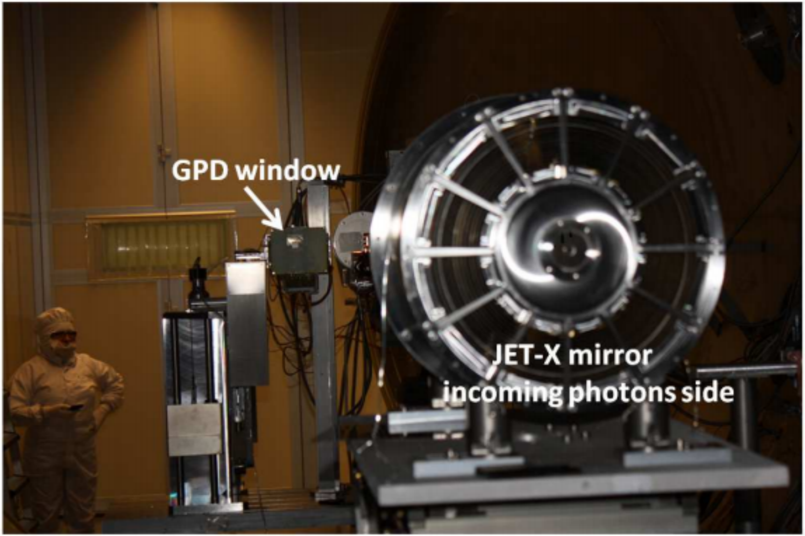


10^{14}G; 60-degree inclination

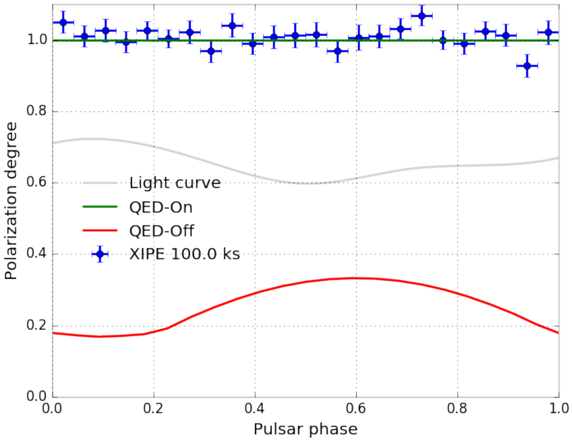
XIPE


Drift plane
Auger electron track
Photoelectron track
GEM
Pixel anode
V_{drift}
V_{top}
V_{bottom}

XIPE

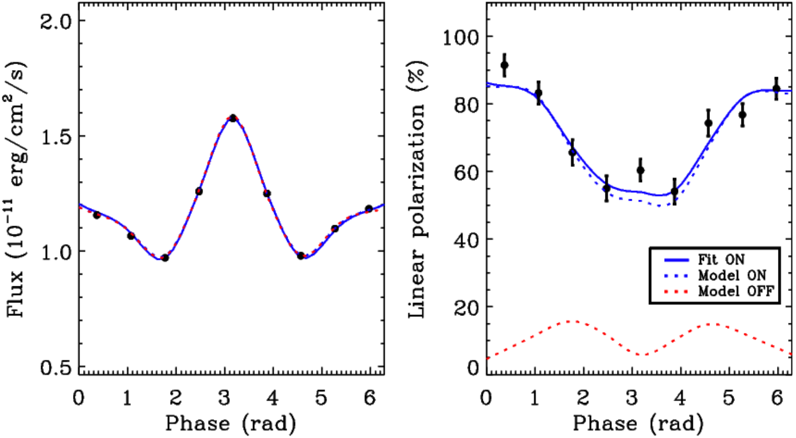


Magnetar Thermal Emission (4U 0142+61)



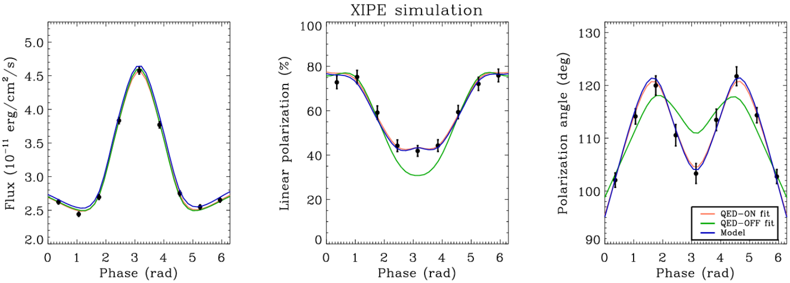
Caiazzo & Heyl 2016; 100ks with XIPE

Magnetar Thermal Emission (SGR 1806-20)



350ks with XIPE; Taverna & Turolla 2016

Magnetar RICS Emission



AXP 1RXS J170849.0400910; 250ks with XIPE; Taverna et al. 2014

Vacuum-Plasma Resonance

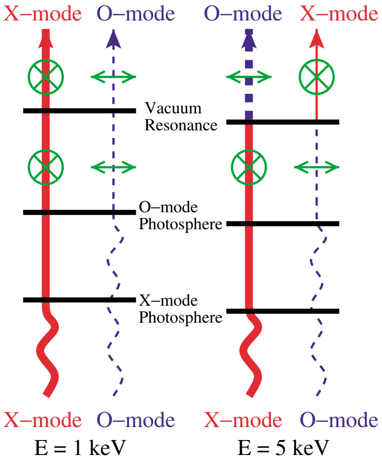
Deep in the atmosphere of the neutron star the plasma dominates, while outside the vacuum dominates.

$$E \lesssim E_{\rm ad} = 2.52\,(f\tan\theta)^{2/3}\,H_{\rho,1}^{-1/3}\ \mathrm{keV}$$

where

$$H_\rho = 1.65\frac{T_6}{g_{14}\mu}\ \mathrm{cm}$$

Ho, Lai 03



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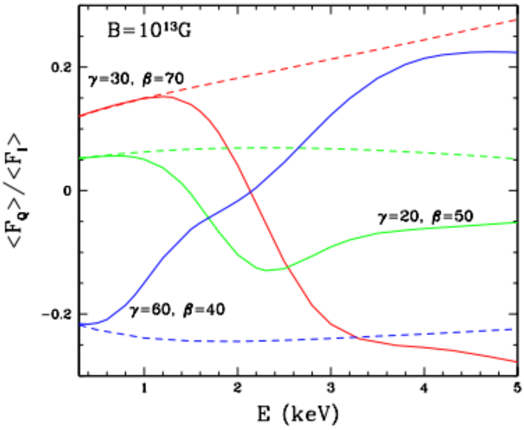
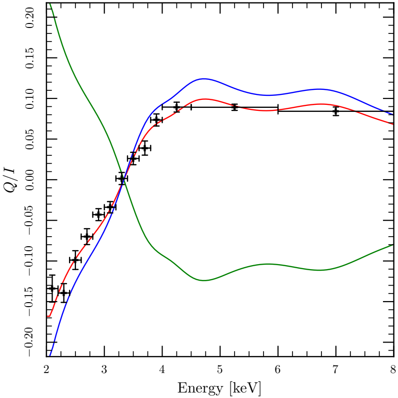
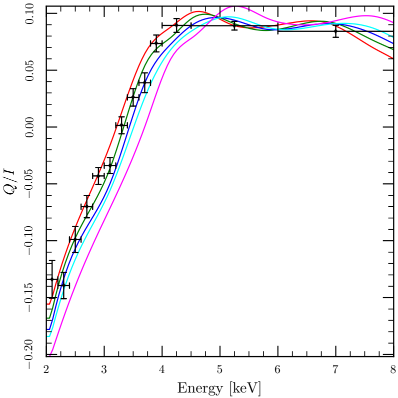


Lai, Ho 03

Accreting X-ray Pulsar (Her X-1)





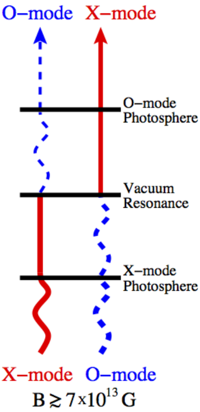
Caiazzo & Heyl 2016; 100ks with XIPE; Meszaros & Nagel 1985

Vacuum-Plasma Resonance

Deep in the atmosphere of the neutron star the plasma dominates, while outside the vacuum dominates.
For large strengths of the magnetic field, the vacuum resonance may lie between the photospheres

$$B \gtrsim B_l \approx 6.6 \times 10^{13} T_6^{-1/8} E_1^{-1/4} S^{-1/4} \mathrm{G}$$

where $S = 1 - e^{-E/kT}$.
This can have a strong effect on the appearance of spectral features and the high-energy slope. Ho, Lai 04



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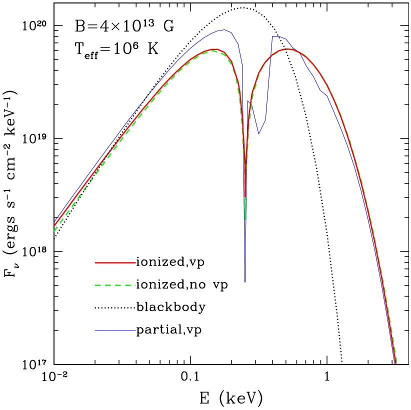
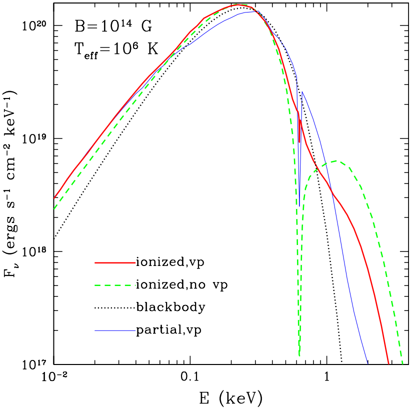


$B = 4 \times 10^{13}$G



$B = 10^{14}$G

Ho, Lai 04

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Future directions

- Observations of x-ray polarization from magnetars will verify QED.
- Observations of x-ray polarization from x-ray pulsars could constrain the radius of the neutron star.
- Soft x-ray polarization from x-ray pulsars could measure the surface gravity of the star.